Computer algebra independent integration tests

`1_Algebraic_functions/1.2_Trinomial_products/1.2.2Quartic/1.2.2.2(dx)^m(a+bx^2+cx^4)`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

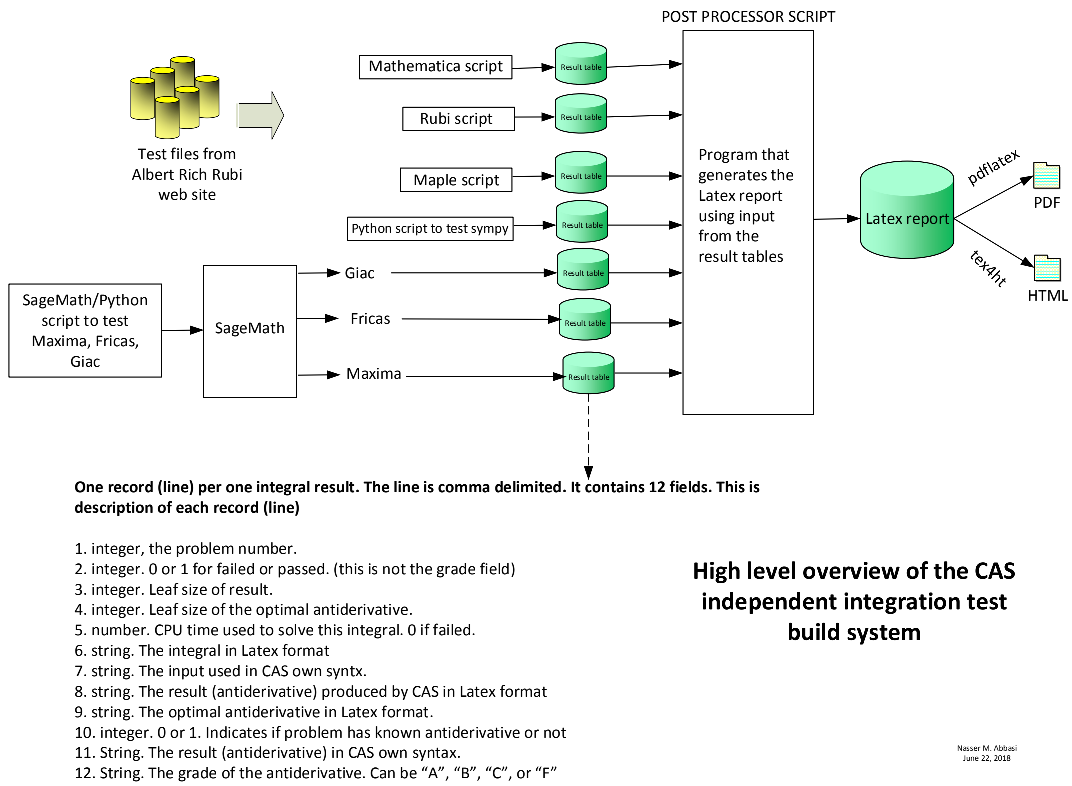
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (1126)	% 0. (0)
Rubi in Sympy	% 82.95 (934)	% 17.05 (192)
Mathematica	% 100. (1126)	% 0. (0)
Maple	% 94.32 (1062)	% 5.68 (64)
Maxima	% 35.17 (396)	% 64.83 (730)
Fricas	% 75.31 (848)	% 24.69 (278)
Sympy	% 42.45 (478)	% 57.55 (648)
Giac	% 68.21 (768)	% 31.79 (358)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

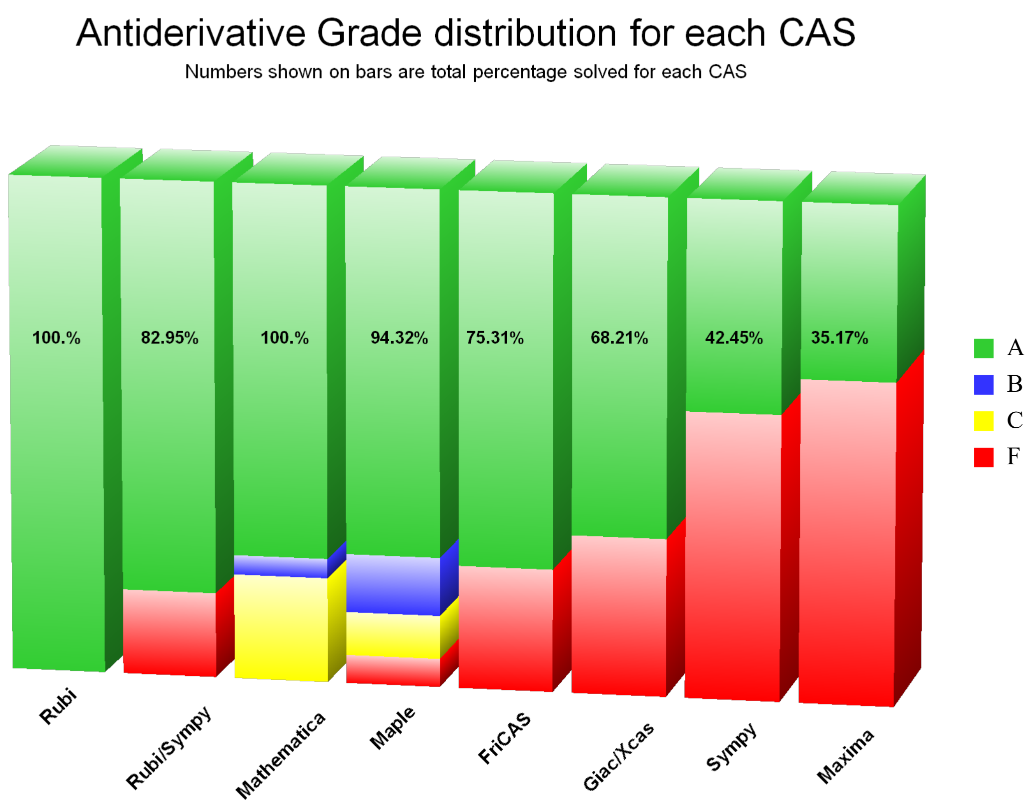
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

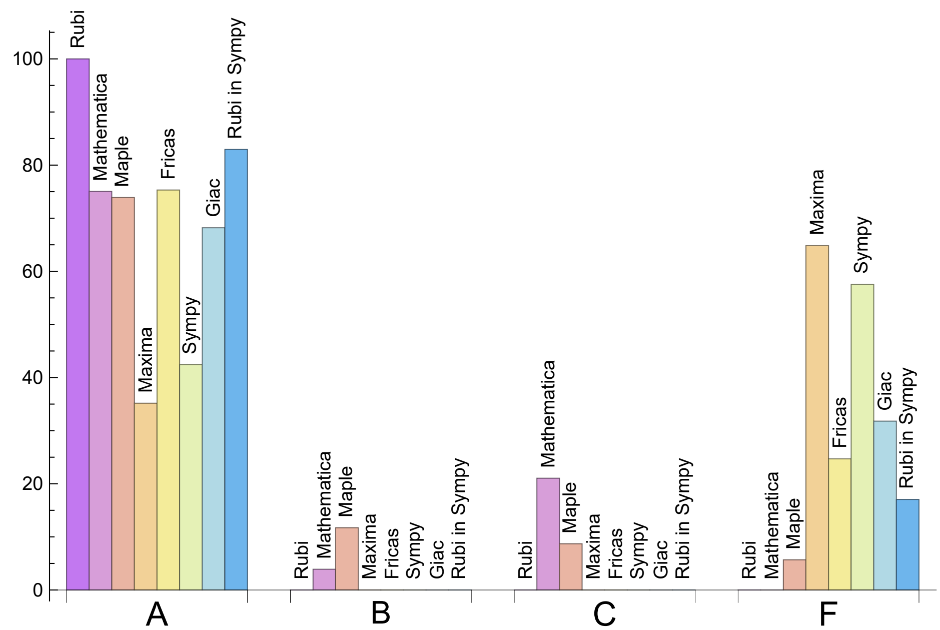
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Rubi in Sympy	82.95	0.	0.	17.05
Mathematica	75.04	3.91	21.05	0.
Maple	73.89	11.72	8.7	5.68
Maxima	35.17	0.	0.	64.83
Fricas	75.31	0.	0.	24.69
Sympy	42.45	0.	0.	57.55
Giac	68.21	0.	0.	31.79

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.32	144.71	1.	90.	1.
Rubi in Sympy	29.07	115.97	0.92	87.	0.92
Mathematica	0.25	126.28	1.04	78.	0.92
Maple	0.02	173.13	1.11	78.	0.87
Maxima	0.7	65.15	1.15	58.	1.12
Fricas	0.32	375.38	1.56	50.	0.97
Sympy	9.33	119.32	1.46	53.	0.97
Giac	0.61	135.57	1.18	85.	1.15

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {137, 138, 139, 140, 146, 149, 150, 152, 153, 162, 163, 165, 167, 168, 174, 175, 176, 177, 178, 179, 190, 191, 192, 193, 205, 206, 412, 413, 415, 416, 428, 430, 432, 434, 435, 453, 455, 457, 459, 461, 463, 464, 475, 476, 477, 478, 484, 485, 486, 493, 494, 502, 503, 512, 513, 523, 524, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 624, 625, 627, 628, 629, 630, 631, 632, 633, 640, 641, 642, 643, 644, 653, 654, 655, 656, 657, 658, 696, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 815, 816, 817, 818, 819, 820, 826, 827, 828, 830, 831, 832, 834, 835, 842, 843, 844, 846, 848, 861, 867, 873, 881, 886, 887, 1033, 1034, 1070, 1071, 1072, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088}

Not solved by Mathematica {}

Not solved by Maple {3, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Not solved by Maxima {1, 2, 3, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 174, 176, 178, 180, 182, 184, 186, 188, 190, 192, 194, 196, 198, 200, 202, 204, 205, 207, 209, 211, 213, 215, 217, 219, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 278, 279, 280, 283, 284, 285, 290, 291, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 484, 485, 486, 487, 488, 489, 490, 491, 492, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 541, 542, 543, 544, 545, 546, 547, 548, 549, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 627, 628, 629, 630, 631, 632, 633, 638, 639, 640, 641, 642, 643, 644, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 995, 997, 998, 999, 1000, 1001, 1002, 1003, 1005, 1006, 1007, 1008, 1009, 1011, 1012, 1013, 1014, 1016, 1017, 1018, 1019, 1020, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Not solved by Fricas {16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 930, 931, 932, 933, 934, 935, 947, 948, 949, 950, 951, 952, 953, 954, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 989, 990, 991, 992, 993, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1071, 1079, 1080, 1081, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Not solved by Sympy {1, 2, 3, 4, 5, 6, 7, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 308, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 686, 687, 688, 689, 690, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 866, 879, 880, 887, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1024, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Not solved by Giac {2, 5, 6, 7, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 260, 261, 268, 271, 272, 273, 274, 278, 279, 280, 283, 285, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 406, 407, 408, 659, 660, 661, 662, 663, 664, 788, 789, 790, 794, 795, 796, 797, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 882, 885, 887, 902, 905, 911, 914, 918, 919, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 959, 960, 961, 962, 963, 964, 965, 966, 967, 976, 977, 978, 979, 980, 986, 987, 988, 989, 990, 991, 992, 993, 994, 1000, 1002, 1003, 1005, 1007, 1009, 1011, 1039, 1040, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {930, 947, 948, 949, 953}

Mathematica {14, 17, 18, 19, 23, 24, 25, 27, 28, 30, 31, 32, 36, 37, 38, 40, 41, 43, 44, 45, 46, 49, 50, 51, 53, 54, 56, 57, 58, 62, 63, 64, 78, 79, 81, 124, 125, 126, 128, 129, 130, 132, 133, 918, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	89	77	0	1	0	117	124
normalized size	1	1.	0.7	0.6	0.	0.01	0.	0.91	0.97
time (sec)	N/A	0.084	0.079	0.056	0.	0.29	0.	0.285	24.604

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	58	0	1	0	0	99
normalized size	1	1.	0.65	0.64	0.	0.01	0.	0.	1.09
time (sec)	N/A	0.058	0.058	0.02	0.	0.288	0.	0.	21.93

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	49	0	0	1	0	36	54
normalized size	1	1.	0.82	0.	0.	0.02	0.	0.6	0.9
time (sec)	N/A	0.035	0.026	0.011	0.	0.285	0.	0.281	19.958

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	33	19	46	0	32	36
normalized size	1	1.	0.74	0.97	0.56	1.35	0.	0.94	1.06
time (sec)	N/A	0.023	0.024	0.004	0.709	0.273	0.	0.28	15.422

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	40	44	0	78	0	0	66
normalized size	1	1.03	0.59	0.65	0.	1.15	0.	0.	0.97
time (sec)	N/A	0.043	0.028	0.004	0.	0.277	0.	0.	16.068

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	51	55	0	108	0	0	107
normalized size	1	1.02	0.49	0.52	0.	1.03	0.	0.	1.02
time (sec)	N/A	0.065	0.032	0.004	0.	0.288	0.	0.	31.678

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	148	62	66	0	138	0	0	138
normalized size	1	1.1	0.46	0.49	0.	1.02	0.	0.	1.02
time (sec)	N/A	0.089	0.039	0.006	0.	0.305	0.	0.	32.726

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	81	1099	0	787	63	1	264
normalized size	1	1.	0.27	3.68	0.	2.63	0.21	0.	0.88
time (sec)	N/A	0.663	0.075	0.109	0.	0.263	2.022	2.805	56.629

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	43	32	0	1	257	42	37
normalized size	1	1.	0.91	0.68	0.	0.02	5.47	0.89	0.79
time (sec)	N/A	0.052	0.039	0.015	0.	0.278	1.463	0.286	8.923

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	52	1073	0	1048	48	0	264
normalized size	1	1.	0.17	3.59	0.	3.51	0.16	0.	0.88
time (sec)	N/A	0.575	0.047	0.095	0.	0.29	1.38	0.	54.447

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	37	26	34	34	26	39	10
normalized size	1	1.	2.18	1.53	2.	2.	1.53	2.29	0.59
time (sec)	N/A	0.016	0.009	0.012	0.68	0.261	0.481	0.269	10.871

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	18	23	28	20	23	20
normalized size	1	1.	1.	0.75	0.96	1.17	0.83	0.96	0.83
time (sec)	N/A	0.019	0.018	0.01	0.764	0.26	0.395	0.268	2.824

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	91	54	72	77	70	72	63
normalized size	1	1.	1.36	0.81	1.07	1.15	1.04	1.07	0.94
time (sec)	N/A	0.096	0.122	0.009	0.764	0.26	0.591	0.27	12.713

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	77	57	0	143	63	0	63
normalized size	1	1.	1.04	0.77	0.	1.93	0.85	0.	0.85
time (sec)	N/A	0.09	0.122	0.035	0.	0.267	0.526	0.	16.722

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	41	386	0	630	20	0	165
normalized size	1	1.	0.23	2.19	0.	3.58	0.11	0.	0.94
time (sec)	N/A	0.393	0.062	0.074	0.	0.277	1.761	0.	30.1

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0	12
normalized size	1	1.	6.5	5.1	0.	0.	0.	0.	1.2
time (sec)	N/A	0.04	0.044	0.063	0.	0.	0.	0.	7.627

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	0	0	0	73
normalized size	1	1.	1.02	1.75	0.	0.	0.	0.	1.52
time (sec)	N/A	0.272	0.1	0.179	0.	0.	0.	0.	19.496

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	80	0	0	0	0	51
normalized size	1	1.	1.1	1.67	0.	0.	0.	0.	1.06
time (sec)	N/A	0.21	0.103	0.11	0.	0.	0.	0.	12.644

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	0	0	0	71
normalized size	1	1.	1.11	1.91	0.	0.	0.	0.	1.61
time (sec)	N/A	0.155	0.075	0.122	0.	0.	0.	0.	20.171

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	63	41	0	0	0	0	14
normalized size	1	1.	5.25	3.42	0.	0.	0.	0.	1.17
time (sec)	N/A	0.041	0.04	0.018	0.	0.	0.	0.	8.542

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	0	37	0	24
normalized size	1	1.	1.	3.	0.	0.	2.06	0.	1.33
time (sec)	N/A	0.02	0.035	0.063	0.	0.	1.838	0.	1.169

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	49	0	0	0	0	20
normalized size	1	1.	1.	2.45	0.	0.	0.	0.	1.
time (sec)	N/A	0.041	0.045	0.038	0.	0.	0.	0.	7.243

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	0	0	0	70
normalized size	1	1.	1.21	2.	0.	0.	0.	0.	1.67
time (sec)	N/A	0.138	0.077	0.099	0.	0.	0.	0.	20.848

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	55	80	0	0	0	0	48
normalized size	1	1.	1.2	1.74	0.	0.	0.	0.	1.04
time (sec)	N/A	0.224	0.112	0.097	0.	0.	0.	0.	12.888

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	84	0	0	0	0	71
normalized size	1	1.	1.02	1.75	0.	0.	0.	0.	1.48
time (sec)	N/A	0.242	0.102	0.099	0.	0.	0.	0.	19.848

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	0	0	0	19
normalized size	1	1.	3.	2.78	0.	0.	0.	0.	1.06
time (sec)	N/A	0.044	0.043	0.031	0.	0.	0.	0.	7.93

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	0	0	0	58
normalized size	1	1.	1.16	1.87	0.	0.	0.	0.	1.29
time (sec)	N/A	0.142	0.076	0.117	0.	0.	0.	0.	12.884

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	84	0	0	0	0	68
normalized size	1	1.	0.98	1.91	0.	0.	0.	0.	1.55
time (sec)	N/A	0.265	0.092	0.119	0.	0.	0.	0.	19.628

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0	12
normalized size	1	1.	6.5	5.1	0.	0.	0.	0.	1.2
time (sec)	N/A	0.037	0.043	0.033	0.	0.	0.	0.	7.658

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	0	0	0	73
normalized size	1	1.	1.16	1.91	0.	0.	0.	0.	1.66
time (sec)	N/A	0.216	0.097	0.098	0.	0.	0.	0.	25.479

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	84	0	0	0	0	56
normalized size	1	1.	1.11	1.87	0.	0.	0.	0.	1.24
time (sec)	N/A	0.18	0.089	0.101	0.	0.	0.	0.	12.717

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	49	84	0	0	0	0	71
normalized size	1	1.	1.11	1.91	0.	0.	0.	0.	1.61
time (sec)	N/A	0.152	0.074	0.094	0.	0.	0.	0.	20.257

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	47	0	0	0	0	20
normalized size	1	1.	1.	2.35	0.	0.	0.	0.	1.
time (sec)	N/A	0.04	0.046	0.034	0.	0.	0.	0.	7.248

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	54	0	0	37	0	24
normalized size	1	1.	1.	3.	0.	0.	2.06	0.	1.33
time (sec)	N/A	0.019	0.036	0.05	0.	0.	1.838	0.	1.163

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	0	0	0	14
normalized size	1	1.	5.42	3.58	0.	0.	0.	0.	1.17
time (sec)	N/A	0.037	0.038	0.024	0.	0.	0.	0.	8.491

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	51	84	0	0	0	0	70
normalized size	1	1.	1.21	2.	0.	0.	0.	0.	1.67
time (sec)	N/A	0.134	0.075	0.093	0.	0.	0.	0.	20.438

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	52	84	0	0	0	0	53
normalized size	1	1.	1.21	1.95	0.	0.	0.	0.	1.23
time (sec)	N/A	0.149	0.095	0.093	0.	0.	0.	0.	13.316

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	84	0	0	0	0	71
normalized size	1	1.	1.16	1.91	0.	0.	0.	0.	1.61
time (sec)	N/A	0.22	0.101	0.091	0.	0.	0.	0.	19.694

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	54	50	0	0	0	0	19
normalized size	1	1.	3.	2.78	0.	0.	0.	0.	1.06
time (sec)	N/A	0.042	0.043	0.026	0.	0.	0.	0.	7.833

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	84	0	0	0	0	66
normalized size	1	1.	1.07	2.	0.	0.	0.	0.	1.57
time (sec)	N/A	0.254	0.099	0.101	0.	0.	0.	0.	19.766

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	84	0	0	0	0	54
normalized size	1	1.	1.16	1.87	0.	0.	0.	0.	1.2
time (sec)	N/A	0.123	0.077	0.108	0.	0.	0.	0.	12.996

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0	70
normalized size	1	1.	0.81	0.79	0.	0.	0.	0.	1.04
time (sec)	N/A	0.029	0.041	0.013	0.	0.	0.	0.	3.844

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	81	84	0	0	0	0	126
normalized size	1	1.	0.57	0.6	0.	0.	0.	0.	0.89
time (sec)	N/A	0.121	0.115	0.041	0.	0.	0.	0.	4.169

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	83	84	0	0	0	0	126
normalized size	1	1.	0.57	0.58	0.	0.	0.	0.	0.86
time (sec)	N/A	0.118	0.123	0.041	0.	0.	0.	0.	4.017

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	83	84	0	0	0	0	126
normalized size	1	1.	0.59	0.6	0.	0.	0.	0.	0.89
time (sec)	N/A	0.09	0.091	0.042	0.	0.	0.	0.	4.616

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	43	0	0	0	0	68
normalized size	1	1.	0.76	0.68	0.	0.	0.	0.	1.08
time (sec)	N/A	0.026	0.051	0.012	0.	0.	0.	0.	2.972

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	40	56	0	0	34	0	51
normalized size	1	1.	0.35	0.49	0.	0.	0.3	0.	0.44
time (sec)	N/A	0.058	0.043	0.032	0.	0.	1.835	0.	1.557

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	53	0	0	0	0	68
normalized size	1	1.	0.92	0.82	0.	0.	0.	0.	1.05
time (sec)	N/A	0.025	0.043	0.035	0.	0.	0.	0.	3.124

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0	128
normalized size	1	1.	0.55	0.57	0.	0.	0.	0.	0.86
time (sec)	N/A	0.094	0.085	0.042	0.	0.	0.	0.	3.938

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	81	84	0	0	0	0	128
normalized size	1	1.	0.53	0.55	0.	0.	0.	0.	0.84
time (sec)	N/A	0.124	0.116	0.068	0.	0.	0.	0.	3.982

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	81	84	0	0	0	0	128
normalized size	1	1.	0.55	0.57	0.	0.	0.	0.	0.86
time (sec)	N/A	0.133	0.115	0.042	0.	0.	0.	0.	4.041

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0	70
normalized size	1	1.	1.03	0.84	0.	0.	0.	0.	1.11
time (sec)	N/A	0.025	0.043	0.02	0.	0.	0.	0.	3.715

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	80	84	0	0	0	0	128
normalized size	1	1.	0.54	0.57	0.	0.	0.	0.	0.86
time (sec)	N/A	0.097	0.087	0.042	0.	0.	0.	0.	4.183

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	77	84	0	0	0	0	136
normalized size	1	1.	0.52	0.57	0.	0.	0.	0.	0.92
time (sec)	N/A	0.128	0.111	0.04	0.	0.	0.	0.	4.149

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	0	0	0	0	70
normalized size	1	1.	0.81	0.79	0.	0.	0.	0.	1.04
time (sec)	N/A	0.028	0.042	0.009	0.	0.	0.	0.	3.813

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	83	84	0	0	0	0	136
normalized size	1	1.	0.56	0.57	0.	0.	0.	0.	0.92
time (sec)	N/A	0.11	0.116	0.04	0.	0.	0.	0.	4.176

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	80	84	0	0	0	0	126
normalized size	1	1.	0.55	0.58	0.	0.	0.	0.	0.86
time (sec)	N/A	0.113	0.108	0.04	0.	0.	0.	0.	4.666

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	83	84	0	0	0	0	131
normalized size	1	1.	0.58	0.59	0.	0.	0.	0.	0.92
time (sec)	N/A	0.091	0.092	0.093	0.	0.	0.	0.	4.1

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	51	0	0	0	0	68
normalized size	1	1.	1.	0.81	0.	0.	0.	0.	1.08
time (sec)	N/A	0.027	0.038	0.035	0.	0.	0.	0.	2.949

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	40	56	0	0	34	0	51
normalized size	1	1.	0.36	0.5	0.	0.	0.3	0.	0.46
time (sec)	N/A	0.06	0.043	0.03	0.	0.	1.873	0.	1.5

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	45	0	0	0	0	68
normalized size	1	1.	0.78	0.69	0.	0.	0.	0.	1.05
time (sec)	N/A	0.025	0.043	0.012	0.	0.	0.	0.	3.079

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	81	84	0	0	0	0	133
normalized size	1	1.	0.54	0.56	0.	0.	0.	0.	0.89
time (sec)	N/A	0.093	0.086	0.041	0.	0.	0.	0.	4.018

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	78	84	0	0	0	0	128
normalized size	1	1.	0.51	0.55	0.	0.	0.	0.	0.84
time (sec)	N/A	0.116	0.098	0.041	0.	0.	0.	0.	3.905

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	83	84	0	0	0	0	138
normalized size	1	1.	0.54	0.54	0.	0.	0.	0.	0.89
time (sec)	N/A	0.135	0.112	0.041	0.	0.	0.	0.	4.066

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	53	0	0	0	0	70
normalized size	1	1.	1.03	0.84	0.	0.	0.	0.	1.11
time (sec)	N/A	0.029	0.042	0.022	0.	0.	0.	0.	3.707

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	0	0	0	46
normalized size	1	1.	1.12	0.85	0.	0.	0.	0.	0.88
time (sec)	N/A	0.024	0.043	0.102	0.	0.	0.	0.	3.688

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	88
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	0.98
time (sec)	N/A	0.07	0.154	0.168	0.	0.	0.	0.	3.822

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	88
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	0.96
time (sec)	N/A	0.058	0.207	0.159	0.	0.	0.	0.	3.768

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	88
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	0.96
time (sec)	N/A	0.057	0.143	0.177	0.	0.	0.	0.	3.659

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	0	0	0	85
normalized size	1	1.	1.61	0.97	0.	0.	0.	0.	0.97
time (sec)	N/A	0.059	0.145	0.154	0.	0.	0.	0.	2.995

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	0	36	0	71
normalized size	1	1.	0.35	0.92	0.	0.	0.5	0.	0.99
time (sec)	N/A	0.034	0.041	0.08	0.	0.	1.74	0.	1.459

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	85
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	0.94
time (sec)	N/A	0.056	0.14	0.144	0.	0.	0.	0.	3.423

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	88
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	0.98
time (sec)	N/A	0.052	0.136	0.141	0.	0.	0.	0.	4.522

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	88
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	0.98
time (sec)	N/A	0.054	0.201	0.159	0.	0.	0.	0.	3.966

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	88
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.	1.
time (sec)	N/A	0.046	0.145	0.143	0.	0.	0.	0.	4.058

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	0	0	0	90
normalized size	1	1.	0.58	0.46	0.	0.	0.	0.	0.98
time (sec)	N/A	0.053	0.041	0.012	0.	0.	0.	0.	4.052

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	82	0	0	0	0	88
normalized size	1	1.	0.94	0.91	0.	0.	0.	0.	0.98
time (sec)	N/A	0.055	0.144	0.223	0.	0.	0.	0.	3.989

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	82	0	0	0	0	102
normalized size	1	1.	0.88	0.75	0.	0.	0.	0.	0.93
time (sec)	N/A	0.183	0.127	0.236	0.	0.	0.	0.	4.984

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	82	0	0	0	0	107
normalized size	1	1.	0.89	0.75	0.	0.	0.	0.	0.97
time (sec)	N/A	0.185	0.14	0.234	0.	0.	0.	0.	5.067

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	61	50	0	0	0	0	56
normalized size	1	1.	1.02	0.83	0.	0.	0.	0.	0.93
time (sec)	N/A	0.028	0.042	0.105	0.	0.	0.	0.	3.699

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	90	82	0	0	0	0	102
normalized size	1	1.	0.87	0.79	0.	0.	0.	0.	0.98
time (sec)	N/A	0.139	0.098	0.221	0.	0.	0.	0.	4.966

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	50	0	0	0	0	49
normalized size	1	1.	1.12	0.96	0.	0.	0.	0.	0.94
time (sec)	N/A	0.022	0.042	0.049	0.	0.	0.	0.	3.638

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	88
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	0.98
time (sec)	N/A	0.073	0.156	0.124	0.	0.	0.	0.	3.832

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	0	0	0	88
normalized size	1	1.	1.54	0.95	0.	0.	0.	0.	0.96
time (sec)	N/A	0.058	0.182	0.125	0.	0.	0.	0.	3.769

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	88
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	0.96
time (sec)	N/A	0.054	0.14	0.125	0.	0.	0.	0.	3.739

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	0	0	0	85
normalized size	1	1.	1.59	0.97	0.	0.	0.	0.	0.97
time (sec)	N/A	0.055	0.125	0.128	0.	0.	0.	0.	3.005

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	25	66	0	0	36	0	71
normalized size	1	1.	0.35	0.92	0.	0.	0.5	0.	0.99
time (sec)	N/A	0.033	0.043	0.063	0.	0.	1.764	0.	1.446

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	85
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.	0.94
time (sec)	N/A	0.053	0.12	0.117	0.	0.	0.	0.	3.376

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	88
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	0.98
time (sec)	N/A	0.053	0.136	0.116	0.	0.	0.	0.	3.939

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	88
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.	0.98
time (sec)	N/A	0.052	0.175	0.119	0.	0.	0.	0.	4.503

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	88
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.	1.
time (sec)	N/A	0.046	0.148	0.12	0.	0.	0.	0.	3.983

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	53	42	0	0	0	0	90
normalized size	1	1.	0.58	0.46	0.	0.	0.	0.	0.98
time (sec)	N/A	0.055	0.04	0.043	0.	0.	0.	0.	4.005

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	81	82	0	0	0	0	88
normalized size	1	1.	0.9	0.91	0.	0.	0.	0.	0.98
time (sec)	N/A	0.053	0.139	0.173	0.	0.	0.	0.	3.991

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	58	49	0	0	0	0	90
normalized size	1	1.	0.63	0.53	0.	0.	0.	0.	0.98
time (sec)	N/A	0.057	0.041	0.047	0.	0.	0.	0.	3.989

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	58	48	0	0	0	0	20
normalized size	1	1.	3.05	2.53	0.	0.	0.	0.	1.05
time (sec)	N/A	0.044	0.039	0.041	0.	0.	0.	0.	8.108

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	81	82	0	0	0	0	63
normalized size	1	1.	1.84	1.86	0.	0.	0.	0.	1.43
time (sec)	N/A	0.157	0.063	0.096	0.	0.	0.	0.	21.669

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	53	50	0	0	0	0	12
normalized size	1	1.	3.79	3.57	0.	0.	0.	0.	0.86
time (sec)	N/A	0.04	0.04	0.048	0.	0.	0.	0.	9.433

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	90
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.	1.02
time (sec)	N/A	0.045	0.132	0.062	0.	0.	0.	0.	3.88

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	90
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.	1.
time (sec)	N/A	0.05	0.167	0.059	0.	0.	0.	0.	4.293

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	90
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	1.
time (sec)	N/A	0.053	0.132	0.058	0.	0.	0.	0.	3.806

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	140	85	0	0	0	0	87
normalized size	1	1.	1.59	0.97	0.	0.	0.	0.	0.99
time (sec)	N/A	0.051	0.115	0.071	0.	0.	0.	0.	3.184

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	0	39	0	76
normalized size	1	1.	0.65	0.92	0.	0.	0.54	0.	1.06
time (sec)	N/A	0.036	0.046	0.031	0.	0.	1.801	0.	1.435

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	142	87	0	0	0	0	90
normalized size	1	1.	1.58	0.97	0.	0.	0.	0.	1.
time (sec)	N/A	0.059	0.123	0.067	0.	0.	0.	0.	2.882

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	94
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	1.02
time (sec)	N/A	0.053	0.141	0.066	0.	0.	0.	0.	3.608

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	142	87	0	0	0	0	94
normalized size	1	1.	1.54	0.95	0.	0.	0.	0.	1.02
time (sec)	N/A	0.054	0.17	0.068	0.	0.	0.	0.	3.529

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	94
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	1.04
time (sec)	N/A	0.049	0.157	0.059	0.	0.	0.	0.	3.615

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	63	44	0	0	0	0	54
normalized size	1	1.	1.19	0.83	0.	0.	0.	0.	1.02
time (sec)	N/A	0.055	0.042	0.049	0.	0.	0.	0.	8.391

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	85	82	0	0	0	0	63
normalized size	1	1.	2.02	1.95	0.	0.	0.	0.	1.5
time (sec)	N/A	0.137	0.146	0.099	0.	0.	0.	0.	27.435

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	53	42	0	0	0	0	5
normalized size	1	1.	8.83	7.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.038	0.044	0.011	0.	0.	0.	0.	9.434

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	144	87	0	0	0	0	90
normalized size	1	1.	1.64	0.99	0.	0.	0.	0.	1.02
time (sec)	N/A	0.057	0.148	0.073	0.	0.	0.	0.	3.86

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	90
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	1.
time (sec)	N/A	0.053	0.2	0.062	0.	0.	0.	0.	3.807

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	90
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	1.
time (sec)	N/A	0.054	0.142	0.06	0.	0.	0.	0.	4.285

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	142	85	0	0	0	0	87
normalized size	1	1.	1.61	0.97	0.	0.	0.	0.	0.99
time (sec)	N/A	0.052	0.144	0.06	0.	0.	0.	0.	3.174

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	47	66	0	0	39	0	76
normalized size	1	1.	0.65	0.92	0.	0.	0.54	0.	1.06
time (sec)	N/A	0.034	0.048	0.032	0.	0.	1.887	0.	1.458

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	90
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	1.
time (sec)	N/A	0.06	0.144	0.061	0.	0.	0.	0.	2.904

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	94
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	1.02
time (sec)	N/A	0.052	0.142	0.067	0.	0.	0.	0.	3.544

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	94
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	1.02
time (sec)	N/A	0.061	0.207	0.063	0.	0.	0.	0.	3.594

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	144	87	0	0	0	0	94
normalized size	1	1.	1.6	0.97	0.	0.	0.	0.	1.04
time (sec)	N/A	0.052	0.158	0.07	0.	0.	0.	0.	3.663

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	63	50	0	0	0	0	56
normalized size	1	1.	1.21	0.96	0.	0.	0.	0.	1.08
time (sec)	N/A	0.055	0.043	0.051	0.	0.	0.	0.	8.279

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	144	87	0	0	0	0	88
normalized size	1	1.	1.57	0.95	0.	0.	0.	0.	0.96
time (sec)	N/A	0.085	0.206	0.151	0.	0.	0.	0.	3.719

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	87	0	0	0	0	80
normalized size	1	1.	1.63	0.97	0.	0.	0.	0.	0.89
time (sec)	N/A	0.053	0.155	0.161	0.	0.	0.	0.	3.747

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	58	44	0	0	0	0	46
normalized size	1	1.	1.12	0.85	0.	0.	0.	0.	0.88
time (sec)	N/A	0.024	0.034	0.	0.	0.	0.	0.	3.659

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	48	0	0	0	0	56
normalized size	1	1.	1.	0.83	0.	0.	0.	0.	0.97
time (sec)	N/A	0.025	0.036	0.081	0.	0.	0.	0.	3.493

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	103	76	0	0	0	0	90
normalized size	1	1.	0.95	0.7	0.	0.	0.	0.	0.83
time (sec)	N/A	0.099	0.145	0.238	0.	0.	0.	0.	4.146

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	55	80	0	0	0	0	48
normalized size	1	1.	1.15	1.67	0.	0.	0.	0.	1.
time (sec)	N/A	0.222	0.112	0.115	0.	0.	0.	0.	12.417

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	0	0	0	53
normalized size	1	1.	1.16	1.69	0.	0.	0.	0.	1.18
time (sec)	N/A	0.136	0.076	0.109	0.	0.	0.	0.	15.028

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	65	51	0	0	0	0	12
normalized size	1	1.	6.5	5.1	0.	0.	0.	0.	1.2
time (sec)	N/A	0.041	0.04	0.	0.	0.	0.	0.	7.779

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0	48
normalized size	1	1.	1.14	1.63	0.	0.	0.	0.	0.98
time (sec)	N/A	0.223	0.105	0.115	0.	0.	0.	0.	13.332

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	52	80	0	0	0	0	51
normalized size	1	1.	1.08	1.67	0.	0.	0.	0.	1.06
time (sec)	N/A	0.225	0.1	0.117	0.	0.	0.	0.	13.376

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0	53
normalized size	1	1.	1.14	1.63	0.	0.	0.	0.	1.08
time (sec)	N/A	0.171	0.085	0.111	0.	0.	0.	0.	13.771

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	65	43	0	0	0	0	14
normalized size	1	1.	5.42	3.58	0.	0.	0.	0.	1.17
time (sec)	N/A	0.042	0.044	0.018	0.	0.	0.	0.	10.081

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	76	0	0	0	0	53
normalized size	1	1.	1.16	1.69	0.	0.	0.	0.	1.18
time (sec)	N/A	0.151	0.072	0.111	0.	0.	0.	0.	15.235

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	56	80	0	0	0	0	48
normalized size	1	1.	1.14	1.63	0.	0.	0.	0.	0.98
time (sec)	N/A	0.169	0.082	0.12	0.	0.	0.	0.	13.852

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	1	12	18	12
normalized size	1	1.	1.	0.82	1.06	0.06	0.71	1.06	0.71
time (sec)	N/A	0.015	0.003	0.001	0.679	0.233	0.07	0.269	4.79

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	1	12	18	12
normalized size	1	1.	1.	0.82	1.06	0.06	0.71	1.06	0.71
time (sec)	N/A	0.015	0.002	0.001	0.686	0.234	0.07	0.268	4.624

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	1	12	18	12
normalized size	1	1.	1.	0.82	1.06	0.06	0.71	1.06	0.71
time (sec)	N/A	0.011	0.	0.001	0.679	0.231	0.068	0.268	1.752

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	12	18	0
normalized size	1	1.	1.	0.82	1.06	1.06	0.71	1.06	0.
time (sec)	N/A	0.014	0.002	0.002	0.677	0.244	0.065	0.269	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	14	8	14	0
normalized size	1	1.	1.	0.92	1.17	1.17	0.67	1.17	0.
time (sec)	N/A	0.013	0.001	0.002	0.688	0.247	0.07	0.269	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	15	10	19	0
normalized size	1	1.	1.	0.92	1.46	1.15	0.77	1.46	0.
time (sec)	N/A	0.014	0.002	0.003	0.672	0.254	0.157	0.27	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	18	5	14	0
normalized size	1	1.	1.	1.1	1.4	1.8	0.5	1.4	0.
time (sec)	N/A	0.013	0.002	0.008	0.691	0.247	1.001	0.268	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	19	23	10	27	10
normalized size	1	1.	1.	0.92	1.46	1.77	0.77	2.08	0.77
time (sec)	N/A	0.014	0.004	0.007	0.689	0.254	1.116	0.27	4.538

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	14	18	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.93	1.2	0.67
time (sec)	N/A	0.015	0.004	0.007	0.682	0.249	1.083	0.268	4.677

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	18	14	18	14
normalized size	1	1.	1.	0.82	1.06	1.06	0.82	1.06	0.82
time (sec)	N/A	0.015	0.004	0.007	0.686	0.245	1.141	0.269	4.704

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	20	20	15	20	14
normalized size	1	1.	1.	0.82	1.18	1.18	0.88	1.18	0.82
time (sec)	N/A	0.015	0.004	0.006	0.687	0.247	1.145	0.268	4.659

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	26	32	26
normalized size	1	1.	1.	0.83	1.07	0.03	0.87	1.07	0.87
time (sec)	N/A	0.035	0.003	0.001	0.686	0.233	0.09	0.266	10.835

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	32	24	32	0
normalized size	1	1.	1.	0.83	1.07	1.07	0.8	1.07	0.
time (sec)	N/A	0.057	0.001	0.001	0.677	0.249	0.095	0.269	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	32	26	32	26
normalized size	1	1.	1.	0.83	1.07	1.07	0.87	1.07	0.87
time (sec)	N/A	0.04	0.001	0.001	0.684	0.247	0.096	0.267	8.116

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	32	32	24	32	10
normalized size	1	1.	1.	1.56	2.	2.	1.5	2.	0.62
time (sec)	N/A	0.018	0.004	0.001	0.703	0.245	0.096	0.268	4.068

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	28	22	28	0
normalized size	1	1.	1.	0.88	1.12	1.12	0.88	1.12	0.
time (sec)	N/A	0.026	0.002	0.001	0.685	0.246	0.098	0.268	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	32	28	20	32	0
normalized size	1	1.	1.	0.96	1.39	1.22	0.87	1.39	0.
time (sec)	N/A	0.043	0.002	0.003	0.686	0.256	1.03	0.27	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	34	19	30	19
normalized size	1	1.	1.	0.96	1.25	1.42	0.79	1.25	0.79
time (sec)	N/A	0.036	0.001	0.005	0.691	0.248	1.032	0.27	7.758

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	32	36	24	43	0
normalized size	1	1.	1.	0.89	1.19	1.33	0.89	1.59	0.
time (sec)	N/A	0.046	0.002	0.009	0.685	0.253	1.137	0.27	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	30	35	20	30	0
normalized size	1	1.	1.	0.96	1.3	1.52	0.87	1.3	0.
time (sec)	N/A	0.034	0.001	0.008	0.688	0.248	1.183	0.268	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	35	38	22	46	24
normalized size	1	1.	1.	0.96	1.46	1.58	0.92	1.92	1.
time (sec)	N/A	0.041	0.001	0.008	0.676	0.255	1.296	0.27	9.358

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	35	35	27	35	24
normalized size	1	1.	1.	0.89	1.25	1.25	0.96	1.25	0.86
time (sec)	N/A	0.036	0.001	0.007	0.688	0.247	1.335	0.267	7.727

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	32	32	26	32	15
normalized size	1	1.	1.58	1.32	1.68	1.68	1.37	1.68	0.79
time (sec)	N/A	0.023	0.002	0.007	0.692	0.248	1.35	0.267	5.073

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	35	27	35	27
normalized size	1	1.	1.	0.83	1.17	1.17	0.9	1.17	0.9
time (sec)	N/A	0.036	0.002	0.008	0.683	0.251	1.365	0.269	7.703

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	47	37	47	37
normalized size	1	1.	1.	0.84	1.09	1.09	0.86	1.09	0.86
time (sec)	N/A	0.053	0.003	0.001	0.678	0.246	0.111	0.268	9.81

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	43	36	47	47	37	47	27
normalized size	1	1.	1.26	1.06	1.38	1.38	1.09	1.38	0.79
time (sec)	N/A	0.09	0.003	0.001	0.682	0.245	0.113	0.268	11.673

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	47	47	39	47	39
normalized size	1	1.	1.	0.84	1.09	1.09	0.91	1.09	0.91
time (sec)	N/A	0.051	0.004	0.003	0.685	0.247	0.111	0.268	10.021

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	36	47	47	37	47	10
normalized size	1	1.	1.	2.25	2.94	2.94	2.31	2.94	0.62
time (sec)	N/A	0.018	0.004	0.002	0.675	0.251	0.105	0.269	4.066

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	42	32	42	0
normalized size	1	1.	1.	0.91	1.2	1.2	0.91	1.2	0.
time (sec)	N/A	0.036	0.002	0.001	0.681	0.247	0.103	0.268	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	49	45	37	49	0
normalized size	1	1.	1.	0.87	1.26	1.15	0.95	1.26	0.
time (sec)	N/A	0.056	0.007	0.003	0.692	0.254	1.088	0.27	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	43	49	29	43	29
normalized size	1	1.	1.	0.97	1.26	1.44	0.85	1.26	0.85
time (sec)	N/A	0.045	0.006	0.005	0.688	0.247	1.074	0.268	9.32

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	49	51	37	62	0
normalized size	1	1.	1.	0.88	1.22	1.27	0.92	1.55	0.
time (sec)	N/A	0.065	0.011	0.007	0.687	0.254	1.184	0.27	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	46	49	34	46	32
normalized size	1	1.	1.	0.92	1.24	1.32	0.92	1.24	0.86
time (sec)	N/A	0.045	0.006	0.008	0.679	0.247	1.202	0.269	9.541

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	50	53	36	62	0
normalized size	1	1.	1.	0.88	1.25	1.32	0.9	1.55	0.
time (sec)	N/A	0.06	0.007	0.009	0.687	0.255	1.353	0.269	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	45	50	32	45	0
normalized size	1	1.	1.	0.97	1.32	1.47	0.94	1.32	0.
time (sec)	N/A	0.044	0.009	0.007	0.678	0.246	1.405	0.268	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	34	53	53	36	63	41
normalized size	1	1.	1.	0.87	1.36	1.36	0.92	1.62	1.05
time (sec)	N/A	0.056	0.007	0.009	0.684	0.252	1.532	0.27	11.544

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	50	50	39	50	34
normalized size	1	1.	1.	0.92	1.28	1.28	1.	1.28	0.87
time (sec)	N/A	0.045	0.007	0.007	0.692	0.246	1.559	0.268	9.885

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	47	47	37	47	15
normalized size	1	1.	2.26	1.89	2.47	2.47	1.95	2.47	0.79
time (sec)	N/A	0.023	0.011	0.007	0.696	0.246	1.636	0.268	5.161

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	50	50	39	50	41
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.95
time (sec)	N/A	0.045	0.007	0.007	0.692	0.249	1.67	0.269	9.896

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	36	50	50	39	50	39
normalized size	1	1.	1.08	0.9	1.25	1.25	0.98	1.25	0.98
time (sec)	N/A	0.06	0.007	0.008	0.687	0.245	1.714	0.269	11.838

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	0	1	107	88	0
normalized size	1	1.	1.	0.88	0.	0.01	1.57	1.29	0.
time (sec)	N/A	0.087	0.047	0.008	0.	0.26	1.335	0.269	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	46	62	61	44	63	0
normalized size	1	1.	1.	0.87	1.17	1.15	0.83	1.19	0.
time (sec)	N/A	0.101	0.01	0.004	0.683	0.25	1.262	0.271	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	0	1	95	74	0
normalized size	1	1.	1.	0.89	0.	0.02	1.73	1.35	0.
time (sec)	N/A	0.073	0.043	0.004	0.	0.263	1.302	0.269	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	46	45	32	47	0
normalized size	1	1.	1.	0.88	1.15	1.12	0.8	1.18	0.
time (sec)	N/A	0.074	0.009	0.004	0.688	0.252	1.214	0.27	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	1	80	54	0
normalized size	1	1.	1.	0.9	0.	0.02	1.9	1.29	0.
time (sec)	N/A	0.062	0.033	0.003	0.	0.258	1.313	0.27	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	30	20	32	0
normalized size	1	1.	1.	0.89	1.15	1.11	0.74	1.19	0.
time (sec)	N/A	0.056	0.007	0.004	0.673	0.255	1.174	0.27	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	0	1	56	35	26
normalized size	1	1.	1.	0.87	0.	0.03	1.81	1.13	0.84
time (sec)	N/A	0.042	0.015	0.004	0.	0.26	1.184	0.268	8.568

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	10	19	10
normalized size	1	1.	1.	0.93	1.2	1.2	0.67	1.27	0.67
time (sec)	N/A	0.02	0.003	0.002	0.688	0.25	0.237	0.271	4.186

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	53	20	22
normalized size	1	1.	1.	0.67	0.	0.04	2.21	0.83	0.92
time (sec)	N/A	0.023	0.008	0.002	0.	0.262	0.295	0.269	4.498

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	31	24	15	30	19
normalized size	1	1.	1.	0.95	1.41	1.09	0.68	1.36	0.86
time (sec)	N/A	0.038	0.008	0.007	0.68	0.256	0.505	0.271	8.414

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	0	1	65	39	29
normalized size	1	1.	1.	0.88	0.	0.03	1.91	1.15	0.85
time (sec)	N/A	0.037	0.022	0.005	0.	0.259	1.272	0.269	11.274

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	45	45	31	58	34
normalized size	1	1.	1.	0.91	1.29	1.29	0.89	1.66	0.97
time (sec)	N/A	0.066	0.011	0.009	0.682	0.257	1.571	0.27	11.35

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	0	1	87	54	37
normalized size	1	1.	1.	0.91	0.	0.02	2.02	1.26	0.86
time (sec)	N/A	0.059	0.037	0.008	0.	0.263	1.464	0.27	11.838

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	63	61	42	77	48
normalized size	1	1.	1.	0.9	1.29	1.24	0.86	1.57	0.98
time (sec)	N/A	0.078	0.012	0.009	0.696	0.255	1.796	0.27	14.213

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	52	0	1	100	70	49
normalized size	1	1.	1.	0.9	0.	0.02	1.72	1.21	0.84
time (sec)	N/A	0.081	0.043	0.01	0.	0.26	1.76	0.27	16.308

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	78	78	56	95	60
normalized size	1	1.	1.	0.89	1.24	1.24	0.89	1.51	0.95
time (sec)	N/A	0.093	0.012	0.01	0.703	0.261	2.191	0.271	16.57

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	0	1	124	99	0
normalized size	1	1.	0.9	0.86	0.	0.01	1.57	1.25	0.
time (sec)	N/A	0.096	0.087	0.011	0.	0.26	1.721	0.269	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	73	95	53	90	0
normalized size	1	1.	0.86	0.91	1.28	1.67	0.93	1.58	0.
time (sec)	N/A	0.113	0.03	0.014	0.697	0.251	1.574	0.271	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	0	1	107	82	0
normalized size	1	1.	0.91	0.86	0.	0.02	1.62	1.24	0.
time (sec)	N/A	0.082	0.075	0.011	0.	0.261	1.656	0.269	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	58	76	39	66	0
normalized size	1	1.	0.86	0.93	1.32	1.73	0.89	1.5	0.
time (sec)	N/A	0.087	0.026	0.013	0.696	0.251	1.529	0.271	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	0	1	83	57	48
normalized size	1	1.	0.93	0.78	0.	0.02	1.51	1.04	0.87
time (sec)	N/A	0.06	0.066	0.011	0.	0.26	1.549	0.27	12.702

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	43	47	29	43	26
normalized size	1	1.	0.82	0.91	1.3	1.42	0.88	1.3	0.79
time (sec)	N/A	0.067	0.013	0.012	0.7	0.251	1.343	0.27	10.787

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.045	0.036	0.01	0.	0.262	1.37	0.27	8.532

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	15	20	20	15	19	12
normalized size	1	1.	0.7	0.65	0.87	0.87	0.65	0.83	0.52
time (sec)	N/A	0.014	0.004	0.001	0.703	0.245	1.217	0.268	4.043

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.037	0.041	0.005	0.	0.26	1.407	0.27	5.937

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	50	63	34	49	34
normalized size	1	1.	0.87	0.92	1.32	1.66	0.89	1.29	0.89
time (sec)	N/A	0.072	0.022	0.017	0.703	0.261	1.66	0.272	11.967

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	1	90	63	48
normalized size	1	1.	0.95	0.81	0.	0.02	1.58	1.11	0.84
time (sec)	N/A	0.062	0.067	0.013	0.	0.261	1.753	0.27	12.18

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	70	99	49	68	46
normalized size	1	1.	0.84	0.94	1.43	2.02	1.	1.39	0.94
time (sec)	N/A	0.092	0.062	0.019	0.702	0.257	2.038	0.271	14.749

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	1	114	80	61
normalized size	1	1.	0.99	0.87	0.	0.01	1.68	1.18	0.9
time (sec)	N/A	0.075	0.072	0.016	0.	0.264	2.055	0.268	19.203

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	95	122	68	116	66
normalized size	1	1.	0.86	0.92	1.44	1.85	1.03	1.76	1.
time (sec)	N/A	0.112	0.098	0.019	0.698	0.258	2.534	0.273	18.9

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	0	1	126	95	75
normalized size	1	1.	0.99	0.86	0.	0.01	1.56	1.17	0.93
time (sec)	N/A	0.102	0.086	0.016	0.	0.263	2.772	0.269	21.131

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	77	0	1	131	99	0
normalized size	1	1.	0.91	0.91	0.	0.01	1.54	1.16	0.
time (sec)	N/A	0.106	0.099	0.014	0.	0.261	2.222	0.27	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	48	58	89	123	66	84	0
normalized size	1	1.	0.74	0.89	1.37	1.89	1.02	1.29	0.
time (sec)	N/A	0.119	0.105	0.015	0.699	0.254	2.068	0.274	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	63	0	1	107	73	66
normalized size	1	1.	0.89	0.85	0.	0.01	1.45	0.99	0.89
time (sec)	N/A	0.086	0.089	0.013	0.	0.261	2.108	0.272	17.631

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	39	46	74	93	53	57	41
normalized size	1	1.	0.8	0.94	1.51	1.9	1.08	1.16	0.84
time (sec)	N/A	0.094	0.027	0.012	0.702	0.252	1.831	0.272	15.226

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	55	47	0	1	109	61	56
normalized size	1	1.	0.86	0.73	0.	0.02	1.7	0.95	0.88
time (sec)	N/A	0.067	0.077	0.01	0.	0.263	1.879	0.27	12.727

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	49	49	36	30	14
normalized size	1	1.	1.26	1.63	2.58	2.58	1.89	1.58	0.74
time (sec)	N/A	0.025	0.012	0.011	0.709	0.25	1.65	0.269	5.231

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	58	49	0	1	110	68	51
normalized size	1	1.	0.89	0.75	0.	0.02	1.69	1.05	0.78
time (sec)	N/A	0.062	0.05	0.011	0.	0.262	1.783	0.272	10.933

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	16	15	35	35	27	19	14
normalized size	1	1.	0.7	0.65	1.52	1.52	1.17	0.83	0.61
time (sec)	N/A	0.014	0.005	0.001	0.689	0.247	1.539	0.272	4.072

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	0	1	105	61	54
normalized size	1	1.	0.89	0.82	0.	0.02	1.69	0.98	0.87
time (sec)	N/A	0.051	0.068	0.005	0.	0.263	1.828	0.271	8.079

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	49	81	122	56	80	49
normalized size	1	1.	0.8	0.91	1.5	2.26	1.04	1.48	0.91
time (sec)	N/A	0.094	0.054	0.016	0.68	0.259	2.18	0.273	15.198

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	66	0	1	114	77	65
normalized size	1	1.	0.89	0.87	0.	0.01	1.5	1.01	0.86
time (sec)	N/A	0.086	0.075	0.016	0.	0.264	2.397	0.272	16.885

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	62	104	161	78	89	66
normalized size	1	1.	0.88	0.93	1.55	2.4	1.16	1.33	0.99
time (sec)	N/A	0.121	0.098	0.019	0.677	0.258	2.994	0.275	19.15

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	79	0	1	138	96	80
normalized size	1	1.	0.91	0.91	0.	0.01	1.59	1.1	0.92
time (sec)	N/A	0.103	0.084	0.018	0.	0.264	3.322	0.273	21.295

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	74	79	124	181	90	107	85
normalized size	1	1.	0.86	0.92	1.44	2.1	1.05	1.24	0.99
time (sec)	N/A	0.15	0.087	0.02	0.688	0.257	4.306	0.273	23.754

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	90	89	0	1	150	108	94
normalized size	1	1.	0.9	0.89	0.	0.01	1.5	1.08	0.94
time (sec)	N/A	0.13	0.101	0.018	0.	0.266	5.513	0.27	29.951

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	90	139	196	104	149	95
normalized size	1	1.	0.89	0.95	1.46	2.06	1.09	1.57	1.
time (sec)	N/A	0.177	0.123	0.021	0.727	0.256	7.502	0.272	26.24

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	114	124	0	1	0	138	109
normalized size	1	1.	0.96	1.04	0.	0.01	0.	1.16	0.92
time (sec)	N/A	0.265	0.157	0.015	0.	0.293	0.	0.278	24.263

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	103	104	0	1	0	116	78
normalized size	1	1.	1.13	1.14	0.	0.01	0.	1.27	0.86
time (sec)	N/A	0.198	0.108	0.011	0.	0.284	0.	0.277	18.218

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	90	82	0	1	0	95	58
normalized size	1	1.	1.32	1.21	0.	0.01	0.	1.4	0.85
time (sec)	N/A	0.126	0.093	0.01	0.	0.278	0.	0.279	10.789

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	64	64	0	1	0	72	46
normalized size	1	1.	1.16	1.16	0.	0.02	0.	1.31	0.84
time (sec)	N/A	0.141	0.048	0.006	0.	0.272	0.	0.275	12.833

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	66	84	0	1	0	82	44
normalized size	1	1.	1.27	1.62	0.	0.02	0.	1.58	0.85
time (sec)	N/A	0.146	0.049	0.009	0.	0.277	0.	0.293	12.663

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	38	0	85	20
normalized size	1	1.	1.	1.16	0.	1.52	0.	3.4	0.8
time (sec)	N/A	0.067	0.025	0.006	0.	0.262	0.	0.287	7.732

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	46	39	0	57	0	162	44
normalized size	1	1.	0.88	0.75	0.	1.1	0.	3.12	0.85
time (sec)	N/A	0.136	0.022	0.007	0.	0.265	0.	0.295	13.61

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	57	50	0	72	0	200	71
normalized size	1	1.	0.71	0.62	0.	0.9	0.	2.5	0.89
time (sec)	N/A	0.208	0.028	0.007	0.	0.277	0.	0.3	20.934

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	68	61	0	86	0	240	99
normalized size	1	1.	0.63	0.56	0.	0.8	0.	2.22	0.92
time (sec)	N/A	0.286	0.033	0.008	0.	0.294	0.	0.302	28.57

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	79	72	0	101	0	278	126
normalized size	1	1.	0.58	0.53	0.	0.74	0.	2.04	0.93
time (sec)	N/A	0.364	0.037	0.009	0.	0.333	0.	0.307	36.921

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	57	50	62	72	0	76	68
normalized size	1	1.	0.73	0.64	0.79	0.92	0.	0.97	0.87
time (sec)	N/A	0.169	0.029	0.007	0.734	0.27	0.	0.272	20.398

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	45	39	46	55	0	57	42
normalized size	1	1.	0.87	0.75	0.88	1.06	0.	1.1	0.81
time (sec)	N/A	0.088	0.023	0.007	0.724	0.264	0.	0.271	12.897

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	19	38	0	36	19
normalized size	1	1.	1.	1.16	0.76	1.52	0.	1.44	0.76
time (sec)	N/A	0.016	0.01	0.004	0.701	0.259	0.	0.27	5.158

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	75	65	0	1	0	92	41
normalized size	1	1.	1.5	1.3	0.	0.02	0.	1.84	0.82
time (sec)	N/A	0.09	0.071	0.008	0.	0.278	0.	0.275	12.979

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	89	85	0	1	0	61	49
normalized size	1	1.	1.59	1.52	0.	0.02	0.	1.09	0.88
time (sec)	N/A	0.092	0.065	0.009	0.	0.275	0.	0.29	12.937

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	102	106	0	1	0	86	71
normalized size	1	1.	1.21	1.26	0.	0.01	0.	1.02	0.85
time (sec)	N/A	0.175	0.108	0.011	0.	0.278	0.	0.29	21.118

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	128	0	1	0	111	97
normalized size	1	1.	1.03	1.14	0.	0.01	0.	0.99	0.87
time (sec)	N/A	0.254	0.113	0.014	0.	0.287	0.	0.304	29.114

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	125	142	0	1	0	157	112
normalized size	1	1.	1.01	1.15	0.	0.01	0.	1.27	0.9
time (sec)	N/A	0.25	0.173	0.015	0.	0.336	0.	0.28	21.532

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	114	122	0	1	0	135	92
normalized size	1	1.	1.13	1.21	0.	0.01	0.	1.34	0.91
time (sec)	N/A	0.168	0.145	0.011	0.	0.297	0.	0.279	14.404

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	103	102	0	1	0	115	75
normalized size	1	1.	1.17	1.16	0.	0.01	0.	1.31	0.85
time (sec)	N/A	0.189	0.119	0.011	0.	0.287	0.	0.278	16.273

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	75	84	0	1	0	93	71
normalized size	1	1.	0.94	1.05	0.	0.01	0.	1.16	0.89
time (sec)	N/A	0.19	0.085	0.006	0.	0.278	0.	0.279	17.089

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	81	107	0	1	0	107	68
normalized size	1	1.	1.07	1.41	0.	0.01	0.	1.41	0.89
time (sec)	N/A	0.186	0.083	0.009	0.	0.276	0.	0.305	16.913

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	87	129	0	1	0	165	65
normalized size	1	1.	1.16	1.72	0.	0.01	0.	2.2	0.87
time (sec)	N/A	0.186	0.066	0.009	0.	0.275	0.	0.37	16.544

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	0	53	0	124	20
normalized size	1	1.	1.	1.16	0.	2.12	0.	4.96	0.8
time (sec)	N/A	0.066	0.036	0.005	0.	0.268	0.	0.293	8.192

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	39	0	72	0	240	44
normalized size	1	1.	0.67	0.75	0.	1.38	0.	4.62	0.85
time (sec)	N/A	0.137	0.038	0.008	0.	0.273	0.	0.304	14.087

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	0	86	0	278	71
normalized size	1	1.	0.57	0.62	0.	1.08	0.	3.48	0.89
time (sec)	N/A	0.209	0.041	0.006	0.	0.296	0.	0.308	21.067

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	0	101	0	319	99
normalized size	1	1.	0.53	0.56	0.	0.94	0.	2.95	0.92
time (sec)	N/A	0.294	0.048	0.008	0.	0.342	0.	0.311	28.819

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	68	72	0	116	0	356	126
normalized size	1	1.	0.5	0.53	0.	0.85	0.	2.62	0.93
time (sec)	N/A	0.386	0.052	0.009	0.	0.412	0.	0.315	37.577

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	75	72	107	116	0	240	122
normalized size	1	1.	0.56	0.54	0.8	0.87	0.	1.79	0.91
time (sec)	N/A	0.35	0.049	0.009	0.693	0.27	0.	0.276	38.394

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	64	61	92	101	0	201	95
normalized size	1	1.	0.6	0.58	0.87	0.95	0.	1.9	0.9
time (sec)	N/A	0.253	0.045	0.008	0.705	0.267	0.	0.276	28.495

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	50	77	86	0	163	70
normalized size	1	1.	0.66	0.62	0.96	1.08	0.	2.04	0.88
time (sec)	N/A	0.174	0.04	0.008	0.696	0.265	0.	0.273	20.344

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	39	61	70	0	126	44
normalized size	1	1.	0.81	0.75	1.17	1.35	0.	2.42	0.85
time (sec)	N/A	0.09	0.035	0.007	0.706	0.263	0.	0.273	12.592

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	43	53	0	78	19
normalized size	1	1.	1.	1.16	1.72	2.12	0.	3.12	0.76
time (sec)	N/A	0.071	0.017	0.003	0.692	0.267	0.	0.273	7.859

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	89	78	0	1	0	119	61
normalized size	1	1.	1.22	1.07	0.	0.01	0.	1.63	0.84
time (sec)	N/A	0.167	0.108	0.009	0.	0.278	0.	0.273	19.842

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	99	102	0	1	0	80	70
normalized size	1	1.	1.25	1.29	0.	0.01	0.	1.01	0.89
time (sec)	N/A	0.17	0.09	0.01	0.	0.277	0.	0.296	20.2

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	105	125	0	1	0	85	75
normalized size	1	1.	1.3	1.54	0.	0.01	0.	1.05	0.93
time (sec)	N/A	0.169	0.095	0.01	0.	0.28	0.	0.297	20.5

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	115	145	0	1	0	111	94
normalized size	1	1.	1.06	1.33	0.	0.01	0.	1.02	0.86
time (sec)	N/A	0.253	0.113	0.014	0.	0.285	0.	0.304	29.452

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	127	165	0	1	0	130	122
normalized size	1	1.	0.93	1.2	0.	0.01	0.	0.95	0.89
time (sec)	N/A	0.342	0.143	0.024	0.	0.299	0.	0.326	38.271

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	137	186	0	1	0	149	150
normalized size	1	1.	0.83	1.13	0.	0.01	0.	0.9	0.91
time (sec)	N/A	0.435	0.271	0.043	0.	0.331	0.	0.344	47.779

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	103	105	0	1	0	0	104
normalized size	1	1.	0.9	0.92	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.271	0.076	0.013	0.	0.284	0.	0.	23.146

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	85	0	1	0	0	76
normalized size	1	1.	1.07	0.99	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.201	0.066	0.014	0.	0.278	0.	0.	18.265

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	76	64	0	1	0	80	48
normalized size	1	1.	1.31	1.1	0.	0.02	0.	1.38	0.83
time (sec)	N/A	0.145	0.051	0.011	0.	0.276	0.	0.297	13.585

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	0	1	0	53	27
normalized size	1	1.	1.68	1.42	0.	0.03	0.	1.71	0.87
time (sec)	N/A	0.084	0.025	0.006	0.	0.269	0.	0.291	8.293

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	0	28	0	19	19
normalized size	1	1.	1.	1.13	0.	1.22	0.	0.83	0.83
time (sec)	N/A	0.069	0.023	0.005	0.	0.259	0.	0.277	8.162

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	0	42	0	36	44
normalized size	1	1.	0.67	0.71	0.	0.81	0.	0.69	0.85
time (sec)	N/A	0.141	0.038	0.006	0.	0.265	0.	0.278	14.284

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	50	0	57	0	58	71
normalized size	1	1.	0.57	0.62	0.	0.71	0.	0.72	0.89
time (sec)	N/A	0.217	0.039	0.007	0.	0.265	0.	0.278	21.312

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	57	61	0	72	0	77	99
normalized size	1	1.	0.53	0.56	0.	0.67	0.	0.71	0.92
time (sec)	N/A	0.294	0.041	0.008	0.	0.273	0.	0.279	29.278

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	46	41	0	0	41
normalized size	1	1.	0.68	0.74	0.92	0.82	0.	0.	0.82
time (sec)	N/A	0.093	0.027	0.007	0.704	0.266	0.	0.	15.189

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	18	27	0	42	15
normalized size	1	1.	1.	1.18	0.82	1.23	0.	1.91	0.68
time (sec)	N/A	0.014	0.01	0.004	0.742	0.259	0.	0.276	8.371

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	58	50	0	1	0	62	27
normalized size	1	1.	1.93	1.67	0.	0.03	0.	2.07	0.9
time (sec)	N/A	0.024	0.044	0.011	0.	0.266	0.	0.273	5.716

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	97	73	0	1	0	0	49
normalized size	1	1.	1.64	1.24	0.	0.02	0.	0.	0.83
time (sec)	N/A	0.097	0.073	0.011	0.	0.277	0.	0.	13.587

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	114	94	0	1	0	0	78
normalized size	1	1.	1.31	1.08	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.175	0.084	0.011	0.	0.278	0.	0.	20.585

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	92	87	0	1	0	0	99
normalized size	1	1.	0.84	0.8	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.257	0.076	0.015	0.	0.281	0.	0.	23.458

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	74	0	1	0	0	71
normalized size	1	1.	0.96	0.91	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.198	0.057	0.01	0.	0.292	0.	0.	18.103

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	65	62	0	1	0	55	46
normalized size	1	1.	1.18	1.13	0.	0.02	0.	1.	0.84
time (sec)	N/A	0.156	0.046	0.011	0.	0.276	0.	0.285	13.939

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	28	27	35	0	47	17
normalized size	1	1.	1.	1.27	1.23	1.59	0.	2.14	0.77
time (sec)	N/A	0.076	0.02	0.004	0.694	0.258	0.	0.294	8.037

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	29	37	55	55	0	38	29
normalized size	1	1.	1.04	1.32	1.96	1.96	0.	1.36	1.04
time (sec)	N/A	0.078	0.029	0.007	0.694	0.262	0.	0.292	7.324

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	46	45	0	73	0	0	68
normalized size	1	1.	0.62	0.61	0.	0.99	0.	0.	0.92
time (sec)	N/A	0.216	0.034	0.007	0.	0.268	0.	0.	21.742

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	59	0	85	0	0	95
normalized size	1	1.	0.56	0.58	0.	0.83	0.	0.	0.93
time (sec)	N/A	0.296	0.041	0.007	0.	0.271	0.	0.	29.922

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	68	72	0	103	0	0	122
normalized size	1	1.	0.52	0.55	0.	0.79	0.	0.	0.94
time (sec)	N/A	0.38	0.049	0.008	0.	0.285	0.	0.	38.302

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	29	37	30	53	0	70	37
normalized size	1	1.	0.62	0.79	0.64	1.13	0.	1.49	0.79
time (sec)	N/A	0.094	0.026	0.007	0.736	0.266	0.	0.282	15.662

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	29	19	39	0	23	17
normalized size	1	1.	1.	1.38	0.9	1.86	0.	1.1	0.81
time (sec)	N/A	0.015	0.012	0.004	0.736	0.26	0.	0.283	8.405

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	75	67	0	1	0	0	42
normalized size	1	1.	1.47	1.31	0.	0.02	0.	0.	0.82
time (sec)	N/A	0.101	0.059	0.008	0.	0.276	0.	0.	13.814

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	99	79	0	1	0	4	73
normalized size	1	1.	1.22	0.98	0.	0.01	0.	0.05	0.9
time (sec)	N/A	0.12	0.069	0.01	0.	0.278	0.	0.64	18.811

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	115	94	0	1	0	0	102
normalized size	1	1.	1.06	0.86	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.26	0.1	0.012	0.	0.281	0.	0.	29.925

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	58	48	35	50	0	35	27
normalized size	1	1.	1.71	1.41	1.03	1.47	0.	1.03	0.79
time (sec)	N/A	0.099	0.033	0.009	0.771	0.265	0.	0.281	11.083

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	52	54	35	80	0	36	31
normalized size	1	1.	1.53	1.59	1.03	2.35	0.	1.06	0.91
time (sec)	N/A	0.103	0.034	0.013	0.766	0.264	0.	0.287	10.77

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	51	48	55	61	0	55	37
normalized size	1	1.	1.13	1.07	1.22	1.36	0.	1.22	0.82
time (sec)	N/A	0.104	0.024	0.009	0.759	0.259	0.	0.279	10.541

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	58	60	55	61	0	57	37
normalized size	1	1.	1.29	1.33	1.22	1.36	0.	1.27	0.82
time (sec)	N/A	0.101	0.021	0.01	0.765	0.262	0.	0.282	10.737

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	76	64	0	1	0	80	48
normalized size	1	1.	1.31	1.1	0.	0.02	0.	1.38	0.83
time (sec)	N/A	0.154	0.062	0.01	0.	0.277	0.	0.297	13.463

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	67	0	1	0	92	48
normalized size	1	1.	1.28	1.12	0.	0.02	0.	1.53	0.8
time (sec)	N/A	0.158	0.084	0.014	0.	0.274	0.	0.298	13.781

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.009	0.004	0.678	0.257	34.927	0.269	4.083

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.008	0.004	0.684	0.26	17.024	0.266	4.152

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.007	0.004	0.696	0.257	8.137	0.268	4.078

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.008	0.004	0.734	0.257	2.505	0.268	4.103

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	24	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.14	0.9	0.86	0.9
time (sec)	N/A	0.014	0.008	0.003	0.684	0.257	2.042	0.267	4.092

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	18	22	19	18	19
normalized size	1	1.	1.	0.76	0.86	1.05	0.9	0.86	0.9
time (sec)	N/A	0.014	0.007	0.004	0.688	0.258	2.388	0.267	4.125

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	18	19	17	18	17
normalized size	1	1.	1.	0.79	0.95	1.	0.89	0.95	0.89
time (sec)	N/A	0.014	0.007	0.004	0.687	0.257	3.234	0.269	4.024

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	18	19	17	18	17
normalized size	1	1.	1.	0.84	0.95	1.	0.89	0.95	0.89
time (sec)	N/A	0.014	0.008	0.004	0.684	0.257	6.146	0.269	4.119

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.035	0.013	0.007	0.685	0.259	113.024	0.268	6.743

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.034	0.013	0.007	0.685	0.268	65.733	0.268	6.763

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.036	0.012	0.008	0.68	0.264	35.244	0.267	6.748

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.034	0.012	0.007	0.688	0.266	7.831	0.267	6.737

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.033	0.012	0.007	0.671	0.266	14.29	0.267	6.721

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.034	0.012	0.007	0.689	0.266	15.813	0.268	6.751

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	39	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.08	0.94	0.89	0.94
time (sec)	N/A	0.035	0.012	0.008	0.688	0.271	18.652	0.268	6.673

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	27	32	36	34	32	34
normalized size	1	1.	0.83	0.75	0.89	1.	0.94	0.89	0.94
time (sec)	N/A	0.034	0.012	0.008	0.72	0.269	26.205	0.268	6.692

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	47	54	0	47	49
normalized size	1	1.	1.	0.75	0.92	1.06	0.	0.92	0.96
time (sec)	N/A	0.047	0.017	0.007	0.71	0.271	0.	0.269	8.191

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	47	54	49	47	49
normalized size	1	1.	1.	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.045	0.017	0.008	0.681	0.272	175.417	0.268	8.029

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	38	47	54	49	47	49
normalized size	1	1.	1.	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.045	0.016	0.008	0.679	0.269	113.632	0.268	8.02

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.044	0.014	0.007	0.686	0.269	24.503	0.269	8.048

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.045	0.014	0.008	0.682	0.271	57.546	0.268	7.955

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.047	0.015	0.007	0.684	0.275	62.838	0.268	8.008

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.046	0.014	0.008	0.686	0.261	72.218	0.268	7.982

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	38	47	54	49	47	49
normalized size	1	1.	0.8	0.75	0.92	1.06	0.96	0.92	0.96
time (sec)	N/A	0.046	0.014	0.008	0.676	0.262	90.628	0.267	8.077

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	203	158	0	223	0	266	206
normalized size	1	1.	0.94	0.73	0.	1.03	0.	1.23	0.95
time (sec)	N/A	0.45	0.118	0.039	0.	0.283	0.	0.278	71.328

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	0	207	0	265	204
normalized size	1	1.	0.94	0.71	0.	0.96	0.	1.23	0.95
time (sec)	N/A	0.405	0.073	0.01	0.	0.281	0.	0.276	72.07

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	190	143	0	200	0	240	192
normalized size	1	1.	0.93	0.7	0.	0.98	0.	1.18	0.94
time (sec)	N/A	0.352	0.063	0.009	0.	0.283	0.	0.277	66.265

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	0	144	0	240	190
normalized size	1	1.	0.94	0.69	0.	0.71	0.	1.19	0.94
time (sec)	N/A	0.348	0.047	0.01	0.	0.281	0.	0.276	65.447

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	0	159	0	246	182
normalized size	1	1.	0.76	0.69	0.	0.83	0.	1.28	0.95
time (sec)	N/A	0.293	0.043	0.009	0.	0.282	0.	0.278	60.154

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	146	132	0	147	170	246	182
normalized size	1	1.	0.76	0.69	0.	0.77	0.89	1.28	0.95
time (sec)	N/A	0.289	0.037	0.008	0.	0.281	153.089	0.278	58.728

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	189	140	0	180	175	257	190
normalized size	1	1.	0.94	0.69	0.	0.89	0.87	1.27	0.94
time (sec)	N/A	0.338	0.118	0.012	0.	0.284	86.717	0.278	65.831

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	190	143	0	197	184	240	192
normalized size	1	1.	0.93	0.7	0.	0.97	0.9	1.18	0.94
time (sec)	N/A	0.341	0.116	0.012	0.	0.282	140.023	0.274	64.682

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	203	152	0	230	0	270	204
normalized size	1	1.	0.94	0.71	0.	1.07	0.	1.26	0.95
time (sec)	N/A	0.392	0.174	0.015	0.	0.285	0.	0.276	71.794

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	221	158	0	224	0	259	206
normalized size	1	1.	1.02	0.73	0.	1.03	0.	1.19	0.95
time (sec)	N/A	0.387	0.097	0.014	0.	0.294	0.	0.275	71.928

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	234	169	0	244	0	269	219
normalized size	1	1.	1.02	0.73	0.	1.06	0.	1.17	0.95
time (sec)	N/A	0.431	0.104	0.016	0.	0.285	0.	0.276	80.373

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	227	172	0	284	0	292	230
normalized size	1	1.	0.93	0.71	0.	1.17	0.	1.2	0.95
time (sec)	N/A	0.441	0.377	0.018	0.	0.285	0.	0.279	79.408

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	161	0	288	0	265	216
normalized size	1	1.	0.92	0.7	0.	1.25	0.	1.15	0.94
time (sec)	N/A	0.394	0.284	0.018	0.	0.285	0.	0.279	72.967

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	158	0	239	0	265	216
normalized size	1	1.	0.92	0.69	0.	1.04	0.	1.15	0.94
time (sec)	N/A	0.384	0.27	0.019	0.	0.282	0.	0.278	71.692

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	0	234	0	269	204
normalized size	1	1.	0.91	0.68	0.	1.07	0.	1.23	0.94
time (sec)	N/A	0.347	0.219	0.016	0.	0.278	0.	0.28	66.243

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	0	225	0	269	197
normalized size	1	1.	0.91	0.72	0.	1.03	0.	1.23	0.9
time (sec)	N/A	0.348	0.227	0.017	0.	0.286	0.	0.279	64.996

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	198	158	0	235	0	269	197
normalized size	1	1.	0.91	0.72	0.	1.08	0.	1.23	0.9
time (sec)	N/A	0.342	0.264	0.012	0.	0.285	0.	0.28	65.804

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	199	149	0	221	0	269	204
normalized size	1	1.	0.91	0.68	0.	1.01	0.	1.23	0.94
time (sec)	N/A	0.33	0.226	0.012	0.	0.284	0.	0.278	63.71

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	158	0	259	0	284	218
normalized size	1	1.	0.92	0.69	0.	1.13	0.	1.23	0.95
time (sec)	N/A	0.386	0.415	0.02	0.	0.293	0.	0.281	71.963

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	212	161	0	282	0	265	218
normalized size	1	1.	0.92	0.7	0.	1.23	0.	1.15	0.95
time (sec)	N/A	0.379	0.383	0.019	0.	0.287	0.	0.28	70.957

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	227	172	0	325	0	297	231
normalized size	1	1.	0.93	0.71	0.	1.34	0.	1.22	0.95
time (sec)	N/A	0.431	0.405	0.025	0.	0.284	0.	0.28	79.702

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	227	178	0	316	0	286	231
normalized size	1	1.	0.93	0.73	0.	1.3	0.	1.18	0.95
time (sec)	N/A	0.434	0.429	0.022	0.	0.289	0.	0.276	78.554

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	242	189	0	340	0	296	246
normalized size	1	1.	0.94	0.73	0.	1.32	0.	1.15	0.95
time (sec)	N/A	0.488	0.487	0.024	0.	0.289	0.	0.278	86.963

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	236	178	0	313	0	281	236
normalized size	1	1.	0.94	0.71	0.	1.25	0.	1.12	0.94
time (sec)	N/A	0.439	0.19	0.024	0.	0.286	0.	0.279	79.549

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	161	0	319	0	282	224
normalized size	1	1.	0.92	0.67	0.	1.33	0.	1.18	0.94
time (sec)	N/A	0.392	0.208	0.02	0.	0.285	0.	0.282	72.885

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	221	170	0	319	0	282	224
normalized size	1	1.	0.92	0.71	0.	1.33	0.	1.18	0.94
time (sec)	N/A	0.392	0.201	0.022	0.	0.285	0.	0.279	72.782

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	0	335	0	286	224
normalized size	1	1.	0.92	0.7	0.	1.38	0.	1.18	0.93
time (sec)	N/A	0.391	0.216	0.02	0.	0.283	0.	0.281	72.724

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	223	169	0	323	0	285	223
normalized size	1	1.	0.92	0.7	0.	1.33	0.	1.18	0.92
time (sec)	N/A	0.398	0.224	0.022	0.	0.284	0.	0.278	71.756

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	220	175	0	321	0	282	224
normalized size	1	1.	0.92	0.73	0.	1.34	0.	1.18	0.94
time (sec)	N/A	0.394	0.187	0.013	0.	0.286	0.	0.28	72.136

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	220	166	0	305	0	282	224
normalized size	1	1.	0.92	0.69	0.	1.28	0.	1.18	0.94
time (sec)	N/A	0.382	0.179	0.012	0.	0.285	0.	0.281	71.215

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	234	178	0	333	0	297	238
normalized size	1	1.	0.93	0.71	0.	1.33	0.	1.18	0.95
time (sec)	N/A	0.437	0.221	0.025	0.	0.29	0.	0.28	80.45

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	234	181	0	356	0	281	238
normalized size	1	1.	0.93	0.72	0.	1.42	0.	1.12	0.95
time (sec)	N/A	0.431	0.21	0.025	0.	0.291	0.	0.28	81.442

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	251	192	0	400	0	313	252
normalized size	1	1.	0.95	0.73	0.	1.52	0.	1.19	0.95
time (sec)	N/A	0.498	0.238	0.027	0.	0.289	0.	0.282	97.451

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	251	198	0	390	0	302	252
normalized size	1	1.	0.95	0.75	0.	1.48	0.	1.14	0.95
time (sec)	N/A	0.491	0.237	0.027	0.	0.292	0.	0.281	99.51

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	209	0	414	0	312	267
normalized size	1	1.	0.95	0.75	0.	1.48	0.	1.12	0.96
time (sec)	N/A	0.548	0.282	0.029	0.	0.291	0.	0.282	104.615

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	266	209	0	405	0	328	267
normalized size	1	1.	0.95	0.75	0.	1.45	0.	1.18	0.96
time (sec)	N/A	0.539	0.263	0.028	0.	0.293	0.	0.277	102.195

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	201	237	0	0	0	0	304
normalized size	1	1.	0.62	0.73	0.	0.	0.	0.	0.94
time (sec)	N/A	0.745	0.342	0.095	0.	0.	0.	0.	67.796

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	133	157	0	0	0	0	168
normalized size	1	1.	0.76	0.89	0.	0.	0.	0.	0.95
time (sec)	N/A	0.459	0.601	0.033	0.	0.	0.	0.	42.236

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	190	226	0	0	0	0	275
normalized size	1	1.	0.65	0.77	0.	0.	0.	0.	0.94
time (sec)	N/A	0.622	0.315	0.033	0.	0.	0.	0.	56.512

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	120	145	0	0	0	0	139
normalized size	1	1.	0.82	0.99	0.	0.	0.	0.	0.95
time (sec)	N/A	0.355	0.427	0.032	0.	0.	0.	0.	32.417

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	176	213	0	0	0	0	248
normalized size	1	1.	0.67	0.81	0.	0.	0.	0.	0.94
time (sec)	N/A	0.506	0.27	0.032	0.	0.	0.	0.	45.293

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	130	0	0	0	0	114
normalized size	1	1.	0.86	1.1	0.	0.	0.	0.	0.97
time (sec)	N/A	0.266	0.229	0.031	0.	0.	0.	0.	24.6

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	175	202	0	0	0	0	240
normalized size	1	1.	0.69	0.8	0.	0.	0.	0.	0.94
time (sec)	N/A	0.507	0.257	0.039	0.	0.	0.	0.	47.661

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	125	0	0	0	0	114
normalized size	1	1.	0.88	1.06	0.	0.	0.	0.	0.97
time (sec)	N/A	0.267	0.317	0.036	0.	0.	0.	0.	24.887

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	196	224	0	0	0	0	274
normalized size	1	1.	0.67	0.76	0.	0.	0.	0.	0.94
time (sec)	N/A	0.619	0.364	0.042	0.	0.	0.	0.	57.372

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	122	142	0	0	0	0	141
normalized size	1	1.	0.84	0.97	0.	0.	0.	0.	0.97
time (sec)	N/A	0.359	0.429	0.039	0.	0.	0.	0.	32.599

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	209	239	0	0	0	0	304
normalized size	1	1.	0.65	0.74	0.	0.	0.	0.	0.94
time (sec)	N/A	0.74	0.312	0.042	0.	0.	0.	0.	68.199

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	133	156	0	0	0	0	168
normalized size	1	1.	0.76	0.89	0.	0.	0.	0.	0.95
time (sec)	N/A	0.453	0.516	0.04	0.	0.	0.	0.	41.206

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	212	248	0	0	0	0	330
normalized size	1	1.	0.61	0.71	0.	0.	0.	0.	0.94
time (sec)	N/A	0.865	0.436	0.034	0.	0.	0.	0.	81.908

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	164	168	0	0	0	0	194
normalized size	1	1.	0.81	0.83	0.	0.	0.	0.	0.96
time (sec)	N/A	0.55	0.321	0.036	0.	0.	0.	0.	52.073

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	201	237	0	0	0	0	301
normalized size	1	1.	0.63	0.74	0.	0.	0.	0.	0.94
time (sec)	N/A	0.723	0.326	0.018	0.	0.	0.	0.	67.419

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	153	157	0	0	0	0	165
normalized size	1	1.	0.88	0.91	0.	0.	0.	0.	0.95
time (sec)	N/A	0.447	0.298	0.018	0.	0.	0.	0.	39.923

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	190	226	0	0	0	0	274
normalized size	1	1.	0.66	0.78	0.	0.	0.	0.	0.94
time (sec)	N/A	0.61	0.307	0.018	0.	0.	0.	0.	55.39

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	119	145	0	0	0	0	138
normalized size	1	1.	0.83	1.01	0.	0.	0.	0.	0.97
time (sec)	N/A	0.349	0.388	0.018	0.	0.	0.	0.	31.586

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	190	216	0	0	0	0	270
normalized size	1	1.	0.66	0.76	0.	0.	0.	0.	0.94
time (sec)	N/A	0.604	0.272	0.024	0.	0.	0.	0.	56.29

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	111	130	0	0	0	0	138
normalized size	1	1.	0.78	0.91	0.	0.	0.	0.	0.97
time (sec)	N/A	0.353	0.337	0.021	0.	0.	0.	0.	36.233

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	193	221	0	0	0	0	270
normalized size	1	1.	0.67	0.77	0.	0.	0.	0.	0.94
time (sec)	N/A	0.604	0.344	0.023	0.	0.	0.	0.	54.452

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	120	140	0	0	0	0	138
normalized size	1	1.	0.84	0.98	0.	0.	0.	0.	0.97
time (sec)	N/A	0.35	0.476	0.021	0.	0.	0.	0.	31.549

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	209	239	0	0	0	0	299
normalized size	1	1.	0.65	0.75	0.	0.	0.	0.	0.93
time (sec)	N/A	0.731	0.476	0.024	0.	0.	0.	0.	65.895

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	154	156	0	0	0	0	167
normalized size	1	1.	0.89	0.9	0.	0.	0.	0.	0.97
time (sec)	N/A	0.447	0.322	0.021	0.	0.	0.	0.	42.688

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	220	250	0	0	0	0	330
normalized size	1	1.	0.63	0.71	0.	0.	0.	0.	0.94
time (sec)	N/A	0.865	0.448	0.048	0.	0.	0.	0.	83.499

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	165	167	0	0	0	0	194
normalized size	1	1.	0.81	0.82	0.	0.	0.	0.	0.96
time (sec)	N/A	0.552	0.351	0.045	0.	0.	0.	0.	59.339

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	153	148	0	0	0	0	172
normalized size	1	1.	0.85	0.83	0.	0.	0.	0.	0.96
time (sec)	N/A	0.463	0.187	0.036	0.	0.	0.	0.	45.103

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	190	217	0	0	0	0	279
normalized size	1	1.	0.64	0.73	0.	0.	0.	0.	0.94
time (sec)	N/A	0.629	0.253	0.036	0.	0.	0.	0.	60.457

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	137	0	0	0	0	143
normalized size	1	1.	0.97	0.92	0.	0.	0.	0.	0.96
time (sec)	N/A	0.364	0.161	0.018	0.	0.	0.	0.	36.582

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	176	206	0	0	0	0	250
normalized size	1	1.	0.66	0.77	0.	0.	0.	0.	0.94
time (sec)	N/A	0.51	0.245	0.019	0.	0.	0.	0.	54.376

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	126	123	0	0	0	0	114
normalized size	1	1.	1.04	1.02	0.	0.	0.	0.	0.94
time (sec)	N/A	0.271	0.103	0.016	0.	0.	0.	0.	27.418

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	112	131	0	0	0	0	218
normalized size	1	1.	0.48	0.57	0.	0.	0.	0.	0.94
time (sec)	N/A	0.402	0.088	0.018	0.	0.	0.	0.	38.044

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	106	0	0	0	0	88
normalized size	1	1.	0.94	1.18	0.	0.	0.	0.	0.98
time (sec)	N/A	0.181	0.045	0.022	0.	0.	0.	0.	16.98

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	177	195	0	0	0	0	241
normalized size	1	1.	0.68	0.75	0.	0.	0.	0.	0.93
time (sec)	N/A	0.502	0.19	0.024	0.	0.	0.	0.	49.343

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	110	119	0	0	0	0	116
normalized size	1	1.	0.91	0.98	0.	0.	0.	0.	0.96
time (sec)	N/A	0.278	0.273	0.022	0.	0.	0.	0.	26.04

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	199	215	0	0	0	0	279
normalized size	1	1.	0.67	0.73	0.	0.	0.	0.	0.94
time (sec)	N/A	0.625	0.24	0.024	0.	0.	0.	0.	58.546

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	134	0	0	0	0	143
normalized size	1	1.	0.97	0.9	0.	0.	0.	0.	0.96
time (sec)	N/A	0.366	0.162	0.022	0.	0.	0.	0.	34.905

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	210	230	0	0	0	0	308
normalized size	1	1.	0.64	0.71	0.	0.	0.	0.	0.94
time (sec)	N/A	0.747	0.26	0.025	0.	0.	0.	0.	76.492

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	134	147	0	0	0	0	172
normalized size	1	1.	0.75	0.82	0.	0.	0.	0.	0.96
time (sec)	N/A	0.454	0.336	0.022	0.	0.	0.	0.	45.671

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	141	144	0	0	0	0	165
normalized size	1	1.	0.81	0.83	0.	0.	0.	0.	0.95
time (sec)	N/A	0.457	0.15	0.051	0.	0.	0.	0.	46.21

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	179	213	0	0	0	0	272
normalized size	1	1.	0.62	0.73	0.	0.	0.	0.	0.93
time (sec)	N/A	0.625	0.209	0.046	0.	0.	0.	0.	59.285

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	128	131	0	0	0	0	138
normalized size	1	1.	0.88	0.9	0.	0.	0.	0.	0.95
time (sec)	N/A	0.355	0.137	0.023	0.	0.	0.	0.	33.841

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	167	200	0	0	0	0	241
normalized size	1	1.	0.64	0.77	0.	0.	0.	0.	0.93
time (sec)	N/A	0.504	0.178	0.025	0.	0.	0.	0.	47.222

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	120	0	0	0	0	110
normalized size	1	1.	0.97	1.01	0.	0.	0.	0.	0.92
time (sec)	N/A	0.269	0.093	0.018	0.	0.	0.	0.	25.06

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	168	203	0	0	0	0	238
normalized size	1	1.	0.65	0.78	0.	0.	0.	0.	0.92
time (sec)	N/A	0.511	0.17	0.019	0.	0.	0.	0.	48.659

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	114	123	0	0	0	0	110
normalized size	1	1.	0.97	1.04	0.	0.	0.	0.	0.93
time (sec)	N/A	0.27	0.085	0.026	0.	0.	0.	0.	24.949

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	181	203	0	0	0	0	269
normalized size	1	1.	0.63	0.71	0.	0.	0.	0.	0.94
time (sec)	N/A	0.624	0.204	0.028	0.	0.	0.	0.	58.855

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	110	127	0	0	0	0	139
normalized size	1	1.	0.76	0.88	0.	0.	0.	0.	0.96
time (sec)	N/A	0.357	0.24	0.029	0.	0.	0.	0.	33.262

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	198	222	0	0	0	0	303
normalized size	1	1.	0.62	0.69	0.	0.	0.	0.	0.95
time (sec)	N/A	0.727	0.207	0.028	0.	0.	0.	0.	70.86

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	143	141	0	0	0	0	167
normalized size	1	1.	0.83	0.82	0.	0.	0.	0.	0.97
time (sec)	N/A	0.462	0.14	0.027	0.	0.	0.	0.	43.375

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	210	237	0	0	0	0	332
normalized size	1	1.	0.6	0.68	0.	0.	0.	0.	0.95
time (sec)	N/A	0.86	0.238	0.03	0.	0.	0.	0.	83.225

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	59	181	103	217	758	400	65
normalized size	1	1.	0.8	2.45	1.39	2.93	10.24	5.41	0.88
time (sec)	N/A	0.128	0.057	0.008	0.7	0.272	16.167	0.277	24.357

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	43	96	74	120	352	215	46
normalized size	1	1.	0.8	1.78	1.37	2.22	6.52	3.98	0.85
time (sec)	N/A	0.092	0.033	0.008	0.7	0.271	7.017	0.274	18.214

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	27	39	46	53	119	86	27
normalized size	1	1.	0.79	1.15	1.35	1.56	3.5	2.53	0.79
time (sec)	N/A	0.036	0.028	0.003	0.696	0.272	1.964	0.271	10.109

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	56	0	0	0	0	0	34
normalized size	1	1.	1.24	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.063	0.071	0.058	0.	0.	0.	0.	8.41

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	109	0	0	0	0	0	37
normalized size	1	1.	2.32	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.06	0.131	0.074	0.	0.	0.	0.	8.532

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	164	0	0	0	0	0	37
normalized size	1	1.	3.49	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.059	0.209	0.06	0.	0.	0.	0.	8.483

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	24	32	27
normalized size	1	1.	1.	0.83	1.07	0.03	0.8	1.07	0.9
time (sec)	N/A	0.027	0.003	0.001	0.695	0.238	0.077	0.268	14.142

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	1	26	32	26
normalized size	1	1.	1.	0.83	1.07	0.03	0.87	1.07	0.87
time (sec)	N/A	0.025	0.002	0.001	0.683	0.241	0.071	0.267	12.025

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	16	25	32	1	24	32	10
normalized size	1	1.	0.53	0.83	1.07	0.03	0.8	1.07	0.33
time (sec)	N/A	0.025	0.003	0.001	0.678	0.241	0.075	0.268	5.914

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	1	22	28	0
normalized size	1	1.	1.	0.88	1.12	0.04	0.88	1.12	0.
time (sec)	N/A	0.016	0.	0.001	0.693	0.24	0.071	0.269	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	32	28	20	32	0
normalized size	1	1.	1.	0.96	1.39	1.22	0.87	1.39	0.
time (sec)	N/A	0.021	0.002	0.004	0.682	0.261	0.953	0.269	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	34	19	30	19
normalized size	1	1.	1.	0.96	1.25	1.42	0.79	1.25	0.79
time (sec)	N/A	0.023	0.002	0.005	0.696	0.253	0.951	0.268	11.229

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	32	36	24	43	0
normalized size	1	1.	1.	0.89	1.19	1.33	0.89	1.59	0.
time (sec)	N/A	0.025	0.002	0.008	0.692	0.264	1.058	0.27	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	30	35	20	30	0
normalized size	1	1.	1.	0.96	1.3	1.52	0.87	1.3	0.
time (sec)	N/A	0.024	0.002	0.008	0.687	0.256	1.078	0.268	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	35	38	22	46	24
normalized size	1	1.	1.	0.96	1.46	1.58	0.92	1.92	1.
time (sec)	N/A	0.023	0.002	0.008	0.691	0.266	1.223	0.27	13.929

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	35	35	27	35	24
normalized size	1	1.	1.	0.89	1.25	1.25	0.96	1.25	0.86
time (sec)	N/A	0.024	0.002	0.007	0.681	0.258	1.232	0.269	11.915

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	32	26	32	15
normalized size	1	1.	1.	0.83	1.07	1.07	0.87	1.07	0.5
time (sec)	N/A	0.024	0.002	0.007	0.683	0.254	1.259	0.267	7.703

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	35	35	27	35	27
normalized size	1	1.	1.	0.83	1.17	1.17	0.9	1.17	0.9
time (sec)	N/A	0.024	0.002	0.007	0.69	0.258	1.291	0.268	11.631

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	1	53	62	53
normalized size	1	1.	1.	0.84	1.11	0.02	0.95	1.11	0.95
time (sec)	N/A	0.079	0.004	0.001	0.688	0.242	0.112	0.269	17.515

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	56	47	62	1	49	62	44
normalized size	1	1.	1.06	0.89	1.17	0.02	0.92	1.17	0.83
time (sec)	N/A	0.166	0.003	0.001	0.686	0.242	0.105	0.267	20.088

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	1	53	62	53
normalized size	1	1.	1.	0.84	1.11	0.02	0.95	1.11	0.95
time (sec)	N/A	0.074	0.004	0.002	0.682	0.242	0.106	0.268	17.54

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	56	47	62	1	53	62	27
normalized size	1	1.	1.65	1.38	1.82	0.03	1.56	1.82	0.79
time (sec)	N/A	0.103	0.004	0.001	0.688	0.242	0.108	0.268	16.519

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	62	1	53	62	53
normalized size	1	1.	1.	0.84	1.11	0.02	0.95	1.11	0.95
time (sec)	N/A	0.074	0.003	0.002	0.687	0.242	0.106	0.266	17.88

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	45	59	1	44	59	10
normalized size	1	1.	1.	2.81	3.69	0.06	2.75	3.69	0.62
time (sec)	N/A	0.017	0.004	0.001	0.703	0.242	0.108	0.268	6.661

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	74	1	49	58	49
normalized size	1	1.	1.	0.86	1.45	0.02	0.96	1.14	0.96
time (sec)	N/A	0.055	0.002	0.001	0.704	0.242	0.112	0.268	25.131

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	63	59	49	63	0
normalized size	1	1.	1.	0.9	1.26	1.18	0.98	1.26	0.
time (sec)	N/A	0.082	0.007	0.003	0.707	0.268	1.114	0.27	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	59	65	44	59	44
normalized size	1	1.	1.	0.94	1.23	1.35	0.92	1.23	0.92
time (sec)	N/A	0.064	0.013	0.005	0.694	0.259	1.055	0.268	17.472

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	62	66	46	76	0
normalized size	1	1.	1.	0.94	1.29	1.38	0.96	1.58	0.
time (sec)	N/A	0.092	0.008	0.008	0.703	0.264	1.156	0.271	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	61	65	48	61	46
normalized size	1	1.	1.	0.9	1.22	1.3	0.96	1.22	0.92
time (sec)	N/A	0.066	0.011	0.008	0.704	0.256	1.176	0.268	17.356

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	65	66	48	80	0
normalized size	1	1.	1.	0.94	1.33	1.35	0.98	1.63	0.
time (sec)	N/A	0.09	0.008	0.009	0.698	0.254	1.331	0.27	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	63	65	48	63	46
normalized size	1	1.	1.	0.9	1.26	1.3	0.96	1.26	0.92
time (sec)	N/A	0.067	0.014	0.008	0.696	0.248	1.356	0.268	17.142

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	46	65	68	48	77	0
normalized size	1	1.	1.	0.94	1.33	1.39	0.98	1.57	0.
time (sec)	N/A	0.087	0.008	0.009	0.709	0.257	1.519	0.269	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	62	65	46	62	0
normalized size	1	1.	1.	0.94	1.32	1.38	0.98	1.32	0.
time (sec)	N/A	0.068	0.009	0.009	0.698	0.248	1.561	0.267	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	68	68	48	78	53
normalized size	1	1.	1.	0.9	1.36	1.36	0.96	1.56	1.06
time (sec)	N/A	0.084	0.008	0.009	0.694	0.255	1.74	0.269	20.161

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	47	65	65	51	65	51
normalized size	1	1.	1.	0.87	1.2	1.2	0.94	1.2	0.94
time (sec)	N/A	0.07	0.014	0.008	0.704	0.252	1.751	0.268	17.491

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	52	47	62	62	49	62	15
normalized size	1	1.	2.74	2.47	3.26	3.26	2.58	3.26	0.79
time (sec)	N/A	0.024	0.007	0.008	0.699	0.252	1.86	0.268	8.255

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	65	51	65	54
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.96
time (sec)	N/A	0.068	0.012	0.008	0.706	0.251	1.89	0.269	18.07

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	47	65	65	51	65	54
normalized size	1	1.	1.4	1.18	1.62	1.62	1.27	1.62	1.35
time (sec)	N/A	0.072	0.007	0.008	0.703	0.255	1.98	0.269	21.002

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	65	51	65	54
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.96
time (sec)	N/A	0.069	0.014	0.011	0.696	0.255	1.968	0.266	17.699

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	65	51	65	51
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.91
time (sec)	N/A	0.088	0.007	0.008	0.695	0.258	2.077	0.268	20.827

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	65	65	51	65	54
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	0.96
time (sec)	N/A	0.067	0.012	0.008	0.695	0.251	2.095	0.268	17.671

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	92	1	80	92	80
normalized size	1	1.	1.	0.84	1.12	0.01	0.98	1.12	0.98
time (sec)	N/A	0.116	0.005	0.001	0.703	0.236	0.132	0.267	23.295

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	82	69	92	1	78	92	63
normalized size	1	1.	1.14	0.96	1.28	0.01	1.08	1.28	0.88
time (sec)	N/A	0.255	0.004	0.002	0.705	0.236	0.139	0.268	27.596

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	90	1	76	90	76
normalized size	1	1.	1.	0.86	1.14	0.01	0.96	1.14	0.96
time (sec)	N/A	0.105	0.004	0.001	0.695	0.236	0.134	0.268	23.157

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	82	69	92	1	80	92	44
normalized size	1	1.	1.55	1.3	1.74	0.02	1.51	1.74	0.83
time (sec)	N/A	0.19	0.004	0.001	0.704	0.237	0.134	0.268	22.329

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	92	1	80	92	80
normalized size	1	1.	1.	0.84	1.12	0.01	0.98	1.12	0.98
time (sec)	N/A	0.104	0.004	0.001	0.679	0.235	0.128	0.269	24.6

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	77	68	90	1	75	90	27
normalized size	1	1.	2.26	2.	2.65	0.03	2.21	2.65	0.79
time (sec)	N/A	0.118	0.005	0.002	0.693	0.235	0.128	0.269	17.942

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	92	1	80	92	80
normalized size	1	1.	1.	0.84	1.12	0.01	0.98	1.12	0.98
time (sec)	N/A	0.103	0.004	0.002	0.689	0.232	0.13	0.269	23.482

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	69	92	1	78	92	10
normalized size	1	1.	1.	4.31	5.75	0.06	4.88	5.75	0.62
time (sec)	N/A	0.018	0.003	0.001	0.684	0.232	0.127	0.267	6.493

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	66	135	1	73	88	73
normalized size	1	1.	1.	0.9	1.85	0.01	1.	1.21	1.
time (sec)	N/A	0.084	0.002	0.001	0.69	0.232	0.121	0.269	32.673

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	93	89	76	93	0
normalized size	1	1.	1.	0.88	1.22	1.17	1.	1.22	0.
time (sec)	N/A	0.116	0.007	0.004	0.684	0.255	1.159	0.27	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	89	95	70	89	70
normalized size	1	1.	1.	0.93	1.24	1.32	0.97	1.24	0.97
time (sec)	N/A	0.093	0.015	0.005	0.689	0.248	1.135	0.267	23.527

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	93	97	76	107	0
normalized size	1	1.	1.	0.88	1.21	1.26	0.99	1.39	0.
time (sec)	N/A	0.136	0.013	0.008	0.688	0.257	1.261	0.271	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	90	95	73	90	71
normalized size	1	1.	1.	0.91	1.22	1.28	0.99	1.22	0.96
time (sec)	N/A	0.094	0.015	0.008	0.689	0.255	1.273	0.268	23.068

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	93	96	71	108	0
normalized size	1	1.	1.	0.93	1.29	1.33	0.99	1.5	0.
time (sec)	N/A	0.129	0.009	0.009	0.684	0.259	1.453	0.27	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	90	95	71	90	70
normalized size	1	1.	1.	0.93	1.25	1.32	0.99	1.25	0.97
time (sec)	N/A	0.096	0.011	0.008	0.692	0.25	1.538	0.268	22.109

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	68	95	96	75	109	0
normalized size	1	1.	1.	0.86	1.2	1.22	0.95	1.38	0.
time (sec)	N/A	0.13	0.009	0.01	0.689	0.261	1.657	0.271	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	67	93	95	71	93	70
normalized size	1	1.	1.	0.93	1.29	1.32	0.99	1.29	0.97
time (sec)	N/A	0.094	0.016	0.009	0.692	0.254	1.679	0.268	22.38

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	68	95	97	71	109	0
normalized size	1	1.	1.	0.93	1.3	1.33	0.97	1.49	0.
time (sec)	N/A	0.125	0.014	0.01	0.689	0.255	1.921	0.27	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	67	93	95	71	93	71
normalized size	1	1.	1.	0.91	1.26	1.28	0.96	1.26	0.96
time (sec)	N/A	0.094	0.015	0.009	0.695	0.247	1.961	0.27	22.412

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	68	97	97	73	109	0
normalized size	1	1.	1.	0.88	1.26	1.26	0.95	1.42	0.
time (sec)	N/A	0.122	0.009	0.011	0.691	0.257	2.241	0.271	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	66	92	95	70	92	0
normalized size	1	1.	1.	0.93	1.3	1.34	0.99	1.3	0.
time (sec)	N/A	0.098	0.01	0.009	0.686	0.249	2.239	0.269	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	67	97	97	71	108	80
normalized size	1	1.	1.	0.88	1.28	1.28	0.93	1.42	1.05
time (sec)	N/A	0.12	0.008	0.01	0.685	0.256	2.52	0.271	27.247

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	69	95	95	75	95	75
normalized size	1	1.	1.	0.91	1.25	1.25	0.99	1.25	0.99
time (sec)	N/A	0.099	0.016	0.008	0.69	0.25	2.497	0.268	23.114

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	82	69	92	92	73	92	15
normalized size	1	1.	4.32	3.63	4.84	4.84	3.84	4.84	0.79
time (sec)	N/A	0.025	0.013	0.008	0.686	0.249	2.625	0.27	8.454

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	95	95	75	95	82
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	1.
time (sec)	N/A	0.096	0.016	0.008	0.682	0.248	2.627	0.268	23.463

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	78	69	95	95	75	95	32
normalized size	1	1.	1.95	1.72	2.38	2.38	1.88	2.38	0.8
time (sec)	N/A	0.072	0.008	0.008	0.696	0.248	2.821	0.269	13.793

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	95	95	75	95	82
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	1.
time (sec)	N/A	0.097	0.011	0.008	0.688	0.248	2.81	0.268	23.261

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	69	95	95	75	95	53
normalized size	1	1.	1.32	1.11	1.53	1.53	1.21	1.53	0.85
time (sec)	N/A	0.107	0.008	0.007	0.679	0.251	2.954	0.27	17.584

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	69	95	95	75	95	78
normalized size	1	1.	1.	0.86	1.19	1.19	0.94	1.19	0.98
time (sec)	N/A	0.1	0.016	0.009	0.687	0.248	2.965	0.269	22.977

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	69	95	95	75	95	80
normalized size	1	1.	0.98	0.82	1.13	1.13	0.89	1.13	0.95
time (sec)	N/A	0.145	0.013	0.008	0.685	0.247	3.162	0.269	26.77

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	69	95	95	75	95	82
normalized size	1	1.	1.	0.84	1.16	1.16	0.91	1.16	1.
time (sec)	N/A	0.1	0.016	0.008	0.69	0.252	3.136	0.27	23.009

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	74	104	126	80	124	0
normalized size	1	1.	0.87	0.89	1.25	1.52	0.96	1.49	0.
time (sec)	N/A	0.177	0.036	0.01	0.681	0.255	1.591	0.271	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	60	63	88	109	66	108	0
normalized size	1	1.	0.86	0.9	1.26	1.56	0.94	1.54	0.
time (sec)	N/A	0.145	0.037	0.01	0.691	0.254	1.533	0.27	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	52	73	95	53	90	0
normalized size	1	1.	0.86	0.91	1.28	1.67	0.93	1.58	0.
time (sec)	N/A	0.122	0.027	0.01	0.683	0.254	1.471	0.27	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	38	41	58	76	39	66	0
normalized size	1	1.	0.86	0.93	1.32	1.73	0.89	1.5	0.
time (sec)	N/A	0.095	0.026	0.01	0.681	0.257	1.405	0.27	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	30	43	47	29	41	26
normalized size	1	1.	0.82	0.91	1.3	1.42	0.88	1.24	0.79
time (sec)	N/A	0.071	0.015	0.009	0.689	0.252	1.264	0.271	14.905

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	20	20	15	19	12
normalized size	1	1.	1.	0.94	1.25	1.25	0.94	1.19	0.75
time (sec)	N/A	0.015	0.005	0.005	0.697	0.246	1.15	0.27	6.742

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	50	63	34	63	34
normalized size	1	1.	0.87	0.92	1.32	1.66	0.89	1.66	0.89
time (sec)	N/A	0.088	0.022	0.017	0.685	0.26	1.613	0.273	19.349

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	70	99	49	69	46
normalized size	1	1.	0.84	0.94	1.43	2.02	1.	1.41	0.94
time (sec)	N/A	0.106	0.064	0.018	0.685	0.259	1.928	0.27	19.277

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	61	95	122	68	116	66
normalized size	1	1.	0.86	0.92	1.44	1.85	1.03	1.76	1.
time (sec)	N/A	0.12	0.117	0.02	0.689	0.259	2.43	0.27	22.275

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	82	78	0	1	134	113	0
normalized size	1	1.	0.89	0.85	0.	0.01	1.46	1.23	0.
time (sec)	N/A	0.129	0.095	0.017	0.	0.262	1.719	0.269	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	71	68	0	1	124	99	0
normalized size	1	1.	0.9	0.86	0.	0.01	1.57	1.25	0.
time (sec)	N/A	0.11	0.085	0.012	0.	0.263	1.631	0.27	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	57	0	1	107	82	0
normalized size	1	1.	0.91	0.86	0.	0.02	1.62	1.24	0.
time (sec)	N/A	0.093	0.072	0.013	0.	0.263	1.563	0.268	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	43	0	1	83	57	48
normalized size	1	1.	0.93	0.78	0.	0.02	1.51	1.04	0.87
time (sec)	N/A	0.071	0.059	0.011	0.	0.263	1.456	0.271	19.434

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.048	0.036	0.01	0.	0.263	1.283	0.269	13.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.043	0.043	0.005	0.	0.26	1.34	0.268	19.551

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	0	1	90	63	48
normalized size	1	1.	0.95	0.81	0.	0.02	1.58	1.11	0.84
time (sec)	N/A	0.072	0.065	0.014	0.	0.264	1.652	0.27	19.625

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	0	1	114	80	61
normalized size	1	1.	0.99	0.87	0.	0.01	1.68	1.18	0.9
time (sec)	N/A	0.094	0.075	0.016	0.	0.268	1.956	0.27	27.699

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	80	70	0	1	126	95	75
normalized size	1	1.	0.99	0.86	0.	0.01	1.56	1.17	0.93
time (sec)	N/A	0.119	0.08	0.017	0.	0.265	2.626	0.27	33.994

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	78	86	134	185	100	123	0
normalized size	1	1.	0.86	0.95	1.47	2.03	1.1	1.35	0.
time (sec)	N/A	0.208	0.054	0.017	0.684	0.253	2.78	0.272	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	74	119	167	88	99	0
normalized size	1	1.	0.77	0.96	1.55	2.17	1.14	1.29	0.
time (sec)	N/A	0.168	0.088	0.016	0.697	0.257	2.642	0.272	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	50	64	104	138	76	72	63
normalized size	1	1.	0.7	0.9	1.46	1.94	1.07	1.01	0.89
time (sec)	N/A	0.145	0.032	0.013	0.687	0.252	2.287	0.271	26.14

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	35	48	78	78	60	45	14
normalized size	1	1.	1.84	2.53	4.11	4.11	3.16	2.37	0.74
time (sec)	N/A	0.025	0.022	0.012	0.684	0.249	2.123	0.27	9.019

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	63	63	48	30	29
normalized size	1	1.	0.71	0.91	1.85	1.85	1.41	0.88	0.85
time (sec)	N/A	0.073	0.012	0.011	0.678	0.249	1.984	0.27	16.569

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	50	50	39	19	14
normalized size	1	1.	1.	0.94	3.12	3.12	2.44	1.19	0.88
time (sec)	N/A	0.015	0.005	0.006	0.692	0.259	1.892	0.269	7.011

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	54	63	111	181	80	95	65
normalized size	1	1.	0.77	0.9	1.59	2.59	1.14	1.36	0.93
time (sec)	N/A	0.156	0.075	0.018	0.693	0.269	2.957	0.271	28.902

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	70	77	134	220	100	126	82
normalized size	1	1.	0.83	0.92	1.6	2.62	1.19	1.5	0.98
time (sec)	N/A	0.186	0.119	0.023	0.688	0.27	4.835	0.271	31.866

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	85	96	154	240	116	146	100
normalized size	1	1.	0.84	0.95	1.52	2.38	1.15	1.45	0.99
time (sec)	N/A	0.215	0.109	0.022	0.697	0.272	9.002	0.271	37.915

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	99	108	0	1	172	130	0
normalized size	1	1.	0.85	0.92	0.	0.01	1.47	1.11	0.
time (sec)	N/A	0.185	0.105	0.015	0.	0.273	2.909	0.27	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	89	97	0	1	155	113	0
normalized size	1	1.	0.86	0.93	0.	0.01	1.49	1.09	0.
time (sec)	N/A	0.158	0.083	0.015	0.	0.271	2.784	0.27	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	77	83	0	1	131	88	85
normalized size	1	1.	0.83	0.89	0.	0.01	1.41	0.95	0.91
time (sec)	N/A	0.135	0.083	0.014	0.	0.279	2.623	0.269	30.937

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	66	58	0	1	133	76	75
normalized size	1	1.	0.8	0.7	0.	0.01	1.6	0.92	0.9
time (sec)	N/A	0.112	0.069	0.013	0.	0.274	2.311	0.271	23.956

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	69	58	0	1	143	84	70
normalized size	1	1.	0.82	0.69	0.	0.01	1.7	1.	0.83
time (sec)	N/A	0.106	0.081	0.012	0.	0.274	2.201	0.27	21.843

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	69	58	0	1	139	84	68
normalized size	1	1.	0.81	0.68	0.	0.01	1.64	0.99	0.8
time (sec)	N/A	0.101	0.076	0.012	0.	0.275	2.16	0.272	20.325

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	66	66	0	1	129	76	71
normalized size	1	1.	0.84	0.84	0.	0.01	1.63	0.96	0.9
time (sec)	N/A	0.086	0.064	0.006	0.	0.274	2.208	0.269	22.887

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	79	86	0	1	138	92	82
normalized size	1	1.	0.83	0.91	0.	0.01	1.45	0.97	0.86
time (sec)	N/A	0.138	0.078	0.017	0.	0.278	3.621	0.271	32.732

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	91	99	0	1	162	111	99
normalized size	1	1.	0.86	0.93	0.	0.01	1.53	1.05	0.93
time (sec)	N/A	0.168	0.088	0.02	0.	0.279	6.309	0.271	39.43

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	101	110	0	1	173	126	112
normalized size	1	1.	0.85	0.92	0.	0.01	1.45	1.06	0.94
time (sec)	N/A	0.199	0.095	0.02	0.	0.279	12.69	0.27	46.335

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	120	193	274	150	153	0
normalized size	1	1.	0.86	0.9	1.45	2.06	1.13	1.15	0.
time (sec)	N/A	0.308	0.046	0.024	0.714	0.265	5.283	0.273	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	101	109	178	257	136	128	0
normalized size	1	1.	0.86	0.92	1.51	2.18	1.15	1.08	0.
time (sec)	N/A	0.256	0.051	0.02	0.7	0.264	5.145	0.272	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	72	98	163	227	124	101	100
normalized size	1	1.	0.66	0.9	1.5	2.08	1.14	0.93	0.92
time (sec)	N/A	0.222	0.043	0.016	0.733	0.268	4.63	0.272	36.114

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	57	81	138	138	107	74	14
normalized size	1	1.	3.	4.26	7.26	7.26	5.63	3.89	0.74
time (sec)	N/A	0.025	0.028	0.012	0.703	0.26	4.334	0.273	8.094

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	65	123	123	95	59	31
normalized size	1	1.	1.18	1.67	3.15	3.15	2.44	1.51	0.79
time (sec)	N/A	0.076	0.024	0.012	0.689	0.26	4.106	0.273	14.65

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	35	48	108	108	83	45	46
normalized size	1	1.	0.66	0.91	2.04	2.04	1.57	0.85	0.87
time (sec)	N/A	0.113	0.024	0.011	0.701	0.262	3.962	0.27	19.433

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	24	31	93	93	71	30	29
normalized size	1	1.	0.71	0.91	2.74	2.74	2.09	0.88	0.85
time (sec)	N/A	0.075	0.013	0.011	0.697	0.258	3.778	0.272	14.93

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	80	80	63	19	14
normalized size	1	1.	1.	0.94	5.	5.	3.94	1.19	0.88
time (sec)	N/A	0.014	0.005	0.006	0.69	0.248	3.648	0.27	6.315

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	76	91	170	300	128	124	95
normalized size	1	1.	0.75	0.89	1.67	2.94	1.25	1.22	0.93
time (sec)	N/A	0.24	0.095	0.021	0.693	0.266	11.293	0.273	35.963

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	107	193	339	148	155	110
normalized size	1	1.	0.79	0.92	1.66	2.92	1.28	1.34	0.95
time (sec)	N/A	0.283	0.154	0.024	0.7	0.265	26.855	0.273	43.268

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	107	129	213	359	165	176	139
normalized size	1	1.	0.76	0.92	1.52	2.56	1.18	1.26	0.99
time (sec)	N/A	0.32	0.11	0.025	0.705	0.264	59.909	0.272	56.704

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	122	148	0	1	218	158	0
normalized size	1	1.	0.79	0.95	0.	0.01	1.41	1.02	0.
time (sec)	N/A	0.27	0.117	0.021	0.	0.268	5.441	0.275	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	111	137	0	1	202	143	0
normalized size	1	1.	0.78	0.96	0.	0.01	1.42	1.01	0.
time (sec)	N/A	0.242	0.104	0.019	0.	0.269	5.284	0.271	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	100	123	0	1	178	117	122
normalized size	1	1.	0.76	0.94	0.	0.01	1.36	0.89	0.93
time (sec)	N/A	0.204	0.097	0.02	0.	0.268	5.027	0.272	42.899

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	88	80	0	1	180	105	112
normalized size	1	1.	0.73	0.66	0.	0.01	1.49	0.87	0.93
time (sec)	N/A	0.185	0.088	0.017	0.	0.273	4.614	0.271	36.342

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	91	80	0	1	194	113	112
normalized size	1	1.	0.75	0.66	0.	0.01	1.59	0.93	0.92
time (sec)	N/A	0.182	0.106	0.015	0.	0.27	4.491	0.272	35.072

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	91	78	0	1	196	113	109
normalized size	1	1.	0.74	0.63	0.	0.01	1.59	0.92	0.89
time (sec)	N/A	0.18	0.105	0.015	0.	0.273	4.264	0.271	33.961

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	91	78	0	1	196	113	112
normalized size	1	1.	0.73	0.63	0.	0.01	1.58	0.91	0.9
time (sec)	N/A	0.177	0.097	0.015	0.	0.267	4.147	0.27	32.56

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	91	80	0	1	190	113	109
normalized size	1	1.	0.73	0.64	0.	0.01	1.52	0.9	0.87
time (sec)	N/A	0.167	0.098	0.015	0.	0.267	4.07	0.27	31.425

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	89	96	0	1	177	105	105
normalized size	1	1.	0.79	0.85	0.	0.01	1.57	0.93	0.93
time (sec)	N/A	0.146	0.08	0.006	0.	0.266	4.128	0.27	27.425

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	101	126	0	1	185	122	116
normalized size	1	1.	0.76	0.95	0.	0.01	1.39	0.92	0.87
time (sec)	N/A	0.229	0.101	0.021	0.	0.27	17.716	0.271	50.214

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	113	139	0	1	209	140	136
normalized size	1	1.	0.78	0.97	0.	0.01	1.45	0.97	0.94
time (sec)	N/A	0.257	0.115	0.024	0.	0.272	40.187	0.271	58.86

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	150	0	1	221	155	150
normalized size	1	1.	0.78	0.96	0.	0.01	1.41	0.99	0.96
time (sec)	N/A	0.292	0.118	0.025	0.	0.273	88.025	0.272	67.147

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	20	26	12	20	12
normalized size	1	1.	0.84	0.84	1.05	1.37	0.63	1.05	0.63
time (sec)	N/A	0.011	0.01	0.005	0.764	0.257	0.191	0.269	2.544

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	12	8	12	8
normalized size	1	1.	1.	0.91	1.09	1.09	0.73	1.09	0.73
time (sec)	N/A	0.008	0.002	0.004	0.697	0.249	0.141	0.268	2.744

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	28	12	20	12
normalized size	1	1.	1.	0.84	1.05	1.47	0.63	1.05	0.63
time (sec)	N/A	0.017	0.013	0.009	0.759	0.254	0.187	0.27	4.049

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	24	31	15	24	15
normalized size	1	1.	0.82	0.86	1.09	1.41	0.68	1.09	0.68
time (sec)	N/A	0.029	0.007	0.008	0.69	0.252	0.154	0.271	4.956

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	12	12	8	12	7
normalized size	1	1.	0.85	0.77	0.92	0.92	0.62	0.92	0.54
time (sec)	N/A	0.008	0.003	0.005	0.695	0.248	0.145	0.269	2.867

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	20	19	24	31	14	26	14
normalized size	1	1.	0.83	0.79	1.	1.29	0.58	1.08	0.58
time (sec)	N/A	0.038	0.009	0.009	0.686	0.255	0.164	0.27	5.312

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	18	12	39	0
normalized size	1	1.	0.49	0.46	0.	0.23	0.15	0.49	0.
time (sec)	N/A	0.165	0.021	0.007	0.	0.255	0.169	0.269	0.

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	39	36	0	18	12	31	0
normalized size	1	1.	0.58	0.54	0.	0.27	0.18	0.46	0.
time (sec)	N/A	0.131	0.015	0.004	0.	0.255	0.166	0.27	0.

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	18	12	30	0
normalized size	1	1.	1.06	0.97	0.	0.5	0.33	0.83	0.
time (sec)	N/A	0.066	0.012	0.062	0.	0.259	0.167	0.269	0.

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	37	34	0	15	10	41	0
normalized size	1	1.	0.49	0.45	0.	0.2	0.13	0.55	0.
time (sec)	N/A	0.068	0.018	0.012	0.	0.259	0.214	0.27	0.

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	39	38	0	23	10	61	0
normalized size	1	1.	0.52	0.51	0.	0.31	0.13	0.81	0.
time (sec)	N/A	0.068	0.016	0.012	0.	0.26	1.071	0.27	0.

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	37	34	0	18	14	41	0
normalized size	1	1.	0.95	0.87	0.	0.46	0.36	1.05	0.
time (sec)	N/A	0.101	0.014	0.013	0.	0.257	1.118	0.269	0.

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	39	36	0	20	15	42	0
normalized size	1	1.	0.54	0.5	0.	0.28	0.21	0.58	0.
time (sec)	N/A	0.057	0.017	0.005	0.	0.256	1.175	0.269	0.

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	20	15	42	0
normalized size	1	1.	0.49	0.46	0.	0.25	0.19	0.53	0.
time (sec)	N/A	0.145	0.014	0.005	0.	0.256	1.246	0.27	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	0	20	15	42	0
normalized size	1	1.	0.49	0.46	0.	0.25	0.19	0.53	0.
time (sec)	N/A	0.146	0.014	0.008	0.	0.257	1.281	0.269	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	18	12	39	0
normalized size	1	1.	0.49	0.46	0.23	0.23	0.15	0.49	0.
time (sec)	N/A	0.073	0.012	0.005	0.695	0.256	0.165	0.269	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	18	18	12	39	0
normalized size	1	1.	0.49	0.46	0.23	0.23	0.15	0.49	0.
time (sec)	N/A	0.074	0.012	0.004	0.692	0.256	0.165	0.269	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	36	33	14	14	8	27	0
normalized size	1	1.	0.49	0.45	0.19	0.19	0.11	0.36	0.
time (sec)	N/A	0.04	0.016	0.003	0.703	0.255	0.154	0.269	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	35	34	18	18	5	35	0
normalized size	1	1.	0.49	0.47	0.25	0.25	0.07	0.49	0.
time (sec)	N/A	0.067	0.014	0.003	0.707	0.266	1.012	0.271	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	37	34	18	18	14	41	0
normalized size	1	1.	0.48	0.44	0.23	0.23	0.18	0.53	0.
time (sec)	N/A	0.07	0.013	0.004	0.698	0.265	1.106	0.27	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.07	0.013	0.004	0.698	0.264	1.163	0.27	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.07	0.014	0.005	0.699	0.265	1.214	0.271	0.

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	39	36	20	20	15	42	0
normalized size	1	1.	0.49	0.46	0.25	0.25	0.19	0.53	0.
time (sec)	N/A	0.071	0.014	0.005	0.689	0.266	1.269	0.271	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	47	0	90	133
normalized size	1	1.	0.37	0.35	0.	0.28	0.	0.54	0.8
time (sec)	N/A	0.282	0.032	0.01	0.	0.267	0.	0.269	17.163

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	47	0	90	131
normalized size	1	1.	0.37	0.35	0.	0.28	0.	0.54	0.78
time (sec)	N/A	0.27	0.027	0.011	0.	0.268	0.	0.271	17.127

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	119	61	58	0	47	0	90	107
normalized size	1	1.12	0.58	0.55	0.	0.44	0.	0.85	1.01
time (sec)	N/A	0.208	0.026	0.009	0.	0.266	0.	0.271	20.742

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	58	0	47	0	61	65
normalized size	1	1.	0.91	0.87	0.	0.7	0.	0.91	0.97
time (sec)	N/A	0.128	0.028	0.009	0.	0.265	0.	0.27	13.686

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	57	0	47	0	59	34
normalized size	1	1.	0.75	1.58	0.	1.31	0.	1.64	0.94
time (sec)	N/A	0.065	0.022	0.007	0.	0.265	0.	0.269	8.173

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	60	57	0	45	0	92	117
normalized size	1	1.	0.37	0.35	0.	0.28	0.	0.56	0.72
time (sec)	N/A	0.134	0.035	0.012	0.	0.272	0.	0.27	16.784

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	62	59	0	51	0	117	131
normalized size	1	1.	0.38	0.36	0.	0.31	0.	0.71	0.8
time (sec)	N/A	0.144	0.038	0.017	0.	0.262	0.	0.271	16.601

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	61	60	0	53	0	117	129
normalized size	1	1.	0.37	0.37	0.	0.32	0.	0.71	0.79
time (sec)	N/A	0.14	0.03	0.018	0.	0.263	0.	0.273	16.829

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	63	60	0	53	0	117	138
normalized size	1	1.	0.39	0.37	0.	0.33	0.	0.72	0.85
time (sec)	N/A	0.138	0.042	0.017	0.	0.265	0.	0.272	22.36

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	59	56	0	47	0	92	39
normalized size	1	1.	1.44	1.37	0.	1.15	0.	2.24	0.95
time (sec)	N/A	0.105	0.027	0.01	0.	0.262	0.	0.27	8.197

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	58	0	50	0	93	68
normalized size	1	1.	0.85	0.81	0.	0.69	0.	1.29	0.94
time (sec)	N/A	0.057	0.024	0.01	0.	0.264	0.	0.271	8.492

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	50	0	93	112
normalized size	1	1.	0.37	0.35	0.	0.3	0.	0.56	0.67
time (sec)	N/A	0.254	0.024	0.01	0.	0.264	0.	0.272	15.206

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	50	0	93	133
normalized size	1	1.	0.37	0.35	0.	0.3	0.	0.56	0.8
time (sec)	N/A	0.257	0.028	0.01	0.	0.261	0.	0.27	16.826

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	0	50	0	93	136
normalized size	1	1.	0.37	0.35	0.	0.3	0.	0.56	0.81
time (sec)	N/A	0.254	0.024	0.011	0.	0.258	0.	0.271	16.932

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.138	0.029	0.009	0.701	0.261	0.	0.272	17.047

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.133	0.03	0.01	0.702	0.256	0.	0.27	17.246

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.132	0.028	0.009	0.695	0.261	0.	0.272	17.467

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	47	47	0	90	136
normalized size	1	1.	0.37	0.35	0.28	0.28	0.	0.54	0.81
time (sec)	N/A	0.132	0.026	0.009	0.696	0.266	0.	0.272	17.694

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	59	56	42	42	0	85	129
normalized size	1	1.	0.37	0.35	0.26	0.26	0.	0.53	0.81
time (sec)	N/A	0.092	0.024	0.005	0.698	0.261	0.	0.27	30.405

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	60	58	49	49	0	86	128
normalized size	1	1.	0.38	0.37	0.31	0.31	0.	0.54	0.81
time (sec)	N/A	0.124	0.027	0.009	0.69	0.258	0.	0.271	16.679

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	59	56	49	49	0	90	134
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.125	0.025	0.009	0.688	0.26	0.	0.27	24.247

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	59	56	50	50	0	89	134
normalized size	1	1.	0.37	0.35	0.32	0.32	0.	0.56	0.85
time (sec)	N/A	0.126	0.026	0.008	0.695	0.257	0.	0.269	16.669

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.36	0.31	0.31	0.	0.57	0.85
time (sec)	N/A	0.126	0.028	0.01	0.7	0.258	0.	0.27	16.968

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.129	0.023	0.01	0.699	0.258	0.	0.271	17.003

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.126	0.029	0.01	0.693	0.26	0.	0.271	16.729

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.129	0.024	0.01	0.691	0.258	0.	0.269	16.785

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	61	58	50	50	0	93	138
normalized size	1	1.	0.37	0.35	0.3	0.3	0.	0.56	0.83
time (sec)	N/A	0.129	0.024	0.01	0.697	0.257	0.	0.271	16.784

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	77	0	142	201
normalized size	1	1.	0.33	0.31	0.	0.3	0.	0.56	0.79
time (sec)	N/A	0.398	0.043	0.011	0.	0.26	0.	0.271	27.002

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	77	0	142	199
normalized size	1	1.	0.33	0.31	0.	0.3	0.	0.56	0.78
time (sec)	N/A	0.381	0.035	0.01	0.	0.26	0.	0.27	26.929

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	83	80	0	77	0	142	196
normalized size	1	1.	0.41	0.4	0.	0.38	0.	0.71	0.98
time (sec)	N/A	0.318	0.04	0.01	0.	0.257	0.	0.27	37.317

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	83	80	0	77	0	142	151
normalized size	1	1.	0.52	0.5	0.	0.48	0.	0.89	0.94
time (sec)	N/A	0.293	0.035	0.009	0.	0.259	0.	0.271	28.408

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	83	80	0	76	0	140	107
normalized size	1	1.	0.7	0.67	0.	0.64	0.	1.18	0.9
time (sec)	N/A	0.239	0.037	0.009	0.	0.256	0.	0.271	20.591

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	80	0	76	0	90	65
normalized size	1	1.	1.24	1.19	0.	1.13	0.	1.34	0.97
time (sec)	N/A	0.13	0.04	0.009	0.	0.256	0.	0.27	13.57

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	79	0	77	0	89	34
normalized size	1	1.	0.75	2.19	0.	2.14	0.	2.47	0.94
time (sec)	N/A	0.065	0.031	0.007	0.	0.258	0.	0.269	8.151

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	82	79	0	74	0	143	178
normalized size	1	1.	0.33	0.31	0.	0.29	0.	0.57	0.71
time (sec)	N/A	0.192	0.045	0.013	0.	0.265	0.	0.272	26.053

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	82	0	169	199
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.68	0.8
time (sec)	N/A	0.207	0.049	0.017	0.	0.269	0.	0.272	26.003

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	82	0	171	202
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.68	0.81
time (sec)	N/A	0.204	0.048	0.018	0.	0.273	0.	0.272	26.731

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	82	0	173	207
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.69	0.83
time (sec)	N/A	0.205	0.042	0.018	0.	0.269	0.	0.272	26.667

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	85	82	0	82	0	170	204
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.68	0.82
time (sec)	N/A	0.203	0.043	0.018	0.	0.271	0.	0.272	26.662

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	85	82	0	82	0	169	209
normalized size	1	1.	0.34	0.33	0.	0.33	0.	0.67	0.83
time (sec)	N/A	0.197	0.053	0.019	0.	0.273	0.	0.272	32.149

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	81	78	0	77	0	143	39
normalized size	1	1.	1.98	1.9	0.	1.88	0.	3.49	0.95
time (sec)	N/A	0.105	0.03	0.01	0.	0.267	0.	0.274	8.237

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	83	80	0	80	0	144	68
normalized size	1	1.	1.15	1.11	0.	1.11	0.	2.	0.94
time (sec)	N/A	0.057	0.045	0.011	0.	0.266	0.	0.275	8.485

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	83	80	0	80	0	144	112
normalized size	1	1.	0.65	0.62	0.	0.62	0.	1.12	0.88
time (sec)	N/A	0.229	0.037	0.01	0.	0.268	0.	0.273	15.261

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	80	0	144	158
normalized size	1	1.	0.33	0.31	0.	0.31	0.	0.56	0.62
time (sec)	N/A	0.36	0.031	0.011	0.	0.267	0.	0.272	23.261

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	80	0	144	204
normalized size	1	1.	0.33	0.31	0.	0.31	0.	0.56	0.8
time (sec)	N/A	0.363	0.035	0.011	0.	0.266	0.	0.273	26.324

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	80	0	144	202
normalized size	1	1.	0.33	0.31	0.	0.31	0.	0.56	0.79
time (sec)	N/A	0.365	0.035	0.012	0.	0.268	0.	0.274	26.531

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	0	80	0	144	204
normalized size	1	1.	0.33	0.31	0.	0.31	0.	0.56	0.8
time (sec)	N/A	0.367	0.04	0.013	0.	0.264	0.	0.273	26.406

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.19	0.039	0.011	0.7	0.258	0.	0.273	26.606

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.185	0.036	0.011	0.686	0.259	0.	0.273	26.508

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.186	0.035	0.01	0.7	0.264	0.	0.273	26.647

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.185	0.036	0.01	0.698	0.26	0.	0.272	26.437

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	77	77	0	142	207
normalized size	1	1.	0.33	0.31	0.3	0.3	0.	0.56	0.81
time (sec)	N/A	0.185	0.035	0.01	0.701	0.26	0.	0.271	27.41

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	83	80	76	76	0	140	207
normalized size	1	1.	0.33	0.32	0.3	0.3	0.	0.56	0.82
time (sec)	N/A	0.183	0.035	0.009	0.7	0.261	0.	0.272	27.629

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	81	78	73	73	0	138	197
normalized size	1	1.	0.33	0.31	0.29	0.29	0.	0.56	0.79
time (sec)	N/A	0.141	0.032	0.006	0.693	0.256	0.	0.272	46.428

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	80	0	139	196
normalized size	1	1.	0.34	0.32	0.32	0.32	0.	0.56	0.79
time (sec)	N/A	0.176	0.039	0.009	0.706	0.26	0.	0.271	26.146

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	80	80	0	140	211
normalized size	1	1.	0.34	0.33	0.33	0.33	0.	0.57	0.86
time (sec)	N/A	0.177	0.038	0.009	0.694	0.263	0.	0.272	40.44

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	83	80	80	80	0	143	209
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.84
time (sec)	N/A	0.177	0.045	0.009	0.7	0.262	0.	0.272	26.71

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	83	80	80	80	0	143	209
normalized size	1	1.	0.34	0.32	0.32	0.32	0.	0.58	0.85
time (sec)	N/A	0.178	0.032	0.009	0.7	0.259	0.	0.274	33.366

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	83	80	80	80	0	142	207
normalized size	1	1.	0.34	0.33	0.33	0.33	0.	0.58	0.84
time (sec)	N/A	0.174	0.031	0.008	0.72	0.263	0.	0.272	26.485

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.84
time (sec)	N/A	0.178	0.034	0.01	0.7	0.26	0.	0.272	26.729

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.32	0.32	0.32	0.	0.57	0.83
time (sec)	N/A	0.18	0.03	0.01	0.695	0.259	0.	0.273	26.429

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.178	0.034	0.01	0.709	0.26	0.	0.273	26.535

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.178	0.031	0.011	0.69	0.259	0.	0.274	26.455

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.182	0.035	0.011	0.707	0.259	0.	0.272	26.468

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.179	0.031	0.012	0.699	0.26	0.	0.273	26.362

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	83	80	80	80	0	144	211
normalized size	1	1.	0.33	0.31	0.31	0.31	0.	0.56	0.83
time (sec)	N/A	0.18	0.036	0.012	0.705	0.259	0.	0.272	26.443

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	55	52	62	45	32	80	0
normalized size	1	1.	0.43	0.41	0.49	0.35	0.25	0.63	0.
time (sec)	N/A	0.248	0.041	0.011	0.694	0.258	1.227	0.272	0.

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	44	41	63	30	20	45	0
normalized size	1	1.	0.59	0.55	0.84	0.4	0.27	0.6	0.
time (sec)	N/A	0.147	0.021	0.011	0.705	0.257	1.167	0.27	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	35	32	23	18	10	30	29
normalized size	1	1.	0.8	0.73	0.52	0.41	0.23	0.68	0.66
time (sec)	N/A	0.079	0.012	0.007	0.699	0.259	0.292	0.27	3.984

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	37	0	24	15	45	0
normalized size	1	1.	0.52	0.46	0.	0.3	0.19	0.56	0.
time (sec)	N/A	0.104	0.02	0.012	0.	0.262	0.57	0.271	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	54	52	0	45	31	70	0
normalized size	1	1.	0.44	0.43	0.	0.37	0.25	0.57	0.
time (sec)	N/A	0.136	0.03	0.016	0.	0.263	1.597	0.272	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	66	63	0	1	80	86	0
normalized size	1	1.	0.51	0.49	0.	0.01	0.62	0.67	0.
time (sec)	N/A	0.139	0.049	0.012	0.	0.265	1.279	0.273	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	48	0	1	56	57	0
normalized size	1	1.	0.61	0.54	0.	0.01	0.63	0.64	0.
time (sec)	N/A	0.104	0.033	0.01	0.	0.269	1.2	0.272	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	34	0	1	53	31	0
normalized size	1	1.	0.83	0.64	0.	0.02	1.	0.58	0.
time (sec)	N/A	0.041	0.024	0.006	0.	0.267	0.364	0.273	0.

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	56	50	0	1	65	50	0
normalized size	1	1.	0.61	0.54	0.	0.01	0.71	0.54	0.
time (sec)	N/A	0.103	0.027	0.014	0.	0.271	1.337	0.276	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	70	69	0	1	87	68	0
normalized size	1	1.	0.54	0.53	0.	0.01	0.67	0.52	0.
time (sec)	N/A	0.14	0.044	0.016	0.	0.27	1.495	0.282	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	81	103	198	123	0	4	141
normalized size	1	1.	0.51	0.65	1.25	0.78	0.	0.03	0.89
time (sec)	N/A	0.31	0.05	0.024	0.691	0.261	0.	0.639	24.076

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	61	81	86	93	0	4	112
normalized size	1	1.	0.54	0.72	0.76	0.82	0.	0.04	0.99
time (sec)	N/A	0.248	0.042	0.022	0.699	0.262	0.	0.623	17.478

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	69	39	32	65	49	0	4	37
normalized size	1	1.68	0.95	0.78	1.59	1.2	0.	0.1	0.9
time (sec)	N/A	0.131	0.023	0.01	0.705	0.258	0.	0.66	8.386

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	35	0	4	36
normalized size	1	1.	0.71	0.63	0.63	0.92	0.	0.11	0.95
time (sec)	N/A	0.067	0.014	0.006	0.694	0.257	0.	0.637	8.104

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	74	107	0	122	0	4	144
normalized size	1	1.	0.5	0.73	0.	0.83	0.	0.03	0.98
time (sec)	N/A	0.205	0.044	0.024	0.	0.269	0.	0.64	27.662

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	97	133	0	161	0	4	190
normalized size	1	1.	0.51	0.7	0.	0.85	0.	0.02	1.01
time (sec)	N/A	0.242	0.07	0.027	0.	0.267	0.	0.679	34.745

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	84	97	0	1	0	4	0
normalized size	1	1.	0.66	0.76	0.	0.01	0.	0.03	0.
time (sec)	N/A	0.152	0.061	0.021	0.	0.268	0.	0.695	0.

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	81	97	0	1	0	4	0
normalized size	1	1.	0.63	0.75	0.	0.01	0.	0.03	0.
time (sec)	N/A	0.148	0.055	0.021	0.	0.274	0.	0.651	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	83	97	0	1	0	4	0
normalized size	1	1.	0.61	0.72	0.	0.01	0.	0.03	0.
time (sec)	N/A	0.101	0.049	0.011	0.	0.27	0.	0.622	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	93	119	0	1	0	4	0
normalized size	1	1.	0.55	0.7	0.	0.01	0.	0.02	0.
time (sec)	N/A	0.185	0.063	0.024	0.	0.274	0.	0.617	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	105	139	0	1	0	4	0
normalized size	1	1.	0.5	0.67	0.	0.	0.	0.02	0.
time (sec)	N/A	0.223	0.071	0.027	0.	0.275	0.	0.62	0.

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	103	163	161	212	0	4	216
normalized size	1	1.	0.43	0.68	0.68	0.89	0.	0.02	0.91
time (sec)	N/A	0.45	0.066	0.027	0.706	0.262	0.	0.62	35.09

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	83	141	134	182	0	4	184
normalized size	1	1.	0.42	0.72	0.68	0.93	0.	0.02	0.94
time (sec)	N/A	0.378	0.055	0.024	0.703	0.261	0.	0.62	28.051

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	61	54	197	108	0	4	37
normalized size	1	1.	1.49	1.32	4.8	2.63	0.	0.1	0.9
time (sec)	N/A	0.119	0.03	0.011	0.698	0.258	0.	0.639	8.453

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	50	43	85	93	0	4	68
normalized size	1	1.	0.68	0.58	1.15	1.26	0.	0.05	0.92
time (sec)	N/A	0.061	0.034	0.009	0.691	0.26	0.	0.621	8.626

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	39	32	65	78	0	4	66
normalized size	1	1.	0.57	0.46	0.94	1.13	0.	0.06	0.96
time (sec)	N/A	0.131	0.023	0.01	0.695	0.259	0.	0.611	13.63

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	27	24	24	65	0	4	36
normalized size	1	1.	0.71	0.63	0.63	1.71	0.	0.11	0.95
time (sec)	N/A	0.066	0.019	0.008	0.692	0.256	0.	0.619	8.218

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	96	193	0	240	0	4	211
normalized size	1	1.	0.43	0.87	0.	1.08	0.	0.02	0.95
time (sec)	N/A	0.294	0.074	0.026	0.	0.27	0.	0.621	36.757

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	119	219	0	279	0	4	264
normalized size	1	1.	0.45	0.82	0.	1.04	0.	0.01	0.99
time (sec)	N/A	0.339	0.085	0.03	0.	0.269	0.	0.619	44.903

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	105	172	0	1	0	4	0
normalized size	1	1.	0.5	0.82	0.	0.	0.	0.02	0.
time (sec)	N/A	0.245	0.081	0.023	0.	0.27	0.	0.623	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	105	172	0	1	0	4	0
normalized size	1	1.	0.5	0.81	0.	0.	0.	0.02	0.
time (sec)	N/A	0.236	0.08	0.032	0.	0.273	0.	0.614	0.

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	105	172	0	1	0	4	0
normalized size	1	1.	0.49	0.81	0.	0.	0.	0.02	0.
time (sec)	N/A	0.23	0.068	0.023	0.	0.271	0.	0.612	0.

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	105	169	0	1	0	4	0
normalized size	1	1.	0.49	0.79	0.	0.	0.	0.02	0.
time (sec)	N/A	0.178	0.073	0.012	0.	0.27	0.	0.604	0.

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	115	191	0	1	0	4	0
normalized size	1	1.	0.46	0.76	0.	0.	0.	0.02	0.
time (sec)	N/A	0.286	0.086	0.028	0.	0.279	0.	0.625	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	127	211	0	1	0	4	0
normalized size	1	1.	0.44	0.73	0.	0.	0.	0.01	0.
time (sec)	N/A	0.34	0.116	0.031	0.	0.277	0.	0.619	0.

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	64	0	0	0	0	0	345
normalized size	1	1.	0.21	0.	0.	0.	0.	0.	1.16
time (sec)	N/A	0.518	0.07	0.071	0.	0.	0.	0.	33.442

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	49	0	0	0	0	0	304
normalized size	1	1.	0.19	0.	0.	0.	0.	0.	1.19
time (sec)	N/A	0.28	0.026	0.013	0.	0.	0.	0.	29.553

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	72	0	0	0	0	0	340
normalized size	1	1.	0.25	0.	0.	0.	0.	0.	1.18
time (sec)	N/A	0.384	0.047	0.041	0.	0.	0.	0.	33.111

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	618	618	64	0	0	0	0	0	736
normalized size	1	1.	0.1	0.	0.	0.	0.	0.	1.19
time (sec)	N/A	0.968	0.073	0.031	0.	0.	0.	0.	71.749

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	64	0	0	0	0	0	738
normalized size	1	1.	0.11	0.	0.	0.	0.	0.	1.21
time (sec)	N/A	0.895	0.053	0.018	0.	0.	0.	0.	76.848

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	79	0	0	0	0	0	774
normalized size	1	1.	0.12	0.	0.	0.	0.	0.	1.19
time (sec)	N/A	1.058	0.066	0.089	0.	0.	0.	0.	88.534

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	55	54	49	65	48
normalized size	1	1.	0.65	0.59	1.08	1.06	0.96	1.27	0.94
time (sec)	N/A	0.042	0.022	0.011	0.686	0.257	8.296	0.262	15.566

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	55	46	49	57	48
normalized size	1	1.	0.65	0.59	1.08	0.9	0.96	1.12	0.94
time (sec)	N/A	0.038	0.018	0.01	0.681	0.256	3.818	0.263	15.59

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	33	30	55	39	49	61	48
normalized size	1	1.	0.65	0.59	1.08	0.76	0.96	1.2	0.94
time (sec)	N/A	0.039	0.013	0.009	0.692	0.257	1.255	0.262	15.527

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	55	42	48	55	46
normalized size	1	1.	0.67	0.61	1.12	0.86	0.98	1.12	0.94
time (sec)	N/A	0.039	0.015	0.01	0.698	0.257	2.022	0.262	15.506

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	59	42	48	69	46
normalized size	1	1.	0.67	0.61	1.2	0.86	0.98	1.41	0.94
time (sec)	N/A	0.038	0.018	0.009	0.69	0.256	2.136	0.262	15.561

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	33	30	58	46	48	72	46
normalized size	1	1.	0.67	0.61	1.18	0.94	0.98	1.47	0.94
time (sec)	N/A	0.038	0.02	0.01	0.695	0.257	2.99	0.262	15.594

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	38	30	63	46	48	65	46
normalized size	1	1.	0.78	0.61	1.29	0.94	0.98	1.33	0.94
time (sec)	N/A	0.037	0.02	0.01	0.685	0.257	6.841	0.262	15.609

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	99	92	90	116	88
normalized size	1	1.	0.6	0.57	1.09	1.01	0.99	1.27	0.97
time (sec)	N/A	0.115	0.027	0.011	0.683	0.258	18.714	0.262	25.334

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	99	78	90	103	88
normalized size	1	1.	0.6	0.57	1.09	0.86	0.99	1.13	0.97
time (sec)	N/A	0.104	0.024	0.011	0.682	0.26	8.736	0.266	25.647

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	52	99	69	90	107	88
normalized size	1	1.	0.6	0.57	1.09	0.76	0.99	1.18	0.97
time (sec)	N/A	0.104	0.017	0.01	0.676	0.257	4.084	0.264	25.669

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	122	72	88	99	87
normalized size	1	1.	0.62	0.58	1.37	0.81	0.99	1.11	0.98
time (sec)	N/A	0.107	0.019	0.01	0.691	0.255	4.521	0.263	25.559

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	103	72	88	120	87
normalized size	1	1.	0.62	0.58	1.16	0.81	0.99	1.35	0.98
time (sec)	N/A	0.105	0.022	0.01	0.693	0.26	4.829	0.265	25.86

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	55	52	103	76	88	124	87
normalized size	1	1.	0.62	0.58	1.16	0.85	0.99	1.39	0.98
time (sec)	N/A	0.105	0.025	0.01	0.69	0.258	5.789	0.264	25.423

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	60	52	111	76	87	128	85
normalized size	1	1.	0.69	0.6	1.28	0.87	1.	1.47	0.98
time (sec)	N/A	0.103	0.03	0.01	0.692	0.255	8.698	0.262	25.464

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	142	130	129	167	128
normalized size	1	1.	0.6	0.57	1.1	1.01	1.	1.29	0.99
time (sec)	N/A	0.174	0.035	0.011	0.688	0.257	33.729	0.265	35.981

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	142	111	131	149	129
normalized size	1	1.	0.59	0.56	1.08	0.85	1.	1.14	0.98
time (sec)	N/A	0.152	0.03	0.011	0.705	0.257	19.336	0.265	36.006

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	77	74	142	99	131	153	129
normalized size	1	1.	0.59	0.56	1.08	0.76	1.	1.17	0.98
time (sec)	N/A	0.152	0.024	0.011	0.704	0.257	9.019	0.265	36.054

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	77	74	209	101	129	142	128
normalized size	1	1.	0.6	0.57	1.62	0.78	1.	1.1	0.99
time (sec)	N/A	0.149	0.025	0.011	0.711	0.256	9.934	0.263	36.038

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	74	146	101	126	171	124
normalized size	1	1.	0.62	0.59	1.17	0.81	1.01	1.37	0.99
time (sec)	N/A	0.151	0.027	0.011	0.705	0.258	10.792	0.264	36.307

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	77	74	146	105	128	176	126
normalized size	1	1.	0.61	0.58	1.15	0.83	1.01	1.39	0.99
time (sec)	N/A	0.151	0.03	0.011	0.711	0.256	10.989	0.266	36.531

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	82	74	154	105	128	180	126
normalized size	1	1.	0.65	0.58	1.21	0.83	1.01	1.42	0.99
time (sec)	N/A	0.151	0.029	0.012	0.717	0.256	15.924	0.269	36.404

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	245	242	0	351	0	406	298
normalized size	1	1.	0.78	0.77	0.	1.11	0.	1.28	0.94
time (sec)	N/A	0.792	0.532	0.035	0.	0.288	0.	0.276	149.22

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	227	226	0	351	0	359	279
normalized size	1	1.	0.76	0.76	0.	1.18	0.	1.2	0.94
time (sec)	N/A	0.653	0.351	0.023	0.	0.281	0.	0.276	131.164

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	232	223	0	305	0	362	279
normalized size	1	1.	0.78	0.75	0.	1.02	0.	1.21	0.94
time (sec)	N/A	0.619	0.368	0.021	0.	0.283	0.	0.273	134.323

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	211	209	0	308	0	355	262
normalized size	1	1.	0.75	0.74	0.	1.1	0.	1.26	0.93
time (sec)	N/A	0.559	0.429	0.02	0.	0.286	0.	0.276	117.827

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	210	212	0	279	0	355	255
normalized size	1	1.	0.75	0.75	0.	0.99	0.	1.26	0.91
time (sec)	N/A	0.526	0.471	0.019	0.	0.283	0.	0.273	121.918

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	210	210	0	293	78	367	255
normalized size	1	1.	0.74	0.74	0.	1.04	0.28	1.3	0.9
time (sec)	N/A	0.552	0.476	0.016	0.	0.284	16.8	0.276	117.367

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	211	207	0	288	0	363	262
normalized size	1	1.	0.75	0.73	0.	1.02	0.	1.28	0.93
time (sec)	N/A	0.54	0.429	0.015	0.	0.282	0.	0.27	120.789

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	233	223	0	340	0	397	279
normalized size	1	1.	0.78	0.74	0.	1.13	0.	1.32	0.93
time (sec)	N/A	0.64	0.298	0.027	0.	0.29	0.	0.273	131.919

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	233	226	0	379	0	373	279
normalized size	1	1.	0.78	0.75	0.	1.26	0.	1.24	0.93
time (sec)	N/A	0.645	0.312	0.023	0.	0.289	0.	0.272	135.371

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	251	242	0	423	0	414	298
normalized size	1	1.	0.79	0.76	0.	1.33	0.	1.3	0.94
time (sec)	N/A	0.72	0.645	0.027	0.	0.291	0.	0.274	147.993

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	283	306	0	508	0	459	0
normalized size	1	1.	0.77	0.83	0.	1.38	0.	1.25	0.
time (sec)	N/A	0.879	0.355	0.031	0.	0.293	0.	0.28	0.

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	275	290	0	508	0	412	330
normalized size	1	1.	0.79	0.83	0.	1.45	0.	1.18	0.94
time (sec)	N/A	0.824	0.289	0.031	0.	0.285	0.	0.281	163.661

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	270	287	0	462	0	414	330
normalized size	1	1.	0.77	0.82	0.	1.32	0.	1.18	0.94
time (sec)	N/A	0.804	0.276	0.03	0.	0.285	0.	0.277	166.267

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	260	271	0	474	0	408	313
normalized size	1	1.	0.78	0.81	0.	1.42	0.	1.23	0.94
time (sec)	N/A	0.721	0.27	0.027	0.	0.286	0.	0.278	147.988

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	260	280	0	470	0	412	313
normalized size	1	1.	0.78	0.84	0.	1.41	0.	1.24	0.94
time (sec)	N/A	0.716	0.268	0.027	0.	0.288	0.	0.28	152.628

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	260	277	0	501	0	412	314
normalized size	1	1.	0.77	0.82	0.	1.49	0.	1.23	0.93
time (sec)	N/A	0.736	0.288	0.029	0.	0.286	0.	0.281	149.427

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	260	277	0	494	0	416	314
normalized size	1	1.	0.77	0.82	0.	1.47	0.	1.24	0.93
time (sec)	N/A	0.749	0.326	0.027	0.	0.291	0.	0.28	152.356

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	259	277	0	505	0	409	309
normalized size	1	1.	0.77	0.83	0.	1.51	0.	1.22	0.92
time (sec)	N/A	0.742	0.327	0.026	0.	0.288	0.	0.278	148.37

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	260	271	0	470	0	409	309
normalized size	1	1.	0.78	0.81	0.	1.4	0.	1.22	0.92
time (sec)	N/A	0.737	0.301	0.026	0.	0.287	0.	0.28	151.361

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	253	272	0	460	252	417	309
normalized size	1	1.	0.76	0.81	0.	1.37	0.75	1.24	0.92
time (sec)	N/A	0.73	0.26	0.026	0.	0.289	136.218	0.278	147.626

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	253	269	0	456	0	416	309
normalized size	1	1.	0.76	0.8	0.	1.36	0.	1.24	0.92
time (sec)	N/A	0.741	0.27	0.026	0.	0.286	0.	0.274	151.186

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	273	285	0	500	0	441	326
normalized size	1	1.	0.78	0.81	0.	1.42	0.	1.25	0.93
time (sec)	N/A	0.852	0.255	0.031	0.	0.297	0.	0.278	165.108

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	273	288	0	560	0	416	326
normalized size	1	1.	0.78	0.82	0.	1.59	0.	1.18	0.93
time (sec)	N/A	0.825	0.303	0.032	0.	0.299	0.	0.273	169.256

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	295	304	0	603	0	471	0
normalized size	1	1.	0.8	0.82	0.	1.63	0.	1.27	0.
time (sec)	N/A	0.918	0.298	0.036	0.	0.302	0.	0.275	0.

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	321	370	0	664	0	510	0
normalized size	1	1.	0.76	0.88	0.	1.58	0.	1.21	0.
time (sec)	N/A	1.096	0.432	0.042	0.	0.293	0.	0.28	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	323	354	0	664	0	463	0
normalized size	1	1.	0.8	0.88	0.	1.65	0.	1.15	0.
time (sec)	N/A	1.014	0.388	0.04	0.	0.294	0.	0.281	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	308	351	0	618	0	466	0
normalized size	1	1.	0.77	0.87	0.	1.54	0.	1.16	0.
time (sec)	N/A	0.979	0.408	0.038	0.	0.293	0.	0.279	0.

Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	298	335	0	630	0	459	0
normalized size	1	1.	0.77	0.87	0.	1.64	0.	1.19	0.
time (sec)	N/A	0.924	0.421	0.034	0.	0.29	0.	0.284	0.

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	298	344	0	626	0	463	0
normalized size	1	1.	0.77	0.89	0.	1.63	0.	1.2	0.
time (sec)	N/A	0.895	0.402	0.034	0.	0.292	0.	0.282	0.

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	308	341	0	657	0	463	0
normalized size	1	1.	0.79	0.88	0.	1.69	0.	1.19	0.
time (sec)	N/A	0.922	0.429	0.033	0.	0.293	0.	0.28	0.

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	298	341	0	651	0	467	0
normalized size	1	1.	0.77	0.88	0.	1.68	0.	1.2	0.
time (sec)	N/A	0.947	0.397	0.033	0.	0.288	0.	0.28	0.

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	308	339	0	668	0	463	0
normalized size	1	1.	0.79	0.87	0.	1.71	0.	1.18	0.
time (sec)	N/A	0.934	0.455	0.033	0.	0.293	0.	0.283	0.

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	298	339	0	662	0	467	0
normalized size	1	1.	0.76	0.87	0.	1.69	0.	1.19	0.
time (sec)	N/A	0.92	0.401	0.033	0.	0.291	0.	0.28	0.

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	308	339	0	672	0	463	0
normalized size	1	1.	0.78	0.86	0.	1.71	0.	1.18	0.
time (sec)	N/A	0.944	0.442	0.033	0.	0.288	0.	0.282	0.

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	298	339	0	662	0	467	0
normalized size	1	1.	0.76	0.86	0.	1.68	0.	1.19	0.
time (sec)	N/A	0.974	0.478	0.034	0.	0.292	0.	0.281	0.

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	307	341	0	662	0	460	0
normalized size	1	1.	0.79	0.88	0.	1.7	0.	1.18	0.
time (sec)	N/A	0.964	0.466	0.033	0.	0.293	0.	0.282	0.

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	298	335	0	621	0	460	0
normalized size	1	1.	0.77	0.86	0.	1.6	0.	1.18	0.
time (sec)	N/A	0.957	0.487	0.031	0.	0.291	0.	0.28	0.

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	295	336	0	609	0	468	0
normalized size	1	1.	0.76	0.87	0.	1.57	0.	1.21	0.
time (sec)	N/A	0.952	0.387	0.032	0.	0.289	0.	0.28	0.

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	295	333	0	616	0	467	0
normalized size	1	1.	0.76	0.86	0.	1.59	0.	1.21	0.
time (sec)	N/A	0.96	0.475	0.032	0.	0.289	0.	0.274	0.

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	313	349	0	659	0	493	0
normalized size	1	1.	0.77	0.86	0.	1.63	0.	1.22	0.
time (sec)	N/A	1.083	0.413	0.041	0.	0.327	0.	0.279	0.

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	313	352	0	741	0	481	0
normalized size	1	1.	0.77	0.87	0.	1.83	0.	1.19	0.
time (sec)	N/A	1.08	0.405	0.039	0.	0.327	0.	0.276	0.

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	339	368	0	784	0	489	0
normalized size	1	1.	0.8	0.87	0.	1.86	0.	1.16	0.
time (sec)	N/A	1.187	0.419	0.043	0.	0.343	0.	0.279	0.

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	30	35	0	61	75
normalized size	1	1.	0.47	0.42	0.32	0.38	0.	0.66	0.81
time (sec)	N/A	0.087	0.035	0.004	0.719	0.258	0.	0.264	23.582

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	30	30	0	55	75
normalized size	1	1.	0.47	0.42	0.32	0.32	0.	0.59	0.81
time (sec)	N/A	0.083	0.023	0.004	0.71	0.258	0.	0.265	23.838

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	44	39	30	24	0	59	75
normalized size	1	1.	0.47	0.42	0.32	0.26	0.	0.63	0.81
time (sec)	N/A	0.082	0.022	0.004	0.713	0.26	0.	0.265	24.254

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	43	38	32	26	0	54	75
normalized size	1	1.	0.47	0.42	0.35	0.29	0.	0.59	0.82
time (sec)	N/A	0.081	0.02	0.004	0.71	0.257	0.	0.263	40.723

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	43	39	32	26	0	55	75
normalized size	1	1.	0.47	0.43	0.35	0.29	0.	0.6	0.82
time (sec)	N/A	0.081	0.028	0.004	0.72	0.259	0.	0.267	23.175

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	37	34	31	0	57	73
normalized size	1	1.	0.46	0.41	0.37	0.34	0.	0.63	0.8
time (sec)	N/A	0.081	0.028	0.004	0.728	0.26	0.	0.265	23.609

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	42	37	32	28	0	59	73
normalized size	1	1.	0.46	0.41	0.35	0.31	0.	0.65	0.8
time (sec)	N/A	0.086	0.028	0.005	0.726	0.262	0.	0.267	23.525

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	112	73	0	134	156
normalized size	1	1.	0.34	0.31	0.57	0.37	0.	0.69	0.8
time (sec)	N/A	0.165	0.045	0.009	0.72	0.268	0.	0.268	18.121

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	112	62	0	123	156
normalized size	1	1.	0.34	0.31	0.57	0.32	0.	0.63	0.8
time (sec)	N/A	0.158	0.04	0.009	0.712	0.273	0.	0.267	18.219

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	66	61	112	54	0	127	156
normalized size	1	1.	0.34	0.31	0.57	0.28	0.	0.65	0.8
time (sec)	N/A	0.157	0.035	0.008	0.71	0.271	0.	0.266	18.162

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	66	61	117	57	0	120	156
normalized size	1	1.	0.34	0.32	0.61	0.3	0.	0.62	0.81
time (sec)	N/A	0.159	0.032	0.009	0.711	0.271	0.	0.268	18.288

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	61	117	57	0	138	156
normalized size	1	1.	0.35	0.32	0.61	0.3	0.	0.72	0.82
time (sec)	N/A	0.16	0.041	0.009	0.722	0.275	0.	0.268	18.23

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	66	61	116	61	0	142	156
normalized size	1	1.	0.34	0.32	0.6	0.32	0.	0.74	0.81
time (sec)	N/A	0.159	0.042	0.009	0.713	0.271	0.	0.267	18.134

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	66	61	116	61	0	144	156
normalized size	1	1.	0.35	0.32	0.61	0.32	0.	0.75	0.82
time (sec)	N/A	0.158	0.041	0.009	0.723	0.27	0.	0.269	18.396

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	198	111	0	207	238
normalized size	1	1.	0.3	0.28	0.67	0.37	0.	0.7	0.8
time (sec)	N/A	0.239	0.059	0.008	0.729	0.274	0.	0.268	29.278

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	198	95	0	190	238
normalized size	1	1.	0.3	0.28	0.67	0.32	0.	0.64	0.8
time (sec)	N/A	0.227	0.046	0.008	0.722	0.273	0.	0.27	29.101

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	88	83	198	84	0	194	238
normalized size	1	1.	0.3	0.28	0.67	0.28	0.	0.65	0.8
time (sec)	N/A	0.226	0.047	0.009	0.718	0.269	0.	0.268	29.544

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	88	83	204	86	0	185	238
normalized size	1	1.	0.3	0.28	0.7	0.29	0.	0.63	0.81
time (sec)	N/A	0.23	0.045	0.008	0.724	0.271	0.	0.269	29.391

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	88	83	204	86	0	211	238
normalized size	1	1.	0.3	0.28	0.69	0.29	0.	0.72	0.81
time (sec)	N/A	0.225	0.053	0.009	0.722	0.27	0.	0.269	29.134

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	88	83	204	90	0	215	238
normalized size	1	1.	0.3	0.28	0.7	0.31	0.	0.73	0.81
time (sec)	N/A	0.227	0.052	0.009	0.712	0.271	0.	0.272	28.958

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	88	83	203	90	0	219	238
normalized size	1	1.	0.3	0.28	0.69	0.31	0.	0.74	0.81
time (sec)	N/A	0.23	0.05	0.009	0.725	0.274	0.	0.271	28.897

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	238	241	0	270	0	374	0
normalized size	1	1.	0.52	0.53	0.	0.59	0.	0.82	0.
time (sec)	N/A	0.779	0.185	0.016	0.	0.294	0.	0.276	0.

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	222	220	0	265	0	325	0
normalized size	1	1.	0.54	0.53	0.	0.64	0.	0.79	0.
time (sec)	N/A	0.663	0.145	0.011	0.	0.292	0.	0.278	0.

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	221	216	0	198	0	327	0
normalized size	1	1.	0.54	0.53	0.	0.48	0.	0.8	0.
time (sec)	N/A	0.631	0.126	0.01	0.	0.291	0.	0.276	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	178	182	0	213	0	339	0
normalized size	1	1.	0.48	0.49	0.	0.58	0.	0.92	0.
time (sec)	N/A	0.549	0.09	0.009	0.	0.292	0.	0.276	0.

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	178	184	0	194	0	339	0
normalized size	1	1.	0.48	0.5	0.	0.53	0.	0.92	0.
time (sec)	N/A	0.533	0.071	0.008	0.	0.289	0.	0.272	0.

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	232	223	0	252	0	356	0
normalized size	1	1.	0.56	0.54	0.	0.61	0.	0.86	0.
time (sec)	N/A	0.638	0.155	0.013	0.	0.295	0.	0.278	0.

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	233	241	0	286	0	346	0
normalized size	1	1.	0.56	0.58	0.	0.69	0.	0.84	0.
time (sec)	N/A	0.632	0.162	0.013	0.	0.296	0.	0.273	0.

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	244	250	0	327	0	383	0
normalized size	1	1.	0.53	0.54	0.	0.71	0.	0.83	0.
time (sec)	N/A	0.74	0.195	0.017	0.	0.297	0.	0.275	0.

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	551	551	498	743	0	429	0	571	0
normalized size	1	1.	0.9	1.35	0.	0.78	0.	1.04	0.
time (sec)	N/A	0.917	0.521	0.028	0.	0.299	0.	0.291	0.

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	476	676	0	429	0	524	0
normalized size	1	1.	0.94	1.34	0.	0.85	0.	1.04	0.
time (sec)	N/A	0.832	0.366	0.028	0.	0.299	0.	0.293	0.

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	474	702	0	383	0	527	0
normalized size	1	1.	0.94	1.39	0.	0.76	0.	1.05	0.
time (sec)	N/A	0.819	0.35	0.027	0.	0.295	0.	0.287	0.

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	434	609	0	396	0	497	0
normalized size	1	1.	0.95	1.33	0.	0.86	0.	1.09	0.
time (sec)	N/A	0.726	0.408	0.025	0.	0.298	0.	0.291	0.

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	458	272	672	0	392	0	501	0
normalized size	1	1.	0.59	1.47	0.	0.86	0.	1.09	0.
time (sec)	N/A	0.728	0.408	0.024	0.	0.3	0.	0.289	0.

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	436	614	0	416	0	498	0
normalized size	1	1.	0.95	1.34	0.	0.91	0.	1.08	0.
time (sec)	N/A	0.739	0.783	0.024	0.	0.298	0.	0.291	0.

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	436	674	0	382	0	497	0
normalized size	1	1.	0.95	1.47	0.	0.83	0.	1.08	0.
time (sec)	N/A	0.757	0.406	0.024	0.	0.295	0.	0.287	0.

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	272	614	0	386	0	506	0
normalized size	1	1.	0.59	1.33	0.	0.84	0.	1.1	0.
time (sec)	N/A	0.765	0.383	0.015	0.	0.302	0.	0.292	0.

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	272	644	0	377	0	505	0
normalized size	1	1.	0.59	1.4	0.	0.82	0.	1.1	0.
time (sec)	N/A	0.768	0.389	0.014	0.	0.296	0.	0.282	0.

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	307	642	0	420	0	554	0
normalized size	1	1.	0.61	1.27	0.	0.83	0.	1.09	0.
time (sec)	N/A	0.865	0.385	0.03	0.	0.302	0.	0.289	0.

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	506	506	307	713	0	470	0	541	0
normalized size	1	1.	0.61	1.41	0.	0.93	0.	1.07	0.
time (sec)	N/A	0.843	0.385	0.029	0.	0.306	0.	0.287	0.

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	508	684	0	513	0	583	0
normalized size	1	1.	0.92	1.24	0.	0.93	0.	1.05	0.
time (sec)	N/A	0.953	0.399	0.033	0.	0.301	0.	0.288	0.

Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	647	647	374	1297	0	586	0	622	0
normalized size	1	1.	0.58	2.	0.	0.91	0.	0.96	0.
time (sec)	N/A	1.155	0.657	0.035	0.	0.305	0.	0.296	0.

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	350	1166	0	586	0	575	0
normalized size	1	1.	0.58	1.94	0.	0.98	0.	0.96	0.
time (sec)	N/A	1.074	0.327	0.035	0.	0.306	0.	0.296	0.

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	600	350	1212	0	540	0	578	0
normalized size	1	1.	0.58	2.02	0.	0.9	0.	0.96	0.
time (sec)	N/A	1.049	0.303	0.035	0.	0.303	0.	0.294	0.

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	324	1041	0	552	0	548	0
normalized size	1	1.	0.58	1.88	0.	1.	0.	0.99	0.
time (sec)	N/A	0.968	0.338	0.031	0.	0.304	0.	0.295	0.

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	324	1144	0	548	0	552	0
normalized size	1	1.	0.58	2.06	0.	0.99	0.	1.	0.
time (sec)	N/A	0.951	0.344	0.032	0.	0.301	0.	0.293	0.

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	324	1046	0	579	0	552	0
normalized size	1	1.	0.58	1.88	0.	1.04	0.	0.99	0.
time (sec)	N/A	0.955	0.354	0.031	0.	0.304	0.	0.297	0.

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	325	1146	0	572	0	556	0
normalized size	1	1.	0.58	2.06	0.	1.03	0.	1.	0.
time (sec)	N/A	0.977	0.763	0.03	0.	0.302	0.	0.292	0.

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	324	1046	0	594	0	552	0
normalized size	1	1.	0.58	1.87	0.	1.06	0.	0.99	0.
time (sec)	N/A	0.971	0.353	0.031	0.	0.302	0.	0.297	0.

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	560	560	324	1146	0	583	0	556	0
normalized size	1	1.	0.58	2.05	0.	1.04	0.	0.99	0.
time (sec)	N/A	0.984	0.323	0.03	0.	0.302	0.	0.293	0.

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	324	1046	0	583	0	549	0
normalized size	1	1.	0.58	1.88	0.	1.05	0.	0.99	0.
time (sec)	N/A	0.974	0.36	0.029	0.	0.302	0.	0.296	0.

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	557	557	324	1146	0	545	0	549	0
normalized size	1	1.	0.58	2.06	0.	0.98	0.	0.99	0.
time (sec)	N/A	1.	0.342	0.03	0.	0.304	0.	0.292	0.

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	319	1046	0	535	0	558	0
normalized size	1	1.	0.57	1.88	0.	0.96	0.	1.	0.
time (sec)	N/A	0.984	0.31	0.029	0.	0.304	0.	0.297	0.

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	556	556	319	1143	0	536	0	556	0
normalized size	1	1.	0.57	2.06	0.	0.96	0.	1.	0.
time (sec)	N/A	0.993	0.316	0.029	0.	0.299	0.	0.291	0.

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	347	1076	0	579	0	605	0
normalized size	1	1.	0.58	1.79	0.	0.96	0.	1.	0.
time (sec)	N/A	1.106	0.326	0.036	0.	0.32	0.	0.292	0.

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	602	602	347	1193	0	651	0	593	0
normalized size	1	1.	0.58	1.98	0.	1.08	0.	0.99	0.
time (sec)	N/A	1.112	0.334	0.035	0.	0.324	0.	0.289	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	649	649	375	1124	0	694	0	635	0
normalized size	1	1.	0.58	1.73	0.	1.07	0.	0.98	0.
time (sec)	N/A	1.211	0.338	0.04	0.	0.321	0.	0.294	0.

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	105	602	0	684	3188	1276	138
normalized size	1	1.	0.7	4.01	0.	4.56	21.25	8.51	0.92
time (sec)	N/A	0.292	0.075	0.013	0.	0.291	19.471	0.279	56.651

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	73	292	0	342	1321	628	94
normalized size	1	1.	0.7	2.81	0.	3.29	12.7	6.04	0.9
time (sec)	N/A	0.193	0.047	0.011	0.	0.287	8.463	0.273	39.243

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	41	94	0	117	345	207	49
normalized size	1	1.	0.71	1.62	0.	2.02	5.95	3.57	0.84
time (sec)	N/A	0.062	0.03	0.01	0.	0.285	2.533	0.266	22.528

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	34
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.052	0.027	0.06	0.	0.	0.	0.	10.858

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	34
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.052	0.037	0.042	0.	0.	0.	0.	10.673

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	0	0	34
normalized size	1	1.	0.95	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.05	0.044	0.056	0.	0.	0.	0.	10.667

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	111	453	328	498	0	1312	298
normalized size	1	1.	0.35	1.45	1.05	1.59	0.	4.19	0.95
time (sec)	N/A	0.337	0.116	0.01	0.703	0.29	0.	0.291	64.537

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	79	199	161	215	0	562	182
normalized size	1	1.	0.39	0.97	0.79	1.05	0.	2.74	0.89
time (sec)	N/A	0.22	0.072	0.009	0.691	0.287	0.	0.275	29.901

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	47	56	47	47	0	123	80
normalized size	1	1.	0.48	0.58	0.48	0.48	0.	1.27	0.82
time (sec)	N/A	0.104	0.043	0.005	0.698	0.285	0.	0.267	10.67

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	0	0	0	0	0	61
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.096	0.047	0.075	0.	0.	0.	0.	15.725

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	63
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.096	0.058	0.034	0.	0.	0.	0.	15.414

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	63
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.097	0.069	0.044	0.	0.	0.	0.	15.434

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	66	0	0	0	0	0	66
normalized size	1	1.04	0.89	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.073	0.061	0.155	0.	0.	0.	0.	18.492

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	110	150	155	220	0	536	189
normalized size	1	1.	0.63	0.86	0.89	1.26	0.	3.08	1.09
time (sec)	N/A	0.264	0.078	0.011	0.693	0.289	0.	0.272	43.48

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	77	96	107	146	0	336	128
normalized size	1	1.	0.59	0.74	0.82	1.12	0.	2.58	0.98
time (sec)	N/A	0.193	0.05	0.01	0.696	0.289	0.	0.269	27.538

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	51	60	73	95	0	189	71
normalized size	1	1.	0.61	0.71	0.87	1.13	0.	2.25	0.85
time (sec)	N/A	0.141	0.031	0.009	0.707	0.287	0.	0.268	15.874

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	29	40	41	50	0	84	37
normalized size	1	1.	0.71	0.98	1.	1.22	0.	2.05	0.9
time (sec)	N/A	0.064	0.008	0.005	0.705	0.291	0.	0.271	9.116

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	53	0	0	0	0	0	76
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	1.21
time (sec)	N/A	0.095	0.024	0.053	0.	0.	0.	0.	17.616

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	62	0	0	0	0	0	75
normalized size	1	1.	0.97	0.	0.	0.	0.	0.	1.17
time (sec)	N/A	0.104	0.031	0.055	0.	0.	0.	0.	17.387

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	53
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.062	0.028	0.071	0.	0.	0.	0.	17.296

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	53
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.061	0.019	0.052	0.	0.	0.	0.	17.754

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	46	0	0	0	0	0	49
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.035	0.017	0.02	0.	0.	0.	0.	21.574

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	0	0	0	0	0	53
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.055	0.019	0.05	0.	0.	0.	0.	16.606

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	58
normalized size	1	1.	0.85	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.056	0.026	0.066	0.	0.	0.	0.	16.511

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	60
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.063	0.026	0.017	0.	0.	0.	0.	17.076

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	60
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.063	0.025	0.017	0.	0.	0.	0.	17.115

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	58
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.062	0.024	0.017	0.	0.	0.	0.	17.341

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	54	0	0	0	0	0	61
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.063	0.026	0.016	0.	0.	0.	0.	17.276

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	56	0	0	0	0	0	63
normalized size	1	1.	0.84	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.063	0.03	0.017	0.	0.	0.	0.	17.316

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	19
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.76
time (sec)	N/A	0.019	0.004	0.	0.69	0.23	0.064	0.264	5.384

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	1	19	26	0
normalized size	1	1.	1.	0.8	1.04	0.04	0.76	1.04	0.
time (sec)	N/A	0.02	0.002	0.001	0.689	0.233	0.065	0.261	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	1	15	22	0
normalized size	1	1.	1.	0.85	1.1	0.05	0.75	1.1	0.
time (sec)	N/A	0.012	0.	0.001	0.682	0.233	0.055	0.261	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	27	23	17	27	0
normalized size	1	1.	1.	0.86	1.29	1.1	0.81	1.29	0.
time (sec)	N/A	0.016	0.003	0.003	0.676	0.256	0.144	0.262	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	27	12	22	0
normalized size	1	1.	1.	0.94	1.22	1.5	0.67	1.22	0.
time (sec)	N/A	0.018	0.003	0.005	0.697	0.249	0.928	0.262	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	27	30	17	35	0
normalized size	1	1.	1.	0.86	1.29	1.43	0.81	1.67	0.
time (sec)	N/A	0.019	0.003	0.005	0.698	0.254	1.017	0.263	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	23	28	15	23	0
normalized size	1	1.	1.	0.94	1.28	1.56	0.83	1.28	0.
time (sec)	N/A	0.018	0.007	0.008	0.684	0.246	1.032	0.262	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	28	31	17	36	20
normalized size	1	1.	1.	0.86	1.33	1.48	0.81	1.71	0.95
time (sec)	N/A	0.019	0.005	0.008	0.684	0.255	1.325	0.262	6.737

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	28	28	22	28	17
normalized size	1	1.	1.	0.87	1.22	1.22	0.96	1.22	0.74
time (sec)	N/A	0.02	0.004	0.007	0.688	0.247	1.425	0.261	4.847

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	28	22	28	20
normalized size	1	1.	1.	0.8	1.12	1.12	0.88	1.12	0.8
time (sec)	N/A	0.02	0.004	0.007	0.685	0.247	1.644	0.264	6.94

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	28	28	22	28	20
normalized size	1	1.	1.	0.8	1.12	1.12	0.88	1.12	0.8
time (sec)	N/A	0.019	0.004	0.007	0.682	0.247	1.614	0.262	4.976

Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	1	51	62	51
normalized size	1	1.	1.	0.83	1.09	0.02	0.94	1.15	0.94
time (sec)	N/A	0.08	0.011	0.001	0.69	0.234	0.104	0.263	11.388

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	45	59	1	46	62	0
normalized size	1	1.	0.89	0.83	1.09	0.02	0.85	1.15	0.
time (sec)	N/A	0.095	0.013	0.001	0.692	0.233	0.101	0.261	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	42	61	1	48	58	0
normalized size	1	1.	1.	0.86	1.24	0.02	0.98	1.18	0.
time (sec)	N/A	0.05	0.008	0.002	0.696	0.23	0.096	0.262	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	44	59	55	42	62	0
normalized size	1	1.	1.	0.94	1.26	1.17	0.89	1.32	0.
time (sec)	N/A	0.118	0.019	0.003	0.686	0.255	1.059	0.266	0.

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	57	62	44	59	44
normalized size	1	1.	1.	0.94	1.19	1.29	0.92	1.23	0.92
time (sec)	N/A	0.061	0.03	0.005	0.691	0.248	1.051	0.262	10.975

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	45	59	63	44	72	0
normalized size	1	1.	0.9	0.88	1.16	1.24	0.86	1.41	0.
time (sec)	N/A	0.096	0.025	0.009	0.688	0.254	1.156	0.265	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	42	57	62	44	57	0
normalized size	1	1.	1.	0.89	1.21	1.32	0.94	1.21	0.
time (sec)	N/A	0.063	0.033	0.008	0.682	0.248	1.175	0.263	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	61	63	42	81	0
normalized size	1	1.	0.91	0.96	1.36	1.4	0.93	1.8	0.
time (sec)	N/A	0.111	0.032	0.009	0.692	0.261	1.785	0.263	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	49	43	61	62	46	63	42
normalized size	1	1.	1.02	0.9	1.27	1.29	0.96	1.31	0.88
time (sec)	N/A	0.063	0.038	0.008	0.686	0.248	1.981	0.263	11.072

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	46	61	65	46	73	0
normalized size	1	1.	0.98	0.9	1.2	1.27	0.9	1.43	0.
time (sec)	N/A	0.09	0.03	0.009	0.692	0.254	2.936	0.263	0.

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	49	42	59	62	44	62	0
normalized size	1	1.	1.04	0.89	1.26	1.32	0.94	1.32	0.
time (sec)	N/A	0.062	0.043	0.008	0.691	0.249	2.895	0.262	0.

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	50	45	65	65	46	78	46
normalized size	1	1.	1.04	0.94	1.35	1.35	0.96	1.62	0.96
time (sec)	N/A	0.087	0.047	0.009	0.685	0.255	4.658	0.263	14.98

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	45	62	62	49	65	49
normalized size	1	1.	0.96	0.87	1.19	1.19	0.94	1.25	0.94
time (sec)	N/A	0.063	0.035	0.008	0.691	0.248	4.935	0.261	11.21

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	45	62	62	49	65	48
normalized size	1	1.	0.98	0.83	1.15	1.15	0.91	1.2	0.89
time (sec)	N/A	0.088	0.029	0.007	0.685	0.247	6.521	0.262	15.332

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	45	62	62	49	65	53
normalized size	1	1.	1.04	0.83	1.15	1.15	0.91	1.2	0.98
time (sec)	N/A	0.062	0.043	0.008	0.7	0.25	6.355	0.261	11.152

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	50	45	62	62	49	65	48
normalized size	1	1.	0.93	0.83	1.15	1.15	0.91	1.2	0.89
time (sec)	N/A	0.089	0.028	0.009	0.692	0.249	8.505	0.263	15.313

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	111	109	1	97	117	85
normalized size	1	1.	1.	1.25	1.22	0.01	1.09	1.31	0.96
time (sec)	N/A	0.146	0.023	0.001	0.699	0.232	0.134	0.262	16.489

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	111	109	1	92	117	0
normalized size	1	1.	0.89	1.25	1.22	0.01	1.03	1.31	0.
time (sec)	N/A	0.193	0.029	0.001	0.693	0.233	0.133	0.262	0.

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	107	115	1	87	112	0
normalized size	1	1.	1.	1.32	1.42	0.01	1.07	1.38	0.
time (sec)	N/A	0.111	0.019	0.001	0.69	0.235	0.121	0.261	0.

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	85	111	107	92	117	0
normalized size	1	1.	1.	1.	1.31	1.26	1.08	1.38	0.
time (sec)	N/A	0.181	0.036	0.004	0.686	0.254	1.232	0.263	0.

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	84	105	112	82	112	73
normalized size	1	1.	1.	1.05	1.31	1.4	1.02	1.4	0.91
time (sec)	N/A	0.118	0.042	0.005	0.684	0.245	1.227	0.265	15.684

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	87	111	115	92	132	0
normalized size	1	1.	0.91	1.01	1.29	1.34	1.07	1.53	0.
time (sec)	N/A	0.202	0.057	0.009	0.695	0.253	1.382	0.265	0.

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	84	108	112	88	113	78
normalized size	1	1.	1.	1.01	1.3	1.35	1.06	1.36	0.94
time (sec)	N/A	0.116	0.047	0.009	0.695	0.249	1.382	0.265	18.719

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	93	142	0	1	391	124	0
normalized size	1	1.	0.93	1.42	0.	0.01	3.91	1.24	0.
time (sec)	N/A	0.283	0.18	0.007	0.	0.271	6.643	0.291	0.

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	78	111	0	1	316	101	73
normalized size	1	1.	0.96	1.37	0.	0.01	3.9	1.25	0.9
time (sec)	N/A	0.185	0.075	0.004	0.	0.268	5.312	0.292	26.589

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	60	0	1	223	80	54
normalized size	1	1.	0.98	0.95	0.	0.02	3.54	1.27	0.86
time (sec)	N/A	0.131	0.04	0.004	0.	0.269	2.535	0.291	17.272

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	39	36	0	1	131	47	34
normalized size	1	1.	1.08	1.	0.	0.03	3.64	1.31	0.94
time (sec)	N/A	0.075	0.015	0.003	0.	0.264	1.328	0.291	9.028

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	113	66	0	1	253	92	63
normalized size	1	1.	1.64	0.96	0.	0.01	3.67	1.33	0.91
time (sec)	N/A	0.158	0.128	0.009	0.	0.27	8.841	0.29	23.425

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	135	119	0	1	345	127	87
normalized size	1	1.	1.52	1.34	0.	0.01	3.88	1.43	0.98
time (sec)	N/A	0.295	0.255	0.012	0.	0.282	21.148	0.292	37.337

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	188	159	0	1	423	170	109
normalized size	1	1.	1.65	1.39	0.	0.01	3.71	1.49	0.96
time (sec)	N/A	0.427	0.559	0.013	0.	0.299	26.488	0.292	44.879

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	250	467	0	2111	194	1	212
normalized size	1	1.	1.23	2.3	0.	10.4	0.96	0.	1.04
time (sec)	N/A	1.307	0.284	0.053	0.	0.29	7.63	0.857	67.767

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	202	343	0	1430	129	1	189
normalized size	1	1.	1.13	1.92	0.	7.99	0.72	0.01	1.06
time (sec)	N/A	0.594	0.196	0.026	0.	0.275	5.618	0.791	48.251

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	165	208	0	755	75	1	141
normalized size	1	1.	1.1	1.39	0.	5.03	0.5	0.01	0.94
time (sec)	N/A	0.242	0.17	0.021	0.	0.269	2.404	0.74	21.652

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	129	116	0	828	87	1	138
normalized size	1	1.	0.86	0.77	0.	5.52	0.58	0.01	0.92
time (sec)	N/A	0.235	0.141	0.017	0.	0.266	2.855	0.367	19.676

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	232	0	1507	148	1	177
normalized size	1	1.	1.1	1.33	0.	8.66	0.85	0.01	1.02
time (sec)	N/A	0.495	0.766	0.025	0.	0.276	6.197	0.796	40.618

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	216	368	0	2190	211	1	207
normalized size	1	1.	1.1	1.88	0.	11.17	1.08	0.01	1.06
time (sec)	N/A	0.988	0.245	0.027	0.	0.287	8.522	0.797	80.593

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	121	342	0	1	745	0	0
normalized size	1	1.	0.92	2.59	0.	0.01	5.64	0.	0.
time (sec)	N/A	0.373	0.338	0.021	0.	0.276	12.25	0.	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	104	0	1	282	0	70
normalized size	1	1.	1.19	1.33	0.	0.01	3.62	0.	0.9
time (sec)	N/A	0.138	0.157	0.014	0.	0.265	6.058	0.	17.115

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	77	0	1	267	0	65
normalized size	1	1.	1.05	1.03	0.	0.01	3.56	0.	0.87
time (sec)	N/A	0.132	0.115	0.008	0.	0.267	5.567	0.	15.606

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	79	75	0	1	267	0	66
normalized size	1	1.	1.07	1.01	0.	0.01	3.61	0.	0.89
time (sec)	N/A	0.119	0.141	0.008	0.	0.265	5.409	0.	12.429

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	207	405	0	1	772	0	116
normalized size	1	1.	1.7	3.32	0.	0.01	6.33	0.	0.95
time (sec)	N/A	0.428	0.728	0.028	0.	0.347	104.571	0.	44.701

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	248	569	0	1	0	0	153
normalized size	1	1.	1.53	3.51	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.551	0.497	0.026	0.	0.411	0.	0.	76.904

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	327	2280	0	3856	450	0	0
normalized size	1	1.	0.99	6.89	0.	11.65	1.36	0.	0.
time (sec)	N/A	1.629	1.176	0.112	0.	0.383	21.262	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	282	2158	0	3047	379	0	267
normalized size	1	1.	1.04	7.96	0.	11.24	1.4	0.	0.99
time (sec)	N/A	1.187	0.989	0.116	0.	0.307	15.276	0.	96.299

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	235	1641	0	2252	294	0	218
normalized size	1	1.	0.99	6.92	0.	9.5	1.24	0.	0.92
time (sec)	N/A	0.758	0.767	0.055	0.	0.281	11.28	0.	55.512

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	222	342	0	2268	298	0	201
normalized size	1	1.	1.	1.55	0.	10.26	1.35	0.	0.91
time (sec)	N/A	0.566	0.856	0.106	0.	0.284	11.842	0.	44.249

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	243	733	0	3117	394	0	230
normalized size	1	1.	0.96	2.91	0.	12.37	1.56	0.	0.91
time (sec)	N/A	0.969	0.793	0.072	0.	0.316	14.924	0.	64.23

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	302	2012	0	3931	481	0	282
normalized size	1	1.	0.98	6.53	0.	12.76	1.56	0.	0.92
time (sec)	N/A	2.86	1.142	0.063	0.	0.392	23.268	0.	137.02

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	244	819	0	1	1520	413	0
normalized size	1	1.	1.17	3.92	0.	0.	7.27	1.98	0.
time (sec)	N/A	0.841	0.584	0.033	0.	0.298	54.554	15.661	0.

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	194	267	0	1	554	286	112
normalized size	1	1.	1.6	2.21	0.	0.01	4.58	2.36	0.93
time (sec)	N/A	0.225	0.304	0.021	0.	0.272	39.256	15.651	26.491

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	137	230	0	1	520	231	112
normalized size	1	1.	1.15	1.93	0.	0.01	4.37	1.94	0.94
time (sec)	N/A	0.196	0.373	0.02	0.	0.27	35.86	15.639	26.032

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	145	270	0	1	580	217	119
normalized size	1	1.	1.12	2.08	0.	0.01	4.46	1.67	0.92
time (sec)	N/A	0.25	0.26	0.019	0.	0.272	35.722	15.623	29.849

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	142	0	1	490	193	105
normalized size	1	1.	1.01	1.26	0.	0.01	4.34	1.71	0.93
time (sec)	N/A	0.182	0.179	0.011	0.	0.271	33.355	15.638	21.39

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	141	0	1	481	194	105
normalized size	1	1.	0.94	1.25	0.	0.01	4.26	1.72	0.93
time (sec)	N/A	0.173	0.194	0.009	0.	0.269	33.359	15.728	18.988

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	342	1200	0	1	0	436	196
normalized size	1	1.	1.71	6.	0.	0.	0.	2.18	0.98
time (sec)	N/A	0.675	0.928	0.044	0.	0.713	0.	15.603	79.064

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	402	1486	0	1	0	516	258
normalized size	1	1.	1.58	5.83	0.	0.	0.	2.02	1.01
time (sec)	N/A	0.895	1.127	0.038	0.	0.965	0.	15.653	138.41

Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	455	5425	0	5777	804	1	0
normalized size	1	1.	1.14	13.56	0.	14.44	2.01	0.	0.
time (sec)	N/A	3.341	2.222	0.153	0.	0.552	74.397	35.039	0.

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	381	4840	0	5029	716	0	340
normalized size	1	1.	1.09	13.91	0.	14.45	2.06	0.	0.98
time (sec)	N/A	1.904	1.793	0.143	0.	0.408	56.322	0.	140.816

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	343	4017	0	4223	627	1	286
normalized size	1	1.	1.15	13.48	0.	14.17	2.1	0.	0.96
time (sec)	N/A	1.503	1.602	0.128	0.	0.325	47.667	31.299	76.462

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	285	3657	0	4223	644	1	274
normalized size	1	1.	0.99	12.65	0.	14.61	2.23	0.	0.95
time (sec)	N/A	1.346	1.315	0.075	0.	0.342	48.11	29.294	87.423

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	334	2958	0	5099	733	0	291
normalized size	1	1.	1.07	9.51	0.	16.4	2.36	0.	0.94
time (sec)	N/A	1.506	1.545	0.165	0.	0.423	53.332	0.	81.317

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	372	3360	0	5836	818	1	0
normalized size	1	1.	1.05	9.46	0.	16.44	2.3	0.	0.
time (sec)	N/A	3.626	1.959	0.136	0.	0.603	66.407	16.399	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	454	5263	0	6647	0	0	0
normalized size	1	1.	1.07	12.38	0.	15.64	0.	0.	0.
time (sec)	N/A	1.902	3.27	0.093	0.	0.931	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	80	116	0	1	311	105	73
normalized size	1	1.	0.98	1.41	0.	0.01	3.79	1.28	0.89
time (sec)	N/A	0.201	0.092	0.008	0.	0.268	5.337	0.314	28.631

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	65	63	0	1	223	84	54
normalized size	1	1.	1.02	0.98	0.	0.02	3.48	1.31	0.84
time (sec)	N/A	0.135	0.042	0.005	0.	0.265	2.512	0.295	18.646

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	41	38	0	1	131	50	34
normalized size	1	1.	1.17	1.09	0.	0.03	3.74	1.43	0.97
time (sec)	N/A	0.075	0.014	0.002	0.	0.265	1.331	0.294	9.642

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	117	69	0	1	253	96	63
normalized size	1	1.	1.67	0.99	0.	0.01	3.61	1.37	0.9
time (sec)	N/A	0.16	0.138	0.01	0.	0.272	9.013	0.297	25.909

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	139	123	0	1	350	128	87
normalized size	1	1.	1.56	1.38	0.	0.01	3.93	1.44	0.98
time (sec)	N/A	0.309	0.333	0.012	0.	0.283	21.253	0.297	41.955

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	208	343	0	1419	129	1	189
normalized size	1	1.	1.16	1.92	0.	7.93	0.72	0.01	1.06
time (sec)	N/A	0.754	0.225	0.035	0.	0.279	5.657	0.786	53.868

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	208	0	744	75	1	141
normalized size	1	1.	0.91	1.39	0.	4.96	0.5	0.01	0.94
time (sec)	N/A	0.265	0.215	0.016	0.	0.267	2.404	0.734	24.026

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	137	116	0	817	87	1	138
normalized size	1	1.	0.91	0.77	0.	5.45	0.58	0.01	0.92
time (sec)	N/A	0.183	0.139	0.015	0.	0.265	2.867	0.37	21.844

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	199	232	0	1496	148	1	177
normalized size	1	1.	1.16	1.35	0.	8.7	0.86	0.01	1.03
time (sec)	N/A	0.384	0.789	0.022	0.	0.281	6.258	0.792	43.598

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	86	0	1	138	81	58
normalized size	1	1.	0.9	1.25	0.	0.01	2.	1.17	0.84
time (sec)	N/A	0.183	0.062	0.005	0.	0.279	3.685	0.548	30.799

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	49	0	1	110	62	48
normalized size	1	1.	0.91	0.88	0.	0.02	1.96	1.11	0.86
time (sec)	N/A	0.107	0.03	0.004	0.	0.282	1.477	0.548	19.731

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	1	53	31	29
normalized size	1	1.	1.	0.84	0.	0.03	1.71	1.	0.94
time (sec)	N/A	0.063	0.015	0.002	0.	0.279	0.788	0.547	9.823

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	90	71	0	1	184	96	66
normalized size	1	1.	1.17	0.92	0.	0.01	2.39	1.25	0.86
time (sec)	N/A	0.158	0.084	0.009	0.	0.284	7.446	0.536	31.189

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	146	122	0	1	372	170	83
normalized size	1	1.	1.51	1.26	0.	0.01	3.84	1.75	0.86
time (sec)	N/A	0.308	0.214	0.014	0.	0.29	17.345	0.551	47.232

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	144	210	0	814	105	0	128
normalized size	1	1.	1.26	1.84	0.	7.14	0.92	0.	1.12
time (sec)	N/A	0.338	0.15	0.04	0.	0.285	3.733	0.	43.36

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	128	134	0	360	44	1	94
normalized size	1	1.	1.17	1.23	0.	3.3	0.4	0.01	0.86
time (sec)	N/A	0.129	0.19	0.017	0.	0.278	1.072	2.255	19.696

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	105	74	0	747	63	1	94
normalized size	1	1.	0.96	0.68	0.	6.85	0.58	0.01	0.86
time (sec)	N/A	0.118	0.112	0.014	0.	0.283	1.969	0.687	17.705

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	143	180	0	2176	134	0	128
normalized size	1	1.	1.18	1.49	0.	17.98	1.11	0.	1.06
time (sec)	N/A	0.265	0.28	0.017	0.	0.29	7.416	0.	37.601

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	84	0	1	144	78	58
normalized size	1	1.	0.9	1.22	0.	0.01	2.09	1.13	0.84
time (sec)	N/A	0.177	0.062	0.009	0.	0.284	3.66	0.548	28.631

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	49	47	0	1	117	57	48
normalized size	1	1.	0.91	0.87	0.	0.02	2.17	1.06	0.89
time (sec)	N/A	0.104	0.03	0.003	0.	0.28	1.409	0.55	18.215

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	0	1	60	28	27
normalized size	1	1.	1.	0.84	0.	0.03	1.94	0.9	0.87
time (sec)	N/A	0.061	0.012	0.002	0.	0.278	0.735	0.543	9.065

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	105	63	0	1	194	82	66
normalized size	1	1.	1.52	0.91	0.	0.01	2.81	1.19	0.96
time (sec)	N/A	0.148	0.091	0.009	0.	0.285	7.151	0.545	28.321

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	163	110	0	1	386	169	83
normalized size	1	1.	1.83	1.24	0.	0.01	4.34	1.9	0.93
time (sec)	N/A	0.295	0.171	0.012	0.	0.29	17.281	0.537	43.803

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	432	432	164	1658	0	830	105	0	393
normalized size	1	1.	0.38	3.84	0.	1.92	0.24	0.	0.91
time (sec)	N/A	2.059	0.162	0.128	0.	0.285	3.717	0.	158.426

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	143	724	0	377	44	1	294
normalized size	1	1.	0.43	2.19	0.	1.14	0.13	0.	0.89
time (sec)	N/A	0.62	0.199	0.049	0.	0.28	1.071	2.24	65.346

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	119	913	0	765	63	1	321
normalized size	1	1.	0.33	2.54	0.	2.13	0.18	0.	0.89
time (sec)	N/A	0.69	0.12	0.062	0.	0.283	1.943	0.678	88.36

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	174	3318	0	2136	134	0	382
normalized size	1	1.	0.4	7.66	0.	4.93	0.31	0.	0.88
time (sec)	N/A	1.274	0.247	0.058	0.	0.291	7.304	0.	128.938

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	24	26	24	22
normalized size	1	1.	1.	0.95	1.2	1.2	1.3	1.2	1.1
time (sec)	N/A	0.049	0.009	0.002	0.767	0.274	0.195	0.285	4.468

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	14	10	14	10
normalized size	1	1.	1.	0.79	1.	1.	0.71	1.	0.71
time (sec)	N/A	0.035	0.009	0.004	0.762	0.269	0.187	0.269	4.71

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	19	24	24	20	24	20
normalized size	1	1.	1.	0.83	1.04	1.04	0.87	1.04	0.87
time (sec)	N/A	0.033	0.021	0.012	0.754	0.276	0.362	0.268	5.999

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	94	57	0	143	63	0	63
normalized size	1	1.	1.27	0.77	0.	1.93	0.85	0.	0.85
time (sec)	N/A	0.107	0.244	0.029	0.	0.293	0.493	0.	22.016

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	39	308	0	630	24	0	185
normalized size	1	1.	0.21	1.64	0.	3.35	0.13	0.	0.98
time (sec)	N/A	0.387	0.051	0.055	0.	0.294	1.698	0.	30.044

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	162	296	0	1	0	248	163
normalized size	1	1.	0.95	1.73	0.	0.01	0.	1.45	0.95
time (sec)	N/A	0.387	0.163	0.046	0.	0.308	0.	0.303	27.221

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	247	0	1	0	197	141
normalized size	1	1.	0.88	1.61	0.	0.01	0.	1.29	0.92
time (sec)	N/A	0.305	0.142	0.022	0.	0.3	0.	0.304	25.843

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	99	139	0	1	0	132	97
normalized size	1	1.	0.92	1.29	0.	0.01	0.	1.22	0.9
time (sec)	N/A	0.188	0.105	0.017	0.	0.296	0.	0.289	15.324

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	81	101	0	1	0	103	73
normalized size	1	1.	0.98	1.22	0.	0.01	0.	1.24	0.88
time (sec)	N/A	0.122	0.073	0.01	0.	0.292	0.	0.284	9.085

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	111	91	0	1	0	0	95
normalized size	1	1.	1.02	0.83	0.	0.01	0.	0.	0.87
time (sec)	N/A	0.278	0.306	0.015	0.	0.319	0.	0.	24.07

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	116	140	0	1	0	0	99
normalized size	1	1.	1.04	1.25	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.281	0.258	0.015	0.	0.316	0.	0.	23.357

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	93	193	0	1	0	0	76
normalized size	1	1.	1.06	2.19	0.	0.01	0.	0.	0.86
time (sec)	N/A	0.172	0.234	0.015	0.	0.296	0.	0.	16.054

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	222	0	1	0	0	104
normalized size	1	1.	0.97	1.91	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.236	0.282	0.019	0.	0.304	0.	0.	20.861

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	143	387	0	1	0	0	148
normalized size	1	1.	0.89	2.4	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.386	0.229	0.023	0.	0.31	0.	0.	30.344

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	175	442	0	1	0	0	185
normalized size	1	1.	0.88	2.22	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.583	0.24	0.025	0.	0.326	0.	0.	44.349

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	538	476	0	0	0	0	364
normalized size	1	1.	1.36	1.21	0.	0.	0.	0.	0.92
time (sec)	N/A	0.661	2.879	0.053	0.	0.	0.	0.	67.246

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	479	417	0	0	0	0	314
normalized size	1	1.	1.4	1.22	0.	0.	0.	0.	0.92
time (sec)	N/A	0.391	2.295	0.015	0.	0.	0.	0.	43.909

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	445	379	0	0	0	0	279
normalized size	1	1.	1.44	1.23	0.	0.	0.	0.	0.9
time (sec)	N/A	0.298	1.53	0.011	0.	0.	0.	0.	39.897

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	435	381	0	0	0	0	274
normalized size	1	1.	1.44	1.26	0.	0.	0.	0.	0.9
time (sec)	N/A	0.271	1.459	0.018	0.	0.	0.	0.	35.509

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	459	404	0	0	0	0	304
normalized size	1	1.	1.35	1.18	0.	0.	0.	0.	0.89
time (sec)	N/A	0.465	1.624	0.02	0.	0.	0.	0.	56.977

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	530	452	0	0	0	0	366
normalized size	1	1.	1.34	1.14	0.	0.	0.	0.	0.92
time (sec)	N/A	0.726	2.55	0.023	0.	0.	0.	0.	79.372

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	220	534	0	1	0	374	218
normalized size	1	1.	0.99	2.39	0.	0.	0.	1.68	0.98
time (sec)	N/A	0.507	0.213	0.041	0.	0.325	0.	0.309	35.807

Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	194	432	0	1	0	311	190
normalized size	1	1.	0.95	2.12	0.	0.	0.	1.52	0.93
time (sec)	N/A	0.406	0.196	0.031	0.	0.309	0.	0.305	35.548

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	316	0	1	0	232	141
normalized size	1	1.	0.95	2.11	0.	0.01	0.	1.55	0.94
time (sec)	N/A	0.26	0.141	0.025	0.	0.304	0.	0.289	21.585

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	111	242	0	1	0	182	116
normalized size	1	1.	0.9	1.95	0.	0.01	0.	1.47	0.94
time (sec)	N/A	0.182	0.102	0.019	0.	0.3	0.	0.29	14.045

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	150	192	0	1	0	0	139
normalized size	1	1.	0.97	1.24	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.463	0.622	0.023	0.	0.418	0.	0.	36.378

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	146	170	0	1	0	0	139
normalized size	1	1.	0.97	1.13	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.45	0.524	0.025	0.	0.368	0.	0.	35.579

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	153	174	0	1	0	0	141
normalized size	1	1.	1.01	1.15	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.428	0.632	0.025	0.	0.354	0.	0.	35.965

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	169	202	0	1	0	0	146
normalized size	1	1.	1.04	1.24	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.464	0.597	0.025	0.	0.378	0.	0.	36.199

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	125	260	0	1	0	0	122
normalized size	1	1.	0.94	1.95	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.259	0.235	0.026	0.	0.31	0.	0.	23.768

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	163	337	0	1	0	0	151
normalized size	1	1.	1.01	2.08	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.328	0.218	0.028	0.	0.324	0.	0.	30.194

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	206	457	0	1	0	0	201
normalized size	1	1.	0.95	2.12	0.	0.	0.	0.	0.93
time (sec)	N/A	0.505	0.329	0.036	0.	0.355	0.	0.	42.004

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	657	674	0	0	0	0	469
normalized size	1	1.	1.33	1.36	0.	0.	0.	0.	0.95
time (sec)	N/A	1.083	4.126	0.017	0.	0.	0.	0.	105.571

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	602	545	0	0	0	0	413
normalized size	1	1.	1.36	1.23	0.	0.	0.	0.	0.93
time (sec)	N/A	0.581	3.645	0.016	0.	0.	0.	0.	66.338

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	533	471	0	0	0	0	354
normalized size	1	1.	1.4	1.24	0.	0.	0.	0.	0.93
time (sec)	N/A	0.499	2.79	0.012	0.	0.	0.	0.	62.074

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	505	430	0	0	0	0	328
normalized size	1	1.	1.4	1.19	0.	0.	0.	0.	0.91
time (sec)	N/A	0.423	2.374	0.019	0.	0.	0.	0.	56.803

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	473	428	0	0	0	0	325
normalized size	1	1.	1.34	1.21	0.	0.	0.	0.	0.92
time (sec)	N/A	0.397	1.735	0.022	0.	0.	0.	0.	55.452

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	527	450	0	0	0	0	362
normalized size	1	1.	1.32	1.12	0.	0.	0.	0.	0.9
time (sec)	N/A	0.614	2.556	0.023	0.	0.	0.	0.	77.087

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	447	447	572	495	0	0	0	0	416
normalized size	1	1.	1.28	1.11	0.	0.	0.	0.	0.93
time (sec)	N/A	0.931	3.004	0.028	0.	0.	0.	0.	103.207

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	59	114	0	0	0	0	49
normalized size	1	1.	1.23	2.38	0.	0.	0.	0.	1.02
time (sec)	N/A	0.14	0.107	0.019	0.	0.	0.	0.	22.919

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	162	0	1	0	147	114
normalized size	1	1.	0.84	1.34	0.	0.01	0.	1.21	0.94
time (sec)	N/A	0.252	0.129	0.022	0.	0.304	0.	0.302	22.346

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	86	116	0	1	0	111	94
normalized size	1	1.	0.83	1.12	0.	0.01	0.	1.07	0.9
time (sec)	N/A	0.19	0.075	0.021	0.	0.296	0.	0.303	21.417

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	56	0	1	0	82	58
normalized size	1	1.	0.97	0.82	0.	0.01	0.	1.21	0.85
time (sec)	N/A	0.107	0.037	0.015	0.	0.293	0.	0.295	12.263

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	41	35	0	1	0	54	37
normalized size	1	1.	0.95	0.81	0.	0.02	0.	1.26	0.86
time (sec)	N/A	0.061	0.019	0.01	0.	0.314	0.	0.291	6.963

Problem 959	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	50	39	0	1	0	0	39
normalized size	1	1.	1.14	0.89	0.	0.02	0.	0.	0.89
time (sec)	N/A	0.086	0.12	0.014	0.	0.287	0.	0.	10.801

Problem 960	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	78	63	0	1	0	0	61
normalized size	1	1.	1.08	0.88	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.128	0.124	0.016	0.	0.299	0.	0.	14.634

Problem 961	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	96	127	0	1	0	0	97
normalized size	1	1.	0.89	1.18	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.239	0.253	0.018	0.	0.303	0.	0.	22.249

Problem 962	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	117	176	0	1	0	0	133
normalized size	1	1.	0.81	1.21	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.389	0.439	0.02	0.	0.31	0.	0.	35.414

Problem 963	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	444	388	0	0	0	0	284
normalized size	1	1.	1.42	1.24	0.	0.	0.	0.	0.91
time (sec)	N/A	0.285	1.633	0.016	0.	0.	0.	0.	40.361

Problem 964	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	278	216	0	0	0	0	240
normalized size	1	1.	1.04	0.81	0.	0.	0.	0.	0.9
time (sec)	N/A	0.194	0.262	0.014	0.	0.	0.	0.	32.13

Problem 965	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	186	144	0	0	0	0	102
normalized size	1	1.	1.63	1.26	0.	0.	0.	0.	0.89
time (sec)	N/A	0.049	0.147	0.01	0.	0.	0.	0.	7.381

Problem 966	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	298	239	0	0	0	0	260
normalized size	1	1.	1.01	0.81	0.	0.	0.	0.	0.88
time (sec)	N/A	0.283	0.909	0.017	0.	0.	0.	0.	44.622

Problem 967	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	459	413	0	0	0	0	314
normalized size	1	1.	1.33	1.2	0.	0.	0.	0.	0.91
time (sec)	N/A	0.415	1.7	0.022	0.	0.	0.	0.	55.647

Problem 968	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	112	168	0	1	0	159	116
normalized size	1	1.	0.9	1.35	0.	0.01	0.	1.28	0.94
time (sec)	N/A	0.265	0.24	0.028	0.	0.3	0.	0.305	23.444

Problem 969	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	94	120	0	1	0	123	95
normalized size	1	1.	0.88	1.12	0.	0.01	0.	1.15	0.89
time (sec)	N/A	0.203	0.129	0.021	0.	0.294	0.	0.306	22.703

Problem 970	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	77	58	0	1	0	95	60
normalized size	1	1.	1.1	0.83	0.	0.01	0.	1.36	0.86
time (sec)	N/A	0.114	0.071	0.017	0.	0.292	0.	0.293	12.507

Problem 971	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	51	36	0	1	0	61	39
normalized size	1	1.	1.16	0.82	0.	0.02	0.	1.39	0.89
time (sec)	N/A	0.064	0.028	0.011	0.	0.288	0.	0.293	7.313

Problem 972	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	55	45	0	1	0	59	37
normalized size	1	1.	1.17	0.96	0.	0.02	0.	1.26	0.79
time (sec)	N/A	0.094	0.161	0.016	0.	0.288	0.	0.325	11.666

Problem 973	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	93	74	0	1	0	93	61
normalized size	1	1.	1.21	0.96	0.	0.01	0.	1.21	0.79
time (sec)	N/A	0.142	0.119	0.019	0.	0.294	0.	0.325	15.15

Problem 974	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	118	149	0	1	0	122	97
normalized size	1	1.	1.03	1.3	0.	0.01	0.	1.06	0.84
time (sec)	N/A	0.27	0.19	0.02	0.	0.302	0.	0.523	23.652

Problem 975	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	139	202	0	1	0	158	133
normalized size	1	1.	0.9	1.31	0.	0.01	0.	1.03	0.86
time (sec)	N/A	0.435	0.278	0.023	0.	0.308	0.	0.521	37.004

Problem 976	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	459	391	0	0	0	0	359
normalized size	1	1.	1.12	0.96	0.	0.	0.	0.	0.88
time (sec)	N/A	1.405	1.386	0.057	0.	0.	0.	0.	117.025

Problem 977	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	271	217	0	0	0	0	332
normalized size	1	1.	0.72	0.58	0.	0.	0.	0.	0.88
time (sec)	N/A	0.852	0.215	0.014	0.	0.	0.	0.	96.437

Problem 978	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	177	145	0	0	0	0	151
normalized size	1	1.	1.05	0.86	0.	0.	0.	0.	0.89
time (sec)	N/A	0.196	0.142	0.011	0.	0.	0.	0.	43.766

Problem 979	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	283	241	0	0	0	0	354
normalized size	1	1.	0.69	0.59	0.	0.	0.	0.	0.87
time (sec)	N/A	1.016	0.825	0.019	0.	0.	0.	0.	117.214

Problem 980	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	445	445	472	417	0	0	0	0	393
normalized size	1	1.	1.06	0.94	0.	0.	0.	0.	0.88
time (sec)	N/A	1.33	1.409	0.021	0.	0.	0.	0.	158.343

Problem 981	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	147	354	0	1	0	490	177
normalized size	1	1.	0.77	1.86	0.	0.01	0.	2.58	0.93
time (sec)	N/A	0.538	0.236	0.023	0.	0.358	0.	0.335	40.925

Problem 982	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	127	264	0	1	0	359	124
normalized size	1	1.	0.95	1.97	0.	0.01	0.	2.68	0.93
time (sec)	N/A	0.255	0.182	0.019	0.	0.326	0.	0.332	25.31

Problem 983	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	92	149	0	1	0	255	100
normalized size	1	1.	0.8	1.3	0.	0.01	0.	2.22	0.87
time (sec)	N/A	0.189	0.141	0.019	0.	0.313	0.	0.329	24.

Problem 984	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	38	0	90	0	59	34
normalized size	1	1.	1.	1.06	0.	2.5	0.	1.64	0.94
time (sec)	N/A	0.068	0.035	0.008	0.	0.292	0.	0.312	9.657

Problem 985	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	36	0	90	0	61	36
normalized size	1	1.	1.	1.	0.	2.5	0.	1.69	1.
time (sec)	N/A	0.049	0.026	0.006	0.	0.293	0.	0.314	6.187

Problem 986	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	100	99	0	1	0	0	78
normalized size	1	1.	1.12	1.11	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.171	0.274	0.018	0.	0.317	0.	0.	19.399

Problem 987	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	132	195	0	1	0	0	128
normalized size	1	1.	0.95	1.4	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.3	0.199	0.019	0.	0.324	0.	0.	31.865

Problem 988	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	159	314	0	1	0	0	180
normalized size	1	1.	0.82	1.61	0.	0.01	0.	0.	0.92
time (sec)	N/A	0.484	0.327	0.023	0.	0.354	0.	0.	46.706

Problem 989	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	489	482	0	0	0	0	372
normalized size	1	1.	1.2	1.18	0.	0.	0.	0.	0.91
time (sec)	N/A	0.558	2.39	0.031	0.	0.	0.	0.	66.332

Problem 990	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	452	450	0	0	0	0	316
normalized size	1	1.	1.32	1.32	0.	0.	0.	0.	0.92
time (sec)	N/A	0.35	1.527	0.021	0.	0.	0.	0.	46.6

Problem 991	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	437	446	0	0	0	0	316
normalized size	1	1.	1.28	1.31	0.	0.	0.	0.	0.93
time (sec)	N/A	0.331	1.465	0.021	0.	0.	0.	0.	44.888

Problem 992	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	456	481	0	0	0	0	326
normalized size	1	1.	1.29	1.36	0.	0.	0.	0.	0.92
time (sec)	N/A	0.308	1.578	0.018	0.	0.	0.	0.	49.348

Problem 993	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	428	515	536	0	0	0	0	393
normalized size	1	1.	1.2	1.25	0.	0.	0.	0.	0.92
time (sec)	N/A	0.554	2.438	0.028	0.	0.	0.	0.	74.649

Problem 994	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	37	46	41	0	0	41
normalized size	1	1.	0.68	0.74	0.92	0.82	0.	0.	0.82
time (sec)	N/A	0.089	0.031	0.007	0.711	0.272	0.	0.	13.477

Problem 995	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	76	64	0	1	0	80	48
normalized size	1	1.	1.31	1.1	0.	0.02	0.	1.38	0.83
time (sec)	N/A	0.139	0.058	0.007	0.	0.285	0.	0.298	12.351

Problem 996	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	26	18	27	0	42	15
normalized size	1	1.	1.	1.18	0.82	1.23	0.	1.91	0.68
time (sec)	N/A	0.013	0.009	0.003	0.715	0.272	0.	0.278	7.591

Problem 997	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	52	44	0	1	0	53	27
normalized size	1	1.	1.68	1.42	0.	0.03	0.	1.71	0.87
time (sec)	N/A	0.077	0.024	0.	0.	0.281	0.	0.292	7.557

Problem 998	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	58	50	0	1	0	62	27
normalized size	1	1.	1.93	1.67	0.	0.03	0.	2.07	0.9
time (sec)	N/A	0.023	0.043	0.	0.	0.278	0.	0.274	5.444

Problem 999	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	0	28	0	19	19
normalized size	1	1.	1.	1.13	0.	1.22	0.	0.83	0.83
time (sec)	N/A	0.066	0.022	0.006	0.	0.268	0.	0.276	7.234

Problem 1000	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	97	73	0	1	0	0	49
normalized size	1	1.	1.64	1.24	0.	0.02	0.	0.	0.83
time (sec)	N/A	0.089	0.07	0.006	0.	0.286	0.	0.	12.324

Problem 1001	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	37	0	42	0	36	44
normalized size	1	1.	0.67	0.71	0.	0.81	0.	0.69	0.85
time (sec)	N/A	0.133	0.037	0.006	0.	0.276	0.	0.288	12.598

Problem 1002	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	114	94	0	1	0	0	78
normalized size	1	1.	1.31	1.08	0.	0.01	0.	0.	0.9
time (sec)	N/A	0.168	0.084	0.006	0.	0.291	0.	0.	18.788

Problem 1003	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	91	0	0	37	0	94
normalized size	1	1.	0.85	0.84	0.	0.	0.34	0.	0.87
time (sec)	N/A	0.072	0.215	0.052	0.	0.	2.152	0.	7.278

Problem 1004	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	19	22	19	12
normalized size	1	1.	1.	0.83	1.06	1.06	1.22	1.06	0.67
time (sec)	N/A	0.012	0.008	0.007	0.714	0.271	1.586	0.276	2.123

Problem 1005	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	104	97	0	0	37	0	187
normalized size	1	1.	0.5	0.46	0.	0.	0.18	0.	0.89
time (sec)	N/A	0.141	0.084	0.01	0.	0.	1.998	0.	16.657

Problem 1006	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	0	1	20	34	26
normalized size	1	1.	1.	0.8	0.	0.03	0.67	1.13	0.87
time (sec)	N/A	0.037	0.014	0.009	0.	0.283	3.38	0.286	4.103

Problem 1007	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	74	70	0	0	36	0	78
normalized size	1	1.	0.84	0.8	0.	0.	0.41	0.	0.89
time (sec)	N/A	0.036	0.052	0.007	0.	0.	1.898	0.	3.558

Problem 1008	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	0	1	22	31	24
normalized size	1	1.	1.	1.07	0.	0.04	0.81	1.15	0.89
time (sec)	N/A	0.048	0.064	0.013	0.	0.281	3.611	0.275	4.972

Problem 1009	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	121	115	0	0	39	0	202
normalized size	1	1.	0.52	0.5	0.	0.	0.17	0.	0.87
time (sec)	N/A	0.188	0.511	0.015	0.	0.	2.238	0.	22.187

Problem 1010	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	23	20	19	17
normalized size	1	1.	1.	0.86	1.1	1.1	0.95	0.9	0.81
time (sec)	N/A	0.019	0.016	0.007	0.713	0.274	1.946	0.279	2.654

Problem 1011	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	95	93	0	0	41	0	97
normalized size	1	1.	0.86	0.85	0.	0.	0.37	0.	0.88
time (sec)	N/A	0.069	0.194	0.016	0.	0.	2.584	0.	6.995

Problem 1012	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	67	59	0	1	95	73	66
normalized size	1	1.	0.92	0.81	0.	0.01	1.3	1.	0.9
time (sec)	N/A	0.069	0.055	0.01	0.	0.289	12.429	0.283	8.488

Problem 1013	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	27	25	0	31	44	36	29
normalized size	1	1.	0.75	0.69	0.	0.86	1.22	1.	0.81
time (sec)	N/A	0.06	0.02	0.007	0.	0.274	1.672	0.278	7.471

Problem 1014	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	52	39	0	1	42	54	41
normalized size	1	1.	1.06	0.8	0.	0.02	0.86	1.1	0.84
time (sec)	N/A	0.04	0.028	0.007	0.	0.285	6.957	0.284	5.404

Problem 1015	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	18	20	18	10
normalized size	1	1.	1.	0.93	1.2	1.2	1.33	1.2	0.67
time (sec)	N/A	0.009	0.004	0.004	0.708	0.272	1.366	0.276	2.05

Problem 1016	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	1	17	31	22
normalized size	1	1.	1.	0.84	0.	0.04	0.68	1.24	0.88
time (sec)	N/A	0.017	0.011	0.003	0.	0.281	3.358	0.282	2.326

Problem 1017	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	31	29	0	1	19	30	22
normalized size	1	1.	1.24	1.16	0.	0.04	0.76	1.2	0.88
time (sec)	N/A	0.048	0.027	0.006	0.	0.28	3.472	0.265	5.423

Problem 1018	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	0	23	19	41	14
normalized size	1	1.	1.	0.95	0.	1.21	1.	2.16	0.74
time (sec)	N/A	0.02	0.015	0.005	0.	0.27	1.794	0.268	3.027

Problem 1019	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	48	0	1	42	65	41
normalized size	1	1.	1.28	0.96	0.	0.02	0.84	1.3	0.82
time (sec)	N/A	0.073	0.036	0.008	0.	0.286	7.302	0.264	7.236

Problem 1020	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	29	26	0	36	46	74	36
normalized size	1	1.	0.66	0.59	0.	0.82	1.05	1.68	0.82
time (sec)	N/A	0.041	0.021	0.006	0.	0.274	2.428	0.268	4.971

Problem 1021	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	18	15	18	12
normalized size	1	1.	1.	0.81	1.	1.12	0.94	1.12	0.75
time (sec)	N/A	0.007	0.004	0.004	0.708	0.263	1.887	0.262	1.854

Problem 1022	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	16	15	16	10
normalized size	1	1.	1.	0.81	1.	1.	0.94	1.	0.62
time (sec)	N/A	0.007	0.003	0.003	0.706	0.263	1.643	0.261	1.913

Problem 1023	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	19	14	7	10
normalized size	1	1.	1.	0.92	1.15	1.46	1.08	0.54	0.77
time (sec)	N/A	0.006	0.002	0.006	0.708	0.266	1.523	0.265	2.125

Problem 1024	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	22	0	16	15
normalized size	1	1.	1.	0.93	1.2	1.47	0.	1.07	1.
time (sec)	N/A	0.006	0.003	0.004	0.714	0.27	0.	0.271	1.94

Problem 1025	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	20	14	11	14
normalized size	1	1.	1.	0.92	1.17	1.67	1.17	0.92	1.17
time (sec)	N/A	0.006	0.002	0.003	0.701	0.261	1.407	0.265	1.374

Problem 1026	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	20	15	12	12
normalized size	1	1.	1.	0.77	0.92	1.54	1.15	0.92	0.92
time (sec)	N/A	0.006	0.002	0.004	0.709	0.268	1.52	0.261	2.12

Problem 1027	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	20	17	11	15
normalized size	1	1.	1.	0.81	1.	1.25	1.06	0.69	0.94
time (sec)	N/A	0.008	0.004	0.004	0.709	0.263	1.68	0.262	2.135

Problem 1028	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	16	20	19	11	15
normalized size	1	1.	1.06	0.81	1.	1.25	1.19	0.69	0.94
time (sec)	N/A	0.007	0.006	0.005	0.723	0.264	1.929	0.264	2.121

Problem 1029	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	13	16	20	19	4	15
normalized size	1	1.	0.94	0.81	1.	1.25	1.19	0.25	0.94
time (sec)	N/A	0.006	0.003	0.004	0.717	0.261	2.248	0.61	2.126

Problem 1030	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	8	11	8
normalized size	1	1.	1.	0.75	0.92	0.92	0.67	0.92	0.67
time (sec)	N/A	0.006	0.001	0.	0.711	0.264	0.057	0.263	1.532

Problem 1031	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	8	11	8
normalized size	1	1.	1.	0.75	0.92	0.92	0.67	0.92	0.67
time (sec)	N/A	0.007	0.001	0.001	0.74	0.266	0.061	0.263	1.534

Problem 1032	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	8	11	8
normalized size	1	1.	1.	0.75	0.92	0.92	0.67	0.92	0.67
time (sec)	N/A	0.006	0.001	0.001	0.747	0.269	0.056	0.261	1.529

Problem 1033	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	8	11	0
normalized size	1	1.	1.	0.75	0.92	0.92	0.67	0.92	0.
time (sec)	N/A	0.006	0.001	0.	0.738	0.266	0.052	0.261	0.

Problem 1034	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	7	7	5	7	0
normalized size	1	1.	1.	0.86	1.	1.	0.71	1.	0.
time (sec)	N/A	0.004	0.001	0.	0.738	0.263	0.04	0.261	0.

Problem 1035	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	9	7
normalized size	1	1.	1.	0.88	1.	1.	0.88	1.12	0.88
time (sec)	N/A	0.005	0.001	0.002	0.752	0.269	0.068	0.263	1.486

Problem 1036	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	11	8	11	8
normalized size	1	1.	1.	0.9	1.1	1.1	0.8	1.1	0.8
time (sec)	N/A	0.006	0.001	0.001	0.775	0.268	0.074	0.259	1.507

Problem 1037	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	12	11	12
normalized size	1	1.	1.	0.75	0.92	0.92	1.	0.92	1.
time (sec)	N/A	0.006	0.001	0.001	0.785	0.266	0.08	0.262	1.509

Problem 1038	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	11	12	11	12
normalized size	1	1.	1.	0.75	0.92	0.92	1.	0.92	1.
time (sec)	N/A	0.006	0.001	0.001	0.743	0.268	0.077	0.262	1.506

Problem 1039	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	43	0	0	0	0	14
normalized size	1	1.	1.5	3.58	0.	0.	0.	0.	1.17
time (sec)	N/A	0.032	0.031	0.008	0.	0.	0.	0.	6.827

Problem 1040	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	87	82	0	0	0	0	61
normalized size	1	1.	2.23	2.1	0.	0.	0.	0.	1.56
time (sec)	N/A	0.152	0.194	0.435	0.	0.	0.	0.	12.507

Problem 1041	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	26	32	29	26	29
normalized size	1	1.	0.81	0.71	0.84	1.03	0.94	0.84	0.94
time (sec)	N/A	0.016	0.011	0.005	0.749	0.267	16.173	0.26	4.403

Problem 1042	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	26	32	29	26	29
normalized size	1	1.	0.81	0.71	0.84	1.03	0.94	0.84	0.94
time (sec)	N/A	0.015	0.009	0.004	0.773	0.27	7.808	0.26	4.744

Problem 1043	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	25	22	26	30	29	26	29
normalized size	1	1.	0.81	0.71	0.84	0.97	0.94	0.84	0.94
time (sec)	N/A	0.016	0.009	0.004	0.773	0.269	2.505	0.262	4.406

Problem 1044	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	26	28	27	26	27
normalized size	1	1.	0.86	0.76	0.9	0.97	0.93	0.9	0.93
time (sec)	N/A	0.015	0.009	0.004	0.749	0.27	1.964	0.26	4.434

Problem 1045	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	26	28	27	26	27
normalized size	1	1.	0.86	0.76	0.9	0.97	0.93	0.9	0.93
time (sec)	N/A	0.016	0.012	0.005	0.733	0.273	3.109	0.261	4.414

Problem 1046	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	26	28	27	26	27
normalized size	1	1.	0.86	0.76	0.9	0.97	0.93	0.9	0.93
time (sec)	N/A	0.015	0.012	0.005	0.733	0.271	3.899	0.262	4.471

Problem 1047	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	22	27	28	27	27	27
normalized size	1	1.	0.86	0.76	0.93	0.97	0.93	0.93	0.93
time (sec)	N/A	0.015	0.012	0.005	0.741	0.27	5.885	0.262	4.447

Problem 1048	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	49	59	66	70	62	65
normalized size	1	1.	1.	0.77	0.92	1.03	1.09	0.97	1.02
time (sec)	N/A	0.058	0.03	0.009	0.76	0.272	62.892	0.261	9.143

Problem 1049	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	49	59	66	70	62	65
normalized size	1	1.	1.	0.77	0.92	1.03	1.09	0.97	1.02
time (sec)	N/A	0.052	0.029	0.009	0.749	0.269	34.08	0.262	9.259

Problem 1050	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	49	59	63	63	62	65
normalized size	1	1.	0.78	0.77	0.92	0.98	0.98	0.97	1.02
time (sec)	N/A	0.052	0.031	0.009	0.764	0.268	7.707	0.261	9.111

Problem 1051	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	49	65	62	68	62	63
normalized size	1	1.	1.	0.79	1.05	1.	1.1	1.	1.02
time (sec)	N/A	0.052	0.026	0.008	0.758	0.281	13.933	0.26	9.135

Problem 1052	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	50	49	59	62	68	62	63
normalized size	1	1.	0.81	0.79	0.95	1.	1.1	1.	1.02
time (sec)	N/A	0.054	0.03	0.009	0.758	0.274	15.98	0.262	9.18

Problem 1053	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	50	49	59	62	68	62	63
normalized size	1	1.	0.81	0.79	0.95	1.	1.1	1.	1.02
time (sec)	N/A	0.054	0.029	0.009	0.744	0.278	18.817	0.261	9.228

Problem 1054	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	50	49	61	62	68	63	63
normalized size	1	1.	0.81	0.79	0.98	1.	1.1	1.02	1.02
time (sec)	N/A	0.053	0.037	0.009	0.733	0.275	26.66	0.263	9.253

Problem 1055	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	109	116	129	117	102
normalized size	1	1.	1.	0.87	1.06	1.13	1.25	1.14	0.99
time (sec)	N/A	0.109	0.051	0.009	0.748	0.282	168.084	0.263	14.045

Problem 1056	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	109	116	129	117	102
normalized size	1	1.	1.	0.87	1.06	1.13	1.25	1.14	0.99
time (sec)	N/A	0.101	0.047	0.01	0.775	0.273	109.808	0.262	14.042

Problem 1057	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	103	90	109	113	112	117	102
normalized size	1	1.	1.	0.87	1.06	1.1	1.09	1.14	0.99
time (sec)	N/A	0.099	0.043	0.01	0.768	0.271	23.384	0.261	14.054

Problem 1058	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	119	112	128	117	100
normalized size	1	1.	1.	0.89	1.18	1.11	1.27	1.16	0.99
time (sec)	N/A	0.101	0.043	0.009	0.74	0.276	55.908	0.26	14.02

Problem 1059	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	109	112	126	117	99
normalized size	1	1.	1.	0.91	1.1	1.13	1.27	1.18	1.
time (sec)	N/A	0.1	0.069	0.009	0.77	0.302	61.618	0.264	14.037

Problem 1060	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	90	109	112	128	117	100
normalized size	1	1.	1.	0.89	1.08	1.11	1.27	1.16	0.99
time (sec)	N/A	0.102	0.064	0.009	0.755	0.272	70.233	0.263	14.287

Problem 1061	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	90	111	112	124	119	99
normalized size	1	1.	1.	0.91	1.12	1.13	1.25	1.2	1.
time (sec)	N/A	0.1	0.064	0.009	0.792	0.272	88.824	0.263	13.911

Problem 1062	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	80	65	0	8598	0	0	403
normalized size	1	1.	0.21	0.17	0.	22.1	0.	0.	1.04
time (sec)	N/A	1.716	0.064	0.092	0.	1.301	0.	0.	167.124

Problem 1063	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	80	64	0	5210	0	0	401
normalized size	1	1.	0.21	0.17	0.	13.53	0.	0.	1.04
time (sec)	N/A	1.584	0.061	0.013	0.	0.49	0.	0.	161.23

Problem 1064	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	48	45	0	5339	0	0	304
normalized size	1	1.	0.15	0.14	0.	16.13	0.	0.	0.92
time (sec)	N/A	0.874	0.038	0.011	0.	0.4	0.	0.	104.831

Problem 1065	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	46	45	0	2476	0	0	304
normalized size	1	1.	0.14	0.14	0.	7.48	0.	0.	0.92
time (sec)	N/A	0.728	0.034	0.012	0.	0.329	0.	0.	104.714

Problem 1066	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	47	45	0	3776	0	0	298
normalized size	1	1.	0.14	0.14	0.	11.41	0.	0.	0.9
time (sec)	N/A	0.594	0.036	0.011	0.	0.35	0.	0.	103.769

Problem 1067	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	49	42	0	4016	0	0	298
normalized size	1	1.	0.15	0.13	0.	12.13	0.	0.	0.9
time (sec)	N/A	0.669	0.04	0.01	0.	0.439	0.	0.	103.835

Problem 1068	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	78	65	0	7055	0	0	374
normalized size	1	1.	0.21	0.18	0.	19.02	0.	0.	1.01
time (sec)	N/A	1.026	0.065	0.015	0.	1.108	0.	0.	147.074

Problem 1069	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	82	64	0	6602	0	0	376
normalized size	1	1.	0.22	0.17	0.	17.8	0.	0.	1.01
time (sec)	N/A	0.904	0.07	0.015	0.	0.972	0.	0.	142.524

Problem 1070	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	107	82	0	10322	0	0	0
normalized size	1	1.	0.26	0.2	0.	25.05	0.	0.	0.
time (sec)	N/A	1.77	0.105	0.019	0.	5.932	0.	0.	0.

Problem 1071	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	144	149	0	0	0	0	0
normalized size	1	1.	0.26	0.27	0.	0.	0.	0.	0.
time (sec)	N/A	4.762	0.325	0.078	0.	0.	0.	0.	0.

Problem 1072	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	144	146	0	11709	0	0	0
normalized size	1	1.	0.28	0.28	0.	22.52	0.	0.	0.
time (sec)	N/A	2.777	0.306	0.028	0.	2.358	0.	0.	0.

Problem 1073	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	124	121	0	14438	0	0	428
normalized size	1	1.	0.26	0.26	0.	30.65	0.	0.	0.91
time (sec)	N/A	1.678	0.235	0.074	0.	5.427	0.	0.	174.712

Problem 1074	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	483	483	127	118	0	9086	0	0	442
normalized size	1	1.	0.26	0.24	0.	18.81	0.	0.	0.92
time (sec)	N/A	1.857	0.234	0.025	0.	0.783	0.	0.	172.751

Problem 1075	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	109	121	0	12902	0	0	394
normalized size	1	1.	0.24	0.27	0.	28.67	0.	0.	0.88
time (sec)	N/A	1.22	0.191	0.026	0.	2.228	0.	0.	147.605

Problem 1076	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	111	118	0	10425	0	0	401
normalized size	1	1.	0.25	0.27	0.	23.59	0.	0.	0.91
time (sec)	N/A	1.372	0.258	0.025	0.	1.355	0.	0.	143.845

Problem 1077	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	149	149	0	16226	0	0	0
normalized size	1	1.	0.3	0.3	0.	33.18	0.	0.	0.
time (sec)	N/A	1.973	0.208	0.077	0.	10.394	0.	0.	0.

Problem 1078	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	153	144	0	13199	0	0	0
normalized size	1	1.	0.3	0.29	0.	26.24	0.	0.	0.
time (sec)	N/A	2.373	0.275	0.026	0.	5.137	0.	0.	0.

Problem 1079	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	573	573	190	245	0	0	0	0	0
normalized size	1	1.	0.33	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	4.34	0.409	0.035	0.	0.	0.	0.	0.

Problem 1080	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	621	621	254	275	0	0	0	0	0
normalized size	1	1.	0.41	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	3.758	0.672	0.078	0.	0.	0.	0.	0.

Problem 1081	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	216	242	0	0	0	0	0
normalized size	1	1.	0.38	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	3.787	0.619	0.076	0.	0.	0.	0.	0.

Problem 1082	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	219	241	0	16012	0	0	0
normalized size	1	1.	0.38	0.42	0.	28.14	0.	0.	0.
time (sec)	N/A	3.491	0.594	0.046	0.	4.594	0.	0.	0.

Problem 1083	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	176	244	0	0	0	0	0
normalized size	1	1.	0.33	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	2.701	0.38	0.045	0.	0.	0.	0.	0.

Problem 1084	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	533	533	177	237	0	0	0	0	0
normalized size	1	1.	0.33	0.44	0.	0.	0.	0.	0.
time (sec)	N/A	2.37	0.473	0.045	0.	0.	0.	0.	0.

Problem 1085	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	222	277	0	0	0	0	0
normalized size	1	1.	0.37	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	4.335	0.628	0.077	0.	0.	0.	0.	0.

Problem 1086	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	224	270	0	0	0	0	0
normalized size	1	1.	0.38	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	4.689	0.612	0.048	0.	0.	0.	0.	0.

Problem 1087	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	254	321	0	0	0	0	0
normalized size	1	1.	0.39	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	10.377	0.798	0.077	0.	0.	0.	0.	0.

Problem 1088	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	C	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	258	316	0	0	0	0	0
normalized size	1	1.	0.39	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	10.566	0.764	0.049	0.	0.	0.	0.	0.

Problem 1089	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	1048	0	0	0	0	0	131
normalized size	1	1.	7.13	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.552	5.338	0.063	0.	0.	0.	0.	31.355

Problem 1090	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	706	0	0	0	0	0	131
normalized size	1	1.	4.8	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.44	4.05	0.047	0.	0.	0.	0.	31.187

Problem 1091	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	709	0	0	0	0	0	129
normalized size	1	1.	4.89	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.444	1.115	0.046	0.	0.	0.	0.	31.562

Problem 1092	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	707	0	0	0	0	0	133
normalized size	1	1.	4.88	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.449	1.238	0.05	0.	0.	0.	0.	31.922

Problem 1093	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	1751	0	0	0	0	0	133
normalized size	1	1.	11.83	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.446	5.008	0.058	0.	0.	0.	0.	37.986

Problem 1094	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	1395	0	0	0	0	0	133
normalized size	1	1.	9.43	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.435	3.286	0.052	0.	0.	0.	0.	38.081

Problem 1095	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	1395	0	0	0	0	0	131
normalized size	1	1.	9.55	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.439	3.108	0.054	0.	0.	0.	0.	37.162

Problem 1096	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	1059	0	0	0	0	0	134
normalized size	1	1.	7.25	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.451	2.194	0.058	0.	0.	0.	0.	37.594

Problem 1097	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	386	0	0	0	0	0	129
normalized size	1	1.	2.63	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.435	0.299	0.027	0.	0.	0.	0.	31.908

Problem 1098	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	386	0	0	0	0	0	129
normalized size	1	1.	2.63	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.445	0.329	0.026	0.	0.	0.	0.	31.917

Problem 1099	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	384	0	0	0	0	0	128
normalized size	1	1.	2.65	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.442	0.31	0.029	0.	0.	0.	0.	32.344

Problem 1100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	710	0	0	0	0	0	131
normalized size	1	1.	4.9	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.448	1.239	0.054	0.	0.	0.	0.	32.465

Problem 1101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	720	0	0	0	0	0	131
normalized size	1	1.	4.8	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.456	1.792	0.031	0.	0.	0.	0.	43.909

Problem 1102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	1058	0	0	0	0	0	131
normalized size	1	1.	7.05	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.442	2.208	0.028	0.	0.	0.	0.	43.412

Problem 1103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	1058	0	0	0	0	0	129
normalized size	1	1.	7.15	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.442	2.031	0.044	0.	0.	0.	0.	41.958

Problem 1104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	1600	0	0	0	0	0	133
normalized size	1	1.	10.81	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.446	3.849	0.074	0.	0.	0.	0.	42.739

Problem 1105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	111	782	0	802	4451	1	143
normalized size	1	1.	0.71	5.01	0.	5.14	28.53	0.01	0.92
time (sec)	N/A	0.216	0.153	0.011	0.	0.318	20.648	0.275	35.459

Problem 1106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	301	0	325	1486	687	90
normalized size	1	1.	0.69	2.98	0.	3.22	14.71	6.8	0.89
time (sec)	N/A	0.119	0.079	0.009	0.	0.299	8.396	0.274	23.229

Problem 1107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	78	0	96	314	185	42
normalized size	1	1.	0.67	1.5	0.	1.85	6.04	3.56	0.81
time (sec)	N/A	0.044	0.034	0.004	0.	0.302	2.44	0.266	10.79

Problem 1108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	82	0	0	0	0	0	148
normalized size	1	1.	0.47	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.515	0.088	0.038	0.	0.	0.	0.	28.436

Problem 1109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	376	0	0	0	0	0	264
normalized size	1	1.	1.19	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	1.306	1.729	0.034	0.	0.	0.	0.	101.333

Problem 1110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	1080	0	0	0	0	0	139
normalized size	1	1.	6.84	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.479	10.44	0.014	0.	0.	0.	0.	36.09

Problem 1111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	423	0	0	0	0	0	138
normalized size	1	1.	2.69	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.463	0.291	0.014	0.	0.	0.	0.	31.223

Problem 1112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	425	0	0	0	0	0	136
normalized size	1	1.	2.71	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.476	2.185	0.017	0.	0.	0.	0.	32.169

Problem 1113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	426	0	0	0	0	0	138
normalized size	1	1.	2.66	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.468	2.833	0.013	0.	0.	0.	0.	42.223

Problem 1114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	499	0	0	0	0	0	129
normalized size	1	1.	3.22	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.303	4.992	0.083	0.	0.	0.	0.	29.626

Problem 1115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	440	0	0	0	0	0	230
normalized size	1	1.	1.71	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.695	3.878	0.069	0.	0.	0.	0.	52.428

Problem 1116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	395	0	0	0	0	0	192
normalized size	1	1.	1.77	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.429	3.64	0.054	0.	0.	0.	0.	40.727

Problem 1117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	440	0	0	0	0	0	136
normalized size	1	1.	2.75	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.255	3.776	0.041	0.	0.	0.	0.	19.374

Problem 1118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	135	0	0	0	0	0	112
normalized size	1	1.	1.07	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.155	0.188	0.028	0.	0.	0.	0.	10.361

Problem 1119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	497	0	0	0	0	0	128
normalized size	1	1.	3.27	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.338	2.861	0.021	0.	0.	0.	0.	24.046

Problem 1120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	516	0	0	0	0	0	151
normalized size	1	1.	3.11	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.325	3.25	0.038	0.	0.	0.	0.	24.739

Problem 1121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	504	0	0	0	0	0	150
normalized size	1	1.	3.07	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.327	3.229	0.051	0.	0.	0.	0.	24.847

Problem 1122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	457	0	0	0	0	0	116
normalized size	1	1.	3.31	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.345	3.432	0.044	0.	0.	0.	0.	26.19

Problem 1123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	457	0	0	0	0	0	116
normalized size	1	1.	3.31	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.308	3.365	0.035	0.	0.	0.	0.	27.929

Problem 1124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	487	0	0	0	0	0	112
normalized size	1	1.	3.66	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.179	3.594	0.021	0.	0.	0.	0.	28.802

Problem 1125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	472	0	0	0	0	0	116
normalized size	1	1.	3.47	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.273	3.57	0.036	0.	0.	0.	0.	26.31

Problem 1126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	456	0	0	0	0	0	121
normalized size	1	1.	3.3	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.276	3.184	0.045	0.	0.	0.	0.	26.125

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [327] had the largest ratio of [0.5263]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	22	0.136
2	A	3	3	1.	22	0.136
3	A	2	2	1.	22	0.091
4	A	2	2	1.	22	0.091
5	A	3	3	1.03	22	0.136
6	A	4	3	1.02	22	0.136
7	A	5	3	1.1	22	0.136
8	A	9	5	1.	16	0.312
9	A	3	3	1.	16	0.188
10	A	9	5	1.	16	0.312
11	A	3	2	1.	12	0.167

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
12	A	3	2	1.	12	0.167
13	A	9	5	1.	12	0.417
14	A	9	5	1.	12	0.417
15	A	9	5	1.	12	0.417
16	A	2	2	1.	16	0.125
17	A	2	2	1.	16	0.125
18	A	2	2	1.	16	0.125
19	A	2	2	1.	16	0.125
20	A	2	2	1.	14	0.143
21	A	1	1	1.	11	0.091
22	A	2	2	1.	16	0.125
23	A	2	2	1.	16	0.125
24	A	2	2	1.	16	0.125
25	A	2	2	1.	16	0.125
26	A	2	2	1.	16	0.125
27	A	2	2	1.	16	0.125
28	A	2	2	1.	16	0.125
29	A	2	2	1.	16	0.125
30	A	2	2	1.	16	0.125
31	A	2	2	1.	16	0.125
32	A	2	2	1.	16	0.125
33	A	2	2	1.	14	0.143
34	A	1	1	1.	11	0.091
35	A	2	2	1.	16	0.125
36	A	2	2	1.	16	0.125
37	A	2	2	1.	16	0.125
38	A	2	2	1.	16	0.125
39	A	2	2	1.	16	0.125
40	A	2	2	1.	16	0.125
41	A	2	2	1.	16	0.125
42	A	1	1	1.	16	0.062
43	A	1	1	1.	16	0.062
44	A	1	1	1.	16	0.062
45	A	1	1	1.	16	0.062

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
46	A	1	1	1.	14	0.071
47	A	1	1	1.	11	0.091
48	A	1	1	1.	16	0.062
49	A	1	1	1.	16	0.062
50	A	1	1	1.	16	0.062
51	A	1	1	1.	16	0.062
52	A	1	1	1.	16	0.062
53	A	1	1	1.	16	0.062
54	A	1	1	1.	16	0.062
55	A	1	1	1.	16	0.062
56	A	1	1	1.	16	0.062
57	A	1	1	1.	16	0.062
58	A	1	1	1.	16	0.062
59	A	1	1	1.	14	0.071
60	A	1	1	1.	11	0.091
61	A	1	1	1.	16	0.062
62	A	1	1	1.	16	0.062
63	A	1	1	1.	16	0.062
64	A	1	1	1.	16	0.062
65	A	1	1	1.	16	0.062
66	A	1	1	1.	16	0.062
67	A	1	1	1.	16	0.062
68	A	1	1	1.	16	0.062
69	A	1	1	1.	16	0.062
70	A	1	1	1.	14	0.071
71	A	1	1	1.	11	0.091
72	A	1	1	1.	16	0.062
73	A	1	1	1.	16	0.062
74	A	1	1	1.	16	0.062
75	A	1	1	1.	16	0.062
76	A	1	1	1.	16	0.062
77	A	1	1	1.	16	0.062
78	A	1	1	1.	16	0.062
79	A	1	1	1.	16	0.062

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	1	1	1.	16	0.062
81	A	1	1	1.	16	0.062
82	A	1	1	1.	16	0.062
83	A	1	1	1.	16	0.062
84	A	1	1	1.	16	0.062
85	A	1	1	1.	16	0.062
86	A	1	1	1.	14	0.071
87	A	1	1	1.	11	0.091
88	A	1	1	1.	16	0.062
89	A	1	1	1.	16	0.062
90	A	1	1	1.	16	0.062
91	A	1	1	1.	16	0.062
92	A	1	1	1.	16	0.062
93	A	1	1	1.	16	0.062
94	A	1	1	1.	16	0.062
95	A	2	2	1.	16	0.125
96	A	2	2	1.	16	0.125
97	A	2	2	1.	16	0.125
98	A	1	1	1.	16	0.062
99	A	1	1	1.	16	0.062
100	A	1	1	1.	16	0.062
101	A	1	1	1.	14	0.071
102	A	1	1	1.	11	0.091
103	A	1	1	1.	16	0.062
104	A	1	1	1.	16	0.062
105	A	1	1	1.	16	0.062
106	A	1	1	1.	16	0.062
107	A	2	2	1.	16	0.125
108	A	2	2	1.	16	0.125
109	A	2	2	1.	16	0.125
110	A	1	1	1.	16	0.062
111	A	1	1	1.	16	0.062
112	A	1	1	1.	16	0.062
113	A	1	1	1.	14	0.071

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
114	A	1	1	1.	11	0.091
115	A	1	1	1.	16	0.062
116	A	1	1	1.	16	0.062
117	A	1	1	1.	16	0.062
118	A	1	1	1.	16	0.062
119	A	2	2	1.	16	0.125
120	A	1	1	1.	16	0.062
121	A	1	1	1.	16	0.062
122	A	1	1	1.	16	0.062
123	A	1	1	1.	16	0.062
124	A	1	1	1.	14	0.071
125	A	2	2	1.	16	0.125
126	A	2	2	1.	16	0.125
127	A	2	2	1.	16	0.125
128	A	2	2	1.	16	0.125
129	A	2	2	1.	16	0.125
130	A	2	2	1.	16	0.125
131	A	2	2	1.	16	0.125
132	A	2	2	1.	16	0.125
133	A	2	2	1.	16	0.125
134	A	2	1	1.	15	0.067
135	A	2	1	1.	13	0.077
136	A	1	0	1.	11	0.
137	A	2	1	1.	15	0.067
138	A	2	1	1.	15	0.067
139	A	2	1	1.	15	0.067
140	A	2	1	1.	15	0.067
141	A	2	1	1.	15	0.067
142	A	2	1	1.	15	0.067
143	A	2	1	1.	15	0.067
144	A	2	1	1.	15	0.067
145	A	3	2	1.	13	0.154
146	A	4	3	1.	17	0.176
147	A	3	2	1.	17	0.118

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
148	A	2	2	1.	17	0.118
149	A	3	2	1.	17	0.118
150	A	4	3	1.	17	0.176
151	A	3	2	1.	17	0.118
152	A	4	3	1.	17	0.176
153	A	3	2	1.	17	0.118
154	A	4	3	1.	17	0.176
155	A	3	2	1.	17	0.118
156	A	2	2	1.	17	0.118
157	A	3	2	1.	17	0.118
158	A	3	2	1.	17	0.118
159	A	4	3	1.	17	0.176
160	A	3	2	1.	17	0.118
161	A	2	2	1.	17	0.118
162	A	3	2	1.	17	0.118
163	A	4	3	1.	17	0.176
164	A	3	2	1.	17	0.118
165	A	4	3	1.	17	0.176
166	A	3	2	1.	17	0.118
167	A	4	3	1.	17	0.176
168	A	3	2	1.	17	0.118
169	A	4	3	1.	17	0.176
170	A	3	2	1.	17	0.118
171	A	2	2	1.	17	0.118
172	A	3	2	1.	17	0.118
173	A	4	4	1.	17	0.235
174	A	4	3	1.	17	0.176
175	A	4	3	1.	17	0.176
176	A	4	3	1.	17	0.176
177	A	4	3	1.	17	0.176
178	A	4	3	1.	17	0.176
179	A	4	3	1.	17	0.176
180	A	3	3	1.	17	0.176
181	A	2	2	1.	17	0.118

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	2	2	1.	17	0.118
183	A	5	5	1.	15	0.333
184	A	3	3	1.	13	0.231
185	A	4	3	1.	17	0.176
186	A	4	3	1.	17	0.176
187	A	4	3	1.	17	0.176
188	A	5	3	1.	17	0.176
189	A	4	3	1.	17	0.176
190	A	5	4	1.	17	0.235
191	A	4	3	1.	17	0.176
192	A	5	4	1.	17	0.235
193	A	4	3	1.	17	0.176
194	A	4	4	1.	17	0.235
195	A	4	3	1.	17	0.176
196	A	3	3	1.	17	0.176
197	A	1	1	1.	17	0.059
198	A	3	3	1.	17	0.176
199	A	4	3	1.	17	0.176
200	A	4	4	1.	17	0.235
201	A	4	3	1.	15	0.2
202	A	5	4	1.	13	0.308
203	A	4	3	1.	17	0.176
204	A	6	4	1.	17	0.235
205	A	6	4	1.	17	0.235
206	A	4	3	1.	17	0.176
207	A	5	4	1.	17	0.235
208	A	4	3	1.	17	0.176
209	A	4	3	1.	17	0.176
210	A	2	2	1.	17	0.118
211	A	4	4	1.	17	0.235
212	A	1	1	1.	17	0.059
213	A	4	3	1.	17	0.176
214	A	4	3	1.	17	0.176
215	A	5	4	1.	17	0.235

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	4	3	1.	17	0.176
217	A	6	4	1.	17	0.235
218	A	4	3	1.	15	0.2
219	A	7	4	1.	13	0.308
220	A	4	3	1.	17	0.176
221	A	6	6	1.	19	0.316
222	A	5	5	1.	19	0.263
223	A	4	4	1.	17	0.235
224	A	4	4	1.	19	0.21
225	A	4	4	1.	19	0.21
226	A	1	1	1.	19	0.053
227	A	2	2	1.	19	0.105
228	A	3	2	1.	19	0.105
229	A	4	2	1.	19	0.105
230	A	5	2	1.	19	0.105
231	A	3	2	1.	19	0.105
232	A	2	2	1.	19	0.105
233	A	1	1	1.	15	0.067
234	A	3	3	1.	19	0.158
235	A	3	3	1.	19	0.158
236	A	4	4	1.	19	0.21
237	A	5	4	1.	19	0.21
238	A	6	5	1.	19	0.263
239	A	5	4	1.	17	0.235
240	A	5	5	1.	19	0.263
241	A	5	4	1.	19	0.21
242	A	5	5	1.	19	0.263
243	A	5	4	1.	19	0.21
244	A	1	1	1.	19	0.053
245	A	2	2	1.	19	0.105
246	A	3	2	1.	19	0.105
247	A	4	2	1.	19	0.105
248	A	5	2	1.	19	0.105
249	A	5	3	1.	19	0.158

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
250	A	4	3	1.	19	0.158
251	A	3	3	1.	19	0.158
252	A	2	2	1.	15	0.133
253	A	1	1	1.	19	0.053
254	A	4	3	1.	19	0.158
255	A	4	4	1.	19	0.21
256	A	4	3	1.	19	0.158
257	A	5	4	1.	19	0.21
258	A	6	4	1.	19	0.21
259	A	7	4	1.	19	0.21
260	A	6	5	1.	19	0.263
261	A	5	5	1.	19	0.263
262	A	4	4	1.	19	0.21
263	A	3	3	1.	17	0.176
264	A	1	1	1.	19	0.053
265	A	2	2	1.	19	0.105
266	A	3	2	1.	19	0.105
267	A	4	2	1.	19	0.105
268	A	2	2	1.	19	0.105
269	A	1	1	1.	19	0.053
270	A	2	2	1.	15	0.133
271	A	3	3	1.	19	0.158
272	A	4	3	1.	19	0.158
273	A	6	6	1.	19	0.316
274	A	5	5	1.	19	0.263
275	A	4	4	1.	19	0.21
276	A	1	1	1.	19	0.053
277	A	2	2	1.	17	0.118
278	A	3	3	1.	19	0.158
279	A	4	3	1.	19	0.158
280	A	5	3	1.	19	0.158
281	A	2	2	1.	19	0.105
282	A	1	1	1.	19	0.053
283	A	3	3	1.	19	0.158

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
284	A	4	4	1.	15	0.267
285	A	5	4	1.	19	0.21
286	A	4	4	1.	19	0.21
287	A	4	4	1.	19	0.21
288	A	4	4	1.	19	0.21
289	A	4	4	1.	19	0.21
290	A	4	4	1.	19	0.21
291	A	4	4	1.	20	0.2
292	A	2	1	1.	17	0.059
293	A	2	1	1.	17	0.059
294	A	2	1	1.	17	0.059
295	A	2	1	1.	17	0.059
296	A	2	1	1.	17	0.059
297	A	2	1	1.	17	0.059
298	A	2	1	1.	17	0.059
299	A	2	1	1.	17	0.059
300	A	3	2	1.	19	0.105
301	A	3	2	1.	19	0.105
302	A	3	2	1.	19	0.105
303	A	3	2	1.	19	0.105
304	A	3	2	1.	19	0.105
305	A	3	2	1.	19	0.105
306	A	3	2	1.	19	0.105
307	A	3	2	1.	19	0.105
308	A	3	2	1.	19	0.105
309	A	3	2	1.	19	0.105
310	A	3	2	1.	19	0.105
311	A	3	2	1.	19	0.105
312	A	3	2	1.	19	0.105
313	A	3	2	1.	19	0.105
314	A	3	2	1.	19	0.105
315	A	3	2	1.	19	0.105
316	A	13	9	1.	19	0.474
317	A	13	9	1.	19	0.474

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
318	A	12	9	1.	19	0.474
319	A	12	9	1.	19	0.474
320	A	11	8	1.	19	0.421
321	A	11	8	1.	19	0.421
322	A	12	9	1.	19	0.474
323	A	12	9	1.	19	0.474
324	A	13	9	1.	19	0.474
325	A	13	9	1.	19	0.474
326	A	14	9	1.	19	0.474
327	A	14	10	1.	19	0.526
328	A	13	10	1.	19	0.526
329	A	13	10	1.	19	0.526
330	A	12	9	1.	19	0.474
331	A	12	9	1.	19	0.474
332	A	12	9	1.	19	0.474
333	A	12	9	1.	19	0.474
334	A	13	10	1.	19	0.526
335	A	13	10	1.	19	0.526
336	A	14	10	1.	19	0.526
337	A	14	10	1.	19	0.526
338	A	15	10	1.	19	0.526
339	A	14	10	1.	19	0.526
340	A	13	9	1.	19	0.474
341	A	13	9	1.	19	0.474
342	A	13	10	1.	19	0.526
343	A	13	10	1.	19	0.526
344	A	13	9	1.	19	0.474
345	A	13	9	1.	19	0.474
346	A	14	10	1.	19	0.526
347	A	14	10	1.	19	0.526
348	A	15	10	1.	19	0.526
349	A	15	10	1.	19	0.526
350	A	16	10	1.	19	0.526
351	A	16	10	1.	19	0.526

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	8	7	1.	21	0.333
353	A	6	5	1.	21	0.238
354	A	7	7	1.	21	0.333
355	A	5	5	1.	21	0.238
356	A	6	6	1.	21	0.286
357	A	4	4	1.	21	0.19
358	A	6	6	1.	21	0.286
359	A	4	4	1.	21	0.19
360	A	7	7	1.	21	0.333
361	A	5	5	1.	21	0.238
362	A	8	7	1.	21	0.333
363	A	6	5	1.	21	0.238
364	A	9	7	1.	21	0.333
365	A	7	5	1.	21	0.238
366	A	8	7	1.	21	0.333
367	A	6	5	1.	21	0.238
368	A	7	6	1.	21	0.286
369	A	5	4	1.	21	0.19
370	A	7	7	1.	21	0.333
371	A	5	5	1.	21	0.238
372	A	7	6	1.	21	0.286
373	A	5	4	1.	21	0.19
374	A	8	7	1.	21	0.333
375	A	6	5	1.	21	0.238
376	A	9	7	1.	21	0.333
377	A	7	5	1.	21	0.238
378	A	6	4	1.	21	0.19
379	A	7	6	1.	21	0.286
380	A	5	4	1.	21	0.19
381	A	6	6	1.	21	0.286
382	A	4	4	1.	21	0.19
383	A	5	5	1.	21	0.238
384	A	3	3	1.	21	0.143
385	A	6	6	1.	21	0.286

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
386	A	4	4	1.	21	0.19
387	A	7	6	1.	21	0.286
388	A	5	4	1.	21	0.19
389	A	8	6	1.	21	0.286
390	A	6	4	1.	21	0.19
391	A	6	5	1.	21	0.238
392	A	7	7	1.	21	0.333
393	A	5	5	1.	21	0.238
394	A	6	6	1.	21	0.286
395	A	4	4	1.	21	0.19
396	A	6	6	1.	21	0.286
397	A	4	4	1.	21	0.19
398	A	7	7	1.	21	0.333
399	A	5	5	1.	21	0.238
400	A	8	7	1.	21	0.333
401	A	6	5	1.	21	0.238
402	A	9	7	1.	21	0.333
403	A	4	3	1.	19	0.158
404	A	4	3	1.	19	0.158
405	A	2	1	1.	17	0.059
406	A	3	3	1.	19	0.158
407	A	3	3	1.	19	0.158
408	A	3	3	1.	19	0.158
409	A	2	1	1.	22	0.045
410	A	2	1	1.	22	0.045
411	A	2	1	1.	20	0.05
412	A	1	0	1.	18	0.
413	A	2	1	1.	22	0.045
414	A	2	1	1.	22	0.045
415	A	2	1	1.	22	0.045
416	A	2	1	1.	22	0.045
417	A	2	1	1.	22	0.045
418	A	2	1	1.	22	0.045
419	A	2	1	1.	22	0.045

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
420	A	2	1	1.	22	0.045
421	A	3	2	1.	24	0.083
422	A	4	3	1.	24	0.125
423	A	3	2	1.	24	0.083
424	A	4	3	1.	24	0.125
425	A	3	2	1.	24	0.083
426	A	2	2	1.	22	0.091
427	A	3	2	1.	20	0.1
428	A	4	3	1.	24	0.125
429	A	3	2	1.	24	0.083
430	A	4	3	1.	24	0.125
431	A	3	2	1.	24	0.083
432	A	4	3	1.	24	0.125
433	A	3	2	1.	24	0.083
434	A	4	3	1.	24	0.125
435	A	3	2	1.	24	0.083
436	A	4	3	1.	24	0.125
437	A	3	2	1.	24	0.083
438	A	2	2	1.	24	0.083
439	A	3	2	1.	24	0.083
440	A	4	4	1.	24	0.167
441	A	3	2	1.	24	0.083
442	A	4	3	1.	24	0.125
443	A	3	2	1.	24	0.083
444	A	3	2	1.	24	0.083
445	A	4	3	1.	24	0.125
446	A	3	2	1.	24	0.083
447	A	4	3	1.	24	0.125
448	A	3	2	1.	24	0.083
449	A	4	3	1.	24	0.125
450	A	3	2	1.	24	0.083
451	A	2	2	1.	22	0.091
452	A	3	2	1.	20	0.1
453	A	4	3	1.	24	0.125

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
454	A	3	2	1.	24	0.083
455	A	4	3	1.	24	0.125
456	A	3	2	1.	24	0.083
457	A	4	3	1.	24	0.125
458	A	3	2	1.	24	0.083
459	A	4	3	1.	24	0.125
460	A	3	2	1.	24	0.083
461	A	4	3	1.	24	0.125
462	A	3	2	1.	24	0.083
463	A	4	3	1.	24	0.125
464	A	3	2	1.	24	0.083
465	A	4	3	1.	24	0.125
466	A	3	2	1.	24	0.083
467	A	2	2	1.	24	0.083
468	A	3	2	1.	24	0.083
469	A	4	4	1.	24	0.167
470	A	3	2	1.	24	0.083
471	A	5	4	1.	24	0.167
472	A	3	2	1.	24	0.083
473	A	6	4	1.	24	0.167
474	A	3	2	1.	24	0.083
475	A	4	3	1.	24	0.125
476	A	4	3	1.	24	0.125
477	A	4	3	1.	24	0.125
478	A	4	3	1.	24	0.125
479	A	4	3	1.	24	0.125
480	A	2	2	1.	22	0.091
481	A	4	3	1.	24	0.125
482	A	4	3	1.	24	0.125
483	A	4	3	1.	24	0.125
484	A	5	4	1.	24	0.167
485	A	5	4	1.	24	0.167
486	A	5	4	1.	24	0.167
487	A	4	4	1.	24	0.167

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
488	A	3	3	1.	24	0.125
489	A	3	3	1.	20	0.15
490	A	4	4	1.	24	0.167
491	A	5	4	1.	24	0.167
492	A	6	4	1.	24	0.167
493	A	4	3	1.	24	0.125
494	A	4	3	1.	24	0.125
495	A	4	3	1.	24	0.125
496	A	2	2	1.	24	0.083
497	A	4	3	1.	24	0.125
498	A	2	2	1.	22	0.091
499	A	4	3	1.	24	0.125
500	A	4	3	1.	24	0.125
501	A	4	3	1.	24	0.125
502	A	7	4	1.	24	0.167
503	A	7	4	1.	24	0.167
504	A	6	4	1.	24	0.167
505	A	5	3	1.	24	0.125
506	A	5	4	1.	24	0.167
507	A	5	4	1.	24	0.167
508	A	5	3	1.	20	0.15
509	A	6	4	1.	24	0.167
510	A	7	4	1.	24	0.167
511	A	8	4	1.	24	0.167
512	A	4	3	1.	24	0.125
513	A	4	3	1.	24	0.125
514	A	4	3	1.	24	0.125
515	A	2	2	1.	24	0.083
516	A	4	4	1.	24	0.167
517	A	4	3	1.	24	0.125
518	A	4	3	1.	24	0.125
519	A	2	2	1.	22	0.091
520	A	4	3	1.	24	0.125
521	A	4	3	1.	24	0.125

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
522	A	4	3	1.	24	0.125
523	A	9	4	1.	24	0.167
524	A	9	4	1.	24	0.167
525	A	8	4	1.	24	0.167
526	A	7	3	1.	24	0.125
527	A	7	4	1.	24	0.167
528	A	7	4	1.	24	0.167
529	A	7	4	1.	24	0.167
530	A	7	4	1.	24	0.167
531	A	7	3	1.	20	0.15
532	A	8	4	1.	24	0.167
533	A	9	4	1.	24	0.167
534	A	10	4	1.	24	0.167
535	A	3	3	1.	12	0.25
536	A	2	2	1.	14	0.143
537	A	3	3	1.	16	0.188
538	A	4	3	1.	16	0.188
539	A	2	2	1.	14	0.143
540	A	4	3	1.	16	0.188
541	A	4	3	1.	26	0.115
542	A	3	3	1.	26	0.115
543	A	2	2	1.	24	0.083
544	A	3	2	1.	26	0.077
545	A	3	2	1.	26	0.077
546	A	3	3	1.	26	0.115
547	A	1	1	1.	26	0.038
548	A	4	3	1.	26	0.115
549	A	4	3	1.	26	0.115
550	A	3	2	1.	26	0.077
551	A	3	2	1.	26	0.077
552	A	2	1	1.	22	0.045
553	A	3	2	1.	26	0.077
554	A	3	2	1.	26	0.077
555	A	3	2	1.	26	0.077

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
556	A	3	2	1.	26	0.077
557	A	3	2	1.	26	0.077
558	A	4	3	1.	26	0.115
559	A	4	3	1.	26	0.115
560	A	3	2	1.12	26	0.077
561	A	3	3	1.	26	0.115
562	A	2	2	1.	24	0.083
563	A	4	3	1.	26	0.115
564	A	4	3	1.	26	0.115
565	A	4	3	1.	26	0.115
566	A	4	3	1.	26	0.115
567	A	3	3	1.	26	0.115
568	A	1	1	1.	26	0.038
569	A	4	3	1.	26	0.115
570	A	4	3	1.	26	0.115
571	A	4	3	1.	26	0.115
572	A	3	2	1.	26	0.077
573	A	3	2	1.	26	0.077
574	A	3	2	1.	26	0.077
575	A	3	2	1.	26	0.077
576	A	3	2	1.	22	0.091
577	A	3	2	1.	26	0.077
578	A	3	2	1.	26	0.077
579	A	3	2	1.	26	0.077
580	A	3	2	1.	26	0.077
581	A	3	2	1.	26	0.077
582	A	3	2	1.	26	0.077
583	A	3	2	1.	26	0.077
584	A	3	2	1.	26	0.077
585	A	4	3	1.	26	0.115
586	A	4	3	1.	26	0.115
587	A	3	2	1.	26	0.077
588	A	4	3	1.	26	0.115
589	A	4	3	1.	26	0.115

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
590	A	3	3	1.	26	0.115
591	A	2	2	1.	24	0.083
592	A	4	3	1.	26	0.115
593	A	4	3	1.	26	0.115
594	A	4	3	1.	26	0.115
595	A	4	3	1.	26	0.115
596	A	4	3	1.	26	0.115
597	A	4	3	1.	26	0.115
598	A	3	3	1.	26	0.115
599	A	1	1	1.	26	0.038
600	A	5	4	1.	26	0.154
601	A	4	3	1.	26	0.115
602	A	4	3	1.	26	0.115
603	A	4	3	1.	26	0.115
604	A	4	3	1.	26	0.115
605	A	3	2	1.	26	0.077
606	A	3	2	1.	26	0.077
607	A	3	2	1.	26	0.077
608	A	3	2	1.	26	0.077
609	A	3	2	1.	26	0.077
610	A	3	2	1.	26	0.077
611	A	3	2	1.	22	0.091
612	A	3	2	1.	26	0.077
613	A	3	2	1.	26	0.077
614	A	3	2	1.	26	0.077
615	A	3	2	1.	26	0.077
616	A	3	2	1.	26	0.077
617	A	3	2	1.	26	0.077
618	A	3	2	1.	26	0.077
619	A	3	2	1.	26	0.077
620	A	3	2	1.	26	0.077
621	A	3	2	1.	26	0.077
622	A	3	2	1.	26	0.077
623	A	3	2	1.	26	0.077

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
624	A	4	3	1.	26	0.115
625	A	4	4	1.	26	0.154
626	A	3	3	1.	24	0.125
627	A	5	5	1.	26	0.192
628	A	4	3	1.	26	0.115
629	A	4	3	1.	26	0.115
630	A	3	3	1.	26	0.115
631	A	2	2	1.	22	0.091
632	A	3	3	1.	26	0.115
633	A	4	3	1.	26	0.115
634	A	4	3	1.	26	0.115
635	A	4	3	1.	26	0.115
636	A	3	3	1.68	26	0.115
637	A	2	2	1.	24	0.083
638	A	4	3	1.	26	0.115
639	A	4	3	1.	26	0.115
640	A	4	3	1.	26	0.115
641	A	4	4	1.	26	0.154
642	A	4	3	1.	22	0.136
643	A	5	4	1.	26	0.154
644	A	6	4	1.	26	0.154
645	A	4	3	1.	26	0.115
646	A	4	3	1.	26	0.115
647	A	3	3	1.	26	0.115
648	A	1	1	1.	26	0.038
649	A	3	3	1.	26	0.115
650	A	2	2	1.	24	0.083
651	A	4	3	1.	26	0.115
652	A	4	3	1.	26	0.115
653	A	6	4	1.	26	0.154
654	A	6	4	1.	26	0.154
655	A	6	4	1.	26	0.154
656	A	6	3	1.	22	0.136
657	A	7	4	1.	26	0.154

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
658	A	8	4	1.	26	0.154
659	A	4	4	1.	26	0.154
660	A	3	3	1.	22	0.136
661	A	4	4	1.	26	0.154
662	A	6	6	1.	26	0.231
663	A	6	6	1.	22	0.273
664	A	7	7	1.	26	0.269
665	A	2	1	1.	26	0.038
666	A	2	1	1.	26	0.038
667	A	2	1	1.	26	0.038
668	A	2	1	1.	26	0.038
669	A	2	1	1.	26	0.038
670	A	2	1	1.	26	0.038
671	A	2	1	1.	26	0.038
672	A	3	2	1.	28	0.071
673	A	3	2	1.	28	0.071
674	A	3	2	1.	28	0.071
675	A	3	2	1.	28	0.071
676	A	3	2	1.	28	0.071
677	A	3	2	1.	28	0.071
678	A	3	2	1.	28	0.071
679	A	3	2	1.	28	0.071
680	A	3	2	1.	28	0.071
681	A	3	2	1.	28	0.071
682	A	3	2	1.	28	0.071
683	A	3	2	1.	28	0.071
684	A	3	2	1.	28	0.071
685	A	3	2	1.	28	0.071
686	A	14	10	1.	28	0.357
687	A	13	10	1.	28	0.357
688	A	13	10	1.	28	0.357
689	A	12	9	1.	28	0.321
690	A	12	9	1.	28	0.321
691	A	12	9	1.	28	0.321

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
692	A	12	9	1.	28	0.321
693	A	13	10	1.	28	0.357
694	A	13	10	1.	28	0.357
695	A	14	10	1.	28	0.357
696	A	16	10	1.	28	0.357
697	A	15	10	1.	28	0.357
698	A	15	10	1.	28	0.357
699	A	14	9	1.	28	0.321
700	A	14	9	1.	28	0.321
701	A	14	10	1.	28	0.357
702	A	14	10	1.	28	0.357
703	A	14	10	1.	28	0.357
704	A	14	10	1.	28	0.357
705	A	14	9	1.	28	0.321
706	A	14	9	1.	28	0.321
707	A	15	10	1.	28	0.357
708	A	15	10	1.	28	0.357
709	A	16	10	1.	28	0.357
710	A	18	10	1.	28	0.357
711	A	17	10	1.	28	0.357
712	A	17	10	1.	28	0.357
713	A	16	9	1.	28	0.321
714	A	16	9	1.	28	0.321
715	A	16	10	1.	28	0.357
716	A	16	10	1.	28	0.357
717	A	16	10	1.	28	0.357
718	A	16	10	1.	28	0.357
719	A	16	10	1.	28	0.357
720	A	16	10	1.	28	0.357
721	A	16	10	1.	28	0.357
722	A	16	10	1.	28	0.357
723	A	16	9	1.	28	0.321
724	A	16	9	1.	28	0.321
725	A	17	10	1.	28	0.357

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	17	10	1.	28	0.357
727	A	18	10	1.	28	0.357
728	A	3	2	1.	30	0.067
729	A	3	2	1.	30	0.067
730	A	3	2	1.	30	0.067
731	A	3	2	1.	30	0.067
732	A	3	2	1.	30	0.067
733	A	3	2	1.	30	0.067
734	A	3	2	1.	30	0.067
735	A	3	2	1.	30	0.067
736	A	3	2	1.	30	0.067
737	A	3	2	1.	30	0.067
738	A	3	2	1.	30	0.067
739	A	3	2	1.	30	0.067
740	A	3	2	1.	30	0.067
741	A	3	2	1.	30	0.067
742	A	3	2	1.	30	0.067
743	A	3	2	1.	30	0.067
744	A	3	2	1.	30	0.067
745	A	3	2	1.	30	0.067
746	A	3	2	1.	30	0.067
747	A	3	2	1.	30	0.067
748	A	3	2	1.	30	0.067
749	A	13	9	1.	30	0.3
750	A	12	9	1.	30	0.3
751	A	12	9	1.	30	0.3
752	A	11	8	1.	30	0.267
753	A	11	8	1.	30	0.267
754	A	12	9	1.	30	0.3
755	A	12	9	1.	30	0.3
756	A	13	9	1.	30	0.3
757	A	15	10	1.	30	0.333
758	A	14	10	1.	30	0.333
759	A	14	10	1.	30	0.333

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
760	A	13	9	1.	30	0.3
761	A	13	9	1.	30	0.3
762	A	13	10	1.	30	0.333
763	A	13	10	1.	30	0.333
764	A	13	9	1.	30	0.3
765	A	13	9	1.	30	0.3
766	A	14	10	1.	30	0.333
767	A	14	10	1.	30	0.333
768	A	15	10	1.	30	0.333
769	A	17	10	1.	30	0.333
770	A	16	10	1.	30	0.333
771	A	16	10	1.	30	0.333
772	A	15	9	1.	30	0.3
773	A	15	9	1.	30	0.3
774	A	15	10	1.	30	0.333
775	A	15	10	1.	30	0.333
776	A	15	10	1.	30	0.333
777	A	15	10	1.	30	0.333
778	A	15	10	1.	30	0.333
779	A	15	10	1.	30	0.333
780	A	15	9	1.	30	0.3
781	A	15	9	1.	30	0.3
782	A	16	10	1.	30	0.333
783	A	16	10	1.	30	0.333
784	A	17	10	1.	30	0.333
785	A	3	2	1.	26	0.077
786	A	3	2	1.	26	0.077
787	A	2	1	1.	24	0.042
788	A	2	2	1.	26	0.077
789	A	2	2	1.	26	0.077
790	A	2	2	1.	26	0.077
791	A	3	2	1.	28	0.071
792	A	3	2	1.	28	0.071
793	A	3	2	1.	28	0.071

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
794	A	2	2	1.	28	0.071
795	A	2	2	1.	28	0.071
796	A	2	2	1.	28	0.071
797	A	2	2	1.04	26	0.077
798	A	4	3	1.	24	0.125
799	A	4	3	1.	24	0.125
800	A	4	3	1.	24	0.125
801	A	2	2	1.	22	0.091
802	A	3	3	1.	24	0.125
803	A	3	3	1.	24	0.125
804	A	2	2	1.	24	0.083
805	A	2	2	1.	24	0.083
806	A	2	2	1.	20	0.1
807	A	2	2	1.	24	0.083
808	A	2	2	1.	24	0.083
809	A	2	2	1.	28	0.071
810	A	2	2	1.	28	0.071
811	A	2	2	1.	28	0.071
812	A	2	2	1.	28	0.071
813	A	2	2	1.	28	0.071
814	A	2	1	1.	16	0.062
815	A	2	1	1.	14	0.071
816	A	1	0	1.	12	0.
817	A	2	1	1.	16	0.062
818	A	2	1	1.	16	0.062
819	A	2	1	1.	16	0.062
820	A	2	1	1.	16	0.062
821	A	2	1	1.	16	0.062
822	A	2	1	1.	16	0.062
823	A	2	1	1.	16	0.062
824	A	2	1	1.	16	0.062
825	A	2	1	1.	18	0.056
826	A	3	2	1.	16	0.125
827	A	2	1	1.	14	0.071

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
828	A	3	2	1.	18	0.111
829	A	2	1	1.	18	0.056
830	A	3	2	1.	18	0.111
831	A	2	1	1.	18	0.056
832	A	3	2	1.	18	0.111
833	A	2	1	1.	18	0.056
834	A	3	2	1.	18	0.111
835	A	2	1	1.	18	0.056
836	A	3	2	1.	18	0.111
837	A	2	1	1.	18	0.056
838	A	3	2	1.	18	0.111
839	A	2	1	1.	18	0.056
840	A	3	2	1.	18	0.111
841	A	2	1	1.	18	0.056
842	A	3	2	1.	16	0.125
843	A	2	1	1.	14	0.071
844	A	3	2	1.	18	0.111
845	A	2	1	1.	18	0.056
846	A	3	2	1.	18	0.111
847	A	2	1	1.	18	0.056
848	A	7	6	1.	18	0.333
849	A	6	6	1.	18	0.333
850	A	5	5	1.	18	0.278
851	A	3	3	1.	16	0.188
852	A	7	7	1.	18	0.389
853	A	8	7	1.	18	0.389
854	A	8	7	1.	18	0.389
855	A	5	4	1.	18	0.222
856	A	4	3	1.	18	0.167
857	A	3	2	1.	18	0.111
858	A	3	2	1.	14	0.143
859	A	4	3	1.	18	0.167
860	A	5	4	1.	18	0.222
861	A	7	7	1.	18	0.389

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
862	A	4	4	1.	18	0.222
863	A	4	4	1.	18	0.222
864	A	4	4	1.	16	0.25
865	A	8	7	1.	18	0.389
866	A	8	7	1.	18	0.389
867	A	6	4	1.	18	0.222
868	A	5	4	1.	18	0.222
869	A	4	3	1.	18	0.167
870	A	4	3	1.	18	0.167
871	A	4	3	1.	14	0.214
872	A	5	4	1.	18	0.222
873	A	8	8	1.	18	0.444
874	A	5	4	1.	18	0.222
875	A	5	5	1.	18	0.278
876	A	5	5	1.	18	0.278
877	A	5	5	1.	18	0.278
878	A	5	4	1.	16	0.25
879	A	9	8	1.	18	0.444
880	A	9	8	1.	18	0.444
881	A	7	5	1.	18	0.278
882	A	6	5	1.	18	0.278
883	A	5	4	1.	18	0.222
884	A	5	4	1.	18	0.222
885	A	5	4	1.	18	0.222
886	A	5	4	1.	14	0.286
887	A	6	5	1.	18	0.278
888	A	6	6	1.	19	0.316
889	A	5	5	1.	19	0.263
890	A	3	3	1.	17	0.176
891	A	7	7	1.	19	0.368
892	A	8	7	1.	19	0.368
893	A	4	3	1.	19	0.158
894	A	3	2	1.	19	0.105
895	A	3	2	1.	15	0.133

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
896	A	4	3	1.	19	0.158
897	A	6	6	1.	22	0.273
898	A	5	5	1.	22	0.227
899	A	3	3	1.	20	0.15
900	A	7	7	1.	22	0.318
901	A	8	7	1.	22	0.318
902	A	4	3	1.	22	0.136
903	A	3	2	1.	22	0.091
904	A	3	2	1.	18	0.111
905	A	4	3	1.	22	0.136
906	A	6	6	1.	20	0.3
907	A	5	5	1.	20	0.25
908	A	3	3	1.	18	0.167
909	A	7	7	1.	20	0.35
910	A	8	7	1.	20	0.35
911	A	10	6	1.	20	0.3
912	A	9	5	1.	20	0.25
913	A	9	5	1.	16	0.312
914	A	10	6	1.	20	0.3
915	A	3	3	1.	12	0.25
916	A	3	3	1.	14	0.214
917	A	3	2	1.	16	0.125
918	A	9	6	1.	16	0.375
919	A	9	6	1.	16	0.375
920	A	6	6	1.	20	0.3
921	A	6	6	1.	20	0.3
922	A	5	5	1.	20	0.25
923	A	4	4	1.	18	0.222
924	A	7	6	1.	20	0.3
925	A	7	6	1.	20	0.3
926	A	4	4	1.	20	0.2
927	A	5	5	1.	20	0.25
928	A	6	6	1.	20	0.3
929	A	7	7	1.	20	0.35

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
930	A	5	5	1.	20	0.25
931	A	4	4	1.	20	0.2
932	A	4	4	1.	16	0.25
933	A	4	4	1.	20	0.2
934	A	5	5	1.	20	0.25
935	A	6	5	1.	20	0.25
936	A	7	6	1.	20	0.3
937	A	7	6	1.	20	0.3
938	A	6	5	1.	20	0.25
939	A	5	4	1.	18	0.222
940	A	8	7	1.	20	0.35
941	A	8	7	1.	20	0.35
942	A	8	7	1.	20	0.35
943	A	8	7	1.	20	0.35
944	A	5	4	1.	20	0.2
945	A	6	5	1.	20	0.25
946	A	7	6	1.	20	0.3
947	A	6	6	1.	20	0.3
948	A	5	5	1.	20	0.25
949	A	5	5	1.	16	0.312
950	A	5	5	1.	20	0.25
951	A	5	5	1.	20	0.25
952	A	6	6	1.	20	0.3
953	A	7	6	1.	20	0.3
954	A	5	5	1.	16	0.312
955	A	5	5	1.	20	0.25
956	A	5	5	1.	20	0.25
957	A	4	4	1.	20	0.2
958	A	3	3	1.	18	0.167
959	A	3	3	1.	20	0.15
960	A	4	4	1.	20	0.2
961	A	5	5	1.	20	0.25
962	A	6	6	1.	20	0.3
963	A	4	4	1.	20	0.2

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
964	A	3	3	1.	20	0.15
965	A	1	1	1.	16	0.062
966	A	5	5	1.	20	0.25
967	A	5	5	1.	20	0.25
968	A	5	5	1.	21	0.238
969	A	5	5	1.	21	0.238
970	A	4	4	1.	21	0.19
971	A	3	3	1.	19	0.158
972	A	3	3	1.	22	0.136
973	A	4	4	1.	22	0.182
974	A	5	5	1.	22	0.227
975	A	6	6	1.	22	0.273
976	A	5	5	1.	21	0.238
977	A	4	4	1.	21	0.19
978	A	2	2	1.	17	0.118
979	A	6	6	1.	21	0.286
980	A	6	6	1.	21	0.286
981	A	6	6	1.	20	0.3
982	A	5	5	1.	20	0.25
983	A	5	5	1.	20	0.25
984	A	2	2	1.	20	0.1
985	A	2	2	1.	18	0.111
986	A	5	5	1.	20	0.25
987	A	5	5	1.	20	0.25
988	A	6	6	1.	20	0.3
989	A	5	5	1.	20	0.25
990	A	4	4	1.	20	0.2
991	A	4	4	1.	20	0.2
992	A	4	4	1.	16	0.25
993	A	5	5	1.	20	0.25
994	A	3	3	1.	28	0.107
995	A	5	5	1.	28	0.179
996	A	2	2	1.	28	0.071
997	A	4	4	1.	26	0.154

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
998	A	3	3	1.	24	0.125
999	A	2	2	1.	28	0.071
1000	A	4	4	1.	28	0.143
1001	A	3	3	1.	28	0.107
1002	A	5	4	1.	28	0.143
1003	A	3	3	1.	29	0.103
1004	A	2	2	1.	29	0.069
1005	A	4	4	1.	29	0.138
1006	A	4	4	1.	27	0.148
1007	A	2	2	1.	25	0.08
1008	A	4	4	1.	29	0.138
1009	A	5	5	1.	29	0.172
1010	A	2	2	1.	29	0.069
1011	A	3	3	1.	29	0.103
1012	A	5	4	1.	29	0.138
1013	A	4	3	1.	29	0.103
1014	A	4	4	1.	29	0.138
1015	A	2	2	1.	27	0.074
1016	A	3	3	1.	25	0.12
1017	A	4	4	1.	29	0.138
1018	A	2	2	1.	29	0.069
1019	A	5	5	1.	29	0.172
1020	A	3	3	1.	29	0.103
1021	A	3	3	1.	23	0.13
1022	A	3	3	1.	23	0.13
1023	A	3	3	1.	23	0.13
1024	A	3	3	1.	21	0.143
1025	A	3	3	1.	19	0.158
1026	A	3	3	1.	23	0.13
1027	A	3	3	1.	23	0.13
1028	A	3	3	1.	23	0.13
1029	A	3	3	1.	23	0.13
1030	A	3	3	1.	24	0.125
1031	A	3	3	1.	24	0.125

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1032	A	3	3	1.	24	0.125
1033	A	3	3	1.	22	0.136
1034	A	2	2	1.	20	0.1
1035	A	3	3	1.	24	0.125
1036	A	3	3	1.	24	0.125
1037	A	3	3	1.	24	0.125
1038	A	3	3	1.	24	0.125
1039	A	2	2	1.	16	0.125
1040	A	2	2	1.	16	0.125
1041	A	2	1	1.	18	0.056
1042	A	2	1	1.	18	0.056
1043	A	2	1	1.	18	0.056
1044	A	2	1	1.	18	0.056
1045	A	2	1	1.	18	0.056
1046	A	2	1	1.	18	0.056
1047	A	2	1	1.	18	0.056
1048	A	2	1	1.	20	0.05
1049	A	2	1	1.	20	0.05
1050	A	2	1	1.	20	0.05
1051	A	2	1	1.	20	0.05
1052	A	2	1	1.	20	0.05
1053	A	2	1	1.	20	0.05
1054	A	2	1	1.	20	0.05
1055	A	2	1	1.	20	0.05
1056	A	2	1	1.	20	0.05
1057	A	2	1	1.	20	0.05
1058	A	2	1	1.	20	0.05
1059	A	2	1	1.	20	0.05
1060	A	2	1	1.	20	0.05
1061	A	2	1	1.	20	0.05
1062	A	9	6	1.	20	0.3
1063	A	9	6	1.	20	0.3
1064	A	8	5	1.	20	0.25
1065	A	8	5	1.	20	0.25

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1066	A	8	5	1.	20	0.25
1067	A	8	5	1.	20	0.25
1068	A	9	6	1.	20	0.3
1069	A	9	6	1.	20	0.3
1070	A	10	7	1.	20	0.35
1071	A	10	7	1.	20	0.35
1072	A	10	7	1.	20	0.35
1073	A	9	6	1.	20	0.3
1074	A	9	6	1.	20	0.3
1075	A	9	6	1.	20	0.3
1076	A	9	6	1.	20	0.3
1077	A	9	6	1.	20	0.3
1078	A	9	6	1.	20	0.3
1079	A	10	7	1.	20	0.35
1080	A	11	8	1.	20	0.4
1081	A	10	7	1.	20	0.35
1082	A	10	7	1.	20	0.35
1083	A	10	7	1.	20	0.35
1084	A	10	7	1.	20	0.35
1085	A	10	7	1.	20	0.35
1086	A	10	7	1.	20	0.35
1087	A	10	7	1.	20	0.35
1088	A	10	7	1.	20	0.35
1089	A	2	2	1.	24	0.083
1090	A	2	2	1.	24	0.083
1091	A	2	2	1.	24	0.083
1092	A	2	2	1.	24	0.083
1093	A	2	2	1.	24	0.083
1094	A	2	2	1.	24	0.083
1095	A	2	2	1.	24	0.083
1096	A	2	2	1.	24	0.083
1097	A	2	2	1.	24	0.083
1098	A	2	2	1.	24	0.083
1099	A	2	2	1.	24	0.083

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1100	A	2	2	1.	24	0.083
1101	A	2	2	1.	24	0.083
1102	A	2	2	1.	24	0.083
1103	A	2	2	1.	24	0.083
1104	A	2	2	1.	24	0.083
1105	A	2	1	1.	20	0.05
1106	A	2	1	1.	20	0.05
1107	A	2	1	1.	18	0.056
1108	A	3	2	1.	20	0.1
1109	A	4	3	1.	20	0.15
1110	A	2	2	1.	22	0.091
1111	A	2	2	1.	22	0.091
1112	A	2	2	1.	22	0.091
1113	A	2	2	1.	22	0.091
1114	A	2	2	1.	20	0.1
1115	A	4	4	1.	18	0.222
1116	A	4	4	1.	18	0.222
1117	A	3	3	1.	18	0.167
1118	A	2	2	1.	16	0.125
1119	A	3	3	1.	18	0.167
1120	A	3	3	1.	18	0.167
1121	A	3	3	1.	18	0.167
1122	A	2	2	1.	18	0.111
1123	A	2	2	1.	18	0.111
1124	A	2	2	1.	14	0.143
1125	A	2	2	1.	18	0.111
1126	A	2	2	1.	18	0.111

3 Listing of integrals

3.1 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/4}\,dx$

Optimal. Leaf size=128

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b
*x^2)/a)^(3/2))
```

Rubi [A] time = 0.0843731, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{3/4}}{8\left(a+bx^2\right)}+\frac{1}{4}x\left(a^2+2abx^2+b^2x^4\right)^{3/4}+\frac{3\sqrt{a}\left(a^2+2abx^2+b^2x^4\right)^{3/4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))/4 + (3*a*x*(a^2 + 2*a*b*x^2
 + b^2*x^4)^(3/4))/(8*(a + b*x^2)) + (3*Sqrt[a]*(a^2 + 2*a*b*x^2
+ b^2*x^4)^(3/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[b]*(1 + (b
*x^2)/a)^(3/2))
```

Rubi in Sympy [A] time = 24.6035, size = 124, normalized size = 0.97

$$\frac{3a^2b\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{bx}{\sqrt{ab+b^2x^2}}\right)}{8\left(ab+b^2x^2\right)^{\frac{3}{2}}}+\frac{3ax\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}}{8\left(a+bx^2\right)}+\frac{x\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] 3*a**2*b*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4)*atanh(b*x/sqrt(a*
b + b**2*x**2))/(8*(a*b + b**2*x**2)**(3/2)) + 3*a*x*(a**2 + 2*a*
b*x**2 + b**2*x**4)**(3/4)/(8*(a + b*x**2)) + x*(a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/4)/4
```

Mathematica [A] time = 0.0785485, size = 89, normalized size = 0.7

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/4}\left(3a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)+\sqrt{b}x\sqrt{a+bx^2}\left(5a+2bx^2\right)\right)}{8\sqrt{b}\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4),x]
```

```
[Out] (((a + b*x^2)^2)^(3/4)*(Sqrt[b]*x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2)
 + 3*a^2*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(8*Sqrt[b]*(a + b*x
^2)^(3/2))
```

Maple [A] time = 0.056, size = 77, normalized size = 0.6

$$\frac{x\left(2\,bx^2+5\,a\right)\left(bx^2+a\right)}{8}\frac{1}{\sqrt[4]{\left(bx^2+a\right)^2}}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\sqrt{bx^2+a}\frac{1}{\sqrt{b}}\frac{1}{\sqrt[4]{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] 1/8*x*(2*b*x^2+5*a)*(b*x^2+a)/((b*x^2+a)^2)^(1/4)+3/8*a^2*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))/b^(1/2)/((b*x^2+a)^2)^(1/4)*(b*x^2+a)^(1/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290218, size = 1, normalized size = 0.01

$$\left[\frac{3\,a^2\sqrt{b}\log\left(-2\,\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\,\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,b^2x^3+5\,abx\right)}{16\,b},\right.$$
$$\left.-\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\right)-\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,b^2x^3+5\,abx\right)}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*sqrt(b)*log(-2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x
 - (2*b*x^2 + a)*sqrt(b)) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(
2*b^2*x^3 + 5*a*b*x))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(-b)*x/(
b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(
1/4)*(2*b^2*x^3 + 5*a*b*x))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4), x)
```

GIAC/XCAS [A] time = 0.284736, size = 117, normalized size = 0.91

$$-\frac{1}{8}\left(\frac{x^4\left(\frac{5\sqrt{-bx^2-a}\left(b+\frac{a}{x^2}\right)|x|}{x^2}-\frac{3\sqrt{-bx^2-a}b|x|}{x^2}\right)}{a^2}+\frac{3\arctan\left(\frac{\sqrt{-bx^2-a}|x|}{\sqrt{b}x^2}\right)}{\sqrt{b}}\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/4),x, algorithm="giac")
```

```
[Out] -1/8*(x^4*(5*sqrt(-b*x^2 - a)*(b + a/x^2)*abs(x)/x^2 - 3*sqrt(-b*
x^2 - a)*b*abs(x)/x^2)/a^2 + 3*arctan(sqrt(-b*x^2 - a)*abs(x)/(sq
rt(b)*x^2))/sqrt(b))*a^2
```

3.2 $\int \sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x
^2 + b^2*x^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt
[1 + (b*x^2)/a])
```

Rubi [A] time = 0.0578536, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{1}{2}x\sqrt[4]{a^2+2abx^2+b^2x^4}+\frac{\sqrt{a}\sqrt[4]{a^2+2abx^2+b^2x^4}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))/2 + (Sqrt[a]*(a^2 + 2*a*b*x
^2 + b^2*x^4)^(1/4)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[b]*Sqrt
[1 + (b*x^2)/a])
```

Rubi in Sympy [A] time = 21.9298, size = 99, normalized size = 1.09

$$\frac{\sqrt{2}a\sqrt[4]{a^2+2abx^2+b^2x^4}\operatorname{atanh}\left(\frac{\sqrt{2}bx}{\sqrt{2ab+2b^2x^2}}\right)}{2\sqrt{2ab+2b^2x^2}}+\frac{x\sqrt[4]{a^2+2abx^2+b^2x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] sqrt(2)*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)*atanh(sqrt(2)*b*
x/sqrt(2*a*b + 2*b**2*x**2))/(2*sqrt(2*a*b + 2*b**2*x**2)) + x*(a
**2 + 2*a*b*x**2 + b**2*x**4)**(1/4)/2
```

Mathematica [A] time = 0.0577432, size = 59, normalized size = 0.65

$$\frac{1}{2}\sqrt[4]{(a+bx^2)^2}\left(\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{\sqrt{b}\sqrt{a+bx^2}}+x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4),x]
```

```
[Out] (((a + b*x^2)^2)^(1/4)*(x + (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]
)/(Sqrt[b]*Sqrt[a + b*x^2])))/2
```

Maple [A] time = 0.02, size = 58, normalized size = 0.6

$$\frac{x}{2}\sqrt[4]{(bx^2+a)^2}+\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\sqrt[4]{(bx^2+a)^2}\frac{1}{\sqrt{b}}\frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

```
[Out] 1/2*x*((b*x^2+a)^2)^(1/4)+1/2*a*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(
1/2)*((b*x^2+a)^2)^(1/4)/(b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287826, size = 1, normalized size = 0.01

$$\left[\frac{a\sqrt{b}\log\left(-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx}{4\,b},\right.$$
$$\left.-\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\right)-\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x - (2
*b*x^2 + a)*sqrt(b)) + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b
, -1/2*(a*sqrt(-b)*arctan(sqrt(-b)*x/(b^2*x^4 + 2*a*b*x^2 + a^2)^
(1/4)) - (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x)/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt[4]{a^2+2abx^2+b^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4), x)
```

3.3 $\int \frac{1}{\sqrt[4]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi [A] time = 0.0354016, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a]*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
b]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi in Sympy [A] time = 19.9581, size = 54, normalized size = 0.9

$$\frac{b\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{bx}{\sqrt{ab+b^2x^2}}\right)}{\left(ab+b^2x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] b*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4)*atanh(b*x/sqrt(a*b + b**
2*x**2))/(a*b + b**2*x**2)**(3/2)
```

Mathematica [A] time = 0.0258873, size = 49, normalized size = 0.82

$$\frac{\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt[4]{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/4),x]
```

```
[Out] (Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(Sqrt[b]*(
(a + b*x^2)^2)^(1/4))
```

Maple [F] time = 0.011, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284997, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-2\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)}{2\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*b*x - (2*b*x^2 + a)
*sqrt(b))/sqrt(b), -sqrt(-b)*arctan(sqrt(-b)*x/(b^2*x^4 + 2*a*b*x
^2 + a^2)^(1/4))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a^2 + 2abx^2 + b^2x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/4), x)
```

GIAC/XCAS [A] time = 0.281421, size = 36, normalized size = 0.6

$$-\frac{\arctan\left(\frac{\sqrt{-bx^2-a}\,|x|}{\sqrt{b}x^2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/4),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(-b*x^2 - a)*abs(x)/(sqrt(b)*x^2))/sqrt(b)
```

3.4 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/4}}\,dx$

Optimal. Leaf size=34

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

```
[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rubi [A] time = 0.0234605, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x\left(a+bx^2\right)}{a\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

```
[Out] (x*(a + b*x^2))/(a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rubi in Sympy [A] time = 15.4225, size = 36, normalized size = 1.06

$$\frac{x\left(2a+2bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] x*(2*a + 2*b*x**2)/(2*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4))
```

Mathematica [A] time = 0.0244486, size = 25, normalized size = 0.74

$$\frac{x\left(a+bx^2\right)}{a\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/4),x]
```

```
[Out] (x*(a + b*x^2))/(a*((a + b*x^2)^2)^(3/4))
```

Maple [A] time = 0.004, size = 33, normalized size = 1.

$$\frac{x\left(bx^2+a\right)}{a}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/4),x)
```

```
[Out] x*(b*x^2+a)/a/(b^2*x^4+2*a*b*x^2+a^2)^(3/4)
```

Maxima [A] time = 0.708752, size = 19, normalized size = 0.56

$$\frac{x}{\sqrt{bx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4),x, algorithm="maxima")
```

```
[Out] x/(sqrt(b*x^2 + a)*a)
```

Fricas [A] time = 0.273089, size = 46, normalized size = 1.35

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}x}{abx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4),x, algorithm="fricas")
```

```
[Out] (b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*x/(a*b*x^2 + a^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/4), x)
```

GIAC/XCAS [A] time = 0.280429, size = 32, normalized size = 0.94

$$-\frac{x^2}{\sqrt{-bx^2 - a}a|x|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/4),x, algorithm="giac")
```

```
[Out] -x^2/(sqrt(-b*x^2 - a)*a*abs(x))
```

3.5 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/4}}\,dx$

Optimal. Leaf size=68

$$\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x\left(a+bx^2\right)}{3a\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

```
[Out] (x*(a + b*x^2))/(3*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/4)) + (2*x)/(
3*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi [A] time = 0.0427325, antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x}{3a\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (2*x)/(3*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(3*a*(a + b*x
^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi in Sympy [A] time = 16.0683, size = 66, normalized size = 0.97

$$\frac{x\left(2a+2bx^2\right)}{6a\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{4}}}+\frac{2x}{3a^2\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)
```

```
[Out] x*(2*a + 2*b*x**2)/(6*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/4)) +
 2*x/(3*a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4))
```

Mathematica [A] time = 0.0281799, size = 40, normalized size = 0.59

$$\frac{x\left(3a+2bx^2\right)}{3a^2\left(a+bx^2\right)\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/4),x]
```

```
[Out] (x*(3*a + 2*b*x^2))/(3*a^2*(a + b*x^2)*((a + b*x^2)^2)^(1/4))
```

Maple [A] time = 0.004, size = 44, normalized size = 0.7

$$\frac{\left(bx^2+a\right)x\left(2\,bx^2+3\,a\right)}{3\,a^2}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/4),x)
```

```
[Out] 1/3*(b*x^2+a)*x*(2*b*x^2+3*a)/a^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

Fricas [A] time = 0.276706, size = 78, normalized size = 1.15

$$\frac{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}\left(2\,bx^3+3\,ax\right)}{3\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/4)*(2*b*x^3 + 3*a*x)/(a^2*b^2*
x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/4),x)

[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{5}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4),x, algorithm="giac")

[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/4), x)
```

3.6 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{7/4}}\,dx$

Optimal. Leaf size=105

$$\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{x\left(a+bx^2\right)}{5a\left(a^2+2abx^2+b^2x^4\right)^{7/4}}+\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

```
[Out] (x*(a + b*x^2))/(5*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(7/4)) + (4*x)/(
15*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15
*a^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rubi [A] time = 0.0648442, antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{x}{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{3/4}}+\frac{8x\left(a+bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (4*x)/(15*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + x/(5*a*(a + b*
x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4)) + (8*x*(a + b*x^2))/(15*a
^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/4))
```

Rubi in Sympy [A] time = 31.678, size = 107, normalized size = 1.02

$$\frac{x\left(2a+2bx^2\right)}{10a\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{4}}}+\frac{4x}{15a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}}+\frac{4x\left(2a+2bx^2\right)}{15a^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

```
[Out] x*(2*a + 2*b*x**2)/(10*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(7/4))
+ 4*x/(15*a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4)) + 4*x*(2*a
 + 2*b*x**2)/(15*a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/4))
```

Mathematica [A] time = 0.0321417, size = 51, normalized size = 0.49

$$\frac{x\left(15a^2+20abx^2+8b^2x^4\right)}{15a^3\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-7/4),x]
```

```
[Out] (x*(15*a^2 + 20*a*b*x^2 + 8*b^2*x^4))/(15*a^3*(a + b*x^2)*((a + b
*x^2)^2)^(3/4))
```

Maple [A] time = 0.004, size = 55, normalized size = 0.5

$$\frac{\left(bx^2+a\right)x\left(8\,b^2x^4+20\,abx^2+15\,a^2\right)}{15\,a^3}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(7/4),x)
```

```
[Out] 1/15*(b*x^2+a)*x*(8*b^2*x^4+20*a*b*x^2+15*a^2)/a^3/(b^2*x^4+2*a*b
*x^2+a^2)^(7/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

Fricas [A] time = 0.287516, size = 108, normalized size = 1.03

$$\frac{\left(8\,b^2x^5+20\,abx^3+15\,a^2x\right)\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{1}{4}}}{15\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4),x, algorithm="fricas")
```

```
[Out] 1/15*(8*b^2*x^5 + 20*a*b*x^3 + 15*a^2*x)*(b^2*x^4 + 2*a*b*x^2 + a
^2)^(1/4)/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^2 + 2abx^2 + b^2x^4)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(7/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-7/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-7/4), x)
```

3.7 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{9/4}}\,dx$

Optimal. Leaf size=135

$$\frac{6x}{35a^2\left(a^2+2abx^2+b^2x^4\right)^{5/4}}+\frac{x\left(a+bx^2\right)}{7a\left(a^2+2abx^2+b^2x^4\right)^{9/4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{8x\left(a+bx^2\right)}{35a^3\left(a^2+2abx^2+b^2x^4\right)^{5/4}}$$

```
[Out] (x*(a + b*x^2))/(7*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(9/4)) + (6*x)/(
35*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/4)) + (8*x*(a + b*x^2))/(35
*a^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/4)) + (16*x)/(35*a^4*(a^2 + 2
*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi [A] time = 0.0891053, antiderivative size = 148, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{6x}{35a^2\left(a+bx^2\right)^2\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{x}{7a\left(a+bx^2\right)^3\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}+\frac{8x}{35a^3\left(a+bx^2\right)\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (16*x)/(35*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + x/(7*a*(a + b
*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + (6*x)/(35*a^2*(a + b
*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4)) + (8*x)/(35*a^3*(a + b
*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/4))
```

Rubi in Sympy [A] time = 32.7258, size = 138, normalized size = 1.02

$$\frac{x\left(2a+2bx^2\right)}{14a\left(a^2+2abx^2+b^2x^4\right)^{\frac{9}{4}}}+\frac{6x}{35a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{4}}}+\frac{4x\left(2a+2bx^2\right)}{35a^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{4}}}+\frac{16x}{35a^4\sqrt[4]{a^2+2abx^2+b^2x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

```
[Out] x*(2*a + 2*b*x**2)/(14*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(9/4))
+ 6*x/(35*a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/4)) + 4*x*(2*a
 + 2*b*x**2)/(35*a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/4)) + 1
6*x/(35*a**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/4))
```

Mathematica [A] time = 0.0387042, size = 62, normalized size = 0.46

$$\frac{x\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^3\sqrt[4]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-9/4),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3*x^6))/(35*a^4*(
a + b*x^2)^3*((a + b*x^2)^2)^(1/4))
```

Maple [A] time = 0.006, size = 66, normalized size = 0.5

$$\frac{\left(bx^2+a\right)x\left(16\,b^3x^6+56\,b^2x^4a+70\,a^2bx^2+35\,a^3\right)}{35\,a^4}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{9}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(9/4),x)
```

```
[Out] 1/35*(b*x^2+a)*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^
4/(b^2*x^4+2*a*b*x^2+a^2)^(9/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

Fricas [A] time = 0.305009, size = 138, normalized size = 1.02

$$\frac{\left(16\, b^{3} x^{7} + 56\, a b^{2} x^{5} + 70\, a^{2} b x^{3} + 35\, a^{3} x\right) \left(b^{2} x^{4} + 2\, a b x^{2} + a^{2}\right)^{\frac{1}{4}}}{35 \left(a^{4} b^{4} x^{8} + 4\, a^{5} b^{3} x^{6} + 6\, a^{6} b^{2} x^{4} + 4\, a^{7} b x^{2} + a^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*(b^2*x
^4 + 2*a*b*x^2 + a^2)^(1/4)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*
b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2} + 2abx^{2} + b^{2}x^{4}\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(9/4),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-9/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2} x^{4} + 2\, a b x^{2} + a^{2}\right)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-9/4), x)
```

3.8 $\int \frac{1}{a^2+b+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}$$

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2
+ b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan
[(Sqrt[-a + Sqrt[a^2 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/
(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2
+ b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[
a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^2 + b] + Sqrt[2]*
Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-
a + Sqrt[a^2 + b]])
```

Rubi [A] time = 0.662985, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{\log\left(-\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}+\frac{\log\left(\sqrt{2}x\sqrt{\sqrt{a^2+b}-a}+\sqrt{a^2+b}+x^2\right)}{4\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+b}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+b}+a}}\right)}{2\sqrt{2}\sqrt{a^2+b}\sqrt{\sqrt{a^2+b}+a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```


```
[Out] -ArcTan[(Sqrt[-a + Sqrt[a^2 + b]] - Sqrt[2]*x)/Sqrt[a + Sqrt[a^2
+ b]]]/(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) + ArcTan
[(Sqrt[-a + Sqrt[a^2 + b]] + Sqrt[2]*x)/Sqrt[a + Sqrt[a^2 + b]]]/
(2*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[a + Sqrt[a^2 + b]]) - Log[Sqrt[a^2
+ b] - Sqrt[2]*Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[
a^2 + b]*Sqrt[-a + Sqrt[a^2 + b]]) + Log[Sqrt[a^2 + b] + Sqrt[2]*
Sqrt[-a + Sqrt[a^2 + b]]*x + x^2]/(4*Sqrt[2]*Sqrt[a^2 + b]*Sqrt[-
a + Sqrt[a^2 + b]])
```

Rubi in Sympy [A] time = 56.6286, size = 264, normalized size = 0.88

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2a+2\sqrt{a^2+b}}}{2}\right)}{\sqrt{a+\sqrt{a^2+b}}}\right)}{4\sqrt{a+\sqrt{a^2+b}}\sqrt{a^2+b}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2a+2\sqrt{a^2+b}}}{2}\right)}{\sqrt{a+\sqrt{a^2+b}}}\right)}{4\sqrt{a+\sqrt{a^2+b}}\sqrt{a^2+b}}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x\sqrt{-a+\sqrt{a^2+b}}+\sqrt{a^2+b}\right)}{8\sqrt{-a+\sqrt{a^2+b}}\sqrt{a^2+b}}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x\sqrt{-a+\sqrt{a^2+b}}+\sqrt{a^2+b}\right)}{8\sqrt{-a+\sqrt{a^2+b}}\sqrt{a^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+2*a*x**2+a**2+b),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(x - sqrt(-2*a + 2*sqrt(a**2 + b))/2)/sqrt(a
 + sqrt(a**2 + b)))/(4*sqrt(a + sqrt(a**2 + b))*sqrt(a**2 + b)) +
 sqrt(2)*atan(sqrt(2)*(x + sqrt(-2*a + 2*sqrt(a**2 + b))/2)/sqrt(
a + sqrt(a**2 + b)))/(4*sqrt(a + sqrt(a**2 + b))*sqrt(a**2 + b))
- sqrt(2)*log(x**2 - sqrt(2)*x*sqrt(-a + sqrt(a**2 + b)) + sqrt(a
**2 + b))/(8*sqrt(-a + sqrt(a**2 + b))*sqrt(a**2 + b)) + sqrt(2)*
log(x**2 + sqrt(2)*x*sqrt(-a + sqrt(a**2 + b)) + sqrt(a**2 + b))/
(8*sqrt(-a + sqrt(a**2 + b))*sqrt(a**2 + b))
```

Mathematica [C] time = 0.0753346, size = 81, normalized size = 0.27

$$-\frac{i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{b}}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + b + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ((-I/2)*(ArcTan[x/Sqrt[a - I*Sqrt[b]]]/Sqrt[a - I*Sqrt[b]] - ArcT
an[x/Sqrt[a + I*Sqrt[b]]]/Sqrt[a + I*Sqrt[b]]))/Sqrt[b]
```

Maple [B] time = 0.109, size = 1099, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+2*a*x^2+a^2+b),x)
```

```
[Out] 1/8/b/(a^2+b)*ln(x^2+(2*(a^2+b)^(1/2)-2*a)^(1/2)*x+(a^2+b)^(1/2))
*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^2+1/8/b/(a^2+b)^(3/2)*ln(x^2+(2*(a
^2+b)^(1/2)-2*a)^(1/2)*x+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/
2)*a^3+1/8/(a^2+b)*ln(x^2+(2*(a^2+b)^(1/2)-2*a)^(1/2)*x+(a^2+b)^(
1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)+1/8/(a^2+b)^(3/2)*ln(x^2+(2*(a^
2+b)^(1/2)-2*a)^(1/2)*x+(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2
)*a-1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(
2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2+1/2/
b/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b
)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+b)^
(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*
a)^(1/2))/(2*(a^2+b)^(1/2)+2*a)^(1/2))+3/2/(a^2+b)^(3/2)/(2*(a^2+
b)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(
a^2+b)^(1/2)+2*a)^(1/2))*a^2+b/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a
)^(1/2)*arctan((2*x+(2*(a^2+b)^(1/2)-2*a)^(1/2))/(2*(a^2+b)^(1/2)
+2*a)^(1/2))-1/8/b/(a^2+b)*ln((2*(a^2+b)^(1/2)-2*a)^(1/2)*x-x^2-(
a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)*a^2-1/8/b/(a^2+b)^(3/2)
*ln((2*(a^2+b)^(1/2)-2*a)^(1/2)*x-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(
1/2)-2*a)^(1/2)*a^3-1/8/(a^2+b)*ln((2*(a^2+b)^(1/2)-2*a)^(1/2)*x-
x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1/2)-2*a)^(1/2)-1/8/(a^2+b)^(3/2)*
ln((2*(a^2+b)^(1/2)-2*a)^(1/2)*x-x^2-(a^2+b)^(1/2))*(2*(a^2+b)^(1
/2)-2*a)^(1/2)*a+1/2/b/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*
arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1
/2))*a^2-1/2/b/(a^2+b)^(3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan((
(2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^4
+1/2/(a^2+b)^(1/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)
^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))-3/2/(a^2+b)^(
3/2)/(2*(a^2+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1
/2)-2*x)/(2*(a^2+b)^(1/2)+2*a)^(1/2))*a^2-b/(a^2+b)^(3/2)/(2*(a^2
+b)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+b)^(1/2)-2*a)^(1/2)-2*x)/(2*
(a^2+b)^(1/2)+2*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 + b),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + b), x)
```

Fricas [A] time = 0.262627, size = 787, normalized size = 2.63

$$\frac{1}{4}\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+b\right)\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}\log\left(-\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+b\right)\sqrt{\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}+a}{a^2b+b^2}}+x\right)$$
$$-\frac{1}{4}\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}\log\left(\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-b\right)\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}+x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}\log\left(-\left((a^3b+ab^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-b\right)\sqrt{-\frac{(a^2b+b^2)\sqrt{-\frac{1}{a^4b+2a^2b^2+b^3}}-a}{a^2b+b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 + b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(
a^2*b + b^2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b
^3)) + b)*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3))
+ a)/(a^2*b + b^2)) + x) - 1/4*sqrt(((a^2*b + b^2)*sqrt(-1/(a^4*b
 + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2))*log(-((a^3*b + a*b^2)*sq
rt(-1/(a^4*b + 2*a^2*b^2 + b^3)) + b)*sqrt(((a^2*b + b^2)*sqrt(-1
/(a^4*b + 2*a^2*b^2 + b^3)) + a)/(a^2*b + b^2)) + x) - 1/4*sqrt(-
((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b
^2))*log(((a^3*b + a*b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - b)
```

```
*sqrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^2*b^2 + b^3)) - a)/(a^
2*b + b^2)) + x) + 1/4*sqrt(-((a^2*b + b^2)*sqrt(-1/(a^4*b + 2*a^
2*b^2 + b^3)) - a)/(a^2*b + b^2))*log(-((a^3*b + a*b^2)*sqrt(-1/(
a^4*b + 2*a^2*b^2 + b^3)) - b)*sqrt(-((a^2*b + b^2)*sqrt(-1/(a^4*
b + 2*a^2*b^2 + b^3)) - a)/(a^2*b + b^2)) + x)
```

Sympy [A] time = 2.02185, size = 63, normalized size = 0.21

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2+256b^3\right)-32t^2ab+1,\left(t\mapsto t\log\left(64t^3a^3b+64t^3ab^2-4ta^2+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+2*a*x**2+a**2+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*b**3) - 32*_t**2*a*b + 1, Lamb
da(_t, _t*log(64*_t**3*a**3*b + 64*_t**3*a*b**2 - 4*_t*a**2 + 4*_
t*b + x)))
```

GIAC/XCAS [A] time = 2.80541, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 + b),x, algorithm="giac")
```

```
[Out] Done
```

3.9 $\int \frac{1}{-1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

```
[Out] -ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 + a]) - ArcTanh[x/Sqrt[1 - a]]/(
2*Sqrt[1 - a])
```

Rubi [A] time = 0.0516353, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{1-a}}\right)}{2\sqrt{1-a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/Sqrt[1 + a]]/(2*Sqrt[1 + a]) - ArcTanh[x/Sqrt[1 - a]]/(
2*Sqrt[1 - a])
```

Rubi in Sympy [A] time = 8.92252, size = 37, normalized size = 0.79

$$-\frac{\operatorname{atan}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}-\frac{\operatorname{atanh}\left(\frac{x}{\sqrt{-a+1}}\right)}{2\sqrt{-a+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

```
[Out] -atan(x/sqrt(a + 1))/(2*sqrt(a + 1)) - atanh(x/sqrt(-a + 1))/(2*s
qrt(-a + 1))
```

Mathematica [A] time = 0.0388376, size = 43, normalized size = 0.91

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(-1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[-1 + a]]/(2*Sqrt[-1 + a]) - ArcTan[x/Sqrt[1 + a]]/(
2*Sqrt[1 + a])
```

Maple [A] time = 0.015, size = 32, normalized size = 0.7

$$-\frac{1}{2}\arctan\left(x\frac{1}{\sqrt{1+a}}\right)\frac{1}{\sqrt{1+a}}+\frac{1}{2}\arctan\left(x\frac{1}{\sqrt{a-1}}\right)\frac{1}{\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+2*a*x^2+a^2-1),x)
```

```
[Out] -1/2*arctan(x/(1+a)^(1/2))/(1+a)^(1/2)+1/2/(a-1)^(1/2)*arctan(x/(
a-1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 - 1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278277, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{-a+1}\log\left(-\frac{2\,(a+1)x-\left(x^2-a-1\right)\sqrt{-a-1}}{x^2+a+1}\right)+\sqrt{-a-1}\log\left(\frac{2\,(a-1)x+\left(x^2-a+1\right)\sqrt{-a+1}}{x^2+a-1}\right)}{4\,\sqrt{-a+1}\sqrt{-a-1}},\ \frac{2\,\sqrt{-a-1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)+\sqrt{a-1}\log\left(-\cdots\right.}{4\,\sqrt{a-1}\sqrt{-a-1}}\right.$$

$$-\frac{2\,\sqrt{-a+1}\arctan\left(\frac{x}{\sqrt{a+1}}\right)-\sqrt{a+1}\log\left(\frac{2\,(a-1)x+\left(x^2-a+1\right)\sqrt{-a+1}}{x^2+a-1}\right)}{4\,\sqrt{a+1}\sqrt{-a+1}},$$

$$\left.-\frac{\sqrt{a-1}\arctan\left(\frac{x}{\sqrt{a+1}}\right)-\sqrt{a+1}\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\,\sqrt{a+1}\sqrt{a-1}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 - 1),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a + 1)*log(-(2*(a + 1)*x - (x^2 - a - 1)*sqrt(-a - 1)
)/(x^2 + a + 1)) + sqrt(-a - 1)*log((2*(a - 1)*x + (x^2 - a + 1)*
sqrt(-a + 1))/(x^2 + a - 1)))/(sqrt(-a + 1)*sqrt(-a - 1)), 1/4*(2
*sqrt(-a - 1)*arctan(x/sqrt(a - 1)) + sqrt(a - 1)*log(-(2*(a + 1)
*x - (x^2 - a - 1)*sqrt(-a - 1))/(x^2 + a + 1)))/(sqrt(a - 1)*sqr
t(-a - 1)), -1/4*(2*sqrt(-a + 1)*arctan(x/sqrt(a + 1)) - sqrt(a +
 1)*log((2*(a - 1)*x + (x^2 - a + 1)*sqrt(-a + 1))/(x^2 + a - 1))
)/(sqrt(a + 1)*sqrt(-a + 1)), -1/2*(sqrt(a - 1)*arctan(x/sqrt(a +
 1)) - sqrt(a + 1)*arctan(x/sqrt(a - 1)))/(sqrt(a + 1)*sqrt(a - 1
))]
```

Sympy [A] time = 1.46282, size = 257, normalized size = 5.47

$$\frac{\sqrt{-\frac{1}{a-1}}\log\left(-a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a-1}}+a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a-1}}\right)}{4} - \frac{\sqrt{-\frac{1}{a-1}}\log\left(a^3\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a-1}}-a\left(-\frac{1}{a-1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a-1}}\right)}{4} + \frac{\sqrt{-\frac{1}{a+1}}\log\left(-a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}-a^2\sqrt{-\frac{1}{a+1}}+a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x-\sqrt{-\frac{1}{a+1}}\right)}{4} - \frac{\sqrt{-\frac{1}{a+1}}\log\left(a^3\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+a^2\sqrt{-\frac{1}{a+1}}-a\left(-\frac{1}{a+1}\right)^{\frac{3}{2}}+x+\sqrt{-\frac{1}{a+1}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+2*a*x**2+a**2-1),x)
```

```
[Out] sqrt(-1/(a - 1))*log(-a**3*(-1/(a - 1))**(3/2) - a**2*sqrt(-1/(a
- 1)) + a*(-1/(a - 1))**(3/2) + x - sqrt(-1/(a - 1)))/4 - sqrt(-1
/(a - 1))*log(a**3*(-1/(a - 1))**(3/2) + a**2*sqrt(-1/(a - 1)) -
a*(-1/(a - 1))**(3/2) + x + sqrt(-1/(a - 1)))/4 + sqrt(-1/(a + 1)
)*log(-a**3*(-1/(a + 1))**(3/2) - a**2*sqrt(-1/(a + 1)) + a*(-1/(
a + 1))**(3/2) + x - sqrt(-1/(a + 1)))/4 - sqrt(-1/(a + 1))*log(a
**3*(-1/(a + 1))**(3/2) + a**2*sqrt(-1/(a + 1)) - a*(-1/(a + 1))*
*(3/2) + x + sqrt(-1/(a + 1)))/4
```

GIAC/XCAS [A] time = 0.286117, size = 42, normalized size = 0.89

$$-\frac{\arctan\left(\frac{x}{\sqrt{a+1}}\right)}{2\sqrt{a+1}}+\frac{\arctan\left(\frac{x}{\sqrt{a-1}}\right)}{2\sqrt{a-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 - 1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/sqrt(a + 1))/sqrt(a + 1) + 1/2*arctan(x/sqrt(a - 1)
)/sqrt(a - 1)
```

3.10 $\int \frac{1}{1+a^2+2ax^2+x^4}\,dx$

Optimal. Leaf size=299

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}$$

```
[Out] -ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 +
a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan
[(Sqrt[-a + Sqrt[1 + a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/
(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) - Log[Sqrt[1 +
a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[
1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1 + a^2] + Sqrt[2]*
Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-
a + Sqrt[1 + a^2]])
```

Rubi [A] time = 0.574841, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$-\frac{\log\left(-\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}+\frac{\log\left(\sqrt{2}\sqrt{\sqrt{a^2+1}-a}\,x+\sqrt{a^2+1}+x^2\right)}{4\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}-a}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}-\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+1}-a}+\sqrt{2}x}{\sqrt{\sqrt{a^2+1}+a}}\right)}{2\sqrt{2}\sqrt{a^2+1}\sqrt{\sqrt{a^2+1}+a}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```


```
[Out] -ArcTan[(Sqrt[-a + Sqrt[1 + a^2]] - Sqrt[2]*x)/Sqrt[a + Sqrt[1 +
a^2]]]/(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) + ArcTan
[(Sqrt[-a + Sqrt[1 + a^2]] + Sqrt[2]*x)/Sqrt[a + Sqrt[1 + a^2]]]/
(2*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[a + Sqrt[1 + a^2]]) - Log[Sqrt[1 +
a^2] - Sqrt[2]*Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[
1 + a^2]*Sqrt[-a + Sqrt[1 + a^2]]) + Log[Sqrt[1 + a^2] + Sqrt[2]*
Sqrt[-a + Sqrt[1 + a^2]]*x + x^2]/(4*Sqrt[2]*Sqrt[1 + a^2]*Sqrt[-
a + Sqrt[1 + a^2]])
```

Rubi in Sympy [A] time = 54.4471, size = 264, normalized size = 0.88

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2a+2\sqrt{a^2+1}}}{2}\right)}{\sqrt{a+\sqrt{a^2+1}}}\right)}{4\sqrt{a+\sqrt{a^2+1}}\sqrt{a^2+1}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2a+2\sqrt{a^2+1}}}{2}\right)}{\sqrt{a+\sqrt{a^2+1}}}\right)}{4\sqrt{a+\sqrt{a^2+1}}\sqrt{a^2+1}}-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x\sqrt{-a+\sqrt{a^2+1}}+\sqrt{a^2+1}\right)}{8\sqrt{-a+\sqrt{a^2+1}}\sqrt{a^2+1}}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x\sqrt{-a+\sqrt{a^2+1}}+\sqrt{a^2+1}\right)}{8\sqrt{-a+\sqrt{a^2+1}}\sqrt{a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(x - sqrt(-2*a + 2*sqrt(a**2 + 1))/2)/sqrt(a
 + sqrt(a**2 + 1)))/(4*sqrt(a + sqrt(a**2 + 1))*sqrt(a**2 + 1)) +
 sqrt(2)*atan(sqrt(2)*(x + sqrt(-2*a + 2*sqrt(a**2 + 1))/2)/sqrt(
a + sqrt(a**2 + 1)))/(4*sqrt(a + sqrt(a**2 + 1))*sqrt(a**2 + 1))
- sqrt(2)*log(x**2 - sqrt(2)*x*sqrt(-a + sqrt(a**2 + 1)) + sqrt(a
**2 + 1))/(8*sqrt(-a + sqrt(a**2 + 1))*sqrt(a**2 + 1)) + sqrt(2)*
log(x**2 + sqrt(2)*x*sqrt(-a + sqrt(a**2 + 1)) + sqrt(a**2 + 1))/
(8*sqrt(-a + sqrt(a**2 + 1))*sqrt(a**2 + 1))
```

Mathematica [C] time = 0.0471354, size = 52, normalized size = 0.17

$$-\frac{1}{2}i\left(\frac{\tan^{-1}\left(\frac{x}{\sqrt{a-i}}\right)}{\sqrt{a-i}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{a+i}}\right)}{\sqrt{a+i}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + a^2 + 2*a*x^2 + x^4)^(-1),x]
```

```
[Out] (-I/2)*(ArcTan[x/Sqrt[-I + a]]/Sqrt[-I + a] - ArcTan[x/Sqrt[I + a
]]/Sqrt[I + a])
```

Maple [B] time = 0.095, size = 1073, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+2*a*x^2+a^2+1),x)
```

```
[Out] 1/8/(a^2+1)*ln(x^2+(2*(a^2+1)^(1/2)-2*a)^(1/2)*x+(a^2+1)^(1/2))*(
2*(a^2+1)^(1/2)-2*a)^(1/2)*a^2+1/8/(a^2+1)^(3/2)*ln(x^2+(2*(a^2+1
)^(1/2)-2*a)^(1/2)*x+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a
^3+1/8/(a^2+1)*ln(x^2+(2*(a^2+1)^(1/2)-2*a)^(1/2)*x+(a^2+1)^(1/2)
)*(2*(a^2+1)^(1/2)-2*a)^(1/2)+1/8/(a^2+1)^(3/2)*ln(x^2+(2*(a^2+1)
^(1/2)-2*a)^(1/2)*x+(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a-
1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2
+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2+1/2/(a^2+1
)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-
2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^4-1/2/(a^2+1)^(1/2)/(2
*(a^2+1)^(1/2)+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2)
)/(2*(a^2+1)^(1/2)+2*a)^(1/2))+3/2/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)
+2*a)^(1/2)*arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(
1/2)+2*a)^(1/2))*a^2+1/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*
arctan((2*x+(2*(a^2+1)^(1/2)-2*a)^(1/2))/(2*(a^2+1)^(1/2)+2*a)^(1
/2))-1/8/(a^2+1)*ln((2*(a^2+1)^(1/2)-2*a)^(1/2)*x-x^2-(a^2+1)^(1/
2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)*a^2-1/8/(a^2+1)^(3/2)*ln((2*(a^2+
1)^(1/2)-2*a)^(1/2)*x-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1
/2)*a^3-1/8/(a^2+1)*ln((2*(a^2+1)^(1/2)-2*a)^(1/2)*x-x^2-(a^2+1)^
(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/2)-1/8/(a^2+1)^(3/2)*ln((2*(a^2+1
)^(1/2)-2*a)^(1/2)*x-x^2-(a^2+1)^(1/2))*(2*(a^2+1)^(1/2)-2*a)^(1/
2)*a+1/2/(a^2+1)^(1/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^
2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^2-1/2/(
a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)
-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))*a^4+1/2/(a^2+1)^(1/
2)/(2*(a^2+1)^(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2
)-2*x)/(2*(a^2+1)^(1/2)+2*a)^(1/2))-3/2/(a^2+1)^(3/2)/(2*(a^2+1)^
(1/2)+2*a)^(1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2
+1)^(1/2)+2*a)^(1/2))*a^2-1/(a^2+1)^(3/2)/(2*(a^2+1)^(1/2)+2*a)^(
1/2)*arctan(((2*(a^2+1)^(1/2)-2*a)^(1/2)-2*x)/(2*(a^2+1)^(1/2)+2*
a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

Fricas [A] time = 0.28996, size = 1048, normalized size = 3.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*((a + sqrt(a^2 + 1))*log(x^2 + sqrt(2)*x*sqrt((a^2 +
(a^3 + a)/sqrt(a^2 + 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1) +
 1))/(a^2 + 1)^(1/4) + sqrt(a^2 + 1))/(a^2 + 1)^(1/4) - (a + sqrt
(a^2 + 1))*log(x^2 - sqrt(2)*x*sqrt((a^2 + (a^3 + a)/sqrt(a^2 + 1
) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1) + 1))/(a^2 + 1)^(1/4) +
 sqrt(a^2 + 1))/(a^2 + 1)^(1/4) - 4*(a^2 + 1)^(3/4)*arctan((sqrt(
a^4 + 2*a^2 + 1) + (a^5 + 2*a^3 + a)/(sqrt(a^4 + 2*a^2 + 1)*sqrt(
a^2 + 1)))/((sqrt(2)*sqrt(x^2 + sqrt(2)*x*sqrt((a^2 + (a^3 + a)/s
qrt(a^2 + 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1) + 1))/(a^2 +
 1)^(1/4) + sqrt(a^2 + 1))*(a + sqrt(a^2 + 1))*sqrt((a^2 + (a^3 +
 a)/sqrt(a^2 + 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1) + 1)) +
 sqrt(2)*(a*x + sqrt(a^2 + 1)*x)*sqrt((a^2 + (a^3 + a)/sqrt(a^2 +
 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1) + 1)) + (a^2 + 1)^(1/
4))*(a^2 + 1)^(1/4)))/sqrt(a^4 + 2*a^2 + 1) - 4*(a^2 + 1)^(3/4)*a
rctan((sqrt(a^4 + 2*a^2 + 1) + (a^5 + 2*a^3 + a)/(sqrt(a^4 + 2*a^
2 + 1)*sqrt(a^2 + 1)))/((sqrt(2)*sqrt(x^2 - sqrt(2)*x*sqrt((a^2 +
 (a^3 + a)/sqrt(a^2 + 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1)
+ 1))/(a^2 + 1)^(1/4) + sqrt(a^2 + 1))*(a + sqrt(a^2 + 1))*sqrt((
a^2 + (a^3 + a)/sqrt(a^2 + 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2
+ 1) + 1)) + sqrt(2)*(a*x + sqrt(a^2 + 1)*x)*sqrt((a^2 + (a^3 + a
)/sqrt(a^2 + 1) + 1)/(2*a^2 + 2*(a^3 + a)/sqrt(a^2 + 1) + 1)) - (
a^2 + 1)^(1/4))*(a^2 + 1)^(1/4)))/sqrt(a^4 + 2*a^2 + 1))/((a + sq
rt(a^2 + 1))*sqrt((a^2 + (a^3 + a)/sqrt(a^2 + 1) + 1)/(2*a^2 + 2*
(a^3 + a)/sqrt(a^2 + 1) + 1)))
```

Sympy [A] time = 1.37955, size = 48, normalized size = 0.16

$$\operatorname{RootSum}\left(t^4\left(256a^2+256\right)-32t^2a+1,\left(t\mapsto t\log\left(64t^3a^3+64t^3a-4ta^2+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+2*a*x**2+a**2+1),x)
```

```
[Out] RootSum(_t**4*(256*a**2 + 256) - 32*_t**2*a + 1, Lambda(_t, _t*lo
g(64*_t**3*a**3 + 64*_t**3*a - 4*_t*a**2 + 4*_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 2\,ax^2 + a^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*a*x^2 + a^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 + 2*a*x^2 + a^2 + 1), x)
```

3.11 $\int \frac{1}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTanh[x/2]/6 + ArcTanh[x]/3
```

Rubi [A] time = 0.0159435, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{3}\tanh^{-1}(x)-\frac{1}{6}\tanh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 5*x^2 + x^4)^(-1),x]

[Out] -ArcTanh[x/2]/6 + ArcTanh[x]/3
```

Rubi in Sympy [A] time = 10.8714, size = 10, normalized size = 0.59

$$-\frac{\operatorname{atanh}\left(\frac{x}{2}\right)}{6}+\frac{\operatorname{atanh}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-5*x**2+4),x)

[Out] -atanh(x/2)/6 + atanh(x)/3
```

Mathematica [B] time = 0.00898096, size = 37, normalized size = 2.18

$$-\frac{1}{6}\log(1-x)+\frac{1}{12}\log(2-x)+\frac{1}{6}\log(x+1)-\frac{1}{12}\log(x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 5*x^2 + x^4)^(-1),x]
```

```
[Out] -Log[1 - x]/6 + Log[2 - x]/12 + Log[1 + x]/6 - Log[2 + x]/12
```

Maple [B] time = 0.012, size = 26, normalized size = 1.5

$$-\frac{\ln(2+x)}{12} - \frac{\ln(-1+x)}{6} + \frac{\ln(1+x)}{6} + \frac{\ln(x-2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-5*x^2+4),x)
```

```
[Out] -1/12*ln(2+x)-1/6*ln(-1+x)+1/6*ln(1+x)+1/12*ln(x-2)
```

Maxima [A] time = 0.679632, size = 34, normalized size = 2.

$$-\frac{1}{12}\log(x+2) + \frac{1}{6}\log(x+1) - \frac{1}{6}\log(x-1) + \frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 5*x^2 + 4),x, algorithm="maxima")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x -
 2)
```

Fricas [A] time = 0.261078, size = 34, normalized size = 2.

$$-\frac{1}{12}\log(x+2) + \frac{1}{6}\log(x+1) - \frac{1}{6}\log(x-1) + \frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 5*x^2 + 4),x, algorithm="fricas")
```

```
[Out] -1/12*log(x + 2) + 1/6*log(x + 1) - 1/6*log(x - 1) + 1/12*log(x -
 2)
```

Sympy [A] time = 0.480528, size = 26, normalized size = 1.53

$$\frac{\log(x-2)}{12}-\frac{\log(x-1)}{6}+\frac{\log(x+1)}{6}-\frac{\log(x+2)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/12 - log(x - 1)/6 + log(x + 1)/6 - log(x + 2)/12
```

GIAC/XCAS [A] time = 0.269179, size = 39, normalized size = 2.29

$$-\frac{1}{12}\ln(|x+2|)+\frac{1}{6}\ln(|x+1|)-\frac{1}{6}\ln(|x-1|)+\frac{1}{12}\ln(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - 5*x^2 + 4),x, algorithm="giac")
```

```
[Out] -1/12*ln(abs(x + 2)) + 1/6*ln(abs(x + 1)) - 1/6*ln(abs(x - 1)) +
1/12*ln(abs(x - 2))
```

3.12 $\int \frac{1}{3+4x^2+x^4}\,dx$

Optimal. Leaf size=24

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0193395, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x^2 + x^4)^(-1),x]

[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Rubi in Sympy [A] time = 2.8243, size = 20, normalized size = 0.83

$$\frac{\operatorname{atan}(x)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+4*x**2+3),x)

[Out] atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

Mathematica [A] time = 0.0182419, size = 24, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x]/2 - ArcTan[x/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.01, size = 18, normalized size = 0.8

$$\frac{\arctan(x)}{2} - \frac{\sqrt{3}}{6}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+4*x^2+3),x)
```

```
[Out] 1/2*arctan(x)-1/6*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.763653, size = 23, normalized size = 0.96

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^2 + 3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

Fricas [A] time = 0.260195, size = 28, normalized size = 1.17

$$\frac{1}{6}\sqrt{3}\left(\sqrt{3}\arctan(x) - \arctan\left(\frac{1}{3}\sqrt{3}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^2 + 3),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(sqrt(3)*arctan(x) - arctan(1/3*sqrt(3)*x))
```

Sympy [A] time = 0.394925, size = 20, normalized size = 0.83

$$\frac{\operatorname{atan}(x)}{2} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+4*x**2+3),x)
```

```
[Out] atan(x)/2 - sqrt(3)*atan(sqrt(3)*x/3)/6
```

GIAC/XCAS [A] time = 0.268327, size = 23, normalized size = 0.96

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 4*x^2 + 3),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/2*arctan(x)
```

3.13 $\int \frac{1}{9+5x^2+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[11]]/(6*Sqrt[11]) + ArcTan[(1 + 2*x)/Sqrt[
11]]/(6*Sqrt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rubi [A] time = 0.0964589, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{1}{12}\log\left(x^2-x+3\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{11}}\right)}{6\sqrt{11}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{11}}\right)}{6\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In]  Int[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[11]]/(6*Sqrt[11]) + ArcTan[(1 + 2*x)/Sqrt[
11]]/(6*Sqrt[11]) - Log[3 - x + x^2]/12 + Log[3 + x + x^2]/12
```

Rubi in Sympy [A] time = 12.7132, size = 63, normalized size = 0.94

$$-\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\,\mathrm{atan}\left(\sqrt{11}\left(\frac{2x}{11}-\frac{1}{11}\right)\right)}{66}+\frac{\sqrt{11}\,\mathrm{atan}\left(\sqrt{11}\left(\frac{2x}{11}+\frac{1}{11}\right)\right)}{66}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+5*x**2+9),x)
```

```
[Out] -log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(sqrt
(11)*(2*x/11 - 1/11))/66 + sqrt(11)*atan(sqrt(11)*(2*x/11 + 1/11)
)/66
```

Mathematica [C] time = 0.121902, size = 91, normalized size = 1.36

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5+i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5+i\sqrt{11}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(5-i\sqrt{11}\right)}}\right)}{\sqrt{\frac{11}{2}\left(5-i\sqrt{11}\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(9 + 5*x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(5 - I*Sqrt[11])/2]])/Sqrt[(11*(5 - I*Sqrt[11
]))/2] + (I*ArcTan[x/Sqrt[(5 + I*Sqrt[11])/2]])/Sqrt[(11*(5 + I*S
qrt[11]))/2]
```

Maple [A] time = 0.009, size = 54, normalized size = 0.8

$$\frac{\ln\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}}{66}\arctan\left(\frac{(1+2x)\sqrt{11}}{11}\right)-\frac{\ln\left(x^2-x+3\right)}{12}+\frac{\sqrt{11}}{66}\arctan\left(\frac{(2x-1)\sqrt{11}}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+5*x^2+9),x)
```

```
[Out] 1/12*ln(x^2+x+3)+1/66*arctan(1/11*(1+2*x)*11^(1/2))*11^(1/2)-1/12
*ln(x^2-x+3)+1/66*11^(1/2)*arctan(1/11*(2*x-1)*11^(1/2))
```

Maxima [A] time = 0.763677, size = 72, normalized size = 1.07

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\log\left(x^2+x+3\right)-\frac{1}{12}\log\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 5*x^2 + 9),x, algorithm="maxima")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arc
tan(1/11*sqrt(11)*(2*x - 1)) + 1/12*log(x^2 + x + 3) - 1/12*log(x
```

```
^2 - x + 3)
```

Fricas [A] time = 0.260302, size = 77, normalized size = 1.15

$$\frac{1}{132}\sqrt{11}\left(\sqrt{11}\log\left(x^2+x+3\right)-\sqrt{11}\log\left(x^2-x+3\right)+2\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+2\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 5*x^2 + 9),x, algorithm="fricas")
```

```
[Out] 1/132*sqrt(11)*(sqrt(11)*log(x^2 + x + 3) - sqrt(11)*log(x^2 - x
+ 3) + 2*arctan(1/11*sqrt(11)*(2*x + 1)) + 2*arctan(1/11*sqrt(11)
*(2*x - 1)))
```

Sympy [A] time = 0.590901, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+3\right)}{12}+\frac{\log\left(x^2+x+3\right)}{12}+\frac{\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11}-\frac{\sqrt{11}}{11}\right)}{66}+\frac{\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11}+\frac{\sqrt{11}}{11}\right)}{66}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+5*x**2+9),x)
```

```
[Out] -log(x**2 - x + 3)/12 + log(x**2 + x + 3)/12 + sqrt(11)*atan(2*sq
rt(11)*x/11 - sqrt(11)/11)/66 + sqrt(11)*atan(2*sqrt(11)*x/11 + s
qrt(11)/11)/66
```

GIAC/XCAS [A] time = 0.270492, size = 72, normalized size = 1.07

$$\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+1)\right)+\frac{1}{66}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x-1)\right)+\frac{1}{12}\ln\left(x^2+x+3\right)-\frac{1}{12}\ln\left(x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 5*x^2 + 9),x, algorithm="giac")
```

```
[Out] 1/66*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 1)) + 1/66*sqrt(11)*arc
tan(1/11*sqrt(11)*(2*x - 1)) + 1/12*ln(x^2 + x + 3) - 1/12*ln(x^2
 - x + 3)
```

3.14 $\int \frac{1}{1-x^2+x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt
[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.0896653, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417

$$-\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}+\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 - Log[1 - Sqrt
[3]*x + x^2]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 16.7216, size = 63, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3
)*x + 1)/12 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Mathematica [C] time = 0.121835, size = 77, normalized size = 1.04

$$\frac{i\left(\sqrt{-1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)-\sqrt{-1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 - x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[-1 - I*Sqrt[3]]*ArcTan[((1 - I*Sqrt[3])*x)/2] - Sqrt[-1
+ I*Sqrt[3]]*ArcTan[((1 + I*Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] time = 0.035, size = 57, normalized size = 0.8

$$\frac{\arctan\left(2x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2x+\sqrt{3}\right)}{2}-\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))-1/12*ln(1+x^2-x*3
^(1/2))*3^(1/2)+1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}-x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.267376, size = 143, normalized size = 1.93

$$-\frac{1}{12}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^{2}+\sqrt{3}x+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^{2}-\sqrt{3}x+1}-3}\right)-\log\left(x^{2}+\sqrt{3}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*(4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x + 2*sqrt(3)*
sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 4*sqrt(3)*arctan(sqrt(3)/(2*sqr
t(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) - log(x^2 + sq
rt(3)*x + 1) + log(x^2 - sqrt(3)*x + 1))
```

Sympy [A] time = 0.526391, size = 63, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}+\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4-x**2+1),x)
```

```
[Out] -sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 + sqrt(3)*log(x**2 + sqrt(3
)*x + 1)/12 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 - x^2 + 1), x)
```

3.15 $\int \frac{1}{2+2x^2+x^4}\,dx$

Optimal. Leaf size=176

$$-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2
*(1 + Sqrt[2])]])/4 + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[2])] + 2*x)/Sqrt[2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(
-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sq
rt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1 + Sqrt[2]])
```

Rubi [A] time = 0.393229, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$

$$-\frac{\log\left(x^2-\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}+\frac{\log\left(x^2+\sqrt{2\left(\sqrt{2}-1\right)}x+\sqrt{2}\right)}{8\sqrt{\sqrt{2}-1}}-\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{\sqrt{2\left(\sqrt{2}-1\right)}-2x}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)+\frac{1}{4}\sqrt{\sqrt{2}-1}\tan^{-1}\left(\frac{2x+\sqrt{2\left(\sqrt{2}-1\right)}}{\sqrt{2\left(1+\sqrt{2}\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sqrt[2])] - 2*x)/Sqrt[2
*(1 + Sqrt[2])]])/4 + (Sqrt[-1 + Sqrt[2]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[2])] + 2*x)/Sqrt[2*(1 + Sqrt[2])]])/4 - Log[Sqrt[2] - Sqrt[2*(
-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1 + Sqrt[2]]) + Log[Sqrt[2] + Sq
rt[2*(-1 + Sqrt[2])]*x + x^2]/(8*Sqrt[-1 + Sqrt[2]])
```

Rubi in Sympy [A] time = 30.1003, size = 165, normalized size = 0.94

$$-\frac{\log\left(x^2-\sqrt{2}x\sqrt{-1+\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{-1+\sqrt{2}}}+\frac{\log\left(x^2+\sqrt{2}x\sqrt{-1+\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{-1+\sqrt{2}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{-2+2\sqrt{2}}}{2}\right)}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}+\frac{\operatorname{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{-2+2\sqrt{2}}}{2}\right)}{\sqrt{1+\sqrt{2}}}\right)}{4\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+2*x**2+2),x)
```

```
[Out] -log(x**2 - sqrt(2)*x*sqrt(-1 + sqrt(2)) + sqrt(2))/(8*sqrt(-1 +
sqrt(2))) + log(x**2 + sqrt(2)*x*sqrt(-1 + sqrt(2)) + sqrt(2))/(8
*sqrt(-1 + sqrt(2))) + atan(sqrt(2)*(x - sqrt(-2 + 2*sqrt(2))/2)/
sqrt(1 + sqrt(2)))/(4*sqrt(1 + sqrt(2))) + atan(sqrt(2)*(x + sqrt
(-2 + 2*sqrt(2))/2)/sqrt(1 + sqrt(2)))/(4*sqrt(1 + sqrt(2)))
```

Mathematica [C] time = 0.062061, size = 41, normalized size = 0.23

$$\frac{1}{4}\left((1-i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1-i}}\right)+(1+i)^{3/2}\tan^{-1}\left(\frac{x}{\sqrt{1+i}}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] ((1 - I)^(3/2)*ArcTan[x/Sqrt[1 - I]] + (1 + I)^(3/2)*ArcTan[x/Sqr
t[1 + I]])/4
```

Maple [B] time = 0.074, size = 386, normalized size = 2.2

$$-\frac{\ln\left(x^{2}+\sqrt{2}-x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}\sqrt{2}}{16}-\frac{\ln\left(x^{2}+\sqrt{2}-x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}}{8}-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{8\sqrt{2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)-\frac{-2+2\sqrt{2}}{4\sqrt{2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)+\frac{\ln\left(x^{2}+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}\sqrt{2}}{16}+\frac{\ln\left(x^{2}+\sqrt{2}+x\sqrt{-2+2\sqrt{2}}\right)\sqrt{-2+2\sqrt{2}}}{8}-\frac{\left(-2+2\sqrt{2}\right)\sqrt{2}}{8\sqrt{2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)-\frac{-2+2\sqrt{2}}{4\sqrt{2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)+\frac{\sqrt{2}}{2\sqrt{2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{-2+2\sqrt{2}}}{\sqrt{2+2\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+2*x^2+2),x)
```

```
[Out] -1/16*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)
*2^(1/2)-1/8*ln(x^2+2^(1/2)-x*(-2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2)
)^(1/2)-1/8/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))
/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))*2^(1/2)-1/4/(2+2*2^(1/2))^(1
/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*
2^(1/2))+1/2/(2+2*2^(1/2))^(1/2)*arctan((2*x-(-2+2*2^(1/2))^(1/2)
)/(2+2*2^(1/2))^(1/2))*2^(1/2)+1/16*ln(x^2+2^(1/2)+x*(-2+2*2^(1/2
))^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/2)+1/8*ln(x^2+2^(1/2)+x*(-2+2
*2^(1/2))^(1/2))*(-2+2*2^(1/2))^(1/2)-1/8/(2+2*2^(1/2))^(1/2)*arc
tan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2)
)*2^(1/2)-1/4/(2+2*2^(1/2))^(1/2)*arctan((2*x+(-2+2*2^(1/2))^(1/2
))/(2+2*2^(1/2))^(1/2))*(-2+2*2^(1/2))+1/2/(2+2*2^(1/2))^(1/2)*ar
ctan((2*x+(-2+2*2^(1/2))^(1/2))/(2+2*2^(1/2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{4}+2\,x^{2}+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

Fricas [A] time = 0.276849, size = 630, normalized size = 3.58

$$\sqrt{2}\left(2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\log\left(24\sqrt{2}x^2+2^{\frac{3}{4}}\left(17\sqrt{2}x+24x\right)\sqrt{\frac{\sqrt{2}+2}{2\sqrt{2}+3}}+34x^2+2\sqrt{2}\left(12\sqrt{2}+17\right)\right)-2^{\frac{1}{4}}\left(\sqrt{2}+1\right)\log\left(24\sqrt{2}x^2-2^{\frac{3}{4}}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2^(1/4)*(sqrt(2) + 1)*log(24*sqrt(2)*x^2 + 2^(3/4)*(
17*sqrt(2)*x + 24*x)*sqrt((sqrt(2) + 2)/(2*sqrt(2) + 3)) + 34*x^2
 + 2*sqrt(2)*(12*sqrt(2) + 17)) - 2^(1/4)*(sqrt(2) + 1)*log(24*sq
rt(2)*x^2 - 2^(3/4)*(17*sqrt(2)*x + 24*x)*sqrt((sqrt(2) + 2)/(2*s
qrt(2) + 3)) + 34*x^2 + 2*sqrt(2)*(12*sqrt(2) + 17)) - 4*2^(1/4)*
arctan(2^(1/4)*(sqrt(2) + 2)/(sqrt(2)*sqrt(1/2)*(sqrt(2) + 2)*sqr
t((24*sqrt(2)*x^2 + 2^(3/4)*(17*sqrt(2)*x + 24*x)*sqrt((sqrt(2) +
 2)/(2*sqrt(2) + 3)) + 34*x^2 + 2*sqrt(2)*(12*sqrt(2) + 17))/(12*
sqrt(2) + 17))*sqrt((sqrt(2) + 2)/(2*sqrt(2) + 3)) + sqrt(2)*(sqr
t(2)*x + 2*x)*sqrt((sqrt(2) + 2)/(2*sqrt(2) + 3)) + 2^(3/4))) - 4
*2^(1/4)*arctan(2^(1/4)*(sqrt(2) + 2)/(sqrt(2)*sqrt(1/2)*(sqrt(2)
 + 2)*sqrt((24*sqrt(2)*x^2 - 2^(3/4)*(17*sqrt(2)*x + 24*x)*sqrt((
sqrt(2) + 2)/(2*sqrt(2) + 3)) + 34*x^2 + 2*sqrt(2)*(12*sqrt(2) +
17))/(12*sqrt(2) + 17))*sqrt((sqrt(2) + 2)/(2*sqrt(2) + 3)) + sqr
t(2)*(sqrt(2)*x + 2*x)*sqrt((sqrt(2) + 2)/(2*sqrt(2) + 3)) - 2^(3
/4))))/((sqrt(2) + 2)*sqrt((sqrt(2) + 2)/(2*sqrt(2) + 3)))
```

Sympy [A] time = 1.76124, size = 20, normalized size = 0.11

$$\mathrm{RootSum}\left(512t^4-32t^2+1,\left(t\mapsto t\log\left(128t^3+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+2*x**2+2),x)
```

```
[Out] RootSum(512*_t**4 - 32*_t**2 + 1, Lambda(_t, _t*log(128*_t**3 + x
)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 2x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 + 2*x^2 + 2), x)
```

3.16 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rubi [A] time = 0.040331, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rubi in Sympy [A] time = 7.62704, size = 12, normalized size = 1.2

$$F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)

[Out] elliptic_f(asin(sqrt(2)*x/2), -6)
```

Mathematica [C] time = 0.0435526, size = 65, normalized size = 6.5

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}F\left(i\sinh^{-1}\left(\sqrt{3}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]
*x], -1/6])/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Maple [B] time = 0.063, size = 51, normalized size = 5.1

$$\frac{\sqrt{2}}{2}\sqrt{-2\,x^{2}+4}\sqrt{3\,x^{2}+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{6}\right)\frac{1}{\sqrt{-3\,x^{4}+5\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/
2)*EllipticF(1/2*2^(1/2)*x,I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^{4}+5\,x^{2}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

3.17 $\int \frac{1}{\sqrt{2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

```
[Out] Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x
], (-7 - 2*Sqrt[10])/3]
```

Rubi [A] time = 0.271582, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{1}{6}\left(2+\sqrt{10}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(-2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(-2 + Sqrt[10])/2]*x
], (-7 - 2*Sqrt[10])/3]
```

Rubi in Sympy [A] time = 19.4964, size = 73, normalized size = 1.52

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x\sqrt{-2+\sqrt{10}}}{2}\right)\middle|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\sqrt{-2+\sqrt{10}}\sqrt{4+2\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(2)*x*sqrt(-2 + sqrt(10))/2), -7/3
- 2*sqrt(10)/3)/(sqrt(-4 + 2*sqrt(10))*sqrt(-2 + sqrt(10))*sqrt(4
 + 2*sqrt(10)))
```

Mathematica [C] time = 0.100209, size = 49, normalized size = 1.02

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/2]]*x], (-7 + 2*Sqrt[10
])/3])/Sqrt[2 + Sqrt[10]]
```

Maple [B] time = 0.179, size = 84, normalized size = 1.8

$$2\,\frac{\sqrt{1-\left(-1+1/2\,\sqrt{10}\right)x^{2}}\sqrt{1-\left(-1-1/2\,\sqrt{10}\right)x^{2}}EllipticF\left(1/2\,x\sqrt{-4+2\,\sqrt{10}},i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{-4+2\,\sqrt{10}}\sqrt{-3\,x^{4}+4\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+4*x^2+2)^(1/2),x)
```

```
[Out] 2/(-4+2*10^(1/2))^(1/2)*(1-(-1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1-1/
2*10^(1/2))*x^2)^(1/2)/(-3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(-4
+2*10^(1/2))^(1/2),1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}+4\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 4*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+4\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 4*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 4*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 + 2), x)
```

3.18 $\int \frac{1}{\sqrt{2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{33}-3}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x
], (-7 - Sqrt[33])/4]
```

Rubi [A] time = 0.209574, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{33}-3}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(3 + Sqrt[33])]*x
], (-7 - Sqrt[33])/4]
```

Rubi in Sympy [A] time = 12.6443, size = 51, normalized size = 1.06

$$-\frac{4\sqrt{3}F\left(\operatorname{asin}\left(\frac{x\sqrt{-3+\sqrt{33}}}{2}\right)\middle|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{3+\sqrt{33}}\left(-\sqrt{33}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] -4*sqrt(3)*elliptic_f(asin(x*sqrt(-3 + sqrt(33))/2), -7/4 - sqrt(
33)/4)/(sqrt(3 + sqrt(33))*(-sqrt(33) + 3))
```

Mathematica [C] time = 0.103329, size = 53, normalized size = 1.1

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sqrt
[33])]*x], (-7 + Sqrt[33])/4]
```

Maple [B] time = 0.11, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-3/4+1/4\,\sqrt{33}\right)x^{2}}\sqrt{1-\left(-3/4-1/4\,\sqrt{33}\right)x^{2}}\,EllipticF\left(1/2\,x\sqrt{-3+\sqrt{33}},i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{-3+\sqrt{33}}\sqrt{-3\,x^{4}+3\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 2/(-3+33^(1/2))^(1/2)*(1-(-3/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4-
1/4*33^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2+2)^(1/2)*EllipticF(1/2*x*(
-3+33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}+3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 3*x^2 + 2),x, algorithm="fricas")

[Out] integral(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+3*x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 3*x^2 + 2),x, algorithm="giac")

[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 + 2), x)
```

3.19 $\int \frac{1}{\sqrt{2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt
[-1 + Sqrt[7]]
```

Rubi [A] time = 0.155244, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt
[-1 + Sqrt[7]]
```

Rubi in Sympy [A] time = 20.1713, size = 71, normalized size = 1.61

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x\sqrt{-1+\sqrt{7}}}{2}\right)\middle|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-2+2\sqrt{7}}\sqrt{-1+\sqrt{7}}\sqrt{2+2\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(2)*x*sqrt(-1 + sqrt(7))/2), -4/3 -
 sqrt(7)/3)/(sqrt(-2 + 2*sqrt(7))*sqrt(-1 + sqrt(7))*sqrt(2 + 2*s
qrt(7)))
```

Mathematica [C] time = 0.0748245, size = 49, normalized size = 1.11

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7
])/3])/Sqrt[1 + Sqrt[7]]
```

Maple [B] time = 0.122, size = 84, normalized size = 1.9

$$2\,\frac{\sqrt{1-\left(-1/2+1/2\,\sqrt{7}\right)x^{2}}\sqrt{1-\left(-1/2-1/2\,\sqrt{7}\right)x^{2}}EllipticF\left(1/2\,x\sqrt{-2+2\,\sqrt{7}},i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{-2+2\,\sqrt{7}}\sqrt{-3\,x^{4}+2\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 2/(-2+2*7^(1/2))^(1/2)*(1-(-1/2+1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2-
1/2*7^(1/2))*x^2)^(1/2)/(-3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2*x*(-
2+2*7^(1/2))^(1/2),1/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^{4}+2\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 + 2), x)
```

3.20 $\int \frac{1}{\sqrt{2+x^2-3x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rubi [A] time = 0.041274, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{F\left(\sin^{-1}(x)|-\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[x], -3/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 8.54244, size = 14, normalized size = 1.17

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+x**2+2)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), -3/2)/2
```

Mathematica [C] time = 0.0395598, size = 63, normalized size = 5.25

$$-\frac{i\sqrt{1-x^2}\sqrt{3x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3}\sqrt{-3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]
*x], -2/3])/(Sqrt[3]*Sqrt[2 + x^2 - 3*x^4])
```

Maple [B] time = 0.018, size = 41, normalized size = 3.4

$$\frac{EllipticF\left(x, \frac{i}{2}\sqrt{6}\right)}{2}\sqrt{-x^2+1}\sqrt{6\,x^2+4}\frac{1}{\sqrt{-3\,x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(-3*x^4+x^2+2)^(1/2)*EllipticF
(x,1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 + 2), x)
```

3.21 $\int \frac{1}{\sqrt{2-3x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Rubi [A] time = 0.0195727, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 3*x^4],x]

[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Rubi in Sympy [A] time = 1.16947, size = 24, normalized size = 1.33

$$\frac{6^{\frac{3}{4}}F\left(\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+2)**(1/2),x)

[Out] 6**(3/4)*elliptic_f(asin(2**(3/4)*3**(1/4)*x/2), -1)/6
```

Mathematica [A] time = 0.0346641, size = 18, normalized size = 1.

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[(3/2)^(1/4)*x], -1]/6^(1/4)
```

Maple [B] time = 0.063, size = 54, normalized size = 3.

$$\frac{\sqrt{2}6^{\frac{3}{4}}}{24}\sqrt{4-2\,x^{2}\sqrt{6}}\sqrt{4+2\,x^{2}\sqrt{6}}EllipticF\left(\frac{x\sqrt{2}\sqrt[4]{6}}{2},i\right)\frac{1}{\sqrt{-3\,x^{4}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+2)^(1/2),x)
```

```
[Out] 1/24*2^(1/2)*6^(3/4)*(4-2*x^2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1
/2)/(-3*x^4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*6^(1/4),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-3\,x^{4}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 2), x)
```

Sympy [A] time = 1.83751, size = 37, normalized size = 2.06

$$\frac{\sqrt{2}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\right|\frac{3x^4e^{2i\pi}}{2}\right)}{8\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(2
*I*pi)/2)/(8*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2), x)
```

3.22 $\int \frac{1}{\sqrt{2-x^2-3x^4}} \, dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0412452, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 7.24327, size = 20, normalized size = 1.

$$\frac{\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{2}\right)\middle|-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-x**2+2)**(1/2),x)

[Out] sqrt(3)*elliptic_f(asin(sqrt(6)*x/2), -2/3)/3
```

Mathematica [A] time = 0.0446491, size = 20, normalized size = 1.

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)\middle|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]/Sqrt[3]
```

Maple [B] time = 0.038, size = 49, normalized size = 2.5

$$\frac{\sqrt{6}}{6}\sqrt{-6\,x^{2}+4}\sqrt{x^{2}+1}\mathit{EllipticF}\left(\frac{x\sqrt{6}}{2},\frac{i}{3}\sqrt{6}\right)\frac{1}{\sqrt{-3\,x^{4}-x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-x^2+2)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-x^2+2)^(1/2)*E
llipticF(1/2*x*6^(1/2),1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}-x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-3\,x^{4}-x^{2}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 - x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 + 2), x)
```

3.23 $\int \frac{1}{\sqrt{2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqr
t[1 + Sqrt[7]]
```

Rubi [A] time = 0.13774, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[3/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqr
t[1 + Sqrt[7]]
```

Rubi in Sympy [A] time = 20.848, size = 70, normalized size = 1.67

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x\sqrt{1+\sqrt{7}}}{2}\right)\middle|-\frac{4}{3}+\frac{\sqrt{7}}{3}\right)}{\sqrt{-2+2\sqrt{7}}\sqrt{1+\sqrt{7}}\sqrt{2+2\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(2)*x*sqrt(1 + sqrt(7))/2), -4/3 +
sqrt(7)/3)/(sqrt(-2 + 2*sqrt(7))*sqrt(1 + sqrt(7))*sqrt(2 + 2*sqr
t(7)))
```

Mathematica [C] time = 0.0774692, size = 51, normalized size = 1.21

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)\middle|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7
]/3])/Sqrt[-1 + Sqrt[7]]
```

Maple [B] time = 0.099, size = 84, normalized size = 2.

$$2\,\frac{\sqrt{1-\left(1/2\sqrt{7}+1/2\right)x^2}\sqrt{1-\left(-1/2\sqrt{7}+1/2\right)x^2}\,EllipticF\left(1/2\,x\sqrt{2+2\sqrt{7}},i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{2+2\sqrt{7}}\sqrt{-3\,x^4-2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-2*x^2+2)^(1/2),x)
```

```
[Out] 2/(2+2*7^(1/2))^(1/2)*(1-(1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(-1/2*7^
(1/2)+1/2)*x^2)^(1/2)/(-3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*(2+2
*7^(1/2))^(1/2),1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4 - 2\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 - 2x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - 2\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 + 2), x)
```

3.24 $\int \frac{1}{\sqrt{2-3x^2-3x^4}}\,dx$

Optimal. Leaf size=46

$$\sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x
], (-7 + Sqrt[33])/4]
```

Rubi [A] time = 0.223873, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[6/(-3 + Sqrt[33])]*x
], (-7 + Sqrt[33])/4]
```

Rubi in Sympy [A] time = 12.8878, size = 48, normalized size = 1.04

$$\frac{4\sqrt{3}F\left(\operatorname{asin}\left(\frac{x\sqrt{3+\sqrt{33}}}{2}\right)\middle|-\frac{7}{4}+\frac{\sqrt{33}}{4}\right)}{\sqrt{-3+\sqrt{33}}\left(3+\sqrt{33}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] 4*sqrt(3)*elliptic_f(asin(x*sqrt(3 + sqrt(33))/2), -7/4 + sqrt(33
)/4)/(sqrt(-3 + sqrt(33))*(3 + sqrt(33)))
```

Mathematica [C] time = 0.112015, size = 55, normalized size = 1.2

$$-i\sqrt{\frac{2}{\sqrt{33}-3}}F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 - 3*x^2 - 3*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqrt
[33])]*x], -7/4 - Sqrt[33]/4]
```

Maple [B] time = 0.097, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(1/4\,\sqrt{33}+3/4\right)x^{2}}\sqrt{1-\left(-1/4\,\sqrt{33}+3/4\right)x^{2}}EllipticF\left(1/2\,x\sqrt{3+\sqrt{33}},i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{3+\sqrt{33}}\sqrt{-3\,x^{4}-3\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-3*x^2+2)^(1/2),x)
```

```
[Out] 2/(3+33^(1/2))^(1/2)*(1-(1/4*33^(1/2)+3/4)*x^2)^(1/2)*(1-(-1/4*33
^(1/2)+3/4)*x^2)^(1/2)/(-3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x*(3+
33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}-3\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4-3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-3x^2+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-3\,x^2+2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 + 2), x)
```

3.25 $\int \frac{1}{\sqrt{2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

```
[Out] Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x
], (-7 + 2*Sqrt[10])/3]
```

Rubi [A] time = 0.242111, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{1}{6}\left(\sqrt{10}-2\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(2+\sqrt{10}\right)}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] Sqrt[(-2 + Sqrt[10])/6]*EllipticF[ArcSin[Sqrt[(2 + Sqrt[10])/2]*x
], (-7 + 2*Sqrt[10])/3]
```

Rubi in Sympy [A] time = 19.8485, size = 71, normalized size = 1.48

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x\sqrt{2+\sqrt{10}}}{2}\right)\middle|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\sqrt{2+\sqrt{10}}\sqrt{4+2\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(2)*x*sqrt(2 + sqrt(10))/2), -7/3 +
 2*sqrt(10)/3)/(sqrt(-4 + 2*sqrt(10))*sqrt(2 + sqrt(10))*sqrt(4 +
 2*sqrt(10)))
```

Mathematica [C] time = 0.102091, size = 49, normalized size = 1.02

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/2]]*x], (-7 - 2*Sqrt[1
0])/3])/Sqrt[-2 + Sqrt[10]]
```

Maple [B] time = 0.099, size = 84, normalized size = 1.8

$$2\,\frac{\sqrt{1-\left(1+1/2\,\sqrt{10}\right)x^2}\sqrt{1-\left(-1/2\,\sqrt{10}+1\right)x^2}\,EllipticF\left(1/2\,x\sqrt{4+2\,\sqrt{10}},i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{4+2\,\sqrt{10}}\sqrt{-3\,x^4-4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-4*x^2+2)^(1/2),x)
```

```
[Out] 2/(4+2*10^(1/2))^(1/2)*(1-(1+1/2*10^(1/2))*x^2)^(1/2)*(1-(-1/2*10
^(1/2)+1)*x^2)^(1/2)/(-3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2*
10^(1/2))^(1/2),1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 4*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4-4\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 4*x^2 + 2),x, algorithm="fricas")

[Out] integral(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-4*x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 4*x^2 + 2),x, algorithm="giac")

[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 + 2), x)
```

3.26 $\int \frac{1}{\sqrt{2-5x^2-3x^4}} \, dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Rubi [A] time = 0.043815, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 5*x^2 - 3*x^4],x]

[Out] EllipticF[ArcSin[Sqrt[3]*x], -1/6]/Sqrt[6]
```

Rubi in Sympy [A] time = 7.93026, size = 19, normalized size = 1.06

$$\frac{\sqrt{6}F\left(\operatorname{asin}\left(\sqrt{3}x\right)\middle|-\frac{1}{6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-5*x**2+2)**(1/2),x)

[Out] sqrt(6)*elliptic_f(asin(sqrt(3)*x), -1/6)/6
```

Mathematica [B] time = 0.0433603, size = 54, normalized size = 3.

$$\frac{\sqrt{1-3x^2}\sqrt{x^2+2}F\left(\sin^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{-3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6]
)/(Sqrt[6]*Sqrt[2 - 5*x^2 - 3*x^4])
```

Maple [B] time = 0.031, size = 50, normalized size = 2.8

$$\frac{\sqrt{3}EllipticF\left(x\sqrt{3},\frac{i}{6}\sqrt{6}\right)}{6}\sqrt{-3\,x^{2}+1}\sqrt{2\,x^{2}+4}\frac{1}{\sqrt{-3\,x^{4}-5\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-5*x^2+2)^(1/2),x)
```

```
[Out] 1/6*3^(1/2)*(-3*x^2+1)^(1/2)*(2*x^2+4)^(1/2)/(-3*x^4-5*x^2+2)^(1/
2)*EllipticF(x*3^(1/2),1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^{4}-5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-3\,x^{4}-5\,x^{2}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 - 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 - 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 + 2), x)
```

3.27 $\int \frac{1}{\sqrt{3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{73}-7}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

```
[Out] Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]
], (-61 - 7*Sqrt[73])/12]
```

Rubi [A] time = 0.142173, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{73}-7}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[7 + Sqrt[73]]
], (-61 - 7*Sqrt[73])/12]
```

Rubi in Sympy [A] time = 12.8836, size = 58, normalized size = 1.29

$$-\frac{4\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{-7+\sqrt{73}}}{6}\right)\middle|-\frac{61}{12}-\frac{7\sqrt{73}}{12}\right)}{\sqrt{7+\sqrt{73}}\left(-\sqrt{73}+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] -4*sqrt(3)*elliptic_f(asin(sqrt(6)*x*sqrt(-7 + sqrt(73))/6), -61/
12 - 7*sqrt(73)/12)/(sqrt(7 + sqrt(73))*(-sqrt(73) + 7))
```

Mathematica [C] time = 0.0760104, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{7+\sqrt{73}}}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-7 + S
qrt[73]]], (-61 + 7*Sqrt[73])/12]
```

Maple [B] time = 0.117, size = 84, normalized size = 1.9

$$6\,\frac{\sqrt{1-\left(-7/6+1/6\,\sqrt{73}\right)x^{2}}\sqrt{1-\left(-1/6\,\sqrt{73}-7/6\right)x^{2}}EllipticF\left(1/6\,x\sqrt{-42+6\,\sqrt{73}},\frac{7\,i}{12}\sqrt{6}+i/12\sqrt{438}\right)}{\sqrt{-42+6\,\sqrt{73}}\sqrt{-2\,x^{4}+7\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+7*x^2+3)^(1/2),x)
```

```
[Out] 6/(-42+6*73^(1/2))^(1/2)*(1-(-7/6+1/6*73^(1/2))*x^2)^(1/2)*(1-(-1
/6*73^(1/2)-7/6)*x^2)^(1/2)/(-2*x^4+7*x^2+3)^(1/2)*EllipticF(1/6*
x*(-42+6*73^(1/2))^(1/2),7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^{4}+7\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 7*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+7\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 7*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 7*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 + 3), x)
```

3.28 $\int \frac{1}{\sqrt{3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right)\middle|-4-\sqrt{15}\right)$$

```
[Out] Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x
], -4 - Sqrt[15]]
```

Rubi [A] time = 0.264769, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{1}{6}\left(3+\sqrt{15}\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(-3+\sqrt{15}\right)}x\right)\middle|-4-\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(-3 + Sqrt[15])/3]*x
], -4 - Sqrt[15]]
```

Rubi in Sympy [A] time = 19.6277, size = 68, normalized size = 1.55

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{3}x\sqrt{-3+\sqrt{15}}}{3}\right)\middle|-4-\sqrt{15}\right)}{\sqrt{-6+2\sqrt{15}}\sqrt{-3+\sqrt{15}}\sqrt{6+2\sqrt{15}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(3)*x*sqrt(-3 + sqrt(15))/3), -4 -
sqrt(15))/(sqrt(-6 + 2*sqrt(15))*sqrt(-3 + sqrt(15))*sqrt(6 + 2*s
qrt(15)))
```

Mathematica [C] time = 0.0916383, size = 43, normalized size = 0.98

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{3}}x}\right)|-4+\sqrt{15}\right)}{\sqrt{3+\sqrt{15}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5/3]]*x], -4 + Sqrt[15]])
/Sqrt[3 + Sqrt[15]]
```

Maple [B] time = 0.119, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(-1+1/3\sqrt{15}\right)x^2}\sqrt{1-\left(-1-1/3\sqrt{15}\right)x^2}\,EllipticF\left(1/3\,x\sqrt{-9+3\sqrt{15}},\,i/2\sqrt{6}+i/2\sqrt{10}\right)}{\sqrt{-9+3\sqrt{15}}\sqrt{-2\,x^4+6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+6*x^2+3)^(1/2),x)
```

```
[Out] 3/(-9+3*15^(1/2))^(1/2)*(1-(-1+1/3*15^(1/2))*x^2)^(1/2)*(1-(-1-1/
3*15^(1/2))*x^2)^(1/2)/(-2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9
+3*15^(1/2))^(1/2),1/2*I*6^(1/2)+1/2*I*10^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 6*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 6*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4+6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 6*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 + 3), x)
```

3.29 $\int \frac{1}{\sqrt{3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Rubi [A] time = 0.037351, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 5*x^2 - 2*x^4],x]

[Out] EllipticF[ArcSin[x/Sqrt[3]], -6]
```

Rubi in Sympy [A] time = 7.65777, size = 12, normalized size = 1.2

$$F\left(\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)\middle|-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+5*x**2+3)**(1/2),x)

[Out] elliptic_f(asin(sqrt(3)*x/3), -6)
```

Mathematica [C] time = 0.0428563, size = 65, normalized size = 6.5

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{2}\sqrt{-2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]
*x], -1/6])/(Sqrt[2]*Sqrt[3 + 5*x^2 - 2*x^4])
```

Maple [B] time = 0.033, size = 51, normalized size = 5.1

$$\frac{\sqrt{3}}{3}\sqrt{-3\,x^{2}+9}\sqrt{2\,x^{2}+1}EllipticF\left(\frac{x\sqrt{3}}{3},i\sqrt{6}\right)\frac{1}{\sqrt{-2\,x^{4}+5\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(-2*x^4+5*x^2+3)^(1/
2)*EllipticF(1/3*x*3^(1/2),I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^{4}+5\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^{4}+5\,x^{2}+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 3), x)
```

3.30 $\int \frac{1}{\sqrt{3+4x^2-2x^4}} \, dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/
Sqrt[-2 + Sqrt[10]]
```

Rubi [A] time = 0.216075, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(2 + Sqrt[10])]*x], (-7 - 2*Sqrt[10])/3]/
Sqrt[-2 + Sqrt[10]]
```

Rubi in Sympy [A] time = 25.4788, size = 73, normalized size = 1.66

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{3}x\sqrt{-2+\sqrt{10}}}{3}\right)\middle|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\sqrt{-2+\sqrt{10}}\sqrt{4+2\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(3)*x*sqrt(-2 + sqrt(10))/3), -7/3
- 2*sqrt(10)/3)/(sqrt(-4 + 2*sqrt(10))*sqrt(-2 + sqrt(10))*sqrt(4
 + 2*sqrt(10)))
```

Mathematica [C] time = 0.0972393, size = 51, normalized size = 1.16

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sqrt[10])]*x], -7/3 + (2*S
qrt[10])/3])/Sqrt[2 + Sqrt[10]]
```

Maple [B] time = 0.098, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(-2/3+1/3\,\sqrt{10}\right)x^{2}}\sqrt{1-\left(-2/3-1/3\,\sqrt{10}\right)x^{2}}EllipticF\left(1/3\,x\sqrt{-6+3\,\sqrt{10}},i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{-6+3\,\sqrt{10}}\sqrt{-2\,x^{4}+4\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+4*x^2+3)^(1/2),x)
```

```
[Out] 3/(-6+3*10^(1/2))^(1/2)*(1-(-2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/
3-1/3*10^(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3*x
*(-6+3*10^(1/2))^(1/2),1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^{4}+4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 4*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^{4}+4\,x^{2}+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 4*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^{4}+4x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^{4}+4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 4*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 + 3), x)
```

3.31 $\int \frac{1}{\sqrt{3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{33}-3}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]
], (-7 - Sqrt[33])/4]
```

Rubi [A] time = 0.179597, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{33}-3}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7-\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[3 + Sqrt[33]]
], (-7 - Sqrt[33])/4]
```

Rubi in Sympy [A] time = 12.7166, size = 56, normalized size = 1.24

$$-\frac{4\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{-3+\sqrt{33}}}{6}\right)\middle|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{3+\sqrt{33}}\left(-\sqrt{33}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] -4*sqrt(3)*elliptic_f(asin(sqrt(6)*x*sqrt(-3 + sqrt(33))/6), -7/4
 - sqrt(33)/4)/(sqrt(3 + sqrt(33))*(-sqrt(33) + 3))
```

Mathematica [C] time = 0.0889822, size = 50, normalized size = 1.11

$$-i\sqrt{\frac{2}{3+\sqrt{33}}}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 + S
qrt[33]]], (-7 + Sqrt[33])/4]
```

Maple [B] time = 0.101, size = 84, normalized size = 1.9

$$6\,\frac{\sqrt{1-\left(-1/2+1/6\,\sqrt{33}\right)x^2}\sqrt{1-\left(-1/2-1/6\,\sqrt{33}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-18+6\,\sqrt{33}},\,i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{-18+6\,\sqrt{33}}\sqrt{-2\,x^4+3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 6/(-18+6*33^(1/2))^(1/2)*(1-(-1/2+1/6*33^(1/2))*x^2)^(1/2)*(1-(-1
/2-1/6*33^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2+3)^(1/2)*EllipticF(1/6*
x*(-18+6*33^(1/2))^(1/2),1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 + 3), x)
```

3.32 $\int \frac{1}{\sqrt{3+2x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)\middle|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt
[-1 + Sqrt[7]]
```

Rubi [A] time = 0.152414, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)\middle|\frac{1}{3}\left(-4-\sqrt{7}\right)\right)}{\sqrt{\sqrt{7}-1}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(1 + Sqrt[7])]*x], (-4 - Sqrt[7])/3]/Sqrt
[-1 + Sqrt[7]]
```

Rubi in Sympy [A] time = 20.2571, size = 71, normalized size = 1.61

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{3}x\sqrt{-1+\sqrt{7}}}{3}\right)\middle|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{-2+2\sqrt{7}}\sqrt{-1+\sqrt{7}}\sqrt{2+2\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(3)*x*sqrt(-1 + sqrt(7))/3), -4/3 -
 sqrt(7)/3)/(sqrt(-2 + 2*sqrt(7))*sqrt(-1 + sqrt(7))*sqrt(2 + 2*s
qrt(7)))
```

Mathematica [C] time = 0.0741948, size = 49, normalized size = 1.11

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7
])/3])/Sqrt[1 + Sqrt[7]]
```

Maple [B] time = 0.094, size = 84, normalized size = 1.9

$$3\,\frac{\sqrt{1-\left(-1/3+1/3\,\sqrt{7}\right)x^2}\sqrt{1-\left(-1/3-1/3\,\sqrt{7}\right)x^2}EllipticF\left(1/3\,x\sqrt{-3+3\,\sqrt{7}},i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{-3+3\,\sqrt{7}}\sqrt{-2\,x^4+2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 3/(-3+3*7^(1/2))^(1/2)*(1-(-1/3+1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3-
1/3*7^(1/2))*x^2)^(1/2)/(-2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3*x*(-
3+3*7^(1/2))^(1/2),1/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 + 3), x)
```

3.33 $\int \frac{1}{\sqrt{3+x^2-2x^4}} \, dx$

Optimal. Leaf size=20

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right) \mid -\frac{3}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rubi [A] time = 0.0399358, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right) \mid -\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + x^2 - 2*x^4],x]
```


```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 7.2481, size = 20, normalized size = 1.

$$\frac{\sqrt{2}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x}{3}\right)\middle| -\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+x**2+3)**(1/2),x)
```

```
[Out] sqrt(2)*elliptic_f(asin(sqrt(6)*x/3), -3/2)/2
```

Mathematica [A] time = 0.0455195, size = 20, normalized size = 1.

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right) \mid -\frac{3}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/3]*x], -3/2]/Sqrt[2]
```

Maple [B] time = 0.034, size = 47, normalized size = 2.4

$$\frac{\sqrt{6}}{6}\sqrt{-6\,x^2+9}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{x\sqrt{6}}{3},\frac{i}{2}\sqrt{6}\right)\frac{1}{\sqrt{-2\,x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+x^2+3)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(-2*x^4+x^2+3)^(1/2)*E
llipticF(1/3*x*6^(1/2),1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-2\,x^4+x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 + x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 + 3), x)
```

3.34 $\int \frac{1}{\sqrt{3-2x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Rubi [A] time = 0.0193289, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 2*x^4],x]

[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Rubi in Sympy [A] time = 1.16261, size = 24, normalized size = 1.33

$$\frac{6^{\frac{3}{4}}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+3)**(1/2),x)

[Out] 6**(3/4)*elliptic_f(asin(2**(1/4)*3**(3/4)*x/3), -1)/6
```

Mathematica [A] time = 0.0358055, size = 18, normalized size = 1.

$$\frac{F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[(2/3)^(1/4)*x], -1]/6^(1/4)
```

Maple [B] time = 0.05, size = 54, normalized size = 3.

$$\frac{\sqrt{3}6^{\frac{3}{4}}}{54}\sqrt{9-3\,x^2\sqrt{6}}\sqrt{9+3\,x^2\sqrt{6}}EllipticF\left(\frac{x\sqrt{3}\sqrt[4]{6}}{3},i\right)\frac{1}{\sqrt{-2\,x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+3)^(1/2),x)
```

```
[Out] 1/54*3^(1/2)*6^(3/4)*(9-3*x^2*6^(1/2))^(1/2)*(9+3*x^2*6^(1/2))^(1
/2)/(-2*x^4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*6^(1/4),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-2\,x^4+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 3), x)
```

Sympy [A] time = 1.83836, size = 37, normalized size = 2.06

$$\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{2i\pi}}{3}\right)}{12\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(2
*I*pi)/3)/(12*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3), x)
```

3.35 $\int \frac{1}{\sqrt{3-x^2-2x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)|-\frac{2}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Rubi [A] time = 0.0372534, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}(x)|-\frac{2}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - x^2 - 2*x^4],x]

[Out] EllipticF[ArcSin[x], -2/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 8.49061, size = 14, normalized size = 1.17

$$\frac{\sqrt{3}F\left(\operatorname{asin}(x)\middle|-\frac{2}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-x**2+3)**(1/2),x)

[Out] sqrt(3)*elliptic_f(asin(x), -2/3)/3
```

Mathematica [C] time = 0.0377868, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)}{\sqrt{2}\sqrt{-2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]
*x], -3/2])/(Sqrt[2]*Sqrt[3 - x^2 - 2*x^4])
```

Maple [B] time = 0.024, size = 43, normalized size = 3.6

$$\frac{EllipticF\left(x, \frac{i}{3}\sqrt{6}\right)}{3}\sqrt{-x^2+1}\sqrt{6\,x^2+9}\frac{1}{\sqrt{-2\,x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-x^2+3)^(1/2)*EllipticF
(x,1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 - x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 - x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 - x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 + 3), x)
```

3.36 $\int \frac{1}{\sqrt{3-2x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqr
t[1 + Sqrt[7]]
```

Rubi [A] time = 0.134194, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-1 + Sqrt[7])]*x], (-4 + Sqrt[7])/3]/Sqr
t[1 + Sqrt[7]]
```

Rubi in Sympy [A] time = 20.438, size = 70, normalized size = 1.67

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{3}x\sqrt{1+\sqrt{7}}}{3}\right)\middle|-\frac{4}{3}+\frac{\sqrt{7}}{3}\right)}{\sqrt{-2+2\sqrt{7}}\sqrt{1+\sqrt{7}}\sqrt{2+2\sqrt{7}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(3)*x*sqrt(1 + sqrt(7))/3), -4/3 +
sqrt(7)/3)/(sqrt(-2 + 2*sqrt(7))*sqrt(1 + sqrt(7))*sqrt(2 + 2*sqr
t(7)))
```

Mathematica [C] time = 0.0747362, size = 51, normalized size = 1.21

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqrt[7])]*x], -4/3 - Sqrt[7
]/3])/Sqrt[-1 + Sqrt[7]]
```

Maple [B] time = 0.093, size = 84, normalized size = 2.

$$3\,\frac{\sqrt{1-\left(1/3\,\sqrt{7}+1/3\right)x^{2}}\sqrt{1-\left(-1/3\,\sqrt{7}+1/3\right)x^{2}}\mathit{EllipticF}\left(1/3\,x\sqrt{3+3\,\sqrt{7}},i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{3+3\,\sqrt{7}}\sqrt{-2\,x^{4}-2\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 3/(3+3*7^(1/2))^(1/2)*(1-(1/3*7^(1/2)+1/3)*x^2)^(1/2)*(1-(-1/3*7^
(1/2)+1/3)*x^2)^(1/2)/(-2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*(3+3
*7^(1/2))^(1/2),1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^{4}-2\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 2*x^2 + 3),x, algorithm="fricas")

[Out] integral(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-2*x**2+3)**(1/2),x)

[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 2*x^2 + 3),x, algorithm="giac")

[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 + 3), x)
```

3.37 $\int \frac{1}{\sqrt{3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]
], (-7 + Sqrt[33])/4]
```

Rubi [A] time = 0.148967, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{3+\sqrt{33}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(3 + Sqrt[33])]*EllipticF[ArcSin[(2*x)/Sqrt[-3 + Sqrt[33]]
], (-7 + Sqrt[33])/4]
```

Rubi in Sympy [A] time = 13.3158, size = 53, normalized size = 1.23

$$\frac{4\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{3+\sqrt{33}}}{6}\right)\middle|-\frac{7}{4}+\frac{\sqrt{33}}{4}\right)}{\sqrt{-3+\sqrt{33}}\left(3+\sqrt{33}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] 4*sqrt(3)*elliptic_f(asin(sqrt(6)*x*sqrt(3 + sqrt(33))/6), -7/4 +
 sqrt(33)/4)/(sqrt(-3 + sqrt(33))*(3 + sqrt(33)))
```

Mathematica [C] time = 0.0950743, size = 52, normalized size = 1.21

$$-i\sqrt{\frac{2}{\sqrt{33}-3}}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 - 3*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-3 + Sqrt[33])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 + S
qrt[33]]], -7/4 - Sqrt[33]/4]
```

Maple [B] time = 0.093, size = 84, normalized size = 2.

$$6\,\frac{\sqrt{1-\left(1/6\,\sqrt{33}+1/2\right)x^2}\sqrt{1-\left(-1/6\,\sqrt{33}+1/2\right)x^2}\,EllipticF\left(1/6\,x\sqrt{18+6\,\sqrt{33}},i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{18+6\,\sqrt{33}}\sqrt{-2\,x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 6/(18+6*33^(1/2))^(1/2)*(1-(1/6*33^(1/2)+1/2)*x^2)^(1/2)*(1-(-1/6
*33^(1/2)+1/2)*x^2)^(1/2)/(-2*x^4-3*x^2+3)^(1/2)*EllipticF(1/6*x*
(18+6*33^(1/2))^(1/2),1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3*x^2 + 3),x, algorithm="fricas")

[Out] integral(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-3*x**2+3)**(1/2),x)

[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3*x^2 + 3),x, algorithm="giac")

[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 + 3), x)
```

3.38 $\int \frac{1}{\sqrt{3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)\middle|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]
/Sqrt[2 + Sqrt[10]]
```

Rubi [A] time = 0.219771, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)\middle|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] EllipticF[ArcSin[Sqrt[2/(-2 + Sqrt[10])]*x], (-7 + 2*Sqrt[10])/3]
/Sqrt[2 + Sqrt[10]]
```

Rubi in Sympy [A] time = 19.6938, size = 71, normalized size = 1.61

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{3}x\sqrt{2+\sqrt{10}}}{3}\right)\middle|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{-4+2\sqrt{10}}\sqrt{2+\sqrt{10}}\sqrt{4+2\sqrt{10}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(3)*x*sqrt(2 + sqrt(10))/3), -7/3 +
 2*sqrt(10)/3)/(sqrt(-4 + 2*sqrt(10))*sqrt(2 + sqrt(10))*sqrt(4 +
 2*sqrt(10)))
```

Mathematica [C] time = 0.101334, size = 51, normalized size = 1.16

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)\middle|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{\sqrt{10}-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqrt[10])]*x], -7/3 - (2*Sq
rt[10])/3])/Sqrt[-2 + Sqrt[10]]
```

Maple [B] time = 0.091, size = 84, normalized size = 1.9

$$3\,{\frac{\sqrt{1-\left(2/3+1/3\,\sqrt{10}\right)x^{2}}\sqrt{1-\left(2/3-1/3\,\sqrt{10}\right)x^{2}}{\it EllipticF}\left(1/3\,x\sqrt{6+3\,\sqrt{10}},i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{6+3\,\sqrt{10}}\sqrt{-2\,x^{4}-4\,x^{2}+3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-4*x^2+3)^(1/2),x)
```

```
[Out] 3/(6+3*10^(1/2))^(1/2)*(1-(2/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3-1
/3*10^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*(6
+3*10^(1/2))^(1/2),1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^{4}-4\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 4*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 4*x^2 + 3),x, algorithm="fricas")

[Out] integral(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-4*x**2+3)**(1/2),x)

[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 4*x^2 + 3),x, algorithm="giac")

[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 + 3), x)
```

3.39 $\int \frac{1}{\sqrt{3-5x^2-2x^4}} dx$

Optimal. Leaf size=18

$$\frac{F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{6}}$$

```
[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Rubi [A] time = 0.0419082, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 5*x^2 - 2*x^4],x]

[Out] EllipticF[ArcSin[Sqrt[2]*x], -1/6]/Sqrt[6]
```

Rubi in Sympy [A] time = 7.83337, size = 19, normalized size = 1.06

$$\frac{\sqrt{6}F\left(\operatorname{asin}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-5*x**2+3)**(1/2),x)

[Out] sqrt(6)*elliptic_f(asin(sqrt(2)*x), -1/6)/6
```

Mathematica [B] time = 0.043255, size = 54, normalized size = 3.

$$\frac{\sqrt{1-2x^2}\sqrt{x^2+3}F\left(\sin^{-1}\left(\sqrt{2}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{-2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6]
)/(Sqrt[6]*Sqrt[3 - 5*x^2 - 2*x^4])
```

Maple [B] time = 0.026, size = 50, normalized size = 2.8

$$\frac{\sqrt{2}EllipticF\left(\sqrt{2}x, \frac{i}{6}\sqrt{6}\right)}{6}\sqrt{-2\,x^2+1}\sqrt{3\,x^2+9}\frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-5*x^2+3)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-2*x^4-5*x^2+3)^(1/
2)*EllipticF(2^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-5\,x^2+3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 - 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 - 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 + 3), x)
```

3.40 $\int \frac{1}{\sqrt{3-6x^2-2x^4}}\,dx$

Optimal. Leaf size=42

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)|-4+\sqrt{15}\right)$$

```
[Out] Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x
], -4 + Sqrt[15]]
```

Rubi [A] time = 0.254014, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{1}{6}\left(\sqrt{15}-3\right)}F\left(\sin^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{15}\right)}x\right)|-4+\sqrt{15}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[(-3 + Sqrt[15])/6]*EllipticF[ArcSin[Sqrt[(3 + Sqrt[15])/3]*x
], -4 + Sqrt[15]]
```

Rubi in Sympy [A] time = 19.7659, size = 66, normalized size = 1.57

$$\frac{2\sqrt{6}F\left(\operatorname{asin}\left(\frac{\sqrt{3}x\sqrt{3+\sqrt{15}}}{3}\right)\middle|-4+\sqrt{15}\right)}{\sqrt{-6+2\sqrt{15}}\sqrt{3+\sqrt{15}}\sqrt{6+2\sqrt{15}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] 2*sqrt(6)*elliptic_f(asin(sqrt(3)*x*sqrt(3 + sqrt(15))/3), -4 + s
qrt(15))/(sqrt(-6 + 2*sqrt(15))*sqrt(3 + sqrt(15))*sqrt(6 + 2*sqr
t(15)))
```

Mathematica [C] time = 0.0993957, size = 45, normalized size = 1.07

$$-\frac{iF\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{3}}}x\right)\middle|-4-\sqrt{15}\right)}{\sqrt{\sqrt{15}-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 - 6*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[5/3]]*x], -4 - Sqrt[15]]
)/Sqrt[-3 + Sqrt[15]]
```

Maple [B] time = 0.101, size = 84, normalized size = 2.

$$3\,\frac{\sqrt{1-\left(1+1/3\,\sqrt{15}\right)x^2}\sqrt{1-\left(1-1/3\,\sqrt{15}\right)x^2}\,EllipticF\left(1/3\,x\sqrt{9+3\,\sqrt{15}},i/2\sqrt{10}-i/2\sqrt{6}\right)}{\sqrt{9+3\,\sqrt{15}}\sqrt{-2\,x^4-6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-6*x^2+3)^(1/2),x)
```

```
[Out] 3/(9+3*15^(1/2))^(1/2)*(1-(1+1/3*15^(1/2))*x^2)^(1/2)*(1-(1-1/3*1
5^(1/2))*x^2)^(1/2)/(-2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*1
5^(1/2))^(1/2),1/2*I*10^(1/2)-1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 6*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 6*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4-6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 6*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 6*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 6*x^2 + 3), x)
```

3.41 $\int \frac{1}{\sqrt{3-7x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{7+\sqrt{73}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

```
[Out] Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]
], (-61 + 7*Sqrt[73])/12]
```

Rubi [A] time = 0.122729, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{7+\sqrt{73}}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{-7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61+7\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(7 + Sqrt[73])]*EllipticF[ArcSin[(2*x)/Sqrt[-7 + Sqrt[73]]
], (-61 + 7*Sqrt[73])/12]
```

Rubi in Sympy [A] time = 12.9955, size = 54, normalized size = 1.2

$$\frac{4\sqrt{3}F\left(\operatorname{asin}\left(\frac{\sqrt{6}x\sqrt{7+\sqrt{73}}}{6}\right)\middle|-\frac{61}{12}+\frac{7\sqrt{73}}{12}\right)}{\sqrt{-7+\sqrt{73}}\left(7+\sqrt{73}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] 4*sqrt(3)*elliptic_f(asin(sqrt(6)*x*sqrt(7 + sqrt(73))/6), -61/12
 + 7*sqrt(73)/12)/(sqrt(-7 + sqrt(73))*(7 + sqrt(73)))
```

Mathematica [C] time = 0.0766011, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{\sqrt{73}-7}}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 - 7*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(-7 + Sqrt[73])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 + S
qrt[73]]], (-61 - 7*Sqrt[73])/12]
```

Maple [B] time = 0.108, size = 84, normalized size = 1.9

$$6\,\frac{\sqrt{1-\left(1/6\,\sqrt{73}+7/6\right)x^2}\sqrt{1-\left(7/6-1/6\,\sqrt{73}\right)x^2}EllipticF\left(1/6\,x\sqrt{42+6\,\sqrt{73}},i/12\sqrt{438}-\frac{7\,i}{12}\sqrt{6}\right)}{\sqrt{42+6\,\sqrt{73}}\sqrt{-2\,x^4-7\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-7*x^2+3)^(1/2),x)
```

```
[Out] 6/(42+6*73^(1/2))^(1/2)*(1-(1/6*73^(1/2)+7/6)*x^2)^(1/2)*(1-(7/6-
1/6*73^(1/2))*x^2)^(1/2)/(-2*x^4-7*x^2+3)^(1/2)*EllipticF(1/6*x*(
42+6*73^(1/2))^(1/2),1/12*I*438^(1/2)-7/12*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 7*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4-7\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 7*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 7*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 7*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 7*x^2 + 3), x)
```

3.42 $\int \frac{1}{\sqrt{-2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{x^2+2}\sqrt{3x^2-1}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{3x^4+5x^2-2}}$$

```
[Out] (Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sq
rt[-1 + 3*x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.028874, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{x^2+2}\sqrt{3x^2-1}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{2}}x}{\sqrt{3x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 + x^2]*Sqrt[-1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7/2]*x)/Sq
rt[-1 + 3*x^2]], 6/7])/(Sqrt[7]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.84405, size = 70, normalized size = 1.04

$$\frac{\sqrt{2}\sqrt{\frac{12x^2}{7}-\frac{4}{7}}\sqrt{2x^2+4}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{12x^2}{7}-\frac{4}{7}}}\right)\middle|\frac{6}{7}\right)}{4\sqrt{3x^4+5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(12*x**2/7 - 4/7)*sqrt(2*x**2 + 4)*elliptic_f(asin(sq
rt(2)*x/sqrt(12*x**2/7 - 4/7)), 6/7)/(4*sqrt(3*x**4 + 5*x**2 - 2)
)
```

Mathematica [A] time = 0.0411476, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-3x^2}\sqrt{x^2+2}F\left(\sin^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{3x^4+5x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 - 3*x^2]*Sqrt[2 + x^2]*EllipticF[ArcSin[Sqrt[3]*x], -1/6]
)/(Sqrt[6]*Sqrt[-2 + 5*x^2 + 3*x^4])
```

Maple [C] time = 0.013, size = 53, normalized size = 0.8

$$-\frac{i}{2}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x, i\sqrt{6}\right)\sqrt{2x^2+4}\sqrt{-3x^2+1}\frac{1}{\sqrt{3x^4+5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+5*x^2-2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(-3*x^2+1)^(1/2)/(3*x^4+5*x^2-2)^(
1/2)*EllipticF(1/2*I*2^(1/2)*x,I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4+5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3x^4+5x^2-2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 + 5x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + 5\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 - 2), x)
```

3.43 $\int \frac{1}{\sqrt{-2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

```
[Out] (Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2
+ (2 + Sqrt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
2 + (2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2
 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

Rubi [A] time = 0.121361, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{2-\left(2-\sqrt{10}\right)x^2}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-2}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{2-\left(2+\sqrt{10}\right)x^2}}\sqrt{3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (2 - Sqrt[10])*x^2)/(2 - (2 + Sqrt[10])*x^2)]*Sqrt[-2
+ (2 + Sqrt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
2 + (2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2
 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-2 + 4*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.16877, size = 126, normalized size = 0.89

$$\frac{10^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{10}+4\right)-4}{x^2\left(4+2\sqrt{10}\right)-4}}\sqrt{x^2\left(4+2\sqrt{10}\right)-4}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{10}x}{\sqrt{x^2\left(4+2\sqrt{10}\right)-4}}\right)\middle|\frac{\sqrt{10}}{10}+\frac{1}{2}\right)}{40\sqrt{-\frac{1}{x^2\left(4+2\sqrt{10}\right)-4}}\sqrt{3x^4+4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] 10**(3/4)*sqrt((x**2*(-2*sqrt(10) + 4) - 4)/(x**2*(4 + 2*sqrt(10)
) - 4))*sqrt(x**2*(4 + 2*sqrt(10)) - 4)*elliptic_f(asin(2*10**(1/
4)*x/sqrt(x**2*(4 + 2*sqrt(10)) - 4)), sqrt(10)/10 + 1/2)/(40*sqr
t(-1/(x**2*(4 + 2*sqrt(10)) - 4))*sqrt(3*x**4 + 4*x**2 - 2))
```

Mathematica [C] time = 0.114596, size = 81, normalized size = 0.57

$$-\frac{i\sqrt{-3x^4-4x^2+2}F\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7-2\sqrt{10}\right)\right)}{\sqrt{\sqrt{10}-2}\sqrt{3x^4+4x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[
5/2]]*x], (-7 - 2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-2 + 4*
x^2 + 3*x^4])
```

Maple [C] time = 0.041, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/2\,\sqrt{10}+1\right)x^2}\sqrt{1-\left(1+1/2\,\sqrt{10}\right)x^2}EllipticF\left(1/2\,\sqrt{4-2\,\sqrt{10}}x,i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{4-2\,\sqrt{10}}\sqrt{3\,x^4+4\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+4*x^2-2)^(1/2),x)
```

```
[Out] 2/(4-2*10^(1/2))^(1/2)*(1-(-1/2*10^(1/2)+1)*x^2)^(1/2)*(1-(1+1/2*
10^(1/2))*x^2)^(1/2)/(3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*10^
(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 4*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+4\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 4*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 4*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 - 2), x)
```

3.44 $\int \frac{1}{\sqrt{-2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

```
[Out] (Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4
+ (3 + Sqrt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-4 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/
4)*Sqrt[(4 - (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rubi [A] time = 0.117822, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{\frac{4-\left(3-\sqrt{33}\right)x^2}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-4}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{4-\left(3+\sqrt{33}\right)x^2}}\sqrt{3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(4 - (3 - Sqrt[33])*x^2)/(4 - (3 + Sqrt[33])*x^2)]*Sqrt[-4
+ (3 + Sqrt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-4 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(2*Sqrt[2]*33^(1/
4)*Sqrt[(4 - (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-2 + 3*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.01665, size = 126, normalized size = 0.86

$$\frac{\sqrt{2}\cdot 33^{\frac{3}{4}}\sqrt{\frac{x^2\left(-\sqrt{33}+3\right)-4}{x^2\left(3+\sqrt{33}\right)-4}}\sqrt{x^2\left(3+\sqrt{33}\right)-4}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{x^2\left(3+\sqrt{33}\right)-4}}\right)\middle|\frac{\sqrt{33}}{22}+\frac{1}{2}\right)}{132\sqrt{-\frac{1}{x^2\left(3+\sqrt{33}\right)-4}}\sqrt{3x^4+3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] sqrt(2)*33**(3/4)*sqrt((x**2*(-sqrt(33) + 3) - 4)/(x**2*(3 + sqrt
(33)) - 4))*sqrt(x**2*(3 + sqrt(33)) - 4)*elliptic_f(asin(sqrt(2)
*33**(1/4)*x/sqrt(x**2*(3 + sqrt(33)) - 4)), sqrt(33)/22 + 1/2)/(
132*sqrt(-1/(x**2*(3 + sqrt(33)) - 4))*sqrt(3*x**4 + 3*x**2 - 2))
```

Mathematica [C] time = 0.123249, size = 83, normalized size = 0.57

$$-\frac{i\sqrt{-6x^4-6x^2+4}F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{3+\sqrt{33}}}x\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{\sqrt{33}-3}\sqrt{3x^4+3x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 - 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(3 + Sqr
t[33])]*x], -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-2 + 3*
x^2 + 3*x^4])
```

Maple [C] time = 0.041, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/4\,\sqrt{33}+3/4\right)x^2}\sqrt{1-\left(1/4\,\sqrt{33}+3/4\right)x^2}EllipticF\left(1/2\,\sqrt{3-\sqrt{33}}x,i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{3-\sqrt{33}}\sqrt{3\,x^4+3\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+3*x^2-2)^(1/2),x)
```

```
[Out] 2/(3-33^(1/2))^(1/2)*(1-(-1/4*33^(1/2)+3/4)*x^2)^(1/2)*(1-(1/4*33
^(1/2)+3/4)*x^2)^(1/2)/(3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*(3-33^
(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 3*x^2 - 2),x, algorithm="maxima")

[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4 + 3\,x^2 - 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 3*x^2 - 2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 + 3x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+3*x**2-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 + 3*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + 3\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 3*x^2 - 2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 + 3*x^2 - 2), x)
```

3.45 $\int \frac{1}{\sqrt{-2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=141

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}$$

```
[Out] (Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 +
(1 + Sqrt[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 +
 (1 + Sqrt[7])*x^2]], (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1
+ Sqrt[7])*x^2)^(-1)]*Sqrt[-2 + 2*x^2 + 3*x^4])
```

Rubi [A] time = 0.0896327, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{\frac{2-\left(1-\sqrt{7}\right)x^2}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{2-\left(1+\sqrt{7}\right)x^2}}\sqrt{3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2 - (1 - Sqrt[7])*x^2)/(2 - (1 + Sqrt[7])*x^2)]*Sqrt[-2 +
(1 + Sqrt[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 +
 (1 + Sqrt[7])*x^2]], (7 + Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 - (1
+ Sqrt[7])*x^2)^(-1)]*Sqrt[-2 + 2*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.61628, size = 126, normalized size = 0.89

$$\frac{7^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{7}+2\right)-4}{x^2\left(2+2\sqrt{7}\right)-4}}\sqrt{x^2\left(2+2\sqrt{7}\right)-4}\,F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{7}x}{\sqrt{x^2\left(2+2\sqrt{7}\right)-4}}\right)\middle|\frac{\sqrt{7}}{14}+\frac{1}{2}\right)}{28\sqrt{-\frac{1}{x^2\left(2+2\sqrt{7}\right)-4}}\sqrt{3x^4+2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] 7**(3/4)*sqrt((x**2*(-2*sqrt(7) + 2) - 4)/(x**2*(2 + 2*sqrt(7)) -
 4))*sqrt(x**2*(2 + 2*sqrt(7)) - 4)*elliptic_f(asin(2*7**(1/4)*x/
sqrt(x**2*(2 + 2*sqrt(7)) - 4)), sqrt(7)/14 + 1/2)/(28*sqrt(-1/(x
**2*(2 + 2*sqrt(7)) - 4))*sqrt(3*x**4 + 2*x**2 - 2))
```

Mathematica [C] time = 0.0905174, size = 83, normalized size = 0.59

$$-\frac{i\sqrt{-3x^4-2x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{1+\sqrt{7}}}x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}\sqrt{3x^4+2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 - 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(1 + Sqr
t[7])]*x], -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-2 + 2*x^2
 + 3*x^4])
```

Maple [C] time = 0.042, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/2\,\sqrt{7}+1/2\right)x^2}\sqrt{1-\left(1/2\,\sqrt{7}+1/2\right)x^2}\,EllipticF\left(1/2\,\sqrt{2-2\,\sqrt{7}}x,i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{2-2\,\sqrt{7}}\sqrt{3\,x^4+2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+2*x^2-2)^(1/2),x)
```

```
[Out] 2/(2-2*7^(1/2))^(1/2)*(1-(-1/2*7^(1/2)+1/2)*x^2)^(1/2)*(1-(1/2*7^
(1/2)+1/2)*x^2)^(1/2)/(3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(2-2*7^
(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2*x^2 - 2),x, algorithm="maxima")

[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2\,x^2-2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2*x^2 - 2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+2*x**2-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 + 2*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2*x^2 - 2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 + 2*x^2 - 2), x)
```

3.46 $\int \frac{1}{\sqrt{-2+x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2+1}\sqrt{3x^2-2}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{3x^4+x^2-2}}$$

```
[Out] (Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt
[-2 + 3*x^2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Rubi [A] time = 0.0258684, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\sqrt{x^2+1}\sqrt{3x^2-2}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{3x^2-2}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt
[-2 + 3*x^2]], 3/5])/(Sqrt[5]*Sqrt[-2 + x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 2.97203, size = 68, normalized size = 1.08

$$\frac{\sqrt{2}\sqrt{\frac{6x^2}{5}-\frac{4}{5}}\sqrt{4x^2+4}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{6x^2}{5}-\frac{4}{5}}}\right)\middle|\frac{3}{5}\right)}{4\sqrt{3x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+x**2-2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(6*x**2/5 - 4/5)*sqrt(4*x**2 + 4)*elliptic_f(asin(sqr
t(2)*x/sqrt(6*x**2/5 - 4/5)), 3/5)/(4*sqrt(3*x**4 + x**2 - 2))
```

Mathematica [A] time = 0.0505583, size = 48, normalized size = 0.76

$$\frac{\sqrt{\left(\frac{2}{3}-x^2\right)\left(x^2+1\right)}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{3x^4+x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 + x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[(2/3 - x^2)*(1 + x^2)]*EllipticF[ArcSin[Sqrt[3/2]*x], -2/3]
)/Sqrt[-2 + x^2 + 3*x^4]
```

Maple [C] time = 0.012, size = 43, normalized size = 0.7

$$-\frac{i}{2}EllipticF\left(ix,\frac{i}{2}\sqrt{6}\right)\sqrt{x^2+1}\sqrt{-6\,x^2+4}\frac{1}{\sqrt{3\,x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+x^2-2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4+x^2-2)^(1/2)*Ellipti
cF(I*x,1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 + x^2 - 2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + x^2 - 2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 - 2), x)
```

3.47 $\int \frac{1}{\sqrt{-2+3x^4}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{\sqrt{6}x^2-2}\sqrt{\frac{\sqrt{6}x^2+2}{2-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{3x^4-2}}$$

```
[Out] (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]
*EllipticF[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/
2])/(2*6^(1/4)*Sqrt[(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Rubi [A] time = 0.0582782, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\sqrt{\sqrt{6}x^2-2}\sqrt{\frac{\sqrt{6}x^2+2}{2-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-2}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{\frac{1}{2-\sqrt{6}x^2}}\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + Sqrt[6]*x^2]*Sqrt[(2 + Sqrt[6]*x^2)/(2 - Sqrt[6]*x^2)]
*EllipticF[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-2 + Sqrt[6]*x^2]], 1/
2])/(2*6^(1/4)*Sqrt[(2 - Sqrt[6]*x^2)^(-1)]*Sqrt[-2 + 3*x^4])
```

Rubi in Sympy [A] time = 1.55652, size = 51, normalized size = 0.44

$$\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}\sqrt{-\frac{3x^4}{2}+1}F\left(\operatorname{asin}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-1\right)}{3\sqrt{3x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-2)**(1/2),x)
```

```
[Out] 2**(1/4)*3**(3/4)*sqrt(-3*x**4/2 + 1)*elliptic_f(asin(2**(3/4)*3*
*(1/4)*x/2), -1)/(3*sqrt(3*x**4 - 2))
```

Mathematica [A] time = 0.0430768, size = 40, normalized size = 0.35

$$\frac{\sqrt{2-3x^4}F\left(\sin^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{3x^4-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^4]*EllipticF[ArcSin[(3/2)^(1/4)*x], -1])/(6^(1/4)*S
qrt[-2 + 3*x^4])
```

Maple [C] time = 0.032, size = 56, normalized size = 0.5

$$\frac{1}{2\sqrt{-2\sqrt{6}}}\sqrt{4+2x^2\sqrt{6}}\sqrt{4-2x^2\sqrt{6}}EllipticF\left(\frac{\sqrt{-2\sqrt{6}}x}{2}, i\right)\frac{1}{\sqrt{3x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-2)^(1/2),x)
```

```
[Out] 1/2/(-2*6^(1/2))^(1/2)*(4+2*x^2*6^(1/2))^(1/2)*(4-2*x^2*6^(1/2))^
(1/2)/(3*x^4-2)^(1/2)*EllipticF(1/2*(-2*6^(1/2))^(1/2)*x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2), x)
```

Sympy [A] time = 1.83453, size = 34, normalized size = 0.3

$$-\frac{\sqrt{2}ix\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4}{2}\right)}{8\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-2)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4/2)/(8*ga
mma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2), x)
```

3.48 $\int \frac{1}{\sqrt{-2-x^2+3x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2-1}\sqrt{3x^2+2}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{3x^4-x^2-2}}$$

```
[Out] (Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sq
rt[-1 + x^2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Rubi [A] time = 0.0247881, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{x^2-1}\sqrt{3x^2+2}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{2}}x}{\sqrt{x^2-1}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[ArcSin[(Sqrt[5/2]*x)/Sq
rt[-1 + x^2]], 2/5])/(Sqrt[5]*Sqrt[-2 - x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.12424, size = 68, normalized size = 1.05

$$\frac{\sqrt{2}\sqrt{\frac{4x^2}{5}-\frac{4}{5}}\sqrt{6x^2+4}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{4x^2}{5}-\frac{4}{5}}}\right)\middle|\frac{2}{5}\right)}{4\sqrt{3x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-x**2-2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(4*x**2/5 - 4/5)*sqrt(6*x**2 + 4)*elliptic_f(asin(sqr
t(2)*x/sqrt(4*x**2/5 - 4/5)), 2/5)/(4*sqrt(3*x**4 - x**2 - 2))
```

Mathematica [C] time = 0.0427372, size = 60, normalized size = 0.92

$$-\frac{i\sqrt{1-x^2}\sqrt{3x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|-\frac{2}{3}\right)}{\sqrt{9x^4-3x^2-6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]
*x], -2/3])/Sqrt[-6 - 3*x^2 + 9*x^4]
```

Maple [C] time = 0.035, size = 53, normalized size = 0.8

$$-\frac{i}{6}\sqrt{6}EllipticF\left(\frac{i}{2}x\sqrt{6},\frac{i}{3}\sqrt{6}\right)\sqrt{6\,x^2+4}\sqrt{-x^2+1}\frac{1}{\sqrt{3\,x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(-x^2+1)^(1/2)/(3*x^4-x^2-2)^(1/2)
*EllipticF(1/2*I*x*6^(1/2),1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - x^2 - 2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*x^4 - x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-x**2-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 - x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\, x^4 - x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - x^2 - 2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 - x^2 - 2), x)
```

3.49 $\int \frac{1}{\sqrt{-2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

```
[Out] (Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (
1 - Sqrt[7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 -
 (1 - Sqrt[7])*x^2]], (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1
- Sqrt[7])*x^2)^(-1)]*Sqrt[-2 - 2*x^2 + 3*x^4])
```

Rubi [A] time = 0.0942734, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+2}{\left(1-\sqrt{7}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-2}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{2\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+2}}\sqrt{3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```


```
[Out] (Sqrt[-2 - (1 - Sqrt[7])*x^2]*Sqrt[(2 + (1 + Sqrt[7])*x^2)/(2 + (
1 - Sqrt[7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-2 -
 (1 - Sqrt[7])*x^2]], (7 - Sqrt[7])/14])/(2*7^(1/4)*Sqrt[(2 + (1
- Sqrt[7])*x^2)^(-1)]*Sqrt[-2 - 2*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.93774, size = 128, normalized size = 0.86

$$\frac{7^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{7}-2\right)-4}{x^2\left(-2+2\sqrt{7}\right)-4}}\sqrt{x^2\left(-2+2\sqrt{7}\right)-4}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{7}x}{\sqrt{x^2\left(-2+2\sqrt{7}\right)-4}}\right)\middle|-\frac{\sqrt{7}}{14}+\frac{1}{2}\right)}{28\sqrt{-\frac{1}{x^2\left(-2+2\sqrt{7}\right)-4}}\sqrt{3x^4-2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] 7**(3/4)*sqrt((x**2*(-2*sqrt(7) - 2) - 4)/(x**2*(-2 + 2*sqrt(7))
- 4))*sqrt(x**2*(-2 + 2*sqrt(7)) - 4)*elliptic_f(asin(2*7**(1/4)*
x/sqrt(x**2*(-2 + 2*sqrt(7)) - 4)), -sqrt(7)/14 + 1/2)/(28*sqrt(-
1/(x**2*(-2 + 2*sqrt(7)) - 4))*sqrt(3*x**4 - 2*x**2 - 2))
```

Mathematica [C] time = 0.0850432, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{-3x^4+2x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{3x^4-2x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 2*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[3/(-1 + Sq
rt[7])]*x], (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-2 - 2*x^2
 + 3*x^4])
```

Maple [C] time = 0.042, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1/2-1/2\,\sqrt{7}\right)x^2}\sqrt{1-\left(-1/2+1/2\,\sqrt{7}\right)x^2}\mathit{EllipticF}\left(1/2\,\sqrt{-2-2\,\sqrt{7}}x,i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{-2-2\,\sqrt{7}}\sqrt{3\,x^4-2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-2*x^2-2)^(1/2),x)
```

```
[Out] 2/(-2-2*7^(1/2))^(1/2)*(1-(-1/2-1/2*7^(1/2))*x^2)^(1/2)*(1-(-1/2+
1/2*7^(1/2))*x^2)^(1/2)/(3*x^4-2*x^2-2)^(1/2)*EllipticF(1/2*(-2-2
*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-2\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4-2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 - 2), x)
```

3.50 $\int \frac{1}{\sqrt{-2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

```
[Out] (Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 +
 (3 - Sqrt[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-4 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/
4)*Sqrt[(4 + (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rubi [A] time = 0.124423, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+4}{\left(3-\sqrt{33}\right)x^2+4}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-4}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\sqrt{2}\sqrt[4]{33}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+4}}\sqrt{3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-4 - (3 - Sqrt[33])*x^2]*Sqrt[(4 + (3 + Sqrt[33])*x^2)/(4 +
 (3 - Sqrt[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-4 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(2*Sqrt[2]*33^(1/
4)*Sqrt[(4 + (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-2 - 3*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.9817, size = 128, normalized size = 0.84

$$\frac{\sqrt{2}\cdot 33^{\frac{3}{4}}\sqrt{\frac{x^2\left(-\sqrt{33}-3\right)-4}{x^2\left(-3+\sqrt{33}\right)-4}}\sqrt{x^2\left(-3+\sqrt{33}\right)-4}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{x^2\left(-3+\sqrt{33}\right)-4}}\right)\middle|-\frac{\sqrt{33}}{22}+\frac{1}{2}\right)}{132\sqrt{-\frac{1}{x^2\left(-3+\sqrt{33}\right)-4}}\sqrt{3x^4-3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] sqrt(2)*33**(3/4)*sqrt((x**2*(-sqrt(33) - 3) - 4)/(x**2*(-3 + sqr
t(33)) - 4))*sqrt(x**2*(-3 + sqrt(33)) - 4)*elliptic_f(asin(sqrt(
2)*33**(1/4)*x/sqrt(x**2*(-3 + sqrt(33)) - 4)), -sqrt(33)/22 + 1/
2)/(132*sqrt(-1/(x**2*(-3 + sqrt(33)) - 4))*sqrt(3*x**4 - 3*x**2
- 2))
```

Mathematica [C] time = 0.115668, size = 81, normalized size = 0.53

$$-\frac{i\sqrt{-6x^4+6x^2+4}F\left(i\sinh^{-1}\left(\sqrt{\frac{6}{-3+\sqrt{33}}}x\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{3x^4-3x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[4 + 6*x^2 - 6*x^4]*EllipticF[I*ArcSinh[Sqrt[6/(-3 + Sq
rt[33])]*x], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-2 - 3*
x^2 + 3*x^4])
```

Maple [C] time = 0.068, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-3/4-1/4\,\sqrt{33}\right)x^2}\sqrt{1-\left(-3/4+1/4\,\sqrt{33}\right)x^2}\mathit{EllipticF}\left(1/2\,\sqrt{-\sqrt{33}-3}x,\,i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{-\sqrt{33}-3}\sqrt{3\,x^4-3\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-3*x^2-2)^(1/2),x)
```

```
[Out] 2/(-33^(1/2)-3)^(1/2)*(1-(-3/4-1/4*33^(1/2))*x^2)^(1/2)*(1-(-3/4+
1/4*33^(1/2))*x^2)^(1/2)/(3*x^4-3*x^2-2)^(1/2)*EllipticF(1/2*(-33
^(1/2)-3)^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 3*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-3\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 3*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4-3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 3*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 - 2), x)
```

3.51 $\int \frac{1}{\sqrt{-2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

```
[Out] (Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 +
 (2 - Sqrt[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
2 - (2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2
 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

Rubi [A] time = 0.133067, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+2}{\left(2-\sqrt{10}\right)x^2+2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-2}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+2}}\sqrt{3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 - (2 - Sqrt[10])*x^2]*Sqrt[(2 + (2 + Sqrt[10])*x^2)/(2 +
 (2 - Sqrt[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
2 - (2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/(2*10^(1/4)*Sqrt[(2
 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-2 - 4*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.04081, size = 128, normalized size = 0.86

$$\frac{10^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{10}-4\right)-4}{x^2\left(-4+2\sqrt{10}\right)-4}}\sqrt{x^2\left(-4+2\sqrt{10}\right)-4}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{10}x}{\sqrt{x^2\left(-4+2\sqrt{10}\right)-4}}\right)\middle|-\frac{\sqrt{10}}{10}+\frac{1}{2}\right)}{40\sqrt{-\frac{1}{x^2\left(-4+2\sqrt{10}\right)-4}}\sqrt{3x^4-4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] 10**(3/4)*sqrt((x**2*(-2*sqrt(10) - 4) - 4)/(x**2*(-4 + 2*sqrt(10
)) - 4))*sqrt(x**2*(-4 + 2*sqrt(10)) - 4)*elliptic_f(asin(2*10**(
1/4)*x/sqrt(x**2*(-4 + 2*sqrt(10)) - 4)), -sqrt(10)/10 + 1/2)/(40
*sqrt(-1/(x**2*(-4 + 2*sqrt(10)) - 4))*sqrt(3*x**4 - 4*x**2 - 2))
```

Mathematica [C] time = 0.114784, size = 81, normalized size = 0.55

$$-\frac{i\sqrt{-3x^4+4x^2+2}F\left(i\sinh^{-1}\left(\sqrt{1+\sqrt{\frac{5}{2}}}x\right)|\frac{1}{3}\left(-7+2\sqrt{10}\right)\right)}{\sqrt{2+\sqrt{10}}\sqrt{3x^4-4x^2-2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[2 + 4*x^2 - 3*x^4]*EllipticF[I*ArcSinh[Sqrt[1 + Sqrt[5
/2]]*x], (-7 + 2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-2 - 4*x^
2 + 3*x^4])
```

Maple [C] time = 0.042, size = 84, normalized size = 0.6

$$2\,\frac{\sqrt{1-\left(-1-1/2\,\sqrt{10}\right)x^2}\sqrt{1-\left(-1+1/2\,\sqrt{10}\right)x^2}EllipticF\left(1/2\,\sqrt{-4-2\,\sqrt{10}}x,i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{-4-2\,\sqrt{10}}\sqrt{3\,x^4-4\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-4*x^2-2)^(1/2),x)
```

```
[Out] 2/(-4-2*10^(1/2))^(1/2)*(1-(-1-1/2*10^(1/2))*x^2)^(1/2)*(1-(-1+1/
2*10^(1/2))*x^2)^(1/2)/(3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-4-2*
10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 4*x^2 - 2),x, algorithm="maxima")

[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{3\,x^{4}-4\,x^{2}-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 4*x^2 - 2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^{4}-4x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-4*x**2-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 - 4*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^{4}-4\,x^{2}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 4*x^2 - 2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 - 4*x^2 - 2), x)
```

3.52 $\int \frac{1}{\sqrt{-2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-2}\sqrt{3x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{3x^4-5x^2-2}}$$

```
[Out] (Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt
[-2 + x^2]], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0254713, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{x^2-2}\sqrt{3x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-2}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[-2 + x^2]*Sqrt[1 + 3*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt
[-2 + x^2]], 1/7])/(Sqrt[7]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.71465, size = 70, normalized size = 1.11

$$\frac{\sqrt{2}\sqrt{\frac{2x^2}{7}-\frac{4}{7}}\sqrt{12x^2+4}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{2x^2}{7}-\frac{4}{7}}}\right)\middle|\frac{1}{7}\right)}{4\sqrt{3x^4-5x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(2*x**2/7 - 4/7)*sqrt(12*x**2 + 4)*elliptic_f(asin(sq
rt(2)*x/sqrt(2*x**2/7 - 4/7)), 1/7)/(4*sqrt(3*x**4 - 5*x**2 - 2))
```

Mathematica [C] time = 0.0429417, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}F\left(i\sinh^{-1}\left(\sqrt{3}x\right)|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]
*x], -1/6])/(Sqrt[3]*Sqrt[-2 - 5*x^2 + 3*x^4])
```

Maple [C] time = 0.02, size = 53, normalized size = 0.8

$$-\frac{i}{6}\sqrt{3}EllipticF\left(i\sqrt{3}x,\frac{i}{6}\sqrt{6}\right)\sqrt{3\,x^2+1}\sqrt{-2\,x^2+4}\frac{1}{\sqrt{3\,x^4-5\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-5*x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*3^(1/2)*(3*x^2+1)^(1/2)*(-2*x^2+4)^(1/2)/(3*x^4-5*x^2-2)^(
1/2)*EllipticF(I*3^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 5*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-5\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 5*x^2 - 2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - 5x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-5*x**2-2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 - 5*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - 5\,x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 5*x^2 - 2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 - 5*x^2 - 2), x)
```

3.53 $\int \frac{1}{\sqrt{-3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

```
[Out] (Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6
+ (7 + Sqrt[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[
-6 + (7 + Sqrt[73])*x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^
(1/4)*Sqrt[(6 - (7 + Sqrt[73])*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4
])
```

Rubi [A] time = 0.0966281, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{\frac{6-\left(7-\sqrt{73}\right)x^2}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{\left(7+\sqrt{73}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{\left(7+\sqrt{73}\right)x^2-6}}\right)|\frac{1}{146}\left(73+7\sqrt{73}\right)\right)}{2\sqrt{3}\sqrt[4]{73}\sqrt{\frac{1}{6-\left(7+\sqrt{73}\right)x^2}}\sqrt{2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (7 - Sqrt[73])*x^2)/(6 - (7 + Sqrt[73])*x^2)]*Sqrt[-6
+ (7 + Sqrt[73])*x^2]*EllipticF[ArcSin[(Sqrt[2]*73^(1/4)*x)/Sqrt[
-6 + (7 + Sqrt[73])*x^2]], (73 + 7*Sqrt[73])/146])/(2*Sqrt[3]*73^
(1/4)*Sqrt[(6 - (7 + Sqrt[73])*x^2)^(-1)]*Sqrt[-3 + 7*x^2 + 2*x^4
])
```

Rubi in Sympy [A] time = 4.18286, size = 128, normalized size = 0.86

$$\frac{\sqrt{3}\cdot 73^{\frac{3}{4}}\sqrt{\frac{x^2\left(-\sqrt{73}+7\right)-6}{x^2\left(7+\sqrt{73}\right)-6}}\sqrt{x^2\left(7+\sqrt{73}\right)-6}\,F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{73}x}{\sqrt{x^2\left(7+\sqrt{73}\right)-6}}\right)\middle|\frac{7\sqrt{73}}{146}+\frac{1}{2}\right)}{438\sqrt{-\frac{1}{x^2\left(7+\sqrt{73}\right)-6}}\sqrt{2x^4+7x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] sqrt(3)*73**(3/4)*sqrt((x**2*(-sqrt(73) + 7) - 6)/(x**2*(7 + sqrt
(73)) - 6))*sqrt(x**2*(7 + sqrt(73)) - 6)*elliptic_f(asin(sqrt(2)
*73**(1/4)*x/sqrt(x**2*(7 + sqrt(73)) - 6)), 7*sqrt(73)/146 + 1/2
)/(438*sqrt(-1/(x**2*(7 + sqrt(73)) - 6))*sqrt(2*x**4 + 7*x**2 -
3))
```

Mathematica **[C]** time = 0.0866178, size = 80, normalized size = 0.54

$$-\frac{i\sqrt{-4x^4-14x^2+6}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{7+\sqrt{73}}}\right)|\frac{1}{12}\left(-61-7\sqrt{73}\right)\right)}{\sqrt{\sqrt{73}-7}\sqrt{2x^4+7x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 14*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[7 +
 Sqrt[73]]], (-61 - 7*Sqrt[73])/12])/(Sqrt[-7 + Sqrt[73]]*Sqrt[-3
 + 7*x^2 + 2*x^4])
```

Maple **[C]** time = 0.042, size = 84, normalized size = 0.6

$$6\,\frac{\sqrt{1-\left(7/6-1/6\,\sqrt{73}\right)x^2}\sqrt{1-\left(1/6\,\sqrt{73}+7/6\right)x^2}EllipticF\left(1/6\,\sqrt{42-6\,\sqrt{73}}x,\frac{7\,i}{12}\sqrt{6}+i/12\sqrt{438}\right)}{\sqrt{42-6\,\sqrt{73}}\sqrt{2\,x^4+7\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+7*x^2-3)^(1/2),x)
```

```
[Out] 6/(42-6*73^(1/2))^(1/2)*(1-(7/6-1/6*73^(1/2))*x^2)^(1/2)*(1-(1/6*
73^(1/2)+7/6)*x^2)^(1/2)/(2*x^4+7*x^2-3)^(1/2)*EllipticF(1/6*(42-
6*73^(1/2))^(1/2)*x,7/12*I*6^(1/2)+1/12*I*438^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 7*x^2 - 3),x, algorithm="maxima")

[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+7\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 7*x^2 - 3),x, algorithm="fricas")

[Out] integral(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+7x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+7*x**2-3)**(1/2),x)

[Out] Integral(1/sqrt(2*x**4 + 7*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 7*x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 + 7*x^2 - 3), x)
```

3.54 $\int \frac{1}{\sqrt{-3+6x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

```
[Out] (Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3
+ (3 + Sqrt[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[
-3 + (3 + Sqrt[15])*x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5
^(1/4)*Sqrt[(3 - (3 + Sqrt[15])*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^
4])
```

Rubi [A] time = 0.128343, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{3-\left(3-\sqrt{15}\right)x^2}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{\left(3+\sqrt{15}\right)x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{15}x}{\sqrt{\left(3+\sqrt{15}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{15}\right)\right)}{\sqrt{2}3^{3/4}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(3+\sqrt{15}\right)x^2}}\sqrt{2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (3 - Sqrt[15])*x^2)/(3 - (3 + Sqrt[15])*x^2)]*Sqrt[-3
+ (3 + Sqrt[15])*x^2]*EllipticF[ArcSin[(Sqrt[2]*15^(1/4)*x)/Sqrt[
-3 + (3 + Sqrt[15])*x^2]], (5 + Sqrt[15])/10])/(Sqrt[2]*3^(3/4)*5
^(1/4)*Sqrt[(3 - (3 + Sqrt[15])*x^2)^(-1)]*Sqrt[-3 + 6*x^2 + 2*x^
4])
```

Rubi in Sympy [A] time = 4.14925, size = 136, normalized size = 0.92

$$\frac{\sqrt{2}\sqrt[4]{3}\cdot 5^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{15}+6\right)-6}{x^2\left(6+2\sqrt{15}\right)-6}}\sqrt{x^2\left(6+2\sqrt{15}\right)-6}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{15}x}{\sqrt{x^2\left(6+2\sqrt{15}\right)-6}}\right)\middle|\frac{\sqrt{15}}{10}+\frac{1}{2}\right)}{60\sqrt{-\frac{1}{x^2\left(6+2\sqrt{15}\right)-6}}\sqrt{2x^4+6x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] sqrt(2)*3**(1/4)*5**(3/4)*sqrt((x**2*(-2*sqrt(15) + 6) - 6)/(x**2
*(6 + 2*sqrt(15)) - 6))*sqrt(x**2*(6 + 2*sqrt(15)) - 6)*elliptic_
f(asin(2*15**(1/4)*x/sqrt(x**2*(6 + 2*sqrt(15)) - 6)), sqrt(15)/1
0 + 1/2)/(60*sqrt(-1/(x**2*(6 + 2*sqrt(15)) - 6))*sqrt(2*x**4 + 6
*x**2 - 3))
```

Mathematica [C] time = 0.110972, size = 77, normalized size = 0.52

$$-\frac{i\sqrt{-2x^4-6x^2+3}F\left(i\sinh^{-1}\left(\sqrt{-1+\sqrt{\frac{5}{3}}x}\right)|-4-\sqrt{15}\right)}{\sqrt{\sqrt{15}-3}\sqrt{2x^4+6x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 6*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[-1 + Sqrt[
5/3]]*x], -4 - Sqrt[15]])/(Sqrt[-3 + Sqrt[15]]*Sqrt[-3 + 6*x^2 +
2*x^4])
```

Maple [C] time = 0.04, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(1-1/3\,\sqrt{15}\right)x^2}\sqrt{1-\left(1+1/3\,\sqrt{15}\right)x^2}EllipticF\left(1/3\,\sqrt{9-3\,\sqrt{15}}x,i/2\sqrt{6}+i/2\sqrt{10}\right)}{\sqrt{9-3\,\sqrt{15}}\sqrt{2\,x^4+6\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+6*x^2-3)^(1/2),x)
```

```
[Out] 3/(9-3*15^(1/2))^(1/2)*(1-(1-1/3*15^(1/2))*x^2)^(1/2)*(1-(1+1/3*1
5^(1/2))*x^2)^(1/2)/(2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*(9-3*15^(
1/2))^(1/2)*x,1/2*I*6^(1/2)+1/2*I*10^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 6*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+6\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 6*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+6x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 6*x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 + 6*x^2 - 3), x)
```

3.55 $\int \frac{1}{\sqrt{-3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{\sqrt{x^2+3}\sqrt{2x^2-1}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{2x^4+5x^2-3}}$$

```
[Out] (Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sq
rt[-1 + 2*x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0277492, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{x^2+3}\sqrt{2x^2-1}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{7}{3}}x}{\sqrt{2x^2-1}}\right)|\frac{6}{7}\right)}{\sqrt{7}\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 + x^2]*Sqrt[-1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7/3]*x)/Sq
rt[-1 + 2*x^2]], 6/7])/(Sqrt[7]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.81343, size = 70, normalized size = 1.04

$$\frac{\sqrt{3}\sqrt{\frac{12x^2}{7}-\frac{6}{7}}\sqrt{2x^2+6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{12x^2}{7}-\frac{6}{7}}}\right)\middle|\frac{6}{7}\right)}{6\sqrt{2x^4+5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(12*x**2/7 - 6/7)*sqrt(2*x**2 + 6)*elliptic_f(asin(sq
rt(2)*x/sqrt(12*x**2/7 - 6/7)), 6/7)/(6*sqrt(2*x**4 + 5*x**2 - 3)
)
```

Mathematica [A] time = 0.0416743, size = 54, normalized size = 0.81

$$\frac{\sqrt{1-2x^2}\sqrt{x^2+3}F\left(\sin^{-1}\left(\sqrt{2}x\right)|-\frac{1}{6}\right)}{\sqrt{6}\sqrt{2x^4+5x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[3 + x^2]*EllipticF[ArcSin[Sqrt[2]*x], -1/6]
)/(Sqrt[6]*Sqrt[-3 + 5*x^2 + 2*x^4])
```

Maple [C] time = 0.009, size = 53, normalized size = 0.8

$$-\frac{i}{3}\sqrt{3}EllipticF\left(\frac{i}{3}\sqrt{3}x, i\sqrt{6}\right)\sqrt{3\,x^2+9}\sqrt{-2\,x^2+1}\frac{1}{\sqrt{2\,x^4+5\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+5*x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(2*x^4+5*x^2-3)^(
1/2)*EllipticF(1/3*I*3^(1/2)*x,I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2-3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 - 3), x)
```

3.56 $\int \frac{1}{\sqrt{-3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=148

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}$$

```
[Out] (Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3
+ (2 + Sqrt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
3 + (2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^
(1/4)*Sqrt[(3 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4
])
```

Rubi [A] time = 0.109734, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{\frac{3-\left(2-\sqrt{10}\right)x^2}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{\left(2+\sqrt{10}\right)x^2-3}\,F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{\left(2+\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5+\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{3-\left(2+\sqrt{10}\right)x^2}}\sqrt{2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (2 - Sqrt[10])*x^2)/(3 - (2 + Sqrt[10])*x^2)]*Sqrt[-3
+ (2 + Sqrt[10])*x^2]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
3 + (2 + Sqrt[10])*x^2]], (5 + Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^
(1/4)*Sqrt[(3 - (2 + Sqrt[10])*x^2)^(-1)]*Sqrt[-3 + 4*x^2 + 2*x^4
])
```

Rubi in Sympy [A] time = 4.17556, size = 136, normalized size = 0.92

$$\frac{\sqrt[4]{2}\sqrt{3}\cdot 5^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{10}+4\right)-6}{x^2\left(4+2\sqrt{10}\right)-6}}\sqrt{x^2\left(4+2\sqrt{10}\right)-6}\,F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{10}x}{\sqrt{x^2\left(4+2\sqrt{10}\right)-6}}\right)\middle|\frac{\sqrt{10}}{10}+\frac{1}{2}\right)}{60\sqrt{-\frac{1}{x^2\left(4+2\sqrt{10}\right)-6}}\sqrt{2x^4+4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] 2**(1/4)*sqrt(3)*5**(3/4)*sqrt((x**2*(-2*sqrt(10) + 4) - 6)/(x**2
*(4 + 2*sqrt(10)) - 6))*sqrt(x**2*(4 + 2*sqrt(10)) - 6)*elliptic_
f(asin(2*10**(1/4)*x/sqrt(x**2*(4 + 2*sqrt(10)) - 6)), sqrt(10)/1
0 + 1/2)/(60*sqrt(-1/(x**2*(4 + 2*sqrt(10)) - 6))*sqrt(2*x**4 + 4
*x**2 - 3))
```

Mathematica [C] time = 0.116333, size = 83, normalized size = 0.56

$$-\frac{i\sqrt{-2x^4-4x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{2+\sqrt{10}}}x\right)|-\frac{7}{3}-\frac{2\sqrt{10}}{3}\right)}{\sqrt{\sqrt{10}-2}\sqrt{2x^4+4x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(2 + Sqr
t[10])]*x], -7/3 - (2*Sqrt[10])/3])/(Sqrt[-2 + Sqrt[10]]*Sqrt[-3
+ 4*x^2 + 2*x^4])
```

Maple [C] time = 0.04, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(2/3-1/3\,\sqrt{10}\right)x^2}\sqrt{1-\left(2/3+1/3\,\sqrt{10}\right)x^2}\,EllipticF\left(1/3\,\sqrt{6-3\,\sqrt{10}}x,\,i/3\sqrt{6}+i/3\sqrt{15}\right)}{\sqrt{6-3\,\sqrt{10}}\sqrt{2\,x^4+4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+4*x^2-3)^(1/2),x)
```

```
[Out] 3/(6-3*10^(1/2))^(1/2)*(1-(2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(2/3+1
/3*10^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(6-3*
10^(1/2))^(1/2)*x,1/3*I*6^(1/2)+1/3*I*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 4*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+4\,x^2-3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 4*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 4*x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 + 4*x^2 - 3), x)
```

3.57 $\int \frac{1}{\sqrt{-3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=146

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}$$

```
[Out] (Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6
+ (3 + Sqrt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-6 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/
4)*Sqrt[(6 - (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rubi [A] time = 0.113252, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{\frac{6-\left(3-\sqrt{33}\right)x^2}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{\left(3+\sqrt{33}\right)x^2-6}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{\left(3+\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11+\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{6-\left(3+\sqrt{33}\right)x^2}}\sqrt{2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 - (3 - Sqrt[33])*x^2)/(6 - (3 + Sqrt[33])*x^2)]*Sqrt[-6
+ (3 + Sqrt[33])*x^2]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-6 + (3 + Sqrt[33])*x^2]], (11 + Sqrt[33])/22])/(2*3^(3/4)*11^(1/
4)*Sqrt[(6 - (3 + Sqrt[33])*x^2)^(-1)]*Sqrt[-3 + 3*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 4.6663, size = 126, normalized size = 0.86

$$\frac{11^{\frac{3}{4}}\sqrt[4]{3}\sqrt{\frac{x^2\left(-\sqrt{33}+3\right)-6}{x^2\left(3+\sqrt{33}\right)-6}}\sqrt{x^2\left(3+\sqrt{33}\right)-6}\,F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{x^2\left(3+\sqrt{33}\right)-6}}\right)\middle|\frac{\sqrt{33}}{22}+\frac{1}{2}\right)}{66\sqrt{-\frac{1}{x^2\left(3+\sqrt{33}\right)-6}}\sqrt{2x^4+3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] 11**(3/4)*3**(1/4)*sqrt((x**2*(-sqrt(33) + 3) - 6)/(x**2*(3 + sqr
t(33)) - 6))*sqrt(x**2*(3 + sqrt(33)) - 6)*elliptic_f(asin(sqrt(2
)*33**(1/4)*x/sqrt(x**2*(3 + sqrt(33)) - 6)), sqrt(33)/22 + 1/2)/
(66*sqrt(-1/(x**2*(3 + sqrt(33)) - 6))*sqrt(2*x**4 + 3*x**2 - 3))
```

Mathematica [C] time = 0.10763, size = 80, normalized size = 0.55

$$-\frac{i\sqrt{-4x^4-6x^2+6}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{3+\sqrt{33}}}\right)|-\frac{7}{4}-\frac{\sqrt{33}}{4}\right)}{\sqrt{\sqrt{33}-3}\sqrt{2x^4+3x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 - 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[3 +
Sqrt[33]]], -7/4 - Sqrt[33]/4])/(Sqrt[-3 + Sqrt[33]]*Sqrt[-3 + 3*
x^2 + 2*x^4])
```

Maple [C] time = 0.04, size = 84, normalized size = 0.6

$$6\,\frac{\sqrt{1-\left(-1/6\,\sqrt{33}+1/2\right)x^2}\sqrt{1-\left(1/6\,\sqrt{33}+1/2\right)x^2}EllipticF\left(1/6\,\sqrt{18-6\,\sqrt{33}}x,i/4\sqrt{6}+i/4\sqrt{22}\right)}{\sqrt{18-6\,\sqrt{33}}\sqrt{2\,x^4+3\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+3*x^2-3)^(1/2),x)
```

```
[Out] 6/(18-6*33^(1/2))^(1/2)*(1-(-1/6*33^(1/2)+1/2)*x^2)^(1/2)*(1-(1/6
*33^(1/2)+1/2)*x^2)^(1/2)/(2*x^4+3*x^2-3)^(1/2)*EllipticF(1/6*(18
-6*33^(1/2))^(1/2)*x,1/4*I*6^(1/2)+1/4*I*22^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 - 3), x)
```

3.58 $\int \frac{1}{\sqrt{-3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=143

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

```
[Out] (Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 +
(1 + Sqrt[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 +
 (1 + Sqrt[7])*x^2]], (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3
 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.0911654, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{3-\left(1-\sqrt{7}\right)x^2}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{\left(1+\sqrt{7}\right)x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{\left(1+\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7+\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{3-\left(1+\sqrt{7}\right)x^2}}\sqrt{2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 - (1 - Sqrt[7])*x^2)/(3 - (1 + Sqrt[7])*x^2)]*Sqrt[-3 +
(1 + Sqrt[7])*x^2]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 +
 (1 + Sqrt[7])*x^2]], (7 + Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3
 - (1 + Sqrt[7])*x^2)^(-1)]*Sqrt[-3 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 4.09994, size = 131, normalized size = 0.92

$$\frac{\sqrt{6}\cdot 7^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{7}+2\right)-6}{x^2\left(2+2\sqrt{7}\right)-6}}\sqrt{x^2\left(2+2\sqrt{7}\right)-6}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{7}x}{\sqrt{x^2\left(2+2\sqrt{7}\right)-6}}\right)\middle|\frac{\sqrt{7}}{14}+\frac{1}{2}\right)}{84\sqrt{-\frac{1}{x^2\left(2+2\sqrt{7}\right)-6}}\sqrt{2x^4+2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] sqrt(6)*7**(3/4)*sqrt((x**2*(-2*sqrt(7) + 2) - 6)/(x**2*(2 + 2*sq
rt(7)) - 6))*sqrt(x**2*(2 + 2*sqrt(7)) - 6)*elliptic_f(asin(2*7**
(1/4)*x/sqrt(x**2*(2 + 2*sqrt(7)) - 6)), sqrt(7)/14 + 1/2)/(84*sq
rt(-1/(x**2*(2 + 2*sqrt(7)) - 6))*sqrt(2*x**4 + 2*x**2 - 3))
```

Mathematica [C] time = 0.0917014, size = 83, normalized size = 0.58

$$-\frac{i\sqrt{-2x^4-2x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{7}}}x\right)|-\frac{4}{3}-\frac{\sqrt{7}}{3}\right)}{\sqrt{\sqrt{7}-1}\sqrt{2x^4+2x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 - 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(1 + Sqr
t[7])]*x], -4/3 - Sqrt[7]/3])/(Sqrt[-1 + Sqrt[7]]*Sqrt[-3 + 2*x^2
 + 2*x^4])
```

Maple [C] time = 0.093, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(-1/3\,\sqrt{7}+1/3\right)x^2}\sqrt{1-\left(1/3\,\sqrt{7}+1/3\right)x^2}EllipticF\left(1/3\,\sqrt{3-3\,\sqrt{7}}x,i/6\sqrt{6}+i/6\sqrt{42}\right)}{\sqrt{3-3\,\sqrt{7}}\sqrt{2\,x^4+2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+2*x^2-3)^(1/2),x)
```

```
[Out] 3/(3-3*7^(1/2))^(1/2)*(1-(-1/3*7^(1/2)+1/3)*x^2)^(1/2)*(1-(1/3*7^
(1/2)+1/3)*x^2)^(1/2)/(2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(3-3*7^
(1/2))^(1/2)*x,1/6*I*6^(1/2)+1/6*I*42^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 2*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+2\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 2*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4+2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 2*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 - 3), x)
```

3.59 $\int \frac{1}{\sqrt{-3+x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-1}\sqrt{2x^2+3}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{2x^4+x^2-3}}$$

```
[Out] (Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sq
rt[-1 + x^2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Rubi [A] time = 0.026796, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\sqrt{x^2-1}\sqrt{2x^2+3}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{x^2-1}}\right)|\frac{3}{5}\right)}{\sqrt{5}\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + x^2 + 2*x^4],x]
```


```
[Out] (Sqrt[-1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5/3]*x)/Sq
rt[-1 + x^2]], 3/5])/(Sqrt[5]*Sqrt[-3 + x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 2.94922, size = 68, normalized size = 1.08

$$\frac{\sqrt{3}\sqrt{\frac{6x^2}{5}-\frac{6}{5}}\sqrt{4x^2+6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{6x^2}{5}-\frac{6}{5}}}\right)\middle|\frac{3}{5}\right)}{6\sqrt{2x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+x**2-3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(6*x**2/5 - 6/5)*sqrt(4*x**2 + 6)*elliptic_f(asin(sqr
t(2)*x/sqrt(6*x**2/5 - 6/5)), 3/5)/(6*sqrt(2*x**4 + x**2 - 3))
```

Mathematica [C] time = 0.0384274, size = 63, normalized size = 1.

$$-\frac{i\sqrt{1-x^2}\sqrt{2x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right)\left|-\frac{3}{2}\right.\right)}{\sqrt{2}\sqrt{2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]
*x], -3/2])/(Sqrt[2]*Sqrt[-3 + x^2 + 2*x^4])
```

Maple [C] time = 0.035, size = 51, normalized size = 0.8

$$-\frac{i}{6}\sqrt{6}EllipticF\left(\frac{i}{3}x\sqrt{6},\frac{i}{2}\sqrt{6}\right)\sqrt{6\,x^2+9}\sqrt{-x^2+1}\frac{1}{\sqrt{2\,x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+x^2-3)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(2*x^4+x^2-3)^(1/2)
*EllipticF(1/3*I*x*6^(1/2),1/2*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 + x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 + x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 - 3), x)
```

3.60 $\int \frac{1}{\sqrt{-3+2x^4}}\,dx$

Optimal. Leaf size=112

$$\frac{\sqrt{\sqrt{6}x^2-3}\sqrt{\frac{\sqrt{6}x^2+3}{3-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-3}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{2x^4-3}}$$

```
[Out] (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]
*EllipticF[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/
2])/(6^(3/4)*Sqrt[(3 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Rubi [A] time = 0.0596717, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt{\sqrt{6}x^2-3}\sqrt{\frac{\sqrt{6}x^2+3}{3-\sqrt{6}x^2}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{3}x}{\sqrt{\sqrt{6}x^2-3}}\right)|\frac{1}{2}\right)}{6^{3/4}\sqrt{\frac{1}{3-\sqrt{6}x^2}}\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 2*x^4],x]
```


```
[Out] (Sqrt[-3 + Sqrt[6]*x^2]*Sqrt[(3 + Sqrt[6]*x^2)/(3 - Sqrt[6]*x^2)]
*EllipticF[ArcSin[(2^(3/4)*3^(1/4)*x)/Sqrt[-3 + Sqrt[6]*x^2]], 1/
2])/(6^(3/4)*Sqrt[(3 - Sqrt[6]*x^2)^(-1)]*Sqrt[-3 + 2*x^4])
```

Rubi in Sympy [A] time = 1.49997, size = 51, normalized size = 0.46

$$\frac{2^{\frac{3}{4}}\sqrt[4]{3}\sqrt{-\frac{2x^4}{3}+1}F\left(\operatorname{asin}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-1\right)}{2\sqrt{2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-3)**(1/2),x)
```

```
[Out] 2**(3/4)*3**(1/4)*sqrt(-2*x**4/3 + 1)*elliptic_f(asin(2**(1/4)*3*
*(3/4)*x/3), -1)/(2*sqrt(2*x**4 - 3))
```

Mathematica [A] time = 0.0433267, size = 40, normalized size = 0.36

$$\frac{\sqrt{3-2x^4}F\left(\sin^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt[4]{6}\sqrt{2x^4-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^4]*EllipticF[ArcSin[(2/3)^(1/4)*x], -1])/(6^(1/4)*S
qrt[-3 + 2*x^4])
```

Maple [C] time = 0.03, size = 56, normalized size = 0.5

$$\frac{1}{3\sqrt{-3\sqrt{6}}}\sqrt{9+3x^2\sqrt{6}}\sqrt{9-3x^2\sqrt{6}}EllipticF\left(\frac{\sqrt{-3\sqrt{6}}x}{3},i\right)\frac{1}{\sqrt{2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-3)^(1/2),x)
```

```
[Out] 1/3/(-3*6^(1/2))^(1/2)*(9+3*x^2*6^(1/2))^(1/2)*(9-3*x^2*6^(1/2))^
(1/2)/(2*x^4-3)^(1/2)*EllipticF(1/3*(-3*6^(1/2))^(1/2)*x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3), x)
```

Sympy [A] time = 1.87267, size = 34, normalized size = 0.3

$$-\frac{\sqrt{3}ix\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{2x^4}{3}\right)}{12\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-3)**(1/2),x)
```

```
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4/3)/(12*g
amma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3), x)
```

3.61 $\int \frac{1}{\sqrt{-3-x^2+2x^4}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2+1}\sqrt{2x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{2x^4-x^2-3}}$$

```
[Out] (Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt
[-3 + 2*x^2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Rubi [A] time = 0.0249164, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{x^2+1}\sqrt{2x^2-3}F\left(\sin^{-1}\left(\frac{\sqrt{5}x}{\sqrt{2x^2-3}}\right)|\frac{2}{5}\right)}{\sqrt{5}\sqrt{2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[-3 + 2*x^2]*EllipticF[ArcSin[(Sqrt[5]*x)/Sqrt
[-3 + 2*x^2]], 2/5])/(Sqrt[5]*Sqrt[-3 - x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.07947, size = 68, normalized size = 1.05

$$\frac{\sqrt{3}\sqrt{\frac{4x^2}{5}-\frac{6}{5}}\sqrt{6x^2+6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{4x^2}{5}-\frac{6}{5}}}\right)\middle|\frac{2}{5}\right)}{6\sqrt{2x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-x**2-3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(4*x**2/5 - 6/5)*sqrt(6*x**2 + 6)*elliptic_f(asin(sqr
t(2)*x/sqrt(4*x**2/5 - 6/5)), 2/5)/(6*sqrt(2*x**4 - x**2 - 3))
```

Mathematica [A] time = 0.0425312, size = 51, normalized size = 0.78

$$\frac{\sqrt{3-2x^2}\sqrt{x^2+1}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right)|-\frac{3}{2}\right)}{\sqrt{4x^4-2x^2-6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 + x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], -3/
2])/Sqrt[-6 - 2*x^2 + 4*x^4]
```

Maple [C] time = 0.012, size = 45, normalized size = 0.7

$$-\frac{i}{3}EllipticF\left(ix,\frac{i}{3}\sqrt{6}\right)\sqrt{x^2+1}\sqrt{-6x^2+9}\frac{1}{\sqrt{2x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-x^2-3)^(1/2)*Ellipti
cF(I*x,1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2x^4-x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - x^2 - 3),x, algorithm="fricas")

[Out] integral(1/sqrt(2*x^4 - x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-x**2-3)**(1/2),x)

[Out] Integral(1/sqrt(2*x**4 - x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 - x^2 - 3), x)
```

3.62 $\int \frac{1}{\sqrt{-3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

```
[Out] (Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (
1 - Sqrt[7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 -
 (1 - Sqrt[7])*x^2]], (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3
 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.0930872, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}\sqrt{\frac{\left(1+\sqrt{7}\right)x^2+3}{\left(1-\sqrt{7}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{7}x}{\sqrt{-\left(1-\sqrt{7}\right)x^2-3}}\right)|\frac{1}{14}\left(7-\sqrt{7}\right)\right)}{\sqrt{6}\sqrt[4]{7}\sqrt{\frac{1}{\left(1-\sqrt{7}\right)x^2+3}}\sqrt{2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```



```
[Out] (Sqrt[-3 - (1 - Sqrt[7])*x^2]*Sqrt[(3 + (1 + Sqrt[7])*x^2)/(3 + (
1 - Sqrt[7])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*7^(1/4)*x)/Sqrt[-3 -
 (1 - Sqrt[7])*x^2]], (7 - Sqrt[7])/14])/(Sqrt[6]*7^(1/4)*Sqrt[(3
 + (1 - Sqrt[7])*x^2)^(-1)]*Sqrt[-3 - 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 4.01815, size = 133, normalized size = 0.89

$$\frac{\sqrt{6}\cdot 7^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{7}-2\right)-6}{x^2\left(-2+2\sqrt{7}\right)-6}}\sqrt{x^2\left(-2+2\sqrt{7}\right)-6}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{7}x}{\sqrt{x^2\left(-2+2\sqrt{7}\right)-6}}\right)\middle|-\frac{\sqrt{7}}{14}+\frac{1}{2}\right)}{84\sqrt{-\frac{1}{x^2\left(-2+2\sqrt{7}\right)-6}}\sqrt{2x^4-2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] sqrt(6)*7**(3/4)*sqrt((x**2*(-2*sqrt(7) - 2) - 6)/(x**2*(-2 + 2*s
qrt(7)) - 6))*sqrt(x**2*(-2 + 2*sqrt(7)) - 6)*elliptic_f(asin(2*7
**(1/4)*x/sqrt(x**2*(-2 + 2*sqrt(7)) - 6)), -sqrt(7)/14 + 1/2)/(8
4*sqrt(-1/(x**2*(-2 + 2*sqrt(7)) - 6))*sqrt(2*x**4 - 2*x**2 - 3))
```

Mathematica [C]   time = 0.0861445, size = 81, normalized size = 0.54

$$-\frac{i\sqrt{-2x^4+2x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-1+\sqrt{7}}}x\right)|\frac{1}{3}\left(-4+\sqrt{7}\right)\right)}{\sqrt{1+\sqrt{7}}\sqrt{2x^4-2x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 2*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-1 + Sq
rt[7])]*x], (-4 + Sqrt[7])/3])/(Sqrt[1 + Sqrt[7]]*Sqrt[-3 - 2*x^2
 + 2*x^4])
```

Maple [C]   time = 0.041, size = 84, normalized size = 0.6

$$3\,\frac{\sqrt{1-\left(-1/3-1/3\,\sqrt{7}\right)x^2}\sqrt{1-\left(-1/3+1/3\,\sqrt{7}\right)x^2}EllipticF\left(1/3\,\sqrt{-3-3\,\sqrt{7}}x,i/6\sqrt{42}-i/6\sqrt{6}\right)}{\sqrt{-3-3\,\sqrt{7}}\sqrt{2\,x^4-2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-2*x^2-3)^(1/2),x)
```

```
[Out] 3/(-3-3*7^(1/2))^(1/2)*(1-(-1/3-1/3*7^(1/2))*x^2)^(1/2)*(1-(-1/3+
1/3*7^(1/2))*x^2)^(1/2)/(2*x^4-2*x^2-3)^(1/2)*EllipticF(1/3*(-3-3
*7^(1/2))^(1/2)*x,1/6*I*42^(1/2)-1/6*I*6^(1/2))
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 2*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-2\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 2*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 2*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 - 3), x)
```

3.63 $\int \frac{1}{\sqrt{-3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=153

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

```
[Out] (Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 +
 (3 - Sqrt[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-6 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/
4)*Sqrt[(6 + (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rubi [A] time = 0.116393, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}\sqrt{\frac{\left(3+\sqrt{33}\right)x^2+6}{\left(3-\sqrt{33}\right)x^2+6}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{-\left(3-\sqrt{33}\right)x^2-6}}\right)|\frac{1}{22}\left(11-\sqrt{33}\right)\right)}{2\,3^{3/4}\sqrt[4]{11}\sqrt{\frac{1}{\left(3-\sqrt{33}\right)x^2+6}}\sqrt{2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-6 - (3 - Sqrt[33])*x^2]*Sqrt[(6 + (3 + Sqrt[33])*x^2)/(6 +
 (3 - Sqrt[33])*x^2)]*EllipticF[ArcSin[(Sqrt[2]*33^(1/4)*x)/Sqrt[
-6 - (3 - Sqrt[33])*x^2]], (11 - Sqrt[33])/22])/(2*3^(3/4)*11^(1/
4)*Sqrt[(6 + (3 - Sqrt[33])*x^2)^(-1)]*Sqrt[-3 - 3*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.90481, size = 128, normalized size = 0.84

$$\frac{11^{\frac{3}{4}}\sqrt[4]{3}\sqrt{\frac{x^2\left(-\sqrt{33}-3\right)-6}{x^2\left(-3+\sqrt{33}\right)-6}}\sqrt{x^2\left(-3+\sqrt{33}\right)-6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt[4]{33}x}{\sqrt{x^2\left(-3+\sqrt{33}\right)-6}}\right)\middle|-\frac{\sqrt{33}}{22}+\frac{1}{2}\right)}{66\sqrt{-\frac{1}{x^2\left(-3+\sqrt{33}\right)-6}}\sqrt{2x^4-3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] 11**(3/4)*3**(1/4)*sqrt((x**2*(-sqrt(33) - 3) - 6)/(x**2*(-3 + sq
rt(33)) - 6))*sqrt(x**2*(-3 + sqrt(33)) - 6)*elliptic_f(asin(sqrt
(2)*33**(1/4)*x/sqrt(x**2*(-3 + sqrt(33)) - 6)), -sqrt(33)/22 + 1
/2)/(66*sqrt(-1/(x**2*(-3 + sqrt(33)) - 6))*sqrt(2*x**4 - 3*x**2
- 3))
```

Mathematica [C] time = 0.0979657, size = 78, normalized size = 0.51

$$-\frac{i\sqrt{-4x^4+6x^2+6}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-3+\sqrt{33}}}\right)|\frac{1}{4}\left(-7+\sqrt{33}\right)\right)}{\sqrt{3+\sqrt{33}}\sqrt{2x^4-3x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[6 + 6*x^2 - 4*x^4]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-3 +
 Sqrt[33]]], (-7 + Sqrt[33])/4])/(Sqrt[3 + Sqrt[33]]*Sqrt[-3 - 3*
x^2 + 2*x^4])
```

Maple [C] time = 0.041, size = 84, normalized size = 0.6

$$6\,\frac{\sqrt{1-\left(-1/2-1/6\,\sqrt{33}\right)x^2}\sqrt{1-\left(-1/2+1/6\,\sqrt{33}\right)x^2}\mathit{EllipticF}\left(1/6\,\sqrt{-18-6\,\sqrt{33}}x,i/4\sqrt{22}-i/4\sqrt{6}\right)}{\sqrt{-18-6\,\sqrt{33}}\sqrt{2\,x^4-3\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-3*x^2-3)^(1/2),x)
```

```
[Out] 6/(-18-6*33^(1/2))^(1/2)*(1-(-1/2-1/6*33^(1/2))*x^2)^(1/2)*(1-(-1
/2+1/6*33^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2-3)^(1/2)*EllipticF(1/6*(
-18-6*33^(1/2))^(1/2)*x,1/4*I*22^(1/2)-1/4*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 - 3), x)
```

3.64 $\int \frac{1}{\sqrt{-3-4x^2+2x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

```
[Out] (Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 +
 (2 - Sqrt[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
3 - (2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^
(1/4)*Sqrt[(3 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4
])
```

Rubi [A] time = 0.134667, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}\sqrt{\frac{\left(2+\sqrt{10}\right)x^2+3}{\left(2-\sqrt{10}\right)x^2+3}}F\left(\sin^{-1}\left(\frac{2^{3/4}\sqrt[4]{5}x}{\sqrt{-\left(2-\sqrt{10}\right)x^2-3}}\right)|\frac{1}{10}\left(5-\sqrt{10}\right)\right)}{2^{3/4}\sqrt{3}\sqrt[4]{5}\sqrt{\frac{1}{\left(2-\sqrt{10}\right)x^2+3}}\sqrt{2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 - (2 - Sqrt[10])*x^2]*Sqrt[(3 + (2 + Sqrt[10])*x^2)/(3 +
 (2 - Sqrt[10])*x^2)]*EllipticF[ArcSin[(2^(3/4)*5^(1/4)*x)/Sqrt[-
3 - (2 - Sqrt[10])*x^2]], (5 - Sqrt[10])/10])/(2^(3/4)*Sqrt[3]*5^
(1/4)*Sqrt[(3 + (2 - Sqrt[10])*x^2)^(-1)]*Sqrt[-3 - 4*x^2 + 2*x^4
])
```

Rubi in Sympy [A] time = 4.06602, size = 138, normalized size = 0.89

$$\frac{\sqrt[4]{2}\sqrt{3}\cdot 5^{\frac{3}{4}}\sqrt{\frac{x^2\left(-2\sqrt{10}-4\right)-6}{x^2\left(-4+2\sqrt{10}\right)-6}}\sqrt{x^2\left(-4+2\sqrt{10}\right)-6}F\left(\operatorname{asin}\left(\frac{2\sqrt[4]{10}x}{\sqrt{x^2\left(-4+2\sqrt{10}\right)-6}}\right)\middle|-\frac{\sqrt{10}}{10}+\frac{1}{2}\right)}{60\sqrt{-\frac{1}{x^2\left(-4+2\sqrt{10}\right)-6}}\sqrt{2x^4-4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] 2**(1/4)*sqrt(3)*5**(3/4)*sqrt((x**2*(-2*sqrt(10) - 4) - 6)/(x**2
*(-4 + 2*sqrt(10)) - 6))*sqrt(x**2*(-4 + 2*sqrt(10)) - 6)*ellipti
c_f(asin(2*10**(1/4)*x/sqrt(x**2*(-4 + 2*sqrt(10)) - 6)), -sqrt(1
0)/10 + 1/2)/(60*sqrt(-1/(x**2*(-4 + 2*sqrt(10)) - 6))*sqrt(2*x**
4 - 4*x**2 - 3))
```

Mathematica [C] time = 0.112247, size = 83, normalized size = 0.54

$$-\frac{i\sqrt{-2x^4+4x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-2+\sqrt{10}}}x\right)|-\frac{7}{3}+\frac{2\sqrt{10}}{3}\right)}{\sqrt{2+\sqrt{10}}\sqrt{2x^4-4x^2-3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[-3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + 4*x^2 - 2*x^4]*EllipticF[I*ArcSinh[Sqrt[2/(-2 + Sq
rt[10])]*x], -7/3 + (2*Sqrt[10])/3])/(Sqrt[2 + Sqrt[10]]*Sqrt[-3
- 4*x^2 + 2*x^4])
```

Maple [C] time = 0.041, size = 84, normalized size = 0.5

$$3\,\frac{\sqrt{1-\left(-2/3-1/3\,\sqrt{10}\right)x^2}\sqrt{1-\left(-2/3+1/3\,\sqrt{10}\right)x^2}\mathit{EllipticF}\left(1/3\,\sqrt{-6-3\,\sqrt{10}}x,i/3\sqrt{15}-i/3\sqrt{6}\right)}{\sqrt{-6-3\,\sqrt{10}}\sqrt{2\,x^4-4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-4*x^2-3)^(1/2),x)
```

```
[Out] 3/(-6-3*10^(1/2))^(1/2)*(1-(-2/3-1/3*10^(1/2))*x^2)^(1/2)*(1-(-2/
3+1/3*10^(1/2))*x^2)^(1/2)/(2*x^4-4*x^2-3)^(1/2)*EllipticF(1/3*(-
6-3*10^(1/2))^(1/2)*x,1/3*I*15^(1/2)-1/3*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 4*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 4*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 4x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 4\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 4*x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 - 4*x^2 - 3), x)
```

3.65 $\int \frac{1}{\sqrt{-3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{x^2-3}\sqrt{2x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{2x^4-5x^2-3}}$$

```
[Out] (Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt
[-3 + x^2]], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0285873, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{x^2-3}\sqrt{2x^2+1}F\left(\sin^{-1}\left(\frac{\sqrt{7}x}{\sqrt{x^2-3}}\right)|\frac{1}{7}\right)}{\sqrt{7}\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[-3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[ArcSin[(Sqrt[7]*x)/Sqrt
[-3 + x^2]], 1/7])/(Sqrt[7]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.70724, size = 70, normalized size = 1.11

$$\frac{\sqrt{3}\sqrt{\frac{2x^2}{7}-\frac{6}{7}}\sqrt{12x^2+6}F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{\sqrt{\frac{2x^2}{7}-\frac{6}{7}}}\right)\middle|\frac{1}{7}\right)}{6\sqrt{2x^4-5x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt(2*x**2/7 - 6/7)*sqrt(12*x**2 + 6)*elliptic_f(asin(sq
rt(2)*x/sqrt(2*x**2/7 - 6/7)), 1/7)/(6*sqrt(2*x**4 - 5*x**2 - 3))
```

Mathematica [C] time = 0.0416615, size = 65, normalized size = 1.03

$$-\frac{i\sqrt{1-\frac{x^2}{3}}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|-\frac{1}{6}\right)}{\sqrt{2}\sqrt{2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/3]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]
*x], -1/6])/(Sqrt[2]*Sqrt[-3 - 5*x^2 + 2*x^4])
```

Maple [C] time = 0.022, size = 53, normalized size = 0.8

$$-\frac{i}{6}\sqrt{2}EllipticF\left(i\sqrt{2}x,\frac{i}{6}\sqrt{6}\right)\sqrt{2\,x^2+1}\sqrt{-3\,x^2+9}\frac{1}{\sqrt{2\,x^4-5\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-5*x^2-3)^(1/2),x)
```

```
[Out] -1/6*I*2^(1/2)*(2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-5*x^2-3)^(
1/2)*EllipticF(I*2^(1/2)*x,1/6*I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 5*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-5\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 5*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 5x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 5*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 - 3), x)
```

3.66 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2]
)/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0242899, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2]
)/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.68821, size = 46, normalized size = 0.88

$$\frac{\sqrt{\frac{6x^2+4}{x^2+1}}\,(4x^2+4)\,F\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{8\sqrt{3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] sqrt((6*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), -1
/2)/(8*sqrt(3*x**4 + 5*x**2 + 2))
```

Mathematica [C] time = 0.0427037, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\sqrt{3x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{9x^4+15x^2+6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]
*x], 2/3])/Sqrt[6 + 15*x^2 + 9*x^4]
```

Maple [A] time = 0.102, size = 44, normalized size = 0.9

$$-\frac{i}{2}\sqrt{x^2+1}\sqrt{6\,x^2+4}EllipticF\left(ix,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ellipt
icF(I*x,1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 + 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

3.67 $\int \frac{1}{\sqrt{2+4x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[2 + 4*x^2 + 3*x^4])
```

Rubi [A] time = 0.0696523, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[2 + 4*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.82159, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4+4x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{3x^4+4x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 + 4*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqrt(
```

```
6)/6 + 1/2)/(12*sqrt(3*x**4 + 4*x**2 + 2))
```

Mathematica [C] time = 0.15356, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{3x^4+4x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I
*Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2
- I*Sqrt[2])/(-2 + I*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(
-1)]*Sqrt[2 + 4*x^2 + 3*x^4])
```

Maple [C] time = 0.168, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1+i/2\sqrt{2}\right)x^2}\sqrt{1-\left(-1-i/2\sqrt{2}\right)x^2}\,EllipticF\left(1/2\,x\sqrt{-4+2\,i\sqrt{2}},1/3\,\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{-4+2\,i\sqrt{2}}\sqrt{3\,x^4+4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+4*x^2+2)^(1/2),x)
```

```
[Out] 2/(-4+2*I*2^(1/2))^(1/2)*(1-(-1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1-
1/2*I*2^(1/2))*x^2)^(1/2)/(3*x^4+4*x^2+2)^(1/2)*EllipticF(1/2*x*(
-4+2*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 4*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+4\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 4*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+4x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 4*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 4*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 4*x^2 + 2), x)
```

3.68 $\int \frac{1}{\sqrt{2+3x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[2 + 3*x^2 + 3*x^4])
```

Rubi [A] time = 0.0580158, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[2 + 3*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.76831, size = 88, normalized size = 0.96

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4+3x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{3x^4+3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 + 3*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqrt(
```

```
6)/8 + 1/2)/(12*sqrt(3*x**4 + 3*x**2 + 2))
```

Mathematica [C] time = 0.207376, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{3x^4+3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 +
I*Sqrt[15])]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (
-3 - I*Sqrt[15])/(-3 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[
15])^(-1)]*Sqrt[2 + 3*x^2 + 3*x^4])
```

Maple [C] time = 0.159, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-3/4+i/4\sqrt{15}\right)x^2}\sqrt{1-\left(-3/4-i/4\sqrt{15}\right)x^2}\,EllipticF\left(1/2\,x\sqrt{-3+i\sqrt{15}},1/2\,\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{-3+i\sqrt{15}}\sqrt{3\,x^4+3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+3*x^2+2)^(1/2),x)
```

```
[Out] 2/(-3+I*15^(1/2))^(1/2)*(1-(-3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-
3/4-1/4*I*15^(1/2))*x^2)^(1/2)/(3*x^4+3*x^2+2)^(1/2)*EllipticF(1/
2*x*(-3+I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+3\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4+3x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 3*x^2 + 2), x)
```

3.69 $\int \frac{1}{\sqrt{2+2x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[2 + 2*x^2 + 3*x^4])
```

Rubi [A] time = 0.0568264, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[2 + 2*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.65911, size = 88, normalized size = 0.96

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4+2x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{3x^4+2x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+2*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 + 2*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqrt(
```

```
6)/12 + 1/2)/(12*sqrt(3*x**4 + 2*x**2 + 2))
```

Mathematica [C] time = 0.142799, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{3x^4+2x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I
*Sqrt[5])]*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1
- I*Sqrt[5])/(-1 + I*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(
-1)]*Sqrt[2 + 2*x^2 + 3*x^4])
```

Maple [C] time = 0.177, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{5}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{5}\right)x^2}EllipticF\left(1/2\,x\sqrt{-2+2\,i\sqrt{5}},1/3\,\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{-2+2\,i\sqrt{5}}\sqrt{3\,x^4+2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+2*x^2+2)^(1/2),x)
```

```
[Out] 2/(-2+2*I*5^(1/2))^(1/2)*(1-(-1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-
1/2-1/2*I*5^(1/2))*x^2)^(1/2)/(3*x^4+2*x^2+2)^(1/2)*EllipticF(1/2
*x*(-2+2*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2*x^2 + 2),x, algorithm="fricas")

[Out] integral(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+2x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+2*x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(3*x**4 + 2*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2*x^2 + 2),x, algorithm="giac")

[Out] integrate(1/sqrt(3*x^4 + 2*x^2 + 2), x)
```

3.70 $\int \frac{1}{\sqrt{2+x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[2 + x^2 + 3*x^4])
```

Rubi [A] time = 0.0588196, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + x^2 + 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[2 + x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 2.99494, size = 85, normalized size = 0.97

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4+x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{3x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 + x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqrt(
6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqrt(6)
```

```
/24 + 1/2)/(12*sqrt(3*x**4 + x**2 + 2))
```

Mathematica [C] time = 0.144619, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{3x^4+x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 +
I*Sqrt[23])]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (
-1 - I*Sqrt[23])/(-1 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[
23])^(-1)]*Sqrt[2 + x^2 + 3*x^4])
```

Maple [C] time = 0.154, size = 85, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/4+i/4\sqrt{23}\right)x^2}\sqrt{1-\left(-1/4-i/4\sqrt{23}\right)x^2}\,EllipticF\left(1/2\,x\sqrt{-1+i\sqrt{23}},1/6\,\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{-1+i\sqrt{23}}\sqrt{3\,x^4+x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+x^2+2)^(1/2),x)
```

```
[Out] 2/(-1+I*23^(1/2))^(1/2)*(1-(-1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-
1/4-1/4*I*23^(1/2))*x^2)^(1/2)/(3*x^4+x^2+2)^(1/2)*EllipticF(1/2*
x*(-1+I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + x^2 + 2), x)
```

3.71 $\int \frac{1}{\sqrt{2+3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3x^4+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Rubi [A] time = 0.0339236, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{3x^4+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[2 + 3*x^4])
```

Rubi in Sympy [A] time = 1.45892, size = 71, normalized size = 0.99

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{1}{2}\right)}{12\sqrt{3x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqrt(6)*x**2
/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), 1/2)/(12*sqrt(3
*x**4 + 2))
```

Mathematica [C] time = 0.0411105, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 3*x^4],x]
```

```
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-3/2)^(1/4)*x], -1])
```

Maple [C] time = 0.08, size = 66, normalized size = 0.9

$$\frac{\sqrt{2}}{4\sqrt{i\sqrt{6}}}\sqrt{4-2\,i\sqrt{6}x^2}\sqrt{4+2\,i\sqrt{6}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{2}\sqrt{i\sqrt{6}}}{2},i\right)\frac{1}{\sqrt{3\,x^4+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+2)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/(I*6^(1/2))^(1/2)*(4-2*I*6^(1/2)*x^2)^(1/2)*(4+2*I*6^
(1/2)*x^2)^(1/2)/(3*x^4+2)^(1/2)*EllipticF(1/2*x*2^(1/2)*(I*6^(1/
2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 2), x)
```

Sympy [A] time = 1.74014, size = 36, normalized size = 0.5

$$\frac{\sqrt{2}x\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+2)**(1/2),x)
```

```
[Out] sqrt(2)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_polar(I
*pi)/2)/(8*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 2), x)
```

3.72 $\int \frac{1}{\sqrt{2-x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[2 - x^2 + 3*x^4])
```

Rubi [A] time = 0.0556495, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[2 - x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.4228, size = 85, normalized size = 0.94

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4-x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{3x^4-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 - x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqrt(
6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt(6)/
```

```
24 + 1/2)/(12*sqrt(3*x**4 - x**2 + 2))
```

Mathematica [C] time = 0.14007, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{3x^4-x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*
Sqrt[23])]*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 -
 I*Sqrt[23])/(1 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(
-1)]*Sqrt[2 - x^2 + 3*x^4])
```

Maple [C] time = 0.144, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/4+i/4\sqrt{23}\right)x^2}\sqrt{1-\left(1/4-i/4\sqrt{23}\right)x^2}EllipticF\left(1/2\,x\sqrt{i\sqrt{23}+1},1/6\,\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{i\sqrt{23}+1}\sqrt{3\,x^4-x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-x^2+2)^(1/2),x)
```

```
[Out] 2/(I*23^(1/2)+1)^(1/2)*(1-(1/4+1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4
-1/4*I*23^(1/2))*x^2)^(1/2)/(3*x^4-x^2+2)^(1/2)*EllipticF(1/2*x*(
I*23^(1/2)+1)^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4 - x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - x^2 + 2), x)
```

3.73 $\int \frac{1}{\sqrt{2-2x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[2 - 2*x^2 + 3*x^4])
```

Rubi [A] time = 0.0516584, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[2 - 2*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.52214, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4-2x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{3x^4-2x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 - 2*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt(6
```

```
)/12 + 1/2)/(12*sqrt(3*x**4 - 2*x**2 + 2))
```

Mathematica [C] time = 0.136237, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{3x^4-2x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 2*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*S
qrt[5])]*EllipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*
Sqrt[5])/(1 + I*Sqrt[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*S
qrt[2 - 2*x^2 + 3*x^4])
```

Maple [C] time = 0.141, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/2+i/2\sqrt{5}\right)x^2}\sqrt{1-\left(1/2-i/2\sqrt{5}\right)x^2}\,EllipticF\left(1/2\,x\sqrt{2+2\,i\sqrt{5}},1/3\,\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{2+2\,i\sqrt{5}}\sqrt{3\,x^4-2\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-2*x^2+2)^(1/2),x)
```

```
[Out] 2/(2+2*I*5^(1/2))^(1/2)*(1-(1/2+1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2
-1/2*I*5^(1/2))*x^2)^(1/2)/(3*x^4-2*x^2+2)^(1/2)*EllipticF(1/2*x*
(2+2*I*5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-2\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 2*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^{4}-2\,x^{2}+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(3*x^4 - 2*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^{4}-2x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-2*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 2*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^{4}-2\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(3*x^4 - 2*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 2*x^2 + 2), x)
```

3.74 $\int \frac{1}{\sqrt{2-3x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[2 - 3*x^2 + 3*x^4])
```

Rubi [A] time = 0.0540099, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```



```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[2 - 3*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.96561, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4-3x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{3x^4-3x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 - 3*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt(6
```

```
)/8 + 1/2)/(12*sqrt(3*x**4 - 3*x**2 + 2))
```

Mathematica [C] time = 0.20103, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{3x^4-3x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 3*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*
Sqrt[15])]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 -
 I*Sqrt[15])/(3 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(
-1)]*Sqrt[2 - 3*x^2 + 3*x^4])
```

Maple [C] time = 0.159, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(3/4+i/4\sqrt{15}\right)x^2}\sqrt{1-\left(3/4-i/4\sqrt{15}\right)x^2}EllipticF\left(1/2\,x\sqrt{i\sqrt{15}+3},1/2\,\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{i\sqrt{15}+3}\sqrt{3\,x^4-3\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-3*x^2+2)^(1/2),x)
```

```
[Out] 2/(I*15^(1/2)+3)^(1/2)*(1-(3/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4
-1/4*I*15^(1/2))*x^2)^(1/2)/(3*x^4-3*x^2+2)^(1/2)*EllipticF(1/2*x
*(I*15^(1/2)+3)^(1/2),1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-3\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 3*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4 - 3\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(3*x^4 - 3*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - 3x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4-3*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 3*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - 3\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(3*x^4 - 3*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 3*x^2 + 2), x)
```

3.75 $\int \frac{1}{\sqrt{2-4x^2+3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[2 - 4*x^2 + 3*x^4])
```

Rubi [A] time = 0.0461396, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[2 - 4*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.05793, size = 88, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4-4x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{3x^4-4x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 - 4*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt(6
```

```
)/6 + 1/2)/(12*sqrt(3*x**4 - 4*x**2 + 2))
```

Mathematica [C] time = 0.144884, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{3x^4-4x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 4*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*S
qrt[2])]*EllipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*
Sqrt[2])/(2 + I*Sqrt[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*S
qrt[2 - 4*x^2 + 3*x^4])
```

Maple [C] time = 0.143, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1+i/2\sqrt{2}\right)x^2}\sqrt{1-\left(1-i/2\sqrt{2}\right)x^2}EllipticF\left(1/2\,x\sqrt{4+2\,i\sqrt{2}},1/3\,\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{4+2\,i\sqrt{2}}\sqrt{3\,x^4-4\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-4*x^2+2)^(1/2),x)
```

```
[Out] 2/(4+2*I*2^(1/2))^(1/2)*(1-(1+1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1-1/2
*I*2^(1/2))*x^2)^(1/2)/(3*x^4-4*x^2+2)^(1/2)*EllipticF(1/2*x*(4+2
*I*2^(1/2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4-4\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 4*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4 - 4\,x^2 + 2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 4*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 - 4x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-4*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 4*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 - 4\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 4*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 4*x^2 + 2), x)
```

3.76 $\int \frac{1}{\sqrt{2-5x^2+3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/
4)*Sqrt[2 - 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0526538, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-5x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 5*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/
4)*Sqrt[2 - 5*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 4.0521, size = 90, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4-5x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{1}{2}+\frac{5\sqrt{6}}{24}\right)}{12\sqrt{3x^4-5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 - 5*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), 1/2 +
```

```
5*sqrt(6)/24)/(12*sqrt(3*x**4 - 5*x**2 + 2))
```

Mathematica [A] time = 0.0414906, size = 53, normalized size = 0.58

$$\frac{\sqrt{2-3x^2}\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{9x^4-15x^2+6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 5*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3
])/Sqrt[6 - 15*x^2 + 9*x^4]
```

Maple [A] time = 0.012, size = 42, normalized size = 0.5

$$\frac{1}{2}\sqrt{-x^2+1}\sqrt{-6\,x^2+4}EllipticF\left(x,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{3\,x^4-5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(3*x^4-5*x^2+2)^(1/2)*Ellipti
cF(x,1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4-5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 5*x^2 + 2), x)
```

3.77 $\int \frac{1}{\sqrt{2-6x^2+3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*S
qrt[2 - 6*x^2 + 3*x^4])
```

Rubi [A] time = 0.0547033, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-6x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 6*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*S
qrt[2 - 6*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.98896, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{3x^4-6x^2+2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{1}{2}+\frac{\sqrt{6}}{4}\right)}{12\sqrt{3x^4-6x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4-6*x**2+2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((3*x**4 - 6*x**2 + 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), 1/2 +
```

```
sqrt(6)/4)/(12*sqrt(3*x**4 - 6*x**2 + 2))
```

Mathematica [A] time = 0.144108, size = 85, normalized size = 0.94

$$\frac{\sqrt{-3x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+2}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right)|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{3x^4-6x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 - 6*x^2 + 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF
[ArcSin[Sqrt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[2 -
 6*x^2 + 3*x^4])
```

Maple [A] time = 0.223, size = 82, normalized size = 0.9

$$2\,\frac{\sqrt{1-\left(1/2\,\sqrt{3}+3/2\right)x^2}\sqrt{1-\left(-1/2\,\sqrt{3}+3/2\right)x^2}\mathit{EllipticF}\left(1/2\,x\sqrt{6+2\,\sqrt{3}},1/2\,\sqrt{6}-1/2\,\sqrt{2}\right)}{\sqrt{6+2\,\sqrt{3}}\sqrt{3\,x^4-6\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4-6*x^2+2)^(1/2),x)
```

```
[Out] 2/(6+2*3^(1/2))^(1/2)*(1-(1/2*3^(1/2)+3/2)*x^2)^(1/2)*(1-(-1/2*3^
(1/2)+3/2)*x^2)^(1/2)/(3*x^4-6*x^2+2)^(1/2)*EllipticF(1/2*x*(6+2*
3^(1/2))^(1/2),1/2*6^(1/2)-1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 6*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4-6\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 6*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3x^4-6x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4-6*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 - 6*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4-6\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 - 6*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 - 6*x^2 + 2), x)
```

3.78 $\int \frac{1}{\sqrt{3+9x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{\left(9-\sqrt{57}\right)x^2+6}{\left(9+\sqrt{57}\right)x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

```
[Out] (Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9
+ Sqrt[57])*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19
 + 3*Sqrt[57])/4])/(Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4
])
```

Rubi [A] time = 0.18325, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{\left(9-\sqrt{57}\right)x^2+6}{\left(9+\sqrt{57}\right)x^2+6}}\left(\left(9+\sqrt{57}\right)x^2+6\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(9+\sqrt{57}\right)}x\right)|\frac{1}{4}\left(-19+3\sqrt{57}\right)\right)}{\sqrt{6\left(9+\sqrt{57}\right)}\sqrt{2x^4+9x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(6 + (9 - Sqrt[57])*x^2)/(6 + (9 + Sqrt[57])*x^2)]*(6 + (9
+ Sqrt[57])*x^2)*EllipticF[ArcTan[Sqrt[(9 + Sqrt[57])/6]*x], (-19
 + 3*Sqrt[57])/4])/(Sqrt[6*(9 + Sqrt[57])]*Sqrt[3 + 9*x^2 + 2*x^4
])
```

Rubi in Sympy [A] time = 4.98422, size = 102, normalized size = 0.93

$$\frac{\sqrt{6}\sqrt{\frac{x^2\left(-\sqrt{57}+9\right)+6}{x^2\left(\sqrt{57}+9\right)+6}}\left(x^2\left(\sqrt{57}+9\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{6}x\sqrt{\sqrt{57}+9}}{6}\right)\middle|-\frac{19}{4}+\frac{3\sqrt{57}}{4}\right)}{6\sqrt{\sqrt{57}+9}\sqrt{2x^4+9x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+9*x**2+3)**(1/2),x)
```

```
[Out] sqrt(6)*sqrt((x**2*(-sqrt(57) + 9) + 6)/(x**2*(sqrt(57) + 9) + 6)
)*(x**2*(sqrt(57) + 9) + 6)*elliptic_f(atan(sqrt(6)*x*sqrt(sqrt(5
7) + 9)/6), -19/4 + 3*sqrt(57)/4)/(6*sqrt(sqrt(57) + 9)*sqrt(2*x*
*4 + 9*x**2 + 3))
```

Mathematica [C] time = 0.127315, size = 97, normalized size = 0.88

$$-\frac{i\sqrt{\frac{-4x^2+\sqrt{57}-9}{\sqrt{57}-9}}\sqrt{4x^2+\sqrt{57}+9}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{9+\sqrt{57}}}\right)|\frac{23}{4}+\frac{3\sqrt{57}}{4}\right)}{2\sqrt{2x^4+9x^2+3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 9*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[(-9 + Sqrt[57] - 4*x^2)/(-9 + Sqrt[57])]*Sqrt[9 + Sq
rt[57] + 4*x^2]*EllipticF[I*ArcSinh[(2*x)/Sqrt[9 + Sqrt[57]]], 23
/4 + (3*Sqrt[57])/4])/Sqrt[3 + 9*x^2 + 2*x^4]
```

Maple [A] time = 0.236, size = 82, normalized size = 0.8

$$6\,\frac{\sqrt{1-\left(-3/2+1/6\,\sqrt{57}\right)x^2}\sqrt{1-\left(-3/2-1/6\,\sqrt{57}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-54+6\,\sqrt{57}},3/4\,\sqrt{6}+1/4\,\sqrt{38}\right)}{\sqrt{-54+6\,\sqrt{57}}\sqrt{2\,x^4+9\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+9*x^2+3)^(1/2),x)
```

```
[Out] 6/(-54+6*57^(1/2))^(1/2)*(1-(-3/2+1/6*57^(1/2))*x^2)^(1/2)*(1-(-3
/2-1/6*57^(1/2))*x^2)^(1/2)/(2*x^4+9*x^2+3)^(1/2)*EllipticF(1/6*x
*(-54+6*57^(1/2))^(1/2),3/4*6^(1/2)+1/4*38^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 9*x^2 + 3),x, algorithm="maxima")

[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+9\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 9*x^2 + 3),x, algorithm="fricas")

[Out] integral(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+9x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+9*x**2+3)**(1/2),x)

[Out] Integral(1/sqrt(2*x**4 + 9*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+9\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 9*x^2 + 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 + 9*x^2 + 3), x)
```

3.79 $\int \frac{1}{\sqrt{3+8x^2+2x^4}}\,dx$

Optimal. Leaf size=110

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

```
[Out] (Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4
+ Sqrt[10])*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*
(5 - 2*Sqrt[10]))/3])/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*
x^4])
```

Rubi [A] time = 0.184747, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{\left(4-\sqrt{10}\right)x^2+3}{\left(4+\sqrt{10}\right)x^2+3}}\left(\left(4+\sqrt{10}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(4+\sqrt{10}\right)}x\right)|-\frac{2}{3}\left(5-2\sqrt{10}\right)\right)}{\sqrt{3\left(4+\sqrt{10}\right)}\sqrt{2x^4+8x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (4 - Sqrt[10])*x^2)/(3 + (4 + Sqrt[10])*x^2)]*(3 + (4
+ Sqrt[10])*x^2)*EllipticF[ArcTan[Sqrt[(4 + Sqrt[10])/3]*x], (-2*
(5 - 2*Sqrt[10]))/3])/(Sqrt[3*(4 + Sqrt[10])]*Sqrt[3 + 8*x^2 + 2*
x^4])
```

Rubi in Sympy [A] time = 5.06709, size = 107, normalized size = 0.97

$$\frac{\sqrt{3}\sqrt{\frac{x^2\left(-2\sqrt{10}+8\right)+6}{x^2\left(2\sqrt{10}+8\right)+6}}\left(x^2\left(2\sqrt{10}+8\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{3}x\sqrt{\sqrt{10}+4}}{3}\right)\middle|-\frac{10}{3}+\frac{4\sqrt{10}}{3}\right)}{6\sqrt{\sqrt{10}+4}\sqrt{2x^4+8x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+8*x**2+3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt((x**2*(-2*sqrt(10) + 8) + 6)/(x**2*(2*sqrt(10) + 8)
+ 6))*(x**2*(2*sqrt(10) + 8) + 6)*elliptic_f(atan(sqrt(3)*x*sqrt(
sqrt(10) + 4)/3), -10/3 + 4*sqrt(10)/3)/(6*sqrt(sqrt(10) + 4)*sqr
t(2*x**4 + 8*x**2 + 3))
```

Mathematica [C] time = 0.140025, size = 98, normalized size = 0.89

$$-\frac{i\sqrt{\frac{-2x^2+\sqrt{10}-4}{\sqrt{10}-4}}\sqrt{2x^2+\sqrt{10}+4}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{4+\sqrt{10}}}x\right)|\frac{13}{3}+\frac{4\sqrt{10}}{3}\right)}{\sqrt{4x^4+16x^2+6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 8*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-4 + Sqrt[10] - 2*x^2)/(-4 + Sqrt[10])]*Sqrt[4 + Sqrt
[10] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(4 + Sqrt[10])]*x], 13/3
 + (4*Sqrt[10])/3])/Sqrt[6 + 16*x^2 + 4*x^4]
```

Maple [A] time = 0.234, size = 82, normalized size = 0.8

$$3\,\frac{\sqrt{1-\left(-4/3+1/3\,\sqrt{10}\right)x^2}\sqrt{1-\left(-4/3-1/3\,\sqrt{10}\right)x^2}\,EllipticF\left(1/3\,x\sqrt{-12+3\,\sqrt{10}},2/3\,\sqrt{6}+1/3\,\sqrt{15}\right)}{\sqrt{-12+3\,\sqrt{10}}\sqrt{2\,x^4+8\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+8*x^2+3)^(1/2),x)
```

```
[Out] 3/(-12+3*10^(1/2))^(1/2)*(1-(-4/3+1/3*10^(1/2))*x^2)^(1/2)*(1-(-4
/3-1/3*10^(1/2))*x^2)^(1/2)/(2*x^4+8*x^2+3)^(1/2)*EllipticF(1/3*x
*(-12+3*10^(1/2))^(1/2),2/3*6^(1/2)+1/3*15^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 8*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+8\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 8*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+8x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+8*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 8*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+8\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 8*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 8*x^2 + 3), x)
```

3.80 $\int \frac{1}{\sqrt{3+7x^2+2x^4}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\,(2x^2+1)\,F\left(\tan^{-1}\left(\sqrt{2}x\right)\,|\frac{5}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

```
[Out] (Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]
*x], 5/6])/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rubi [A] time = 0.0275477, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{x^2+3}{2x^2+1}}\,(2x^2+1)\,F\left(\tan^{-1}\left(\sqrt{2}x\right)\,|\frac{5}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]
*x], 5/6])/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.69853, size = 56, normalized size = 0.93

$$\frac{\sqrt{2}\sqrt{\frac{2x^2+6}{12x^2+6}}\,(12x^2+6)\,F\left(\operatorname{atan}\left(\sqrt{2}x\right)\middle|\frac{5}{6}\right)}{12\sqrt{2x^4+7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt((2*x**2 + 6)/(12*x**2 + 6))*(12*x**2 + 6)*elliptic_f
(atan(sqrt(2)*x), 5/6)/(12*sqrt(2*x**4 + 7*x**2 + 3))
```

Mathematica [C] time = 0.0422, size = 61, normalized size = 1.02

$$-\frac{i\sqrt{x^2+3}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{6}\right)}{\sqrt{6}\sqrt{2x^4+7x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 7*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[3 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]*x
], 1/6])/(Sqrt[6]*Sqrt[3 + 7*x^2 + 2*x^4])
```

Maple [C] time = 0.105, size = 50, normalized size = 0.8

$$-\frac{i}{3}\sqrt{3}EllipticF\left(\frac{i}{3}\sqrt{3}x,\sqrt{6}\right)\sqrt{3x^2+9}\sqrt{2x^2+1}\frac{1}{\sqrt{2x^4+7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+7*x^2+3)^(1/2),x)
```

```
[Out] -1/3*I*3^(1/2)*(3*x^2+9)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+7*x^2+3)^(1
/2)*EllipticF(1/3*I*3^(1/2)*x,6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 7*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2x^4+7x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 7*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 + 7x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 7*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 + 7\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 7*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 7*x^2 + 3), x)
```

3.81 $\int \frac{1}{\sqrt{3+6x^2+2x^4}}\,dx$

Optimal. Leaf size=104

$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+3}{\left(3+\sqrt{3}\right)x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

```
[Out] (Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 +
Sqrt[3])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqr
t[3]])/(Sqrt[3*(3 + Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rubi [A] time = 0.139387, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\sqrt{\frac{\left(3-\sqrt{3}\right)x^2+3}{\left(3+\sqrt{3}\right)x^2+3}}\left(\left(3+\sqrt{3}\right)x^2+3\right)F\left(\tan^{-1}\left(\sqrt{\frac{1}{3}\left(3+\sqrt{3}\right)}x\right)|-1+\sqrt{3}\right)}{\sqrt{3\left(3+\sqrt{3}\right)}\sqrt{2x^4+6x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(3 + (3 - Sqrt[3])*x^2)/(3 + (3 + Sqrt[3])*x^2)]*(3 + (3 +
Sqrt[3])*x^2)*EllipticF[ArcTan[Sqrt[(3 + Sqrt[3])/3]*x], -1 + Sqr
t[3]])/(Sqrt[3*(3 + Sqrt[3])]*Sqrt[3 + 6*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 4.96566, size = 102, normalized size = 0.98

$$\frac{\sqrt{3}\sqrt{\frac{x^2\left(-2\sqrt{3}+6\right)+6}{x^2\left(2\sqrt{3}+6\right)+6}}\left(x^2\left(2\sqrt{3}+6\right)+6\right)F\left(\operatorname{atan}\left(\frac{\sqrt{3}x\sqrt{\sqrt{3}+3}}{3}\right)\middle|-1+\sqrt{3}\right)}{6\sqrt{\sqrt{3}+3}\sqrt{2x^4+6x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] sqrt(3)*sqrt((x**2*(-2*sqrt(3) + 6) + 6)/(x**2*(2*sqrt(3) + 6) +
6))*(x**2*(2*sqrt(3) + 6) + 6)*elliptic_f(atan(sqrt(3)*x*sqrt(sqr
t(3) + 3)/3), -1 + sqrt(3))/(6*sqrt(sqrt(3) + 3)*sqrt(2*x**4 + 6*
x**2 + 3))
```

Mathematica [C] time = 0.0977932, size = 90, normalized size = 0.87

$$-\frac{i\sqrt{\frac{-2x^2+\sqrt{3}-3}{\sqrt{3}-3}}\sqrt{2x^2+\sqrt{3}+3}F\left(i\sinh^{-1}\left(\sqrt{1-\frac{1}{\sqrt{3}}}x\right)|2+\sqrt{3}\right)}{\sqrt{4x^4+12x^2+6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[3 + 6*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[(-3 + Sqrt[3] - 2*x^2)/(-3 + Sqrt[3])]*Sqrt[3 + Sqrt[3
] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - 1/Sqrt[3]]*x], 2 + Sqrt[3
]])/Sqrt[6 + 12*x^2 + 4*x^4]
```

Maple [A] time = 0.221, size = 82, normalized size = 0.8

$$3\,\frac{\sqrt{1-\left(-1+1/3\,\sqrt{3}\right)x^2}\sqrt{1-\left(-1-1/3\,\sqrt{3}\right)x^2}EllipticF\left(1/3\,x\sqrt{-9+3\,\sqrt{3}},1/2\,\sqrt{6}+1/2\,\sqrt{2}\right)}{\sqrt{-9+3\,\sqrt{3}}\sqrt{2\,x^4+6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+6*x^2+3)^(1/2),x)
```

```
[Out] 3/(-9+3*3^(1/2))^(1/2)*(1-(-1+1/3*3^(1/2))*x^2)^(1/2)*(1-(-1-1/3*
3^(1/2))*x^2)^(1/2)/(2*x^4+6*x^2+3)^(1/2)*EllipticF(1/3*x*(-9+3*3
^(1/2))^(1/2),1/2*6^(1/2)+1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 6*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 6*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 6*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 6*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 6*x^2 + 3), x)
```

3.82 $\int \frac{1}{\sqrt{3+5x^2+2x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{\left(x^2+1\right)\sqrt{\frac{2x^2+3}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

```
[Out] ((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])
/(Sqrt[3]*Sqrt[3 + 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0222564, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(x^2+1\right)\sqrt{\frac{2x^2+3}{x^2+1}}F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\sqrt{2x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(3 + 2*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], 1/3])
/(Sqrt[3]*Sqrt[3 + 5*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.63815, size = 49, normalized size = 0.94

$$\frac{\sqrt{6}\sqrt{\frac{4x^2+6}{x^2+1}}\left(6x^2+6\right)F\left(\operatorname{atan}(x)\middle|\frac{1}{3}\right)}{36\sqrt{2x^4+5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+5*x**2+3)**(1/2),x)
```

```
[Out] sqrt(6)*sqrt((4*x**2 + 6)/(x**2 + 1))*(6*x**2 + 6)*elliptic_f(ata
n(x), 1/3)/(36*sqrt(2*x**4 + 5*x**2 + 3))
```

Mathematica [C] time = 0.0421946, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\sqrt{2x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)}{\sqrt{4x^4+10x^2+6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]
*x], 3/2])/Sqrt[6 + 10*x^2 + 4*x^4]
```

Maple [C] time = 0.049, size = 50, normalized size = 1.

$$-\frac{i}{6}\sqrt{6}\sqrt{6\,x^2+9}\sqrt{x^2+1}\mathit{EllipticF}\left(\frac{i}{3}x\sqrt{6},\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{2\,x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+5*x^2+3)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+9)^(1/2)*(x^2+1)^(1/2)/(2*x^4+5*x^2+3)^(1/2
)*EllipticF(1/3*I*x*6^(1/2),1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 + 3),x, algorithm="fricas")

[Out] integral(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+5*x**2+3)**(1/2),x)

[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 + 3),x, algorithm="giac")

[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 3), x)
```

3.83 $\int \frac{1}{\sqrt{3+4x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[3 + 4*x^2 + 2*x^4])
```

Rubi [A] time = 0.0727126, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[3 + 4*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.83202, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4+4x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{2x^4+4x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 + 4*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqrt(
```

```
6)/6 + 1/2)/(12*sqrt(2*x**4 + 4*x**2 + 3))
```

Mathematica [C] time = 0.156345, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right)\mid\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{2x^4+4x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I
*Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2
- I*Sqrt[2])/(-2 + I*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(
-1)]*Sqrt[3 + 4*x^2 + 2*x^4])
```

Maple [C] time = 0.124, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-2/3+i/3\sqrt{2}\right)x^2}\sqrt{1-\left(-2/3-i/3\sqrt{2}\right)x^2}\,EllipticF\left(1/3\,x\sqrt{-6+3\,i\sqrt{2}},1/3\,\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{-6+3\,i\sqrt{2}}\sqrt{2\,x^4+4\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+4*x^2+3)^(1/2),x)
```

```
[Out] 3/(-6+3*I*2^(1/2))^(1/2)*(1-(-2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-
2/3-1/3*I*2^(1/2))*x^2)^(1/2)/(2*x^4+4*x^2+3)^(1/2)*EllipticF(1/3
*x*(-6+3*I*2^(1/2))^(1/2),1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 4*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 4*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4+4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 4*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 4*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 4*x^2 + 3), x)
```

3.84 $\int \frac{1}{\sqrt{3+3x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[3 + 3*x^2 + 2*x^4])
```

Rubi [A] time = 0.0584123, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[3 + 3*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.76889, size = 88, normalized size = 0.96

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4+3x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{2x^4+3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 + 3*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqrt(
```

```
6)/8 + 1/2)/(12*sqrt(2*x**4 + 3*x**2 + 3))
```

Mathematica [C] time = 0.181709, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{2x^4+3x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3
+ I*Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)
]*x], (-3 - I*Sqrt[15])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[1
5])^(-1)]*Sqrt[3 + 3*x^2 + 2*x^4])
```

Maple [C] time = 0.125, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/2+i/6\sqrt{15}\right)x^2}\sqrt{1-\left(-1/2-i/6\sqrt{15}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{-18+6\,i\sqrt{15}},1/2\,\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{-18+6\,i\sqrt{15}}\sqrt{2\,x^4+3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+3*x^2+3)^(1/2),x)
```

```
[Out] 6/(-18+6*I*15^(1/2))^(1/2)*(1-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1
-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)/(2*x^4+3*x^2+3)^(1/2)*EllipticF
(1/6*x*(-18+6*I*15^(1/2))^(1/2),1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 3*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3*x^2 + 3), x)
```

3.85 $\int \frac{1}{\sqrt{3+2x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[3 + 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.053937, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[3 + 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.73867, size = 88, normalized size = 0.96

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4+2x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{2x^4+2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 + 2*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqrt(
```

```
6)/12 + 1/2)/(12*sqrt(2*x**4 + 2*x**2 + 3))
```

Mathematica [C] time = 0.139533, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{2x^4+2x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I
*Sqrt[5])]*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1
- I*Sqrt[5])/(-1 + I*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(
-1)]*Sqrt[3 + 2*x^2 + 2*x^4])
```

Maple [C] time = 0.125, size = 87, normalized size = 1.

$$3\frac{\sqrt{1-\left(-1/3+i/3\sqrt{5}\right)x^2}\sqrt{1-\left(-1/3-i/3\sqrt{5}\right)x^2}EllipticF\left(1/3\,x\sqrt{-3+3\,i\sqrt{5}},1/3\,\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{-3+3\,i\sqrt{5}}\sqrt{2\,x^4+2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+2*x^2+3)^(1/2),x)
```

```
[Out] 3/(-3+3*I*5^(1/2))^(1/2)*(1-(-1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-
1/3-1/3*I*5^(1/2))*x^2)^(1/2)/(2*x^4+2*x^2+3)^(1/2)*EllipticF(1/3
*x*(-3+3*I*5^(1/2))^(1/2),1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+2\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4+2x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 2*x^2 + 3), x)
```

3.86 $\int \frac{1}{\sqrt{3+x^2+2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[3 + x^2 + 2*x^4])
```

Rubi [A] time = 0.0553382, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[3 + x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.00467, size = 85, normalized size = 0.97

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4+x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{2x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 + x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqrt(
6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqrt(6)
```

```
/24 + 1/2)/(12*sqrt(2*x**4 + x**2 + 3))
```

Mathematica [C] time = 0.125209, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{2x^4+x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1
+ I*Sqrt[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)
]*x], (-1 - I*Sqrt[23])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[2
3])^(-1)]*Sqrt[3 + x^2 + 2*x^4])
```

Maple [C] time = 0.128, size = 85, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/6+i/6\sqrt{23}\right)x^2}\sqrt{1-\left(-1/6-i/6\sqrt{23}\right)x^2}EllipticF\left(1/6\,x\sqrt{-6+6\,i\sqrt{23}},1/6\,\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{-6+6\,i\sqrt{23}}\sqrt{2\,x^4+x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+x^2+3)^(1/2),x)
```

```
[Out] 6/(-6+6*I*23^(1/2))^(1/2)*(1-(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-
(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)/(2*x^4+x^2+3)^(1/2)*EllipticF(1/
6*x*(-6+6*I*23^(1/2))^(1/2),1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4+x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + x^2 + 3), x)
```

3.87 $\int \frac{1}{\sqrt{3+2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\mid\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2x^4+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Rubi [A] time = 0.0325061, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\mid\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{2x^4+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[3 + 2*x^4])
```

Rubi in Sympy [A] time = 1.44617, size = 71, normalized size = 0.99

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{1}{2}\right)}{12\sqrt{2x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqrt(6)*x**2
/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), 1/2)/(12*sqrt(2
*x**4 + 3))
```

Mathematica [C] time = 0.0425801, size = 25, normalized size = 0.35

$$-\sqrt[4]{-\frac{1}{6}}F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}x\right)\middle|-1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 + 2*x^4],x]
```

```
[Out] -((-1/6)^(1/4)*EllipticF[I*ArcSinh[(-2/3)^(1/4)*x], -1])
```

Maple [C] time = 0.063, size = 66, normalized size = 0.9

$$\frac{\sqrt{3}}{9\sqrt{i\sqrt{6}}}\sqrt{9-3\,i\sqrt{6}x^2}\sqrt{9+3\,i\sqrt{6}x^2}\mathit{EllipticF}\left(\frac{x\sqrt{3}\sqrt{i\sqrt{6}}}{3},i\right)\frac{1}{\sqrt{2\,x^4+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+3)^(1/2),x)
```

```
[Out] 1/9*3^(1/2)/(I*6^(1/2))^(1/2)*(9-3*I*6^(1/2)*x^2)^(1/2)*(9+3*I*6^
(1/2)*x^2)^(1/2)/(2*x^4+3)^(1/2)*EllipticF(1/3*x*3^(1/2)*(I*6^(1/
2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 3), x)
```

Sympy [A] time = 1.76405, size = 36, normalized size = 0.5

$$\frac{\sqrt{3}x\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\,\frac{2x^{4}e^{i\pi}}{3}\right)}{12\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+3)**(1/2),x)
```

```
[Out] sqrt(3)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_polar(I
*pi)/3)/(12*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^{4}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 3), x)
```

3.88 $\int \frac{1}{\sqrt{3-x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[3 - x^2 + 2*x^4])
```

Rubi [A] time = 0.0529713, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[3 - x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.37596, size = 85, normalized size = 0.94

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{2x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqrt(
6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt(6)/
```

```
24 + 1/2)/(12*sqrt(2*x**4 - x**2 + 3))
```

Mathematica [C] time = 0.120266, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{2x^4-x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 +
I*Sqrt[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x
], (1 - I*Sqrt[23])/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-
1)]*Sqrt[3 - x^2 + 2*x^4])
```

Maple [C] time = 0.117, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/6+i/6\sqrt{23}\right)x^2}\sqrt{1-\left(1/6-i/6\sqrt{23}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{6+6\,i\sqrt{23}},1/6\,\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{6+6\,i\sqrt{23}}\sqrt{2\,x^4-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-x^2+3)^(1/2),x)
```

```
[Out] 6/(6+6*I*23^(1/2))^(1/2)*(1-(1/6+1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1
/6-1/6*I*23^(1/2))*x^2)^(1/2)/(2*x^4-x^2+3)^(1/2)*EllipticF(1/6*x
*(6+6*I*23^(1/2))^(1/2),1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - x^2 + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - x^2 + 3), x)
```

3.89 $\int \frac{1}{\sqrt{3-2x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[3 - 2*x^2 + 2*x^4])
```

Rubi [A] time = 0.0526938, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[3 - 2*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.93937, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-2x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{2x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - 2*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt(6
```

```
)/12 + 1/2)/(12*sqrt(2*x**4 - 2*x**2 + 3))
```

Mathematica [C] time = 0.136139, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{2x^4-2x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 2*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*S
qrt[5])]*EllipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*
Sqrt[5])/(1 + I*Sqrt[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*S
qrt[3 - 2*x^2 + 2*x^4])
```

Maple [C] time = 0.116, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(1/3+i/3\sqrt{5}\right)x^2}\sqrt{1-\left(1/3-i/3\sqrt{5}\right)x^2}EllipticF\left(1/3\,x\sqrt{3+3\,i\sqrt{5}},1/3\,\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{3+3\,i\sqrt{5}}\sqrt{2\,x^4-2\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 3/(3+3*I*5^(1/2))^(1/2)*(1-(1/3+1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3
-1/3*I*5^(1/2))*x^2)^(1/2)/(2*x^4-2*x^2+3)^(1/2)*EllipticF(1/3*x*
(3+3*I*5^(1/2))^(1/2),1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-2\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4 - 2\,x^2 + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 2x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 2\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 2*x^2 + 3), x)
```

3.90 $\int \frac{1}{\sqrt{3-3x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\mid\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[3 - 3*x^2 + 2*x^4])
```

Rubi [A] time = 0.0519937, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)\mid\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[3 - 3*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 4.50281, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-3x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{2x^4-3x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - 3*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt(6
```

```
)/8 + 1/2)/(12*sqrt(2*x**4 - 3*x**2 + 3))
```

Mathematica [C] time = 0.174557, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{2x^4-3x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 3*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 +
I*Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x
], (3 - I*Sqrt[15])/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-
1)]*Sqrt[3 - 3*x^2 + 2*x^4])
```

Maple [C] time = 0.119, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/2+i/6\sqrt{15}\right)x^2}\sqrt{1-\left(1/2-i/6\sqrt{15}\right)x^2}\,EllipticF\left(1/6\,x\sqrt{18+6\,i\sqrt{15}},1/2\,\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{18+6\,i\sqrt{15}}\sqrt{2\,x^4-3\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-3*x^2+3)^(1/2),x)
```

```
[Out] 6/(18+6*I*15^(1/2))^(1/2)*(1-(1/2+1/6*I*15^(1/2))*x^2)^(1/2)*(1-(
1/2-1/6*I*15^(1/2))*x^2)^(1/2)/(2*x^4-3*x^2+3)^(1/2)*EllipticF(1/
6*x*(18+6*I*15^(1/2))^(1/2),1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-3\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4-3x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-3*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 3*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-3\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 3*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 3*x^2 + 3), x)
```

3.91 $\int \frac{1}{\sqrt{3-4x^2+2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[3 - 4*x^2 + 2*x^4])
```

Rubi [A] time = 0.0456795, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[3 - 4*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.98345, size = 88, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-4x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\text{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{2x^4-4x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - 4*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt(6
```

```
)/6 + 1/2)/(12*sqrt(2*x**4 - 4*x**2 + 3))
```

Mathematica [C] time = 0.148492, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{2x^4-4x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 4*x^2 + 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*S
qrt[2])]*EllipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*
Sqrt[2])/(2 + I*Sqrt[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*S
qrt[3 - 4*x^2 + 2*x^4])
```

Maple [C] time = 0.12, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(2/3+i/3\sqrt{2}\right)x^2}\sqrt{1-\left(2/3-i/3\sqrt{2}\right)x^2}\mathit{EllipticF}\left(1/3\,x\sqrt{6+3\,i\sqrt{2}},1/3\,\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{6+3\,i\sqrt{2}}\sqrt{2\,x^4-4\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-4*x^2+3)^(1/2),x)
```

```
[Out] 3/(6+3*I*2^(1/2))^(1/2)*(1-(2/3+1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3
-1/3*I*2^(1/2))*x^2)^(1/2)/(2*x^4-4*x^2+3)^(1/2)*EllipticF(1/3*x*
(6+3*I*2^(1/2))^(1/2),1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 4*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-4\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 4*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4-4x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-4*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 4*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-4\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 4*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 4*x^2 + 3), x)
```

3.92 $\int \frac{1}{\sqrt{3-5x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/
4)*Sqrt[3 - 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.054992, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-5x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+5\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-5x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 5*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + 5*Sqrt[6])/24])/(2*6^(1/
4)*Sqrt[3 - 5*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 4.00519, size = 90, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-5x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{1}{2}+\frac{5\sqrt{6}}{24}\right)}{12\sqrt{2x^4-5x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - 5*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), 1/2 +
```

```
5*sqrt(6)/24)/(12*sqrt(2*x**4 - 5*x**2 + 3))
```

Mathematica [A] time = 0.0395205, size = 53, normalized size = 0.58

$$\frac{\sqrt{3-2x^2}\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)}{\sqrt{4x^4-10x^2+6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2
])/Sqrt[6 - 10*x^2 + 4*x^4]
```

Maple [A] time = 0.043, size = 42, normalized size = 0.5

$$\frac{1}{3}\sqrt{-x^2+1}\sqrt{-6\,x^2+9}EllipticF\left(x,\frac{\sqrt{6}}{3}\right)\frac{1}{\sqrt{2\,x^4-5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-5*x^2+3)^(1/2),x)
```

```
[Out] 1/3*(-x^2+1)^(1/2)*(-6*x^2+9)^(1/2)/(2*x^4-5*x^2+3)^(1/2)*Ellipti
cF(x,1/3*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 5*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4 - 5\,x^2 + 3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 5*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 - 5x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-5*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 5*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 - 5\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 5*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 5*x^2 + 3), x)
```

3.93 $\int \frac{1}{\sqrt{3-6x^2+2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*S
qrt[3 - 6*x^2 + 2*x^4])
```

Rubi [A] time = 0.0532912, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-6x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{4}\left(2+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 6*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (2 + Sqrt[6])/4])/(2*6^(1/4)*S
qrt[3 - 6*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.99093, size = 88, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-6x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{1}{2}+\frac{\sqrt{6}}{4}\right)}{12\sqrt{2x^4-6x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - 6*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), 1/2 +
```

```
sqrt(6)/4)/(12*sqrt(2*x**4 - 6*x**2 + 3))
```

Mathematica [A] time = 0.138904, size = 81, normalized size = 0.9

$$\frac{\sqrt{-2x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+3}F\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}x\right)\middle|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{2x^4-6x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 6*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF
[ArcSin[Sqrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[3 - 6
*x^2 + 2*x^4])
```

Maple [A] time = 0.173, size = 82, normalized size = 0.9

$$3\,\frac{\sqrt{1-\left(1+1/3\,\sqrt{3}\right)x^2}\sqrt{1-\left(1-1/3\,\sqrt{3}\right)x^2}\mathit{EllipticF}\left(1/3\,x\sqrt{9+3\,\sqrt{3}},1/2\,\sqrt{6}-1/2\,\sqrt{2}\right)}{\sqrt{9+3\,\sqrt{3}}\sqrt{2\,x^4-6\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-6*x^2+3)^(1/2),x)
```

```
[Out] 3/(9+3*3^(1/2))^(1/2)*(1-(1+1/3*3^(1/2))*x^2)^(1/2)*(1-(1-1/3*3^(
1/2))*x^2)^(1/2)/(2*x^4-6*x^2+3)^(1/2)*EllipticF(1/3*x*(9+3*3^(1/
2))^(1/2),1/2*6^(1/2)-1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 6*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-6\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 6*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^4-6x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-6*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 6*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-6\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 6*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 6*x^2 + 3), x)
```

3.94 $\int \frac{1}{\sqrt{3-7x^2+2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/
4)*Sqrt[3 - 7*x^2 + 2*x^4])
```

Rubi [A] time = 0.0568219, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-7x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+7\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 7*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + 7*Sqrt[6])/24])/(2*6^(1/
4)*Sqrt[3 - 7*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.98947, size = 90, normalized size = 0.98

$$\frac{6^{\frac{3}{4}}\sqrt{\frac{2x^4-7x^2+3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{1}{2}+\frac{7\sqrt{6}}{24}\right)}{12\sqrt{2x^4-7x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt((2*x**4 - 7*x**2 + 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), 1/2 +
```

```
7*sqrt(6)/24)/(12*sqrt(2*x**4 - 7*x**2 + 3))
```

Mathematica [A] time = 0.0414007, size = 58, normalized size = 0.63

$$\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}F\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{6}\right)}{\sqrt{2}\sqrt{2x^4-7x^2+3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 7*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6
])/(Sqrt[2]*Sqrt[3 - 7*x^2 + 2*x^4])
```

Maple [A] time = 0.047, size = 49, normalized size = 0.5

$$\frac{\sqrt{2}}{6}\sqrt{-2\,x^2+1}\sqrt{-3\,x^2+9}EllipticF\left(\sqrt{2}x,\frac{\sqrt{6}}{6}\right)\frac{1}{\sqrt{2\,x^4-7\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4-7*x^2+3)^(1/2),x)
```

```
[Out] 1/6*2^(1/2)*(-2*x^2+1)^(1/2)*(-3*x^2+9)^(1/2)/(2*x^4-7*x^2+3)^(1/
2)*EllipticF(2^(1/2)*x,1/6*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 7*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2\,x^4-7\,x^2+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 7*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4-7x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4-7*x**2+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 - 7*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4-7\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 - 7*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 - 7*x^2 + 3), x)
```

3.95 $\int \frac{1}{\sqrt{-3+7x^2-2x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

```
[Out] -(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Rubi [A] time = 0.0443061, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{F\left(\cos^{-1}\left(\frac{x}{\sqrt{3}}\right)|\frac{6}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]

[Out] -(EllipticF[ArcCos[x/Sqrt[3]], 6/5]/Sqrt[5])
```

Rubi in Sympy [A] time = 8.108, size = 20, normalized size = 1.05

$$-\frac{\sqrt{5}F\left(\operatorname{acos}\left(\frac{\sqrt{3}x}{3}\right)\middle|\frac{6}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+7*x**2-3)**(1/2),x)

[Out] -sqrt(5)*elliptic_f(acos(sqrt(3)*x/3), 6/5)/5
```

Mathematica [B] time = 0.0389487, size = 58, normalized size = 3.05

$$\frac{\sqrt{1-2x^2}\sqrt{1-\frac{x^2}{3}}F\left(\sin^{-1}\left(\sqrt{2}x\right)|\frac{1}{6}\right)}{\sqrt{2}\sqrt{-2x^4+7x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 7*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[1 - 2*x^2]*Sqrt[1 - x^2/3]*EllipticF[ArcSin[Sqrt[2]*x], 1/6
])/(Sqrt[2]*Sqrt[-3 + 7*x^2 - 2*x^4])
```

Maple [A] time = 0.041, size = 48, normalized size = 2.5

$$\frac{\sqrt{3}}{3}\sqrt{-3\,x^2+9}\sqrt{-2\,x^2+1}\mathit{EllipticF}\left(\frac{x\sqrt{3}}{3},\sqrt{6}\right)\frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+7*x^2-3)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*(-3*x^2+9)^(1/2)*(-2*x^2+1)^(1/2)/(-2*x^4+7*x^2-3)^(1
/2)*EllipticF(1/3*x*3^(1/2),6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+7\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 7*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+7\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 7*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 + 7x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+7*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 7*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 7\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 7*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 7*x^2 - 3), x)
```

3.96 $\int \frac{1}{\sqrt{-3+6x^2-2x^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sq
rt[2]*3^(1/4)))
```

Rubi [A] time = 0.156819, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{1}{3}\left(3-\sqrt{3}\right)}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[(3 - Sqrt[3])/3]*x], (1 + Sqrt[3])/2]/(Sq
rt[2]*3^(1/4)))
```

Rubi in Sympy [A] time = 21.6694, size = 63, normalized size = 1.43

$$-\frac{\sqrt{2}\sqrt[4]{3}F\left(\operatorname{acos}\left(\frac{\sqrt{3}x\sqrt{-\sqrt{3}+3}}{3}\right)\middle|\frac{1}{2}+\frac{\sqrt{3}}{2}\right)}{\sqrt{-\sqrt{3}+3}\sqrt{2\sqrt{3}+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+6*x**2-3)**(1/2),x)
```

```
[Out] -sqrt(2)*3**(1/4)*elliptic_f(acos(sqrt(3)*x*sqrt(-sqrt(3) + 3)/3)
, 1/2 + sqrt(3)/2)/(sqrt(-sqrt(3) + 3)*sqrt(2*sqrt(3) + 6))
```

Mathematica [A] time = 0.0631317, size = 81, normalized size = 1.84

$$\frac{\sqrt{-2x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+3}F\left(\sin^{-1}\left(\sqrt{1+\frac{1}{\sqrt{3}}}x\right)|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-2x^4+6x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 6*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 2*x^2]*Sqrt[3 + (-3 + Sqrt[3])*x^2]*EllipticF
[ArcSin[Sqrt[1 + 1/Sqrt[3]]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-3 +
6*x^2 - 2*x^4])
```

Maple [A] time = 0.096, size = 82, normalized size = 1.9

$$3\,{\frac{\sqrt{1-\left(1-1/3\,\sqrt{3}\right)x^2}\sqrt{1-\left(1+1/3\,\sqrt{3}\right)x^2}{\it EllipticF}\left(1/3\,x\sqrt{9-3\,\sqrt{3}},1/2\,\sqrt{6}+1/2\,\sqrt{2}\right)}{\sqrt{9-3\,\sqrt{3}}\sqrt{-2\,x^4+6\,x^2-3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+6*x^2-3)^(1/2),x)
```

```
[Out] 3/(9-3*3^(1/2))^(1/2)*(1-(1-1/3*3^(1/2))*x^2)^(1/2)*(1-(1+1/3*3^(
1/2))*x^2)^(1/2)/(-2*x^4+6*x^2-3)^(1/2)*EllipticF(1/3*x*(9-3*3^(1
/2))^(1/2),1/2*6^(1/2)+1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 6*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+6\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 6*x^2 - 3),x, algorithm="fricas")

[Out] integral(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+6x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+6*x**2-3)**(1/2),x)

[Out] Integral(1/sqrt(-2*x**4 + 6*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+6\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 6*x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(-2*x^4 + 6*x^2 - 3), x)
```

3.97 $\int \frac{1}{\sqrt{-3+5x^2-2x^4}}\,dx$

Optimal. Leaf size=14

$$-F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}x\right)\middle|3\right)$$

```
[Out] -EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Rubi [A] time = 0.0401121, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-F\left(\cos^{-1}\left(\sqrt{\frac{2}{3}}x\right)\middle|3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]

[Out] -EllipticF[ArcCos[Sqrt[2/3]*x], 3]
```

Rubi in Sympy [A] time = 9.43334, size = 12, normalized size = 0.86

$$-F\left(\operatorname{acos}\left(\frac{\sqrt{6}x}{3}\right)\middle|3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+5*x**2-3)**(1/2),x)

[Out] -elliptic_f(acos(sqrt(6)*x/3), 3)
```

Mathematica [B] time = 0.0395096, size = 53, normalized size = 3.79

$$\frac{\sqrt{3-2x^2}\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)}{\sqrt{-4x^4+10x^2-6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 - 2*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[2/3]*x], 3/2
])/Sqrt[-6 + 10*x^2 - 4*x^4]
```

Maple [A] time = 0.048, size = 50, normalized size = 3.6

$$\frac{\sqrt{6}}{6}\sqrt{-6\,x^2+9}\sqrt{-x^2+1}EllipticF\left(\frac{x\sqrt{6}}{3},\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+5*x^2-3)^(1/2),x)
```

```
[Out] 1/6*6^(1/2)*(-6*x^2+9)^(1/2)*(-x^2+1)^(1/2)/(-2*x^4+5*x^2-3)^(1/2
)*EllipticF(1/3*x*6^(1/2),1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-2\,x^4+5\,x^2-3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 + 5x^2 - 3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**4+5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 + 5\,x^2 - 3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 - 3), x)
```

3.98 $\int \frac{1}{\sqrt{-3+4x^2-2x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-3 + 4*x^2 - 2*x^4])
```

Rubi [A] time = 0.0447378, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-3 + 4*x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 3.88004, size = 90, normalized size = 1.02

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4+4x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\text{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{-2x^4+4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 + 4*x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(s
qrt(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt
```

```
(6)/6 + 1/2)/(12*sqrt(-2*x**4 + 4*x**2 - 3))
```

Mathematica [C] time = 0.131955, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{2x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-2x^4+4x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(2 + I*S
qrt[2])]*EllipticF[I*ArcSinh[Sqrt[-2/(2 - I*Sqrt[2])]*x], (2 - I*
Sqrt[2])/(2 + I*Sqrt[2])])/(Sqrt[2]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*S
qrt[-3 + 4*x^2 - 2*x^4])
```

Maple [C] time = 0.062, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(2/3-i/3\sqrt{2}\right)x^2}\sqrt{1-\left(2/3+i/3\sqrt{2}\right)x^2}EllipticF\left(1/3\sqrt{6-3\,i\sqrt{2}}x,1/3\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{6-3\,i\sqrt{2}}\sqrt{-2\,x^4+4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+4*x^2-3)^(1/2),x)
```

```
[Out] 3/(6-3*I*2^(1/2))^(1/2)*(1-(2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(2/3
+1/3*I*2^(1/2))*x^2)^(1/2)/(-2*x^4+4*x^2-3)^(1/2)*EllipticF(1/3*(
6-3*I*2^(1/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 4*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+4\,x^2-3}}{2\,x^4-4\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 4*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 4*x^2 - 3)/(2*x^4 - 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 4*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 4*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 4*x^2 - 3), x)
```

3.99 $\int \frac{1}{\sqrt{-3+3x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-3 + 3*x^2 - 2*x^4])
```

Rubi [A] time = 0.0501209, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-3 + 3*x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 4.29287, size = 90, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4+3x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{-2x^4+3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 + 3*x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(s
qrt(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt
```

```
(6)/8 + 1/2)/(12*sqrt(-2*x**4 + 3*x**2 - 3))
```

Mathematica [C] time = 0.166947, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-2x^4+3x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 3*x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(3 +
I*Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(3 - I*Sqrt[15])^(-1)]*x
], (3 - I*Sqrt[15])/(3 + I*Sqrt[15])])/(Sqrt[-(3 - I*Sqrt[15])^(-
1)]*Sqrt[-3 + 3*x^2 - 2*x^4])
```

Maple [C] time = 0.059, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/2-i/6\sqrt{15}\right)x^2}\sqrt{1-\left(1/2+i/6\sqrt{15}\right)x^2}\,EllipticF\left(1/6\sqrt{18-6\,i\sqrt{15}}x,1/2\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{18-6\,i\sqrt{15}}\sqrt{-2\,x^4+3\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+3*x^2-3)^(1/2),x)
```

```
[Out] 6/(18-6*I*15^(1/2))^(1/2)*(1-(1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1-(
1/2+1/6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4+3*x^2-3)^(1/2)*EllipticF(1
/6*(18-6*I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+3\,x^2-3}}{2\,x^4-3\,x^2+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 3*x^2 - 3)/(2*x^4 - 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 3*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 3*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 3*x^2 - 3), x)
```

3.100 $\int \frac{1}{\sqrt{-3+2x^2-2x^4}} \, dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-3 + 2*x^2 - 2*x^4])
```

Rubi [A] time = 0.0530369, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-3 + 2*x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 3.80601, size = 90, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4+2x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{-2x^4+2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 + 2*x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(s
qrt(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt
```

```
(6)/12 + 1/2)/(12*sqrt(-2*x**4 + 2*x**2 - 3))
```

Mathematica [C] time = 0.13158, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{1-i\sqrt{5}}}x\right)|\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-2x^4+2x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(1 + I*S
qrt[5])]*EllipticF[I*ArcSinh[Sqrt[-2/(1 - I*Sqrt[5])]*x], (1 - I*
Sqrt[5])/(1 + I*Sqrt[5])])/(Sqrt[2]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*S
qrt[-3 + 2*x^2 - 2*x^4])
```

Maple [C] time = 0.058, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(1/3-i/3\sqrt{5}\right)x^2}\sqrt{1-\left(1/3+i/3\sqrt{5}\right)x^2}\mathit{EllipticF}\left(1/3\sqrt{3-3\,i\sqrt{5}}x,1/3\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{3-3\,i\sqrt{5}}\sqrt{-2\,x^4+2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+2*x^2-3)^(1/2),x)
```

```
[Out] 3/(3-3*I*5^(1/2))^(1/2)*(1-(1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(1/3
+1/3*I*5^(1/2))*x^2)^(1/2)/(-2*x^4+2*x^2-3)^(1/2)*EllipticF(1/3*(
3-3*I*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 2*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4+2\,x^2-3}}{2\,x^4-2\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 2*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + 2*x^2 - 3)/(2*x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4+2x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+2*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + 2*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4+2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 2*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 2*x^2 - 3), x)
```

3.101 $\int \frac{1}{\sqrt{-3+x^2-2x^4}} \, dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-3 + x^2 - 2*x^4])
```

Rubi [A] time = 0.051155, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4-x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 - x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-3 + x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 3.18364, size = 87, normalized size = 0.99

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4+x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{-2x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 + x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), sqrt(6
```

```
)/24 + 1/2)/(12*sqrt(-2*x**4 + x**2 - 3))
```

Mathematica [C] time = 0.114842, size = 140, normalized size = 1.59

$$-\frac{i\sqrt{1-\frac{4x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-2x^4+x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 + x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(1 +
I*Sqrt[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(1 - I*Sqrt[23])^(-1)]*x
], (1 - I*Sqrt[23])/(1 + I*Sqrt[23])])/(Sqrt[-(1 - I*Sqrt[23])^(-
1)]*Sqrt[-3 + x^2 - 2*x^4])
```

Maple [C] time = 0.071, size = 85, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(1/6-i/6\sqrt{23}\right)x^2}\sqrt{1-\left(1/6+i/6\sqrt{23}\right)x^2}EllipticF\left(1/6\sqrt{6-6\,i\sqrt{23}}x,1/6\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{6-6\,i\sqrt{23}}\sqrt{-2\,x^4+x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+x^2-3)^(1/2),x)
```

```
[Out] 6/(6-6*I*23^(1/2))^(1/2)*(1-(1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-(1
/6+1/6*I*23^(1/2))*x^2)^(1/2)/(-2*x^4+x^2-3)^(1/2)*EllipticF(1/6*
(6-6*I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4+x^2-3}}{2\,x^4-x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 + x^2 - 3)/(2*x^4 - x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 + x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 + x^2 - 3), x)
```

3.102 $\int \frac{1}{\sqrt{-3-2x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Rubi [A] time = 0.0363142, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(2/3)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-3 - 2*x^4])
```

Rubi in Sympy [A] time = 1.43513, size = 76, normalized size = 1.06

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|\frac{1}{2}\right)}{12\sqrt{-2x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqrt(6)*x*
*2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), 1/2)/(12*sqrt
(-2*x**4 - 3))
```

Mathematica [C] time = 0.0464711, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{2x^4+3}F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{2}{3}}x\right)\middle|-1\right)}{\sqrt{-2x^4-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - 2*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[3 + 2*x^4]*EllipticF[I*ArcSinh[(-2/3)^(1/4)*
x], -1])/Sqrt[-3 - 2*x^4])
```

Maple [C] time = 0.031, size = 66, normalized size = 0.9

$$\frac{\sqrt{3}}{9\sqrt{-i\sqrt{6}}}\sqrt{9+3\,i\sqrt{6}x^2}\sqrt{9-3\,i\sqrt{6}x^2}EllipticF\left(\frac{\sqrt{3}\sqrt{-i\sqrt{6}}x}{3},i\right)\frac{1}{\sqrt{-2\,x^4-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-3)^(1/2),x)
```

```
[Out] 1/9*3^(1/2)/(-I*6^(1/2))^(1/2)*(9+3*I*6^(1/2)*x^2)^(1/2)*(9-3*I*6
^(1/2)*x^2)^(1/2)/(-2*x^4-3)^(1/2)*EllipticF(1/3*3^(1/2)*(-I*6^(1
/2))^(1/2)*x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3}}{2\,x^4+3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3)/(2*x^4 + 3), x)
```

Sympy [A] time = 1.80139, size = 39, normalized size = 0.54

$$-\frac{\sqrt{3}ix\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^4e^{i\pi}}{3}\right)}{12\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-3)**(1/2),x)
```

```
[Out] -sqrt(3)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**4*exp_pola
r(I*pi)/3)/(12*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3), x)
```

3.103 $\int \frac{1}{\sqrt{-3-x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-3 - x^2 - 2*x^4])
```

Rubi [A] time = 0.0589022, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(2/3)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-3 - x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 2.88222, size = 90, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4-x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{-2x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 - x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(sqr
t(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqrt(
```

```
6)/24 + 1/2)/(12*sqrt(-2*x**4 - x**2 - 3))
```

Mathematica [C] time = 0.123415, size = 142, normalized size = 1.58

$$-\frac{i\sqrt{1-\frac{4x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{4x^2}{-1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-1-i\sqrt{23}}}x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{2\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-2x^4-x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (4*x^2)/(-1
+ I*Sqrt[23])]*EllipticF[I*ArcSinh[2*Sqrt[-(-1 - I*Sqrt[23])^(-1)
]*x], (-1 - I*Sqrt[23])/(-1 + I*Sqrt[23])])/(Sqrt[-(-1 - I*Sqrt[2
3])^(-1)]*Sqrt[-3 - x^2 - 2*x^4])
```

Maple [C] time = 0.067, size = 87, normalized size = 1.

$$6\,\frac{\sqrt{1-\left(-1/6-i/6\sqrt{23}\right)x^2}\sqrt{1-\left(-1/6+i/6\sqrt{23}\right)x^2}EllipticF\left(1/6\sqrt{-6-6\,i\sqrt{23}}x,1/6\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{-6-6\,i\sqrt{23}}\sqrt{-2\,x^4-x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-x^2-3)^(1/2),x)
```

```
[Out] 6/(-6-6*I*23^(1/2))^(1/2)*(1-(-1/6-1/6*I*23^(1/2))*x^2)^(1/2)*(1-
(-1/6+1/6*I*23^(1/2))*x^2)^(1/2)/(-2*x^4-x^2-3)^(1/2)*EllipticF(1
/6*(-6-6*I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^{4}-x^{2}-3}}{2\,x^{4}+x^{2}+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - x^2 - 3)/(2*x^4 + x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^{4}-x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^{4}-x^{2}-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - x^2 - 3), x)
```

3.104 $\int \frac{1}{\sqrt{-3-2x^2-2x^4}} dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-2x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-3 - 2*x^2 - 2*x^4])
```

Rubi [A] time = 0.0529185, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+2x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 2*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-3 - 2*x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 3.60811, size = 94, normalized size = 1.02

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4-2x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{-2x^4-2x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-2*x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 - 2*x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(s
qrt(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqr
```

```
t(6)/12 + 1/2)/(12*sqrt(-2*x**4 - 2*x**2 - 3))
```

Mathematica [C] time = 0.141384, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{2x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{2x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-2x^4-2x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - 2*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (2*x^2)/(-1 + I
*Sqrt[5])]*EllipticF[I*ArcSinh[Sqrt[-2/(-1 - I*Sqrt[5])]*x], (-1
- I*Sqrt[5])/(-1 + I*Sqrt[5])])/(Sqrt[2]*Sqrt[-(-1 - I*Sqrt[5])^(
-1)]*Sqrt[-3 - 2*x^2 - 2*x^4])
```

Maple [C] time = 0.066, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-1/3-i/3\sqrt{5}\right)x^2}\sqrt{1-\left(-1/3+i/3\sqrt{5}\right)x^2}EllipticF\left(1/3\sqrt{-3-3\,i\sqrt{5}}x,1/3\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{-3-3\,i\sqrt{5}}\sqrt{-2\,x^4-2\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-2*x^2-3)^(1/2),x)
```

```
[Out] 3/(-3-3*I*5^(1/2))^(1/2)*(1-(-1/3-1/3*I*5^(1/2))*x^2)^(1/2)*(1-(-
1/3+1/3*I*5^(1/2))*x^2)^(1/2)/(-2*x^4-2*x^2-3)^(1/2)*EllipticF(1/
3*(-3-3*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-2\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 2*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4 - 2\,x^2 - 3}}{2\,x^4 + 2\,x^2 + 3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 2*x^2 - 3),x, algorithm="fricas")

[Out] integral(-sqrt(-2*x^4 - 2*x^2 - 3)/(2*x^4 + 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 - 2x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-2*x**2-3)**(1/2),x)

[Out] Integral(1/sqrt(-2*x**4 - 2*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 - 2\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 2*x^2 - 3),x, algorithm="giac")

[Out] integrate(1/sqrt(-2*x^4 - 2*x^2 - 3), x)
```

3.105 $\int \frac{1}{\sqrt{-3-3x^2-2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-3 - 3*x^2 - 2*x^4])
```

Rubi [A] time = 0.0543651, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+3x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 3*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-3 - 3*x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 3.52942, size = 94, normalized size = 1.02

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4-3x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{-2x^4-3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 - 3*x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(s
qrt(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqr
```

```
t(6)/8 + 1/2)/(12*sqrt(-2*x**4 - 3*x**2 - 3))
```

Mathematica [C] time = 0.170028, size = 142, normalized size = 1.54

$$-\frac{i\sqrt{1-\frac{4x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{4x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{1}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{2\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-2x^4-3x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - 3*x^2 - 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (4*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (4*x^2)/(-3
+ I*Sqrt[15])]*EllipticF[I*ArcSinh[2*Sqrt[-(-3 - I*Sqrt[15])^(-1)
]*x], (-3 - I*Sqrt[15])/(-3 + I*Sqrt[15])])/(Sqrt[-(-3 - I*Sqrt[1
5])^(-1)]*Sqrt[-3 - 3*x^2 - 2*x^4])
```

Maple [C] time = 0.068, size = 87, normalized size = 1.

$$6\frac{\sqrt{1-\left(-1/2-i/6\sqrt{15}\right)x^2}\sqrt{1-\left(-1/2+i/6\sqrt{15}\right)x^2}EllipticF\left(1/6\sqrt{-18-6i\sqrt{15}}x,1/2\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{-18-6i\sqrt{15}}\sqrt{-2x^4-3x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-3*x^2-3)^(1/2),x)
```

```
[Out] 6/(-18-6*I*15^(1/2))^(1/2)*(1-(-1/2-1/6*I*15^(1/2))*x^2)^(1/2)*(1
-(-1/2+1/6*I*15^(1/2))*x^2)^(1/2)/(-2*x^4-3*x^2-3)^(1/2)*Elliptic
F(1/6*(-18-6*I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-3\,x^2-3}}{2\,x^4+3\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 3*x^2 - 3)/(2*x^4 + 3*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-3x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-3*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 3*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-3\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 3*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 3*x^2 - 3), x)
```

3.106 $\int \frac{1}{\sqrt{-3-4x^2-2x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-4x^2-3}}$$

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-3 - 4*x^2 - 2*x^4])
```

Rubi [A] time = 0.0490624, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+3\right)\sqrt{\frac{2x^4+4x^2+3}{\left(\sqrt{6}x^2+3\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{2}{3}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((3 + Sqrt[6]*x^2)*Sqrt[(3 + 4*x^2 + 2*x^4)/(3 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(2/3)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-3 - 4*x^2 - 2*x^4])
```

Rubi in Sympy [A] time = 3.61516, size = 94, normalized size = 1.04

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-2x^4-4x^2-3}{\left(\frac{\sqrt{6}x^2}{3}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{3}+1\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{2}\cdot 3^{\frac{3}{4}}x}{3}\right)\middle|-\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{-2x^4-4x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-2*x**4 - 4*x**2 - 3)/(sqrt(6)*x**2/3 + 1)**2)*(s
qrt(6)*x**2/3 + 1)*elliptic_f(2*atan(2**(1/4)*3**(3/4)*x/3), -sqr
```

```
t(6)/6 + 1/2)/(12*sqrt(-2*x**4 - 4*x**2 - 3))
```

Mathematica [C] time = 0.157449, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{2x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{2x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{2}{-2-i\sqrt{2}}}x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{2}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-2x^4-4x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - 4*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (2*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (2*x^2)/(-2 + I
*Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[-2/(-2 - I*Sqrt[2])]*x], (-2
- I*Sqrt[2])/(-2 + I*Sqrt[2])])/(Sqrt[2]*Sqrt[-(-2 - I*Sqrt[2])^(
-1)]*Sqrt[-3 - 4*x^2 - 2*x^4])
```

Maple [C] time = 0.059, size = 87, normalized size = 1.

$$3\,\frac{\sqrt{1-\left(-2/3-i/3\sqrt{2}\right)x^2}\sqrt{1-\left(-2/3+i/3\sqrt{2}\right)x^2}\,EllipticF\left(1/3\sqrt{-6-3\,i\sqrt{2}}x,1/3\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{-6-3\,i\sqrt{2}}\sqrt{-2\,x^4-4\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-4*x^2-3)^(1/2),x)
```

```
[Out] 3/(-6-3*I*2^(1/2))^(1/2)*(1-(-2/3-1/3*I*2^(1/2))*x^2)^(1/2)*(1-(-
2/3+1/3*I*2^(1/2))*x^2)^(1/2)/(-2*x^4-4*x^2-3)^(1/2)*EllipticF(1/
3*(-6-3*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 4*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-2\,x^4-4\,x^2-3}}{2\,x^4+4\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 4*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 4*x^2 - 3)/(2*x^4 + 4*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4-4x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-4*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 4*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4-4\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 4*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 4*x^2 - 3), x)
```

3.107 $\int \frac{1}{\sqrt{-3-5x^2-2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{2x^2+3}F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}$$

```
[Out] (Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^
2]*Sqrt[(3 + 2*x^2)/(1 + x^2)])
```

Rubi [A] time = 0.0547891, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\sqrt{2x^2+3}F\left(\tan^{-1}(x)\middle|\frac{1}{3}\right)}{\sqrt{3}\sqrt{-x^2-1}\sqrt{\frac{2x^2+3}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] (Sqrt[3 + 2*x^2]*EllipticF[ArcTan[x], 1/3])/(Sqrt[3]*Sqrt[-1 - x^
2]*Sqrt[(3 + 2*x^2)/(1 + x^2)])
```

Rubi in Sympy [A] time = 8.39057, size = 54, normalized size = 1.02

$$\frac{4\sqrt{3}\sqrt{4x^2+6}F\left(\operatorname{atan}(x)\middle|\frac{1}{3}\right)}{3\sqrt{-\frac{-16x^2-24}{x^2+1}}\sqrt{-4x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] 4*sqrt(3)*sqrt(4*x**2 + 6)*elliptic_f(atan(x), 1/3)/(3*sqrt(-(-16
*x**2 - 24)/(x**2 + 1))*sqrt(-4*x**2 - 4))
```

Mathematica [C] time = 0.0417511, size = 63, normalized size = 1.19

$$-\frac{i\sqrt{x^2+1}\sqrt{2x^2+3}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{3}}x\right)|\frac{3}{2}\right)}{\sqrt{2}\sqrt{-2x^4-5x^2-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-3 - 5*x^2 - 2*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[3 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/3]
*x], 3/2])/(Sqrt[2]*Sqrt[-3 - 5*x^2 - 2*x^4])
```

Maple [C] time = 0.049, size = 44, normalized size = 0.8

$$-\frac{i}{3}\sqrt{x^2+1}\sqrt{6\,x^2+9}EllipticF\left(ix,\frac{\sqrt{6}}{3}\right)\frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4-5*x^2-3)^(1/2),x)
```

```
[Out] -1/3*I*(x^2+1)^(1/2)*(6*x^2+9)^(1/2)/(-2*x^4-5*x^2-3)^(1/2)*Ellip
ticF(I*x,1/3*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2\,x^4-5\,x^2-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 5*x^2 - 3),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-2\,x^4-5\,x^2-3}}{2\,x^4+5\,x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 5*x^2 - 3),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-2*x^4 - 5*x^2 - 3)/(2*x^4 + 5*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4 - 5x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4-5*x**2-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**4 - 5*x**2 - 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4 - 5\,x^2 - 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 - 5*x^2 - 3),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-2*x^4 - 5*x^2 - 3), x)
```

3.108 $\int \frac{1}{\sqrt{-2+6x^2-3x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

```
[Out] -(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sq
rt[2]*3^(1/4)))
```

Rubi [A] time = 0.13712, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{3}{3+\sqrt{3}}}x\right)|\frac{1}{2}\left(1+\sqrt{3}\right)\right)}{\sqrt{2}\sqrt[4]{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[3/(3 + Sqrt[3])]*x], (1 + Sqrt[3])/2]/(Sq
rt[2]*3^(1/4)))
```

Rubi in Sympy [A] time = 27.4348, size = 63, normalized size = 1.5

$$-\frac{\sqrt{2}\sqrt[4]{3}F\left(\operatorname{acos}\left(\frac{\sqrt{2}x\sqrt{-\sqrt{3}+3}}{2}\right)\middle|\frac{1}{2}+\frac{\sqrt{3}}{2}\right)}{\sqrt{-\sqrt{3}+3}\sqrt{2\sqrt{3}+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+6*x**2-2)**(1/2),x)
```

```
[Out] -sqrt(2)*3**(1/4)*elliptic_f(acos(sqrt(2)*x*sqrt(-sqrt(3) + 3)/2)
, 1/2 + sqrt(3)/2)/(sqrt(-sqrt(3) + 3)*sqrt(2*sqrt(3) + 6))
```

Mathematica [B] time = 0.145924, size = 85, normalized size = 2.02

$$\frac{\sqrt{-3x^2-\sqrt{3}+3}\sqrt{\left(\sqrt{3}-3\right)x^2+2}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{3}\right)}x\right)|2-\sqrt{3}\right)}{\sqrt{6}\sqrt{-3x^4+6x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 6*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[3 - Sqrt[3] - 3*x^2]*Sqrt[2 + (-3 + Sqrt[3])*x^2]*EllipticF
[ArcSin[Sqrt[(3 + Sqrt[3])/2]*x], 2 - Sqrt[3]])/(Sqrt[6]*Sqrt[-2
+ 6*x^2 - 3*x^4])
```

Maple [A] time = 0.099, size = 82, normalized size = 2.

$$2\,\frac{\sqrt{1-\left(-1/2\,\sqrt{3}+3/2\right)x^2}\sqrt{1-\left(1/2\,\sqrt{3}+3/2\right)x^2}EllipticF\left(1/2\,\sqrt{6-2\,\sqrt{3}}x,1/2\,\sqrt{6}+1/2\,\sqrt{2}\right)}{\sqrt{6-2\,\sqrt{3}}\sqrt{-3\,x^4+6\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+6*x^2-2)^(1/2),x)
```

```
[Out] 2/(6-2*3^(1/2))^(1/2)*(1-(-1/2*3^(1/2)+3/2)*x^2)^(1/2)*(1-(1/2*3^
(1/2)+3/2)*x^2)^(1/2)/(-3*x^4+6*x^2-2)^(1/2)*EllipticF(1/2*(6-2*3
^(1/2))^(1/2)*x,1/2*6^(1/2)+1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 6*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+6\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 6*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+6x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+6*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 6*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+6\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 6*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 6*x^2 - 2), x)
```

3.109 $\int \frac{1}{\sqrt{-2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=6

$$-F\left(\cos^{-1}(x)\middle|3\right)$$

```
[Out] -EllipticF[ArcCos[x], 3]
```

Rubi [A] time = 0.0381273, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-F\left(\cos^{-1}(x)\middle|3\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]

[Out] -EllipticF[ArcCos[x], 3]
```

Rubi in Sympy [A] time = 9.43408, size = 5, normalized size = 0.83

$$-F(\operatorname{acos}(x)|3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+5*x**2-2)**(1/2),x)

[Out] -elliptic_f(acos(x), 3)
```

Mathematica [B] time = 0.0437238, size = 53, normalized size = 8.83

$$\frac{\sqrt{2-3x^2}\sqrt{1-x^2}F\left(\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{-9x^4+15x^2-6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 5*x^2 - 3*x^4],x]
```

```
[Out] (Sqrt[2 - 3*x^2]*Sqrt[1 - x^2]*EllipticF[ArcSin[Sqrt[3/2]*x], 2/3
])/Sqrt[-6 + 15*x^2 - 9*x^4]
```

Maple [A] time = 0.011, size = 42, normalized size = 7.

$$\frac{1}{2}\sqrt{-x^2+1}\sqrt{-6\,x^2+4}EllipticF\left(x,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+5*x^2-2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(-6*x^2+4)^(1/2)/(-3*x^4+5*x^2-2)^(1/2)*Ellipt
icF(x,1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+5\,x^2-2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 + 5x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 + 5\,x^2 - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 - 2), x)
```

3.110 $\int \frac{1}{\sqrt{-2+4x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-2 + 4*x^2 - 3*x^4])
```

Rubi [A] time = 0.0573656, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}+\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 + 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-2 + 4*x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 3.86046, size = 90, normalized size = 1.02

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4+4x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{-3x^4+4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 + 4*x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(s
qrt(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt
```

```
(6)/6 + 1/2)/(12*sqrt(-3*x**4 + 4*x**2 - 2))
```

Mathematica [C] time = 0.14756, size = 144, normalized size = 1.64

$$-\frac{i\sqrt{1-\frac{3x^2}{2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{2-i\sqrt{2}}}x\right)|\frac{2-i\sqrt{2}}{2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{2-i\sqrt{2}}}\sqrt{-3x^4+4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(2 + I*S
qrt[2])]*EllipticF[I*ArcSinh[Sqrt[-3/(2 - I*Sqrt[2])]*x], (2 - I*
Sqrt[2])/(2 + I*Sqrt[2])])/(Sqrt[3]*Sqrt[-(2 - I*Sqrt[2])^(-1)]*S
qrt[-2 + 4*x^2 - 3*x^4])
```

Maple [C] time = 0.073, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1-i/2\sqrt{2}\right)x^2}\sqrt{1-\left(1+i/2\sqrt{2}\right)x^2}EllipticF\left(1/2\sqrt{4-2\,i\sqrt{2}}x,1/3\sqrt{3+6\,i\sqrt{2}}\right)}{\sqrt{4-2\,i\sqrt{2}}\sqrt{-3\,x^4+4\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+4*x^2-2)^(1/2),x)
```

```
[Out] 2/(4-2*I*2^(1/2))^(1/2)*(1-(1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(1+1/2
*I*2^(1/2))*x^2)^(1/2)/(-3*x^4+4*x^2-2)^(1/2)*EllipticF(1/2*(4-2*
I*2^(1/2))^(1/2)*x,1/3*(3+6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 4*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+4\,x^2-2}}{3\,x^4-4\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 4*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 4*x^2 - 2)/(3*x^4 - 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 4*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 4*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 4*x^2 - 2), x)
```

3.111 $\int \frac{1}{\sqrt{-2+3x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-2 + 3*x^2 - 3*x^4])
```

Rubi [A] time = 0.0529409, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 + Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-2 + 3*x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 3.80665, size = 90, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4+3x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{-3x^4+3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 + 3*x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(s
qrt(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt
```

```
(6)/8 + 1/2)/(12*sqrt(-3*x**4 + 3*x**2 - 2))
```

Mathematica [C] time = 0.20036, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{3-i\sqrt{15}}}x\right)|\frac{3-i\sqrt{15}}{3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{3-i\sqrt{15}}}\sqrt{-3x^4+3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(3 + I*
Sqrt[15])]*EllipticF[I*ArcSinh[Sqrt[-6/(3 - I*Sqrt[15])]*x], (3 -
 I*Sqrt[15])/(3 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(3 - I*Sqrt[15])^(
-1)]*Sqrt[-2 + 3*x^2 - 3*x^4])
```

Maple [C] time = 0.062, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(3/4-i/4\sqrt{15}\right)x^2}\sqrt{1-\left(3/4+i/4\sqrt{15}\right)x^2}\,EllipticF\left(1/2\,\sqrt{3-i\sqrt{15}}x,1/2\,\sqrt{-1+i\sqrt{15}}\right)}{\sqrt{3-i\sqrt{15}}\sqrt{-3\,x^4+3\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+3*x^2-2)^(1/2),x)
```

```
[Out] 2/(3-I*15^(1/2))^(1/2)*(1-(3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(3/4
+1/4*I*15^(1/2))*x^2)^(1/2)/(-3*x^4+3*x^2-2)^(1/2)*EllipticF(1/2*
(3-I*15^(1/2))^(1/2)*x,1/2*(-1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 3*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+3\,x^2-2}}{3\,x^4-3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 3*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 3*x^2 - 2)/(3*x^4 - 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 3*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 3*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 3*x^2 - 2), x)
```

3.112 $\int \frac{1}{\sqrt{-2+2x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-2 + 2*x^2 - 3*x^4])
```

Rubi [A] time = 0.0541219, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 + Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-2 + 2*x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 4.28513, size = 90, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4+2x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{-3x^4+2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 + 2*x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(s
qrt(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt
```

```
(6)/12 + 1/2)/(12*sqrt(-3*x**4 + 2*x**2 - 2))
```

Mathematica [C] time = 0.142276, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{1-i\sqrt{5}}}x\right)\mid\frac{1-i\sqrt{5}}{1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{1-i\sqrt{5}}}\sqrt{-3x^4+2x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(1 + I*S
qrt[5])]*EllipticF[I*ArcSinh[Sqrt[-3/(1 - I*Sqrt[5])]*x], (1 - I*
Sqrt[5])/(1 + I*Sqrt[5])])/(Sqrt[3]*Sqrt[-(1 - I*Sqrt[5])^(-1)]*S
qrt[-2 + 2*x^2 - 3*x^4])
```

Maple [C] time = 0.06, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/2-i/2\sqrt{5}\right)x^2}\sqrt{1-\left(1/2+i/2\sqrt{5}\right)x^2}\mathit{EllipticF}\left(1/2\,\sqrt{2-2\,i\sqrt{5}}x,1/3\,\sqrt{-6+3\,i\sqrt{5}}\right)}{\sqrt{2-2\,i\sqrt{5}}\sqrt{-3\,x^4+2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+2*x^2-2)^(1/2),x)
```

```
[Out] 2/(2-2*I*5^(1/2))^(1/2)*(1-(1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(1/2
+1/2*I*5^(1/2))*x^2)^(1/2)/(-3*x^4+2*x^2-2)^(1/2)*EllipticF(1/2*(
2-2*I*5^(1/2))^(1/2)*x,1/3*(-6+3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4+2\,x^2-2}}{3\,x^4-2\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + 2*x^2 - 2)/(3*x^4 - 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4+2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 2*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 2*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 2*x^2 - 2), x)
```

3.113 $\int \frac{1}{\sqrt{-2+x^2-3x^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-2 + x^2 - 3*x^4])
```

Rubi [A] time = 0.0517739, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4-x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12+\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 - x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 + Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-2 + x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 3.17359, size = 87, normalized size = 0.99

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4+x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{-3x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 + x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), sqrt(6
```

```
)/24 + 1/2)/(12*sqrt(-3*x**4 + x**2 - 2))
```

Mathematica [C] time = 0.144128, size = 142, normalized size = 1.61

$$-\frac{i\sqrt{1-\frac{6x^2}{1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{1-i\sqrt{23}}}x\right)|\frac{1-i\sqrt{23}}{1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{1-i\sqrt{23}}}\sqrt{-3x^4+x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(1 + I*
Sqrt[23])]*EllipticF[I*ArcSinh[Sqrt[-6/(1 - I*Sqrt[23])]*x], (1 -
 I*Sqrt[23])/(1 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(1 - I*Sqrt[23])^(
-1)]*Sqrt[-2 + x^2 - 3*x^4])
```

Maple [C] time = 0.06, size = 85, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(1/4-i/4\sqrt{23}\right)x^2}\sqrt{1-\left(1/4+i/4\sqrt{23}\right)x^2}\mathit{EllipticF}\left(1/2\sqrt{1-i\sqrt{23}}x,1/6\sqrt{-33+3\,i\sqrt{23}}\right)}{\sqrt{1-i\sqrt{23}}\sqrt{-3\,x^4+x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+x^2-2)^(1/2),x)
```

```
[Out] 2/(1-I*23^(1/2))^(1/2)*(1-(1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(1/4
+1/4*I*23^(1/2))*x^2)^(1/2)/(-3*x^4+x^2-2)^(1/2)*EllipticF(1/2*(1
-I*23^(1/2))^(1/2)*x,1/6*(-33+3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4+x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4 + x^2 - 2}}{3\,x^4 - x^2 + 2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 + x^2 - 2)/(3*x^4 - x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 + x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 + x^2 - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + x^2 - 2), x)
```

3.114 $\int \frac{1}{\sqrt{-2-3x^4}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Rubi [A] time = 0.0340756, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 3*x^4],x]
```


```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*Elliptic
F[2*ArcTan[(3/2)^(1/4)*x], 1/2])/(2*6^(1/4)*Sqrt[-2 - 3*x^4])
```

Rubi in Sympy [A] time = 1.45783, size = 76, normalized size = 1.06

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\text{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|\frac{1}{2}\right)}{12\sqrt{-3x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqrt(6)*x*
*2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), 1/2)/(12*sqrt
(-3*x**4 - 2))
```

Mathematica [C] time = 0.0481606, size = 47, normalized size = 0.65

$$-\frac{\sqrt[4]{-\frac{1}{6}}\sqrt{3x^4+2}F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{3}{2}}x\right)\middle|-1\right)}{\sqrt{-3x^4-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 3*x^4],x]
```

```
[Out] -(((-1/6)^(1/4)*Sqrt[2 + 3*x^4]*EllipticF[I*ArcSinh[(-3/2)^(1/4)*
x], -1])/Sqrt[-2 - 3*x^4])
```

Maple [C] time = 0.032, size = 66, normalized size = 0.9

$$\frac{\sqrt{2}}{4\sqrt{-i\sqrt{6}}}\sqrt{4+2\,i\sqrt{6}x^2}\sqrt{4-2\,i\sqrt{6}x^2}\,EllipticF\left(\frac{\sqrt{2}\sqrt{-i\sqrt{6}}x}{2},i\right)\frac{1}{\sqrt{-3\,x^4-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-2)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/(-I*6^(1/2))^(1/2)*(4+2*I*6^(1/2)*x^2)^(1/2)*(4-2*I*6
^(1/2)*x^2)^(1/2)/(-3*x^4-2)^(1/2)*EllipticF(1/2*2^(1/2)*(-I*6^(1
/2))^(1/2)*x,I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-2}}{3\,x^4+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2)/(3*x^4 + 2), x)
```

Sympy [A] time = 1.88742, size = 39, normalized size = 0.54

$$-\frac{\sqrt{2}ix\,\Gamma\left(\frac{1}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{3x^4e^{i\pi}}{2}\right)}{8\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-2)**(1/2),x)
```

```
[Out] -sqrt(2)*I*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 3*x**4*exp_pola
r(I*pi)/2)/(8*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2), x)
```

3.115 $\int \frac{1}{\sqrt{-2-x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-2 - x^2 - 3*x^4])
```

Rubi [A] time = 0.0602272, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{24}\left(12-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - x^2 - 3*x^4],x]
```



```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*El
lipticF[2*ArcTan[(3/2)^(1/4)*x], (12 - Sqrt[6])/24])/(2*6^(1/4)*S
qrt[-2 - x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 2.90383, size = 90, normalized size = 1.

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4-x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{24}+\frac{1}{2}\right)}{12\sqrt{-3x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 - x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(sqr
t(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqrt(
```

```
6)/24 + 1/2)/(12*sqrt(-3*x**4 - x**2 - 2))
```

Mathematica [C] time = 0.143913, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{6x^2}{-1-i\sqrt{23}}}\sqrt{1-\frac{6x^2}{-1+i\sqrt{23}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-1-i\sqrt{23}}}x\right)|\frac{-1-i\sqrt{23}}{-1+i\sqrt{23}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-1-i\sqrt{23}}}\sqrt{-3x^4-x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-1 - I*Sqrt[23])]*Sqrt[1 - (6*x^2)/(-1 +
I*Sqrt[23])]*EllipticF[I*ArcSinh[Sqrt[-6/(-1 - I*Sqrt[23])]*x], (
-1 - I*Sqrt[23])/(-1 + I*Sqrt[23])])/(Sqrt[6]*Sqrt[-(-1 - I*Sqrt[
23])^(-1)]*Sqrt[-2 - x^2 - 3*x^4])
```

Maple [C] time = 0.061, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/4-i/4\sqrt{23}\right)x^2}\sqrt{1-\left(-1/4+i/4\sqrt{23}\right)x^2}\,EllipticF\left(1/2\,\sqrt{-1-i\sqrt{23}}x,1/6\,\sqrt{-33-3\,i\sqrt{23}}\right)}{\sqrt{-1-i\sqrt{23}}\sqrt{-3\,x^4-x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-x^2-2)^(1/2),x)
```

```
[Out] 2/(-1-I*23^(1/2))^(1/2)*(1-(-1/4-1/4*I*23^(1/2))*x^2)^(1/2)*(1-(-
1/4+1/4*I*23^(1/2))*x^2)^(1/2)/(-3*x^4-x^2-2)^(1/2)*EllipticF(1/2
*(-1-I*23^(1/2))^(1/2)*x,1/6*(-33-3*I*23^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-x^2-2}}{3\,x^4+x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - x^2 - 2)/(3*x^4 + x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - x^2 - 2), x)
```

3.116 $\int \frac{1}{\sqrt{-2-2x^2-3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-2 - 2*x^2 - 3*x^4])
```

Rubi [A] time = 0.051963, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+2x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{12}\left(6-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 2*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (6 - Sqrt[6])/12])/(2*6^(1/4)*
Sqrt[-2 - 2*x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 3.54447, size = 94, normalized size = 1.02

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4-2x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{12}+\frac{1}{2}\right)}{12\sqrt{-3x^4-2x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 - 2*x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(s
qrt(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqr
```

```
t(6)/12 + 1/2)/(12*sqrt(-3*x**4 - 2*x**2 - 2))
```

Mathematica [C] time = 0.141943, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{3x^2}{-1-i\sqrt{5}}}\sqrt{1-\frac{3x^2}{-1+i\sqrt{5}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-1-i\sqrt{5}}}x\right)|\frac{-1-i\sqrt{5}}{-1+i\sqrt{5}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-1-i\sqrt{5}}}\sqrt{-3x^4-2x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 2*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-1 - I*Sqrt[5])]*Sqrt[1 - (3*x^2)/(-1 + I
*Sqrt[5])]*EllipticF[I*ArcSinh[Sqrt[-3/(-1 - I*Sqrt[5])]*x], (-1
- I*Sqrt[5])/(-1 + I*Sqrt[5])])/(Sqrt[3]*Sqrt[-(-1 - I*Sqrt[5])^(
-1)]*Sqrt[-2 - 2*x^2 - 3*x^4])
```

Maple [C] time = 0.067, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1/2-i/2\sqrt{5}\right)x^2}\sqrt{1-\left(-1/2+i/2\sqrt{5}\right)x^2}\,EllipticF\left(1/2\sqrt{-2-2\,i\sqrt{5}}x,1/3\sqrt{-6-3\,i\sqrt{5}}\right)}{\sqrt{-2-2\,i\sqrt{5}}\sqrt{-3\,x^4-2\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-2*x^2-2)^(1/2),x)
```

```
[Out] 2/(-2-2*I*5^(1/2))^(1/2)*(1-(-1/2-1/2*I*5^(1/2))*x^2)^(1/2)*(1-(-
1/2+1/2*I*5^(1/2))*x^2)^(1/2)/(-3*x^4-2*x^2-2)^(1/2)*EllipticF(1/
2*(-2-2*I*5^(1/2))^(1/2)*x,1/3*(-6-3*I*5^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-2\,x^2-2}}{3\,x^4+2\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 2*x^2 - 2)/(3*x^4 + 2*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^4-2x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-2*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 2*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-2\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 2*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 2*x^2 - 2), x)
```

3.117 $\int \frac{1}{\sqrt{-2-3x^2-3x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-3x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-2 - 3*x^2 - 3*x^4])
```

Rubi [A] time = 0.0609894, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+3x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{6}\right)\right)}{2\sqrt[4]{6}\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 3*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], (4 - Sqrt[6])/8])/(2*6^(1/4)*S
qrt[-2 - 3*x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 3.59409, size = 94, normalized size = 1.02

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4-3x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\,\mathrm{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{8}+\frac{1}{2}\right)}{12\sqrt{-3x^4-3x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 - 3*x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(s
qrt(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqr
```

```
t(6)/8 + 1/2)/(12*sqrt(-3*x**4 - 3*x**2 - 2))
```

Mathematica [C] time = 0.207281, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{6x^2}{-3-i\sqrt{15}}}\sqrt{1-\frac{6x^2}{-3+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{6}{-3-i\sqrt{15}}}x\right)|\frac{-3-i\sqrt{15}}{-3+i\sqrt{15}}\right)}{\sqrt{6}\sqrt{-\frac{1}{-3-i\sqrt{15}}}\sqrt{-3x^4-3x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 3*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (6*x^2)/(-3 - I*Sqrt[15])]*Sqrt[1 - (6*x^2)/(-3 +
I*Sqrt[15])]*EllipticF[I*ArcSinh[Sqrt[-6/(-3 - I*Sqrt[15])]*x], (
-3 - I*Sqrt[15])/(-3 + I*Sqrt[15])])/(Sqrt[6]*Sqrt[-(-3 - I*Sqrt[
15])^(-1)]*Sqrt[-2 - 3*x^2 - 3*x^4])
```

Maple [C] time = 0.063, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-3/4-i/4\sqrt{15}\right)x^2}\sqrt{1-\left(-3/4+i/4\sqrt{15}\right)x^2}\mathit{EllipticF}\left(1/2\sqrt{-3-i\sqrt{15}}x,1/2\sqrt{-1-i\sqrt{15}}\right)}{\sqrt{-3-i\sqrt{15}}\sqrt{-3\,x^4-3\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-3*x^2-2)^(1/2),x)
```

```
[Out] 2/(-3-I*15^(1/2))^(1/2)*(1-(-3/4-1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-
3/4+1/4*I*15^(1/2))*x^2)^(1/2)/(-3*x^4-3*x^2-2)^(1/2)*EllipticF(1
/2*(-3-I*15^(1/2))^(1/2)*x,1/2*(-1-I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 3*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-3\,x^2-2}}{3\,x^4+3\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 3*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 3*x^2 - 2)/(3*x^4 + 3*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^4-3x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-3*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 3*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-3\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 3*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 3*x^2 - 2), x)
```

3.118 $\int \frac{1}{\sqrt{-2-4x^2-3x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}$$

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-2 - 4*x^2 - 3*x^4])
```

Rubi [A] time = 0.0522577, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{6}x^2+2\right)\sqrt{\frac{3x^4+4x^2+2}{\left(\sqrt{6}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{3}{2}}x\right)|\frac{1}{2}-\frac{1}{\sqrt{6}}\right)}{2\sqrt[4]{6}\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((2 + Sqrt[6]*x^2)*Sqrt[(2 + 4*x^2 + 3*x^4)/(2 + Sqrt[6]*x^2)^2]*
EllipticF[2*ArcTan[(3/2)^(1/4)*x], 1/2 - 1/Sqrt[6]])/(2*6^(1/4)*S
qrt[-2 - 4*x^2 - 3*x^4])
```

Rubi in Sympy [A] time = 3.66256, size = 94, normalized size = 1.04

$$\frac{6^{\frac{3}{4}}\sqrt{-\frac{-3x^4-4x^2-2}{\left(\frac{\sqrt{6}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{6}x^2}{2}+1\right)F\left(2\operatorname{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{3}x}{2}\right)\middle|-\frac{\sqrt{6}}{6}+\frac{1}{2}\right)}{12\sqrt{-3x^4-4x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] 6**(3/4)*sqrt(-(-3*x**4 - 4*x**2 - 2)/(sqrt(6)*x**2/2 + 1)**2)*(s
qrt(6)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*3**(1/4)*x/2), -sqr
```

```
t(6)/6 + 1/2)/(12*sqrt(-3*x**4 - 4*x**2 - 2))
```

Mathematica [C] time = 0.158047, size = 144, normalized size = 1.6

$$-\frac{i\sqrt{1-\frac{3x^2}{-2-i\sqrt{2}}}\sqrt{1-\frac{3x^2}{-2+i\sqrt{2}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{3}{-2-i\sqrt{2}}}x\right)|\frac{-2-i\sqrt{2}}{-2+i\sqrt{2}}\right)}{\sqrt{3}\sqrt{-\frac{1}{-2-i\sqrt{2}}}\sqrt{-3x^4-4x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 4*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (3*x^2)/(-2 - I*Sqrt[2])]*Sqrt[1 - (3*x^2)/(-2 + I
*Sqrt[2])]*EllipticF[I*ArcSinh[Sqrt[-3/(-2 - I*Sqrt[2])]*x], (-2
- I*Sqrt[2])/(-2 + I*Sqrt[2])])/(Sqrt[3]*Sqrt[-(-2 - I*Sqrt[2])^(
-1)]*Sqrt[-2 - 4*x^2 - 3*x^4])
```

Maple [C] time = 0.07, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-1-i/2\sqrt{2}\right)x^2}\sqrt{1-\left(-1+i/2\sqrt{2}\right)x^2}\mathit{EllipticF}\left(1/2\sqrt{-4-2\,i\sqrt{2}}x,1/3\sqrt{3-6\,i\sqrt{2}}\right)}{\sqrt{-4-2\,i\sqrt{2}}\sqrt{-3\,x^4-4\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-4*x^2-2)^(1/2),x)
```

```
[Out] 2/(-4-2*I*2^(1/2))^(1/2)*(1-(-1-1/2*I*2^(1/2))*x^2)^(1/2)*(1-(-1+
1/2*I*2^(1/2))*x^2)^(1/2)/(-3*x^4-4*x^2-2)^(1/2)*EllipticF(1/2*(-
4-2*I*2^(1/2))^(1/2)*x,1/3*(3-6*I*2^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 4*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-4\,x^2-2}}{3\,x^4+4\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 4*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 4*x^2 - 2)/(3*x^4 + 4*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3x^4-4x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-4*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 4*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-3\,x^4-4\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 4*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 4*x^2 - 2), x)
```

3.119 $\int \frac{1}{\sqrt{-2-5x^2-3x^4}}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{-3x^2-2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

```
[Out] -((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 +
 x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]))
```

Rubi [A] time = 0.0547113, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{\sqrt{-3x^2-2}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{3x^2+2}{x^2+1}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] -((Sqrt[-2 - 3*x^2]*EllipticF[ArcTan[x], -1/2])/(Sqrt[2]*Sqrt[1 +
 x^2]*Sqrt[(2 + 3*x^2)/(1 + x^2)]))
```

Rubi in Sympy [A] time = 8.2786, size = 56, normalized size = 1.08

$$-\frac{3\sqrt{2}\sqrt{-6x^2-4}F\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{\sqrt{-\frac{-36x^2-24}{x^2+1}}\sqrt{6x^2+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] -3*sqrt(2)*sqrt(-6*x**2 - 4)*elliptic_f(atan(x), -1/2)/(sqrt(-(-3
6*x**2 - 24)/(x**2 + 1))*sqrt(6*x**2 + 6))
```

Mathematica [C] time = 0.0429724, size = 63, normalized size = 1.21

$$-\frac{i\sqrt{x^2+1}\sqrt{3x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{3}\sqrt{-3x^4-5x^2-2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 - 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]
*x], 2/3])/(Sqrt[3]*Sqrt[-2 - 5*x^2 - 3*x^4])
```

Maple [A] time = 0.051, size = 50, normalized size = 1.

$$-\frac{i}{6}\sqrt{6}\sqrt{6\,x^2+4}\sqrt{x^2+1}EllipticF\left(\frac{i}{2}x\sqrt{6},\frac{\sqrt{6}}{3}\right)\frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4-5*x^2-2)^(1/2),x)
```

```
[Out] -1/6*I*6^(1/2)*(6*x^2+4)^(1/2)*(x^2+1)^(1/2)/(-3*x^4-5*x^2-2)^(1/
2)*EllipticF(1/2*I*x*6^(1/2),1/3*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 5*x^2 - 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-3\,x^4-5\,x^2-2}}{3\,x^4+5\,x^2+2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 5*x^2 - 2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-3*x^4 - 5*x^2 - 2)/(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4-5x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4-5*x**2-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 - 5*x**2 - 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4-5\,x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 - 5*x^2 - 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 - 5*x^2 - 2), x)
```

3.120 $\int \frac{1}{\sqrt{2+5x^2+5x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{\left(\sqrt{10}x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

```
[Out] ((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2
]*EllipticF[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/
4)*Sqrt[2 + 5*x^2 + 5*x^4])
```

Rubi [A] time = 0.0849091, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{\left(\sqrt{10}x^2+2\right)\sqrt{\frac{5x^4+5x^2+2}{\left(\sqrt{10}x^2+2\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{\frac{5}{2}}x\right)|\frac{1}{8}\left(4-\sqrt{10}\right)\right)}{2\sqrt[4]{10}\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((2 + Sqrt[10]*x^2)*Sqrt[(2 + 5*x^2 + 5*x^4)/(2 + Sqrt[10]*x^2)^2
]*EllipticF[2*ArcTan[(5/2)^(1/4)*x], (4 - Sqrt[10])/8])/(2*10^(1/
4)*Sqrt[2 + 5*x^2 + 5*x^4])
```

Rubi in Sympy [A] time = 3.71945, size = 88, normalized size = 0.96

$$\frac{10^{\frac{3}{4}}\sqrt{\frac{5x^4+5x^2+2}{\left(\frac{\sqrt{10}x^2}{2}+1\right)^2}}\left(\frac{\sqrt{10}x^2}{2}+1\right)F\left(2\,\text{atan}\left(\frac{2^{\frac{3}{4}}\sqrt[4]{5}x}{2}\right)\middle|-\frac{\sqrt{10}}{8}+\frac{1}{2}\right)}{20\sqrt{5x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] 10**(3/4)*sqrt((5*x**4 + 5*x**2 + 2)/(sqrt(10)*x**2/2 + 1)**2)*(s
qrt(10)*x**2/2 + 1)*elliptic_f(2*atan(2**(3/4)*5**(1/4)*x/2), -sq
```

```
rt(10)/8 + 1/2)/(20*sqrt(5*x**4 + 5*x**2 + 2))
```

Mathematica [C] time = 0.2064, size = 144, normalized size = 1.57

$$-\frac{i\sqrt{1-\frac{10x^2}{-5-i\sqrt{15}}}\sqrt{1-\frac{10x^2}{-5+i\sqrt{15}}}F\left(i\sinh^{-1}\left(\sqrt{-\frac{10}{-5-i\sqrt{15}}}x\right)|\frac{-5-i\sqrt{15}}{-5+i\sqrt{15}}\right)}{\sqrt{10}\sqrt{-\frac{1}{-5-i\sqrt{15}}}\sqrt{5x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 + 5*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - (10*x^2)/(-5 - I*Sqrt[15])]*Sqrt[1 - (10*x^2)/(-5
+ I*Sqrt[15])]*EllipticF[I*ArcSinh[Sqrt[-10/(-5 - I*Sqrt[15])]*x]
, (-5 - I*Sqrt[15])/(-5 + I*Sqrt[15])])/(Sqrt[10]*Sqrt[-(-5 - I*S
qrt[15])^(-1)]*Sqrt[2 + 5*x^2 + 5*x^4])
```

Maple [C] time = 0.151, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-5/4+i/4\sqrt{15}\right)x^2}\sqrt{1-\left(-5/4-i/4\sqrt{15}\right)x^2}\mathit{EllipticF}\left(1/2\,x\sqrt{-5+i\sqrt{15}},1/2\,\sqrt{1+i\sqrt{15}}\right)}{\sqrt{-5+i\sqrt{15}}\sqrt{5\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(5*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+I*15^(1/2))^(1/2)*(1-(-5/4+1/4*I*15^(1/2))*x^2)^(1/2)*(1-(-
5/4-1/4*I*15^(1/2))*x^2)^(1/2)/(5*x^4+5*x^2+2)^(1/2)*EllipticF(1/
2*x*(-5+I*15^(1/2))^(1/2),1/2*(1+I*15^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(5*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{5\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(5*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(5*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(5*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(5*x^4 + 5*x^2 + 2), x)
```

3.121 $\int \frac{1}{\sqrt{2+5x^2+4x^4}}\,dx$

Optimal. Leaf size=90

$$\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\mid\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

```
[Out] ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*
EllipticF[2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sq
rt[2 + 5*x^2 + 4*x^4])
```

Rubi [A] time = 0.0527348, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\tan^{-1}\left(\sqrt[4]{2}x\right)\mid\frac{1}{16}\left(8-5\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((1 + Sqrt[2]*x^2)*Sqrt[(2 + 5*x^2 + 4*x^4)/(1 + Sqrt[2]*x^2)^2]*
EllipticF[2*ArcTan[2^(1/4)*x], (8 - 5*Sqrt[2])/16])/(2*2^(3/4)*Sq
rt[2 + 5*x^2 + 4*x^4])
```

Rubi in Sympy [A] time = 3.74721, size = 80, normalized size = 0.89

$$\frac{\sqrt[4]{2}\sqrt{\frac{4x^4+5x^2+2}{\left(\sqrt{2}x^2+1\right)^2}}\left(\sqrt{2}x^2+1\right)F\left(2\,\mathrm{atan}\left(\sqrt[4]{2}x\right)\middle|-\frac{5\sqrt{2}}{16}+\frac{1}{2}\right)}{4\sqrt{4x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] 2**(1/4)*sqrt((4*x**4 + 5*x**2 + 2)/(sqrt(2)*x**2 + 1)**2)*(sqrt(
2)*x**2 + 1)*elliptic_f(2*atan(2**(1/4)*x), -5*sqrt(2)/16 + 1/2)/
(4*sqrt(4*x**4 + 5*x**2 + 2))
```

Mathematica [C] time = 0.154818, size = 147, normalized size = 1.63

$$-\frac{i\sqrt{1-\frac{8x^2}{-5-i\sqrt{7}}}\sqrt{1-\frac{8x^2}{-5+i\sqrt{7}}}F\left(i\sinh^{-1}\left(2\sqrt{-\frac{2}{-5-i\sqrt{7}}}x\right)|\frac{-5-i\sqrt{7}}{-5+i\sqrt{7}}\right)}{2\sqrt{2}\sqrt{-\frac{1}{-5-i\sqrt{7}}}\sqrt{4x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 + 4*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[1 - (8*x^2)/(-5 - I*Sqrt[7])]*Sqrt[1 - (8*x^2)/(-5 +
 I*Sqrt[7])]*EllipticF[I*ArcSinh[2*Sqrt[-2/(-5 - I*Sqrt[7])]*x],
(-5 - I*Sqrt[7])/(-5 + I*Sqrt[7])])/(Sqrt[2]*Sqrt[-(-5 - I*Sqrt[7
])^(-1)]*Sqrt[2 + 5*x^2 + 4*x^4])
```

Maple [C] time = 0.161, size = 87, normalized size = 1.

$$2\,\frac{\sqrt{1-\left(-5/4+i/4\sqrt{7}\right)x^2}\sqrt{1-\left(-5/4-i/4\sqrt{7}\right)x^2}EllipticF\left(1/2\,x\sqrt{-5+i\sqrt{7}},1/4\,\sqrt{9+5\,i\sqrt{7}}\right)}{\sqrt{-5+i\sqrt{7}}\sqrt{4\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+I*7^(1/2))^(1/2)*(1-(-5/4+1/4*I*7^(1/2))*x^2)^(1/2)*(1-(-5/
4-1/4*I*7^(1/2))*x^2)^(1/2)/(4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x
*(-5+I*7^(1/2))^(1/2),1/4*(9+5*I*7^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{4\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^4 + 5*x^2 + 2),x, algorithm="fricas")

[Out] integral(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**4+5*x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(4*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^4 + 5*x^2 + 2),x, algorithm="giac")

[Out] integrate(1/sqrt(4*x^4 + 5*x^2 + 2), x)
```

3.122 $\int \frac{1}{\sqrt{2+5x^2+3x^4}}\,dx$

Optimal. Leaf size=52

$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2]
)/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi [A] time = 0.0237831, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{(x^2+1)\sqrt{\frac{3x^2+2}{x^2+1}}F\left(\tan^{-1}(x)\middle|-\frac{1}{2}\right)}{\sqrt{2}\sqrt{3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((1 + x^2)*Sqrt[(2 + 3*x^2)/(1 + x^2)]*EllipticF[ArcTan[x], -1/2]
)/(Sqrt[2]*Sqrt[2 + 5*x^2 + 3*x^4])
```

Rubi in Sympy [A] time = 3.65851, size = 46, normalized size = 0.88

$$\frac{\sqrt{\frac{6x^2+4}{x^2+1}}\,(4x^2+4)\,F\left(\operatorname{atan}(x)\middle|-\frac{1}{2}\right)}{8\sqrt{3x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] sqrt((6*x**2 + 4)/(x**2 + 1))*(4*x**2 + 4)*elliptic_f(atan(x), -1
/2)/(8*sqrt(3*x**4 + 5*x**2 + 2))
```

Mathematica [C] time = 0.034497, size = 58, normalized size = 1.12

$$-\frac{i\sqrt{x^2+1}\sqrt{3x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{3}{2}}x\right)|\frac{2}{3}\right)}{\sqrt{9x^4+15x^2+6}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 + 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + x^2]*Sqrt[2 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3/2]
*x], 2/3])/Sqrt[6 + 15*x^2 + 9*x^4]
```

Maple [A] time = 0., size = 44, normalized size = 0.9

$$-\frac{i}{2}\sqrt{x^2+1}\sqrt{6\,x^2+4}EllipticF\left(ix,\frac{\sqrt{6}}{2}\right)\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(6*x^2+4)^(1/2)/(3*x^4+5*x^2+2)^(1/2)*Ellipt
icF(I*x,1/2*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^4 + 5x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3\,x^4 + 5\,x^2 + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(3*x^4 + 5*x^2 + 2), x)
```

3.123 $\int \frac{1}{\sqrt{2+5x^2+2x^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\,(2x^2+1)\,F\left(\tan^{-1}\left(\sqrt{2}x\right)\,|\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

```
[Out] (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]
*x], 3/4])/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rubi [A] time = 0.0245325, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\sqrt{\frac{x^2+2}{2x^2+1}}\,(2x^2+1)\,F\left(\tan^{-1}\left(\sqrt{2}x\right)\,|\frac{3}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] (Sqrt[(2 + x^2)/(1 + 2*x^2)]*(1 + 2*x^2)*EllipticF[ArcTan[Sqrt[2]
*x], 3/4])/(2*Sqrt[2 + 5*x^2 + 2*x^4])
```

Rubi in Sympy [A] time = 3.4931, size = 56, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{\frac{2x^2+4}{8x^2+4}}\,(8x^2+4)\,F\left(\operatorname{atan}\left(\sqrt{2}x\right)\middle|\frac{3}{4}\right)}{8\sqrt{2x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt((2*x**2 + 4)/(8*x**2 + 4))*(8*x**2 + 4)*elliptic_f(a
tan(sqrt(2)*x), 3/4)/(8*sqrt(2*x**4 + 5*x**2 + 2))
```

Mathematica [C] time = 0.036071, size = 58, normalized size = 1.

$$-\frac{i\sqrt{x^2+2}\sqrt{2x^2+1}F\left(i\sinh^{-1}\left(\sqrt{2}x\right)|\frac{1}{4}\right)}{2\sqrt{2x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 + 2*x^4],x]
```

```
[Out] ((-I/2)*Sqrt[2 + x^2]*Sqrt[1 + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2]
*x], 1/4])/Sqrt[2 + 5*x^2 + 2*x^4]
```

Maple [C] time = 0.081, size = 48, normalized size = 0.8

$$-\frac{i}{2}\sqrt{2}EllipticF\left(\frac{i}{2}\sqrt{2}x,2\right)\sqrt{2\,x^2+4}\sqrt{2\,x^2+1}\frac{1}{\sqrt{2\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(2*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] -1/2*I*2^(1/2)*(2*x^2+4)^(1/2)*(2*x^2+1)^(1/2)/(2*x^4+5*x^2+2)^(1
/2)*EllipticF(1/2*I*2^(1/2)*x,2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{2\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^4 + 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(2*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2\,x^4 + 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(2*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(2*x^4 + 5*x^2 + 2), x)
```

3.124 $\int \frac{1}{\sqrt{2+5x^2+x^4}}\,dx$

Optimal. Leaf size=108

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

```
[Out] (Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5
+ Sqrt[17])*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17
 + 5*Sqrt[17])/4])/(2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Rubi [A] time = 0.0988901, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\sqrt{\frac{\left(5-\sqrt{17}\right)x^2+4}{\left(5+\sqrt{17}\right)x^2+4}}\left(\left(5+\sqrt{17}\right)x^2+4\right)F\left(\tan^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{17}}x\right)|\frac{1}{4}\left(-17+5\sqrt{17}\right)\right)}{2\sqrt{5+\sqrt{17}}\sqrt{x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[(4 + (5 - Sqrt[17])*x^2)/(4 + (5 + Sqrt[17])*x^2)]*(4 + (5
+ Sqrt[17])*x^2)*EllipticF[ArcTan[(Sqrt[5 + Sqrt[17]]*x)/2], (-17
 + 5*Sqrt[17])/4])/(2*Sqrt[5 + Sqrt[17]]*Sqrt[2 + 5*x^2 + x^4])
```

Rubi in Sympy [A] time = 4.14588, size = 90, normalized size = 0.83

$$\frac{\sqrt{\frac{x^2\left(-\sqrt{17}+5\right)+4}{x^2\left(\sqrt{17}+5\right)+4}}\left(x^2\left(\sqrt{17}+5\right)+4\right)F\left(\operatorname{atan}\left(\frac{x\sqrt{\sqrt{17}+5}}{2}\right)\middle|-\frac{17}{4}+\frac{5\sqrt{17}}{4}\right)}{2\sqrt{\sqrt{17}+5}\sqrt{x^4+5x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+5*x**2+2)**(1/2),x)
```

```
[Out] sqrt((x**2*(-sqrt(17) + 5) + 4)/(x**2*(sqrt(17) + 5) + 4))*(x**2*
(sqrt(17) + 5) + 4)*elliptic_f(atan(x*sqrt(sqrt(17) + 5)/2), -17/
```

```
4 + 5*sqrt(17)/4)/(2*sqrt(sqrt(17) + 5)*sqrt(x**4 + 5*x**2 + 2))
```

Mathematica [C] time = 0.144596, size = 103, normalized size = 0.95

$$-\frac{i\sqrt{2x^2-\sqrt{17}+5}\sqrt{2x^2+\sqrt{17}+5}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{17}}}x\right)|\frac{21}{4}+\frac{5\sqrt{17}}{4}\right)}{\sqrt{2\left(5-\sqrt{17}\right)}\sqrt{x^4+5x^2+2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 + x^4],x]
```

```
[Out] ((-I)*Sqrt[5 - Sqrt[17] + 2*x^2]*Sqrt[5 + Sqrt[17] + 2*x^2]*Ellip
ticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[17])]*x], 21/4 + (5*Sqrt[17])/4])
/(Sqrt[2*(5 - Sqrt[17])]*Sqrt[2 + 5*x^2 + x^4])
```

Maple [A] time = 0.238, size = 76, normalized size = 0.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{17}\right)x^2}\sqrt{1-\left(-5/4-1/4\,\sqrt{17}\right)x^2}EllipticF\left(1/2\,x\sqrt{-5+\sqrt{17}},5/4\,\sqrt{2}+1/4\,\sqrt{34}\right)}{\sqrt{-5+\sqrt{17}}\sqrt{x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+17^(1/2))^(1/2)*(1-(-5/4+1/4*17^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*17^(1/2))*x^2)^(1/2)/(x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5+
17^(1/2))^(1/2),5/4*2^(1/2)+1/4*34^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sqrt(x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(x^4 + 5*x^2 + 2), x)
```

3.125 $\int \frac{1}{\sqrt{2+5x^2-x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{33}-5}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x
], (-29 - 5*Sqrt[33])/4]
```

Rubi [A] time = 0.222049, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{33}-5}}F\left(\sin^{-1}\left(\sqrt{\frac{2}{5+\sqrt{33}}}x\right)|\frac{1}{4}\left(-29-5\sqrt{33}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[33])]*EllipticF[ArcSin[Sqrt[2/(5 + Sqrt[33])]*x
], (-29 - 5*Sqrt[33])/4]
```

Rubi in Sympy [A] time = 12.4172, size = 48, normalized size = 1.

$$-\frac{4F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{33}}}{2}\right)\middle|-\frac{29}{4}-\frac{5\sqrt{33}}{4}\right)}{\sqrt{5+\sqrt{33}}\left(-\sqrt{33}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -4*elliptic_f(asin(x*sqrt(-5 + sqrt(33))/2), -29/4 - 5*sqrt(33)/4
)/(sqrt(5 + sqrt(33))*(-sqrt(33) + 5))
```

Mathematica [C] time = 0.111864, size = 55, normalized size = 1.15

$$-i\sqrt{\frac{2}{5+\sqrt{33}}}F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{-5+\sqrt{33}}}x\right)|-\frac{29}{4}+\frac{5\sqrt{33}}{4}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[33])]*EllipticF[I*ArcSinh[Sqrt[2/(-5 + Sqrt
[33])]*x], -29/4 + (5*Sqrt[33])/4]
```

Maple [B] time = 0.115, size = 80, normalized size = 1.7

$$2\,{\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{33}\right)x^{2}}\sqrt{1-\left(-5/4-1/4\,\sqrt{33}\right)x^{2}}{\it EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{33}},5/4\,i\sqrt{2}+i/4\sqrt{66}\right)}{\sqrt{-5+\sqrt{33}}\sqrt{-x^{4}+5\,x^{2}+2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+33^(1/2))^(1/2)*(1-(-5/4+1/4*33^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*33^(1/2))*x^2)^(1/2)/(-x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(-5
+33^(1/2))^(1/2),5/4*I*2^(1/2)+1/4*I*66^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 5*x^2 + 2),x, algorithm="fricas")

[Out] integral(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+5*x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(-x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 5*x^2 + 2),x, algorithm="giac")

[Out] integrate(1/sqrt(-x^4 + 5*x^2 + 2), x)
```

3.126 $\int \frac{1}{\sqrt{2+5x^2-2x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{41}-5}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]
], (-33 - 5*Sqrt[41])/8]
```

Rubi [A] time = 0.135698, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{41}-5}}F\left(\sin^{-1}\left(\frac{2x}{\sqrt{5+\sqrt{41}}}\right)|\frac{1}{8}\left(-33-5\sqrt{41}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[41])]*EllipticF[ArcSin[(2*x)/Sqrt[5 + Sqrt[41]]
], (-33 - 5*Sqrt[41])/8]
```

Rubi in Sympy [A] time = 15.0284, size = 53, normalized size = 1.18

$$-\frac{4\sqrt{2}F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{41}}}{2}\right)\middle|-\frac{33}{8}-\frac{5\sqrt{41}}{8}\right)}{\sqrt{5+\sqrt{41}}\left(-\sqrt{41}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-2*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -4*sqrt(2)*elliptic_f(asin(x*sqrt(-5 + sqrt(41))/2), -33/8 - 5*sq
rt(41)/8)/(sqrt(5 + sqrt(41))*(-sqrt(41) + 5))
```

Mathematica [C] time = 0.0757928, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{41}}}F\left(i\sinh^{-1}\left(\frac{2x}{\sqrt{-5+\sqrt{41}}}\right)|-\frac{33}{8}+\frac{5\sqrt{41}}{8}\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 2*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[41])]*EllipticF[I*ArcSinh[(2*x)/Sqrt[-5 + S
qrt[41]]], -33/8 + (5*Sqrt[41])/8]
```

Maple [B] time = 0.109, size = 76, normalized size = 1.7

$$2\,{\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{41}\right)x^{2}}\sqrt{1-\left(-5/4-1/4\,\sqrt{41}\right)x^{2}}{\it EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{41}},5/4\,i+i/4\sqrt{41}\right)}{\sqrt{-5+\sqrt{41}}\sqrt{-2\,x^{4}+5\,x^{2}+2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-2*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+41^(1/2))^(1/2)*(1-(-5/4+1/4*41^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*41^(1/2))*x^2)^(1/2)/(-2*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(
-5+41^(1/2))^(1/2),5/4*I+1/4*I*41^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-2\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 + 2),x, algorithm="fricas")

[Out] integral(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-2*x**4+5*x**2+2)**(1/2),x)

[Out] Integral(1/sqrt(-2*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-2\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-2*x^4 + 5*x^2 + 2),x, algorithm="giac")

[Out] integrate(1/sqrt(-2*x^4 + 5*x^2 + 2), x)
```

3.127 $\int \frac{1}{\sqrt{2+5x^2-3x^4}}\,dx$

Optimal. Leaf size=10

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rubi [A] time = 0.0408244, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$F\left(\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-6\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] EllipticF[ArcSin[x/Sqrt[2]], -6]
```

Rubi in Sympy [A] time = 7.77856, size = 12, normalized size = 1.2

$$F\left(\operatorname{asin}\left(\frac{\sqrt{2}x}{2}\right)\middle|-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] elliptic_f(asin(sqrt(2)*x/2), -6)
```

Mathematica [C] time = 0.0401972, size = 65, normalized size = 6.5

$$-\frac{i\sqrt{1-\frac{x^2}{2}}\sqrt{3x^2+1}F\left(i\sinh^{-1}\left(\sqrt{3}x\right)\middle|-\frac{1}{6}\right)}{\sqrt{3}\sqrt{-3x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 3*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2/2]*Sqrt[1 + 3*x^2]*EllipticF[I*ArcSinh[Sqrt[3]
*x], -1/6])/(Sqrt[3]*Sqrt[2 + 5*x^2 - 3*x^4])
```

Maple [B] time = 0., size = 51, normalized size = 5.1

$$\frac{\sqrt{2}}{2}\sqrt{-2\,x^2+4}\sqrt{3\,x^2+1}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2},i\sqrt{6}\right)\frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*(-2*x^2+4)^(1/2)*(3*x^2+1)^(1/2)/(-3*x^4+5*x^2+2)^(1/
2)*EllipticF(1/2*2^(1/2)*x,I*6^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-3\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^4 + 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3\,x^4 + 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-3*x^4 + 5*x^2 + 2), x)
```

3.128 $\int \frac{1}{\sqrt{2+5x^2-4x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{57}-5}}F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]
*x], (-41 - 5*Sqrt[57])/16]
```

Rubi [A] time = 0.223283, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{57}-5}}F\left(\sin^{-1}\left(2\sqrt{\frac{2}{5+\sqrt{57}}}x\right)|\frac{1}{16}\left(-41-5\sqrt{57}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[57])]*EllipticF[ArcSin[2*Sqrt[2/(5 + Sqrt[57])]
*x], (-41 - 5*Sqrt[57])/16]
```

Rubi in Sympy [A] time = 13.3316, size = 48, normalized size = 0.98

$$-\frac{8F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{57}}}{2}\right)\middle| -\frac{41}{16}-\frac{5\sqrt{57}}{16}\right)}{\sqrt{5+\sqrt{57}}\left(-\sqrt{57}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -8*elliptic_f(asin(x*sqrt(-5 + sqrt(57))/2), -41/16 - 5*sqrt(57)/
16)/(sqrt(5 + sqrt(57))*(-sqrt(57) + 5))
```

Mathematica [C] time = 0.105043, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{57}}}F\left(i\sinh^{-1}\left(2\sqrt{\frac{2}{-5+\sqrt{57}}}x\right)|\frac{1}{16}\left(-41+5\sqrt{57}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 4*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[57])]*EllipticF[I*ArcSinh[2*Sqrt[2/(-5 + Sq
rt[57])]*x], (-41 + 5*Sqrt[57])/16]
```

Maple [B] time = 0.115, size = 80, normalized size = 1.6

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{57}\right)x^2}\sqrt{1-\left(-5/4-1/4\,\sqrt{57}\right)x^2}EllipticF\left(1/2\,x\sqrt{-5+\sqrt{57}},5/8\,i\sqrt{2}+i/8\sqrt{114}\right)}{\sqrt{-5+\sqrt{57}}\sqrt{-4\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+57^(1/2))^(1/2)*(1-(-5/4+1/4*57^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*57^(1/2))*x^2)^(1/2)/(-4*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(
-5+57^(1/2))^(1/2),5/8*I*2^(1/2)+1/8*I*114^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-4\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-4*x^4 + 5*x^2 + 2), x)
```

3.129 $\int \frac{1}{\sqrt{2+5x^2-5x^4}}\,dx$

Optimal. Leaf size=48

$$\sqrt{\frac{2}{\sqrt{65}-5}}F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*
x], (-9 - Sqrt[65])/4]
```

Rubi [A] time = 0.224895, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{65}-5}}F\left(\sin^{-1}\left(\sqrt{\frac{10}{5+\sqrt{65}}}x\right)|\frac{1}{4}\left(-9-\sqrt{65}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[65])]*EllipticF[ArcSin[Sqrt[10/(5 + Sqrt[65])]*
x], (-9 - Sqrt[65])/4]
```

Rubi in Sympy [A] time = 13.3757, size = 51, normalized size = 1.06

$$-\frac{4\sqrt{5}F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{65}}}{2}\right)\middle|-\frac{9}{4}-\frac{\sqrt{65}}{4}\right)}{\sqrt{5+\sqrt{65}}\left(-\sqrt{65}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -4*sqrt(5)*elliptic_f(asin(x*sqrt(-5 + sqrt(65))/2), -9/4 - sqrt(
65)/4)/(sqrt(5 + sqrt(65))*(-sqrt(65) + 5))
```

Mathematica [C] time = 0.100152, size = 52, normalized size = 1.08

$$-i\sqrt{\frac{2}{5+\sqrt{65}}}F\left(i\sinh^{-1}\left(\frac{1}{2}\sqrt{5+\sqrt{65}}x\right)|\frac{1}{4}\left(-9+\sqrt{65}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 5*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[65])]*EllipticF[I*ArcSinh[(Sqrt[5 + Sqrt[65
]]*x)/2], (-9 + Sqrt[65])/4]
```

Maple [B] time = 0.117, size = 80, normalized size = 1.7

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{65}\right)x^{2}}\sqrt{1-\left(-5/4-1/4\,\sqrt{65}\right)x^{2}}EllipticF\left(1/2\,x\sqrt{-5+\sqrt{65}},i/4\sqrt{10}+i/4\sqrt{26}\right)}{\sqrt{-5+\sqrt{65}}\sqrt{-5\,x^{4}+5\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-5*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+65^(1/2))^(1/2)*(1-(-5/4+1/4*65^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*65^(1/2))*x^2)^(1/2)/(-5*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(
-5+65^(1/2))^(1/2),1/4*I*10^(1/2)+1/4*I*26^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-5*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-5\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-5*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-5*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-5*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-5\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-5*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-5*x^4 + 5*x^2 + 2), x)
```

3.130 $\int \frac{1}{\sqrt{2+5x^2-6x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{73}-5}}F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]
*x], (-49 - 5*Sqrt[73])/24]
```

Rubi [A] time = 0.170699, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{73}-5}}F\left(\sin^{-1}\left(2\sqrt{\frac{3}{5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49-5\sqrt{73}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[73])]*EllipticF[ArcSin[2*Sqrt[3/(5 + Sqrt[73])]
*x], (-49 - 5*Sqrt[73])/24]
```

Rubi in Sympy [A] time = 13.7715, size = 53, normalized size = 1.08

$$-\frac{4\sqrt{6}F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{73}}}{2}\right)\middle|-\frac{49}{24}-\frac{5\sqrt{73}}{24}\right)}{\sqrt{5+\sqrt{73}}\left(-\sqrt{73}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-6*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -4*sqrt(6)*elliptic_f(asin(x*sqrt(-5 + sqrt(73))/2), -49/24 - 5*s
qrt(73)/24)/(sqrt(5 + sqrt(73))*(-sqrt(73) + 5))
```

Mathematica [C] time = 0.0847728, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{73}}}F\left(i\sinh^{-1}\left(2\sqrt{\frac{3}{-5+\sqrt{73}}}x\right)|\frac{1}{24}\left(-49+5\sqrt{73}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 6*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[73])]*EllipticF[I*ArcSinh[2*Sqrt[3/(-5 + Sq
rt[73])]*x], (-49 + 5*Sqrt[73])/24]
```

Maple [B] time = 0.111, size = 80, normalized size = 1.6

$$2\,\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{73}\right)x^{2}}\sqrt{1-\left(-5/4-1/4\,\sqrt{73}\right)x^{2}}EllipticF\left(1/2\,x\sqrt{-5+\sqrt{73}},\frac{5\,i}{12}\sqrt{3}+i/12\sqrt{219}\right)}{\sqrt{-5+\sqrt{73}}\sqrt{-6\,x^{4}+5\,x^{2}+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-6*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+73^(1/2))^(1/2)*(1-(-5/4+1/4*73^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*73^(1/2))*x^2)^(1/2)/(-6*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(
-5+73^(1/2))^(1/2),5/12*I*3^(1/2)+1/12*I*219^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-6\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-6*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-6\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-6*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-6x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-6*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-6*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-6\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-6*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-6*x^4 + 5*x^2 + 2), x)
```

3.131 $\int \frac{1}{\sqrt{2+5x^2-7x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

```
[Out] EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Rubi [A] time = 0.042135, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{7}{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 7*x^4],x]

[Out] EllipticF[ArcSin[x], -7/2]/Sqrt[2]
```

Rubi in Sympy [A] time = 10.0808, size = 14, normalized size = 1.17

$$\frac{\sqrt{2}F\left(\operatorname{asin}(x)\middle|-\frac{7}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-7*x**4+5*x**2+2)**(1/2),x)

[Out] sqrt(2)*elliptic_f(asin(x), -7/2)/2
```

Mathematica [C] time = 0.0444309, size = 65, normalized size = 5.42

$$-\frac{i\sqrt{1-x^2}\sqrt{7x^2+2}F\left(i\sinh^{-1}\left(\sqrt{\frac{7}{2}}x\right)\middle|-\frac{2}{7}\right)}{\sqrt{7}\sqrt{-7x^4+5x^2+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 7*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 - x^2]*Sqrt[2 + 7*x^2]*EllipticF[I*ArcSinh[Sqrt[7/2]
*x], -2/7])/(Sqrt[7]*Sqrt[2 + 5*x^2 - 7*x^4])
```

Maple [B] time = 0.018, size = 43, normalized size = 3.6

$$\frac{EllipticF\left(x, \frac{i}{2}\sqrt{14}\right)}{2}\sqrt{-x^2+1}\sqrt{14\,x^2+4}\frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-7*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*(14*x^2+4)^(1/2)/(-7*x^4+5*x^2+2)^(1/2)*Ellipt
icF(x,1/2*I*14^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-7*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-7\,x^4+5\,x^2+2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-7*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-7x^4 + 5x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-7*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-7*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-7\,x^4 + 5\,x^2 + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-7*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-7*x^4 + 5*x^2 + 2), x)
```

3.132 $\int \frac{1}{\sqrt{2+5x^2-8x^4}}\,dx$

Optimal. Leaf size=45

$$\sqrt{\frac{2}{\sqrt{89}-5}}F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]
], (-57 - 5*Sqrt[89])/32]
```

Rubi [A] time = 0.151065, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{89}-5}}F\left(\sin^{-1}\left(\frac{4x}{\sqrt{5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57-5\sqrt{89}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[89])]*EllipticF[ArcSin[(4*x)/Sqrt[5 + Sqrt[89]]
], (-57 - 5*Sqrt[89])/32]
```

Rubi in Sympy [A] time = 15.2349, size = 53, normalized size = 1.18

$$-\frac{8\sqrt{2}F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{89}}}{2}\right)\middle|-\frac{57}{32}-\frac{5\sqrt{89}}{32}\right)}{\sqrt{5+\sqrt{89}}\left(-\sqrt{89}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-8*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -8*sqrt(2)*elliptic_f(asin(x*sqrt(-5 + sqrt(89))/2), -57/32 - 5*s
qrt(89)/32)/(sqrt(5 + sqrt(89))*(-sqrt(89) + 5))
```

Mathematica [C] time = 0.0724016, size = 52, normalized size = 1.16

$$-i\sqrt{\frac{2}{5+\sqrt{89}}}F\left(i\sinh^{-1}\left(\frac{4x}{\sqrt{-5+\sqrt{89}}}\right)|\frac{1}{32}\left(-57+5\sqrt{89}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 8*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[89])]*EllipticF[I*ArcSinh[(4*x)/Sqrt[-5 + S
qrt[89]]], (-57 + 5*Sqrt[89])/32]
```

Maple [B] time = 0.111, size = 76, normalized size = 1.7

$$2\,{\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{89}\right){x}^{2}}\sqrt{1-\left(-5/4-1/4\,\sqrt{89}\right){x}^{2}}{\it EllipticF}\left(1/2\,x\sqrt{-5+\sqrt{89}},5/8\,i+i/8\sqrt{89}\right)}{\sqrt{-5+\sqrt{89}}\sqrt{-8\,{x}^{4}+5\,{x}^{2}+2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-8*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+89^(1/2))^(1/2)*(1-(-5/4+1/4*89^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*89^(1/2))*x^2)^(1/2)/(-8*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(
-5+89^(1/2))^(1/2),5/8*I+1/8*I*89^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-8*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-8\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-8*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-8x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-8*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-8*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-8\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-8*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-8*x^4 + 5*x^2 + 2), x)
```

3.133 $\int \frac{1}{\sqrt{2+5x^2-9x^4}}\,dx$

Optimal. Leaf size=49

$$\sqrt{\frac{2}{\sqrt{97}-5}}F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

```
[Out] Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]
*x], (-61 - 5*Sqrt[97])/36]
```

Rubi [A] time = 0.168765, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{\frac{2}{\sqrt{97}-5}}F\left(\sin^{-1}\left(3\sqrt{\frac{2}{5+\sqrt{97}}}x\right)|\frac{1}{36}\left(-61-5\sqrt{97}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] Sqrt[2/(-5 + Sqrt[97])]*EllipticF[ArcSin[3*Sqrt[2/(5 + Sqrt[97])]
*x], (-61 - 5*Sqrt[97])/36]
```

Rubi in Sympy [A] time = 13.8525, size = 48, normalized size = 0.98

$$-\frac{12F\left(\operatorname{asin}\left(\frac{x\sqrt{-5+\sqrt{97}}}{2}\right)\middle|-\frac{61}{36}-\frac{5\sqrt{97}}{36}\right)}{\sqrt{5+\sqrt{97}}\left(-\sqrt{97}+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-9*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] -12*elliptic_f(asin(x*sqrt(-5 + sqrt(97))/2), -61/36 - 5*sqrt(97)
/36)/(sqrt(5 + sqrt(97))*(-sqrt(97) + 5))
```

Mathematica [C] time = 0.08241, size = 56, normalized size = 1.14

$$-i\sqrt{\frac{2}{5+\sqrt{97}}}F\left(i\sinh^{-1}\left(3\sqrt{\frac{2}{-5+\sqrt{97}}}x\right)|\frac{1}{36}\left(-61+5\sqrt{97}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/Sqrt[2 + 5*x^2 - 9*x^4],x]
```

```
[Out] (-I)*Sqrt[2/(5 + Sqrt[97])]*EllipticF[I*ArcSinh[3*Sqrt[2/(-5 + Sq
rt[97])]*x], (-61 + 5*Sqrt[97])/36]
```

Maple [B] time = 0.12, size = 80, normalized size = 1.6

$$2\,{\frac{\sqrt{1-\left(-5/4+1/4\,\sqrt{97}\right)x^{2}}\sqrt{1-\left(-5/4-1/4\,\sqrt{97}\right)x^{2}}EllipticF\left(1/2\,x\sqrt{-5+\sqrt{97}},\frac{5\,i}{12}\sqrt{2}+i/12\sqrt{194}\right)}{\sqrt{-5+\sqrt{97}}\sqrt{-9\,x^{4}+5\,x^{2}+2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-9*x^4+5*x^2+2)^(1/2),x)
```

```
[Out] 2/(-5+97^(1/2))^(1/2)*(1-(-5/4+1/4*97^(1/2))*x^2)^(1/2)*(1-(-5/4-
1/4*97^(1/2))*x^2)^(1/2)/(-9*x^4+5*x^2+2)^(1/2)*EllipticF(1/2*x*(
-5+97^(1/2))^(1/2),5/12*I*2^(1/2)+1/12*I*194^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-9\,x^{4}+5\,x^{2}+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-9*x^4 + 5*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-9\,x^4+5\,x^2+2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-9*x^4 + 5*x^2 + 2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-9x^4+5x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-9*x**4+5*x**2+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-9*x**4 + 5*x**2 + 2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-9\,x^4+5\,x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-9*x^4 + 5*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-9*x^4 + 5*x^2 + 2), x)
```

3.134 $\int x^2 \left(bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=17

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] time = 0.0154341, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Rubi in Sympy [A] time = 4.79005, size = 12, normalized size = 0.71

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2),x)
```

```
[Out] b*x**5/5 + c*x**7/7
```

Mathematica [A] time = 0.00322095, size = 17, normalized size = 1.

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^5)/5 + (c*x^7)/7
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2),x)
```

```
[Out] 1/5*b*x^5+1/7*c*x^7
```

Maxima [A] time = 0.678668, size = 18, normalized size = 1.06

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

Fricas [A] time = 0.232872, size = 1, normalized size = 0.06

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b
```

Sympy [A] time = 0.069703, size = 12, normalized size = 0.71

$$\frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2),x)
```

```
[Out] b*x**5/5 + c*x**7/7
```

GIAC/XCAS [A] time = 0.269173, size = 18, normalized size = 1.06

$$\frac{1}{7}\,cx^{7}+\frac{1}{5}\,bx^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5
```

3.135 $\int x\left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=17

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] time = 0.014973, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Rubi in Sympy [A] time = 4.62429, size = 12, normalized size = 0.71

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2),x)
```

```
[Out] b*x**4/4 + c*x**6/6
```

Mathematica [A] time = 0.00244627, size = 17, normalized size = 1.

$$\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b*x^2 + c*x^4),x]
```

```
[Out] (b*x^4)/4 + (c*x^6)/6
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2),x)
```

```
[Out] 1/4*b*x^4+1/6*c*x^6
```

Maxima [A] time = 0.685813, size = 18, normalized size = 1.06

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

Fricas [A] time = 0.234449, size = 1, normalized size = 0.06

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c + 1/4*x^4*b
```

Sympy [A] time = 0.069724, size = 12, normalized size = 0.71

$$\frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2),x)
```

```
[Out] b*x**4/4 + c*x**6/6
```

GIAC/XCAS [A] time = 0.267695, size = 18, normalized size = 1.06

$$\frac{1}{6}cx^6 + \frac{1}{4}bx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6 + 1/4*b*x^4
```

3.136 $\int \left(bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=17

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] time = 0.01069, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[b*x^2 + c*x^4,x]

[Out] (b*x^3)/3 + (c*x^5)/5
```

Rubi in Sympy [A] time = 1.75229, size = 12, normalized size = 0.71

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c*x**4+b*x**2,x)

[Out] b*x**3/3 + c*x**5/5
```

Mathematica [A] time = 0.000075516, size = 17, normalized size = 1.

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[b*x^2 + c*x^4,x]
```

```
[Out] (b*x^3)/3 + (c*x^5)/5
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c*x^4+b*x^2,x)
```

```
[Out] 1/3*b*x^3+1/5*c*x^5
```

Maxima [A] time = 0.679489, size = 18, normalized size = 1.06

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

Fricas [A] time = 0.231198, size = 1, normalized size = 0.06

$$\frac{1}{5}x^5c + \frac{1}{3}x^3b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/3*x^3*b
```

Sympy [A] time = 0.0684, size = 12, normalized size = 0.71

$$\frac{bx^3}{3} + \frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**4+b*x**2,x)
```

```
[Out] b*x**3/3 + c*x**5/5
```

GIAC/XCAS [A] time = 0.268235, size = 18, normalized size = 1.06

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^2,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3
```

3.137 $\int \frac{bx^2+cx^4}{x} \, dx$

Optimal. Leaf size=17

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Rubi [A] time = 0.0137548, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x,x]

[Out] (b*x^2)/2 + (c*x^4)/4
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$b \int x \, dx + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x,x)

[Out] b*Integral(x, x) + c*x**4/4
```

Mathematica [A] time = 0.00202357, size = 17, normalized size = 1.

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$\frac{bx^2}{2}+\frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x,x)
```

```
[Out] 1/2*b*x^2+1/4*c*x^4
```

Maxima [A] time = 0.677132, size = 18, normalized size = 1.06

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Fricas [A] time = 0.243969, size = 18, normalized size = 1.06

$$\frac{1}{4}\,cx^4+\frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

Sympy [A] time = 0.064731, size = 12, normalized size = 0.71

$$\frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x,x)
```

```
[Out] b*x**2/2 + c*x**4/4
```

GIAC/XCAS [A] time = 0.268679, size = 18, normalized size = 1.06

$$\frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2
```

3.138 $\int \frac{bx^2+cx^4}{x^2} dx$

Optimal. Leaf size=12

$$bx + \frac{cx^3}{3}$$

```
[Out] b*x + (c*x^3)/3
```

Rubi [A] time = 0.0125875, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^2,x]

[Out] b*x + (c*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3}{3} + \int b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2)/x**2,x)

[Out] c*x**3/3 + Integral(b, x)
```

Mathematica [A] time = 0.000687643, size = 12, normalized size = 1.

$$bx + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^2,x]
```

```
[Out] b*x + (c*x^3)/3
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^2,x)
```

```
[Out] b*x+1/3*c*x^3
```

Maxima [A] time = 0.688338, size = 14, normalized size = 1.17

$$\frac{1}{3}\,cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + b*x
```

Fricas [A] time = 0.24677, size = 14, normalized size = 1.17

$$\frac{1}{3}\,cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*c*x^3 + b*x
```

Sympy [A] time = 0.069957, size = 8, normalized size = 0.67

$$bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**2,x)
```

```
[Out] b*x + c*x**3/3
```

GIAC/XCAS [A] time = 0.26914, size = 14, normalized size = 1.17

$$\frac{1}{3}\,cx^3 + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x
```

3.139 $\int \frac{bx^2+cx^4}{x^3} dx$

Optimal. Leaf size=13

$$b\log(x) + \frac{cx^2}{2}$$

```
[Out] (c*x^2)/2 + b*Log[x]
```

Rubi [A] time = 0.014277, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^3,x]

[Out] (c*x^2)/2 + b*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$b\log(x) + c\int x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**3,x)

[Out] b*log(x) + c*Integral(x, x)
```

Mathematica [A] time = 0.0019599, size = 13, normalized size = 1.

$$b\log(x) + \frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^3,x]
```

```
[Out] (c*x^2)/2 + b*Log[x]
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{cx^2}{2} + b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^3,x)
```

```
[Out] 1/2*c*x^2+b*ln(x)
```

Maxima [A] time = 0.672426, size = 19, normalized size = 1.46

$$\frac{1}{2}cx^2 + \frac{1}{2}b\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*c*x^2 + 1/2*b*log(x^2)
```

Fricas [A] time = 0.254013, size = 15, normalized size = 1.15

$$\frac{1}{2}cx^2 + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*c*x^2 + b*log(x)
```

Sympy [A] time = 0.156971, size = 10, normalized size = 0.77

$$b\log(x) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**3,x)
```

```
[Out] b*log(x) + c*x**2/2
```

GIAC/XCAS [A] time = 0.270472, size = 19, normalized size = 1.46

$$\frac{1}{2}\,cx^2 + \frac{1}{2}\,b\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c*x^2 + 1/2*b*ln(x^2)
```

3.140 $\int \frac{bx^2+cx^4}{x^4} \, dx$

Optimal. Leaf size=10

$$cx - \frac{b}{x}$$

```
[Out] -(b/x) + c*x
```

Rubi [A] time = 0.0132703, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$cx - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^4,x]

[Out] -(b/x) + c*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b}{x} + \int c \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**4,x)

[Out] -b/x + Integral(c, x)
```

Mathematica [A] time = 0.00164119, size = 10, normalized size = 1.

$$cx - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -(b/x) + c*x
```

Maple [A] time = 0.008, size = 11, normalized size = 1.1

$$-\frac{b}{x} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^4,x)

[Out] -b/x+c*x
```

Maxima [A] time = 0.691197, size = 14, normalized size = 1.4

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^4,x, algorithm="maxima")

[Out] c*x - b/x
```

Fricas [A] time = 0.246565, size = 18, normalized size = 1.8

$$\frac{cx^2 - b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^4,x, algorithm="fricas")

[Out] (c*x^2 - b)/x
```

Sympy [A] time = 1.00118, size = 5, normalized size = 0.5

$$-\frac{b}{x} + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**4,x)
```

```
[Out] -b/x + c*x
```

GIAC/XCAS [A] time = 0.268033, size = 14, normalized size = 1.4

$$cx - \frac{b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^4,x, algorithm="giac")
```

```
[Out] c*x - b/x
```

3.141 $\int \frac{bx^2+cx^4}{x^5}\,dx$

Optimal. Leaf size=13

$$c\log(x) - \frac{b}{2x^2}$$

```
[Out] -b/(2*x^2) + c*Log[x]
```

Rubi [A] time = 0.0143272, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$c\log(x) - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^5,x]

[Out] -b/(2*x^2) + c*Log[x]
```

Rubi in Sympy [A] time = 4.53769, size = 10, normalized size = 0.77

$$-\frac{b}{2x^2} + c\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2)/x**5,x)

[Out] -b/(2*x**2) + c*log(x)
```

Mathematica [A] time = 0.00405706, size = 13, normalized size = 1.

$$c\log(x) - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -b/(2*x^2) + c*Log[x]
```

Maple [A] time = 0.007, size = 12, normalized size = 0.9

$$-\frac{b}{2\,x^{2}}+c\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^5,x)
```

```
[Out] -1/2*b/x^2+c*ln(x)
```

Maxima [A] time = 0.689465, size = 19, normalized size = 1.46

$$\frac{1}{2}\,c\log\left(x^{2}\right)-\frac{b}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/2*b/x^2
```

Fricas [A] time = 0.253534, size = 23, normalized size = 1.77

$$\frac{2\,cx^{2}\log\left(x\right)-b}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(2*c*x^2*log(x) - b)/x^2
```

Sympy [A] time = 1.11603, size = 10, normalized size = 0.77

$$-\frac{b}{2x^{2}}+c\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**5,x)
```

```
[Out] -b/(2*x**2) + c*log(x)
```

GIAC/XCAS [A] time = 0.269925, size = 27, normalized size = 2.08

$$\frac{1}{2}\,c\ln\left(x^2\right) - \frac{cx^2 + b}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*c*ln(x^2) - 1/2*(c*x^2 + b)/x^2
```

3.142 $\int \frac{bx^2+cx^4}{x^6} dx$

Optimal. Leaf size=15

$$-\frac{b}{3x^3} - \frac{c}{x}$$

```
[Out] -b/(3*x^3) - c/x
```

Rubi [A] time = 0.0149077, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{b}{3x^3} - \frac{c}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^6,x]

[Out] -b/(3*x^3) - c/x
```

Rubi in Sympy [A] time = 4.67684, size = 10, normalized size = 0.67

$$-\frac{b}{3x^3} - \frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**6,x)

[Out] -b/(3*x**3) - c/x
```

Mathematica [A] time = 0.00358957, size = 15, normalized size = 1.

$$-\frac{b}{3x^3} - \frac{c}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -b/(3*x^3) - c/x
```

Maple [A] time = 0.007, size = 14, normalized size = 0.9

$$-\frac{b}{3\,x^3}-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^6,x)

[Out] -1/3*b/x^3-c/x
```

Maxima [A] time = 0.682489, size = 18, normalized size = 1.2

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^6,x, algorithm="maxima")

[Out] -1/3*(3*c*x^2 + b)/x^3
```

Fricas [A] time = 0.248548, size = 18, normalized size = 1.2

$$-\frac{3\,cx^2+b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^6,x, algorithm="fricas")

[Out] -1/3*(3*c*x^2 + b)/x^3
```

Sympy [A] time = 1.08279, size = 14, normalized size = 0.93

$$-\frac{b+3cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**6,x)
```

```
[Out] -(b + 3*c*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.267519, size = 18, normalized size = 1.2

$$-\frac{3\,cx^2 + b}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/3*(3*c*x^2 + b)/x^3
```

3.143 $\int \frac{bx^2+cx^4}{x^7} \, dx$

Optimal. Leaf size=17

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -b/(4*x^4) - c/(2*x^2)
```

Rubi [A] time = 0.0149013, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^7,x]

[Out] -b/(4*x^4) - c/(2*x^2)
```

Rubi in Sympy [A] time = 4.70439, size = 14, normalized size = 0.82

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**7,x)

[Out] -b/(4*x**4) - c/(2*x**2)
```

Mathematica [A] time = 0.00352525, size = 17, normalized size = 1.

$$-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -b/(4*x^4) - c/(2*x^2)
```

Maple [A] time = 0.007, size = 14, normalized size = 0.8

$$-\frac{b}{4\,x^4}-\frac{c}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^7,x)
```

```
[Out] -1/4*b/x^4-1/2*c/x^2
```

Maxima [A] time = 0.685818, size = 18, normalized size = 1.06

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Fricas [A] time = 0.245329, size = 18, normalized size = 1.06

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

Sympy [A] time = 1.14135, size = 14, normalized size = 0.82

$$-\frac{b+2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**7,x)
```

```
[Out] -(b + 2*c*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.268598, size = 18, normalized size = 1.06

$$-\frac{2\,cx^2+b}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/x^4
```

3.144 $\int \frac{bx^2+cx^4}{x^8} dx$

Optimal. Leaf size=17

$$-\frac{b}{5x^5} - \frac{c}{3x^3}$$

```
[Out] -b/(5*x^5) - c/(3*x^3)
```

Rubi [A] time = 0.0147413, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$-\frac{b}{5x^5} - \frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^8,x]

[Out] -b/(5*x^5) - c/(3*x^3)
```

Rubi in Sympy [A] time = 4.65891, size = 14, normalized size = 0.82

$$-\frac{b}{5x^5} - \frac{c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**8,x)

[Out] -b/(5*x**5) - c/(3*x**3)
```

Mathematica [A] time = 0.00361293, size = 17, normalized size = 1.

$$-\frac{b}{5x^5} - \frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -b/(5*x^5) - c/(3*x^3)
```

Maple [A] time = 0.006, size = 14, normalized size = 0.8

$$-\frac{b}{5\,x^5}-\frac{c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^8,x)

[Out] -1/5*b/x^5-1/3*c/x^3
```

Maxima [A] time = 0.686661, size = 20, normalized size = 1.18

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^8,x, algorithm="maxima")

[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Fricas [A] time = 0.246537, size = 20, normalized size = 1.18

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^8,x, algorithm="fricas")

[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

Sympy [A] time = 1.14532, size = 15, normalized size = 0.88

$$-\frac{3b+5cx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**8,x)
```

```
[Out] -(3*b + 5*c*x**2)/(15*x**5)
```

GIAC/XCAS [A] time = 0.268346, size = 20, normalized size = 1.18

$$-\frac{5\,cx^2+3\,b}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/15*(5*c*x^2 + 3*b)/x^5
```

3.145 $\int \left(bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi [A] time = 0.0350977, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2,x]

[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Rubi in Sympy [A] time = 10.835, size = 26, normalized size = 0.87

$$\frac{b^2x^5}{5}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2,x)

[Out] b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

Mathematica [A] time = 0.00288497, size = 30, normalized size = 1.

$$\frac{b^2x^5}{5}+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^5)/5 + (2*b*c*x^7)/7 + (c^2*x^9)/9
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{b^2x^5}{5}+\frac{2\,bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*b^2*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.685842, size = 32, normalized size = 1.07

$$\frac{1}{9}\,c^2x^9+\frac{2}{7}\,bcx^7+\frac{1}{5}\,b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

Fricas [A] time = 0.232586, size = 1, normalized size = 0.03

$$\frac{1}{9}x^9c^2+\frac{2}{7}x^7cb+\frac{1}{5}x^5b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2
```

Sympy [A] time = 0.089564, size = 26, normalized size = 0.87

$$\frac{b^2x^5}{5}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2,x)
```

```
[Out] b**2*x**5/5 + 2*b*c*x**7/7 + c**2*x**9/9
```

GIAC/XCAS [A] time = 0.266483, size = 32, normalized size = 1.07

$$\frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5
```

3.146 $\int \frac{(bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=30

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

```
[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rubi [A] time = 0.0571499, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x,x]

[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2 \int^{x^2} x\,dx}{2} + \frac{bcx^6}{3} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x,x)

[Out] b**2*Integral(x, (x, x**2))/2 + b*c*x**6/3 + c**2*x**8/8
```

Mathematica [A] time = 0.00142008, size = 30, normalized size = 1.

$$\frac{b^2x^4}{4} + \frac{1}{3}bcx^6 + \frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x,x]

[Out] (b^2*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{b^2x^4}{4} + \frac{bcx^6}{3} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x,x)

[Out] 1/4*b^2*x^4+1/3*b*c*x^6+1/8*c^2*x^8
```

Maxima [A] time = 0.676879, size = 32, normalized size = 1.07

$$\frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x,x, algorithm="maxima")

[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Fricas [A] time = 0.249335, size = 32, normalized size = 1.07

$$\frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x,x, algorithm="fricas")

[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

Sympy [A] time = 0.095225, size = 24, normalized size = 0.8

$$\frac{b^2x^4}{4} + \frac{bcx^6}{3} + \frac{c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x,x)
```

```
[Out] b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8
```

GIAC/XCAS [A] time = 0.269007, size = 32, normalized size = 1.07

$$\frac{1}{8}\,c^2x^8 + \frac{1}{3}\,bcx^6 + \frac{1}{4}\,b^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4
```

3.147 $\int \frac{(bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=30

$$\frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

```
[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rubi [A] time = 0.0402167, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^2,x]

[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Rubi in Sympy [A] time = 8.11645, size = 26, normalized size = 0.87

$$\frac{b^2x^3}{3}+\frac{2bcx^5}{5}+\frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**2,x)

[Out] b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

Mathematica [A] time = 0.00125369, size = 30, normalized size = 1.

$$\frac{b^2x^3}{3}+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^2,x]

[Out] (b^2*x^3)/3 + (2*b*c*x^5)/5 + (c^2*x^7)/7
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{b^2x^3}{3} + \frac{2\,bcx^5}{5} + \frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^2,x)

[Out] 1/3*b^2*x^3+2/5*b*c*x^5+1/7*c^2*x^7
```

Maxima [A] time = 0.684196, size = 32, normalized size = 1.07

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^2,x, algorithm="maxima")

[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Fricas [A] time = 0.246899, size = 32, normalized size = 1.07

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^2,x, algorithm="fricas")

[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

Sympy [A] time = 0.096337, size = 26, normalized size = 0.87

$$\frac{b^2x^3}{3} + \frac{2bcx^5}{5} + \frac{c^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**2,x)
```

```
[Out] b**2*x**3/3 + 2*b*c*x**5/5 + c**2*x**7/7
```

GIAC/XCAS [A] time = 0.266901, size = 32, normalized size = 1.07

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3
```

3.148 $\int \frac{(bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^3}{6c}$$

```
[Out] (b + c*x^2)^3/(6*c)
```

Rubi [A] time = 0.0178013, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^3,x]

[Out] (b + c*x^2)^3/(6*c)
```

Rubi in Sympy [A] time = 4.06761, size = 10, normalized size = 0.62

$$\frac{(b+cx^2)^3}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**3,x)

[Out] (b + c*x**2)**3/(6*c)
```

Mathematica [A] time = 0.00375564, size = 16, normalized size = 1.

$$\frac{(b+cx^2)^3}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^3,x]

[Out] (b + c*x^2)^3/(6*c)
```

Maple [A] time = 0.001, size = 25, normalized size = 1.6

$$\frac{c^2x^6}{6} + \frac{bcx^4}{2} + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^3,x)

[Out] 1/6*c^2*x^6+1/2*b*c*x^4+1/2*b^2*x^2
```

Maxima [A] time = 0.703019, size = 32, normalized size = 2.

$$\frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^3,x, algorithm="maxima")

[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Fricas [A] time = 0.245141, size = 32, normalized size = 2.

$$\frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^3,x, algorithm="fricas")

[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

Sympy [A] time = 0.09585, size = 24, normalized size = 1.5

$$\frac{b^2x^2}{2} + \frac{bcx^4}{2} + \frac{c^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**3,x)
```

```
[Out] b**2*x**2/2 + b*c*x**4/2 + c**2*x**6/6
```

GIAC/XCAS [A] time = 0.267556, size = 32, normalized size = 2.

$$\frac{1}{6}\,c^2x^6 + \frac{1}{2}\,bcx^4 + \frac{1}{2}\,b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2
```

3.149 $\int \frac{(bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=25

$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

```
[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] time = 0.0260684, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^4,x]

[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2bcx^3}{3}+\frac{c^2x^5}{5}+\int b^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**4,x)

[Out] 2*b*c*x**3/3 + c**2*x**5/5 + Integral(b**2, x)
```

Mathematica [A] time = 0.00166583, size = 25, normalized size = 1.

$$b^2x+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^4,x]

[Out] b^2*x + (2*b*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A] time = 0.001, size = 22, normalized size = 0.9

$$b^2x + \frac{2\,bcx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^4,x)

[Out] b^2*x+2/3*b*c*x^3+1/5*c^2*x^5
```

Maxima [A] time = 0.685341, size = 28, normalized size = 1.12

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^4,x, algorithm="maxima")

[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Fricas [A] time = 0.245947, size = 28, normalized size = 1.12

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^4,x, algorithm="fricas")

[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

Sympy [A] time = 0.098394, size = 22, normalized size = 0.88

$$b^2x + \frac{2bcx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**4,x)
```

```
[Out] b**2*x + 2*b*c*x**3/3 + c**2*x**5/5
```

GIAC/XCAS [A] time = 0.26784, size = 28, normalized size = 1.12

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x
```

3.150 $\int \frac{(bx^2+cx^4)^2}{x^5}\,dx$

Optimal. Leaf size=23

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rubi [A] time = 0.0430028, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^5,x]

[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2\log\left(x^2\right)}{2}+bcx^2+\frac{c^2\int^{x^2}x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**5,x)

[Out] b**2*log(x**2)/2 + b*c*x**2 + c**2*Integral(x, (x, x**2))/2
```

Mathematica [A] time = 0.0015484, size = 23, normalized size = 1.

$$b^2\log(x)+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] b*c*x^2 + (c^2*x^4)/4 + b^2*Log[x]
```

Maple [A] time = 0.003, size = 22, normalized size = 1.

$$bcx^2 + \frac{c^2x^4}{4} + b^2 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^5,x)
```

```
[Out] b*c*x^2+1/4*c^2*x^4+b^2*ln(x)
```

Maxima [A] time = 0.685945, size = 32, normalized size = 1.39

$$\frac{1}{4}c^2x^4 + bcx^2 + \frac{1}{2}b^2 \log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*log(x^2)
```

Fricas [A] time = 0.256377, size = 28, normalized size = 1.22

$$\frac{1}{4}c^2x^4 + bcx^2 + b^2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + b^2*log(x)
```

Sympy [A] time = 1.0301, size = 20, normalized size = 0.87

$$b^2 \log(x) + bcx^2 + \frac{c^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**5,x)
```

```
[Out] b**2*log(x) + b*c*x**2 + c**2*x**4/4
```

GIAC/XCAS [A] time = 0.270257, size = 32, normalized size = 1.39

$$\frac{1}{4}\, c^2x^4 + bcx^2 + \frac{1}{2}\, b^2 \ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*b^2*ln(x^2)
```

3.151 $\int \frac{(bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

```
[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rubi [A] time = 0.035577, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^6,x]

[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Rubi in Sympy [A] time = 7.75805, size = 19, normalized size = 0.79

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**6,x)

[Out] -b**2/x + 2*b*c*x + c**2*x**3/3
```

Mathematica [A] time = 0.00136793, size = 24, normalized size = 1.

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^6,x]

[Out] -(b^2/x) + 2*b*c*x + (c^2*x^3)/3
```

Maple [A] time = 0.005, size = 23, normalized size = 1.

$$-\frac{b^2}{x} + 2\,bcx + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^6,x)

[Out] -b^2/x+2*b*c*x+1/3*c^2*x^3
```

Maxima [A] time = 0.6914, size = 30, normalized size = 1.25

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^6,x, algorithm="maxima")

[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

Fricas [A] time = 0.248451, size = 34, normalized size = 1.42

$$\frac{c^2x^4 + 6\,bcx^2 - 3\,b^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^6,x, algorithm="fricas")

[Out] 1/3*(c^2*x^4 + 6*b*c*x^2 - 3*b^2)/x
```

Sympy [A] time = 1.03231, size = 19, normalized size = 0.79

$$-\frac{b^2}{x} + 2bcx + \frac{c^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**6,x)
```

```
[Out] -b**2/x + 2*b*c*x + c**2*x**3/3
```

GIAC/XCAS [A] time = 0.270072, size = 30, normalized size = 1.25

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - b^2/x
```

3.152 $\int \frac{(bx^2+cx^4)^2}{x^7} dx$

Optimal. Leaf size=27

$$-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}$$

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rubi [A] time = 0.0463815, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^7,x]

[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2}{2x^2} + bc\log\left(x^2\right) + \frac{\int^{x^2} c^2\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**7,x)

[Out] -b**2/(2*x**2) + b*c*log(x**2) + Integral(c**2, (x, x**2))/2
```

Mathematica [A] time = 0.00160311, size = 27, normalized size = 1.

$$-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -b^2/(2*x^2) + (c^2*x^2)/2 + 2*b*c*Log[x]
```

Maple [A] time = 0.009, size = 24, normalized size = 0.9

$$-\frac{b^2}{2\,x^2}+\frac{c^2x^2}{2}+2\,bc\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^7,x)
```

```
[Out] -1/2*b^2/x^2+1/2*c^2*x^2+2*b*c*ln(x)
```

Maxima [A] time = 0.685174, size = 32, normalized size = 1.19

$$\frac{1}{2}\,c^2x^2+bc\log\left(x^2\right)-\frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/2*b^2/x^2
```

Fricas [A] time = 0.253075, size = 36, normalized size = 1.33

$$\frac{c^2x^4+4\,bcx^2\log\left(x\right)-b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + 4*b*c*x^2*log(x) - b^2)/x^2
```

Sympy [A] time = 1.13703, size = 24, normalized size = 0.89

$$-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**7,x)
```

```
[Out] -b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2
```

GIAC/XCAS [A] time = 0.269794, size = 43, normalized size = 1.59

$$\frac{1}{2}c^2x^2 + bc\ln\left(x^2\right) - \frac{2\,bcx^2 + b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*ln(x^2) - 1/2*(2*b*c*x^2 + b^2)/x^2
```

3.153 $\int \frac{(bx^2+cx^4)^2}{x^8}\,dx$

Optimal. Leaf size=23

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Rubi [A] time = 0.0341991, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+\int c^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] -b**2/(3*x**3) - 2*b*c/x + Integral(c**2, x)
```

Mathematica [A] time = 0.00136025, size = 23, normalized size = 1.

$$-\frac{b^2}{3x^3}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -b^2/(3*x^3) - (2*b*c)/x + c^2*x
```

Maple [A] time = 0.008, size = 22, normalized size = 1.

$$-\frac{b^2}{3\,x^3}-2\,\frac{bc}{x}+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^8,x)
```

```
[Out] -1/3*b^2/x^3-2*b*c/x+c^2*x
```

Maxima [A] time = 0.687831, size = 30, normalized size = 1.3

$$c^2x-\frac{6\,bcx^2+b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

Fricas [A] time = 0.248095, size = 35, normalized size = 1.52

$$\frac{3\,c^2x^4-6\,bcx^2-b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/3*(3*c^2*x^4 - 6*b*c*x^2 - b^2)/x^3
```

Sympy [A] time = 1.18282, size = 20, normalized size = 0.87

$$c^2x - \frac{b^2 + 6bcx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**8,x)
```

```
[Out] c**2*x - (b**2 + 6*b*c*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.268128, size = 30, normalized size = 1.3

$$c^2x - \frac{6\,bcx^2 + b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/3*(6*b*c*x^2 + b^2)/x^3
```

3.154 $\int \frac{(bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=24

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

```
[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Rubi [A] time = 0.0414948, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^9,x]

[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Rubi in Sympy [A] time = 9.35779, size = 24, normalized size = 1.

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + \frac{c^2 \log(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**9,x)

[Out] -b**2/(4*x**4) - b*c/x**2 + c**2*log(x**2)/2
```

Mathematica [A] time = 0.00144792, size = 24, normalized size = 1.

$$-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -b^2/(4*x^4) - (b*c)/x^2 + c^2*Log[x]
```

Maple [A] time = 0.008, size = 23, normalized size = 1.

$$-\frac{b^2}{4\,x^4}-\frac{bc}{x^2}+c^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^9,x)
```

```
[Out] -1/4*b^2/x^4-b*c/x^2+c^2*ln(x)
```

Maxima [A] time = 0.67625, size = 35, normalized size = 1.46

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{4\,bcx^2+b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/4*(4*b*c*x^2 + b^2)/x^4
```

Fricas [A] time = 0.254783, size = 38, normalized size = 1.58

$$\frac{4\,c^2x^4\log\left(x\right)-4\,bcx^2-b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(4*c^2*x^4*log(x) - 4*b*c*x^2 - b^2)/x^4
```

Sympy [A] time = 1.29588, size = 22, normalized size = 0.92

$$c^2 \log(x) - \frac{b^2 + 4bcx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**9,x)
```

```
[Out] c**2*log(x) - (b**2 + 4*b*c*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.270006, size = 46, normalized size = 1.92

$$\frac{1}{2}\, c^2 \ln\left(x^2\right) - \frac{3\, c^2 x^4 + 4\, bcx^2 + b^2}{4\, x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*ln(x^2) - 1/4*(3*c^2*x^4 + 4*b*c*x^2 + b^2)/x^4
```

3.155 $\int \frac{(bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=28

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rubi [A] time = 0.0356362, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^2/x^10,x]

[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Rubi in Sympy [A] time = 7.72666, size = 24, normalized size = 0.86

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2)**2/x**10,x)

[Out] -b**2/(5*x**5) - 2*b*c/(3*x**3) - c**2/x
```

Mathematica [A] time = 0.00142648, size = 28, normalized size = 1.

$$-\frac{b^2}{5x^5}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^10,x]

[Out] -b^2/(5*x^5) - (2*b*c)/(3*x^3) - c^2/x
```

Maple [A] time = 0.007, size = 25, normalized size = 0.9

$$-\frac{b^2}{5\,x^5}-\frac{2\,bc}{3\,x^3}-\frac{c^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^10,x)

[Out] -1/5*b^2/x^5-2/3*b*c/x^3-c^2/x
```

Maxima [A] time = 0.688104, size = 35, normalized size = 1.25

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^10,x, algorithm="maxima")

[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Fricas [A] time = 0.246754, size = 35, normalized size = 1.25

$$-\frac{15\,c^2x^4+10\,bcx^2+3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^10,x, algorithm="fricas")

[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

Sympy [A] time = 1.33471, size = 27, normalized size = 0.96

$$-\frac{3b^2 + 10bcx^2 + 15c^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**10,x)
```

```
[Out] -(3*b**2 + 10*b*c*x**2 + 15*c**2*x**4)/(15*x**5)
```

GIAC/XCAS [A] time = 0.267281, size = 35, normalized size = 1.25

$$-\frac{15\,c^2x^4 + 10\,bcx^2 + 3\,b^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/15*(15*c^2*x^4 + 10*b*c*x^2 + 3*b^2)/x^5
```

3.156 $\int \frac{(bx^2+cx^4)^2}{x^{11}} \, dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^3}{6bx^6}$$

```
[Out] -(b + c*x^2)^3/(6*b*x^6)
```

Rubi [A] time = 0.0228202, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -(b + c*x^2)^3/(6*b*x^6)
```

Rubi in Sympy [A] time = 5.07302, size = 15, normalized size = 0.79

$$-\frac{(b+cx^2)^3}{6bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] -(b + c*x**2)**3/(6*b*x**6)
```

Mathematica [A] time = 0.00161207, size = 30, normalized size = 1.58

$$-\frac{b^2}{6x^6}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -b^2/(6*x^6) - (b*c)/(2*x^4) - c^2/(2*x^2)
```

Maple [A] time = 0.007, size = 25, normalized size = 1.3

$$-\frac{b^2}{6\,x^6}-\frac{c^2}{2\,x^2}-\frac{bc}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^11,x)
```

```
[Out] -1/6*b^2/x^6-1/2*c^2/x^2-1/2*b*c/x^4
```

Maxima [A] time = 0.692191, size = 32, normalized size = 1.68

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Fricas [A] time = 0.248345, size = 32, normalized size = 1.68

$$-\frac{3\,c^2x^4+3\,bcx^2+b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

Sympy [A] time = 1.35014, size = 26, normalized size = 1.37

$$-\frac{b^2 + 3bcx^2 + 3c^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**11,x)
```

```
[Out] -(b**2 + 3*b*c*x**2 + 3*c**2*x**4)/(6*x**6)
```

GIAC/XCAS [A] time = 0.267, size = 32, normalized size = 1.68

$$-\frac{3\,c^2x^4 + 3\,bcx^2 + b^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/6*(3*c^2*x^4 + 3*b*c*x^2 + b^2)/x^6
```

3.157 $\int \frac{(bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=30

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rubi [A] time = 0.0360403, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^12,x]

[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Rubi in Sympy [A] time = 7.70317, size = 27, normalized size = 0.9

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**12,x)

[Out] -b**2/(7*x**7) - 2*b*c/(5*x**5) - c**2/(3*x**3)
```

Mathematica [A] time = 0.0015932, size = 30, normalized size = 1.

$$-\frac{b^2}{7x^7}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^12,x]

[Out] -b^2/(7*x^7) - (2*b*c)/(5*x^5) - c^2/(3*x^3)
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$-\frac{b^2}{7\,x^7}-\frac{2\,bc}{5\,x^5}-\frac{c^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^12,x)

[Out] -1/7*b^2/x^7-2/5*b*c/x^5-1/3*c^2/x^3
```

Maxima [A] time = 0.682683, size = 35, normalized size = 1.17

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^12,x, algorithm="maxima")

[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Fricas [A] time = 0.251217, size = 35, normalized size = 1.17

$$-\frac{35\,c^2x^4+42\,bcx^2+15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^12,x, algorithm="fricas")

[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

Sympy [A] time = 1.36472, size = 27, normalized size = 0.9

$$-\frac{15b^2 + 42bcx^2 + 35c^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**12,x)
```

```
[Out] -(15*b**2 + 42*b*c*x**2 + 35*c**2*x**4)/(105*x**7)
```

GIAC/XCAS [A] time = 0.268928, size = 35, normalized size = 1.17

$$-\frac{35\,c^2x^4 + 42\,bcx^2 + 15\,b^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/105*(35*c^2*x^4 + 42*b*c*x^2 + 15*b^2)/x^7
```

3.158 $\int \frac{(bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi [A] time = 0.0534906, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^2,x]

[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi in Sympy [A] time = 9.8099, size = 37, normalized size = 0.86

$$\frac{b^3x^5}{5}+\frac{3b^2cx^7}{7}+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**2,x)

[Out] b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

Mathematica [A] time = 0.00326927, size = 43, normalized size = 1.

$$\frac{b^3x^5}{5}+\frac{3}{7}b^2cx^7+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^2,x]

[Out] (b^3*x^5)/5 + (3*b^2*c*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{b^3x^5}{5}+\frac{3\,b^2cx^7}{7}+\frac{bc^2x^9}{3}+\frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^2,x)

[Out] 1/5*b^3*x^5+3/7*b^2*c*x^7+1/3*b*c^2*x^9+1/11*c^3*x^11
```

Maxima [A] time = 0.678027, size = 47, normalized size = 1.09

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^2,x, algorithm="maxima")

[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Fricas [A] time = 0.246307, size = 47, normalized size = 1.09

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\,b^2cx^7+\frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^2,x, algorithm="fricas")

[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

Sympy [A] time = 0.111335, size = 37, normalized size = 0.86

$$\frac{b^3x^5}{5} + \frac{3b^2cx^7}{7} + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**2,x)
```

```
[Out] b**3*x**5/5 + 3*b**2*c*x**7/7 + b*c**2*x**9/3 + c**3*x**11/11
```

GIAC/XCAS [A] time = 0.267969, size = 47, normalized size = 1.09

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{1}{5}\,b^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 1/5*b^3*x^5
```

3.159 $\int \frac{(bx^2+cx^4)^3}{x^3} dx$

Optimal. Leaf size=34

$$\frac{(b+cx^2)^5}{10c^2} - \frac{b(b+cx^2)^4}{8c^2}$$

```
[Out] -(b*(b + c*x^2)^4)/(8*c^2) + (b + c*x^2)^5/(10*c^2)
```

Rubi [A] time = 0.0897248, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{(b+cx^2)^5}{10c^2} - \frac{b(b+cx^2)^4}{8c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^3,x]

[Out] -(b*(b + c*x^2)^4)/(8*c^2) + (b + c*x^2)^5/(10*c^2)
```

Rubi in Sympy [A] time = 11.6727, size = 27, normalized size = 0.79

$$-\frac{b(b+cx^2)^4}{8c^2} + \frac{(b+cx^2)^5}{10c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**3,x)

[Out] -b*(b + c*x**2)**4/(8*c**2) + (b + c*x**2)**5/(10*c**2)
```

Mathematica [A] time = 0.00323695, size = 43, normalized size = 1.26

$$\frac{b^3x^4}{4} + \frac{1}{2}b^2cx^6 + \frac{3}{8}bc^2x^8 + \frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] (b^3*x^4)/4 + (b^2*c*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10
```

Maple [A] time = 0.001, size = 36, normalized size = 1.1

$$\frac{c^3x^{10}}{10}+\frac{3\,bc^2x^8}{8}+\frac{b^2cx^6}{2}+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*b^2*c*x^6+1/4*b^3*x^4
```

Maxima [A] time = 0.681888, size = 47, normalized size = 1.38

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Fricas [A] time = 0.244644, size = 47, normalized size = 1.38

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\,b^2cx^6+\frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

Sympy [A] time = 0.113337, size = 37, normalized size = 1.09

$$\frac{b^3x^4}{4} + \frac{b^2cx^6}{2} + \frac{3bc^2x^8}{8} + \frac{c^3x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**3,x)
```

```
[Out] b**3*x**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10
```

GIAC/XCAS [A] time = 0.267796, size = 47, normalized size = 1.38

$$\frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,b^2cx^6 + \frac{1}{4}\,b^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/4*b^3*x^4
```

3.160 $\int \frac{(bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=43

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi [A] time = 0.0510402, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^4,x]

[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi in Sympy [A] time = 10.0215, size = 39, normalized size = 0.91

$$\frac{b^3x^3}{3}+\frac{3b^2cx^5}{5}+\frac{3bc^2x^7}{7}+\frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**4,x)

[Out] b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

Mathematica [A] time = 0.00358765, size = 43, normalized size = 1.

$$\frac{b^3x^3}{3}+\frac{3}{5}b^2cx^5+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] (b^3*x^3)/3 + (3*b^2*c*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$\frac{b^3x^3}{3}+\frac{3\,b^2cx^5}{5}+\frac{3\,bc^2x^7}{7}+\frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^4,x)
```

```
[Out] 1/3*b^3*x^3+3/5*b^2*c*x^5+3/7*b*c^2*x^7+1/9*c^3*x^9
```

Maxima [A] time = 0.684866, size = 47, normalized size = 1.09

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Fricas [A] time = 0.246852, size = 47, normalized size = 1.09

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\,b^2cx^5+\frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

Sympy [A] time = 0.111177, size = 39, normalized size = 0.91

$$\frac{b^3x^3}{3} + \frac{3b^2cx^5}{5} + \frac{3bc^2x^7}{7} + \frac{c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**4,x)
```

```
[Out] b**3*x**3/3 + 3*b**2*c*x**5/5 + 3*b*c**2*x**7/7 + c**3*x**9/9
```

GIAC/XCAS [A] time = 0.268141, size = 47, normalized size = 1.09

$$\frac{1}{9}\,c^3x^9 + \frac{3}{7}\,bc^2x^7 + \frac{3}{5}\,b^2cx^5 + \frac{1}{3}\,b^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 1/3*b^3*x^3
```

3.161 $\int \frac{(bx^2+cx^4)^3}{x^5} \, dx$

Optimal. Leaf size=16

$$\frac{(b+cx^2)^4}{8c}$$

```
[Out] (b + c*x^2)^4/(8*c)
```

Rubi [A] time = 0.0182227, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^5,x]

[Out] (b + c*x^2)^4/(8*c)
```

Rubi in Sympy [A] time = 4.06611, size = 10, normalized size = 0.62

$$\frac{(b+cx^2)^4}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**5,x)

[Out] (b + c*x**2)**4/(8*c)
```

Mathematica [A] time = 0.00378252, size = 16, normalized size = 1.

$$\frac{(b+cx^2)^4}{8c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^5,x]
```

```
[Out] (b + c*x^2)^4/(8*c)
```

Maple [B] time = 0.002, size = 36, normalized size = 2.3

$$\frac{x^8c^3}{8}+\frac{bc^2x^6}{2}+\frac{3\,b^2cx^4}{4}+\frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^5,x)
```

```
[Out] 1/8*x^8*c^3+1/2*b*c^2*x^6+3/4*b^2*c*x^4+1/2*b^3*x^2
```

Maxima [A] time = 0.675355, size = 47, normalized size = 2.94

$$\frac{1}{8}\,c^3x^8+\frac{1}{2}\,bc^2x^6+\frac{3}{4}\,b^2cx^4+\frac{1}{2}\,b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Fricas [A] time = 0.250904, size = 47, normalized size = 2.94

$$\frac{1}{8}\,c^3x^8+\frac{1}{2}\,bc^2x^6+\frac{3}{4}\,b^2cx^4+\frac{1}{2}\,b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

Sympy [A] time = 0.104572, size = 37, normalized size = 2.31

$$\frac{b^3x^2}{2} + \frac{3b^2cx^4}{4} + \frac{bc^2x^6}{2} + \frac{c^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**5,x)

[Out] b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8
```

GIAC/XCAS [A] time = 0.26868, size = 47, normalized size = 2.94

$$\frac{1}{8}\,c^3x^8 + \frac{1}{2}\,bc^2x^6 + \frac{3}{4}\,b^2cx^4 + \frac{1}{2}\,b^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^5,x, algorithm="giac")

[Out] 1/8*c^3*x^8 + 1/2*b*c^2*x^6 + 3/4*b^2*c*x^4 + 1/2*b^3*x^2
```

3.162 $\int \frac{(bx^2+cx^4)^3}{x^6}\,dx$

Optimal. Leaf size=35

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

```
[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi [A] time = 0.0360736, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^6,x]

[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$b^2cx^3 + \frac{3bc^2x^5}{5} + \frac{c^3x^7}{7} + \int b^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**6,x)

[Out] b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7 + Integral(b**3, x)
```

Mathematica [A] time = 0.0019519, size = 35, normalized size = 1.

$$b^3x + b^2cx^3 + \frac{3}{5}bc^2x^5 + \frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^6,x]

[Out] b^3*x + b^2*c*x^3 + (3*b*c^2*x^5)/5 + (c^3*x^7)/7
```

Maple [A] time = 0.001, size = 32, normalized size = 0.9

$$b^3x + b^2cx^3 + \frac{3\,bc^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^6,x)

[Out] b^3*x+b^2*c*x^3+3/5*b*c^2*x^5+1/7*c^3*x^7
```

Maxima [A] time = 0.681346, size = 42, normalized size = 1.2

$$\frac{1}{7}\,c^3x^7 + \frac{3}{5}\,bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^6,x, algorithm="maxima")

[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Fricas [A] time = 0.246832, size = 42, normalized size = 1.2

$$\frac{1}{7}\,c^3x^7 + \frac{3}{5}\,bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^6,x, algorithm="fricas")

[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

Sympy [A] time = 0.102678, size = 32, normalized size = 0.91

$$b^3x + b^2cx^3 + \frac{3bc^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**6,x)
```

```
[Out] b**3*x + b**2*c*x**3 + 3*b*c**2*x**5/5 + c**3*x**7/7
```

GIAC/XCAS [A] time = 0.268406, size = 42, normalized size = 1.2

$$\frac{1}{7}c^3x^7 + \frac{3}{5}bc^2x^5 + b^2cx^3 + b^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*b*c^2*x^5 + b^2*c*x^3 + b^3*x
```

3.163 $\int \frac{(bx^2+cx^4)^3}{x^7} \, dx$

Optimal. Leaf size=39

$$b^3 \log(x) + \frac{3}{2} b^2 c x^2 + \frac{3}{4} b c^2 x^4 + \frac{c^3 x^6}{6}$$

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rubi [A] time = 0.0561289, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$b^3 \log(x) + \frac{3}{2} b^2 c x^2 + \frac{3}{4} b c^2 x^4 + \frac{c^3 x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^3 \log\left(x^2\right)}{2} + \frac{3b^2 c x^2}{2} + \frac{3bc^2 \int^{x^2} x \, dx}{2} + \frac{c^3 x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] b**3*log(x**2)/2 + 3*b**2*c*x**2/2 + 3*b*c**2*Integral(x, (x, x**
2))/2 + c**3*x**6/6
```

Mathematica [A] time = 0.00723354, size = 39, normalized size = 1.

$$b^3 \log(x) + \frac{3}{2} b^2 c x^2 + \frac{3}{4} b c^2 x^4 + \frac{c^3 x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^7,x]
```

```
[Out] (3*b^2*c*x^2)/2 + (3*b*c^2*x^4)/4 + (c^3*x^6)/6 + b^3*Log[x]
```

Maple [A] time = 0.003, size = 34, normalized size = 0.9

$$\frac{3\,b^2cx^2}{2}+\frac{3\,bc^2x^4}{4}+\frac{c^3x^6}{6}+b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^7,x)
```

```
[Out] 3/2*b^2*c*x^2+3/4*b*c^2*x^4+1/6*c^3*x^6+b^3*ln(x)
```

Maxima [A] time = 0.692434, size = 49, normalized size = 1.26

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+\frac{1}{2}\,b^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*log(x^2)
```

Fricas [A] time = 0.254333, size = 45, normalized size = 1.15

$$\frac{1}{6}\,c^3x^6+\frac{3}{4}\,bc^2x^4+\frac{3}{2}\,b^2cx^2+b^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + b^3*log(x)
```

Sympy [A] time = 1.0877, size = 37, normalized size = 0.95

$$b^3 \log(x) + \frac{3b^2cx^2}{2} + \frac{3bc^2x^4}{4} + \frac{c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**7,x)
```

```
[Out] b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*x**6/6
```

GIAC/XCAS [A] time = 0.269656, size = 49, normalized size = 1.26

$$\frac{1}{6}c^3x^6 + \frac{3}{4}bc^2x^4 + \frac{3}{2}b^2cx^2 + \frac{1}{2}b^3\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*c^3*x^6 + 3/4*b*c^2*x^4 + 3/2*b^2*c*x^2 + 1/2*b^3*ln(x^2)
```

3.164 $\int \frac{(bx^2+cx^4)^3}{x^8}\,dx$

Optimal. Leaf size=34

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

```
[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rubi [A] time = 0.0452978, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^8,x]

[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Rubi in Sympy [A] time = 9.32005, size = 29, normalized size = 0.85

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**8,x)

[Out] -b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

Mathematica [A] time = 0.00635934, size = 34, normalized size = 1.

$$-\frac{b^3}{x}+3b^2cx+bc^2x^3+\frac{c^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^8,x]

[Out] -(b^3/x) + 3*b^2*c*x + b*c^2*x^3 + (c^3*x^5)/5
```

Maple [A] time = 0.005, size = 33, normalized size = 1.

$$-\frac{b^3}{x} + 3\,b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^8,x)

[Out] -b^3/x+3*b^2*c*x+b*c^2*x^3+1/5*c^3*x^5
```

Maxima [A] time = 0.687566, size = 43, normalized size = 1.26

$$\frac{1}{5}\,c^3x^5 + bc^2x^3 + 3\,b^2cx - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^8,x, algorithm="maxima")

[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

Fricas [A] time = 0.246761, size = 49, normalized size = 1.44

$$\frac{c^3x^6 + 5\,bc^2x^4 + 15\,b^2cx^2 - 5\,b^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^8,x, algorithm="fricas")

[Out] 1/5*(c^3*x^6 + 5*b*c^2*x^4 + 15*b^2*c*x^2 - 5*b^3)/x
```

Sympy [A] time = 1.074, size = 29, normalized size = 0.85

$$-\frac{b^3}{x} + 3b^2cx + bc^2x^3 + \frac{c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**8,x)
```

```
[Out] -b**3/x + 3*b**2*c*x + b*c**2*x**3 + c**3*x**5/5
```

GIAC/XCAS [A] time = 0.26797, size = 43, normalized size = 1.26

$$\frac{1}{5}\,c^3x^5 + bc^2x^3 + 3\,b^2cx - \frac{b^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*c^3*x^5 + b*c^2*x^3 + 3*b^2*c*x - b^3/x
```

3.165 $\int \frac{(bx^2+cx^4)^3}{x^9}\,dx$

Optimal. Leaf size=40

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rubi [A] time = 0.0650173, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^9,x]

[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3}{2x^2}+\frac{3b^2c\log\left(x^2\right)}{2}+\frac{3bc^2x^2}{2}+\frac{c^3\int^{x^2}x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**9,x)

[Out] -b**3/(2*x**2) + 3*b**2*c*log(x**2)/2 + 3*b*c**2*x**2/2 + c**3*In
tegral(x, (x, x**2))/2
```

Mathematica [A] time = 0.01137, size = 40, normalized size = 1.

$$-\frac{b^3}{2x^2}+3b^2c\log(x)+\frac{3}{2}bc^2x^2+\frac{c^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^9,x]
```

```
[Out] -b^3/(2*x^2) + (3*b*c^2*x^2)/2 + (c^3*x^4)/4 + 3*b^2*c*Log[x]
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$-\frac{b^3}{2\,x^2}+\frac{3\,bc^2x^2}{2}+\frac{c^3x^4}{4}+3\,b^2c\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^9,x)
```

```
[Out] -1/2*b^3/x^2+3/2*b*c^2*x^2+1/4*c^3*x^4+3*b^2*c*ln(x)
```

Maxima [A] time = 0.687417, size = 49, normalized size = 1.22

$$\frac{1}{4}\,c^3x^4+\frac{3}{2}\,bc^2x^2+\frac{3}{2}\,b^2c\log\left(x^2\right)-\frac{b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*log(x^2) - 1/2*b^3/x^2
```

Fricas [A] time = 0.253846, size = 51, normalized size = 1.27

$$\frac{c^3x^6+6\,bc^2x^4+12\,b^2cx^2\log\left(x\right)-2\,b^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 + 6*b*c^2*x^4 + 12*b^2*c*x^2*log(x) - 2*b^3)/x^2
```

Sympy [A] time = 1.18373, size = 37, normalized size = 0.92

$$-\frac{b^3}{2x^2} + 3b^2c\log(x) + \frac{3bc^2x^2}{2} + \frac{c^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**9,x)
```

```
[Out] -b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4
```

GIAC/XCAS [A] time = 0.269652, size = 62, normalized size = 1.55

$$\frac{1}{4}c^3x^4 + \frac{3}{2}bc^2x^2 + \frac{3}{2}b^2c\ln\left(x^2\right) - \frac{3\,b^2cx^2 + b^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*c^3*x^4 + 3/2*b*c^2*x^2 + 3/2*b^2*c*ln(x^2) - 1/2*(3*b^2*c*x^
2 + b^3)/x^2
```

3.166 $\int \frac{(bx^2+cx^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

```
[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rubi [A] time = 0.0446987, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^10,x]

[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Rubi in Sympy [A] time = 9.54112, size = 32, normalized size = 0.86

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**10,x)

[Out] -b**3/(3*x**3) - 3*b**2*c/x + 3*b*c**2*x + c**3*x**3/3
```

Mathematica [A] time = 0.00629311, size = 37, normalized size = 1.

$$-\frac{b^3}{3x^3}-\frac{3b^2c}{x}+3bc^2x+\frac{c^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^10,x]

[Out] -b^3/(3*x^3) - (3*b^2*c)/x + 3*b*c^2*x + (c^3*x^3)/3
```

Maple [A] time = 0.008, size = 34, normalized size = 0.9

$$-\frac{b^3}{3\,x^3} - 3\,\frac{b^2c}{x} + 3\,bc^2x + \frac{c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^10,x)

[Out] -1/3*b^3/x^3-3*b^2*c/x+3*b*c^2*x+1/3*c^3*x^3
```

Maxima [A] time = 0.678911, size = 46, normalized size = 1.24

$$\frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^10,x, algorithm="maxima")

[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

Fricas [A] time = 0.246847, size = 49, normalized size = 1.32

$$\frac{c^3x^6 + 9\,bc^2x^4 - 9\,b^2cx^2 - b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^10,x, algorithm="fricas")

[Out] 1/3*(c^3*x^6 + 9*b*c^2*x^4 - 9*b^2*c*x^2 - b^3)/x^3
```

Sympy [A] time = 1.20223, size = 34, normalized size = 0.92

$$3bc^2x + \frac{c^3x^3}{3} - \frac{b^3 + 9b^2cx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**10,x)
```

```
[Out] 3*b*c**2*x + c**3*x**3/3 - (b**3 + 9*b**2*c*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.269295, size = 46, normalized size = 1.24

$$\frac{1}{3}\,c^3x^3 + 3\,bc^2x - \frac{9\,b^2cx^2 + b^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*c^3*x^3 + 3*b*c^2*x - 1/3*(9*b^2*c*x^2 + b^3)/x^3
```

3.167 $\int \frac{(bx^2+cx^4)^3}{x^{11}}\,dx$

Optimal. Leaf size=40

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rubi [A] time = 0.060033, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^11,x]

[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+\frac{3bc^2\log\left(x^2\right)}{2}+\frac{\int^{x^2} c^3\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**11,x)

[Out] -b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*log(x**2)/2 + Integ
ral(c**3, (x, x**2))/2
```

Mathematica [A] time = 0.00723674, size = 40, normalized size = 1.

$$-\frac{b^3}{4x^4}-\frac{3b^2c}{2x^2}+3bc^2\log(x)+\frac{c^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^11,x]
```

```
[Out] -b^3/(4*x^4) - (3*b^2*c)/(2*x^2) + (c^3*x^2)/2 + 3*b*c^2*Log[x]
```

Maple [A] time = 0.009, size = 35, normalized size = 0.9

$$-\frac{b^3}{4\,x^4}-\frac{3\,b^2c}{2\,x^2}+\frac{c^3x^2}{2}+3\,bc^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^11,x)
```

```
[Out] -1/4*b^3/x^4-3/2*b^2*c/x^2+1/2*c^3*x^2+3*b*c^2*ln(x)
```

Maxima [A] time = 0.687281, size = 50, normalized size = 1.25

$$\frac{1}{2}\,c^3x^2+\frac{3}{2}\,bc^2\log\left(x^2\right)-\frac{6\,b^2cx^2+b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*log(x^2) - 1/4*(6*b^2*c*x^2 + b^3)/x^4
```

Fricas [A] time = 0.254501, size = 53, normalized size = 1.32

$$\frac{2\,c^3x^6+12\,bc^2x^4\log(x)-6\,b^2cx^2-b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 12*b*c^2*x^4*log(x) - 6*b^2*c*x^2 - b^3)/x^4
```

Sympy [A] time = 1.35278, size = 36, normalized size = 0.9

$$3bc^2\log(x) + \frac{c^3x^2}{2} - \frac{b^3 + 6b^2cx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**11,x)
```

```
[Out] 3*b*c**2*log(x) + c**3*x**2/2 - (b**3 + 6*b**2*c*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.268968, size = 62, normalized size = 1.55

$$\frac{1}{2}\,c^3x^2 + \frac{3}{2}\,bc^2\ln\left(x^2\right) - \frac{9\,bc^2x^4 + 6\,b^2cx^2 + b^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*c^3*x^2 + 3/2*b*c^2*ln(x^2) - 1/4*(9*b*c^2*x^4 + 6*b^2*c*x^2
+ b^3)/x^4
```

3.168 $\int \frac{(bx^2+cx^4)^3}{x^{12}}\,dx$

Optimal. Leaf size=34

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rubi [A] time = 0.0441125, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^12,x]

[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+\int c^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**12,x)

[Out] -b**3/(5*x**5) - b**2*c/x**3 - 3*b*c**2/x + Integral(c**3, x)
```

Mathematica [A] time = 0.00885713, size = 34, normalized size = 1.

$$-\frac{b^3}{5x^5}-\frac{b^2c}{x^3}-\frac{3bc^2}{x}+c^3x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^12,x]
```

```
[Out] -b^3/(5*x^5) - (b^2*c)/x^3 - (3*b*c^2)/x + c^3*x
```

Maple [A] time = 0.007, size = 33, normalized size = 1.

$$-\frac{b^3}{5\,x^5}-\frac{b^2c}{x^3}-3\,\frac{bc^2}{x}+c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^12,x)
```

```
[Out] -1/5*b^3/x^5-b^2*c/x^3-3*b*c^2/x+c^3*x
```

Maxima [A] time = 0.678231, size = 45, normalized size = 1.32

$$c^3x-\frac{15\,bc^2x^4+5\,b^2cx^2+b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

Fricas [A] time = 0.245991, size = 50, normalized size = 1.47

$$\frac{5\,c^3x^6-15\,bc^2x^4-5\,b^2cx^2-b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/5*(5*c^3*x^6 - 15*b*c^2*x^4 - 5*b^2*c*x^2 - b^3)/x^5
```

Sympy [A] time = 1.4051, size = 32, normalized size = 0.94

$$c^3x - \frac{b^3 + 5b^2cx^2 + 15bc^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**12,x)
```

```
[Out] c**3*x - (b**3 + 5*b**2*c*x**2 + 15*b*c**2*x**4)/(5*x**5)
```

GIAC/XCAS [A] time = 0.267714, size = 45, normalized size = 1.32

$$c^3x - \frac{15\,bc^2x^4 + 5\,b^2cx^2 + b^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^12,x, algorithm="giac")
```

```
[Out] c^3*x - 1/5*(15*b*c^2*x^4 + 5*b^2*c*x^2 + b^3)/x^5
```

3.169 $\int \frac{(bx^2+cx^4)^3}{x^{13}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rubi [A] time = 0.0555436, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Rubi in Sympy [A] time = 11.5437, size = 41, normalized size = 1.05

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+\frac{c^3\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] -b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log
(x**2)/2
```

Mathematica [A] time = 0.00726585, size = 39, normalized size = 1.

$$-\frac{b^3}{6x^6}-\frac{3b^2c}{4x^4}-\frac{3bc^2}{2x^2}+c^3\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^13,x]
```

```
[Out] -b^3/(6*x^6) - (3*b^2*c)/(4*x^4) - (3*b*c^2)/(2*x^2) + c^3*Log[x]
```

Maple [A] time = 0.009, size = 34, normalized size = 0.9

$$-\frac{b^3}{6\,x^6}-\frac{3\,b^2c}{4\,x^4}-\frac{3\,bc^2}{2\,x^2}+c^3\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^13,x)
```

```
[Out] -1/6*b^3/x^6-3/4*b^2*c/x^4-3/2*b*c^2/x^2+c^3*ln(x)
```

Maxima [A] time = 0.683542, size = 53, normalized size = 1.36

$$\frac{1}{2}\,c^3\log\left(x^2\right)-\frac{18\,bc^2x^4+9\,b^2cx^2+2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(x^2) - 1/12*(18*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3)/x^6
```

Fricas [A] time = 0.252205, size = 53, normalized size = 1.36

$$\frac{12\,c^3x^6\log\left(x\right)-18\,bc^2x^4-9\,b^2cx^2-2\,b^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/12*(12*c^3*x^6*log(x) - 18*b*c^2*x^4 - 9*b^2*c*x^2 - 2*b^3)/x^6
```

Sympy [A] time = 1.53151, size = 36, normalized size = 0.92

$$c^3 \log(x) - \frac{2b^3 + 9b^2cx^2 + 18bc^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**13,x)
```

```
[Out] c**3*log(x) - (2*b**3 + 9*b**2*c*x**2 + 18*b*c**2*x**4)/(12*x**6)
```

GIAC/XCAS [A] time = 0.269707, size = 63, normalized size = 1.62

$$\frac{1}{2}\, c^3 \ln\left(x^2\right) - \frac{11\, c^3 x^6 + 18\, bc^2 x^4 + 9\, b^2 c x^2 + 2\, b^3}{12\, x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*c^3*ln(x^2) - 1/12*(11*c^3*x^6 + 18*b*c^2*x^4 + 9*b^2*c*x^2 +
 2*b^3)/x^6
```

3.170 $\int \frac{(bx^2+cx^4)^3}{x^{14}}\,dx$

Optimal. Leaf size=39

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

```
[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rubi [A] time = 0.0450574, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^14,x]

[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Rubi in Sympy [A] time = 9.885, size = 34, normalized size = 0.87

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**14,x)

[Out] -b**3/(7*x**7) - 3*b**2*c/(5*x**5) - b*c**2/x**3 - c**3/x
```

Mathematica [A] time = 0.006751, size = 39, normalized size = 1.

$$-\frac{b^3}{7x^7}-\frac{3b^2c}{5x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^14,x]

[Out] -b^3/(7*x^7) - (3*b^2*c)/(5*x^5) - (b*c^2)/x^3 - c^3/x
```

Maple [A] time = 0.007, size = 36, normalized size = 0.9

$$-\frac{b^3}{7\,x^7}-\frac{3\,b^2c}{5\,x^5}-\frac{bc^2}{x^3}-\frac{c^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^14,x)

[Out] -1/7*b^3/x^7-3/5*b^2*c/x^5-b*c^2/x^3-c^3/x
```

Maxima [A] time = 0.692415, size = 50, normalized size = 1.28

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^14,x, algorithm="maxima")

[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Fricas [A] time = 0.246176, size = 50, normalized size = 1.28

$$-\frac{35\,c^3x^6+35\,bc^2x^4+21\,b^2cx^2+5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^14,x, algorithm="fricas")

[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

Sympy [A] time = 1.55879, size = 39, normalized size = 1.

$$-\frac{5b^3 + 21b^2cx^2 + 35bc^2x^4 + 35c^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**14,x)
```

```
[Out] -(5*b**3 + 21*b**2*c*x**2 + 35*b*c**2*x**4 + 35*c**3*x**6)/(35*x*
*7)
```

GIAC/XCAS [A] time = 0.267991, size = 50, normalized size = 1.28

$$-\frac{35\,c^3x^6 + 35\,bc^2x^4 + 21\,b^2cx^2 + 5\,b^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^14,x, algorithm="giac")
```

```
[Out] -1/35*(35*c^3*x^6 + 35*b*c^2*x^4 + 21*b^2*c*x^2 + 5*b^3)/x^7
```

3.171 $\int \frac{(bx^2+cx^4)^3}{x^{15}}\,dx$

Optimal. Leaf size=19

$$-\frac{(b+cx^2)^4}{8bx^8}$$

```
[Out] -(b + c*x^2)^4/(8*b*x^8)
```

Rubi [A] time = 0.0228797, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^15,x]

[Out] -(b + c*x^2)^4/(8*b*x^8)
```

Rubi in Sympy [A] time = 5.16102, size = 15, normalized size = 0.79

$$-\frac{(b+cx^2)^4}{8bx^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**15,x)

[Out] -(b + c*x**2)**4/(8*b*x**8)
```

Mathematica [B] time = 0.0113402, size = 43, normalized size = 2.26

$$-\frac{b^3}{8x^8}-\frac{b^2c}{2x^6}-\frac{3bc^2}{4x^4}-\frac{c^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^15,x]

[Out] -b^3/(8*x^8) - (b^2*c)/(2*x^6) - (3*b*c^2)/(4*x^4) - c^3/(2*x^2)
```

Maple [B] time = 0.007, size = 36, normalized size = 1.9

$$-\frac{b^3}{8\,x^8}-\frac{b^2c}{2\,x^6}-\frac{c^3}{2\,x^2}-\frac{3\,bc^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^15,x)

[Out] -1/8*b^3/x^8-1/2*b^2*c/x^6-1/2*c^3/x^2-3/4*b*c^2/x^4
```

Maxima [A] time = 0.695561, size = 47, normalized size = 2.47

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^15,x, algorithm="maxima")

[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Fricas [A] time = 0.246106, size = 47, normalized size = 2.47

$$-\frac{4\,c^3x^6+6\,bc^2x^4+4\,b^2cx^2+b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^15,x, algorithm="fricas")

[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

Sympy [A] time = 1.63611, size = 37, normalized size = 1.95

$$-\frac{b^3 + 4b^2cx^2 + 6bc^2x^4 + 4c^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**15,x)
```

```
[Out] -(b**3 + 4*b**2*c*x**2 + 6*b*c**2*x**4 + 4*c**3*x**6)/(8*x**8)
```

GIAC/XCAS [A] time = 0.268212, size = 47, normalized size = 2.47

$$-\frac{4\,c^3x^6 + 6\,bc^2x^4 + 4\,b^2cx^2 + b^3}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/8*(4*c^3*x^6 + 6*b*c^2*x^4 + 4*b^2*c*x^2 + b^3)/x^8
```

3.172 $\int \frac{(bx^2+cx^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=43

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3
)
```

Rubi [A] time = 0.0454872, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3
)
```

Rubi in Sympy [A] time = 9.89588, size = 41, normalized size = 0.95

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] -b**3/(9*x**9) - 3*b**2*c/(7*x**7) - 3*b*c**2/(5*x**5) - c**3/(3*
x**3)
```

Mathematica [A] time = 0.00667005, size = 43, normalized size = 1.

$$-\frac{b^3}{9x^9}-\frac{3b^2c}{7x^7}-\frac{3bc^2}{5x^5}-\frac{c^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^16,x]
```

```
[Out] -b^3/(9*x^9) - (3*b^2*c)/(7*x^7) - (3*b*c^2)/(5*x^5) - c^3/(3*x^3
)
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$-\frac{b^3}{9\,x^9}-\frac{3\,b^2c}{7\,x^7}-\frac{3\,bc^2}{5\,x^5}-\frac{c^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^16,x)
```

```
[Out] -1/9*b^3/x^9-3/7*b^2*c/x^7-3/5*b*c^2/x^5-1/3*c^3/x^3
```

Maxima [A] time = 0.692134, size = 50, normalized size = 1.16

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Fricas [A] time = 0.249166, size = 50, normalized size = 1.16

$$-\frac{105\,c^3x^6+189\,bc^2x^4+135\,b^2cx^2+35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

Sympy [A] time = 1.66973, size = 39, normalized size = 0.91

$$-\frac{35b^3 + 135b^2cx^2 + 189bc^2x^4 + 105c^3x^6}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**16,x)
```

```
[Out] -(35*b**3 + 135*b**2*c*x**2 + 189*b*c**2*x**4 + 105*c**3*x**6)/(3
15*x**9)
```

GIAC/XCAS [A] time = 0.268588, size = 50, normalized size = 1.16

$$-\frac{105\,c^3x^6 + 189\,bc^2x^4 + 135\,b^2cx^2 + 35\,b^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/315*(105*c^3*x^6 + 189*b*c^2*x^4 + 135*b^2*c*x^2 + 35*b^3)/x^9
```

3.173 $\int \frac{(bx^2+cx^4)^3}{x^{17}}\,dx$

Optimal. Leaf size=40

$$\frac{c\left(b+cx^2\right)^4}{40b^2x^8}-\frac{\left(b+cx^2\right)^4}{10bx^{10}}$$

```
[Out] -(b + c*x^2)^4/(10*b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rubi [A] time = 0.0604499, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{c\left(b+cx^2\right)^4}{40b^2x^8}-\frac{\left(b+cx^2\right)^4}{10bx^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -(b + c*x^2)^4/(10*b*x^10) + (c*(b + c*x^2)^4)/(40*b^2*x^8)
```

Rubi in Sympy [A] time = 11.8384, size = 39, normalized size = 0.98

$$-\frac{b^3}{10x^{10}}-\frac{3b^2c}{8x^8}-\frac{bc^2}{2x^6}-\frac{c^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] -b**3/(10*x**10) - 3*b**2*c/(8*x**8) - b*c**2/(2*x**6) - c**3/(4*
x**4)
```

Mathematica [A] time = 0.00677212, size = 43, normalized size = 1.08

$$-\frac{b^3}{10x^{10}}-\frac{3b^2c}{8x^8}-\frac{bc^2}{2x^6}-\frac{c^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^17,x]
```

```
[Out] -b^3/(10*x^10) - (3*b^2*c)/(8*x^8) - (b*c^2)/(2*x^6) - c^3/(4*x^4
)
```

Maple [A] time = 0.008, size = 36, normalized size = 0.9

$$-\frac{3\,b^2c}{8\,x^8}-\frac{bc^2}{2\,x^6}-\frac{b^3}{10\,x^{10}}-\frac{c^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^17,x)
```

```
[Out] -3/8*b^2*c/x^8-1/2*b*c^2/x^6-1/10*b^3/x^10-1/4*c^3/x^4
```

Maxima [A] time = 0.68679, size = 50, normalized size = 1.25

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Fricas [A] time = 0.245115, size = 50, normalized size = 1.25

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

Sympy [A] time = 1.71445, size = 39, normalized size = 0.98

$$-\frac{4b^3+15b^2cx^2+20bc^2x^4+10c^3x^6}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**17,x)
```

```
[Out] -(4*b**3 + 15*b**2*c*x**2 + 20*b*c**2*x**4 + 10*c**3*x**6)/(40*x*
*10)
```

GIAC/XCAS [A] time = 0.2689, size = 50, normalized size = 1.25

$$-\frac{10\,c^3x^6+20\,bc^2x^4+15\,b^2cx^2+4\,b^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/40*(10*c^3*x^6 + 20*b*c^2*x^4 + 15*b^2*c*x^2 + 4*b^3)/x^10
```

3.174 $\int \frac{x^{10}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c)
+ (b^(7/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rubi [A] time = 0.0867442, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c)
+ (b^(7/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^{\frac{7}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{\frac{9}{2}}}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}-\frac{\int b^3\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(c*x**4+b*x**2),x)
```

```
[Out] b**(7/2)*atan(sqrt(c)*x/sqrt(b))/c**(9/2) + b**2*x**3/(3*c**3) -
b*x**5/(5*c**2) + x**7/(7*c) - Integral(b**3, x)/c**4
```

Mathematica [A] time = 0.0465441, size = 68, normalized size = 1.

$$\frac{b^{7/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{9/2}}-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}+\frac{x^7}{7c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(b*x^2 + c*x^4),x]
```

```
[Out] -((b^3*x)/c^4) + (b^2*x^3)/(3*c^3) - (b*x^5)/(5*c^2) + x^7/(7*c)
+ (b^(7/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(9/2)
```

Maple [A] time = 0.008, size = 60, normalized size = 0.9

$$\frac{x^7}{7\,c}-\frac{bx^5}{5\,c^2}+\frac{b^2x^3}{3\,c^3}-\frac{b^3x}{c^4}+\frac{b^4}{c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(c*x^4+b*x^2),x)
```

```
[Out] 1/7*x^7/c-1/5*b*x^5/c^2+1/3*b^2*x^3/c^3-b^3*x/c^4+b^4/c^4/(b*c)^(
1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260012, size = 1, normalized size = 0.01

$$\left[\frac{30\,c^3x^7-42\,bc^2x^5+70\,b^2cx^3+105\,b^3\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-210\,b^3x}{210\,c^4},\frac{15\,c^3x^7-21\,bc^2x^5+35\,b^2cx^3+105\,b^3\sqrt{\frac{b}{c}}\,\mathrm{arc}}{105\,c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/210*(30*c^3*x^7 - 42*b*c^2*x^5 + 70*b^2*c*x^3 + 105*b^3*sqrt(-
b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)) - 210*b^3*x)
/c^4, 1/105*(15*c^3*x^7 - 21*b*c^2*x^5 + 35*b^2*c*x^3 + 105*b^3*s
qrt(b/c)*arctan(x/sqrt(b/c)) - 105*b^3*x)/c^4]
```

Sympy [A] time = 1.33467, size = 107, normalized size = 1.57

$$-\frac{b^3x}{c^4}+\frac{b^2x^3}{3c^3}-\frac{bx^5}{5c^2}-\frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{\sqrt{-\frac{b^7}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^7}{c^9}}}{b^3}\right)}{2}+\frac{x^7}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*x/c**4 + b**2*x**3/(3*c**3) - b*x**5/(5*c**2) - sqrt(-b**7/
c**9)*log(x - c**4*sqrt(-b**7/c**9)/b**3)/2 + sqrt(-b**7/c**9)*lo
g(x + c**4*sqrt(-b**7/c**9)/b**3)/2 + x**7/(7*c)
```

GIAC/XCAS [A] time = 0.268679, size = 88, normalized size = 1.29

$$\frac{b^4\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^4}+\frac{15\,c^6x^7-21\,bc^5x^5+35\,b^2c^4x^3-105\,b^3c^3x}{105\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] b^4*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/105*(15*c^6*x^7 - 2
1*b*c^5*x^5 + 35*b^2*c^4*x^3 - 105*b^3*c^3*x)/c^7
```

3.175 $\int \frac{x^9}{bx^2+cx^4}\,dx$

Optimal. Leaf size=53

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*
x^2])/(2*c^4)
```

Rubi [A] time = 0.100689, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*
x^2])/(2*c^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}-\frac{b\int^{x^2}x\,dx}{2c^2}+\frac{x^6}{6c}+\frac{\int^{x^2}b^2\,dx}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*log(b + c*x**2)/(2*c**4) - b*Integral(x, (x, x**2))/(2*c**2
) + x**6/(6*c) + Integral(b**2, (x, x**2))/(2*c**3)
```

Mathematica [A] time = 0.00954285, size = 53, normalized size = 1.

$$-\frac{b^3\log\left(b+cx^2\right)}{2c^4}+\frac{b^2x^2}{2c^3}-\frac{bx^4}{4c^2}+\frac{x^6}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x^2)/(2*c^3) - (b*x^4)/(4*c^2) + x^6/(6*c) - (b^3*Log[b + c*
x^2])/(2*c^4)
```

Maple [A] time = 0.004, size = 46, normalized size = 0.9

$$\frac{b^2x^2}{2\,c^3} - \frac{bx^4}{4\,c^2} + \frac{x^6}{6\,c} - \frac{b^3\ln\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*b^2*x^2/c^3-1/4*b*x^4/c^2+1/6*x^6/c-1/2*b^3*ln(c*x^2+b)/c^4
```

Maxima [A] time = 0.682689, size = 62, normalized size = 1.17

$$-\frac{b^3\log\left(cx^2+b\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(c*x^2 + b)/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6*b^2
*x^2)/c^3
```

Fricas [A] time = 0.25041, size = 61, normalized size = 1.15

$$\frac{2\,c^3x^6 - 3\,bc^2x^4 + 6\,b^2cx^2 - 6\,b^3\log\left(cx^2+b\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(2*c^3*x^6 - 3*b*c^2*x^4 + 6*b^2*c*x^2 - 6*b^3*log(c*x^2 + b
))/c^4
```

Sympy [A] time = 1.26219, size = 44, normalized size = 0.83

$$-\frac{b^3 \log \left(b + cx^2\right)}{2c^4} + \frac{b^2x^2}{2c^3} - \frac{bx^4}{4c^2} + \frac{x^6}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**4+b*x**2),x)
```

```
[Out] -b**3*log(b + c*x**2)/(2*c**4) + b**2*x**2/(2*c**3) - b*x**4/(4*c
**2) + x**6/(6*c)
```

GIAC/XCAS [A] time = 0.270823, size = 63, normalized size = 1.19

$$-\frac{b^3 \ln \left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{2\,c^2x^6 - 3\,bcx^4 + 6\,b^2x^2}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*b^3*ln(abs(c*x^2 + b))/c^4 + 1/12*(2*c^2*x^6 - 3*b*c*x^4 + 6
*b^2*x^2)/c^3
```

3.176 $\int \frac{x^8}{bx^2+cx^4} \, dx$

Optimal. Leaf size=55

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/c^(7/2)
```

Rubi [A] time = 0.0725561, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/c^(7/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{\frac{7}{2}}}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}+\frac{\int b^2\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**2),x)
```

```
[Out] -b**(5/2)*atan(sqrt(c)*x/sqrt(b))/c**(7/2) - b*x**3/(3*c**2) + x*
*5/(5*c) + Integral(b**2, x)/c**3
```

Mathematica [A] time = 0.0432015, size = 55, normalized size = 1.

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{7/2}}+\frac{b^2x}{c^3}-\frac{bx^3}{3c^2}+\frac{x^5}{5c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(b*x^2 + c*x^4),x]
```

```
[Out] (b^2*x)/c^3 - (b*x^3)/(3*c^2) + x^5/(5*c) - (b^(5/2)*ArcTan[(Sqrt
[c]*x)/Sqrt[b]])/c^(7/2)
```

Maple [A] time = 0.004, size = 49, normalized size = 0.9

$$\frac{x^5}{5\,c}-\frac{bx^3}{3\,c^2}+\frac{b^2x}{c^3}-\frac{b^3}{c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^2),x)
```

```
[Out] 1/5*x^5/c-1/3*b*x^3/c^2+b^2*x/c^3-b^3/c^3/(b*c)^(1/2)*arctan(c*x/
(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263321, size = 1, normalized size = 0.02

$$\left[\frac{6\,c^2x^5-10\,bcx^3+15\,b^2\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+30\,b^2x}{30\,c^3},\frac{3\,c^2x^5-5\,bcx^3-15\,b^2\sqrt{\frac{b}{c}}\arctan\left(\frac{x}{\sqrt{\frac{b}{c}}}\right)+15\,b^2x}{15\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/30*(6*c^2*x^5 - 10*b*c*x^3 + 15*b^2*sqrt(-b/c)*log((c*x^2 - 2*
c*x*sqrt(-b/c) - b)/(c*x^2 + b)) + 30*b^2*x)/c^3, 1/15*(3*c^2*x^5
 - 5*b*c*x^3 - 15*b^2*sqrt(b/c)*arctan(x/sqrt(b/c)) + 15*b^2*x)/c
^3]
```

Sympy [A] time = 1.30234, size = 95, normalized size = 1.73

$$\frac{b^2x}{c^3} - \frac{bx^3}{3c^2} + \frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x - \frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2} - \frac{\sqrt{-\frac{b^5}{c^7}}\log\left(x + \frac{c^3\sqrt{-\frac{b^5}{c^7}}}{b^2}\right)}{2} + \frac{x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**4+b*x**2),x)
```

```
[Out] b**2*x/c**3 - b*x**3/(3*c**2) + sqrt(-b**5/c**7)*log(x - c**3*sqr
t(-b**5/c**7)/b**2)/2 - sqrt(-b**5/c**7)*log(x + c**3*sqrt(-b**5/
c**7)/b**2)/2 + x**5/(5*c)
```

GIAC/XCAS [A] time = 0.268974, size = 74, normalized size = 1.35

$$-\frac{b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{3\,c^4x^5 - 5\,bc^3x^3 + 15\,b^2c^2x}{15\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + 1/15*(3*c^4*x^5 - 5*
b*c^3*x^3 + 15*b^2*c^2*x)/c^5
```

3.177 $\int \frac{x^7}{bx^2+cx^4}\,dx$

Optimal. Leaf size=40

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rubi [A] time = 0.0743586, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}+\frac{\int^{x^2} x\,dx}{2c}-\frac{\int^{x^2} b\,dx}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2),x)
```

```
[Out] b**2*log(b + c*x**2)/(2*c**3) + Integral(x, (x, x**2))/(2*c) - In
tegral(b, (x, x**2))/(2*c**2)
```

Mathematica [A] time = 0.0093579, size = 40, normalized size = 1.

$$\frac{b^2\log\left(b+cx^2\right)}{2c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(b*x^2 + c*x^4),x]

[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b^2*Log[b + c*x^2])/(2*c^3)
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$-\frac{bx^2}{2\,c^2}+\frac{x^4}{4\,c}+\frac{b^2\ln\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2),x)

[Out] -1/2*b*x^2/c^2+1/4*x^4/c+1/2*b^2*ln(c*x^2+b)/c^3
```

Maxima [A] time = 0.688344, size = 46, normalized size = 1.15

$$\frac{b^2\log\left(cx^2+b\right)}{2\,c^3}+\frac{cx^4-2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] 1/2*b^2*log(c*x^2 + b)/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

Fricas [A] time = 0.251709, size = 45, normalized size = 1.12

$$\frac{c^2x^4-2\,bcx^2+2\,b^2\log\left(cx^2+b\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] 1/4*(c^2*x^4 - 2*b*c*x^2 + 2*b^2*log(c*x^2 + b))/c^3
```

Sympy [A] time = 1.2139, size = 32, normalized size = 0.8

$$\frac{b^2 \log \left(b + cx^2\right)}{2c^3} - \frac{bx^2}{2c^2} + \frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2),x)
```

```
[Out] b**2*log(b + c*x**2)/(2*c**3) - b*x**2/(2*c**2) + x**4/(4*c)
```

GIAC/XCAS [A] time = 0.270056, size = 47, normalized size = 1.18

$$\frac{b^2 \ln \left(\left|cx^2 + b\right|\right)}{2\,c^3} + \frac{cx^4 - 2\,bx^2}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*b^2*ln(abs(c*x^2 + b))/c^3 + 1/4*(c*x^4 - 2*b*x^2)/c^2
```

3.178 $\int \frac{x^6}{bx^2+cx^4} \, dx$

Optimal. Leaf size=42

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
c^(5/2)
```

Rubi [A] time = 0.0619986, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
c^(5/2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{\frac{5}{2}}}+\frac{x^3}{3c}-\frac{\int b\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2),x)
```

```
[Out] b**(3/2)*atan(sqrt(c)*x/sqrt(b))/c**(5/2) + x**3/(3*c) - Integral
(b, x)/c**2
```

Mathematica [A] time = 0.032989, size = 42, normalized size = 1.

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{5/2}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + (b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
c^(5/2)
```

Maple [A] time = 0.003, size = 38, normalized size = 0.9

$$\frac{x^3}{3\,c}-\frac{bx}{c^2}+\frac{b^2}{c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2+b^2/c^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258052, size = 1, normalized size = 0.02

$$\left[\frac{2\,cx^3+3\,b\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)-6\,bx}{6\,c^2},\ \frac{cx^3+3\,b\sqrt{\frac{b}{c}}\arctan\left(\frac{x}{\sqrt{\frac{b}{c}}}\right)-3\,bx}{3\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c*x^3 + 3*b*sqrt(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)
/(c*x^2 + b)) - 6*b*x)/c^2, 1/3*(c*x^3 + 3*b*sqrt(b/c)*arctan(x/s
qrt(b/c)) - 3*b*x)/c^2]
```

Sympy [A] time = 1.3132, size = 80, normalized size = 1.9

$$-\frac{bx}{c^2} - \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x - \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{\sqrt{-\frac{b^3}{c^5}}\log\left(x + \frac{c^2\sqrt{-\frac{b^3}{c^5}}}{b}\right)}{2} + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2),x)
```

```
[Out] -b*x/c**2 - sqrt(-b**3/c**5)*log(x - c**2*sqrt(-b**3/c**5)/b)/2 +
 sqrt(-b**3/c**5)*log(x + c**2*sqrt(-b**3/c**5)/b)/2 + x**3/(3*c)
```

GIAC/XCAS [A] time = 0.269771, size = 54, normalized size = 1.29

$$\frac{b^2 \arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{c^2x^3 - 3\,bcx}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/3*(c^2*x^3 - 3*b*c*
x)/c^3
```

3.179 $\int \frac{x^5}{bx^2+cx^4}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rubi [A] time = 0.0556956, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b\log\left(b+cx^2\right)}{2c^2}+\frac{\int^{x^2}\frac{1}{c}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2),x)
```

```
[Out] -b*log(b + c*x**2)/(2*c**2) + Integral(1/c, (x, x**2))/2
```

Mathematica [A] time = 0.00726489, size = 27, normalized size = 1.

$$\frac{x^2}{2c}-\frac{b\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - (b*Log[b + c*x^2])/(2*c^2)
```

Maple [A] time = 0.004, size = 24, normalized size = 0.9

$$\frac{x^2}{2\,c} - \frac{b\ln\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*x^2/c-1/2*b*ln(c*x^2+b)/c^2
```

Maxima [A] time = 0.673051, size = 31, normalized size = 1.15

$$\frac{x^2}{2\,c} - \frac{b\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/c - 1/2*b*log(c*x^2 + b)/c^2
```

Fricas [A] time = 0.254526, size = 30, normalized size = 1.11

$$\frac{cx^2 - b\log\left(cx^2 + b\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^2 - b*log(c*x^2 + b))/c^2
```

Sympy [A] time = 1.17377, size = 20, normalized size = 0.74

$$-\frac{b\log\left(b+cx^2\right)}{2c^2}+\frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2),x)
```

```
[Out] -b*log(b + c*x**2)/(2*c**2) + x**2/(2*c)
```

GIAC/XCAS [A] time = 0.270228, size = 32, normalized size = 1.19

$$\frac{x^2}{2\,c}-\frac{b\ln\left(\left|cx^2+b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/2*b*ln(abs(c*x^2 + b))/c^2
```

3.180 $\int \frac{x^4}{bx^2+cx^4} \, dx$

Optimal. Leaf size=31

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rubi [A] time = 0.0424931, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(b*x^2 + c*x^4),x]
```

```
[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Rubi in Sympy [A] time = 8.56843, size = 26, normalized size = 0.84

$$-\frac{\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{\frac{3}{2}}} + \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(b)*atan(sqrt(c)*x/sqrt(b))/c**(3/2) + x/c
```

Mathematica [A] time = 0.0151931, size = 31, normalized size = 1.

$$\frac{x}{c} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(b*x^2 + c*x^4),x]

[Out] x/c - (Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/c^(3/2)
```

Maple [A] time = 0.004, size = 27, normalized size = 0.9

$$\frac{x}{c} - \frac{b}{c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2),x)

[Out] x/c-b/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259961, size = 1, normalized size = 0.03

$$\left[\frac{\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+2\,x}{2\,c}, -\frac{\sqrt{\frac{b}{c}}\arctan\left(\frac{x}{\sqrt{\frac{b}{c}}}\right)-x}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] [1/2*(sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b))
+ 2*x)/c, -(sqrt(b/c)*arctan(x/sqrt(b/c)) - x)/c]
```

Sympy [A] time = 1.18445, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{b}{c^3}}\log\left(-c\sqrt{-\frac{b}{c^3}}+x\right)}{2}-\frac{\sqrt{-\frac{b}{c^3}}\log\left(c\sqrt{-\frac{b}{c^3}}+x\right)}{2}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-b/c**3)*log(-c*sqrt(-b/c**3) + x)/2 - sqrt(-b/c**3)*log(c*s
qrt(-b/c**3) + x)/2 + x/c
```

GIAC/XCAS [A] time = 0.268052, size = 35, normalized size = 1.13

$$-\frac{b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}c}+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) + x/c
```

3.181 $\int \frac{x^3}{bx^2+cx^4}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(b+cx^2\right)}{2c}$$

```
[Out] Log[b + c*x^2]/(2*c)
```

Rubi [A] time = 0.0197065, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(b*x^2 + c*x^4),x]

[Out] Log[b + c*x^2]/(2*c)
```

Rubi in Sympy [A] time = 4.18605, size = 10, normalized size = 0.67

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2),x)

[Out] log(b + c*x**2)/(2*c)
```

Mathematica [A] time = 0.0034123, size = 15, normalized size = 1.

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(b*x^2 + c*x^4),x]
```

```
[Out] Log[b + c*x^2]/(2*c)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.9

$$\frac{\ln\left(cx^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2),x)
```

```
[Out] 1/2*ln(c*x^2+b)/c
```

Maxima [A] time = 0.688025, size = 18, normalized size = 1.2

$$\frac{\log\left(cx^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

Fricas [A] time = 0.250013, size = 18, normalized size = 1.2

$$\frac{\log\left(cx^2+b\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b)/c
```

Sympy [A] time = 0.237187, size = 10, normalized size = 0.67

$$\frac{\log\left(b+cx^2\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2),x)

[Out] log(b + c*x**2)/(2*c)
```

GIAC/XCAS [A] time = 0.270813, size = 19, normalized size = 1.27

$$\frac{\ln\left(\left|cx^2+b\right|\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2),x, algorithm="giac")

[Out] 1/2*ln(abs(c*x^2 + b))/c
```

3.182 $\int \frac{x^2}{bx^2+cx^4}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rubi [A] time = 0.0234973, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(b*x^2 + c*x^4),x]

[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Rubi in Sympy [A] time = 4.49833, size = 22, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2),x)

[Out] atan(sqrt(c)*x/sqrt(b))/(sqrt(b)*sqrt(c))
```

Mathematica [A] time = 0.00773879, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(Sqrt[b]*Sqrt[c])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$1\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2),x)
```

```
[Out] 1/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261649, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{2\,bcx+\left(cx^2-b\right)\sqrt{-bc}}{cx^2+b}\right)}{2\,\sqrt{-bc}},\ \frac{\arctan\left(\frac{\sqrt{bc}x}{b}\right)}{\sqrt{bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*b*c*x + (c*x^2 - b)*sqrt(-b*c))/(c*x^2 + b))/sqrt(-b*
c), arctan(sqrt(b*c)*x/b)/sqrt(b*c)]
```

Sympy [A] time = 0.295193, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{bc}}\log\left(-b\sqrt{-\frac{1}{bc}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{bc}}\log\left(b\sqrt{-\frac{1}{bc}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-1/(b*c))*log(-b*sqrt(-1/(b*c)) + x)/2 + sqrt(-1/(b*c))*log
(b*sqrt(-1/(b*c)) + x)/2
```

GIAC/XCAS [A] time = 0.269095, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(b*c))/sqrt(b*c)
```

3.183 $\int \frac{x}{bx^2+cx^4} \, dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{b} - \frac{\log(b + cx^2)}{2b}$$

```
[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rubi [A] time = 0.038038, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{\log(x)}{b} - \frac{\log(b + cx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(b*x^2 + c*x^4),x]

[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Rubi in Sympy [A] time = 8.41414, size = 19, normalized size = 0.86

$$\frac{\log(x^2)}{2b} - \frac{\log(b + cx^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2),x)

[Out] log(x**2)/(2*b) - log(b + c*x**2)/(2*b)
```

Mathematica [A] time = 0.00836916, size = 22, normalized size = 1.

$$\frac{\log(x)}{b} - \frac{\log(b + cx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(b*x^2 + c*x^4),x]

[Out] Log[x]/b - Log[b + c*x^2]/(2*b)
```

Maple [A] time = 0.007, size = 21, normalized size = 1.

$$\frac{\ln(x)}{b} - \frac{\ln\left(cx^2 + b\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2),x)

[Out] ln(x)/b-1/2*ln(c*x^2+b)/b
```

Maxima [A] time = 0.680484, size = 31, normalized size = 1.41

$$-\frac{\log\left(cx^2 + b\right)}{2\,b} + \frac{\log\left(x^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] -1/2*log(c*x^2 + b)/b + 1/2*log(x^2)/b
```

Fricas [A] time = 0.255674, size = 24, normalized size = 1.09

$$-\frac{\log\left(cx^2 + b\right) - 2\,\log(x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] -1/2*(log(c*x^2 + b) - 2*log(x))/b
```

Sympy [A] time = 0.505183, size = 15, normalized size = 0.68

$$\frac{\log(x)}{b} - \frac{\log\left(\frac{b}{c} + x^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2),x)

[Out] log(x)/b - log(b/c + x**2)/(2*b)
```

GIAC/XCAS [A] time = 0.271004, size = 30, normalized size = 1.36

$$-\frac{\ln\left(\left|cx^2 + b\right|\right)}{2\,b} + \frac{\ln\left(|x|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2),x, algorithm="giac")

[Out] -1/2*ln(abs(c*x^2 + b))/b + ln(abs(x))/b
```

3.184 $\int \frac{1}{bx^2+cx^4}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rubi [A] time = 0.0372819, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(-1),x]
```

```
[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Rubi in Sympy [A] time = 11.274, size = 29, normalized size = 0.85

$$-\frac{1}{bx}-\frac{\sqrt{c}\,\text{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2),x)
```

```
[Out] -1/(b*x) - sqrt(c)*atan(sqrt(c)*x/sqrt(b))/b**(3/2)
```

Mathematica [A] time = 0.0217809, size = 34, normalized size = 1.

$$-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{3/2}}-\frac{1}{bx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(-1),x]

[Out] -(1/(b*x)) - (Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/b^(3/2)
```

Maple [A] time = 0.005, size = 30, normalized size = 0.9

$$-\frac{c}{b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2),x)

[Out] -c/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/b/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25905, size = 1, normalized size = 0.03

$$\left[\frac{x\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-2}{2\,bx},\,-\frac{x\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)+1}{bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] [1/2*(x*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)
) - 2)/(b*x), -(x*sqrt(c/b)*arctan(c*x/(b*sqrt(c/b))) + 1)/(b*x)]
```

Sympy [A] time = 1.27202, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{c}{b^3}}\log\left(-\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{\sqrt{-\frac{c}{b^3}}\log\left(\frac{b^2\sqrt{-\frac{c}{b^3}}}{c}+x\right)}{2}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c/b**3)*log(-b**2*sqrt(-c/b**3)/c + x)/2 - sqrt(-c/b**3)*lo
g(b**2*sqrt(-c/b**3)/c + x)/2 - 1/(b*x)
```

GIAC/XCAS [A] time = 0.269403, size = 39, normalized size = 1.15

$$-\frac{c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b}-\frac{1}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) - 1/(b*x)
```

3.185 $\int \frac{1}{x(bx^2+cx^4)}\,dx$

Optimal. Leaf size=35

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rubi [A] time = 0.0660019, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Rubi in Sympy [A] time = 11.35, size = 34, normalized size = 0.97

$$-\frac{1}{2bx^2}-\frac{c\log\left(x^2\right)}{2b^2}+\frac{c\log\left(b+cx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2),x)
```

```
[Out] -1/(2*b*x**2) - c*log(x**2)/(2*b**2) + c*log(b + c*x**2)/(2*b**2)
```

Mathematica [A] time = 0.0114368, size = 35, normalized size = 1.

$$\frac{c\log\left(b+cx^2\right)}{2b^2}-\frac{c\log(x)}{b^2}-\frac{1}{2bx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*b*x^2) - (c*Log[x])/b^2 + (c*Log[b + c*x^2])/(2*b^2)
```

Maple [A]   time = 0.009, size = 32, normalized size = 0.9

$$-\frac{1}{2\,bx^2}-\frac{c\ln(x)}{b^2}+\frac{c\ln(cx^2+b)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2),x)
```

```
[Out] -1/2/b/x^2-c*ln(x)/b^2+1/2*c*ln(c*x^2+b)/b^2
```

Maxima [A]   time = 0.682254, size = 45, normalized size = 1.29

$$\frac{c\log(cx^2+b)}{2\,b^2}-\frac{c\log(x^2)}{2\,b^2}-\frac{1}{2\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x),x, algorithm="maxima")
```

```
[Out] 1/2*c*log(c*x^2 + b)/b^2 - 1/2*c*log(x^2)/b^2 - 1/2/(b*x^2)
```

Fricas [A]   time = 0.256559, size = 45, normalized size = 1.29

$$\frac{cx^2\log(cx^2+b)-2\,cx^2\log(x)-b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x),x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^2*log(c*x^2 + b) - 2*c*x^2*log(x) - b)/(b^2*x^2)
```

Sympy [A] time = 1.5713, size = 31, normalized size = 0.89

$$-\frac{1}{2bx^2} - \frac{c\log(x)}{b^2} + \frac{c\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2),x)
```

```
[Out] -1/(2*b*x**2) - c*log(x)/b**2 + c*log(b/c + x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.269539, size = 58, normalized size = 1.66

$$-\frac{c\ln\left(x^2\right)}{2\,b^2} + \frac{c\ln\left(\left|cx^2 + b\right|\right)}{2\,b^2} + \frac{cx^2 - b}{2\,b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x),x, algorithm="giac")
```

```
[Out] -1/2*c*ln(x^2)/b^2 + 1/2*c*ln(abs(c*x^2 + b))/b^2 + 1/2*(c*x^2 -
b)/(b^2*x^2)
```

3.186 $\int \frac{1}{x^2(bx^2+cx^4)}\,dx$

Optimal. Leaf size=43

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
b^(5/2)
```

Rubi [A] time = 0.0591597, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
b^(5/2)
```

Rubi in Sympy [A] time = 11.8376, size = 37, normalized size = 0.86

$$-\frac{1}{3bx^3}+\frac{c}{b^2x}+\frac{c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2),x)
```

```
[Out] -1/(3*b*x**3) + c/(b**2*x) + c**(3/2)*atan(sqrt(c)*x/sqrt(b))/b**
(5/2)
```

Mathematica [A] time = 0.0368547, size = 43, normalized size = 1.

$$\frac{c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{b^{5/2}}+\frac{c}{b^2x}-\frac{1}{3bx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*b*x^3) + c/(b^2*x) + (c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/
b^(5/2)
```

Maple [A] time = 0.008, size = 39, normalized size = 0.9

$$\frac{c^2}{b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{3\,bx^3}+\frac{c}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2),x)
```

```
[Out] c^2/b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/3/b/x^3+c/b^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262642, size = 1, normalized size = 0.02

$$\left[\frac{3\,cx^3\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)+6\,cx^2-2\,b}{6\,b^2x^3},\ \frac{3\,cx^3\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)+3\,cx^2-b}{3\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*c*x^3*sqrt(-c/b)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^
2 + b)) + 6*c*x^2 - 2*b)/(b^2*x^3), 1/3*(3*c*x^3*sqrt(c/b)*arctan
(c*x/(b*sqrt(c/b))) + 3*c*x^2 - b)/(b^2*x^3)]
```

Sympy [A] time = 1.46433, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2}+\frac{\sqrt{-\frac{c^3}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c^3}{b^5}}}{c^2}+x\right)}{2}+\frac{-b+3cx^2}{3b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(-c**3/b**5)*log(-b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + sqrt(-
c**3/b**5)*log(b**3*sqrt(-c**3/b**5)/c**2 + x)/2 + (-b + 3*c*x**2
)/(3*b**2*x**3)
```

GIAC/XCAS [A] time = 0.269852, size = 54, normalized size = 1.26

$$\frac{c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^2}+\frac{3\,cx^2-b}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^2),x, algorithm="giac")
```

```
[Out] c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/3*(3*c*x^2 - b)/(b^
2*x^3)
```

3.187 $\int \frac{1}{x^3(bx^2+cx^4)}\,dx$

Optimal. Leaf size=49

$$-\frac{c^2 \log \left(b+cx^2\right)}{2b^3}+\frac{c^2 \log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*
x^2])/(2*b^3)
```

Rubi [A] time = 0.0777645, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-\frac{c^2 \log \left(b+cx^2\right)}{2b^3}+\frac{c^2 \log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*
x^2])/(2*b^3)
```

Rubi in Sympy [A] time = 14.2125, size = 48, normalized size = 0.98

$$-\frac{1}{4bx^4}+\frac{c}{2b^2x^2}+\frac{c^2 \log \left(x^2\right)}{2b^3}-\frac{c^2 \log \left(b+cx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2),x)
```

```
[Out] -1/(4*b*x**4) + c/(2*b**2*x**2) + c**2*log(x**2)/(2*b**3) - c**2*
log(b + c*x**2)/(2*b**3)
```

Mathematica [A] time = 0.0124022, size = 49, normalized size = 1.

$$-\frac{c^2 \log \left(b+cx^2\right)}{2b^3}+\frac{c^2 \log(x)}{b^3}+\frac{c}{2b^2x^2}-\frac{1}{4bx^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*b*x^4) + c/(2*b^2*x^2) + (c^2*Log[x])/b^3 - (c^2*Log[b + c*
x^2])/(2*b^3)
```

Maple [A] time = 0.009, size = 44, normalized size = 0.9

$$-\frac{1}{4\,bx^4}+\frac{c}{2\,b^2x^2}+\frac{c^2\ln\left(x\right)}{b^3}-\frac{c^2\ln\left(cx^2+b\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/x^4+1/2*c/b^2/x^2+c^2*ln(x)/b^3-1/2*c^2*ln(c*x^2+b)/b^3
```

Maxima [A] time = 0.696242, size = 63, normalized size = 1.29

$$-\frac{c^2\log\left(cx^2+b\right)}{2\,b^3}+\frac{c^2\log\left(x^2\right)}{2\,b^3}+\frac{2\,cx^2-b}{4\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*log(c*x^2 + b)/b^3 + 1/2*c^2*log(x^2)/b^3 + 1/4*(2*c*x^2
 - b)/(b^2*x^4)
```

Fricas [A] time = 0.25526, size = 61, normalized size = 1.24

$$-\frac{2\,c^2x^4\log\left(cx^2+b\right)-4\,c^2x^4\log\left(x\right)-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(2*c^2*x^4*log(c*x^2 + b) - 4*c^2*x^4*log(x) - 2*b*c*x^2 + b
^2)/(b^3*x^4)
```

Sympy [A] time = 1.79553, size = 42, normalized size = 0.86

$$\frac{-b+2cx^2}{4b^2x^4}+\frac{c^2\log(x)}{b^3}-\frac{c^2\log\left(\frac{b}{c}+x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2),x)
```

```
[Out] (-b + 2*c*x**2)/(4*b**2*x**4) + c**2*log(x)/b**3 - c**2*log(b/c +
 x**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.269841, size = 77, normalized size = 1.57

$$\frac{c^2\ln\left(x^2\right)}{2\,b^3}-\frac{c^2\ln\left(\left|cx^2+b\right|\right)}{2\,b^3}-\frac{3\,c^2x^4-2\,bcx^2+b^2}{4\,b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*c^2*ln(x^2)/b^3 - 1/2*c^2*ln(abs(c*x^2 + b))/b^3 - 1/4*(3*c^2
*x^4 - 2*b*c*x^2 + b^2)/(b^3*x^4)
```

3.188 $\int \frac{1}{x^4(bx^2+cx^4)}\,dx$

Optimal. Leaf size=58

$$-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/b^(7/2)
```

Rubi [A] time = 0.0809778, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/b^(7/2)
```

Rubi in Sympy [A] time = 16.3081, size = 49, normalized size = 0.84

$$-\frac{1}{5bx^5}+\frac{c}{3b^2x^3}-\frac{c^2}{b^3x}-\frac{c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+b*x**2),x)
```

```
[Out] -1/(5*b*x**5) + c/(3*b**2*x**3) - c**2/(b**3*x) - c**(5/2)*atan(s
qrt(c)*x/sqrt(b))/b**(7/2)
```

Mathematica [A] time = 0.0426528, size = 58, normalized size = 1.

$$-\frac{c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{b}}\right)}{b^{7/2}}-\frac{c^2}{b^3x}+\frac{c}{3b^2x^3}-\frac{1}{5bx^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(5*b*x^5) + c/(3*b^2*x^3) - c^2/(b^3*x) - (c^(5/2)*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/b^(7/2)
```

Maple [A] time = 0.01, size = 52, normalized size = 0.9

$$-\frac{c^3}{b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{5\,bx^5}-\frac{c^2}{b^3x}+\frac{c}{3\,b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+b*x^2),x)
```

```
[Out] -c^3/b^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/5/b/x^5-c^2/b^3/x+
1/3*c/b^2/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260171, size = 1, normalized size = 0.02

$$\left[\frac{15\,c^2x^5\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-30\,c^2x^4+10\,bcx^2-6\,b^2}{30\,b^3x^5},\right.$$
$$\left.-\frac{15\,c^2x^5\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)+15\,c^2x^4-5\,bcx^2+3\,b^2}{15\,b^3x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^4),x, algorithm="fricas")
```

```
[Out] [1/30*(15*c^2*x^5*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(
c*x^2 + b)) - 30*c^2*x^4 + 10*b*c*x^2 - 6*b^2)/(b^3*x^5), -1/15*(
15*c^2*x^5*sqrt(c/b)*arctan(c*x/(b*sqrt(c/b))) + 15*c^2*x^4 - 5*b
*c*x^2 + 3*b^2)/(b^3*x^5)]
```

Sympy [A] time = 1.75963, size = 100, normalized size = 1.72

$$\frac{\sqrt{-\frac{c^5}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^5}{b^7}}}{c^3}+x\right)}{2}-\frac{\sqrt{-\frac{c^5}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^5}{b^7}}}{c^3}+x\right)}{2}-\frac{3b^2-5bcx^2+15c^2x^4}{15b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(-c**5/b**7)*log(-b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - sqrt(-c
**5/b**7)*log(b**4*sqrt(-c**5/b**7)/c**3 + x)/2 - (3*b**2 - 5*b*c
*x**2 + 15*c**2*x**4)/(15*b**3*x**5)
```

GIAC/XCAS [A] time = 0.269627, size = 70, normalized size = 1.21

$$-\frac{c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{\sqrt{bc}b^3}-\frac{15\,c^2x^4-5\,bcx^2+3\,b^2}{15\,b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^4),x, algorithm="giac")
```

```
[Out] -c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/15*(15*c^2*x^4 - 5
*b*c*x^2 + 3*b^2)/(b^3*x^5)
```

3.189 $\int \frac{1}{x^5(bx^2+cx^4)}\,dx$

Optimal. Leaf size=63

$$\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4
 + (c^3*Log[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.0929339, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4
 + (c^3*Log[b + c*x^2])/(2*b^4)
```

Rubi in Sympy [A] time = 16.5699, size = 60, normalized size = 0.95

$$-\frac{1}{6bx^6}+\frac{c}{4b^2x^4}-\frac{c^2}{2b^3x^2}-\frac{c^3\log\left(x^2\right)}{2b^4}+\frac{c^3\log\left(b+cx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2),x)
```

```
[Out] -1/(6*b*x**6) + c/(4*b**2*x**4) - c**2/(2*b**3*x**2) - c**3*log(x
**2)/(2*b**4) + c**3*log(b + c*x**2)/(2*b**4)
```

Mathematica [A] time = 0.0122656, size = 63, normalized size = 1.

$$\frac{c^3\log\left(b+cx^2\right)}{2b^4}-\frac{c^3\log(x)}{b^4}-\frac{c^2}{2b^3x^2}+\frac{c}{4b^2x^4}-\frac{1}{6bx^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(b*x^2 + c*x^4)),x]
```

```
[Out] -1/(6*b*x^6) + c/(4*b^2*x^4) - c^2/(2*b^3*x^2) - (c^3*Log[x])/b^4
 + (c^3*Log[b + c*x^2])/(2*b^4)
```

Maple [A] time = 0.01, size = 56, normalized size = 0.9

$$-\frac{1}{6\,bx^6}+\frac{c}{4\,b^2x^4}-\frac{c^2}{2\,b^3x^2}-\frac{c^3\ln(x)}{b^4}+\frac{c^3\ln\left(cx^2+b\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2),x)
```

```
[Out] -1/6/b/x^6+1/4*c/b^2/x^4-1/2*c^2/b^3/x^2-c^3*ln(x)/b^4+1/2*c^3*ln
(c*x^2+b)/b^4
```

Maxima [A] time = 0.70294, size = 78, normalized size = 1.24

$$\frac{c^3\log\left(cx^2+b\right)}{2\,b^4}-\frac{c^3\log\left(x^2\right)}{2\,b^4}-\frac{6\,c^2x^4-3\,bcx^2+2\,b^2}{12\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^5),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*log(c*x^2 + b)/b^4 - 1/2*c^3*log(x^2)/b^4 - 1/12*(6*c^2*x
^4 - 3*b*c*x^2 + 2*b^2)/(b^3*x^6)
```

Fricas [A] time = 0.260537, size = 78, normalized size = 1.24

$$\frac{6\,c^3x^6\log\left(cx^2+b\right)-12\,c^3x^6\log(x)-6\,bc^2x^4+3\,b^2cx^2-2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^5),x, algorithm="fricas")
```

```
[Out] 1/12*(6*c^3*x^6*log(c*x^2 + b) - 12*c^3*x^6*log(x) - 6*b*c^2*x^4
+ 3*b^2*c*x^2 - 2*b^3)/(b^4*x^6)
```

Sympy [A] time = 2.19135, size = 56, normalized size = 0.89

$$-\frac{2b^2 - 3bcx^2 + 6c^2x^4}{12b^3x^6} - \frac{c^3 \log(x)}{b^4} + \frac{c^3 \log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2),x)
```

```
[Out] -(2*b**2 - 3*b*c*x**2 + 6*c**2*x**4)/(12*b**3*x**6) - c**3*log(x)
/b**4 + c**3*log(b/c + x**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.270705, size = 95, normalized size = 1.51

$$-\frac{c^3 \ln\left(x^2\right)}{2\,b^4} + \frac{c^3 \ln\left(\left|cx^2 + b\right|\right)}{2\,b^4} + \frac{11\,c^3x^6 - 6\,bc^2x^4 + 3\,b^2cx^2 - 2\,b^3}{12\,b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^5),x, algorithm="giac")
```

```
[Out] -1/2*c^3*ln(x^2)/b^4 + 1/2*c^3*ln(abs(c*x^2 + b))/b^4 + 1/12*(11*
c^3*x^6 - 6*b*c^2*x^4 + 3*b^2*c*x^2 - 2*b^3)/(b^4*x^6)
```

3.190 $\int \frac{x^{12}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=79

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\,(b+cx^2)}+\frac{7x^5}{10c^2}$$

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2
*c*(b + c*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9
/2))
```

Rubi [A] time = 0.0961293, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}+\frac{7b^2x}{2c^4}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\,(b+cx^2)}+\frac{7x^5}{10c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (7*b^2*x)/(2*c^4) - (7*b*x^3)/(6*c^3) + (7*x^5)/(10*c^2) - x^7/(2
*c*(b + c*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9
/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{\frac{9}{2}}}-\frac{7bx^3}{6c^3}-\frac{x^7}{2c\,(b+cx^2)}+\frac{7x^5}{10c^2}+\frac{7\int b^2\,dx}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12/(c*x**4+b*x**2)**2,x)
```

```
[Out] -7*b**(5/2)*atan(sqrt(c)*x/sqrt(b))/(2*c**(9/2)) - 7*b*x**3/(6*c*
*3) - x**7/(2*c*(b + c*x**2)) + 7*x**5/(10*c**2) + 7*Integral(b**
2, x)/(2*c**4)
```

Mathematica [A] time = 0.0869714, size = 71, normalized size = 0.9

$$\frac{x\left(\frac{15b^3}{b+cx^2}+90b^2-20bcx^2+6c^2x^4\right)}{30c^4}-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^12/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(90*b^2 - 20*b*c*x^2 + 6*c^2*x^4 + (15*b^3)/(b + c*x^2)))/(30*
c^4) - (7*b^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(9/2))
```

Maple [A] time = 0.011, size = 68, normalized size = 0.9

$$\frac{x^5}{5\,c^2}-\frac{2\,bx^3}{3\,c^3}+3\,\frac{b^2x}{c^4}+\frac{b^3x}{2\,c^4\,(cx^2+b)}-\frac{7\,b^3}{2\,c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/5*x^5/c^2-2/3*b*x^3/c^3+3*b^2*x/c^4+1/2/c^4*b^3*x/(c*x^2+b)-7/2
/c^4*b^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260354, size = 1, normalized size = 0.01

$$\left[\frac{12\,c^3x^7-28\,bc^2x^5+140\,b^2cx^3+210\,b^3x+105\left(b^2cx^2+b^3\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{60\,(c^5x^2+bc^4)},\frac{6\,c^3x^7-14\,bc^2x^5+70\,b^2cx^3+105}{30\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*c^3*x^7 - 28*b*c^2*x^5 + 140*b^2*c*x^3 + 210*b^3*x + 10
5*(b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)
/(c*x^2 + b)))/(c^5*x^2 + b*c^4), 1/30*(6*c^3*x^7 - 14*b*c^2*x^5
+ 70*b^2*c*x^3 + 105*b^3*x - 105*(b^2*c*x^2 + b^3)*sqrt(b/c)*arct
an(x/sqrt(b/c)))/(c^5*x^2 + b*c^4)]
```

Sympy [A] time = 1.72063, size = 124, normalized size = 1.57

$$\frac{b^3x}{2bc^4+2c^5x^2}+\frac{3b^2x}{c^4}-\frac{2bx^3}{3c^3}+\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x-\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}-\frac{7\sqrt{-\frac{b^5}{c^9}}\log\left(x+\frac{c^4\sqrt{-\frac{b^5}{c^9}}}{b^2}\right)}{4}+\frac{x^5}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3*x/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*x/c**4 - 2*b*x**3/(3*c**
3) + 7*sqrt(-b**5/c**9)*log(x - c**4*sqrt(-b**5/c**9)/b**2)/4 - 7
*sqrt(-b**5/c**9)*log(x + c**4*sqrt(-b**5/c**9)/b**2)/4 + x**5/(5
*c**2)
```

GIAC/XCAS [A] time = 0.269147, size = 99, normalized size = 1.25

$$-\frac{7\,b^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^4}+\frac{b^3x}{2\,(cx^2+b)c^4}+\frac{3\,c^8x^5-10\,bc^7x^3+45\,b^2c^6x}{15\,c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) + 1/2*b^3*x/((c*x^
2 + b)*c^4) + 1/15*(3*c^8*x^5 - 10*b*c^7*x^3 + 45*b^2*c^6*x)/c^10
```

3.191 $\int \frac{x^{11}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=57

$$\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*L
og[b + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.113125, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}-\frac{bx^2}{c^3}+\frac{x^4}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b*x^2)/c^3) + x^4/(4*c^2) + b^3/(2*c^4*(b + c*x^2)) + (3*b^2*L
og[b + c*x^2])/(2*c^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^3}{2c^4\left(b+cx^2\right)}+\frac{3b^2\log\left(b+cx^2\right)}{2c^4}-\frac{bx^2}{c^3}+\frac{\int^{x^2} x\,dx}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3/(2*c**4*(b + c*x**2)) + 3*b**2*log(b + c*x**2)/(2*c**4) - b*
x**2/c**3 + Integral(x, (x, x**2))/(2*c**2)
```

Mathematica [A] time = 0.0301901, size = 49, normalized size = 0.86

$$\frac{\frac{2b^3}{b+cx^2}+6b^2\log\left(b+cx^2\right)-4bcx^2+c^2x^4}{4c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-4*b*c*x^2 + c^2*x^4 + (2*b^3)/(b + c*x^2) + 6*b^2*Log[b + c*x^2
])/(4*c^4)
```

Maple [A] time = 0.014, size = 52, normalized size = 0.9

$$-\frac{bx^2}{c^3}+\frac{x^4}{4\,c^2}+\frac{b^3}{2\,c^4\,(cx^2+b)}+\frac{3\,b^2\ln\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^4+b*x^2)^2,x)
```

```
[Out] -b*x^2/c^3+1/4*x^4/c^2+1/2*b^3/c^4/(c*x^2+b)+3/2*b^2*ln(c*x^2+b)/
c^4
```

Maxima [A] time = 0.696912, size = 73, normalized size = 1.28

$$\frac{b^3}{2\,(c^5x^2+bc^4)}+\frac{3\,b^2\log\left(cx^2+b\right)}{2\,c^4}+\frac{cx^4-4\,bx^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3/(c^5*x^2 + b*c^4) + 3/2*b^2*log(c*x^2 + b)/c^4 + 1/4*(c*x
^4 - 4*b*x^2)/c^3
```

Fricas [A] time = 0.250712, size = 95, normalized size = 1.67

$$\frac{c^3x^6-3\,bc^2x^4-4\,b^2cx^2+2\,b^3+6\,\left(b^2cx^2+b^3\right)\log\left(cx^2+b\right)}{4\,(c^5x^2+bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(c^3*x^6 - 3*b*c^2*x^4 - 4*b^2*c*x^2 + 2*b^3 + 6*(b^2*c*x^2 +
 b^3)*log(c*x^2 + b))/(c^5*x^2 + b*c^4)
```

Sympy [A] time = 1.57398, size = 53, normalized size = 0.93

$$\frac{b^3}{2bc^4 + 2c^5x^2} + \frac{3b^2\log\left(b + cx^2\right)}{2c^4} - \frac{bx^2}{c^3} + \frac{x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**4+b*x**2)**2,x)
```

```
[Out] b**3/(2*b*c**4 + 2*c**5*x**2) + 3*b**2*log(b + c*x**2)/(2*c**4) -
 b*x**2/c**3 + x**4/(4*c**2)
```

GIAC/XCAS [A] time = 0.27074, size = 90, normalized size = 1.58

$$\frac{3\,b^2\ln\left(\left|cx^2 + b\right|\right)}{2\,c^4} + \frac{c^2x^4 - 4\,bcx^2}{4\,c^4} - \frac{3\,b^2cx^2 + 2\,b^3}{2\,(cx^2 + b)c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/2*b^2*ln(abs(c*x^2 + b))/c^4 + 1/4*(c^2*x^4 - 4*b*c*x^2)/c^4 -
1/2*(3*b^2*c*x^2 + 2*b^3)/((c*x^2 + b)*c^4)
```

3.192 $\int \frac{x^{10}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=66

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b
^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rubi [A] time = 0.0820254, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}-\frac{5bx}{2c^3}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-5*b*x)/(2*c^3) + (5*x^3)/(6*c^2) - x^5/(2*c*(b + c*x^2)) + (5*b
^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{\frac{7}{2}}}-\frac{x^5}{2c\left(b+cx^2\right)}+\frac{5x^3}{6c^2}-\frac{5\int b\,dx}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(c*x**4+b*x**2)**2,x)
```

```
[Out] 5*b**(3/2)*atan(sqrt(c)*x/sqrt(b))/(2*c**(7/2)) - x**5/(2*c*(b +
c*x**2)) + 5*x**3/(6*c**2) - 5*Integral(b, x)/(2*c**3)
```

Mathematica [A] time = 0.0750306, size = 60, normalized size = 0.91

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{7/2}}+\frac{x\left(-\frac{3b^2}{b+cx^2}-12b+2cx^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(-12*b + 2*c*x^2 - (3*b^2)/(b + c*x^2)))/(6*c^3) + (5*b^(3/2)*
ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*c^(7/2))
```

Maple [A] time = 0.011, size = 57, normalized size = 0.9

$$\frac{x^3}{3\,c^2} - 2\,\frac{bx}{c^3} - \frac{b^2x}{2\,c^3\,(cx^2+b)} + \frac{5\,b^2}{2\,c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/3*x^3/c^2-2*b*x/c^3-1/2/c^3*b^2*x/(c*x^2+b)+5/2/c^3*b^2/(b*c)^(
1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260646, size = 1, normalized size = 0.02

$$\left[\frac{4\,c^2x^5 - 20\,bcx^3 - 30\,b^2x + 15\left(bcx^2+b^2\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{12\left(c^4x^2+bc^3\right)}, \frac{2\,c^2x^5 - 10\,bcx^3 - 15\,b^2x + 15\left(bcx^2+b^2\right)\sqrt{\frac{b}{c}}\,\mathrm{arcta}}{6\left(c^4x^2+bc^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*c^2*x^5 - 20*b*c*x^3 - 30*b^2*x + 15*(b*c*x^2 + b^2)*sqr
t(-b/c)*log((c*x^2 + 2*c*x*sqrt(-b/c) - b)/(c*x^2 + b)))/(c^4*x^2
 + b*c^3), 1/6*(2*c^2*x^5 - 10*b*c*x^3 - 15*b^2*x + 15*(b*c*x^2 +
 b^2)*sqrt(b/c)*arctan(x/sqrt(b/c)))/(c^4*x^2 + b*c^3)]
```

Sympy [A] time = 1.65574, size = 107, normalized size = 1.62

$$-\frac{b^2x}{2bc^3+2c^4x^2}-\frac{2bx}{c^3}-\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x-\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{5\sqrt{-\frac{b^3}{c^7}}\log\left(x+\frac{c^3\sqrt{-\frac{b^3}{c^7}}}{b}\right)}{4}+\frac{x^3}{3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2*x/(2*b*c**3 + 2*c**4*x**2) - 2*b*x/c**3 - 5*sqrt(-b**3/c**7
)*log(x - c**3*sqrt(-b**3/c**7)/b)/4 + 5*sqrt(-b**3/c**7)*log(x +
 c**3*sqrt(-b**3/c**7)/b)/4 + x**3/(3*c**2)
```

GIAC/XCAS [A] time = 0.269179, size = 82, normalized size = 1.24

$$\frac{5\,b^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}c^3}-\frac{b^2x}{2\,(cx^2+b)c^3}+\frac{c^4x^3-6\,bc^3x}{3\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) - 1/2*b^2*x/((c*x^2
 + b)*c^3) + 1/3*(c^4*x^3 - 6*b*c^3*x)/c^6
```

3.193 $\int \frac{x^9}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{b^2}{2c^3(b+cx^2)}-\frac{b\log(b+cx^2)}{c^3}+\frac{x^2}{2c^2}$$

```
[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rubi [A] time = 0.0868463, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^2}{2c^3(b+cx^2)}-\frac{b\log(b+cx^2)}{c^3}+\frac{x^2}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(b*x^2 + c*x^4)^2,x]

[Out] x^2/(2*c^2) - b^2/(2*c^3*(b + c*x^2)) - (b*Log[b + c*x^2])/c^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{b^2}{2c^3(b+cx^2)}-\frac{b\log(b+cx^2)}{c^3}+\frac{\int^{x^2}\frac{1}{c^2}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**4+b*x**2)**2,x)

[Out] -b**2/(2*c**3*(b + c*x**2)) - b*log(b + c*x**2)/c**3 + Integral(c
**(-2), (x, x**2))/2
```

Mathematica [A] time = 0.0263484, size = 38, normalized size = 0.86

$$\frac{-\frac{b^2}{b+cx^2}-2b\log(b+cx^2)+cx^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (c*x^2 - b^2/(b + c*x^2) - 2*b*Log[b + c*x^2])/(2*c^3)
```

Maple [A] time = 0.013, size = 41, normalized size = 0.9

$$\frac{x^2}{2\,c^2} - \frac{b^2}{2\,c^3\,(cx^2+b)} - \frac{b\ln\left(cx^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^2/c^2-1/2*b^2/c^3/(c*x^2+b)-b*ln(c*x^2+b)/c^3
```

Maxima [A] time = 0.69572, size = 58, normalized size = 1.32

$$-\frac{b^2}{2\,(c^4x^2+bc^3)} + \frac{x^2}{2\,c^2} - \frac{b\log\left(cx^2+b\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b^2/(c^4*x^2 + b*c^3) + 1/2*x^2/c^2 - b*log(c*x^2 + b)/c^3
```

Fricas [A] time = 0.251434, size = 76, normalized size = 1.73

$$\frac{c^2x^4 + bcx^2 - b^2 - 2\left(bcx^2+b^2\right)\log\left(cx^2+b\right)}{2\,(c^4x^2+bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(c^2*x^4 + b*c*x^2 - b^2 - 2*(b*c*x^2 + b^2)*log(c*x^2 + b))/
(c^4*x^2 + b*c^3)
```

Sympy [A] time = 1.52915, size = 39, normalized size = 0.89

$$-\frac{b^2}{2bc^3+2c^4x^2}-\frac{b\log\left(b+cx^2\right)}{c^3}+\frac{x^2}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**4+b*x**2)**2,x)
```

```
[Out] -b**2/(2*b*c**3 + 2*c**4*x**2) - b*log(b + c*x**2)/c**3 + x**2/(2
*c**2)
```

GIAC/XCAS [A] time = 0.271205, size = 66, normalized size = 1.5

$$\frac{x^2}{2\,c^2}-\frac{b\ln\left(\left|cx^2+b\right|\right)}{c^3}+\frac{2\,bcx^2+b^2}{2\,(cx^2+b)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^2 - b*ln(abs(c*x^2 + b))/c^3 + 1/2*(2*b*c*x^2 + b^2)/((
c*x^2 + b)*c^3)
```

3.194 $\int \frac{x^8}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c(b+cx^2)}+\frac{3x}{2c^2}$$

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c
]*x)/Sqrt[b]])/(2*c^(5/2))
```

Rubi [A] time = 0.0600582, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}-\frac{x^3}{2c(b+cx^2)}+\frac{3x}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (3*x)/(2*c^2) - x^3/(2*c*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c
]*x)/Sqrt[b]])/(2*c^(5/2))
```

Rubi in Sympy [A] time = 12.7016, size = 48, normalized size = 0.87

$$-\frac{3\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{\frac{5}{2}}}-\frac{x^3}{2c(b+cx^2)}+\frac{3x}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**2)**2,x)
```

```
[Out] -3*sqrt(b)*atan(sqrt(c)*x/sqrt(b))/(2*c**(5/2)) - x**3/(2*c*(b +
c*x**2)) + 3*x/(2*c**2)
```

Mathematica [A] time = 0.0659683, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2c^{5/2}}+\frac{bx}{2c^2(b+cx^2)}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/c^2 + (b*x)/(2*c^2*(b + c*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[c]*x)
/Sqrt[b]])/(2*c^(5/2))
```

Maple [A] time = 0.011, size = 43, normalized size = 0.8

$$\frac{x}{c^2}+\frac{bx}{2\,c^2\,(cx^2+b)}-\frac{3\,b}{2\,c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^2)^2,x)
```

```
[Out] x/c^2+1/2/c^2*b*x/(c*x^2+b)-3/2/c^2*b/(b*c)^(1/2)*arctan(c*x/(b*c
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260118, size = 1, normalized size = 0.02

$$\left[\frac{4\,cx^3+3\left(cx^2+b\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)+6\,bx}{4\,(c^3x^2+bc^2)},\frac{2\,cx^3-3\left(cx^2+b\right)\sqrt{\frac{b}{c}}\arctan\left(\frac{x}{\sqrt{\frac{b}{c}}}\right)+3\,bx}{2\,(c^3x^2+bc^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*c*x^3 + 3*(c*x^2 + b)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(
-b/c) - b)/(c*x^2 + b)) + 6*b*x)/(c^3*x^2 + b*c^2), 1/2*(2*c*x^3
- 3*(c*x^2 + b)*sqrt(b/c)*arctan(x/sqrt(b/c)) + 3*b*x)/(c^3*x^2 +
 b*c^2)]
```

Sympy [A] time = 1.5486, size = 83, normalized size = 1.51

$$\frac{bx}{2bc^2 + 2c^3x^2} + \frac{3\sqrt{-\frac{b}{c^5}}\log\left(-c^2\sqrt{-\frac{b}{c^5}} + x\right)}{4} - \frac{3\sqrt{-\frac{b}{c^5}}\log\left(c^2\sqrt{-\frac{b}{c^5}} + x\right)}{4} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**4+b*x**2)**2,x)
```

```
[Out] b*x/(2*b*c**2 + 2*c**3*x**2) + 3*sqrt(-b/c**5)*log(-c**2*sqrt(-b/
c**5) + x)/4 - 3*sqrt(-b/c**5)*log(c**2*sqrt(-b/c**5) + x)/4 + x/
c**2
```

GIAC/XCAS [A] time = 0.270295, size = 57, normalized size = 1.04

$$-\frac{3\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c^2} + \frac{bx}{2\,(cx^2 + b)c^2} + \frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) + 1/2*b*x/((c*x^2 +
b)*c^2) + x/c^2
```

3.195 $\int \frac{x^7}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=33

$$\frac{b}{2c^2\,(b+cx^2)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

```
[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rubi [A] time = 0.0667373, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b}{2c^2\,(b+cx^2)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(b*x^2 + c*x^4)^2,x]

[Out] b/(2*c^2*(b + c*x^2)) + Log[b + c*x^2]/(2*c^2)
```

Rubi in Sympy [A] time = 10.7874, size = 26, normalized size = 0.79

$$\frac{b}{2c^2\,(b+cx^2)}+\frac{\log\left(b+cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2)**2,x)

[Out] b/(2*c**2*(b + c*x**2)) + log(b + c*x**2)/(2*c**2)
```

Mathematica [A] time = 0.0132495, size = 27, normalized size = 0.82

$$\frac{\frac{b}{b+cx^2}+\log\left(b+cx^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(b*x^2 + c*x^4)^2,x]

[Out] (b/(b + c*x^2) + Log[b + c*x^2])/(2*c^2)
```

Maple [A] time = 0.012, size = 30, normalized size = 0.9

$$\frac{b}{2\,c^{2}\left(cx^{2}+b\right)}+\frac{\ln\left(cx^{2}+b\right)}{2\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2)^2,x)

[Out] 1/2*b/c^2/(c*x^2+b)+1/2*ln(c*x^2+b)/c^2
```

Maxima [A] time = 0.699619, size = 43, normalized size = 1.3

$$\frac{b}{2\left(c^{3}x^{2}+bc^{2}\right)}+\frac{\log\left(cx^{2}+b\right)}{2\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^2,x, algorithm="maxima")

[Out] 1/2*b/(c^3*x^2 + b*c^2) + 1/2*log(c*x^2 + b)/c^2
```

Fricas [A] time = 0.251406, size = 47, normalized size = 1.42

$$\frac{\left(cx^{2}+b\right)\log\left(cx^{2}+b\right)+b}{2\left(c^{3}x^{2}+bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^2,x, algorithm="fricas")

[Out] 1/2*((c*x^2 + b)*log(c*x^2 + b) + b)/(c^3*x^2 + b*c^2)
```

Sympy [A] time = 1.34337, size = 29, normalized size = 0.88

$$\frac{b}{2bc^2 + 2c^3x^2} + \frac{\log\left(b + cx^2\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2)**2,x)
```

```
[Out] b/(2*b*c**2 + 2*c**3*x**2) + log(b + c*x**2)/(2*c**2)
```

GIAC/XCAS [A] time = 0.270413, size = 43, normalized size = 1.3

$$-\frac{x^2}{2\,(cx^2 + b)c} + \frac{\ln\left(\left|cx^2 + b\right|\right)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2/((c*x^2 + b)*c) + 1/2*ln(abs(c*x^2 + b))/c^2
```

3.196 $\int \frac{x^6}{(bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}} - \frac{x}{2c\,(b+cx^2)}$$

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(
3/2))
```

Rubi [A] time = 0.0448098, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}} - \frac{x}{2c\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(
3/2))
```

Rubi in Sympy [A] time = 8.53163, size = 36, normalized size = 0.8

$$-\frac{x}{2c\,(b+cx^2)} + \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x/(2*c*(b + c*x**2)) + atan(sqrt(c)*x/sqrt(b))/(2*sqrt(b)*c**(3/
2))
```

Mathematica [A] time = 0.0363123, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2\sqrt{b}c^{3/2}}-\frac{x}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x/(2*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*Sqrt[b]*c^(
3/2))
```

Maple [A] time = 0.01, size = 36, normalized size = 0.8

$$-\frac{x}{2\,c\,(cx^2+b)}+\frac{1}{2\,c}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x/c/(c*x^2+b)+1/2/c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261656, size = 1, normalized size = 0.02

$$\left[\frac{\left(cx^2+b\right)\log\left(\frac{2\,bcx+\left(cx^2-b\right)\sqrt{-bc}}{cx^2+b}\right)-2\,\sqrt{-bc}x}{4\,(c^2x^2+bc)\sqrt{-bc}},\ \frac{\left(cx^2+b\right)\arctan\left(\frac{\sqrt{bc}x}{b}\right)-\sqrt{bc}x}{2\,(c^2x^2+bc)\sqrt{bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((c*x^2 + b)*log((2*b*c*x + (c*x^2 - b)*sqrt(-b*c))/(c*x^2 +
 b)) - 2*sqrt(-b*c)*x)/((c^2*x^2 + b*c)*sqrt(-b*c)), 1/2*((c*x^2
+ b)*arctan(sqrt(b*c)*x/b) - sqrt(b*c)*x)/((c^2*x^2 + b*c)*sqrt(b
*c))]
```

Sympy [A] time = 1.37021, size = 78, normalized size = 1.73

$$-\frac{x}{2bc+2c^2x^2}-\frac{\sqrt{-\frac{1}{bc^3}}\log\left(-bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{bc^3}}\log\left(bc\sqrt{-\frac{1}{bc^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x/(2*b*c + 2*c**2*x**2) - sqrt(-1/(b*c**3))*log(-b*c*sqrt(-1/(b*
c**3)) + x)/4 + sqrt(-1/(b*c**3))*log(b*c*sqrt(-1/(b*c**3)) + x)/
4
```

GIAC/XCAS [A] time = 0.269774, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}c}-\frac{x}{2\,(cx^2+b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c) - 1/2*x/((c*x^2 + b)*c)
```

3.197 $\int \frac{x^5}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=23

$$-\frac{x^2}{2c\,(bx^2+cx^4)}$$

```
[Out] -x^2/(2*c*(b*x^2 + c*x^4))
```

Rubi [A] time = 0.0138063, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$-\frac{x^2}{2c\,(bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(b*x^2 + c*x^4)^2,x]

[Out] -x^2/(2*c*(b*x^2 + c*x^4))
```

Rubi in Sympy [A] time = 4.0434, size = 12, normalized size = 0.52

$$-\frac{1}{2c\,(b+cx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2)**2,x)

[Out] -1/(2*c*(b + c*x**2))
```

Mathematica [A] time = 0.00367212, size = 16, normalized size = 0.7

$$-\frac{1}{2c\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*c*(b + c*x^2))
```

Maple [A] time = 0.001, size = 15, normalized size = 0.7

$$-\frac{1}{\left(2\,c x^{2}+2\,b\right) c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/(c*x^2+b)/c
```

Maxima [A] time = 0.702671, size = 20, normalized size = 0.87

$$-\frac{1}{2\,\left(c^{2} x^{2}+b c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(c^2*x^2 + b*c)
```

Fricas [A] time = 0.24483, size = 20, normalized size = 0.87

$$-\frac{1}{2\,\left(c^{2} x^{2}+b c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2/(c^2*x^2 + b*c)
```

Sympy [A] time = 1.21667, size = 15, normalized size = 0.65

$$-\frac{1}{2bc+2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2)**2,x)

[Out] -1/(2*b*c + 2*c**2*x**2)
```

GIAC/XCAS [A] time = 0.268008, size = 19, normalized size = 0.83

$$-\frac{1}{2\,(cx^2+b)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^2,x, algorithm="giac")

[Out] -1/2/((c*x^2 + b)*c)
```

3.198 $\int \frac{x^4}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt
[c])
```

Rubi [A] time = 0.0367459, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt
[c])
```

Rubi in Sympy [A] time = 5.93658, size = 36, normalized size = 0.8

$$\frac{x}{2b\left(b+cx^2\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{\frac{3}{2}}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2)**2,x)
```

```
[Out] x/(2*b*(b + c*x**2)) + atan(sqrt(c)*x/sqrt(b))/(2*b**(3/2)*sqrt(c
))
```

Mathematica [A] time = 0.0412644, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{3/2}\sqrt{c}}+\frac{x}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(b*x^2 + c*x^4)^2,x]
```

```
[Out] x/(2*b*(b + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[b]]/(2*b^(3/2)*Sqrt
[c])
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$\frac{x}{2\,b\,(cx^2+b)}+\frac{1}{2\,b}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x/b/(c*x^2+b)+1/2/b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260081, size = 1, normalized size = 0.02

$$\left[\frac{\left(cx^2+b\right)\log\left(\frac{2\,bcx+\left(cx^2-b\right)\sqrt{-bc}}{cx^2+b}\right)+2\,\sqrt{-bc}x}{4\,(bcx^2+b^2)\sqrt{-bc}},\frac{\left(cx^2+b\right)\arctan\left(\frac{\sqrt{bc}x}{b}\right)+\sqrt{bc}x}{2\,(bcx^2+b^2)\sqrt{bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((c*x^2 + b)*log((2*b*c*x + (c*x^2 - b)*sqrt(-b*c))/(c*x^2 +
 b)) + 2*sqrt(-b*c)*x)/((b*c*x^2 + b^2)*sqrt(-b*c)), 1/2*((c*x^2
+ b)*arctan(sqrt(b*c)*x/b) + sqrt(b*c)*x)/((b*c*x^2 + b^2)*sqrt(b
*c))]
```

Sympy [A] time = 1.40657, size = 78, normalized size = 1.73

$$\frac{x}{2b^2 + 2bcx^2} - \frac{\sqrt{-\frac{1}{b^3c}}\log\left(-b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{b^3c}}\log\left(b^2\sqrt{-\frac{1}{b^3c}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2)**2,x)
```

```
[Out] x/(2*b**2 + 2*b*c*x**2) - sqrt(-1/(b**3*c))*log(-b**2*sqrt(-1/(b*
*3*c)) + x)/4 + sqrt(-1/(b**3*c))*log(b**2*sqrt(-1/(b**3*c)) + x)
/4
```

GIAC/XCAS [A] time = 0.270274, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\sqrt{bc}b} + \frac{x}{2(cx^2 + b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b) + 1/2*x/((c*x^2 + b)*b)
```

3.199 $\int \frac{x^3}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b\left(b+cx^2\right)}$$

```
[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)
```

Rubi [A] time = 0.0718247, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{\log\left(b+cx^2\right)}{2b^2}+\frac{\log(x)}{b^2}+\frac{1}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] 1/(2*b*(b + c*x^2)) + Log[x]/b^2 - Log[b + c*x^2]/(2*b^2)
```

Rubi in Sympy [A] time = 11.9673, size = 34, normalized size = 0.89

$$\frac{1}{2b\left(b+cx^2\right)}+\frac{\log\left(x^2\right)}{2b^2}-\frac{\log\left(b+cx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*(b + c*x**2)) + log(x**2)/(2*b**2) - log(b + c*x**2)/(2*b*
*2)
```

Mathematica [A] time = 0.0219342, size = 33, normalized size = 0.87

$$\frac{\frac{b}{b+cx^2}-\log\left(b+cx^2\right)+2\log(x)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b/(b + c*x^2) + 2*Log[x] - Log[b + c*x^2])/(2*b^2)
```

Maple [A] time = 0.017, size = 35, normalized size = 0.9

$$\frac{1}{2\,b\,(cx^2+b)}+\frac{\ln(x)}{b^2}-\frac{\ln\left(cx^2+b\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b/(c*x^2+b)+ln(x)/b^2-1/2*ln(c*x^2+b)/b^2
```

Maxima [A] time = 0.702543, size = 50, normalized size = 1.32

$$\frac{1}{2\,(bcx^2+b^2)}-\frac{\log\left(cx^2+b\right)}{2\,b^2}+\frac{\log\left(x^2\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(b*c*x^2 + b^2) - 1/2*log(c*x^2 + b)/b^2 + 1/2*log(x^2)/b^2
```

Fricas [A] time = 0.260859, size = 63, normalized size = 1.66

$$-\frac{\left(cx^2+b\right)\log\left(cx^2+b\right)-2\left(cx^2+b\right)\log(x)-b}{2\,(b^2cx^2+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((c*x^2 + b)*log(c*x^2 + b) - 2*(c*x^2 + b)*log(x) - b)/(b^2
*c*x^2 + b^3)
```

Sympy [A] time = 1.65951, size = 34, normalized size = 0.89

$$\frac{1}{2b^2 + 2bcx^2} + \frac{\log(x)}{b^2} - \frac{\log\left(\frac{b}{c} + x^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b**2 + 2*b*c*x**2) + log(x)/b**2 - log(b/c + x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.271521, size = 49, normalized size = 1.29

$$-\frac{\ln\left(\left|cx^2 + b\right|\right)}{2\,b^2} + \frac{\ln\left(|x|\right)}{b^2} + \frac{1}{2\,(cx^2 + b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(c*x^2 + b))/b^2 + ln(abs(x))/b^2 + 1/2/((c*x^2 + b)*b
)
```

3.200 $\int \frac{x^2}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]
*x)/Sqrt[b]])/(2*b^(5/2))
```

Rubi [A] time = 0.0622492, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{3}{2b^2x}+\frac{1}{2bx\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -3/(2*b^2*x) + 1/(2*b*x*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqrt[c]
*x)/Sqrt[b]])/(2*b^(5/2))
```

Rubi in Sympy [A] time = 12.18, size = 48, normalized size = 0.84

$$\frac{1}{2bx\left(b+cx^2\right)}-\frac{3}{2b^2x}-\frac{3\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*x*(b + c*x**2)) - 3/(2*b**2*x) - 3*sqrt(c)*atan(sqrt(c)*x/
sqrt(b))/(2*b**(5/2))
```

Mathematica [A] time = 0.0665926, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{cx}{2b^2\left(b+cx^2\right)}-\frac{1}{b^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(1/(b^2*x)) - (c*x)/(2*b^2*(b + c*x^2)) - (3*Sqrt[c]*ArcTan[(Sqr
t[c]*x)/Sqrt[b]])/(2*b^(5/2))
```

Maple [A] time = 0.013, size = 46, normalized size = 0.8

$$-\frac{cx}{2\,b^{2}\left(cx^{2}+b\right)}-\frac{3\,c}{2\,b^{2}}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x/(c*x^2+b)-3/2/b^2*c/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/
2))-1/b^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260704, size = 1, normalized size = 0.02

$$\left[-\frac{6\,cx^{2}-3\left(cx^{3}+bx\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)+4\,b}{4\left(b^{2}cx^{3}+b^{3}x\right)},-\frac{3\,cx^{2}+3\left(cx^{3}+bx\right)\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)+2\,b}{2\left(b^{2}cx^{3}+b^{3}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*c*x^2 - 3*(c*x^3 + b*x)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sq
rt(-c/b) - b)/(c*x^2 + b)) + 4*b)/(b^2*c*x^3 + b^3*x), -1/2*(3*c*
x^2 + 3*(c*x^3 + b*x)*sqrt(c/b)*arctan(c*x/(b*sqrt(c/b))) + 2*b)/
(b^2*c*x^3 + b^3*x)]
```

Sympy [A] time = 1.75319, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{c}{b^5}}\log\left(-\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{3\sqrt{-\frac{c}{b^5}}\log\left(\frac{b^3\sqrt{-\frac{c}{b^5}}}{c}+x\right)}{4}-\frac{2b+3cx^2}{2b^3x+2b^2cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 3*sqrt(-c/b**5)*log(-b**3*sqrt(-c/b**5)/c + x)/4 - 3*sqrt(-c/b**5
)*log(b**3*sqrt(-c/b**5)/c + x)/4 - (2*b + 3*c*x**2)/(2*b**3*x +
2*b**2*c*x**3)
```

GIAC/XCAS [A] time = 0.269904, size = 63, normalized size = 1.11

$$-\frac{3\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^2}-\frac{3\,cx^2+2\,b}{2\,(cx^3+bx)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -3/2*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) - 1/2*(3*c*x^2 + 2*b
)/((c*x^3 + b*x)*b^2)
```

3.201 $\int \frac{x}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=49

$$\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{1}{2b^2x^2}$$

```
[Out] -1/(2*b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Lo
g[b + c*x^2])/b^3
```

Rubi [A] time = 0.0922975, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{c\log\left(b+cx^2\right)}{b^3}-\frac{2c\log(x)}{b^3}-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{1}{2b^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/(2*b^2*x^2) - c/(2*b^2*(b + c*x^2)) - (2*c*Log[x])/b^3 + (c*Lo
g[b + c*x^2])/b^3
```

Rubi in Sympy [A] time = 14.7485, size = 46, normalized size = 0.94

$$-\frac{c}{2b^2\left(b+cx^2\right)}-\frac{1}{2b^2x^2}-\frac{c\log\left(x^2\right)}{b^3}+\frac{c\log\left(b+cx^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2)**2,x)
```

```
[Out] -c/(2*b**2*(b + c*x**2)) - 1/(2*b**2*x**2) - c*log(x**2)/b**3 + c
*log(b + c*x**2)/b**3
```

Mathematica [A] time = 0.061829, size = 41, normalized size = 0.84

$$-\frac{b\left(\frac{c}{b+cx^2}+\frac{1}{x^2}\right)-2c\log\left(b+cx^2\right)+4c\log(x)}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*(x^(-2) + c/(b + c*x^2)) + 4*c*Log[x] - 2*c*Log[b + c*x^2])/(
2*b^3)
```

Maple [A] time = 0.019, size = 46, normalized size = 0.9

$$-\frac{1}{2\,b^2x^2}-\frac{c}{2\,b^2\,(cx^2+b)}-2\,\frac{c\ln(x)}{b^3}+\frac{c\ln\left(cx^2+b\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2/x^2-1/2*c/b^2/(c*x^2+b)-2*c*ln(x)/b^3+c*ln(c*x^2+b)/b^3
```

Maxima [A] time = 0.702168, size = 70, normalized size = 1.43

$$-\frac{2\,cx^2+b}{2\,(b^2cx^4+b^3x^2)}+\frac{c\log\left(cx^2+b\right)}{b^3}-\frac{c\log\left(x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2) + c*log(c*x^2 + b)/b^3 -
 c*log(x^2)/b^3
```

Fricas [A] time = 0.25666, size = 99, normalized size = 2.02

$$-\frac{2\,bcx^2+b^2-2\left(c^2x^4+bcx^2\right)\log\left(cx^2+b\right)+4\left(c^2x^4+bcx^2\right)\log(x)}{2\,(b^3cx^4+b^4x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*x^2 + b^2 - 2*(c^2*x^4 + b*c*x^2)*log(c*x^2 + b) + 4*
(c^2*x^4 + b*c*x^2)*log(x))/(b^3*c*x^4 + b^4*x^2)
```

Sympy [A] time = 2.03763, size = 49, normalized size = 1.

$$-\frac{b+2cx^2}{2b^3x^2+2b^2cx^4}-\frac{2c\log(x)}{b^3}+\frac{c\log\left(\frac{b}{c}+x^2\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2)**2,x)
```

```
[Out] -(b + 2*c*x**2)/(2*b**3*x**2 + 2*b**2*c*x**4) - 2*c*log(x)/b**3 +
 c*log(b/c + x**2)/b**3
```

GIAC/XCAS [A] time = 0.271084, size = 68, normalized size = 1.39

$$\frac{c\ln\left(\left|cx^2+b\right|\right)}{b^3}-\frac{2\,c\ln\left(|x|\right)}{b^3}-\frac{2\,cx^2+b}{2\,(cx^4+bx^2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] c*ln(abs(c*x^2 + b))/b^3 - 2*c*ln(abs(x))/b^3 - 1/2*(2*c*x^2 + b)
/((c*x^4 + b*x^2)*b^2)
```

3.202 $\int \frac{1}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\,(b+cx^2)}$$

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c
^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rubi [A] time = 0.0751883, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{5c}{2b^3x}-\frac{5}{6b^2x^3}+\frac{1}{2bx^3\,(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -5/(6*b^2*x^3) + (5*c)/(2*b^3*x) + 1/(2*b*x^3*(b + c*x^2)) + (5*c
^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Rubi in Sympy [A] time = 19.2029, size = 61, normalized size = 0.9

$$\frac{1}{2bx^3\,(b+cx^2)}-\frac{5}{6b^2x^3}+\frac{5c}{2b^3x}+\frac{5c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*x**3*(b + c*x**2)) - 5/(6*b**2*x**3) + 5*c/(2*b**3*x) + 5*
c**(3/2)*atan(sqrt(c)*x/sqrt(b))/(2*b**(7/2))
```

Mathematica [A] time = 0.0722496, size = 67, normalized size = 0.99

$$\frac{5c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{7/2}}+\frac{c^2x}{2b^3(b+cx^2)}+\frac{2c}{b^3x}-\frac{1}{3b^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(-2),x]
```

```
[Out] -1/(3*b^2*x^3) + (2*c)/(b^3*x) + (c^2*x)/(2*b^3*(b + c*x^2)) + (5
*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(7/2))
```

Maple [A] time = 0.016, size = 59, normalized size = 0.9

$$\frac{c^2x}{2\,b^3(cx^2+b)}+\frac{5\,c^2}{2\,b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{3\,b^2x^3}+2\,\frac{c}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x/(c*x^2+b)+5/2/b^3*c^2/(b*c)^(1/2)*arctan(c*x/(b*c)^
(1/2))-1/3/b^2/x^3+2*c/b^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264371, size = 1, normalized size = 0.01

$$\left[\frac{30\,c^2x^4+20\,bcx^2+15\,(c^2x^5+bcx^3)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)-4\,b^2}{12\,(b^3cx^5+b^4x^3)},\frac{15\,c^2x^4+10\,bcx^2+15\,(c^2x^5+bcx^3)\sqrt{\frac{c}{b}}\arctan\left(\frac{c}{b\sqrt{}}\right.}{6\,(b^3cx^5+b^4x^3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-2),x, algorithm="fricas")
```

```
[Out] [1/12*(30*c^2*x^4 + 20*b*c*x^2 + 15*(c^2*x^5 + b*c*x^3)*sqrt(-c/b
)*log((c*x^2 + 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) - 4*b^2)/(b^3*c
*x^5 + b^4*x^3), 1/6*(15*c^2*x^4 + 10*b*c*x^2 + 15*(c^2*x^5 + b*c
*x^3)*sqrt(c/b)*arctan(c*x/(b*sqrt(c/b))) - 2*b^2)/(b^3*c*x^5 + b
^4*x^3)]
```

Sympy [A] time = 2.05535, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(-\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{5\sqrt{-\frac{c^3}{b^7}}\log\left(\frac{b^4\sqrt{-\frac{c^3}{b^7}}}{c^2}+x\right)}{4}+\frac{-2b^2+10bcx^2+15c^2x^4}{6b^4x^3+6b^3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**2,x)
```

```
[Out] -5*sqrt(-c**3/b**7)*log(-b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + 5*sq
rt(-c**3/b**7)*log(b**4*sqrt(-c**3/b**7)/c**2 + x)/4 + (-2*b**2 +
 10*b*c*x**2 + 15*c**2*x**4)/(6*b**4*x**3 + 6*b**3*c*x**5)
```

GIAC/XCAS [A] time = 0.268406, size = 80, normalized size = 1.18

$$\frac{5\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^3}+\frac{c^2x}{2\,(cx^2+b)b^3}+\frac{6\,cx^2-b}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-2),x, algorithm="giac")
```

```
[Out] 5/2*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) + 1/2*c^2*x/((c*x^2
 + b)*b^3) + 1/3*(6*c*x^2 - b)/(b^3*x^3)
```

3.203 $\int \frac{1}{x(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=66

$$-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

```
[Out] -1/(4*b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*L
og[x])/b^4 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.112021, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}+\frac{3c^2\log(x)}{b^4}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{c}{b^3x^2}-\frac{1}{4b^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/(4*b^2*x^4) + c/(b^3*x^2) + c^2/(2*b^3*(b + c*x^2)) + (3*c^2*L
og[x])/b^4 - (3*c^2*Log[b + c*x^2])/(2*b^4)
```

Rubi in Sympy [A] time = 18.9001, size = 66, normalized size = 1.

$$-\frac{1}{4b^2x^4}+\frac{c^2}{2b^3\left(b+cx^2\right)}+\frac{c}{b^3x^2}+\frac{3c^2\log\left(x^2\right)}{2b^4}-\frac{3c^2\log\left(b+cx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] -1/(4*b**2*x**4) + c**2/(2*b**3*(b + c*x**2)) + c/(b**3*x**2) + 3
*c**2*log(x**2)/(2*b**4) - 3*c**2*log(b + c*x**2)/(2*b**4)
```

Mathematica [A] time = 0.0982034, size = 57, normalized size = 0.86

$$\frac{-6c^2\log\left(b+cx^2\right)+b\left(\frac{2c^2}{b+cx^2}-\frac{b}{x^4}+\frac{4c}{x^2}\right)+12c^2\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(b*x^2 + c*x^4)^2),x]
```

```
[Out] (b*(-(b/x^4) + (4*c)/x^2 + (2*c^2)/(b + c*x^2)) + 12*c^2*Log[x] -
 6*c^2*Log[b + c*x^2])/(4*b^4)
```

Maple [A] time = 0.019, size = 61, normalized size = 0.9

$$-\frac{1}{4\,b^2x^4}+\frac{c}{b^3x^2}+\frac{c^2}{2\,b^3\,(cx^2+b)}+3\,\frac{c^2\ln{(x)}}{b^4}-\frac{3\,c^2\ln{\left(cx^2+b\right)}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/4/b^2/x^4+c/b^3/x^2+1/2*c^2/b^3/(c*x^2+b)+3*c^2*ln(x)/b^4-3/2*
c^2*ln(c*x^2+b)/b^4
```

Maxima [A] time = 0.698151, size = 95, normalized size = 1.44

$$\frac{6\,c^2x^4+3\,bcx^2-b^2}{4\,(b^3cx^6+b^4x^4)}-\frac{3\,c^2\log{\left(cx^2+b\right)}}{2\,b^4}+\frac{3\,c^2\log{\left(x^2\right)}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x),x, algorithm="maxima")
```

```
[Out] 1/4*(6*c^2*x^4 + 3*b*c*x^2 - b^2)/(b^3*c*x^6 + b^4*x^4) - 3/2*c^2
*log(c*x^2 + b)/b^4 + 3/2*c^2*log(x^2)/b^4
```

Fricas [A] time = 0.258424, size = 122, normalized size = 1.85

$$\frac{6\,bc^2x^4+3\,b^2cx^2-b^3-6\,\left(c^3x^6+bc^2x^4\right)\log{\left(cx^2+b\right)}+12\,\left(c^3x^6+bc^2x^4\right)\log{(x)}}{4\,(b^4cx^6+b^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x),x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*c^2*x^4 + 3*b^2*c*x^2 - b^3 - 6*(c^3*x^6 + b*c^2*x^4)*lo
g(c*x^2 + b) + 12*(c^3*x^6 + b*c^2*x^4)*log(x))/(b^4*c*x^6 + b^5*
x^4)
```

Sympy [A] time = 2.53419, size = 68, normalized size = 1.03

$$\frac{-b^2+3bcx^2+6c^2x^4}{4b^4x^4+4b^3cx^6}+\frac{3c^2\log(x)}{b^4}-\frac{3c^2\log\left(\frac{b}{c}+x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2)**2,x)
```

```
[Out] (-b**2 + 3*b*c*x**2 + 6*c**2*x**4)/(4*b**4*x**4 + 4*b**3*c*x**6)
+ 3*c**2*log(x)/b**4 - 3*c**2*log(b/c + x**2)/(2*b**4)
```

GIAC/XCAS [A] time = 0.272531, size = 116, normalized size = 1.76

$$\frac{3\,c^2\ln\left(x^2\right)}{2\,b^4}-\frac{3\,c^2\ln\left(\left|cx^2+b\right|\right)}{2\,b^4}+\frac{3\,c^3x^2+4\,bc^2}{2\,(cx^2+b)b^4}-\frac{9\,c^2x^4-4\,bcx^2+b^2}{4\,b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x),x, algorithm="giac")
```

```
[Out] 3/2*c^2*ln(x^2)/b^4 - 3/2*c^2*ln(abs(c*x^2 + b))/b^4 + 1/2*(3*c^3
*x^2 + 4*b*c^2)/((c*x^2 + b)*b^4) - 1/4*(9*c^2*x^4 - 4*b*c*x^2 +
b^2)/(b^4*x^4)
```

3.204 $\int \frac{1}{x^2(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{7c^2}{2b^4x}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*
x^5*(b + c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(
9/2))
```

Rubi [A] time = 0.102282, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{7c^2}{2b^4x}+\frac{7c}{6b^3x^3}-\frac{7}{10b^2x^5}+\frac{1}{2bx^5\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -7/(10*b^2*x^5) + (7*c)/(6*b^3*x^3) - (7*c^2)/(2*b^4*x) + 1/(2*b*
x^5*(b + c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^(
9/2))
```

Rubi in Sympy [A] time = 21.1308, size = 75, normalized size = 0.93

$$\frac{1}{2bx^5\left(b+cx^2\right)}-\frac{7}{10b^2x^5}+\frac{7c}{6b^3x^3}-\frac{7c^2}{2b^4x}-\frac{7c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*x**5*(b + c*x**2)) - 7/(10*b**2*x**5) + 7*c/(6*b**3*x**3)
- 7*c**2/(2*b**4*x) - 7*c**(5/2)*atan(sqrt(c)*x/sqrt(b))/(2*b**(9
/2))
```

Mathematica [A] time = 0.0862012, size = 80, normalized size = 0.99

$$-\frac{7c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{2b^{9/2}}-\frac{c^3x}{2b^4\left(b+cx^2\right)}-\frac{3c^2}{b^4x}+\frac{2c}{3b^3x^3}-\frac{1}{5b^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/(5*b^2*x^5) + (2*c)/(3*b^3*x^3) - (3*c^2)/(b^4*x) - (c^3*x)/(2
*b^4*(b + c*x^2)) - (7*c^(5/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(2*b^
(9/2))
```

Maple [A] time = 0.016, size = 70, normalized size = 0.9

$$-\frac{c^3x}{2\,b^4\left(cx^2+b\right)}-\frac{7\,c^3}{2\,b^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{5\,b^2x^5}-3\,\frac{c^2}{b^4x}+\frac{2\,c}{3\,b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x/(c*x^2+b)-7/2/b^4*c^3/(b*c)^(1/2)*arctan(c*x/(b*c)
^(1/2))-1/5/b^2/x^5-3*c^2/b^4/x+2/3*c/b^3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26337, size = 1, normalized size = 0.01

$$\left[-\frac{210\,c^3x^6+140\,bc^2x^4-28\,b^2cx^2+12\,b^3-105\,\left(c^3x^7+bc^2x^5\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{60\,(b^4cx^7+b^5x^5)},\right.$$
$$\left.-\frac{105\,c^3x^6+70\,bc^2x^4-14\,b^2cx^2+6\,b^3+105\,\left(c^3x^7+bc^2x^5\right)\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)}{30\,(b^4cx^7+b^5x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(210*c^3*x^6 + 140*b*c^2*x^4 - 28*b^2*c*x^2 + 12*b^3 - 105
*(c^3*x^7 + b*c^2*x^5)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) -
 b)/(c*x^2 + b)))/(b^4*c*x^7 + b^5*x^5), -1/30*(105*c^3*x^6 + 70*
b*c^2*x^4 - 14*b^2*c*x^2 + 6*b^3 + 105*(c^3*x^7 + b*c^2*x^5)*sqrt
(c/b)*arctan(c*x/(b*sqrt(c/b))))/(b^4*c*x^7 + b^5*x^5)]
```

Sympy [A] time = 2.77206, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4}-\frac{7\sqrt{-\frac{c^5}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^5}{b^9}}}{c^3}+x\right)}{4}-\frac{6b^3-14b^2cx^2+70bc^2x^4+105c^3x^6}{30b^5x^5+30b^4cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2)**2,x)
```

```
[Out] 7*sqrt(-c**5/b**9)*log(-b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - 7*sqr
t(-c**5/b**9)*log(b**5*sqrt(-c**5/b**9)/c**3 + x)/4 - (6*b**3 - 1
4*b**2*c*x**2 + 70*b*c**2*x**4 + 105*c**3*x**6)/(30*b**5*x**5 + 3
0*b**4*c*x**7)
```

GIAC/XCAS [A] time = 0.269274, size = 95, normalized size = 1.17

$$-\frac{7\,c^3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{2\,\sqrt{bc}b^4}-\frac{c^3x}{2\,(cx^2+b)b^4}-\frac{45\,c^2x^4-10\,bcx^2+3\,b^2}{15\,b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x^2),x, algorithm="giac")
```

```
[Out] -7/2*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) - 1/2*c^3*x/((c*x^
2 + b)*b^4) - 1/15*(45*c^2*x^4 - 10*b*c*x^2 + 3*b^2)/(b^4*x^5)
```

3.205 $\int \frac{x^{14}}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=85

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{35bx}{8c^4}-\frac{7x^5}{8c^2(b+cx^2)}-\frac{x^7}{4c(b+cx^2)^2}+\frac{35x^3}{24c^3}$$

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) -
 (7*x^5)/(8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/(8*c^(9/2))
```

Rubi [A] time = 0.106416, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{35bx}{8c^4}-\frac{7x^5}{8c^2(b+cx^2)}-\frac{x^7}{4c(b+cx^2)^2}+\frac{35x^3}{24c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-35*b*x)/(8*c^4) + (35*x^3)/(24*c^3) - x^7/(4*c*(b + c*x^2)^2) -
 (7*x^5)/(8*c^2*(b + c*x^2)) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqr
t[b]])/(8*c^(9/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{35b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{\frac{9}{2}}}-\frac{x^7}{4c(b+cx^2)^2}-\frac{7x^5}{8c^2(b+cx^2)}+\frac{35x^3}{24c^3}-\frac{35\int b\,dx}{8c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14/(c*x**4+b*x**2)**3,x)
```

```
[Out] 35*b**(3/2)*atan(sqrt(c)*x/sqrt(b))/(8*c**(9/2)) - x**7/(4*c*(b +
 c*x**2)**2) - 7*x**5/(8*c**2*(b + c*x**2)) + 35*x**3/(24*c**3) -
 35*Integral(b, x)/(8*c**4)
```

Mathematica [A] time = 0.0993253, size = 77, normalized size = 0.91

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{9/2}}-\frac{105b^3x+175b^2cx^3+56bc^2x^5-8c^3x^7}{24c^4\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(105*b^3*x + 175*b^2*c*x^3 + 56*b*c^2*x^5 - 8*c^3*x^7)/(24*c^4*(
b + c*x^2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(9/
2))
```

Maple [A] time = 0.014, size = 77, normalized size = 0.9

$$\frac{x^3}{3\,c^3}-3\,\frac{bx}{c^4}-\frac{13\,b^2x^3}{8\,c^3\left(cx^2+b\right)^2}-\frac{11\,b^3x}{8\,c^4\left(cx^2+b\right)^2}+\frac{35\,b^2}{8\,c^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/3*x^3/c^3-3*b*x/c^4-13/8/c^3*b^2/(c*x^2+b)^2*x^3-11/8/c^4*b^3/(
c*x^2+b)^2*x+35/8/c^4*b^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260937, size = 1, normalized size = 0.01

$$\left[\frac{16\,c^{3}x^{7}-112\,bc^{2}x^{5}-350\,b^{2}cx^{3}-210\,b^{3}x+105\,\left(bc^{2}x^{4}+2\,b^{2}cx^{2}+b^{3}\right)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^{2}+2\,cx\sqrt{-\frac{b}{c}}-b}{cx^{2}+b}\right)}{48\,(c^{6}x^{4}+2\,bc^{5}x^{2}+b^{2}c^{4})},\ \frac{8\,c^{3}x^{7}-56\,bc^{2}x^{5}-17\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*c^3*x^7 - 112*b*c^2*x^5 - 350*b^2*c*x^3 - 210*b^3*x + 1
05*(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(-b/c)*log((c*x^2 + 2*c*x*
sqrt(-b/c) - b)/(c*x^2 + b)))/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4),
1/24*(8*c^3*x^7 - 56*b*c^2*x^5 - 175*b^2*c*x^3 - 105*b^3*x + 105*
(b*c^2*x^4 + 2*b^2*c*x^2 + b^3)*sqrt(b/c)*arctan(x/sqrt(b/c)))/(c
^6*x^4 + 2*b*c^5*x^2 + b^2*c^4)]
```

Sympy [A] time = 2.22234, size = 131, normalized size = 1.54

$$-\frac{3bx}{c^{4}}-\frac{35\sqrt{-\frac{b^{3}}{c^{9}}}\log\left(x-\frac{c^{4}\sqrt{-\frac{b^{3}}{c^{9}}}}{b}\right)}{16}+\frac{35\sqrt{-\frac{b^{3}}{c^{9}}}\log\left(x+\frac{c^{4}\sqrt{-\frac{b^{3}}{c^{9}}}}{b}\right)}{16}-\frac{11b^{3}x+13b^{2}cx^{3}}{8b^{2}c^{4}+16bc^{5}x^{2}+8c^{6}x^{4}}+\frac{x^{3}}{3c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*x/c**4 - 35*sqrt(-b**3/c**9)*log(x - c**4*sqrt(-b**3/c**9)/b
)/16 + 35*sqrt(-b**3/c**9)*log(x + c**4*sqrt(-b**3/c**9)/b)/16 -
(11*b**3*x + 13*b**2*c*x**3)/(8*b**2*c**4 + 16*b*c**5*x**2 + 8*c*
*6*x**4) + x**3/(3*c**3)
```

GIAC/XCAS [A] time = 0.270119, size = 99, normalized size = 1.16

$$\frac{35\,b^{2}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^{4}}-\frac{13\,b^{2}cx^{3}+11\,b^{3}x}{8\,(cx^{2}+b)^{2}c^{4}}+\frac{c^{6}x^{3}-9\,bc^{5}x}{3\,c^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*b^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/8*(13*b^2*c*x^
3 + 11*b^3*x)/((c*x^2 + b)^2*c^4) + 1/3*(c^6*x^3 - 9*b*c^5*x)/c^9
```

3.206 $\int \frac{x^{13}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=65

$$\frac{b^3}{4c^4(b+cx^2)^2}-\frac{3b^2}{2c^4(b+cx^2)}-\frac{3b\log(b+cx^2)}{2c^4}+\frac{x^2}{2c^3}$$

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x
^2)) - (3*b*Log[b + c*x^2])/(2*c^4)
```

Rubi [A] time = 0.119345, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{b^3}{4c^4(b+cx^2)^2}-\frac{3b^2}{2c^4(b+cx^2)}-\frac{3b\log(b+cx^2)}{2c^4}+\frac{x^2}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x^2/(2*c^3) + b^3/(4*c^4*(b + c*x^2)^2) - (3*b^2)/(2*c^4*(b + c*x
^2)) - (3*b*Log[b + c*x^2])/(2*c^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b^3}{4c^4(b+cx^2)^2}-\frac{3b^2}{2c^4(b+cx^2)}-\frac{3b\log(b+cx^2)}{2c^4}+\frac{\int^{x^2}\frac{1}{c^3}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**13/(c*x**4+b*x**2)**3,x)
```

```
[Out] b**3/(4*c**4*(b + c*x**2)**2) - 3*b**2/(2*c**4*(b + c*x**2)) - 3*
b*log(b + c*x**2)/(2*c**4) + Integral(c**(-3), (x, x**2))/2
```

Mathematica [A] time = 0.104605, size = 48, normalized size = 0.74

$$-\frac{\frac{b^2(5b+6cx^2)}{(b+cx^2)^2}+6b\log(b+cx^2)-2cx^2}{4c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^13/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(-2*c*x^2 + (b^2*(5*b + 6*c*x^2))/(b + c*x^2)^2 + 6*b*Log[b + c*
x^2])/(4*c^4)
```

Maple [A] time = 0.015, size = 58, normalized size = 0.9

$$\frac{x^2}{2\,c^3}+\frac{b^3}{4\,c^4\,(cx^2+b)^2}-\frac{3\,b^2}{2\,c^4\,(cx^2+b)}-\frac{3\,b\ln\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^13/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/2*x^2/c^3+1/4*b^3/c^4/(c*x^2+b)^2-3/2*b^2/c^4/(c*x^2+b)-3/2*b*l
n(c*x^2+b)/c^4
```

Maxima [A] time = 0.699089, size = 89, normalized size = 1.37

$$-\frac{6\,b^2cx^2+5\,b^3}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}+\frac{x^2}{2\,c^3}-\frac{3\,b\log\left(cx^2+b\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*b^2*c*x^2 + 5*b^3)/(c^6*x^4 + 2*b*c^5*x^2 + b^2*c^4) + 1/
2*x^2/c^3 - 3/2*b*log(c*x^2 + b)/c^4
```

Fricas [A] time = 0.253847, size = 123, normalized size = 1.89

$$\frac{2\,c^3x^6+4\,bc^2x^4-4\,b^2cx^2-5\,b^3-6\left(bc^2x^4+2\,b^2cx^2+b^3\right)\log\left(cx^2+b\right)}{4\,(c^6x^4+2\,bc^5x^2+b^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*c^3*x^6 + 4*b*c^2*x^4 - 4*b^2*c*x^2 - 5*b^3 - 6*(b*c^2*x^4
 + 2*b^2*c*x^2 + b^3)*log(c*x^2 + b))/(c^6*x^4 + 2*b*c^5*x^2 + b^
2*c^4)
```

Sympy [A] time = 2.06777, size = 66, normalized size = 1.02

$$-\frac{3b\log\left(b+cx^2\right)}{2c^4}-\frac{5b^3+6b^2cx^2}{4b^2c^4+8bc^5x^2+4c^6x^4}+\frac{x^2}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**13/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*b*log(b + c*x**2)/(2*c**4) - (5*b**3 + 6*b**2*c*x**2)/(4*b**2*
c**4 + 8*b*c**5*x**2 + 4*c**6*x**4) + x**2/(2*c**3)
```

GIAC/XCAS [A] time = 0.273912, size = 84, normalized size = 1.29

$$\frac{x^2}{2\,c^3}-\frac{3\,b\ln\left(\left|cx^2+b\right|\right)}{2\,c^4}+\frac{9\,bc^2x^4+12\,b^2cx^2+4\,b^3}{4\,{\left(cx^2+b\right)}^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/c^3 - 3/2*b*ln(abs(c*x^2 + b))/c^4 + 1/4*(9*b*c^2*x^4 + 1
2*b^2*c*x^2 + 4*b^3)/((c*x^2 + b)^2*c^4)
```

3.207 $\int \frac{x^{12}}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=74

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*
x^2)) - (15*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rubi [A] time = 0.0857119, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}-\frac{5x^3}{8c^2\left(b+cx^2\right)}-\frac{x^5}{4c\left(b+cx^2\right)^2}+\frac{15x}{8c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*x)/(8*c^3) - x^5/(4*c*(b + c*x^2)^2) - (5*x^3)/(8*c^2*(b + c*
x^2)) - (15*Sqrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Rubi in Sympy [A] time = 17.6312, size = 66, normalized size = 0.89

$$-\frac{15\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{\frac{7}{2}}}-\frac{x^5}{4c\left(b+cx^2\right)^2}-\frac{5x^3}{8c^2\left(b+cx^2\right)}+\frac{15x}{8c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12/(c*x**4+b*x**2)**3,x)
```

```
[Out] -15*sqrt(b)*atan(sqrt(c)*x/sqrt(b))/(8*c**(7/2)) - x**5/(4*c*(b +
 c*x**2)**2) - 5*x**3/(8*c**2*(b + c*x**2)) + 15*x/(8*c**3)
```

Mathematica [A] time = 0.0894832, size = 66, normalized size = 0.89

$$\frac{15b^2x + 25bcx^3 + 8c^2x^5}{8c^3(b + cx^2)^2} - \frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^12/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (15*b^2*x + 25*b*c*x^3 + 8*c^2*x^5)/(8*c^3*(b + c*x^2)^2) - (15*S
qrt[b]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*c^(7/2))
```

Maple [A] time = 0.013, size = 63, normalized size = 0.9

$$\frac{x}{c^3} + \frac{9\,bx^3}{8\,c^2\,(cx^2 + b)^2} + \frac{7\,b^2x}{8\,c^3\,(cx^2 + b)^2} - \frac{15\,b}{8\,c^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12/(c*x^4+b*x^2)^3,x)
```

```
[Out] x/c^3+9/8/c^2*b/(c*x^2+b)^2*x^3+7/8/c^3*b^2/(c*x^2+b)^2*x-15/8/c^
3*b/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261269, size = 1, normalized size = 0.01

$$\left[\frac{16\,c^2x^5+50\,bcx^3+30\,b^2x+15\,(c^2x^4+2\,bcx^2+b^2)\sqrt{-\frac{b}{c}}\log\left(\frac{cx^2-2\,cx\sqrt{-\frac{b}{c}}-b}{cx^2+b}\right)}{16\,(c^5x^4+2\,bc^4x^2+b^2c^3)},\ \frac{8\,c^2x^5+25\,bcx^3+15\,b^2x-15\,(c^2x^4+2\,bc\ldots}{8\,(c^5x^4+2\,bc^4x^2+b\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*c^2*x^5 + 50*b*c*x^3 + 30*b^2*x + 15*(c^2*x^4 + 2*b*c*x
^2 + b^2)*sqrt(-b/c)*log((c*x^2 - 2*c*x*sqrt(-b/c) - b)/(c*x^2 +
b)))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3), 1/8*(8*c^2*x^5 + 25*b*c*x
^3 + 15*b^2*x - 15*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(b/c)*arctan(x
/sqrt(b/c)))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)]
```

Sympy [A] time = 2.10836, size = 107, normalized size = 1.45

$$\frac{15\sqrt{-\frac{b}{c^7}}\log\left(-c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}-\frac{15\sqrt{-\frac{b}{c^7}}\log\left(c^3\sqrt{-\frac{b}{c^7}}+x\right)}{16}+\frac{7b^2x+9bcx^3}{8b^2c^3+16bc^4x^2+8c^5x^4}+\frac{x}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-b/c**7)*log(-c**3*sqrt(-b/c**7) + x)/16 - 15*sqrt(-b/c**
7)*log(c**3*sqrt(-b/c**7) + x)/16 + (7*b**2*x + 9*b*c*x**3)/(8*b*
*2*c**3 + 16*b*c**4*x**2 + 8*c**5*x**4) + x/c**3
```

GIAC/XCAS [A] time = 0.271776, size = 73, normalized size = 0.99

$$-\frac{15\,b\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}c^3}+\frac{x}{c^3}+\frac{9\,bcx^3+7\,b^2x}{8\,(cx^2+b)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*b*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^3) + x/c^3 + 1/8*(9*b*
c*x^3 + 7*b^2*x)/((c*x^2 + b)^2*c^3)
```

3.208 $\int \frac{x^{11}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

```
[Out] -b^2/(4*c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]
/(2*c^3)
```

Rubi [A] time = 0.0942056, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -b^2/(4*c^3*(b + c*x^2)^2) + b/(c^3*(b + c*x^2)) + Log[b + c*x^2]
/(2*c^3)
```

Rubi in Sympy [A] time = 15.2264, size = 41, normalized size = 0.84

$$-\frac{b^2}{4c^3\left(b+cx^2\right)^2}+\frac{b}{c^3\left(b+cx^2\right)}+\frac{\log\left(b+cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**4+b*x**2)**3,x)
```

```
[Out] -b**2/(4*c**3*(b + c*x**2)**2) + b/(c**3*(b + c*x**2)) + log(b +
c*x**2)/(2*c**3)
```

Mathematica [A] time = 0.0265215, size = 39, normalized size = 0.8

$$\frac{\frac{b\left(3b+4cx^2\right)}{\left(b+cx^2\right)^2}+2\log\left(b+cx^2\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(3*b + 4*c*x^2))/(b + c*x^2)^2 + 2*Log[b + c*x^2])/(4*c^3)
```

Maple [A] time = 0.012, size = 46, normalized size = 0.9

$$-\frac{b^2}{4\,c^3\,(cx^2+b)^2}+\frac{b}{c^3\,(cx^2+b)}+\frac{\ln\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4*b^2/c^3/(c*x^2+b)^2+b/c^3/(c*x^2+b)+1/2*ln(c*x^2+b)/c^3
```

Maxima [A] time = 0.702109, size = 74, normalized size = 1.51

$$\frac{4\,bcx^2+3\,b^2}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)}+\frac{\log\left(cx^2+b\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2)/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3) + 1/2*l
og(c*x^2 + b)/c^3
```

Fricas [A] time = 0.251717, size = 93, normalized size = 1.9

$$\frac{4\,bcx^2+3\,b^2+2\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(cx^2+b\right)}{4\,(c^5x^4+2\,bc^4x^2+b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*c*x^2 + 3*b^2 + 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2
+ b))/(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)
```

Sympy [A] time = 1.83135, size = 53, normalized size = 1.08

$$\frac{3b^2 + 4bcx^2}{4b^2c^3 + 8bc^4x^2 + 4c^5x^4} + \frac{\log\left(b + cx^2\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b**2 + 4*b*c*x**2)/(4*b**2*c**3 + 8*b*c**4*x**2 + 4*c**5*x**4)
 + log(b + c*x**2)/(2*c**3)
```

GIAC/XCAS [A] time = 0.271529, size = 57, normalized size = 1.16

$$\frac{\ln\left(\left|cx^2 + b\right|\right)}{2\,c^3} - \frac{3\,cx^4 + 2\,bx^2}{4\,(cx^2 + b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(c*x^2 + b))/c^3 - 1/4*(3*c*x^4 + 2*b*x^2)/((c*x^2 + b)
^2*c^2)
```

3.209 $\int \frac{x^{10}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=64

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}-\frac{3x}{8c^2\left(b+cx^2\right)}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

```
[Out] -x^3/(4*c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rubi [A] time = 0.0666467, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}-\frac{3x}{8c^2\left(b+cx^2\right)}-\frac{x^3}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^3/(4*c*(b + c*x^2)^2) - (3*x)/(8*c^2*(b + c*x^2)) + (3*ArcTan[
(Sqrt[c]*x)/Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Rubi in Sympy [A] time = 12.7275, size = 56, normalized size = 0.88

$$-\frac{x^3}{4c\left(b+cx^2\right)^2}-\frac{3x}{8c^2\left(b+cx^2\right)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(c*x**4+b*x**2)**3,x)
```

```
[Out] -x**3/(4*c*(b + c*x**2)**2) - 3*x/(8*c**2*(b + c*x**2)) + 3*atan(
sqrt(c)*x/sqrt(b))/(8*sqrt(b)*c**(5/2))
```

Mathematica [A] time = 0.0774676, size = 55, normalized size = 0.86

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8\sqrt{b}c^{5/2}}-\frac{3bx+5cx^3}{8c^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(3*b*x + 5*c*x^3)/(8*c^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/
Sqrt[b]])/(8*Sqrt[b]*c^(5/2))
```

Maple [A] time = 0.01, size = 47, normalized size = 0.7

$$\frac{1}{\left(cx^2+b\right)^2}\left(-\frac{5\,x^3}{8\,c}-\frac{3\,bx}{8\,c^2}\right)+\frac{3}{8\,c^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(c*x^4+b*x^2)^3,x)
```

```
[Out] (-5/8*x^3/c-3/8*b*x/c^2)/(c*x^2+b)^2+3/8/c^2/(b*c)^(1/2)*arctan(c
*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263194, size = 1, normalized size = 0.02

$$\left[\frac{3\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(\frac{2\,bcx+\left(cx^2-b\right)\sqrt{-bc}}{cx^2+b}\right)-2\left(5\,cx^3+3\,bx\right)\sqrt{-bc}}{16\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)\sqrt{-bc}},\frac{3\left(c^2x^4+2\,bcx^2+b^2\right)\arctan\left(\frac{\sqrt{bc}x}{b}\right)-\left(5\,cx^3+3\right.}{8\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)\sqrt{bc}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(c^2*x^4 + 2*b*c*x^2 + b^2)*log((2*b*c*x + (c*x^2 - b)*s
qrt(-b*c))/(c*x^2 + b)) - 2*(5*c*x^3 + 3*b*x)*sqrt(-b*c))/((c^4*x
^4 + 2*b*c^3*x^2 + b^2*c^2)*sqrt(-b*c)), 1/8*(3*(c^2*x^4 + 2*b*c*
x^2 + b^2)*arctan(sqrt(b*c)*x/b) - (5*c*x^3 + 3*b*x)*sqrt(b*c))/(
(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*sqrt(b*c))]
```

Sympy [A] time = 1.87917, size = 109, normalized size = 1.7

$$-\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(-bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{bc^5}}\log\left(bc^2\sqrt{-\frac{1}{bc^5}}+x\right)}{16}-\frac{3bx+5cx^3}{8b^2c^2+16bc^3x^2+8c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b*c**5))*log(-b*c**2*sqrt(-1/(b*c**5)) + x)/16 + 3*sq
rt(-1/(b*c**5))*log(b*c**2*sqrt(-1/(b*c**5)) + x)/16 - (3*b*x + 5
*c*x**3)/(8*b**2*c**2 + 16*b*c**3*x**2 + 8*c**4*x**4)
```

GIAC/XCAS [A] time = 0.270292, size = 61, normalized size = 0.95

$$\frac{3\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}c^2}-\frac{5\,cx^3+3\,bx}{8\,(cx^2+b)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*c^2) - 1/8*(5*c*x^3 + 3*b*x)
/((c*x^2 + b)^2*c^2)
```

3.210 $\int \frac{x^9}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b(b+cx^2)^2}$$

```
[Out] x^4/(4*b*(b + c*x^2)^2)
```

Rubi [A] time = 0.0248147, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$\frac{x^4}{4b(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(b*x^2 + c*x^4)^3,x]

[Out] x^4/(4*b*(b + c*x^2)^2)
```

Rubi in Sympy [A] time = 5.23147, size = 14, normalized size = 0.74

$$\frac{x^4}{4b(b+cx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**4+b*x**2)**3,x)

[Out] x**4/(4*b*(b + c*x**2)**2)
```

Mathematica [A] time = 0.0122253, size = 24, normalized size = 1.26

$$-\frac{b+2cx^2}{4c^2(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b + 2*c*x^2)/(4*c^2*(b + c*x^2)^2)
```

Maple [A] time = 0.011, size = 31, normalized size = 1.6

$$-\frac{1}{(2\,cx^2+2\,b)\,c^2}+\frac{b}{4\,c^2\,(cx^2+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2/(c*x^2+b)/c^2+1/4*b/c^2/(c*x^2+b)^2
```

Maxima [A] time = 0.708922, size = 49, normalized size = 2.58

$$-\frac{2\,cx^2+b}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Fricas [A] time = 0.250397, size = 49, normalized size = 2.58

$$-\frac{2\,cx^2+b}{4\,(c^4x^4+2\,bc^3x^2+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*c*x^2 + b)/(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [A] time = 1.6502, size = 36, normalized size = 1.89

$$-\frac{b+2cx^2}{4b^2c^2+8bc^3x^2+4c^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(b + 2*c*x**2)/(4*b**2*c**2 + 8*b*c**3*x**2 + 4*c**4*x**4)
```

GIAC/XCAS [A] time = 0.269287, size = 30, normalized size = 1.58

$$-\frac{2\,cx^2+b}{4\,{\left(cx^2+b\right)}^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*c*x^2 + b)/((c*x^2 + b)^2*c^2)
```

3.211 $\int \frac{x^8}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=65

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\,(b+cx^2)}-\frac{x}{4c\,(b+cx^2)^2}$$

```
[Out] -x/(4*c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*
x)/Sqrt[b]]/(8*b^(3/2)*c^(3/2))
```

Rubi [A] time = 0.0619164, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}+\frac{x}{8bc\,(b+cx^2)}-\frac{x}{4c\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x/(4*c*(b + c*x^2)^2) + x/(8*b*c*(b + c*x^2)) + ArcTan[(Sqrt[c]*
x)/Sqrt[b]]/(8*b^(3/2)*c^(3/2))
```

Rubi in Sympy [A] time = 10.933, size = 51, normalized size = 0.78

$$-\frac{x}{4c\,(b+cx^2)^2}+\frac{x}{8bc\,(b+cx^2)}+\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**2)**3,x)
```

```
[Out] -x/(4*c*(b + c*x**2)**2) + x/(8*b*c*(b + c*x**2)) + atan(sqrt(c)*
x/sqrt(b))/(8*b**(3/2)*c**(3/2))
```

Mathematica [A] time = 0.0500802, size = 58, normalized size = 0.89

$$\frac{\frac{\sqrt{b}\sqrt{c}x(cx^2-b)}{(b+cx^2)^2}+\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{3/2}c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*Sqrt[c]*x*(-b + c*x^2))/(b + c*x^2)^2 + ArcTan[(Sqrt[c]
*x)/Sqrt[b]])/(8*b^(3/2)*c^(3/2))
```

Maple [A] time = 0.011, size = 49, normalized size = 0.8

$$\frac{1}{(cx^2+b)^2}\left(\frac{x^3}{8\,b}-\frac{x}{8\,c}\right)+\frac{1}{8\,bc}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^2)^3,x)
```

```
[Out] (1/8/b*x^3-1/8*x/c)/(c*x^2+b)^2+1/8/c/b/(b*c)^(1/2)*arctan(c*x/(b
*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262037, size = 1, normalized size = 0.02

$$\left[\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(\frac{2\,bcx+\left(cx^2-b\right)\sqrt{-bc}}{cx^2+b}\right)+2\left(cx^3-bx\right)\sqrt{-bc}}{16\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)\sqrt{-bc}},\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\arctan\left(\frac{\sqrt{bc}x}{b}\right)+\left(cx^3-bx\right)\sqrt{bc}}{8\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)\sqrt{bc}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*((c^2*x^4 + 2*b*c*x^2 + b^2)*log((2*b*c*x + (c*x^2 - b)*sqr
t(-b*c))/(c*x^2 + b)) + 2*(c*x^3 - b*x)*sqrt(-b*c))/((b*c^3*x^4 +
 2*b^2*c^2*x^2 + b^3*c)*sqrt(-b*c)), 1/8*((c^2*x^4 + 2*b*c*x^2 +
b^2)*arctan(sqrt(b*c)*x/b) + (c*x^3 - b*x)*sqrt(b*c))/((b*c^3*x^4
 + 2*b^2*c^2*x^2 + b^3*c)*sqrt(b*c))]
```

Sympy [A] time = 1.78342, size = 110, normalized size = 1.69

$$-\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(-b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{b^3c^3}}\log\left(b^2c\sqrt{-\frac{1}{b^3c^3}}+x\right)}{16}+\frac{-bx+cx^3}{8b^3c+16b^2c^2x^2+8bc^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(-1/(b**3*c**3))*log(-b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 +
sqrt(-1/(b**3*c**3))*log(b**2*c*sqrt(-1/(b**3*c**3)) + x)/16 + (-
b*x + c*x**3)/(8*b**3*c + 16*b**2*c**2*x**2 + 8*b*c**3*x**4)
```

GIAC/XCAS [A] time = 0.271923, size = 68, normalized size = 1.05

$$\frac{\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\sqrt{bc}bc}+\frac{cx^3-bx}{8\left(cx^2+b\right)^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b*c) + 1/8*(c*x^3 - b*x)/((c
*x^2 + b)^2*b*c)
```

3.212 $\int \frac{x^7}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=23

$$-\frac{x^4}{4c\,(bx^2+cx^4)^2}$$

```
[Out] -x^4/(4*c*(b*x^2 + c*x^4)^2)
```

Rubi [A] time = 0.0135321, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$-\frac{x^4}{4c\,(bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(b*x^2 + c*x^4)^3,x]

[Out] -x^4/(4*c*(b*x^2 + c*x^4)^2)
```

Rubi in Sympy [A] time = 4.07219, size = 14, normalized size = 0.61

$$-\frac{1}{4c\,(b+cx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2)**3,x)

[Out] -1/(4*c*(b + c*x**2)**2)
```

Mathematica [A] time = 0.00520676, size = 16, normalized size = 0.7

$$-\frac{1}{4c\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*c*(b + c*x^2)^2)
```

Maple [A] time = 0.001, size = 15, normalized size = 0.7

$$-\frac{1}{4\,c\,(cx^2+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/c/(c*x^2+b)^2
```

Maxima [A] time = 0.689434, size = 35, normalized size = 1.52

$$-\frac{1}{4\,(c^3x^4+2\,bc^2x^2+b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Fricas [A] time = 0.247427, size = 35, normalized size = 1.52

$$-\frac{1}{4\,(c^3x^4+2\,bc^2x^2+b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(c^3*x^4 + 2*b*c^2*x^2 + b^2*c)
```

Sympy [A] time = 1.53875, size = 27, normalized size = 1.17

$$-\frac{1}{4b^2c + 8bc^2x^2 + 4c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2)**3,x)
```

```
[Out] -1/(4*b**2*c + 8*b*c**2*x**2 + 4*c**3*x**4)
```

GIAC/XCAS [A] time = 0.271606, size = 19, normalized size = 0.83

$$-\frac{1}{4\,(cx^2 + b)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/4/((c*x^2 + b)^2*c)
```

3.213 $\int \frac{x^6}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{3x}{8b^2(b+cx^2)}+\frac{x}{4b(b+cx^2)^2}$$

```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Rubi [A] time = 0.0512469, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{3x}{8b^2(b+cx^2)}+\frac{x}{4b(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] x/(4*b*(b + c*x^2)^2) + (3*x)/(8*b^2*(b + c*x^2)) + (3*ArcTan[(Sq
rt[c]*x)/Sqrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Rubi in Sympy [A] time = 8.07891, size = 54, normalized size = 0.87

$$\frac{x}{4b(b+cx^2)^2}+\frac{3x}{8b^2(b+cx^2)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{\frac{5}{2}}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2)**3,x)
```

```
[Out] x/(4*b*(b + c*x**2)**2) + 3*x/(8*b**2*(b + c*x**2)) + 3*atan(sqrt
(c)*x/sqrt(b))/(8*b**(5/2)*sqrt(c))
```

Mathematica [A] time = 0.0677497, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{5/2}\sqrt{c}}+\frac{5bx+3cx^3}{8b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (5*b*x + 3*c*x^3)/(8*b^2*(b + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[b]])/(8*b^(5/2)*Sqrt[c])
```

Maple [A] time = 0.005, size = 51, normalized size = 0.8

$$\frac{x}{4\,b\,(cx^2+b)^2}+\frac{3\,x}{8\,b^2\,(cx^2+b)}+\frac{3}{8\,b^2}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x/b/(c*x^2+b)^2+3/8*x/b^2/(c*x^2+b)+3/8/b^2/(b*c)^(1/2)*arcta
n(c*x/(b*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262632, size = 1, normalized size = 0.02

$$\left[\frac{3\left(c^2x^4+2\,bcx^2+b^2\right)\log\left(\frac{2\,bcx+\left(cx^2-b\right)\sqrt{-bc}}{cx^2+b}\right)+2\left(3\,cx^3+5\,bx\right)\sqrt{-bc}}{16\,(b^2c^2x^4+2\,b^3cx^2+b^4)\sqrt{-bc}},\frac{3\left(c^2x^4+2\,bcx^2+b^2\right)\arctan\left(\frac{\sqrt{bc}x}{b}\right)+\left(3\,cx^3+5\,b\right.}{8\,(b^2c^2x^4+2\,b^3cx^2+b^4)\sqrt{bc}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*(c^2*x^4 + 2*b*c*x^2 + b^2)*log((2*b*c*x + (c*x^2 - b)*s
qrt(-b*c))/(c*x^2 + b)) + 2*(3*c*x^3 + 5*b*x)*sqrt(-b*c))/((b^2*c
^2*x^4 + 2*b^3*c*x^2 + b^4)*sqrt(-b*c)), 1/8*(3*(c^2*x^4 + 2*b*c*
x^2 + b^2)*arctan(sqrt(b*c)*x/b) + (3*c*x^3 + 5*b*x)*sqrt(b*c))/(
(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*sqrt(b*c))]
```

Sympy [A] time = 1.82839, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(-b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{b^5c}}\log\left(b^3\sqrt{-\frac{1}{b^5c}}+x\right)}{16}+\frac{5bx+3cx^3}{8b^4+16b^3cx^2+8b^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2)**3,x)
```

```
[Out] -3*sqrt(-1/(b**5*c))*log(-b**3*sqrt(-1/(b**5*c)) + x)/16 + 3*sqrt
(-1/(b**5*c))*log(b**3*sqrt(-1/(b**5*c)) + x)/16 + (5*b*x + 3*c*x
**3)/(8*b**4 + 16*b**3*c*x**2 + 8*b**2*c**2*x**4)
```

GIAC/XCAS [A] time = 0.271452, size = 61, normalized size = 0.98

$$\frac{3\,\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^2}+\frac{3\,cx^3+5\,bx}{8\,(cx^2+b)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^2) + 1/8*(3*c*x^3 + 5*b*x)
/((c*x^2 + b)^2*b^2)
```

3.214 $\int \frac{x^5}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=54

$$-\frac{\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{1}{4b\left(b+cx^2\right)^2}$$

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[
b + c*x^2]/(2*b^3)
```

Rubi [A] time = 0.0943195, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$-\frac{\log\left(b+cx^2\right)}{2b^3}+\frac{\log(x)}{b^3}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{1}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] 1/(4*b*(b + c*x^2)^2) + 1/(2*b^2*(b + c*x^2)) + Log[x]/b^3 - Log[
b + c*x^2]/(2*b^3)
```

Rubi in Sympy [A] time = 15.1976, size = 49, normalized size = 0.91

$$\frac{1}{4b\left(b+cx^2\right)^2}+\frac{1}{2b^2\left(b+cx^2\right)}+\frac{\log\left(x^2\right)}{2b^3}-\frac{\log\left(b+cx^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*(b + c*x**2)**2) + 1/(2*b**2*(b + c*x**2)) + log(x**2)/(2*
b**3) - log(b + c*x**2)/(2*b**3)
```

Mathematica [A] time = 0.0540378, size = 43, normalized size = 0.8

$$\frac{\frac{b\left(3b+2cx^2\right)}{\left(b+cx^2\right)^2}-2\log\left(b+cx^2\right)+4\log(x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(3*b + 2*c*x^2))/(b + c*x^2)^2 + 4*Log[x] - 2*Log[b + c*x^2])
/(4*b^3)
```

Maple [A] time = 0.016, size = 49, normalized size = 0.9

$$\frac{1}{4\,b\left(cx^{2}+b\right)^{2}}+\frac{1}{2\,b^{2}\left(cx^{2}+b\right)}+\frac{\ln\left(x\right)}{b^{3}}-\frac{\ln\left(cx^{2}+b\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4/b/(c*x^2+b)^2+1/2/b^2/(c*x^2+b)+ln(x)/b^3-1/2*ln(c*x^2+b)/b^3
```

Maxima [A] time = 0.680227, size = 81, normalized size = 1.5

$$\frac{2\,cx^{2}+3\,b}{4\left(b^{2}c^{2}x^{4}+2\,b^{3}cx^{2}+b^{4}\right)}-\frac{\log\left(cx^{2}+b\right)}{2\,b^{3}}+\frac{\log\left(x^{2}\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c*x^2 + 3*b)/(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4) - 1/2*log(c
*x^2 + b)/b^3 + 1/2*log(x^2)/b^3
```

Fricas [A] time = 0.258556, size = 122, normalized size = 2.26

$$\frac{2\,bcx^{2}+3\,b^{2}-2\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\log\left(cx^{2}+b\right)+4\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\log\left(x\right)}{4\left(b^{3}c^{2}x^{4}+2\,b^{4}cx^{2}+b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2 + 3*b^2 - 2*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(c*x^2
+ b) + 4*(c^2*x^4 + 2*b*c*x^2 + b^2)*log(x))/(b^3*c^2*x^4 + 2*b^4
*c*x^2 + b^5)
```

Sympy [A] time = 2.17985, size = 56, normalized size = 1.04

$$\frac{3b + 2cx^2}{4b^4 + 8b^3cx^2 + 4b^2c^2x^4} + \frac{\log(x)}{b^3} - \frac{\log\left(\frac{b}{c} + x^2\right)}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2)**3,x)
```

```
[Out] (3*b + 2*c*x**2)/(4*b**4 + 8*b**3*c*x**2 + 4*b**2*c**2*x**4) + lo
g(x)/b**3 - log(b/c + x**2)/(2*b**3)
```

GIAC/XCAS [A] time = 0.272827, size = 80, normalized size = 1.48

$$\frac{\ln\left(x^2\right)}{2\,b^3} - \frac{\ln\left(\left|cx^2 + b\right|\right)}{2\,b^3} + \frac{3\,c^2x^4 + 8\,bcx^2 + 6\,b^2}{4\,(cx^2 + b)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2)/b^3 - 1/2*ln(abs(c*x^2 + b))/b^3 + 1/4*(3*c^2*x^4 + 8
*b*c*x^2 + 6*b^2)/((c*x^2 + b)^2*b^3)
```

3.215 $\int \frac{x^4}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=76

$$-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2))
 - (15*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rubi [A] time = 0.0855407, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{15}{8b^3x}+\frac{5}{8b^2x\left(b+cx^2\right)}+\frac{1}{4bx\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -15/(8*b^3*x) + 1/(4*b*x*(b + c*x^2)^2) + 5/(8*b^2*x*(b + c*x^2))
 - (15*Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Rubi in Sympy [A] time = 16.8851, size = 65, normalized size = 0.86

$$\frac{1}{4bx\left(b+cx^2\right)^2}+\frac{5}{8b^2x\left(b+cx^2\right)}-\frac{15}{8b^3x}-\frac{15\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x*(b + c*x**2)**2) + 5/(8*b**2*x*(b + c*x**2)) - 15/(8*b**
3*x) - 15*sqrt(c)*atan(sqrt(c)*x/sqrt(b))/(8*b**(7/2))
```

Mathematica [A] time = 0.074645, size = 68, normalized size = 0.89

$$-\frac{15\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{7/2}}-\frac{8b^2+25bcx^2+15c^2x^4}{8b^3x\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -(8*b^2 + 25*b*c*x^2 + 15*c^2*x^4)/(8*b^3*x*(b + c*x^2)^2) - (15*
Sqrt[c]*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(7/2))
```

Maple [A] time = 0.016, size = 66, normalized size = 0.9

$$-\frac{7\,c^2x^3}{8\,b^3\left(cx^2+b\right)^2}-\frac{9\,cx}{8\,b^2\left(cx^2+b\right)^2}-\frac{15\,c}{8\,b^3}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2)^3,x)
```

```
[Out] -7/8/b^3*c^2/(c*x^2+b)^2*x^3-9/8/b^2*c/(c*x^2+b)^2*x-15/8/b^3*c/(
b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/b^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264044, size = 1, normalized size = 0.01

$$\left[-\frac{30\,c^{2}x^{4}+50\,bcx^{2}-15\,\left(c^{2}x^{5}+2\,bcx^{3}+b^{2}x\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^{2}-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^{2}+b}\right)+16\,b^{2}}{16\,\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)},\right.$$
$$\left.-\frac{15\,c^{2}x^{4}+25\,bcx^{2}+15\,\left(c^{2}x^{5}+2\,bcx^{3}+b^{2}x\right)\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)+8\,b^{2}}{8\,\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(30*c^2*x^4 + 50*b*c*x^2 - 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x
)*sqrt(-c/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)) + 16
*b^2)/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x), -1/8*(15*c^2*x^4 + 25*
b*c*x^2 + 15*(c^2*x^5 + 2*b*c*x^3 + b^2*x)*sqrt(c/b)*arctan(c*x/(
b*sqrt(c/b))) + 8*b^2)/(b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)]
```

Sympy [A] time = 2.3972, size = 114, normalized size = 1.5

$$\frac{15\sqrt{-\frac{c}{b^{7}}}\log\left(-\frac{b^{4}\sqrt{-\frac{c}{b^{7}}}}{c}+x\right)}{16}-\frac{15\sqrt{-\frac{c}{b^{7}}}\log\left(\frac{b^{4}\sqrt{-\frac{c}{b^{7}}}}{c}+x\right)}{16}-\frac{8b^{2}+25bcx^{2}+15c^{2}x^{4}}{8b^{5}x+16b^{4}cx^{3}+8b^{3}c^{2}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2)**3,x)
```

```
[Out] 15*sqrt(-c/b**7)*log(-b**4*sqrt(-c/b**7)/c + x)/16 - 15*sqrt(-c/b
**7)*log(b**4*sqrt(-c/b**7)/c + x)/16 - (8*b**2 + 25*b*c*x**2 + 1
5*c**2*x**4)/(8*b**5*x + 16*b**4*c*x**3 + 8*b**3*c**2*x**5)
```

GIAC/XCAS [A] time = 0.271848, size = 77, normalized size = 1.01

$$-\frac{15\,c\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^{3}}-\frac{7\,c^{2}x^{3}+9\,bcx}{8\,\left(cx^{2}+b\right)^{2}b^{3}}-\frac{1}{b^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -15/8*c*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^3) - 1/8*(7*c^2*x^3 +
9*b*c*x)/((c*x^2 + b)^2*b^3) - 1/(b^3*x)
```

3.216 $\int \frac{x^3}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=67

$$\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}$$

```
[Out] -1/(2*b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) -
(3*c*Log[x])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rubi [A] time = 0.121368, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{3c\log\left(b+cx^2\right)}{2b^4}-\frac{3c\log(x)}{b^4}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{1}{2b^3x^2}-\frac{c}{4b^2\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(2*b^3*x^2) - c/(4*b^2*(b + c*x^2)^2) - c/(b^3*(b + c*x^2)) -
(3*c*Log[x])/b^4 + (3*c*Log[b + c*x^2])/(2*b^4)
```

Rubi in Sympy [A] time = 19.1503, size = 66, normalized size = 0.99

$$-\frac{c}{4b^2\left(b+cx^2\right)^2}-\frac{c}{b^3\left(b+cx^2\right)}-\frac{1}{2b^3x^2}-\frac{3c\log\left(x^2\right)}{2b^4}+\frac{3c\log\left(b+cx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2)**3,x)
```

```
[Out] -c/(4*b**2*(b + c*x**2)**2) - c/(b**3*(b + c*x**2)) - 1/(2*b**3*x
**2) - 3*c*log(x**2)/(2*b**4) + 3*c*log(b + c*x**2)/(2*b**4)
```

Mathematica [A] time = 0.0976038, size = 59, normalized size = 0.88

$$-\frac{\frac{b\left(2b^2+9bcx^2+6c^2x^4\right)}{x^2\left(b+cx^2\right)^2}-6c\log\left(b+cx^2\right)+12c\log(x)}{4b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b*(2*b^2 + 9*b*c*x^2 + 6*c^2*x^4))/(x^2*(b + c*x^2)^2) + 12*c*
Log[x] - 6*c*Log[b + c*x^2])/(4*b^4)
```

Maple [A] time = 0.019, size = 62, normalized size = 0.9

$$-\frac{1}{2\,b^3x^2}-\frac{c}{4\,b^2\left(cx^2+b\right)^2}-\frac{c}{b^3\left(cx^2+b\right)}-3\,\frac{c\ln\left(x\right)}{b^4}+\frac{3\,c\ln\left(cx^2+b\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/2/b^3/x^2-1/4*c/b^2/(c*x^2+b)^2-c/b^3/(c*x^2+b)-3*c*ln(x)/b^4+
3/2*c*ln(c*x^2+b)/b^4
```

Maxima [A] time = 0.676693, size = 104, normalized size = 1.55

$$-\frac{6\,c^2x^4+9\,bcx^2+2\,b^2}{4\,(b^3c^2x^6+2\,b^4cx^4+b^5x^2)}+\frac{3\,c\log\left(cx^2+b\right)}{2\,b^4}-\frac{3\,c\log\left(x^2\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(6*c^2*x^4 + 9*b*c*x^2 + 2*b^2)/(b^3*c^2*x^6 + 2*b^4*c*x^4 +
 b^5*x^2) + 3/2*c*log(c*x^2 + b)/b^4 - 3/2*c*log(x^2)/b^4
```

Fricas [A] time = 0.25849, size = 161, normalized size = 2.4

$$-\frac{6\,bc^2x^4+9\,b^2cx^2+2\,b^3-6\left(c^3x^6+2\,bc^2x^4+b^2cx^2\right)\log\left(cx^2+b\right)+12\left(c^3x^6+2\,bc^2x^4+b^2cx^2\right)\log\left(x\right)}{4\,(b^4c^2x^6+2\,b^5cx^4+b^6x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*b*c^2*x^4 + 9*b^2*c*x^2 + 2*b^3 - 6*(c^3*x^6 + 2*b*c^2*x^
4 + b^2*c*x^2)*log(c*x^2 + b) + 12*(c^3*x^6 + 2*b*c^2*x^4 + b^2*c
*x^2)*log(x))/(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)
```

Sympy [A] time = 2.9944, size = 78, normalized size = 1.16

$$-\frac{2b^2 + 9bcx^2 + 6c^2x^4}{4b^5x^2 + 8b^4cx^4 + 4b^3c^2x^6} - \frac{3c\log(x)}{b^4} + \frac{3c\log\left(\frac{b}{c} + x^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(2*b**2 + 9*b*c*x**2 + 6*c**2*x**4)/(4*b**5*x**2 + 8*b**4*c*x**4
 + 4*b**3*c**2*x**6) - 3*c*log(x)/b**4 + 3*c*log(b/c + x**2)/(2*b
**4)
```

GIAC/XCAS [A] time = 0.275326, size = 89, normalized size = 1.33

$$\frac{3\,c\ln\left(\left|cx^2 + b\right|\right)}{2\,b^4} - \frac{3\,c\ln\left(|x|\right)}{b^4} - \frac{6\,bc^2x^4 + 9\,b^2cx^2 + 2\,b^3}{4\,(cx^2 + b)^2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/2*c*ln(abs(c*x^2 + b))/b^4 - 3*c*ln(abs(x))/b^4 - 1/4*(6*b*c^2*
x^4 + 9*b^2*c*x^2 + 2*b^3)/((c*x^2 + b)^2*b^4*x^2)
```

3.217 $\int \frac{x^2}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=87

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{7}{8b^2x^3\,(b+cx^2)}+\frac{1}{4bx^3\,(b+cx^2)^2}$$

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) +
 7/(8*b^2*x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(8*b^(9/2))
```

Rubi [A] time = 0.103409, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{35c}{8b^4x}-\frac{35}{24b^3x^3}+\frac{7}{8b^2x^3\,(b+cx^2)}+\frac{1}{4bx^3\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -35/(24*b^3*x^3) + (35*c)/(8*b^4*x) + 1/(4*b*x^3*(b + c*x^2)^2) +
 7/(8*b^2*x^3*(b + c*x^2)) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[
b]])/(8*b^(9/2))
```

Rubi in Sympy [A] time = 21.2951, size = 80, normalized size = 0.92

$$\frac{1}{4bx^3\,(b+cx^2)^2}+\frac{7}{8b^2x^3\,(b+cx^2)}-\frac{35}{24b^3x^3}+\frac{35c}{8b^4x}+\frac{35c^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x**3*(b + c*x**2)**2) + 7/(8*b**2*x**3*(b + c*x**2)) - 35/
(24*b**3*x**3) + 35*c/(8*b**4*x) + 35*c**(3/2)*atan(sqrt(c)*x/sqr
t(b))/(8*b**(9/2))
```

Mathematica [A] time = 0.0837642, size = 79, normalized size = 0.91

$$\frac{35c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{9/2}}+\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^4x^3\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (-8*b^3 + 56*b^2*c*x^2 + 175*b*c^2*x^4 + 105*c^3*x^6)/(24*b^4*x^3
*(b + c*x^2)^2) + (35*c^(3/2)*ArcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(
9/2))
```

Maple [A] time = 0.018, size = 79, normalized size = 0.9

$$\frac{11\,c^3x^3}{8\,b^4\left(cx^2+b\right)^2}+\frac{13\,c^2x}{8\,b^3\left(cx^2+b\right)^2}+\frac{35\,c^2}{8\,b^4}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{3\,b^3x^3}+3\,\frac{c}{b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2)^3,x)
```

```
[Out] 11/8/b^4*c^3/(c*x^2+b)^2*x^3+13/8/b^3*c^2/(c*x^2+b)^2*x+35/8/b^4*
c^2/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/3/b^3/x^3+3*c/b^4/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263777, size = 1, normalized size = 0.01

$$\left[\frac{210\,c^3x^6+350\,bc^2x^4+112\,b^2cx^2-16\,b^3+105\,\left(c^3x^7+2\,bc^2x^5+b^2cx^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2+2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{48\,(b^4c^2x^7+2\,b^5cx^5+b^6x^3)}, \frac{105\,c^3x^6+175\,bc^2x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(210*c^3*x^6 + 350*b*c^2*x^4 + 112*b^2*c*x^2 - 16*b^3 + 105
*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(-c/b)*log((c*x^2 + 2*b*
x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*
x^3), 1/24*(105*c^3*x^6 + 175*b*c^2*x^4 + 56*b^2*c*x^2 - 8*b^3 +
105*(c^3*x^7 + 2*b*c^2*x^5 + b^2*c*x^3)*sqrt(c/b)*arctan(c*x/(b*s
qrt(c/b))))/(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)]
```

Sympy [A] time = 3.3217, size = 138, normalized size = 1.59

$$-\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(-\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{35\sqrt{-\frac{c^3}{b^9}}\log\left(\frac{b^5\sqrt{-\frac{c^3}{b^9}}}{c^2}+x\right)}{16}+\frac{-8b^3+56b^2cx^2+175bc^2x^4+105c^3x^6}{24b^6x^3+48b^5cx^5+24b^4c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2)**3,x)
```

```
[Out] -35*sqrt(-c**3/b**9)*log(-b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + 35
*sqrt(-c**3/b**9)*log(b**5*sqrt(-c**3/b**9)/c**2 + x)/16 + (-8*b*
*3 + 56*b**2*c*x**2 + 175*b*c**2*x**4 + 105*c**3*x**6)/(24*b**6*x
**3 + 48*b**5*c*x**5 + 24*b**4*c**2*x**7)
```

GIAC/XCAS [A] time = 0.272892, size = 96, normalized size = 1.1

$$\frac{35\,c^2\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^4}+\frac{11\,c^3x^3+13\,bc^2x}{8\,(cx^2+b)^2b^4}+\frac{9\,cx^2-b}{3\,b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 35/8*c^2*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^4) + 1/8*(11*c^3*x^3
+ 13*b*c^2*x)/((c*x^2 + b)^2*b^4) + 1/3*(9*c*x^2 - b)/(b^4*x^3)
```

3.218 $\int \frac{x}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=86

$$-\frac{3c^2 \log \left(b+cx^2\right)}{b^5}+\frac{6c^2 \log(x)}{b^5}+\frac{3c^2}{2b^4 \left(b+cx^2\right)}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3 \left(b+cx^2\right)^2}-\frac{1}{4b^3x^4}$$

```
[Out] -1/(4*b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) +
(3*c^2)/(2*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b +
 c*x^2])/b^5
```

Rubi [A] time = 0.149733, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3c^2 \log \left(b+cx^2\right)}{b^5}+\frac{6c^2 \log(x)}{b^5}+\frac{3c^2}{2b^4 \left(b+cx^2\right)}+\frac{3c}{2b^4x^2}+\frac{c^2}{4b^3 \left(b+cx^2\right)^2}-\frac{1}{4b^3x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/(4*b^3*x^4) + (3*c)/(2*b^4*x^2) + c^2/(4*b^3*(b + c*x^2)^2) +
(3*c^2)/(2*b^4*(b + c*x^2)) + (6*c^2*Log[x])/b^5 - (3*c^2*Log[b +
 c*x^2])/b^5
```

Rubi in Sympy [A] time = 23.754, size = 85, normalized size = 0.99

$$\frac{c^2}{4b^3 \left(b+cx^2\right)^2}-\frac{1}{4b^3x^4}+\frac{3c^2}{2b^4 \left(b+cx^2\right)}+\frac{3c}{2b^4x^2}+\frac{3c^2 \log \left(x^2\right)}{b^5}-\frac{3c^2 \log \left(b+cx^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2)**3,x)
```

```
[Out] c**2/(4*b**3*(b + c*x**2)**2) - 1/(4*b**3*x**4) + 3*c**2/(2*b**4*
(b + c*x**2)) + 3*c/(2*b**4*x**2) + 3*c**2*log(x**2)/b**5 - 3*c**
2*log(b + c*x**2)/b**5
```

Mathematica [A] time = 0.0870258, size = 74, normalized size = 0.86

$$\frac{\frac{b(-b^3+4b^2cx^2+18bc^2x^4+12c^3x^6)}{x^4(b+cx^2)^2}-12c^2\log\left(b+cx^2\right)+24c^2\log(x)}{4b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b*(-b^3 + 4*b^2*c*x^2 + 18*b*c^2*x^4 + 12*c^3*x^6))/(x^4*(b + c
*x^2)^2) + 24*c^2*Log[x] - 12*c^2*Log[b + c*x^2])/(4*b^5)
```

Maple [A] time = 0.02, size = 79, normalized size = 0.9

$$-\frac{1}{4\,b^3x^4}+\frac{3\,c}{2\,b^4x^2}+\frac{c^2}{4\,b^3\,(cx^2+b)^2}+\frac{3\,c^2}{2\,b^4\,(cx^2+b)}+6\,\frac{c^2\ln(x)}{b^5}-3\,\frac{c^2\ln\left(cx^2+b\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/4/b^3/x^4+3/2*c/b^4/x^2+1/4*c^2/b^3/(c*x^2+b)^2+3/2*c^2/b^4/(c
*x^2+b)+6*c^2*ln(x)/b^5-3*c^2*ln(c*x^2+b)/b^5
```

Maxima [A] time = 0.687798, size = 124, normalized size = 1.44

$$\frac{12\,c^3x^6+18\,bc^2x^4+4\,b^2cx^2-b^3}{4\,(b^4c^2x^8+2\,b^5cx^6+b^6x^4)}-\frac{3\,c^2\log\left(cx^2+b\right)}{b^5}+\frac{3\,c^2\log\left(x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(12*c^3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/(b^4*c^2*x^8
+ 2*b^5*c*x^6 + b^6*x^4) - 3*c^2*log(c*x^2 + b)/b^5 + 3*c^2*log(x
^2)/b^5
```

Fricas [A] time = 0.257294, size = 181, normalized size = 2.1

$$\frac{12\,bc^3x^6+18\,b^2c^2x^4+4\,b^3cx^2-b^4-12\left(c^4x^8+2\,bc^3x^6+b^2c^2x^4\right)\log\left(cx^2+b\right)+24\left(c^4x^8+2\,bc^3x^6+b^2c^2x^4\right)\log(x)}{4\,(b^5c^2x^8+2\,b^6cx^6+b^7x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(12*b*c^3*x^6 + 18*b^2*c^2*x^4 + 4*b^3*c*x^2 - b^4 - 12*(c^4*
x^8 + 2*b*c^3*x^6 + b^2*c^2*x^4)*log(c*x^2 + b) + 24*(c^4*x^8 + 2
*b*c^3*x^6 + b^2*c^2*x^4)*log(x))/(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^
7*x^4)
```

Sympy [A] time = 4.30609, size = 90, normalized size = 1.05

$$\frac{-b^3 + 4b^2cx^2 + 18bc^2x^4 + 12c^3x^6}{4b^6x^4 + 8b^5cx^6 + 4b^4c^2x^8} + \frac{6c^2\log(x)}{b^5} - \frac{3c^2\log\left(\frac{b}{c} + x^2\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2)**3,x)
```

```
[Out] (-b**3 + 4*b**2*c*x**2 + 18*b*c**2*x**4 + 12*c**3*x**6)/(4*b**6*x
**4 + 8*b**5*c*x**6 + 4*b**4*c**2*x**8) + 6*c**2*log(x)/b**5 - 3*
c**2*log(b/c + x**2)/b**5
```

GIAC/XCAS [A] time = 0.273102, size = 107, normalized size = 1.24

$$-\frac{3\,c^2\ln\left(\left|cx^2 + b\right|\right)}{b^5} + \frac{6\,c^2\ln\left(|x|\right)}{b^5} + \frac{12\,c^3x^6 + 18\,bc^2x^4 + 4\,b^2cx^2 - b^3}{4\,(cx^4 + bx^2)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -3*c^2*ln(abs(c*x^2 + b))/b^5 + 6*c^2*ln(abs(x))/b^5 + 1/4*(12*c^
3*x^6 + 18*b*c^2*x^4 + 4*b^2*c*x^2 - b^3)/((c*x^4 + b*x^2)^2*b^4)
```

3.219 $\int \frac{1}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=100

$$-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{63c^2}{8b^5x}+\frac{21c}{8b^4x^3}-\frac{63}{40b^3x^5}+\frac{9}{8b^2x^5(b+cx^2)}+\frac{1}{4bx^5(b+cx^2)^2}$$

```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4
*b*x^5*(b + c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(11/2))
```

Rubi [A] time = 0.129774, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{63c^2}{8b^5x}+\frac{21c}{8b^4x^3}-\frac{63}{40b^3x^5}+\frac{9}{8b^2x^5(b+cx^2)}+\frac{1}{4bx^5(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -63/(40*b^3*x^5) + (21*c)/(8*b^4*x^3) - (63*c^2)/(8*b^5*x) + 1/(4
*b*x^5*(b + c*x^2)^2) + 9/(8*b^2*x^5*(b + c*x^2)) - (63*c^(5/2)*A
rcTan[(Sqrt[c]*x)/Sqrt[b]])/(8*b^(11/2))
```

Rubi in Sympy [A] time = 29.9508, size = 94, normalized size = 0.94

$$\frac{1}{4bx^5(b+cx^2)^2}+\frac{9}{8b^2x^5(b+cx^2)}-\frac{63}{40b^3x^5}+\frac{21c}{8b^4x^3}-\frac{63c^2}{8b^5x}-\frac{63c^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x**5*(b + c*x**2)**2) + 9/(8*b**2*x**5*(b + c*x**2)) - 63/
(40*b**3*x**5) + 21*c/(8*b**4*x**3) - 63*c**2/(8*b**5*x) - 63*c**
(5/2)*atan(sqrt(c)*x/sqrt(b))/(8*b**(11/2))
```

Mathematica [A] time = 0.101033, size = 90, normalized size = 0.9

$$-\frac{63c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b}}\right)}{8b^{11/2}}-\frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^5x^5(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(-3),x]
```

```
[Out] -(8*b^4 - 24*b^3*c*x^2 + 168*b^2*c^2*x^4 + 525*b*c^3*x^6 + 315*c^
4*x^8)/(40*b^5*x^5*(b + c*x^2)^2) - (63*c^(5/2)*ArcTan[(Sqrt[c]*x
)/Sqrt[b]])/(8*b^(11/2))
```

Maple [A] time = 0.018, size = 89, normalized size = 0.9

$$-\frac{15\,c^4x^3}{8\,b^5\,(cx^2+b)^2}-\frac{17\,c^3x}{8\,b^4\,(cx^2+b)^2}-\frac{63\,c^3}{8\,b^5}\arctan\left(cx\frac{1}{\sqrt{bc}}\right)\frac{1}{\sqrt{bc}}-\frac{1}{5\,b^3x^5}-6\,\frac{c^2}{b^5x}+\frac{c}{b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/8/b^5*c^4/(c*x^2+b)^2*x^3-17/8/b^4*c^3/(c*x^2+b)^2*x-63/8/b^5
*c^3/(b*c)^(1/2)*arctan(c*x/(b*c)^(1/2))-1/5/b^3/x^5-6*c^2/b^5/x+
c/b^4/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265939, size = 1, normalized size = 0.01

$$\left[-\frac{630\,c^4x^8+1050\,bc^3x^6+336\,b^2c^2x^4-48\,b^3cx^2+16\,b^4-315\,\left(c^4x^9+2\,bc^3x^7+b^2c^2x^5\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx^2-2\,bx\sqrt{-\frac{c}{b}}-b}{cx^2+b}\right)}{80\,\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)},\right.$$
$$\left.-\frac{315\,c^4x^8+525\,bc^3x^6+168\,b^2c^2x^4-24\,b^3cx^2+8\,b^4+315\,\left(c^4x^9+2\,bc^3x^7+b^2c^2x^5\right)\sqrt{\frac{c}{b}}\arctan\left(\frac{cx}{b\sqrt{\frac{c}{b}}}\right)}{40\,\left(b^5c^2x^9+2\,b^6cx^7+b^7x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-3),x, algorithm="fricas")
```

```
[Out] [-1/80*(630*c^4*x^8 + 1050*b*c^3*x^6 + 336*b^2*c^2*x^4 - 48*b^3*c
*x^2 + 16*b^4 - 315*(c^4*x^9 + 2*b*c^3*x^7 + b^2*c^2*x^5)*sqrt(-c
/b)*log((c*x^2 - 2*b*x*sqrt(-c/b) - b)/(c*x^2 + b)))/(b^5*c^2*x^9
 + 2*b^6*c*x^7 + b^7*x^5), -1/40*(315*c^4*x^8 + 525*b*c^3*x^6 + 1
68*b^2*c^2*x^4 - 24*b^3*c*x^2 + 8*b^4 + 315*(c^4*x^9 + 2*b*c^3*x^
7 + b^2*c^2*x^5)*sqrt(c/b)*arctan(c*x/(b*sqrt(c/b))))/(b^5*c^2*x^
9 + 2*b^6*c*x^7 + b^7*x^5)]
```

Sympy [A] time = 5.51313, size = 150, normalized size = 1.5

$$\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(-\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}-\frac{63\sqrt{-\frac{c^5}{b^{11}}}\log\left(\frac{b^6\sqrt{-\frac{c^5}{b^{11}}}}{c^3}+x\right)}{16}-\frac{8b^4-24b^3cx^2+168b^2c^2x^4+525bc^3x^6+315c^4x^8}{40b^7x^5+80b^6cx^7+40b^5c^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**3,x)
```

```
[Out] 63*sqrt(-c**5/b**11)*log(-b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - 6
3*sqrt(-c**5/b**11)*log(b**6*sqrt(-c**5/b**11)/c**3 + x)/16 - (8*
b**4 - 24*b**3*c*x**2 + 168*b**2*c**2*x**4 + 525*b*c**3*x**6 + 31
5*c**4*x**8)/(40*b**7*x**5 + 80*b**6*c*x**7 + 40*b**5*c**2*x**9)
```

GIAC/XCAS [A] time = 0.269743, size = 108, normalized size = 1.08

$$-\frac{63\,c^{3}\arctan\left(\frac{cx}{\sqrt{bc}}\right)}{8\,\sqrt{bc}b^{5}}-\frac{15\,c^{4}x^{3}+17\,bc^{3}x}{8\,(cx^{2}+b)^{2}b^{5}}-\frac{30\,c^{2}x^{4}-5\,bcx^{2}+b^{2}}{5\,b^{5}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-3),x, algorithm="giac")
```

```
[Out] -63/8*c^3*arctan(c*x/sqrt(b*c))/(sqrt(b*c)*b^5) - 1/8*(15*c^4*x^3
 + 17*b*c^3*x)/((c*x^2 + b)^2*b^5) - 1/5*(30*c^2*x^4 - 5*b*c*x^2
+ b^2)/(b^5*x^5)
```

3.220 $\int \frac{1}{x(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=95

$$\frac{5c^3\log\left(b+cx^2\right)}{b^6}-\frac{10c^3\log(x)}{b^6}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

```
[Out] -1/(6*b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b
^4*(b + c*x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b
^6 + (5*c^3*Log[b + c*x^2])/b^6
```

Rubi [A] time = 0.177392, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{5c^3\log\left(b+cx^2\right)}{b^6}-\frac{10c^3\log(x)}{b^6}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{3c^2}{b^5x^2}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}+\frac{3c}{4b^4x^4}-\frac{1}{6b^3x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/(6*b^3*x^6) + (3*c)/(4*b^4*x^4) - (3*c^2)/(b^5*x^2) - c^3/(4*b
^4*(b + c*x^2)^2) - (2*c^3)/(b^5*(b + c*x^2)) - (10*c^3*Log[x])/b
^6 + (5*c^3*Log[b + c*x^2])/b^6
```

Rubi in Sympy [A] time = 26.2399, size = 95, normalized size = 1.

$$-\frac{1}{6b^3x^6}-\frac{c^3}{4b^4\left(b+cx^2\right)^2}+\frac{3c}{4b^4x^4}-\frac{2c^3}{b^5\left(b+cx^2\right)}-\frac{3c^2}{b^5x^2}-\frac{5c^3\log\left(x^2\right)}{b^6}+\frac{5c^3\log\left(b+cx^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] -1/(6*b**3*x**6) - c**3/(4*b**4*(b + c*x**2)**2) + 3*c/(4*b**4*x*
*4) - 2*c**3/(b**5*(b + c*x**2)) - 3*c**2/(b**5*x**2) - 5*c**3*lo
g(x**2)/b**6 + 5*c**3*log(b + c*x**2)/b**6
```

Mathematica [A] time = 0.123232, size = 85, normalized size = 0.89

$$-\frac{\frac{b(2b^4-5b^3cx^2+20b^2c^2x^4+90bc^3x^6+60c^4x^8)}{x^6(b+cx^2)^2}-60c^3\log\left(b+cx^2\right)+120c^3\log(x)}{12b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -((b*(2*b^4 - 5*b^3*c*x^2 + 20*b^2*c^2*x^4 + 90*b*c^3*x^6 + 60*c^
4*x^8))/(x^6*(b + c*x^2)^2) + 120*c^3*Log[x] - 60*c^3*Log[b + c*x
^2])/(12*b^6)
```

Maple [A] time = 0.021, size = 90, normalized size = 1.

$$-\frac{1}{6\,b^3x^6}+\frac{3\,c}{4\,b^4x^4}-3\,\frac{c^2}{b^5x^2}-\frac{c^3}{4\,b^4\,(cx^2+b)^2}-2\,\frac{c^3}{b^5\,(cx^2+b)}-10\,\frac{c^3\ln(x)}{b^6}+5\,\frac{c^3\ln\left(cx^2+b\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2)^3,x)
```

```
[Out] -1/6/b^3/x^6+3/4*c/b^4/x^4-3*c^2/b^5/x^2-1/4*c^3/b^4/(c*x^2+b)^2-
2*c^3/b^5/(c*x^2+b)-10*c^3*ln(x)/b^6+5*c^3*ln(c*x^2+b)/b^6
```

Maxima [A] time = 0.727316, size = 139, normalized size = 1.46

$$-\frac{60\,c^4x^8+90\,bc^3x^6+20\,b^2c^2x^4-5\,b^3cx^2+2\,b^4}{12\,(b^5c^2x^{10}+2\,b^6cx^8+b^7x^6)}+\frac{5\,c^3\log\left(cx^2+b\right)}{b^6}-\frac{5\,c^3\log\left(x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^3*x),x, algorithm="maxima")
```

```
[Out] -1/12*(60*c^4*x^8 + 90*b*c^3*x^6 + 20*b^2*c^2*x^4 - 5*b^3*c*x^2 +
 2*b^4)/(b^5*c^2*x^10 + 2*b^6*c*x^8 + b^7*x^6) + 5*c^3*log(c*x^2
+ b)/b^6 - 5*c^3*log(x^2)/b^6
```

Fricas [A] time = 0.2561, size = 196, normalized size = 2.06

$$-\frac{60\,bc^4x^8 + 90\,b^2c^3x^6 + 20\,b^3c^2x^4 - 5\,b^4cx^2 + 2\,b^5 - 60\,\left(c^5x^{10} + 2\,bc^4x^8 + b^2c^3x^6\right)\log\left(cx^2 + b\right) + 120\,\left(c^5x^{10} + 2\,bc^4x^8 + b^2\right.}{12\,(b^6c^2x^{10} + 2\,b^7cx^8 + b^8x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^3*x),x, algorithm="fricas")
```

```
[Out] -1/12*(60*b*c^4*x^8 + 90*b^2*c^3*x^6 + 20*b^3*c^2*x^4 - 5*b^4*c*x
^2 + 2*b^5 - 60*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(c*x^2
+ b) + 120*(c^5*x^10 + 2*b*c^4*x^8 + b^2*c^3*x^6)*log(x))/(b^6*c^
2*x^10 + 2*b^7*c*x^8 + b^8*x^6)
```

Sympy [A] time = 7.50238, size = 104, normalized size = 1.09

$$-\frac{2b^4 - 5b^3cx^2 + 20b^2c^2x^4 + 90bc^3x^6 + 60c^4x^8}{12b^7x^6 + 24b^6cx^8 + 12b^5c^2x^{10}} - \frac{10c^3\log(x)}{b^6} + \frac{5c^3\log\left(\frac{b}{c} + x^2\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2)**3,x)
```

```
[Out] -(2*b**4 - 5*b**3*c*x**2 + 20*b**2*c**2*x**4 + 90*b*c**3*x**6 + 6
0*c**4*x**8)/(12*b**7*x**6 + 24*b**6*c*x**8 + 12*b**5*c**2*x**10)
 - 10*c**3*log(x)/b**6 + 5*c**3*log(b/c + x**2)/b**6
```

GIAC/XCAS [A] time = 0.271741, size = 149, normalized size = 1.57

$$-\frac{5\,c^3\ln\left(x^2\right)}{b^6} + \frac{5\,c^3\ln\left(\left|cx^2 + b\right|\right)}{b^6} - \frac{30\,c^5x^4 + 68\,bc^4x^2 + 39\,b^2c^3}{4\,(cx^2 + b)^2b^6} + \frac{110\,c^3x^6 - 36\,bc^2x^4 + 9\,b^2cx^2 - 2\,b^3}{12\,b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^3*x),x, algorithm="giac")
```

```
[Out] -5*c^3*ln(x^2)/b^6 + 5*c^3*ln(abs(c*x^2 + b))/b^6 - 1/4*(30*c^5*x
^4 + 68*b*c^4*x^2 + 39*b^2*c^3)/((c*x^2 + b)^2*b^6) + 1/12*(110*c
^3*x^6 - 36*b*c^2*x^4 + 9*b^2*c*x^2 - 2*b^3)/(b^6*x^6)
```

3.221 $\int x^5\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=119

$$-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2
 + c*x^4)^(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (
5*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rubi [A] time = 0.265464, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$-\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{7/2}}+\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{3/2}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^3) - (5*b*(b*x^2
 + c*x^4)^(3/2))/(48*c^2) + (x^2*(b*x^2 + c*x^4)^(3/2))/(8*c) - (
5*b^4*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(128*c^(7/2))
```

Rubi in Sympy [A] time = 24.2626, size = 109, normalized size = 0.92

$$-\frac{5b^4\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{\frac{7}{2}}}+\frac{5b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^3}-\frac{5b\left(bx^2+cx^4\right)^{\frac{3}{2}}}{48c^2}+\frac{x^2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -5*b**4*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(128*c**(7/2))
+ 5*b**2*(b + 2*c*x**2)*sqrt(b*x**2 + c*x**4)/(128*c**3) - 5*b*(b
*x**2 + c*x**4)**(3/2)/(48*c**2) + x**2*(b*x**2 + c*x**4)**(3/2)/
(8*c)
```

Mathematica [A] time = 0.156715, size = 114, normalized size = 0.96

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\left(15b^3-10b^2cx^2+8bc^2x^4+48c^3x^6\right)-15b^4\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{384c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^3 - 10*b^2*c*
x^2 + 8*b*c^2*x^4 + 48*c^3*x^6) - 15*b^4*Log[c*x + Sqrt[c]*Sqrt[b
 + c*x^2]]))/(384*c^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.015, size = 124, normalized size = 1.

$$-\frac{1}{384\,x}\sqrt{cx^4+bx^2}\left(-48\,x^5\left(cx^2+b\right)^{3/2}c^{5/2}+40\left(cx^2+b\right)^{3/2}c^{3/2}x^3b-30\left(cx^2+b\right)^{3/2}\sqrt{c}xb^2+15\sqrt{cx^2+b}\sqrt{c}xb^3+15\,\mathrm{ln}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/384*(c*x^4+b*x^2)^(1/2)*(-48*x^5*(c*x^2+b)^(3/2)*c^(5/2)+40*(c
*x^2+b)^(3/2)*c^(3/2)*x^3*b-30*(c*x^2+b)^(3/2)*c^(1/2)*x*b^2+15*(
c*x^2+b)^(1/2)*c^(1/2)*x*b^3+15*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4
)/x/(c*x^2+b)^(1/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292881, size = 1, normalized size = 0.01

$$\left[\frac{15\,b^4\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\,\left(48\,c^4x^6+8\,bc^3x^4-10\,b^2c^2x^2+15\,b^3c\right)\sqrt{cx^4+bx^2}}{768\,c^4}, \frac{15\,b^4\sqrt{-c}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^5,x, algorithm="fricas")
```

```
[Out] [1/768*(15*b^4*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4
+ b*x^2)*c) + 2*(48*c^4*x^6 + 8*b*c^3*x^4 - 10*b^2*c^2*x^2 + 15*b
^3*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/384*(15*b^4*sqrt(-c)*arctan(sqr
t(-c)*x^2/sqrt(c*x^4 + b*x^2)) + (48*c^4*x^6 + 8*b*c^3*x^4 - 10*b
^2*c^2*x^2 + 15*b^3*c)*sqrt(c*x^4 + b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.278297, size = 138, normalized size = 1.16

$$\frac{1}{384}\left(2\left(4\left(6\,x^2\mathrm{sign}\left(x\right)+\frac{b\mathrm{sign}\left(x\right)}{c}\right)x^2-\frac{5\,b^2\mathrm{sign}\left(x\right)}{c^2}\right)x^2+\frac{15\,b^3\mathrm{sign}\left(x\right)}{c^3}\right)\sqrt{cx^2+b}x$$
$$-\frac{5\,b^4\ln\left(\sqrt{b}\right)\,\mathrm{sign}\left(x\right)}{128\,c^{\frac{7}{2}}}+\frac{5\,b^4\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\,\mathrm{sign}\left(x\right)}{128\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^5,x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*x^2*sign(x) + b*sign(x)/c)*x^2 - 5*b^2*sign(x)/c^2
)*x^2 + 15*b^3*sign(x)/c^3)*sqrt(c*x^2 + b)*x - 5/128*b^4*ln(sqrt
(b))*sign(x)/c^(7/2) + 5/128*b^4*ln(abs(-sqrt(c)*x + sqrt(c*x^2 +
 b)))*sign(x)/c^(7/2)
```

3.222 $\int x^3\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=91

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (b*x^2 + c*x^4)
^(3/2)/(6*c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(
16*c^(5/2))
```

Rubi [A] time = 0.19806, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c^2) + (b*x^2 + c*x^4)
^(3/2)/(6*c) + (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(
16*c^(5/2))
```

Rubi in Sympy [A] time = 18.2184, size = 78, normalized size = 0.86

$$\frac{b^3\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{\frac{5}{2}}}-\frac{b\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{16c^2}+\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] b**3*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(16*c**(5/2)) - b*
(b + 2*c*x**2)*sqrt(b*x**2 + c*x**4)/(16*c**2) + (b*x**2 + c*x**4
)**(3/2)/(6*c)
```

Mathematica [A] time = 0.107951, size = 103, normalized size = 1.13

$$\frac{x\sqrt{b+cx^2}\left(3b^3\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)+\sqrt{c}x\sqrt{b+cx^2}\left(-3b^2+2bcx^2+8c^2x^4\right)\right)}{48c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^2 + 2*b*c*x^2
 + 8*c^2*x^4) + 3*b^3*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(48*c^
(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 104, normalized size = 1.1

$$\frac{1}{48\,x}\sqrt{cx^4+bx^2}\left(8\,x^3\left(cx^2+b\right)^{3/2}c^{3/2}-6\left(cx^2+b\right)^{3/2}\sqrt{c}xb+3\sqrt{cx^2+b}\sqrt{c}xb^2+3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^3\right)\frac{1}{\sqrt{cx^2+b}}c^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(1/2)*(8*x^3*(c*x^2+b)^(3/2)*c^(3/2)-6*(c*x^2+
b)^(3/2)*c^(1/2)*x*b+3*(c*x^2+b)^(1/2)*c^(1/2)*x*b^2+3*ln(x*c^(1/
2)+(c*x^2+b)^(1/2))*b^3)/x/(c*x^2+b)^(1/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284228, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^3\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)+2\,\left(8\,c^3x^4+2\,bc^2x^2-3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^3},\right.$$
$$\left.-\frac{3\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)-\left(8\,c^3x^4+2\,bc^2x^2-3\,b^2c\right)\sqrt{cx^4+bx^2}}{48\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 +
b*x^2)*c) + 2*(8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*
x^2))/c^3, -1/48*(3*b^3*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 +
 b*x^2)) - (8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c)*sqrt(c*x^4 + b*x^2
))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.276789, size = 116, normalized size = 1.27

$$\frac{1}{48}\left(2\left(4\,x^2\mathrm{sign}\left(x\right)+\frac{b\mathrm{sign}\left(x\right)}{c}\right)x^2-\frac{3\,b^2\mathrm{sign}\left(x\right)}{c^2}\right)\sqrt{cx^2+b}x$$
$$+\frac{b^3\ln\left(\sqrt{b}\right)\,\mathrm{sign}\left(x\right)}{16\,c^{\frac{5}{2}}}-\frac{b^3\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\,\mathrm{sign}\left(x\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2*sign(x) + b*sign(x)/c)*x^2 - 3*b^2*sign(x)/c^2)*sq
rt(c*x^2 + b)*x + 1/16*b^3*ln(sqrt(b))*sign(x)/c^(5/2) - 1/16*b^3
*ln(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))*sign(x)/c^(5/2)
```

3.223 $\int x\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=68

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rubi [A] time = 0.126336, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}-\frac{b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(8*c) - (b^2*ArcTanh[(Sqrt[c]
*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(3/2))
```

Rubi in Sympy [A] time = 10.7891, size = 58, normalized size = 0.85

$$-\frac{b^2\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{\frac{3}{2}}}+\frac{(b+2cx^2)\sqrt{bx^2+cx^4}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -b**2*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(8*c**(3/2)) + (b
 + 2*c*x**2)*sqrt(b*x**2 + c*x**4)/(8*c)
```

Mathematica [A] time = 0.0930907, size = 90, normalized size = 1.32

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\,(b+2cx^2)-b^2\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{8c^{3/2}\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(b + 2*c*x^2) - b^2
*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(8*c^(3/2)*Sqrt[x^2*(b + c*
x^2)])
```

Maple [A]   time = 0.01, size = 82, normalized size = 1.2

$$-\frac{1}{8\,x}\sqrt{cx^{4}+bx^{2}}\left(-2\,x\left(cx^{2}+b\right)^{3/2}\sqrt{c}+\sqrt{cx^{2}+b}\sqrt{c}xb+\ln\left(x\sqrt{c}+\sqrt{cx^{2}+b}\right)b^{2}\right)\frac{1}{\sqrt{cx^{2}+b}}c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(1/2)*(-2*x*(c*x^2+b)^(3/2)*c^(1/2)+(c*x^2+b)^
(1/2)*c^(1/2)*x*b+ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/x/(c*x^2+b)^
(1/2)/c^(3/2)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.277845, size = 1, normalized size = 0.01

$$\left[\frac{b^{2}\sqrt{c}\log\left(-\left(2\,cx^{2}+b\right)\sqrt{c}+2\,\sqrt{cx^{4}+bx^{2}}c\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(2\,c^{2}x^{2}+bc\right)}{16\,c^{2}},\frac{b^{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^{2}}{\sqrt{cx^{4}+bx^{2}}}\right)+\sqrt{cx^{4}+bx^{2}}\left(2\,c^{2}x^{2}\right.}{8\,c^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x,x, algorithm="fricas")
```

```
[Out] [1/16*(b^2*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4 + b*
x^2)*c) + 2*sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + b*c))/c^2, 1/8*(b^2*
sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 +
b*x^2)*(2*c^2*x^2 + b*c))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.278503, size = 95, normalized size = 1.4

$$\frac{1}{8}\sqrt{cx^2+b}\left(2\,x^2\mathrm{sign}\left(x\right)+\frac{b\mathrm{sign}\left(x\right)}{c}\right)x-\frac{b^2\ln\left(\sqrt{b}\right)\,\mathrm{sign}\left(x\right)}{8\,c^{\frac{3}{2}}}+\frac{b^2\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\,\mathrm{sign}\left(x\right)}{8\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^2 + b)*(2*x^2*sign(x) + b*sign(x)/c)*x - 1/8*b^2*ln(
sqrt(b))*sign(x)/c^(3/2) + 1/8*b^2*ln(abs(-sqrt(c)*x + sqrt(c*x^2
 + b)))*sign(x)/c^(3/2)
```

3.224 $\int \frac{\sqrt{bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(2*Sqrt[c])
```

Rubi [A] time = 0.141039, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{1}{2}\sqrt{bx^2+cx^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/2 + (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x
^4]])/(2*Sqrt[c])
```

Rubi in Sympy [A] time = 12.8331, size = 46, normalized size = 0.84

$$\frac{b\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{c}}+\frac{\sqrt{bx^2+cx^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] b*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(2*sqrt(c)) + sqrt(b*
x**2 + c*x**4)/2
```

Mathematica [A] time = 0.0477283, size = 64, normalized size = 1.16

$$\frac{1}{2}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{b\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)}{\sqrt{c}x\sqrt{b+cx^2}}+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(1 + (b*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]
)/(Sqrt[c]*x*Sqrt[b + c*x^2])))/2
```

Maple [A] time = 0.006, size = 64, normalized size = 1.2

$$\frac{1}{2x}\sqrt{cx^4+bx^2}\left(x\sqrt{cx^2+b}\sqrt{c}+b\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\right)\frac{1}{\sqrt{cx^2+b}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(1/2)*(x*(c*x^2+b)^(1/2)*c^(1/2)+b*ln(x*c^(1/2)
+(c*x^2+b)^(1/2)))/x/(c*x^2+b)^(1/2)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272014, size = 1, normalized size = 0.02

$$\left[\frac{b\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c},-\frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)-\sqrt{cx^4+bx^2}c}{2\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 + b*x^2
)*c) + 2*sqrt(c*x^4 + b*x^2)*c)/c, -1/2*(b*sqrt(-c)*arctan(sqrt(-
c)*x^2/sqrt(c*x^4 + b*x^2)) - sqrt(c*x^4 + b*x^2)*c)/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x, x)
```

GIAC/XCAS [A] time = 0.275128, size = 72, normalized size = 1.31

$$\frac{b\ln\left(\sqrt{b}\right)\operatorname{sign}(x)}{2\sqrt{c}} + \frac{1}{2}\left(\sqrt{cx^2 + b}x - \frac{b\ln\left(\left|-\sqrt{c}x + \sqrt{cx^2 + b}\right|\right)}{\sqrt{c}}\right)\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*ln(sqrt(b))*sign(x)/sqrt(c) + 1/2*(sqrt(c*x^2 + b)*x - b*ln
(abs(-sqrt(c)*x + sqrt(c*x^2 + b)))/sqrt(c))*sign(x)
```

3.225 $\int \frac{\sqrt{bx^2+cx^4}}{x^3}\,dx$

Optimal. Leaf size=52

$$\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]]
```

Rubi [A] time = 0.146377, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -(Sqrt[b*x^2 + c*x^4]/x^2) + Sqrt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b
*x^2 + c*x^4]]
```

Rubi in Sympy [A] time = 12.6633, size = 44, normalized size = 0.85

$$\sqrt{c}\,\mathrm{atanh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)-\frac{\sqrt{bx^2+cx^4}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] sqrt(c)*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4)) - sqrt(b*x**2 +
 c*x**4)/x**2
```

Mathematica [A] time = 0.0490217, size = 66, normalized size = 1.27

$$\frac{\sqrt{c}\,x\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)-b-cx^2}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^3,x]
```

```
[Out] (-b - c*x^2 + Sqrt[c]*x*Sqrt[b + c*x^2]*Log[c*x + Sqrt[c]*Sqrt[b
+ c*x^2]])/Sqrt[x^2*(b + c*x^2)]
```

Maple [A] time = 0.009, size = 84, normalized size = 1.6

$$\frac{1}{bx^2}\sqrt{cx^4+bx^2}\left(\sqrt{cx^2+b}c^{\frac{3}{2}}x^2+\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)xbc-\left(cx^2+b\right)^{\frac{3}{2}}\sqrt{c}\right)\frac{1}{\sqrt{cx^2+b}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^4+b*x^2)^(1/2)*((c*x^2+b)^(1/2)*c^(3/2)*x^2+ln(x*c^(1/2)+(c*
x^2+b)^(1/2))*x*b*c-(c*x^2+b)^(3/2)*c^(1/2))/x^2/(c*x^2+b)^(1/2)/
b/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276618, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}}{2\,x^2},\frac{\sqrt{-c}x^2\arctan\left(\frac{cx^2}{\sqrt{cx^4+bx^2}\sqrt{-c}}\right)-\sqrt{cx^4+bx^2}}{x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt(c
)) - 2*sqrt(c*x^4 + b*x^2))/x^2, (sqrt(-c)*x^2*arctan(c*x^2/(sqrt
(c*x^4 + b*x^2)*sqrt(-c))) - sqrt(c*x^4 + b*x^2))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**3, x)
```

GIAC/XCAS [A] time = 0.293318, size = 82, normalized size = 1.58

$$-\frac{1}{2}\,\sqrt{c}\ln\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sign}\left(x\right)+\frac{2\,b\sqrt{c}\mathrm{sign}\left(x\right)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c)*ln((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sign(x) + 2*b*sq
rt(c)*sign(x)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)
```

3.226 $\int \frac{\sqrt{bx^2+cx^4}}{x^5} \, dx$

Optimal. Leaf size=25

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(3*b*x^6)
```

Rubi [A] time = 0.0672054, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{\left(bx^2+cx^4\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^5,x]

[Out] -(b*x^2 + c*x^4)^(3/2)/(3*b*x^6)
```

Rubi in Sympy [A] time = 7.73192, size = 20, normalized size = 0.8

$$-\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{3bx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**5,x)

[Out] -(b*x**2 + c*x**4)**(3/2)/(3*b*x**6)
```

Mathematica [A] time = 0.0248192, size = 25, normalized size = 1.

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3bx^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -(x^2*(b + c*x^2))^(3/2)/(3*b*x^6)
```

Maple [A] time = 0.006, size = 29, normalized size = 1.2

$$-\frac{cx^2+b}{3\,bx^4}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^5,x)
```

```
[Out] -1/3/x^4*(c*x^2+b)/b*(c*x^4+b*x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262214, size = 38, normalized size = 1.52

$$-\frac{\sqrt{cx^4+bx^2}\,(cx^2+b)}{3\,bx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(b*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**5, x)
```

GIAC/XCAS [A] time = 0.287196, size = 85, normalized size = 3.4

$$\frac{2\left(3\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 c^{\frac{3}{2}} \operatorname{sign}(x) + b^2 c^{\frac{3}{2}} \operatorname{sign}(x)\right)}{3\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^5,x, algorithm="giac")
```

```
[Out] 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b))^4*c^(3/2)*sign(x) + b^2*c^(3
/2)*sign(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

3.227 $\int \frac{\sqrt{bx^2+cx^4}}{x^7} \, dx$

Optimal. Leaf size=52

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(5*b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(1
5*b^2*x^6)
```

Rubi [A] time = 0.136472, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2c\left(bx^2+cx^4\right)^{3/2}}{15b^2x^6}-\frac{\left(bx^2+cx^4\right)^{3/2}}{5bx^8}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(5*b*x^8) + (2*c*(b*x^2 + c*x^4)^(3/2))/(1
5*b^2*x^6)
```

Rubi in Sympy [A] time = 13.6097, size = 44, normalized size = 0.85

$$-\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{5bx^8}+\frac{2c\left(bx^2+cx^4\right)^{\frac{3}{2}}}{15b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] -(b*x**2 + c*x**4)**(3/2)/(5*b*x**8) + 2*c*(b*x**2 + c*x**4)**(3/
2)/(15*b**2*x**6)
```

Mathematica [A] time = 0.0221732, size = 46, normalized size = 0.88

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-3b^2-bcx^2+2c^2x^4\right)}{15b^2x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-3*b^2 - b*c*x^2 + 2*c^2*x^4))/(15*b^2*x^
6)
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$-\frac{(cx^2+b)(-2cx^2+3b)}{15b^2x^6}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^7,x)
```

```
[Out] -1/15*(c*x^2+b)*(-2*c*x^2+3*b)*(c*x^4+b*x^2)^(1/2)/b^2/x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265086, size = 57, normalized size = 1.1

$$\frac{(2c^2x^4-bcx^2-3b^2)\sqrt{cx^4+bx^2}}{15b^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/15*(2*c^2*x^4 - b*c*x^2 - 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^2*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**7, x)
```

GIAC/XCAS [A] time = 0.295162, size = 162, normalized size = 3.12

$$\frac{4\left(15\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^6c^{\frac{5}{2}}\operatorname{sign}(x)+5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4bc^{\frac{5}{2}}\operatorname{sign}(x)+5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2b^2c^{\frac{5}{2}}\operatorname{sign}(x)-b^3c^{\frac{5}{2}}\operatorname{sign}(x)\right)}{15\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^7,x, algorithm="giac")
```

```
[Out] 4/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b))^6*c^(5/2)*sign(x) + 5*(sqr
t(c)*x - sqrt(c*x^2 + b))^4*b*c^(5/2)*sign(x) + 5*(sqrt(c)*x - sq
rt(c*x^2 + b))^2*b^2*c^(5/2)*sign(x) - b^3*c^(5/2)*sign(x))/((sqr
t(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.228 $\int \frac{\sqrt{bx^2+cx^4}}{x^9} \, dx$

Optimal. Leaf size=80

$$-\frac{8c^2 \left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c \left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(7*b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(
35*b^2*x^8) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rubi [A] time = 0.208071, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{8c^2 \left(bx^2+cx^4\right)^{3/2}}{105b^3x^6}+\frac{4c \left(bx^2+cx^4\right)^{3/2}}{35b^2x^8}-\frac{\left(bx^2+cx^4\right)^{3/2}}{7bx^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(7*b*x^10) + (4*c*(b*x^2 + c*x^4)^(3/2))/(
35*b^2*x^8) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^6)
```

Rubi in Sympy [A] time = 20.9341, size = 71, normalized size = 0.89

$$-\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{7bx^{10}}+\frac{4c \left(bx^2+cx^4\right)^{\frac{3}{2}}}{35b^2x^8}-\frac{8c^2 \left(bx^2+cx^4\right)^{\frac{3}{2}}}{105b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] -(b*x**2 + c*x**4)**(3/2)/(7*b*x**10) + 4*c*(b*x**2 + c*x**4)**(3
/2)/(35*b**2*x**8) - 8*c**2*(b*x**2 + c*x**4)**(3/2)/(105*b**3*x*
*6)
```

Mathematica [A] time = 0.0284311, size = 57, normalized size = 0.71

$$-\frac{\sqrt{x^2 \left(b+cx^2\right)} \left(15b^3+3b^2cx^2-4bc^2x^4+8c^3x^6\right)}{105b^3x^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(15*b^3 + 3*b^2*c*x^2 - 4*b*c^2*x^4 + 8*c
^3*x^6))/(105*b^3*x^8)
```

Maple [A] time = 0.007, size = 50, normalized size = 0.6

$$-\frac{(cx^2+b)\left(8\,c^2x^4-12\,bcx^2+15\,b^2\right)}{105\,x^8b^3}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^9,x)
```

```
[Out] -1/105*(c*x^2+b)*(8*c^2*x^4-12*b*c*x^2+15*b^2)*(c*x^4+b*x^2)^(1/2
)/x^8/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276636, size = 72, normalized size = 0.9

$$-\frac{\left(8\,c^3x^6-4\,bc^2x^4+3\,b^2cx^2+15\,b^3\right)\sqrt{cx^4+bx^2}}{105\,b^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/105*(8*c^3*x^6 - 4*b*c^2*x^4 + 3*b^2*c*x^2 + 15*b^3)*sqrt(c*x^
4 + b*x^2)/(b^3*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**9, x)
```

GIAC/XCAS [A] time = 0.299592, size = 200, normalized size = 2.5

$$\frac{16 \left(70 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^8 c^{\frac{7}{2}} \operatorname{sign}(x) + 35 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^6 b c^{\frac{7}{2}} \operatorname{sign}(x) + 21 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 b^2 c^{\frac{7}{2}} \operatorname{sign}(x) - 7 \left(\sqrt{c}x - \cdots\right.\right.}{105 \left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^9,x, algorithm="giac")
```

```
[Out] 16/105*(70*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(7/2)*sign(x) + 35*(
sqrt(c)*x - sqrt(c*x^2 + b))^6*b*c^(7/2)*sign(x) + 21*(sqrt(c)*x
- sqrt(c*x^2 + b))^4*b^2*c^(7/2)*sign(x) - 7*(sqrt(c)*x - sqrt(c*
x^2 + b))^2*b^3*c^(7/2)*sign(x) + b^4*c^(7/2)*sign(x))/((sqrt(c)*
x - sqrt(c*x^2 + b))^2 - b)^7
```

3.229 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(9*b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(
21*b^2*x^10) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*
c^3*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rubi [A] time = 0.285518, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{16c^3\left(bx^2+cx^4\right)^{3/2}}{315b^4x^6}-\frac{8c^2\left(bx^2+cx^4\right)^{3/2}}{105b^3x^8}+\frac{2c\left(bx^2+cx^4\right)^{3/2}}{21b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{9bx^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(9*b*x^12) + (2*c*(b*x^2 + c*x^4)^(3/2))/(
21*b^2*x^10) - (8*c^2*(b*x^2 + c*x^4)^(3/2))/(105*b^3*x^8) + (16*
c^3*(b*x^2 + c*x^4)^(3/2))/(315*b^4*x^6)
```

Rubi in Sympy [A] time = 28.5699, size = 99, normalized size = 0.92

$$-\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{9bx^{12}}+\frac{2c\left(bx^2+cx^4\right)^{\frac{3}{2}}}{21b^2x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{105b^3x^8}+\frac{16c^3\left(bx^2+cx^4\right)^{\frac{3}{2}}}{315b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] -(b*x**2 + c*x**4)**(3/2)/(9*b*x**12) + 2*c*(b*x**2 + c*x**4)**(3
/2)/(21*b**2*x**10) - 8*c**2*(b*x**2 + c*x**4)**(3/2)/(105*b**3*x
**8) + 16*c**3*(b*x**2 + c*x**4)**(3/2)/(315*b**4*x**6)
```

Mathematica [A] time = 0.0328914, size = 68, normalized size = 0.63

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-35b^4-5b^3cx^2+6b^2c^2x^4-8bc^3x^6+16c^4x^8\right)}{315b^4x^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^11,x]

[Out] (Sqrt[x^2*(b + c*x^2)]*(-35*b^4 - 5*b^3*c*x^2 + 6*b^2*c^2*x^4 - 8
*b*c^3*x^6 + 16*c^4*x^8))/(315*b^4*x^10)
```

Maple [A] time = 0.008, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-16\,c^3x^6+24\,bc^2x^4-30\,b^2cx^2+35\,b^3\right)}{315\,x^{10}b^4}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^11,x)

[Out] -1/315*(c*x^2+b)*(-16*c^3*x^6+24*b*c^2*x^4-30*b^2*c*x^2+35*b^3)*(
c*x^4+b*x^2)^(1/2)/x^10/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^11,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294473, size = 86, normalized size = 0.8

$$\frac{\left(16\,c^4x^8-8\,bc^3x^6+6\,b^2c^2x^4-5\,b^3cx^2-35\,b^4\right)\sqrt{cx^4+bx^2}}{315\,b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/315*(16*c^4*x^8 - 8*b*c^3*x^6 + 6*b^2*c^2*x^4 - 5*b^3*c*x^2 - 3
5*b^4)*sqrt(c*x^4 + b*x^2)/(b^4*x^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{11}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**11, x)
```

GIAC/XCAS [A] time = 0.302402, size = 240, normalized size = 2.22

$$\frac{32 \left(315 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^{10} c^{\frac{9}{2}} \operatorname{sign}(x) + 189 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^8 bc^{\frac{9}{2}} \operatorname{sign}(x) + 84 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^6 b^2 c^{\frac{9}{2}} \operatorname{sign}(x) - 36 \left(\sqrt{c}\right.\right.}{315 \left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^11,x, algorithm="giac")
```

```
[Out] 32/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(9/2)*sign(x) + 18
9*(sqrt(c)*x - sqrt(c*x^2 + b))^8*b*c^(9/2)*sign(x) + 84*(sqrt(c)
*x - sqrt(c*x^2 + b))^6*b^2*c^(9/2)*sign(x) - 36*(sqrt(c)*x - sqr
t(c*x^2 + b))^4*b^3*c^(9/2)*sign(x) + 9*(sqrt(c)*x - sqrt(c*x^2 +
 b))^2*b^4*c^(9/2)*sign(x) - b^5*c^(9/2)*sign(x))/((sqrt(c)*x - s
qrt(c*x^2 + b))^2 - b)^9
```

3.230 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13}}\,dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(11*b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/
(99*b^2*x^12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (
64*c^3*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 +
c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rubi [A] time = 0.364311, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{128c^4\left(bx^2+cx^4\right)^{3/2}}{3465b^5x^6}+\frac{64c^3\left(bx^2+cx^4\right)^{3/2}}{1155b^4x^8}-\frac{16c^2\left(bx^2+cx^4\right)^{3/2}}{231b^3x^{10}}+\frac{8c\left(bx^2+cx^4\right)^{3/2}}{99b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{3/2}}{11bx^{14}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(3/2)/(11*b*x^14) + (8*c*(b*x^2 + c*x^4)^(3/2))/
(99*b^2*x^12) - (16*c^2*(b*x^2 + c*x^4)^(3/2))/(231*b^3*x^10) + (
64*c^3*(b*x^2 + c*x^4)^(3/2))/(1155*b^4*x^8) - (128*c^4*(b*x^2 +
c*x^4)^(3/2))/(3465*b^5*x^6)
```

Rubi in Sympy [A] time = 36.9205, size = 126, normalized size = 0.93

$$-\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{11bx^{14}}+\frac{8c\left(bx^2+cx^4\right)^{\frac{3}{2}}}{99b^2x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{231b^3x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{\frac{3}{2}}}{1155b^4x^8}-\frac{128c^4\left(bx^2+cx^4\right)^{\frac{3}{2}}}{3465b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] -(b*x**2 + c*x**4)**(3/2)/(11*b*x**14) + 8*c*(b*x**2 + c*x**4)**(
3/2)/(99*b**2*x**12) - 16*c**2*(b*x**2 + c*x**4)**(3/2)/(231*b**3
*x**10) + 64*c**3*(b*x**2 + c*x**4)**(3/2)/(1155*b**4*x**8) - 128
*c**4*(b*x**2 + c*x**4)**(3/2)/(3465*b**5*x**6)
```

Mathematica [A] time = 0.0368774, size = 79, normalized size = 0.58

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(315b^5+35b^4cx^2-40b^3c^2x^4+48b^2c^3x^6-64bc^4x^8+128c^5x^{10}\right)}{3465b^5x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^13,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(315*b^5 + 35*b^4*c*x^2 - 40*b^3*c^2*x^4
+ 48*b^2*c^3*x^6 - 64*b*c^4*x^8 + 128*c^5*x^10))/(3465*b^5*x^12)
```

Maple [A] time = 0.009, size = 72, normalized size = 0.5

$$-\frac{\left(cx^2+b\right)\left(128\,c^4x^8-192\,c^3x^6b+240\,c^2x^4b^2-280\,cx^2b^3+315\,b^4\right)}{3465\,x^{12}b^5}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^13,x)
```

```
[Out] -1/3465*(c*x^2+b)*(128*c^4*x^8-192*b*c^3*x^6+240*b^2*c^2*x^4-280*
b^3*c*x^2+315*b^4)*(c*x^4+b*x^2)^(1/2)/x^12/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.332517, size = 101, normalized size = 0.74

$$-\frac{\left(128\,c^5x^{10}-64\,bc^4x^8+48\,b^2c^3x^6-40\,b^3c^2x^4+35\,b^4cx^2+315\,b^5\right)\sqrt{cx^4+bx^2}}{3465\,b^5x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/3465*(128*c^5*x^10 - 64*b*c^4*x^8 + 48*b^2*c^3*x^6 - 40*b^3*c^
2*x^4 + 35*b^4*c*x^2 + 315*b^5)*sqrt(c*x^4 + b*x^2)/(b^5*x^12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**13,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**13, x)
```

GIAC/XCAS [A] time = 0.306955, size = 278, normalized size = 2.04

$$\frac{256\left(1386\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{11}{2}}\operatorname{sign}(x)+924\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{11}{2}}\operatorname{sign}(x)+330\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{11}{2}}\operatorname{sign}(x)-1\right.}{3465\left(\left(\sqrt{c}x-\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^13,x, algorithm="giac")
```

```
[Out] 256/3465*(1386*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(11/2)*sign(x)
+ 924*(sqrt(c)*x - sqrt(c*x^2 + b))^10*b*c^(11/2)*sign(x) + 330*(
sqrt(c)*x - sqrt(c*x^2 + b))^8*b^2*c^(11/2)*sign(x) - 165*(sqrt(c
)*x - sqrt(c*x^2 + b))^6*b^3*c^(11/2)*sign(x) + 55*(sqrt(c)*x - s
qrt(c*x^2 + b))^4*b^4*c^(11/2)*sign(x) - 11*(sqrt(c)*x - sqrt(c*x
^2 + b))^2*b^5*c^(11/2)*sign(x) + b^6*c^(11/2)*sign(x))/((sqrt(c)
*x - sqrt(c*x^2 + b))^2 - b)^11
```

3.231 $\int x^4\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=78

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4
)^(3/2))/(35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rubi [A] time = 0.168645, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{8b^2\left(bx^2+cx^4\right)^{3/2}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{3/2}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{3/2}}{7c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(3/2))/(105*c^3*x^3) - (4*b*(b*x^2 + c*x^4
)^(3/2))/(35*c^2*x) + (x*(b*x^2 + c*x^4)^(3/2))/(7*c)
```

Rubi in Sympy [A] time = 20.3978, size = 68, normalized size = 0.87

$$\frac{8b^2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{105c^3x^3}-\frac{4b\left(bx^2+cx^4\right)^{\frac{3}{2}}}{35c^2x}+\frac{x\left(bx^2+cx^4\right)^{\frac{3}{2}}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 8*b**2*(b*x**2 + c*x**4)**(3/2)/(105*c**3*x**3) - 4*b*(b*x**2 + c
*x**4)**(3/2)/(35*c**2*x) + x*(b*x**2 + c*x**4)**(3/2)/(7*c)
```

Mathematica [A] time = 0.028546, size = 57, normalized size = 0.73

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(8b^3-4b^2cx^2+3bc^2x^4+15c^3x^6\right)}{105c^3x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(8*b^3 - 4*b^2*c*x^2 + 3*b*c^2*x^4 + 15*c^
3*x^6))/(105*c^3*x)
```

Maple [A] time = 0.007, size = 50, normalized size = 0.6

$$\frac{\left(cx^2+b\right)\left(15\,c^2x^4-12\,bcx^2+8\,b^2\right)}{105\,c^3x}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/105*(c*x^2+b)*(15*c^2*x^4-12*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(1/2)
/c^3/x
```

Maxima [A] time = 0.734401, size = 62, normalized size = 0.79

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^2+b}}{105\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^2
 + b)/c^3
```

Fricas [A] time = 0.270043, size = 72, normalized size = 0.92

$$\frac{\left(15\,c^3x^6+3\,bc^2x^4-4\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{105\,c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^3*x^6 + 3*b*c^2*x^4 - 4*b^2*c*x^2 + 8*b^3)*sqrt(c*x^4
 + b*x^2)/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{x^2 (b + cx^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.271534, size = 76, normalized size = 0.97

$$-\frac{8\, b^{\frac{7}{2}} \operatorname{sign}(x)}{105\, c^3} + \frac{\left(15\, (cx^2 + b)^{\frac{7}{2}} - 42\, (cx^2 + b)^{\frac{5}{2}} b + 35\, (cx^2 + b)^{\frac{3}{2}} b^2\right) \operatorname{sign}(x)}{105\, c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^4,x, algorithm="giac")
```

```
[Out] -8/105*b^(7/2)*sign(x)/c^3 + 1/105*(15*(c*x^2 + b)^(7/2) - 42*(c*
x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*sign(x)/c^3
```

3.232 $\int x^2\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)
/(5*c*x)
```

Rubi [A] time = 0.0881505, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{5cx}-\frac{2b\left(bx^2+cx^4\right)^{3/2}}{15c^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(3/2))/(15*c^2*x^3) + (b*x^2 + c*x^4)^(3/2)
/(5*c*x)
```

Rubi in Sympy [A] time = 12.8972, size = 42, normalized size = 0.81

$$-\frac{2b\left(bx^2+cx^4\right)^{\frac{3}{2}}}{15c^2x^3}+\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{5cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*b*(b*x**2 + c*x**4)**(3/2)/(15*c**2*x**3) + (b*x**2 + c*x**4)*
*(3/2)/(5*c*x)
```

Mathematica [A] time = 0.0225514, size = 45, normalized size = 0.87

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-2b^2+bcx^2+3c^2x^4\right)}{15c^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4))/(15*c^2*x)
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$-\frac{\left(cx^{2}+b\right)\left(-3\,cx^{2}+2\,b\right)}{15\,c^{2}x}\sqrt{cx^{4}+bx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(-3*c*x^2+2*b)*(c*x^4+b*x^2)^(1/2)/c^2/x
```

Maxima [A] time = 0.723788, size = 46, normalized size = 0.88

$$\frac{\left(3\,c^{2}x^{4}+bcx^{2}-2\,b^{2}\right)\sqrt{cx^{2}+b}}{15\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^2 + b)/c^2
```

Fricas [A] time = 0.263729, size = 55, normalized size = 1.06

$$\frac{\left(3\,c^{2}x^{4}+bcx^{2}-2\,b^{2}\right)\sqrt{cx^{4}+bx^{2}}}{15\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^2*x^4 + b*c*x^2 - 2*b^2)*sqrt(c*x^4 + b*x^2)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{x^2 \left(b + cx^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.270913, size = 57, normalized size = 1.1

$$\frac{2\, b^{\frac{5}{2}} \operatorname{sign}\left(x\right)}{15\, c^2} + \frac{\left(3\, \left(cx^2 + b\right)^{\frac{5}{2}} - 5\, \left(cx^2 + b\right)^{\frac{3}{2}} b\right) \operatorname{sign}\left(x\right)}{15\, c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^2,x, algorithm="giac")
```

```
[Out] 2/15*b^(5/2)*sign(x)/c^2 + 1/15*(3*(c*x^2 + b)^(5/2) - 5*(c*x^2 +
 b)^(3/2)*b)*sign(x)/c^2
```

3.233 $\int \sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=25

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

```
[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Rubi [A] time = 0.0159563, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{\left(bx^2+cx^4\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (b*x^2 + c*x^4)^(3/2)/(3*c*x^3)
```

Rubi in Sympy [A] time = 5.1575, size = 19, normalized size = 0.76

$$\frac{\left(bx^2+cx^4\right)^{\frac{3}{2}}}{3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2),x)
```

```
[Out] (b*x**2 + c*x**4)**(3/2)/(3*c*x**3)
```

Mathematica [A] time = 0.00956013, size = 25, normalized size = 1.

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{3/2}}{3cx^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4],x]

[Out] (x^2*(b + c*x^2))^(3/2)/(3*c*x^3)
```

Maple [A] time = 0.004, size = 29, normalized size = 1.2

$$\frac{cx^2+b}{3\,cx}\sqrt{cx^4+bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2),x)

[Out] 1/3*(c*x^2+b)/c/x*(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.701052, size = 19, normalized size = 0.76

$$\frac{\left(cx^2+b\right)^{\frac{3}{2}}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] 1/3*(c*x^2 + b)^(3/2)/c
```

Fricas [A] time = 0.259146, size = 38, normalized size = 1.52

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{3\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.270496, size = 36, normalized size = 1.44

$$\frac{\left(cx^2 + b\right)^{\frac{3}{2}} \operatorname{sign}(x)}{3\,c} - \frac{b^{\frac{3}{2}} \operatorname{sign}(x)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/3*(c*x^2 + b)^(3/2)*sign(x)/c - 1/3*b^(3/2)*sign(x)/c
```

3.234 $\int \frac{\sqrt{bx^2+cx^4}}{x^2}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 +
c*x^4]]
```

Rubi [A] time = 0.0900042, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\sqrt{bx^2+cx^4}}{x}-\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/x - Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 +
c*x^4]]
```

Rubi in Sympy [A] time = 12.9791, size = 41, normalized size = 0.82

$$-\sqrt{b}\,\text{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)+\frac{\sqrt{bx^2+cx^4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4)) + sqrt(b*x**2 + c
*x**4)/x
```

Mathematica [A] time = 0.0711607, size = 75, normalized size = 1.5

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{b+cx^2}-\sqrt{b}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+\sqrt{b}\log(x)\right)}{\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[b + c*x^2] + Sqrt[b]*Log[x] - Sqrt[b]*Lo
g[b + Sqrt[b]*Sqrt[b + c*x^2]]))/Sqrt[x^2*(b + c*x^2)]
```

Maple [A] time = 0.008, size = 65, normalized size = 1.3

$$-\frac{1}{x}\sqrt{cx^4+bx^2}\left(\sqrt{b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)-\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2)^(1/2)*(b^(1/2)*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)
-(c*x^2+b)^(1/2))/x/(c*x^2+b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278106, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{b}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}}{2\,x},-\frac{\sqrt{-b}x\arctan\left(\frac{bx}{\sqrt{cx^4+bx^2}\sqrt{-b}}\right)-\sqrt{cx^4+bx^2}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqrt(
b))/x^3) + 2*sqrt(c*x^4 + b*x^2))/x, -(sqrt(-b)*x*arctan(b*x/(sqr
t(c*x^4 + b*x^2)*sqrt(-b))) - sqrt(c*x^4 + b*x^2))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**2, x)
```

GIAC/XCAS [A] time = 0.275216, size = 92, normalized size = 1.84

$$\left(\frac{b\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\sqrt{cx^2+b}\right)\mathrm{sign}\left(x\right)-\frac{\left(b\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\sqrt{b}\right)\mathrm{sign}\left(x\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^2,x, algorithm="giac")
```

```
[Out] (b*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + sqrt(c*x^2 + b))*s
ign(x) - (b*arctan(sqrt(b)/sqrt(-b)) + sqrt(-b)*sqrt(b))*sign(x)/
sqrt(-b)
```

3.235 $\int \frac{\sqrt{bx^2+cx^4}}{x^4} \, dx$

Optimal. Leaf size=56

$$-\frac{c \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*x^3) - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(2*Sqrt[b])
```

Rubi [A] time = 0.0920732, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{c \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx^2+cx^4}}{2x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*x^3) - (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2
+ c*x^4]])/(2*Sqrt[b])
```

Rubi in Sympy [A] time = 12.9372, size = 49, normalized size = 0.88

$$-\frac{\sqrt{bx^2+cx^4}}{2x^3}-\frac{c \operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(2*x**3) - c*atanh(sqrt(b)*x/sqrt(b*x**2 +
 c*x**4))/(2*sqrt(b))
```

Mathematica [A] time = 0.0651876, size = 89, normalized size = 1.59

$$-\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}+cx^2\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)-cx^2\log(x)\right)}{2\sqrt{b}x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2] - c*x^2*Log[x] +
 c*x^2*Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(2*Sqrt[b]*x^3*Sqrt[b +
 c*x^2])
```

Maple [A] time = 0.009, size = 85, normalized size = 1.5

$$-\frac{1}{2\,b x^3}\sqrt{cx^4+bx^2}\left(\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{b}x^2c-\sqrt{cx^2+b}x^2c+\left(cx^2+b\right)^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(1/2)*(ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(
1/2)*x^2*c-(c*x^2+b)^(1/2)*x^2*c+(c*x^2+b)^(3/2))/x^3/(c*x^2+b)^(
1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274876, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{b}cx^3\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}-2\,\sqrt{cx^4+bx^2}b}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,bx^3},\frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)-\sqrt{cx^4+bx^2}b}{2\,bx^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c*x^4
+ b*x^2)*b)/x^3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b*x^3), 1/2*(sqrt(-b
)*c*x^3*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2)) - sqrt(c*x^4 + b*x
^2)*b)/(b*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**4, x)
```

GIAC/XCAS [A] time = 0.289783, size = 61, normalized size = 1.09

$$\frac{1}{2}\,c\left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\sqrt{cx^2 + b}}{cx^2}\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*c*(arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - sqrt(c*x^2 + b
)/(c*x^2))*sign(x)
```

3.236 $\int \frac{\sqrt{bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=84

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*x^5) - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3)
+ (c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rubi [A] time = 0.175475, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*x^5) - (c*Sqrt[b*x^2 + c*x^4])/(8*b*x^3)
+ (c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(3/2))
```

Rubi in Sympy [A] time = 21.1176, size = 71, normalized size = 0.85

$$-\frac{\sqrt{bx^2+cx^4}}{4x^5}-\frac{c\sqrt{bx^2+cx^4}}{8bx^3}+\frac{c^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(4*x**5) - c*sqrt(b*x**2 + c*x**4)/(8*b*x*
*3) + c**2*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(8*b**(3/2))
```

Mathematica [A] time = 0.107538, size = 102, normalized size = 1.21

$$-\frac{\sqrt{x^2(b+cx^2)}\left(-c^2x^4\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+\sqrt{b}\sqrt{b+cx^2}\left(2b+cx^2\right)+c^2x^4\log(x)\right)}{8b^{3/2}x^5\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(2*b + c*x^2) +
c^2*x^4*Log[x] - c^2*x^4*Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(8*b^
(3/2)*x^5*Sqrt[b + c*x^2])
```

Maple [A] time = 0.011, size = 106, normalized size = 1.3

$$\frac{1}{8\,b^2x^5}\sqrt{cx^4+bx^2}\left(\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{b}x^4c^2-\sqrt{cx^2+b}x^4c^2+\left(cx^2+b\right)^{\frac{3}{2}}x^2c-2\,\left(cx^2+b\right)^{3/2}b\right)\frac{1}{\sqrt{cx^2+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^6,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(1/2)*(ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b^(1
/2)*x^4*c^2-(c*x^2+b)^(1/2)*x^4*c^2+(c*x^2+b)^(3/2)*x^2*c-2*(c*x^
2+b)^(3/2)*b)/x^5/(c*x^2+b)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278186, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{b}c^2x^5\log\left(-\frac{\left(cx^3+2\,bx\right)\sqrt{b}+2\,\sqrt{cx^4+bx^2}b}{x^3}\right)-2\,\sqrt{cx^4+bx^2}\left(bcx^2+2\,b^2\right)}{16\,b^2x^5},\right.$$
$$\left.-\frac{\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}\left(bcx^2+2\,b^2\right)}{8\,b^2x^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/16*(sqrt(b)*c^2*x^5*log(-((c*x^3 + 2*b*x)*sqrt(b) + 2*sqrt(c*x
^4 + b*x^2)*b)/x^3) - 2*sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b
^2*x^5), -1/8*(sqrt(-b)*c^2*x^5*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*
x^2)) + sqrt(c*x^4 + b*x^2)*(b*c*x^2 + 2*b^2))/(b^2*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**6, x)
```

GIAC/XCAS [A] time = 0.290431, size = 86, normalized size = 1.02

$$-\frac{1}{8}\,c^2\left(\frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{\left(cx^2+b\right)^{\frac{3}{2}}+\sqrt{cx^2+b}b}{bc^2x^4}\right)\operatorname{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/8*c^2*(arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + ((c*x^2
 + b)^(3/2) + sqrt(c*x^2 + b)*b)/(b*c^2*x^4))*sign(x)
```

3.237 $\int \frac{\sqrt{bx^2+cx^4}}{x^8}\,dx$

Optimal. Leaf size=112

$$-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(6*x^7) - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5)
 + (c^2*Sqrt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]
*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(5/2))
```

Rubi [A] time = 0.254379, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{5/2}}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(6*x^7) - (c*Sqrt[b*x^2 + c*x^4])/(24*b*x^5)
 + (c^2*Sqrt[b*x^2 + c*x^4])/(16*b^2*x^3) - (c^3*ArcTanh[(Sqrt[b]
*x)/Sqrt[b*x^2 + c*x^4]])/(16*b^(5/2))
```

Rubi in Sympy [A] time = 29.1142, size = 97, normalized size = 0.87

$$-\frac{\sqrt{bx^2+cx^4}}{6x^7}-\frac{c\sqrt{bx^2+cx^4}}{24bx^5}+\frac{c^2\sqrt{bx^2+cx^4}}{16b^2x^3}-\frac{c^3\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(6*x**7) - c*sqrt(b*x**2 + c*x**4)/(24*b*x
**5) + c**2*sqrt(b*x**2 + c*x**4)/(16*b**2*x**3) - c**3*atanh(sqr
t(b)*x/sqrt(b*x**2 + c*x**4))/(16*b**(5/2))
```

Mathematica [A] time = 0.112827, size = 115, normalized size = 1.03

$$\frac{\sqrt{x^2(b+cx^2)}\left(\sqrt{b}\sqrt{b+cx^2}\left(-8b^2-2bcx^2+3c^2x^4\right)-3c^3x^6\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+3c^3x^6\log(x)\right)}{48b^{5/2}x^7\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^8,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(-8*b^2 - 2*b*c*x
^2 + 3*c^2*x^4) + 3*c^3*x^6*Log[x] - 3*c^3*x^6*Log[b + Sqrt[b]*Sq
rt[b + c*x^2]]))/(48*b^(5/2)*x^7*Sqrt[b + c*x^2])
```

Maple [A] time = 0.014, size = 128, normalized size = 1.1

$$-\frac{1}{48\,x^7b^3}\sqrt{cx^4+bx^2}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{b}x^6c^3-3\,\sqrt{cx^2+b}x^6c^3+3\,\left(cx^2+b\right)^{3/2}x^4c^2-6\,\left(cx^2+b\right)^{3/2}x^2bc+8\,(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^8,x)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*
b^(1/2)*x^6*c^3-3*(c*x^2+b)^(1/2)*x^6*c^3+3*(c*x^2+b)^(3/2)*x^4*c
^2-6*(c*x^2+b)^(3/2)*x^2*b*c+8*(c*x^2+b)^(3/2)*b^2)/x^7/(c*x^2+b)
^(1/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287384, size = 1, normalized size = 0.01

$$\left[\frac{3\,\sqrt{b}c^3x^7\log\left(-\frac{\left(cx^3+2\,bx\right)\sqrt{b}-2\,\sqrt{cx^4+bx^2}b}{x^3}\right)+2\,\left(3\,bc^2x^4-2\,b^2cx^2-8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^3x^7},\ \frac{3\,\sqrt{-b}c^3x^7\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\left(3\,bc^2 \cdots}{48\,b^3x \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c
*x^4 + b*x^2)*b)/x^3) + 2*(3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)*sqr
t(c*x^4 + b*x^2))/(b^3*x^7), 1/48*(3*sqrt(-b)*c^3*x^7*arctan(sqrt
(-b)*x/sqrt(c*x^4 + b*x^2)) + (3*b*c^2*x^4 - 2*b^2*c*x^2 - 8*b^3)
*sqrt(c*x^4 + b*x^2))/(b^3*x^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**8, x)
```

GIAC/XCAS [A] time = 0.304445, size = 111, normalized size = 0.99

$$\frac{1}{48}\,c^3\left(\frac{3\,\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\,\left(cx^2+b\right)^{\frac{5}{2}}-8\,\left(cx^2+b\right)^{\frac{3}{2}}b-3\,\sqrt{cx^2+b}b^2}{b^2c^3x^6}\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^8,x, algorithm="giac")
```

```
[Out] 1/48*c^3*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3*
(c*x^2 + b)^(5/2) - 8*(c*x^2 + b)^(3/2)*b - 3*sqrt(c*x^2 + b)*b^2
)/(b^2*c^3*x^6))*sign(x)
```

3.238 $\int x^3 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=124

$$-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c
*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10
*c) - (3*b^5*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(
7/2))
```

Rubi [A] time = 0.24973, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b^3*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(256*c^3) - (b*(b + 2*c
*x^2)*(b*x^2 + c*x^4)^(3/2))/(32*c^2) + (b*x^2 + c*x^4)^(5/2)/(10
*c) - (3*b^5*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(256*c^(
7/2))
```

Rubi in Sympy [A] time = 21.5317, size = 112, normalized size = 0.9

$$-\frac{3b^5\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{256c^{\frac{7}{2}}}+\frac{3b^3\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{\frac{3}{2}}}{32c^2}+\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{10c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -3*b**5*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(256*c**(7/2))
+ 3*b**3*(b + 2*c*x**2)*sqrt(b*x**2 + c*x**4)/(256*c**3) - b*(b +
 2*c*x**2)*(b*x**2 + c*x**4)**(3/2)/(32*c**2) + (b*x**2 + c*x**4)
```

```
**(5/2)/(10*c)
```

Mathematica [A] time = 0.172593, size = 125, normalized size = 1.01

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\left(15b^4-10b^3cx^2+8b^2c^2x^4+176bc^3x^6+128c^4x^8\right)-15b^5\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{1280c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(15*b^4 - 10*b^3*c*
x^2 + 8*b^2*c^2*x^4 + 176*b*c^3*x^6 + 128*c^4*x^8) - 15*b^5*Log[c
*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(1280*c^(7/2)*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] time = 0.015, size = 142, normalized size = 1.2

$$-\frac{1}{1280\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-128\,x^5\left(cx^2+b\right)^{5/2}c^{5/2}+80\left(cx^2+b\right)^{5/2}c^{3/2}x^3b-40\left(cx^2+b\right)^{5/2}\sqrt{c}xb^2+10\left(cx^2+b\right)^{3/2}\sqrt{c}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/1280*(c*x^4+b*x^2)^(3/2)*(-128*x^5*(c*x^2+b)^(5/2)*c^(5/2)+80*
(c*x^2+b)^(5/2)*c^(3/2)*x^3*b-40*(c*x^2+b)^(5/2)*c^(1/2)*x*b^2+10
*(c*x^2+b)^(3/2)*c^(1/2)*x*b^3+15*(c*x^2+b)^(1/2)*c^(1/2)*x*b^4+1
5*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^5)/x^3/(c*x^2+b)^(3/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.335906, size = 1, normalized size = 0.01

$$\left[\frac{15\,b^5\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\,\left(128\,c^5x^8+176\,bc^4x^6+8\,b^2c^3x^4-10\,b^3c^2x^2+15\,b^4c\right)\sqrt{cx^4+bx^2}}{2560\,c^4}, \frac{15\,b}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*b^5*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4
 + b*x^2)*c) + 2*(128*c^5*x^8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 1
0*b^3*c^2*x^2 + 15*b^4*c)*sqrt(c*x^4 + b*x^2))/c^4, 1/1280*(15*b^
5*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2)) + (128*c^5*x^
8 + 176*b*c^4*x^6 + 8*b^2*c^3*x^4 - 10*b^3*c^2*x^2 + 15*b^4*c)*sq
rt(c*x^4 + b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**3*(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.279853, size = 157, normalized size = 1.27

$$-\frac{3\,b^5\ln\left(\sqrt{b}\right)\mathrm{sign}\left(x\right)}{256\,c^{\frac{7}{2}}}+\frac{3\,b^5\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\mathrm{sign}\left(x\right)}{256\,c^{\frac{7}{2}}}+\frac{1}{1280}\left(2\left(4\left(2\left(8\,cx^2\mathrm{sign}\left(x\right)+11\,b\mathrm{sign}\left(x\right)\right)x^2+\frac{b^2\mathrm{sign}\left(x\right)}{c}\right)x^2-\frac{5\,b^3\mathrm{sign}\left(x\right)}{c^2}\right)x^2+\frac{15\,b^4\mathrm{sign}\left(x\right)}{c^3}\right)\sqrt{cx^2+b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] -3/256*b^5*ln(sqrt(b))*sign(x)/c^(7/2) + 3/256*b^5*ln(abs(-sqrt(c
)*x + sqrt(c*x^2 + b)))*sign(x)/c^(7/2) + 1/1280*(2*(4*(2*(8*c*x^
2*sign(x) + 11*b*sign(x))*x^2 + b^2*sign(x)/c)*x^2 - 5*b^3*sign(x
)/c^2)*x^2 + 15*b^4*sign(x)/c^3)*sqrt(c*x^2 + b)*x
```

3.239 $\int x\left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*
x^2)*(b*x^2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)
/Sqrt[b*x^2 + c*x^4]])/(128*c^(5/2))
```

Rubi [A] time = 0.168027, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235

$$\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{5/2}}-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*b^2*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(128*c^2) + ((b + 2*c*
x^2)*(b*x^2 + c*x^4)^(3/2))/(16*c) + (3*b^4*ArcTanh[(Sqrt[c]*x^2)
/Sqrt[b*x^2 + c*x^4]])/(128*c^(5/2))
```

Rubi in Sympy [A] time = 14.4037, size = 92, normalized size = 0.91

$$\frac{3b^4\operatorname{atanh}\left(\frac{\sqrt{c}\,x^2}{\sqrt{bx^2+cx^4}}\right)}{128c^{\frac{5}{2}}}-\frac{3b^2\left(b+2cx^2\right)\sqrt{bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{\frac{3}{2}}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 3*b**4*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(128*c**(5/2)) -
 3*b**2*(b + 2*c*x**2)*sqrt(b*x**2 + c*x**4)/(128*c**2) + (b + 2*
c*x**2)*(b*x**2 + c*x**4)**(3/2)/(16*c)
```

Mathematica [A] time = 0.144909, size = 114, normalized size = 1.13

$$\frac{x\sqrt{b+cx^2}\left(3b^4\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)+\sqrt{c}x\sqrt{b+cx^2}\left(-3b^3+2b^2cx^2+24bc^2x^4+16c^3x^6\right)\right)}{128c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(-3*b^3 + 2*b^2*c*x
^2 + 24*b*c^2*x^4 + 16*c^3*x^6) + 3*b^4*Log[c*x + Sqrt[c]*Sqrt[b
+ c*x^2]]))/(128*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 122, normalized size = 1.2

$$\frac{1}{128\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(16\,x^3\left(cx^2+b\right)^{5/2}c^{3/2}-8\left(cx^2+b\right)^{5/2}\sqrt{c}xb+2\left(cx^2+b\right)^{3/2}\sqrt{c}xb^2+3\sqrt{cx^2+b}\sqrt{c}xb^3+3\ln\left(x\sqrt{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/128*(c*x^4+b*x^2)^(3/2)*(16*x^3*(c*x^2+b)^(5/2)*c^(3/2)-8*(c*x^
2+b)^(5/2)*c^(1/2)*x*b+2*(c*x^2+b)^(3/2)*c^(1/2)*x*b^2+3*(c*x^2+b
)^(1/2)*c^(1/2)*x*b^3+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^4)/x^3/(c
*x^2+b)^(3/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29652, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^4\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)+2\,\left(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c\right)\sqrt{cx^4+bx^2}}{256\,c^3},\right.$$
$$\left.-\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)-\left(16\,c^4x^6+24\,bc^3x^4+2\,b^2c^2x^2-3\,b^3c\right)\sqrt{cx^4+bx^2}}{128\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/256*(3*b^4*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 +
 b*x^2)*c) + 2*(16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2*c^2*x^2 - 3*b^3
*c)*sqrt(c*x^4 + b*x^2))/c^3, -1/128*(3*b^4*sqrt(-c)*arctan(sqrt(
-c)*x^2/sqrt(c*x^4 + b*x^2)) - (16*c^4*x^6 + 24*b*c^3*x^4 + 2*b^2
*c^2*x^2 - 3*b^3*c)*sqrt(c*x^4 + b*x^2))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.278967, size = 135, normalized size = 1.34

$$\frac{3\,b^4\ln\left(\sqrt{b}\right)\mathrm{sign}\left(x\right)}{128\,c^{\frac{5}{2}}}-\frac{3\,b^4\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\mathrm{sign}\left(x\right)}{128\,c^{\frac{5}{2}}}$$
$$+\frac{1}{128}\left(2\left(4\left(2\,cx^2\mathrm{sign}\left(x\right)+3\,b\mathrm{sign}\left(x\right)\right)x^2+\frac{b^2\mathrm{sign}\left(x\right)}{c}\right)x^2-\frac{3\,b^3\mathrm{sign}\left(x\right)}{c^2}\right)\sqrt{cx^2+b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 3/128*b^4*ln(sqrt(b))*sign(x)/c^(5/2) - 3/128*b^4*ln(abs(-sqrt(c)
*x + sqrt(c*x^2 + b)))*sign(x)/c^(5/2) + 1/128*(2*(4*(2*c*x^2*sig
n(x) + 3*b*sign(x))*x^2 + b^2*sign(x)/c)*x^2 - 3*b^3*sign(x)/c^2)
*sqrt(c*x^2 + b)*x
```

3.240 $\int \frac{(bx^2+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=88

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}(bx^2+cx^4)^{3/2}$$

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3
/2)/6 - (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3
/2))
```

Rubi [A] time = 0.188542, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{3/2}}+\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{1}{6}(bx^2+cx^4)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (b*(b + 2*c*x^2)*Sqrt[b*x^2 + c*x^4])/(16*c) + (b*x^2 + c*x^4)^(3
/2)/6 - (b^3*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(3
/2))
```

Rubi in Sympy [A] time = 16.2732, size = 75, normalized size = 0.85

$$-\frac{b^3\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{\frac{3}{2}}}+\frac{b(b+2cx^2)\sqrt{bx^2+cx^4}}{16c}+\frac{(bx^2+cx^4)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] -b**3*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(16*c**(3/2)) + b
*(b + 2*c*x**2)*sqrt(b*x**2 + c*x**4)/(16*c) + (b*x**2 + c*x**4)*
*(3/2)/6
```

Mathematica [A] time = 0.118815, size = 103, normalized size = 1.17

$$\frac{x\sqrt{b+cx^2}\left(\sqrt{c}x\sqrt{b+cx^2}\left(3b^2+14bcx^2+8c^2x^4\right)-3b^3\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{48c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[c]*x*Sqrt[b + c*x^2]*(3*b^2 + 14*b*c*x^2
 + 8*c^2*x^4) - 3*b^3*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(48*c^
(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 102, normalized size = 1.2

$$-\frac{1}{48\,x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-8\,x\left(cx^2+b\right)^{5/2}\sqrt{c}+2\left(cx^2+b\right)^{3/2}\sqrt{c}xb+3\sqrt{cx^2+b}\sqrt{c}xb^2+3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^3\right)\left(cx^2+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x,x)
```

```
[Out] -1/48*(c*x^4+b*x^2)^(3/2)*(-8*x*(c*x^2+b)^(5/2)*c^(1/2)+2*(c*x^2+
b)^(3/2)*c^(1/2)*x*b+3*(c*x^2+b)^(1/2)*c^(1/2)*x*b^2+3*ln(x*c^(1/
2)+(c*x^2+b)^(1/2))*b^3)/x^3/(c*x^2+b)^(3/2)/c^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286907, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^3\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\left(8\,c^3x^4+14\,bc^2x^2+3\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^2},\frac{3\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)+(8}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/96*(3*b^3*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4 +
b*x^2)*c) + 2*(8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b
*x^2))/c^2, 1/48*(3*b^3*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 +
 b*x^2)) + (8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c)*sqrt(c*x^4 + b*x^
2))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x, x)
```

GIAC/XCAS [A] time = 0.277701, size = 115, normalized size = 1.31

$$-\frac{b^3\ln\left(\sqrt{b}\right)\operatorname{sign}(x)}{16\,c^{\frac{3}{2}}}+\frac{b^3\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+b}\right|\right)\operatorname{sign}(x)}{16\,c^{\frac{3}{2}}}+\frac{1}{48}\left(2\left(4\,cx^2\operatorname{sign}(x)+7\,b\operatorname{sign}(x)\right)x^2+\frac{3\,b^2\operatorname{sign}(x)}{c}\right)\sqrt{cx^2+b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/16*b^3*ln(sqrt(b))*sign(x)/c^(3/2) + 1/16*b^3*ln(abs(-sqrt(c)*
x + sqrt(c*x^2 + b)))*sign(x)/c^(3/2) + 1/48*(2*(4*c*x^2*sign(x)
+ 7*b*sign(x))*x^2 + 3*b^2*sign(x)/c)*sqrt(c*x^2 + b)*x
```

3.241 $\int \frac{(bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=80

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*
b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rubi [A] time = 0.189743, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3}{8}b\sqrt{bx^2+cx^4}+\frac{(bx^2+cx^4)^{3/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*b*Sqrt[b*x^2 + c*x^4])/8 + (b*x^2 + c*x^4)^(3/2)/(4*x^2) + (3*
b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[c])
```

Rubi in Sympy [A] time = 17.0887, size = 71, normalized size = 0.89

$$\frac{3b^2\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{c}}+\frac{3b\sqrt{bx^2+cx^4}}{8}+\frac{(bx^2+cx^4)^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] 3*b**2*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(8*sqrt(c)) + 3*
b*sqrt(b*x**2 + c*x**4)/8 + (b*x**2 + c*x**4)**(3/2)/(4*x**2)
```

Mathematica [A] time = 0.0853203, size = 75, normalized size = 0.94

$$\frac{1}{8}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{3b^2\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)}{\sqrt{c}x\sqrt{b+cx^2}}+5b+2cx^2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(5*b + 2*c*x^2 + (3*b^2*Log[c*x + Sqrt[c]*
Sqrt[b + c*x^2]])/(Sqrt[c]*x*Sqrt[b + c*x^2])))/8
```

Maple [A] time = 0.006, size = 84, normalized size = 1.1

$$\frac{1}{8x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2x\left(cx^2+b\right)^{3/2}\sqrt{c}+3\sqrt{cx^2+b}\sqrt{c}xb+3\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^2\right)\left(cx^2+b\right)^{-\frac{3}{2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^3,x)
```

```
[Out] 1/8*(c*x^4+b*x^2)^(3/2)*(2*x*(c*x^2+b)^(3/2)*c^(1/2)+3*(c*x^2+b)^
(1/2)*c^(1/2)*x*b+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2)/x^3/(c*x^2
+b)^(3/2)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277626, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^{2}\sqrt{c}\log\left(-\left(2\,cx^{2}+b\right)\sqrt{c}-2\,\sqrt{cx^{4}+bx^{2}}c\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(2\,c^{2}x^{2}+5\,bc\right)}{16\,c},\right.$$
$$\left.-\frac{3\,b^{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^{2}}{\sqrt{cx^{4}+bx^{2}}}\right)-\sqrt{cx^{4}+bx^{2}}\left(2\,c^{2}x^{2}+5\,bc\right)}{8\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 +
b*x^2)*c) + 2*sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c, -1/8*(3
*b^2*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2)) - sqrt(c*x
^4 + b*x^2)*(2*c^2*x^2 + 5*b*c))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.279282, size = 93, normalized size = 1.16

$$\frac{3\,b^{2}\ln\left(\sqrt{b}\right)\mathrm{sign}\left(x\right)}{8\,\sqrt{c}}-\frac{3\,b^{2}\ln\left(\left|-\sqrt{c}x+\sqrt{cx^{2}+b}\right|\right)\mathrm{sign}\left(x\right)}{8\,\sqrt{c}}+\frac{1}{8}\left(2\,cx^{2}\mathrm{sign}\left(x\right)+5\,b\mathrm{sign}\left(x\right)\right)\sqrt{cx^{2}+b}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 3/8*b^2*ln(sqrt(b))*sign(x)/sqrt(c) - 3/8*b^2*ln(abs(-sqrt(c)*x +
 sqrt(c*x^2 + b)))*sign(x)/sqrt(c) + 1/8*(2*c*x^2*sign(x) + 5*b*s
```

```
ign(x))*sqrt(c*x^2 + b)*x
```

3.242 $\int \frac{(bx^2+cx^4)^{3/2}}{x^5} dx$

Optimal. Leaf size=76

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi [A] time = 0.186348, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{(bx^2+cx^4)^{3/2}}{x^4}+\frac{3}{2}c\sqrt{bx^2+cx^4}+\frac{3}{2}b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/2 - (b*x^2 + c*x^4)^(3/2)/x^4 + (3*b*Sq
rt[c]*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi in Sympy [A] time = 16.9128, size = 68, normalized size = 0.89

$$\frac{3b\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2}+\frac{3c\sqrt{bx^2+cx^4}}{2}-\frac{(bx^2+cx^4)^{\frac{3}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] 3*b*sqrt(c)*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/2 + 3*c*sqr
t(b*x**2 + c*x**4)/2 - (b*x**2 + c*x**4)**(3/2)/x**4
```

Mathematica [A] time = 0.0831197, size = 81, normalized size = 1.07

$$\frac{-2b^2-bcx^2+3b\sqrt{c}x\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)+c^2x^4}{2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (-2*b^2 - b*c*x^2 + c^2*x^4 + 3*b*Sqrt[c]*x*Sqrt[b + c*x^2]*Log[c
*x + Sqrt[c]*Sqrt[b + c*x^2]])/(2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.009, size = 107, normalized size = 1.4

$$\frac{1}{2\,bx^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\,\left(cx^2+b\right)^{3/2}c^{3/2}x^2-2\,\left(cx^2+b\right)^{5/2}\sqrt{c}+3\,\sqrt{cx^2+b}c^{3/2}x^2b+3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)xb^2c\right)\left(cx^2+b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^5,x)
```

```
[Out] 1/2*(c*x^4+b*x^2)^(3/2)*(2*(c*x^2+b)^(3/2)*c^(3/2)*x^2-2*(c*x^2+b
)^(5/2)*c^(1/2)+3*(c*x^2+b)^(1/2)*c^(3/2)*x^2*b+3*ln(x*c^(1/2)+(c
*x^2+b)^(1/2))*x*b^2*c)/x^4/(c*x^2+b)^(3/2)/b/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275526, size = 1, normalized size = 0.01

$$\left[\frac{3\,b\sqrt{c}x^2\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2-2\,b\right)}{4\,x^2},\frac{3\,b\sqrt{-c}x^2\arctan\left(\frac{cx^2}{\sqrt{cx^4+bx^2}\sqrt{-c}}\right)+\sqrt{cx^4+bx^2}\left(cx\right.}{2\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/4*(3*b*sqrt(c)*x^2*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sq
rt(c)) + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b))/x^2, 1/2*(3*b*sqrt(
-c)*x^2*arctan(c*x^2/(sqrt(c*x^4 + b*x^2)*sqrt(-c))) + sqrt(c*x^4
 + b*x^2)*(c*x^2 - 2*b))/x^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**5, x)
```

GIAC/XCAS [A] time = 0.305079, size = 107, normalized size = 1.41

$$\frac{1}{2}\sqrt{cx^2+b}\,cx\,\mathrm{sign}\left(x\right)-\frac{3}{4}\,b\sqrt{c}\ln\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2\right)\mathrm{sign}\left(x\right)+\frac{2\,b^2\sqrt{c}\,\mathrm{sign}\left(x\right)}{\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + b)*c*x*sign(x) - 3/4*b*sqrt(c)*ln((sqrt(c)*x - s
qrt(c*x^2 + b))^2)*sign(x) + 2*b^2*sqrt(c)*sign(x)/((sqrt(c)*x -
sqrt(c*x^2 + b))^2 - b)
```

3.243 $\int \frac{(bx^2+cx^4)^{3/2}}{x^7} dx$

Optimal. Leaf size=75

$$c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) +
c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.186365, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$c^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{(bx^2+cx^4)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((c*Sqrt[b*x^2 + c*x^4])/x^2) - (b*x^2 + c*x^4)^(3/2)/(3*x^6) +
c^(3/2)*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]
```

Rubi in Sympy [A] time = 16.5436, size = 65, normalized size = 0.87

$$c^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right) - \frac{c\sqrt{bx^2+cx^4}}{x^2} - \frac{(bx^2+cx^4)^{\frac{3}{2}}}{3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] c**(3/2)*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4)) - c*sqrt(b*x**
2 + c*x**4)/x**2 - (b*x**2 + c*x**4)**(3/2)/(3*x**6)
```

Mathematica [A] time = 0.0656077, size = 87, normalized size = 1.16

$$\frac{\sqrt{x^2(b+cx^2)}\left(3c^{3/2}x^3\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)-\sqrt{b+cx^2}\left(b+4cx^2\right)\right)}{3x^4\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b + c*x^2]*(b + 4*c*x^2)) + 3*c^(3
/2)*x^3*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(3*x^4*Sqrt[b + c*x^
2])
```

Maple [B] time = 0.009, size = 129, normalized size = 1.7

$$\frac{1}{3\,b^2x^6}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\,\left(cx^2+b\right)^{3/2}c^{5/2}x^4-2\,\left(cx^2+b\right)^{5/2}c^{3/2}x^2+3\,\sqrt{cx^2+b}c^{5/2}x^4b+3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)x^3b^2c^2-(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^7,x)
```

```
[Out] 1/3*(c*x^4+b*x^2)^(3/2)*(2*(c*x^2+b)^(3/2)*c^(5/2)*x^4-2*(c*x^2+b
)^(5/2)*c^(3/2)*x^2+3*(c*x^2+b)^(1/2)*c^(5/2)*x^4*b+3*ln(x*c^(1/2
)+(c*x^2+b)^(1/2))*x^3*b^2*c^2-(c*x^2+b)^(5/2)*b*c^(1/2))/x^6/(c*
x^2+b)^(3/2)/b^2/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27462, size = 1, normalized size = 0.01

$$\left[\frac{3\,c^{\frac{3}{2}}x^4\log\left(-2\,cx^2-b-2\,\sqrt{cx^4+bx^2}\sqrt{c}\right)-2\,\sqrt{cx^4+bx^2}\left(4\,cx^2+b\right)}{6\,x^4},\frac{3\,\sqrt{-c}cx^4\arctan\left(\frac{cx^2}{\sqrt{cx^4+bx^2}\sqrt{-c}}\right)-\sqrt{cx^4+bx^2}\left(4\,cx\right.}{3\,x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/6*(3*c^(3/2)*x^4*log(-2*c*x^2 - b - 2*sqrt(c*x^4 + b*x^2)*sqrt
(c)) - 2*sqrt(c*x^4 + b*x^2)*(4*c*x^2 + b))/x^4, 1/3*(3*sqrt(-c)*
c*x^4*arctan(c*x^2/(sqrt(c*x^4 + b*x^2)*sqrt(-c))) - sqrt(c*x^4 +
 b*x^2)*(4*c*x^2 + b))/x^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^7}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**7, x)
```

GIAC/XCAS [A] time = 0.370355, size = 165, normalized size = 2.2

$$-\frac{1}{2}\, c^{\frac{3}{2}} \ln\left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2\right) \operatorname{sign}(x) + \frac{4 \left(3 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^4 bc^{\frac{3}{2}} \operatorname{sign}(x) - 3 \left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 b^2 c^{\frac{3}{2}} \operatorname{sign}(x) + 2\, b^3 c^{\frac{3}{2}} \operatorname{sign}(x)\right)}{3 \left(\left(\sqrt{c}x - \sqrt{cx^2 + b}\right)^2 - b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/2*c^(3/2)*ln((sqrt(c)*x - sqrt(c*x^2 + b))^2)*sign(x) + 4/3*(3
*(sqrt(c)*x - sqrt(c*x^2 + b))^4*b*c^(3/2)*sign(x) - 3*(sqrt(c)*x
 - sqrt(c*x^2 + b))^2*b^2*c^(3/2)*sign(x) + 2*b^3*c^(3/2)*sign(x)
)/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^3
```

3.244 $\int \frac{(bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=25

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(5*b*x^10)
```

Rubi [A] time = 0.0662835, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{(bx^2+cx^4)^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)^(3/2)/x^9,x]

[Out] -(b*x^2 + c*x^4)^(5/2)/(5*b*x^10)
```

Rubi in Sympy [A] time = 8.19169, size = 20, normalized size = 0.8

$$-\frac{(bx^2+cx^4)^{\frac{5}{2}}}{5bx^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2)**(3/2)/x**9,x)

[Out] -(b*x**2 + c*x**4)**(5/2)/(5*b*x**10)
```

Mathematica [A] time = 0.0361389, size = 25, normalized size = 1.

$$-\frac{(x^2(b+cx^2))^{5/2}}{5bx^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^9,x]

[Out] -(x^2*(b + c*x^2))^(5/2)/(5*b*x^10)
```

Maple [A] time = 0.005, size = 29, normalized size = 1.2

$$-\frac{cx^{2}+b}{5\,x^{8}b}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^9,x)

[Out] -1/5/x^8*(c*x^2+b)/b*(c*x^4+b*x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^9,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267644, size = 53, normalized size = 2.12

$$-\frac{\left(c^{2}x^{4}+2\,bcx^{2}+b^{2}\right)\sqrt{cx^{4}+bx^{2}}}{5\,bx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^9,x, algorithm="fricas")

[Out] -1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(b*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**9, x)
```

GIAC/XCAS [A] time = 0.293364, size = 124, normalized size = 4.96

$$\frac{2\left(5\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^8 c^{\frac{5}{2}}\operatorname{sign}(x)+10\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^4 b^2c^{\frac{5}{2}}\operatorname{sign}(x)+b^4c^{\frac{5}{2}}\operatorname{sign}(x)\right)}{5\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] 2/5*(5*(sqrt(c)*x - sqrt(c*x^2 + b))^8*c^(5/2)*sign(x) + 10*(sqrt
(c)*x - sqrt(c*x^2 + b))^4*b^2*c^(5/2)*sign(x) + b^4*c^(5/2)*sign
(x))/((sqrt(c)*x - sqrt(c*x^2 + b))^2 - b)^5
```

3.245 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11}} dx$

Optimal. Leaf size=52

$$\frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(7*b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(
35*b^2*x^10)
```

Rubi [A] time = 0.136685, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2c\left(bx^2+cx^4\right)^{5/2}}{35b^2x^{10}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{7bx^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(7*b*x^12) + (2*c*(b*x^2 + c*x^4)^(5/2))/(
35*b^2*x^10)
```

Rubi in Sympy [A] time = 14.0873, size = 44, normalized size = 0.85

$$-\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{7bx^{12}}+\frac{2c\left(bx^2+cx^4\right)^{\frac{5}{2}}}{35b^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] -(b*x**2 + c*x**4)**(5/2)/(7*b*x**12) + 2*c*(b*x**2 + c*x**4)**(5
/2)/(35*b**2*x**10)
```

Mathematica [A] time = 0.037509, size = 35, normalized size = 0.67

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(2cx^2-5b\right)}{35b^2x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-5*b + 2*c*x^2))/(35*b^2*x^12)
```

Maple [A] time = 0.008, size = 39, normalized size = 0.8

$$-\frac{(cx^2+b)(-2cx^2+5b)}{35x^{10}b^2}(cx^4+bx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^11,x)
```

```
[Out] -1/35*(c*x^2+b)*(-2*c*x^2+5*b)*(c*x^4+b*x^2)^(3/2)/x^10/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272948, size = 72, normalized size = 1.38

$$\frac{(2c^3x^6-bc^2x^4-8b^2cx^2-5b^3)\sqrt{cx^4+bx^2}}{35b^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/35*(2*c^3*x^6 - b*c^2*x^4 - 8*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4 + b
*x^2)/(b^2*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**11,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**11, x)
```

GIAC/XCAS [A] time = 0.303902, size = 240, normalized size = 4.62

$$\frac{4\left(35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}c^{\frac{7}{2}}\operatorname{sign}(x)+35\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}bc^{\frac{7}{2}}\operatorname{sign}(x)+70\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{6}b^2c^{\frac{7}{2}}\operatorname{sign}(x)+14\left(\sqrt{c}x-\right.\right.}{35\left(\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^2-b\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] 4/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b))^10*c^(7/2)*sign(x) + 35*(s
qrt(c)*x - sqrt(c*x^2 + b))^8*b*c^(7/2)*sign(x) + 70*(sqrt(c)*x -
 sqrt(c*x^2 + b))^6*b^2*c^(7/2)*sign(x) + 14*(sqrt(c)*x - sqrt(c*
x^2 + b))^4*b^3*c^(7/2)*sign(x) + 7*(sqrt(c)*x - sqrt(c*x^2 + b))
^2*b^4*c^(7/2)*sign(x) - b^5*c^(7/2)*sign(x))/((sqrt(c)*x - sqrt(
c*x^2 + b))^2 - b)^7
```

3.246 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13}} dx$

Optimal. Leaf size=80

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(9*b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(
63*b^2*x^12) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rubi [A] time = 0.209459, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{315b^3x^{10}}+\frac{4c\left(bx^2+cx^4\right)^{5/2}}{63b^2x^{12}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{9bx^{14}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(9*b*x^14) + (4*c*(b*x^2 + c*x^4)^(5/2))/(
63*b^2*x^12) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(315*b^3*x^10)
```

Rubi in Sympy [A] time = 21.0673, size = 71, normalized size = 0.89

$$-\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{9bx^{14}}+\frac{4c\left(bx^2+cx^4\right)^{\frac{5}{2}}}{63b^2x^{12}}-\frac{8c^2\left(bx^2+cx^4\right)^{\frac{5}{2}}}{315b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] -(b*x**2 + c*x**4)**(5/2)/(9*b*x**14) + 4*c*(b*x**2 + c*x**4)**(5
/2)/(63*b**2*x**12) - 8*c**2*(b*x**2 + c*x**4)**(5/2)/(315*b**3*x
**10)
```

Mathematica [A] time = 0.0413076, size = 46, normalized size = 0.57

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(35b^2-20bcx^2+8c^2x^4\right)}{315b^3x^{14}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(35*b^2 - 20*b*c*x^2 + 8*c^2*x^4))/(315
*b^3*x^14)
```

Maple [A] time = 0.006, size = 50, normalized size = 0.6

$$-\frac{(cx^2+b)\,(8\,c^2x^4-20\,bcx^2+35\,b^2)}{315\,x^{12}b^3}\,(cx^4+bx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^13,x)
```

```
[Out] -1/315*(c*x^2+b)*(8*c^2*x^4-20*b*c*x^2+35*b^2)*(c*x^4+b*x^2)^(3/2
)/x^12/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295869, size = 86, normalized size = 1.08

$$-\frac{(8\,c^4x^8-4\,bc^3x^6+3\,b^2c^2x^4+50\,b^3cx^2+35\,b^4)\sqrt{cx^4+bx^2}}{315\,b^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/315*(8*c^4*x^8 - 4*b*c^3*x^6 + 3*b^2*c^2*x^4 + 50*b^3*c*x^2 +
35*b^4)*sqrt(c*x^4 + b*x^2)/(b^3*x^10)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**13,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**13, x)
```

GIAC/XCAS [A] time = 0.307544, size = 278, normalized size = 3.48

$$\frac{16\left(210\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}c^{\frac{9}{2}}\operatorname{sign}(x)+315\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{10}bc^{\frac{9}{2}}\operatorname{sign}(x)+441\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{8}b^2c^{\frac{9}{2}}\operatorname{sign}(x)+126\left(\right.}{315\left(\left(\sqrt{c}x-\sqrt{cx\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] 16/315*(210*(sqrt(c)*x - sqrt(c*x^2 + b))^12*c^(9/2)*sign(x) + 31
5*(sqrt(c)*x - sqrt(c*x^2 + b))^10*b*c^(9/2)*sign(x) + 441*(sqrt(
c)*x - sqrt(c*x^2 + b))^8*b^2*c^(9/2)*sign(x) + 126*(sqrt(c)*x -
sqrt(c*x^2 + b))^6*b^3*c^(9/2)*sign(x) + 36*(sqrt(c)*x - sqrt(c*x
^2 + b))^4*b^4*c^(9/2)*sign(x) - 9*(sqrt(c)*x - sqrt(c*x^2 + b))^
2*b^5*c^(9/2)*sign(x) + b^6*c^(9/2)*sign(x))/((sqrt(c)*x - sqrt(c
*x^2 + b))^2 - b)^9
```

3.247 $\int \frac{\left(bx^2+cx^4\right)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(11*b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/
(33*b^2*x^14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (1
6*c^3*(b*x^2 + c*x^4)^(5/2))/(1155*b^4*x^10)
```

Rubi [A] time = 0.294217, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{16c^3\left(bx^2+cx^4\right)^{5/2}}{1155b^4x^{10}}-\frac{8c^2\left(bx^2+cx^4\right)^{5/2}}{231b^3x^{12}}+\frac{2c\left(bx^2+cx^4\right)^{5/2}}{33b^2x^{14}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{11bx^{16}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(11*b*x^16) + (2*c*(b*x^2 + c*x^4)^(5/2))/
(33*b^2*x^14) - (8*c^2*(b*x^2 + c*x^4)^(5/2))/(231*b^3*x^12) + (1
6*c^3*(b*x^2 + c*x^4)^(5/2))/(1155*b^4*x^10)
```

Rubi in Sympy [A] time = 28.8189, size = 99, normalized size = 0.92

$$-\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{11bx^{16}}+\frac{2c\left(bx^2+cx^4\right)^{\frac{5}{2}}}{33b^2x^{14}}-\frac{8c^2\left(bx^2+cx^4\right)^{\frac{5}{2}}}{231b^3x^{12}}+\frac{16c^3\left(bx^2+cx^4\right)^{\frac{5}{2}}}{1155b^4x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] -(b*x**2 + c*x**4)**(5/2)/(11*b*x**16) + 2*c*(b*x**2 + c*x**4)**(
5/2)/(33*b**2*x**14) - 8*c**2*(b*x**2 + c*x**4)**(5/2)/(231*b**3*
x**12) + 16*c**3*(b*x**2 + c*x**4)**(5/2)/(1155*b**4*x**10)
```

Mathematica [A] time = 0.0477584, size = 57, normalized size = 0.53

$$\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(-105b^3+70b^2cx^2-40bc^2x^4+16c^3x^6\right)}{1155b^4x^{16}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^15,x]
```

```
[Out] ((x^2*(b + c*x^2))^(5/2)*(-105*b^3 + 70*b^2*c*x^2 - 40*b*c^2*x^4
+ 16*c^3*x^6))/(1155*b^4*x^16)
```

Maple [A] time = 0.008, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-16\,c^3x^6+40\,bc^2x^4-70\,b^2cx^2+105\,b^3\right)}{1155\,x^{14}b^4}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^15,x)
```

```
[Out] -1/1155*(c*x^2+b)*(-16*c^3*x^6+40*b*c^2*x^4-70*b^2*c*x^2+105*b^3)
*(c*x^4+b*x^2)^(3/2)/x^14/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.341717, size = 101, normalized size = 0.94

$$\frac{\left(16\,c^5x^{10}-8\,bc^4x^8+6\,b^2c^3x^6-5\,b^3c^2x^4-140\,b^4cx^2-105\,b^5\right)\sqrt{cx^4+bx^2}}{1155\,b^4x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] 1/1155*(16*c^5*x^10 - 8*b*c^4*x^8 + 6*b^2*c^3*x^6 - 5*b^3*c^2*x^4
 - 140*b^4*c*x^2 - 105*b^5)*sqrt(c*x^4 + b*x^2)/(b^4*x^12)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**15,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**15, x)
```

GIAC/XCAS [A] time = 0.311436, size = 319, normalized size = 2.95

$$\frac{32\left(1155\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{14}c^{\frac{11}{2}}\operatorname{sign}(x) + 2079\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{12}bc^{\frac{11}{2}}\operatorname{sign}(x) + 2541\left(\sqrt{c}x - \sqrt{cx^2+b}\right)^{10}b^2c^{\frac{11}{2}}\operatorname{sign}(x) +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] 32/1155*(1155*(sqrt(c)*x - sqrt(c*x^2 + b))^14*c^(11/2)*sign(x) +
 2079*(sqrt(c)*x - sqrt(c*x^2 + b))^12*b*c^(11/2)*sign(x) + 2541*
(sqrt(c)*x - sqrt(c*x^2 + b))^10*b^2*c^(11/2)*sign(x) + 825*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*b^3*c^(11/2)*sign(x) + 165*(sqrt(c)*x
- sqrt(c*x^2 + b))^6*b^4*c^(11/2)*sign(x) - 55*(sqrt(c)*x - sqrt(
c*x^2 + b))^4*b^5*c^(11/2)*sign(x) + 11*(sqrt(c)*x - sqrt(c*x^2 +
 b))^2*b^6*c^(11/2)*sign(x) - b^7*c^(11/2)*sign(x))/((sqrt(c)*x -
 sqrt(c*x^2 + b))^2 - b)^11
```

3.248 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=136

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(13*b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/
(143*b^2*x^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) +
(64*c^3*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2
+ c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rubi [A] time = 0.385765, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{128c^4\left(bx^2+cx^4\right)^{5/2}}{15015b^5x^{10}}+\frac{64c^3\left(bx^2+cx^4\right)^{5/2}}{3003b^4x^{12}}-\frac{16c^2\left(bx^2+cx^4\right)^{5/2}}{429b^3x^{14}}+\frac{8c\left(bx^2+cx^4\right)^{5/2}}{143b^2x^{16}}-\frac{\left(bx^2+cx^4\right)^{5/2}}{13bx^{18}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -(b*x^2 + c*x^4)^(5/2)/(13*b*x^18) + (8*c*(b*x^2 + c*x^4)^(5/2))/
(143*b^2*x^16) - (16*c^2*(b*x^2 + c*x^4)^(5/2))/(429*b^3*x^14) +
(64*c^3*(b*x^2 + c*x^4)^(5/2))/(3003*b^4*x^12) - (128*c^4*(b*x^2
+ c*x^4)^(5/2))/(15015*b^5*x^10)
```

Rubi in Sympy [A] time = 37.577, size = 126, normalized size = 0.93

$$-\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{13bx^{18}}+\frac{8c\left(bx^2+cx^4\right)^{\frac{5}{2}}}{143b^2x^{16}}-\frac{16c^2\left(bx^2+cx^4\right)^{\frac{5}{2}}}{429b^3x^{14}}+\frac{64c^3\left(bx^2+cx^4\right)^{\frac{5}{2}}}{3003b^4x^{12}}-\frac{128c^4\left(bx^2+cx^4\right)^{\frac{5}{2}}}{15015b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] -(b*x**2 + c*x**4)**(5/2)/(13*b*x**18) + 8*c*(b*x**2 + c*x**4)**(
5/2)/(143*b**2*x**16) - 16*c**2*(b*x**2 + c*x**4)**(5/2)/(429*b**
3*x**14) + 64*c**3*(b*x**2 + c*x**4)**(5/2)/(3003*b**4*x**12) - 1
28*c**4*(b*x**2 + c*x**4)**(5/2)/(15015*b**5*x**10)
```

Mathematica [A] time = 0.0515323, size = 68, normalized size = 0.5

$$-\frac{\left(x^2\left(b+cx^2\right)\right)^{5/2}\left(1155b^4-840b^3cx^2+560b^2c^2x^4-320bc^3x^6+128c^4x^8\right)}{15015b^5x^{18}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^17,x]
```

```
[Out] -((x^2*(b + c*x^2))^(5/2)*(1155*b^4 - 840*b^3*c*x^2 + 560*b^2*c^2
*x^4 - 320*b*c^3*x^6 + 128*c^4*x^8))/(15015*b^5*x^18)
```

Maple [A] time = 0.009, size = 72, normalized size = 0.5

$$-\frac{\left(cx^2+b\right)\left(128\,c^4x^8-320\,c^3x^6b+560\,c^2x^4b^2-840\,cx^2b^3+1155\,b^4\right)}{15015\,x^{16}b^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^17,x)
```

```
[Out] -1/15015*(c*x^2+b)*(128*c^4*x^8-320*b*c^3*x^6+560*b^2*c^2*x^4-840
*b^3*c*x^2+1155*b^4)*(c*x^4+b*x^2)^(3/2)/x^16/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.412493, size = 116, normalized size = 0.85

$$-\frac{\left(128\,c^6x^{12}-64\,bc^5x^{10}+48\,b^2c^4x^8-40\,b^3c^3x^6+35\,b^4c^2x^4+1470\,b^5cx^2+1155\,b^6\right)\sqrt{cx^4+bx^2}}{15015\,b^5x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/15015*(128*c^6*x^12 - 64*b*c^5*x^10 + 48*b^2*c^4*x^8 - 40*b^3*
c^3*x^6 + 35*b^4*c^2*x^4 + 1470*b^5*c*x^2 + 1155*b^6)*sqrt(c*x^4
+ b*x^2)/(b^5*x^14)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**17,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**17, x)
```

GIAC/XCAS [A] time = 0.315218, size = 356, normalized size = 2.62

$$\frac{256\left(6006\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{16}c^{\frac{13}{2}}\operatorname{sign}(x)+12012\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{14}bc^{\frac{13}{2}}\operatorname{sign}(x)+13728\left(\sqrt{c}x-\sqrt{cx^2+b}\right)^{12}b^2c^{\frac{13}{2}}\operatorname{sign}(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] 256/15015*(6006*(sqrt(c)*x - sqrt(c*x^2 + b))^16*c^(13/2)*sign(x)
 + 12012*(sqrt(c)*x - sqrt(c*x^2 + b))^14*b*c^(13/2)*sign(x) + 13
728*(sqrt(c)*x - sqrt(c*x^2 + b))^12*b^2*c^(13/2)*sign(x) + 4719*
(sqrt(c)*x - sqrt(c*x^2 + b))^10*b^3*c^(13/2)*sign(x) + 715*(sqrt
(c)*x - sqrt(c*x^2 + b))^8*b^4*c^(13/2)*sign(x) - 286*(sqrt(c)*x
- sqrt(c*x^2 + b))^6*b^5*c^(13/2)*sign(x) + 78*(sqrt(c)*x - sqrt(
c*x^2 + b))^4*b^6*c^(13/2)*sign(x) - 13*(sqrt(c)*x - sqrt(c*x^2 +
 b))^2*b^7*c^(13/2)*sign(x) + b^8*c^(13/2)*sign(x))/((sqrt(c)*x -
 sqrt(c*x^2 + b))^2 - b)^13
```

3.249 $\int x^6 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=134

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2
+ c*x^4)^(5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(
429*c^3*x) - (8*b*x*(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^
2 + c*x^4)^(5/2))/(13*c)
```

Rubi [A] time = 0.350285, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{128b^4\left(bx^2+cx^4\right)^{5/2}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{5/2}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{5/2}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{5/2}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{5/2}}{13c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (128*b^4*(b*x^2 + c*x^4)^(5/2))/(15015*c^5*x^5) - (64*b^3*(b*x^2
+ c*x^4)^(5/2))/(3003*c^4*x^3) + (16*b^2*(b*x^2 + c*x^4)^(5/2))/(
429*c^3*x) - (8*b*x*(b*x^2 + c*x^4)^(5/2))/(143*c^2) + (x^3*(b*x^
2 + c*x^4)^(5/2))/(13*c)
```

Rubi in Sympy [A] time = 38.3936, size = 122, normalized size = 0.91

$$\frac{128b^4\left(bx^2+cx^4\right)^{\frac{5}{2}}}{15015c^5x^5}-\frac{64b^3\left(bx^2+cx^4\right)^{\frac{5}{2}}}{3003c^4x^3}+\frac{16b^2\left(bx^2+cx^4\right)^{\frac{5}{2}}}{429c^3x}-\frac{8bx\left(bx^2+cx^4\right)^{\frac{5}{2}}}{143c^2}+\frac{x^3\left(bx^2+cx^4\right)^{\frac{5}{2}}}{13c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 128*b**4*(b*x**2 + c*x**4)**(5/2)/(15015*c**5*x**5) - 64*b**3*(b*
x**2 + c*x**4)**(5/2)/(3003*c**4*x**3) + 16*b**2*(b*x**2 + c*x**4
)**(5/2)/(429*c**3*x) - 8*b*x*(b*x**2 + c*x**4)**(5/2)/(143*c**2)
 + x**3*(b*x**2 + c*x**4)**(5/2)/(13*c)
```

Mathematica [A] time = 0.0493523, size = 75, normalized size = 0.56

$$\frac{x\left(b+cx^2\right)^3\left(128b^4-320b^3cx^2+560b^2c^2x^4-840bc^3x^6+1155c^4x^8\right)}{15015c^5\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(128*b^4 - 320*b^3*c*x^2 + 560*b^2*c^2*x^4 - 840
*b*c^3*x^6 + 1155*c^4*x^8))/(15015*c^5*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.009, size = 72, normalized size = 0.5

$$\frac{\left(cx^2+b\right)\left(1155\,x^8c^4-840\,bx^6c^3+560\,b^2x^4c^2-320\,b^3x^2c+128\,b^4\right)}{15015\,c^5x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/15015*(c*x^2+b)*(1155*c^4*x^8-840*b*c^3*x^6+560*b^2*c^2*x^4-320
*b^3*c*x^2+128*b^4)*(c*x^4+b*x^2)^(3/2)/c^5/x^3
```

Maxima [A] time = 0.693262, size = 107, normalized size = 0.8

$$\frac{\left(1155\,c^6x^{12}+1470\,bc^5x^{10}+35\,b^2c^4x^8-40\,b^3c^3x^6+48\,b^4c^2x^4-64\,b^5cx^2+128\,b^6\right)\sqrt{cx^2+b}}{15015\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^
3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^2 +
 b)/c^5
```

Fricas [A] time = 0.270143, size = 116, normalized size = 0.87

$$\frac{\left(1155\,c^6x^{12} + 1470\,bc^5x^{10} + 35\,b^2c^4x^8 - 40\,b^3c^3x^6 + 48\,b^4c^2x^4 - 64\,b^5cx^2 + 128\,b^6\right)\sqrt{cx^4 + bx^2}}{15015\,c^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*c^6*x^12 + 1470*b*c^5*x^10 + 35*b^2*c^4*x^8 - 40*b^
3*c^3*x^6 + 48*b^4*c^2*x^4 - 64*b^5*c*x^2 + 128*b^6)*sqrt(c*x^4 +
 b*x^2)/(c^5*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6*(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.276418, size = 240, normalized size = 1.79

$$-\frac{128\,b^{\frac{13}{2}}\,\mathrm{sign}\left(x\right)}{15015\,c^5} + \frac{\frac{13\left(315\left(cx^2+b\right)^{\frac{11}{2}} - 1540\left(cx^2+b\right)^{\frac{9}{2}}b + 2970\left(cx^2+b\right)^{\frac{7}{2}}b^2 - 2772\left(cx^2+b\right)^{\frac{5}{2}}b^3 + 1155\left(cx^2+b\right)^{\frac{3}{2}}b^4\right)b\,\mathrm{sign}(x)}{c^4} + \frac{5\left(693\left(cx^2+b\right)^{\frac{13}{2}} - 4095\left(cx^2+b\right)^{\frac{11}{2}}b + 10010\left(cx^2\cdots\right.\right.}{\cdots}}{45045\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] -128/15015*b^(13/2)*sign(x)/c^5 + 1/45045*(13*(315*(c*x^2 + b)^(1
1/2) - 1540*(c*x^2 + b)^(9/2)*b + 2970*(c*x^2 + b)^(7/2)*b^2 - 27
72*(c*x^2 + b)^(5/2)*b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*b*sign(x)/
c^4 + 5*(693*(c*x^2 + b)^(13/2) - 4095*(c*x^2 + b)^(11/2)*b + 100
10*(c*x^2 + b)^(9/2)*b^2 - 12870*(c*x^2 + b)^(7/2)*b^3 + 9009*(c*
x^2 + b)^(5/2)*b^4 - 3003*(c*x^2 + b)^(3/2)*b^5)*sign(x)/c^4)/c
```

3.250 $\int x^4 \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=106

$$-\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}$$

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 +
c*x^4)^(5/2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2
*x) + (x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```

Rubi [A] time = 0.253039, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{16b^3\left(bx^2+cx^4\right)^{5/2}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{5/2}}{11c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-16*b^3*(b*x^2 + c*x^4)^(5/2))/(1155*c^4*x^5) + (8*b^2*(b*x^2 +
c*x^4)^(5/2))/(231*c^3*x^3) - (2*b*(b*x^2 + c*x^4)^(5/2))/(33*c^2
*x) + (x*(b*x^2 + c*x^4)^(5/2))/(11*c)
```

Rubi in Sympy [A] time = 28.495, size = 95, normalized size = 0.9

$$-\frac{16b^3\left(bx^2+cx^4\right)^{\frac{5}{2}}}{1155c^4x^5}+\frac{8b^2\left(bx^2+cx^4\right)^{\frac{5}{2}}}{231c^3x^3}-\frac{2b\left(bx^2+cx^4\right)^{\frac{5}{2}}}{33c^2x}+\frac{x\left(bx^2+cx^4\right)^{\frac{5}{2}}}{11c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -16*b**3*(b*x**2 + c*x**4)**(5/2)/(1155*c**4*x**5) + 8*b**2*(b*x*
*2 + c*x**4)**(5/2)/(231*c**3*x**3) - 2*b*(b*x**2 + c*x**4)**(5/2
)/(33*c**2*x) + x*(b*x**2 + c*x**4)**(5/2)/(11*c)
```

Mathematica [A] time = 0.0452642, size = 64, normalized size = 0.6

$$\frac{x\left(b+cx^2\right)^3\left(-16b^3+40b^2cx^2-70bc^2x^4+105c^3x^6\right)}{1155c^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-16*b^3 + 40*b^2*c*x^2 - 70*b*c^2*x^4 + 105*c^3
*x^6))/(1155*c^4*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 61, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(-105\,c^3x^6+70\,bc^2x^4-40\,b^2cx^2+16\,b^3\right)}{1155\,c^4x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/1155*(c*x^2+b)*(-105*c^3*x^6+70*b*c^2*x^4-40*b^2*c*x^2+16*b^3)
*(c*x^4+b*x^2)^(3/2)/c^4/x^3
```

Maxima [A] time = 0.705053, size = 92, normalized size = 0.87

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^2+b}}{1155\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*
x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^2 + b)/c^4
```

Fricas [A] time = 0.266583, size = 101, normalized size = 0.95

$$\frac{\left(105\,c^5x^{10}+140\,bc^4x^8+5\,b^2c^3x^6-6\,b^3c^2x^4+8\,b^4cx^2-16\,b^5\right)\sqrt{cx^4+bx^2}}{1155\,c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^5*x^10 + 140*b*c^4*x^8 + 5*b^2*c^3*x^6 - 6*b^3*c^2*
x^4 + 8*b^4*c*x^2 - 16*b^5)*sqrt(c*x^4 + b*x^2)/(c^4*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4*(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.275515, size = 201, normalized size = 1.9

$$\frac{16\, b^{\frac{11}{2}} \operatorname{sign}(x)}{1155\, c^4} + \frac{\frac{11 \left(35 \left(cx^2+b\right)^{\frac{9}{2}} - 135 \left(cx^2+b\right)^{\frac{7}{2}} b + 189 \left(cx^2+b\right)^{\frac{5}{2}} b^2 - 105 \left(cx^2+b\right)^{\frac{3}{2}} b^3\right) b \operatorname{sign}(x)}{c^3} + \frac{\left(315 \left(cx^2+b\right)^{\frac{11}{2}} - 1540 \left(cx^2+b\right)^{\frac{9}{2}} b + 2970 \left(cx^2+b\right)^{\frac{7}{2}} b^2 - 2772 \left(cx^2+b\right)^{\frac{5}{2}} b^3 + \cdots}{c^3}}{3465\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] 16/1155*b^(11/2)*sign(x)/c^4 + 1/3465*(11*(35*(c*x^2 + b)^(9/2) -
 135*(c*x^2 + b)^(7/2)*b + 189*(c*x^2 + b)^(5/2)*b^2 - 105*(c*x^2
 + b)^(3/2)*b^3)*b*sign(x)/c^3 + (315*(c*x^2 + b)^(11/2) - 1540*(
c*x^2 + b)^(9/2)*b + 2970*(c*x^2 + b)^(7/2)*b^2 - 2772*(c*x^2 + b
)^(5/2)*b^3 + 1155*(c*x^2 + b)^(3/2)*b^4)*sign(x)/c^3)/c
```

3.251 $\int x^2 \left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=80

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4
)^(5/2))/(63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rubi [A] time = 0.173749, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{8b^2\left(bx^2+cx^4\right)^{5/2}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{5/2}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{5/2}}{9cx}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (8*b^2*(b*x^2 + c*x^4)^(5/2))/(315*c^3*x^5) - (4*b*(b*x^2 + c*x^4
)^(5/2))/(63*c^2*x^3) + (b*x^2 + c*x^4)^(5/2)/(9*c*x)
```

Rubi in Sympy [A] time = 20.3437, size = 70, normalized size = 0.88

$$\frac{8b^2\left(bx^2+cx^4\right)^{\frac{5}{2}}}{315c^3x^5}-\frac{4b\left(bx^2+cx^4\right)^{\frac{5}{2}}}{63c^2x^3}+\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{9cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 8*b**2*(b*x**2 + c*x**4)**(5/2)/(315*c**3*x**5) - 4*b*(b*x**2 + c
*x**4)**(5/2)/(63*c**2*x**3) + (b*x**2 + c*x**4)**(5/2)/(9*c*x)
```

Mathematica [A] time = 0.0399144, size = 53, normalized size = 0.66

$$\frac{x\left(b+cx^2\right)^3\left(8b^2-20bcx^2+35c^2x^4\right)}{315c^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(8*b^2 - 20*b*c*x^2 + 35*c^2*x^4))/(315*c^3*Sqrt
[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 50, normalized size = 0.6

$$\frac{(cx^2+b)(35c^2x^4-20bcx^2+8b^2)}{315c^3x^3}(cx^4+bx^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/315*(c*x^2+b)*(35*c^2*x^4-20*b*c*x^2+8*b^2)*(c*x^4+b*x^2)^(3/2)
/c^3/x^3
```

Maxima [A] time = 0.695611, size = 77, normalized size = 0.96

$$\frac{(35c^4x^8+50bc^3x^6+3b^2c^2x^4-4b^3cx^2+8b^4)\sqrt{cx^2+b}}{315c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 +
8*b^4)*sqrt(c*x^2 + b)/c^3
```

Fricas [A] time = 0.264651, size = 86, normalized size = 1.08

$$\frac{(35c^4x^8+50bc^3x^6+3b^2c^2x^4-4b^3cx^2+8b^4)\sqrt{cx^4+bx^2}}{315c^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^4*x^8 + 50*b*c^3*x^6 + 3*b^2*c^2*x^4 - 4*b^3*c*x^2 +
8*b^4)*sqrt(c*x^4 + b*x^2)/(c^3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.272949, size = 163, normalized size = 2.04

$$-\frac{8\, b^{\frac{9}{2}} \mathrm{sign}\left(x\right)}{315\, c^3} + \frac{\frac{3\left(15\left(cx^2+b\right)^{\frac{7}{2}} - 42\left(cx^2+b\right)^{\frac{5}{2}} b + 35\left(cx^2+b\right)^{\frac{3}{2}} b^2\right) b \mathrm{sign}(x)}{c^2} + \frac{\left(35\left(cx^2+b\right)^{\frac{9}{2}} - 135\left(cx^2+b\right)^{\frac{7}{2}} b + 189\left(cx^2+b\right)^{\frac{5}{2}} b^2 - 105\left(cx^2+b\right)^{\frac{3}{2}} b^3\right) \mathrm{sign}(x)}{c^2}}{315\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] -8/315*b^(9/2)*sign(x)/c^3 + 1/315*(3*(15*(c*x^2 + b)^(7/2) - 42*
(c*x^2 + b)^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*b*sign(x)/c^2 + (
35*(c*x^2 + b)^(9/2) - 135*(c*x^2 + b)^(7/2)*b + 189*(c*x^2 + b)^
(5/2)*b^2 - 105*(c*x^2 + b)^(3/2)*b^3)*sign(x)/c^2)/c
```

3.252 $\int \left(bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=52

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)
/(7*c*x^3)
```

Rubi [A] time = 0.0896004, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\left(bx^2+cx^4\right)^{5/2}}{7cx^3}-\frac{2b\left(bx^2+cx^4\right)^{5/2}}{35c^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b*x^2 + c*x^4)^(5/2))/(35*c^2*x^5) + (b*x^2 + c*x^4)^(5/2)
/(7*c*x^3)
```

Rubi in Sympy [A] time = 12.5922, size = 44, normalized size = 0.85

$$-\frac{2b\left(bx^2+cx^4\right)^{\frac{5}{2}}}{35c^2x^5}+\frac{\left(bx^2+cx^4\right)^{\frac{5}{2}}}{7cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -2*b*(b*x**2 + c*x**4)**(5/2)/(35*c**2*x**5) + (b*x**2 + c*x**4)*
*(5/2)/(7*c*x**3)
```

Mathematica [A] time = 0.0345354, size = 42, normalized size = 0.81

$$\frac{x\left(b+cx^2\right)^3\left(5cx^2-2b\right)}{35c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(b + c*x^2)^3*(-2*b + 5*c*x^2))/(35*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$-\frac{\left(cx^{2}+b\right)\left(-5\,cx^{2}+2\,b\right)}{35\,c^{2}x^{3}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-5*c*x^2+2*b)*(c*x^4+b*x^2)^(3/2)/c^2/x^3
```

Maxima [A] time = 0.705537, size = 61, normalized size = 1.17

$$\frac{\left(5\,c^{3}x^{6}+8\,bc^{2}x^{4}+b^{2}cx^{2}-2\,b^{3}\right)\sqrt{cx^{2}+b}}{35\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^2 + b
)/c^2
```

Fricas [A] time = 0.262872, size = 70, normalized size = 1.35

$$\frac{\left(5\,c^{3}x^{6}+8\,bc^{2}x^{4}+b^{2}cx^{2}-2\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{35\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*c^3*x^6 + 8*b*c^2*x^4 + b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b
*x^2)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^2 + cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.273338, size = 126, normalized size = 2.42

$$\frac{2\,b^{\frac{7}{2}}\operatorname{sign}(x)}{35\,c^2} + \frac{\frac{7\left(3\left(cx^2+b\right)^{\frac{5}{2}} - 5\left(cx^2+b\right)^{\frac{3}{2}}b\right)b\operatorname{sign}(x)}{c} + \frac{\left(15\left(cx^2+b\right)^{\frac{7}{2}} - 42\left(cx^2+b\right)^{\frac{5}{2}}b + 35\left(cx^2+b\right)^{\frac{3}{2}}b^2\right)\operatorname{sign}(x)}{c}}{105\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/35*b^(7/2)*sign(x)/c^2 + 1/105*(7*(3*(c*x^2 + b)^(5/2) - 5*(c*x
^2 + b)^(3/2)*b)*b*sign(x)/c + (15*(c*x^2 + b)^(7/2) - 42*(c*x^2
+ b)^(5/2)*b + 35*(c*x^2 + b)^(3/2)*b^2)*sign(x)/c)/c
```

3.253 $\int \frac{(bx^2+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=25

$$\frac{(bx^2+cx^4)^{5/2}}{5cx^5}$$

```
[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Rubi [A] time = 0.070811, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{(bx^2+cx^4)^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^2,x]

[Out] (b*x^2 + c*x^4)^(5/2)/(5*c*x^5)
```

Rubi in Sympy [A] time = 7.85927, size = 19, normalized size = 0.76

$$\frac{(bx^2+cx^4)^{\frac{5}{2}}}{5cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**2,x)

[Out] (b*x**2 + c*x**4)**(5/2)/(5*c*x**5)
```

Mathematica [A] time = 0.0173786, size = 25, normalized size = 1.

$$\frac{(x^2(b+cx^2))^{5/2}}{5cx^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^2,x]

[Out] (x^2*(b + c*x^2))^(5/2)/(5*c*x^5)
```

Maple [A] time = 0.003, size = 29, normalized size = 1.2

$$\frac{cx^2+b}{5\,cx^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^2,x)

[Out] 1/5*(c*x^2+b)/c/x^3*(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.691884, size = 43, normalized size = 1.72

$$\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^2+b}}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^2,x, algorithm="maxima")

[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^2 + b)/c
```

Fricas [A] time = 0.266549, size = 53, normalized size = 2.12

$$\frac{\left(c^2x^4+2\,bcx^2+b^2\right)\sqrt{cx^4+bx^2}}{5\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/5*(c^2*x^4 + 2*b*c*x^2 + b^2)*sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.272516, size = 78, normalized size = 3.12

$$-\frac{b^{\frac{5}{2}}\operatorname{sign}(x)}{5\,c}+\frac{5\left(cx^2+b\right)^{\frac{3}{2}}b\operatorname{sign}(x)+\left(3\left(cx^2+b\right)^{\frac{5}{2}}-5\left(cx^2+b\right)^{\frac{3}{2}}b\right)\operatorname{sign}(x)}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/5*b^(5/2)*sign(x)/c + 1/15*(5*(c*x^2 + b)^(3/2)*b*sign(x) + (3
*(c*x^2 + b)^(5/2) - 5*(c*x^2 + b)^(3/2)*b)*sign(x))/c
```

3.254 $\int \frac{(bx^2+cx^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=73

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rubi [A] time = 0.166862, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$b^{3/2}\left(-\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (b*Sqrt[b*x^2 + c*x^4])/x + (b*x^2 + c*x^4)^(3/2)/(3*x^3) - b^(3/
2)*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]
```

Rubi in Sympy [A] time = 19.8416, size = 61, normalized size = 0.84

$$-b^{\frac{3}{2}}\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)+\frac{b\sqrt{bx^2+cx^4}}{x}+\frac{(bx^2+cx^4)^{\frac{3}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] -b**(3/2)*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4)) + b*sqrt(b*x**2
+ c*x**4)/x + (b*x**2 + c*x**4)**(3/2)/(3*x**3)
```

Mathematica [A] time = 0.108358, size = 89, normalized size = 1.22

$$\frac{x\sqrt{b+cx^2}\left(-3b^{3/2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+3b^{3/2}\log(x)+\sqrt{b+cx^2}\left(4b+cx^2\right)\right)}{3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Sqrt[b + c*x^2]*(4*b + c*x^2) + 3*b^(3/2)*Log
[x] - 3*b^(3/2)*Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(3*Sqrt[x^2*(b
 + c*x^2)])
```

Maple [A] time = 0.009, size = 78, normalized size = 1.1

$$-\frac{1}{3x^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b^{3/2}-\left(cx^2+b\right)^{\frac{3}{2}}-3\sqrt{cx^2+b}b\right)\left(cx^2+b\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b
^(3/2)-(c*x^2+b)^(3/2)-3*(c*x^2+b)^(1/2)*b)/x^3/(c*x^2+b)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277891, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^{\frac{3}{2}}x\log\left(-\frac{cx^3+2\,bx-2\,\sqrt{cx^4+bx^2}\sqrt{b}}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{6\,x},\right.$$
$$\left.-\frac{3\,\sqrt{-b}bx\arctan\left(\frac{bx}{\sqrt{cx^4+bx^2}\sqrt{-b}}\right)-\sqrt{cx^4+bx^2}\left(cx^2+4\,b\right)}{3\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/6*(3*b^(3/2)*x*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)*sqr
t(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(c*x^2 + 4*b))/x, -1/3*(3*sqrt
(-b)*b*x*arctan(b*x/(sqrt(c*x^4 + b*x^2)*sqrt(-b))) - sqrt(c*x^4
+ b*x^2)*(c*x^2 + 4*b))/x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**4, x)
```

GIAC/XCAS [A] time = 0.273278, size = 119, normalized size = 1.63

$$\frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+\left(cx^2+b\right)^{\frac{3}{2}}+3\,\sqrt{cx^2+b}b\right)\mathrm{sign}\left(x\right)-\frac{\left(3\,b^2\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\,\sqrt{-b}b^{\frac{3}{2}}\right)\mathrm{sign}\left(x\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + (c*x^2 + b
)^(3/2) + 3*sqrt(c*x^2 + b)*b)*sign(x) - 1/3*(3*b^2*arctan(sqrt(b
)/sqrt(-b)) + 4*sqrt(-b)*b^(3/2))*sign(x)/sqrt(-b)
```

3.255 $\int \frac{(bx^2+cx^4)^{3/2}}{x^6} \, dx$

Optimal. Leaf size=79

$$\frac{3c\sqrt{bx^2+cx^4}}{2x} - \frac{3}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) - \frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) -
 (3*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi [A] time = 0.170232, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3c\sqrt{bx^2+cx^4}}{2x} - \frac{3}{2}\sqrt{b}c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right) - \frac{(bx^2+cx^4)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (3*c*Sqrt[b*x^2 + c*x^4])/(2*x) - (b*x^2 + c*x^4)^(3/2)/(2*x^5) -
 (3*Sqrt[b]*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/2
```

Rubi in Sympy [A] time = 20.2004, size = 70, normalized size = 0.89

$$-\frac{3\sqrt{b}c\,\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2} + \frac{3c\sqrt{bx^2+cx^4}}{2x} - \frac{(bx^2+cx^4)^{\frac{3}{2}}}{2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] -3*sqrt(b)*c*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/2 + 3*c*sqrt(
b*x**2 + c*x**4)/(2*x) - (b*x**2 + c*x**4)**(3/2)/(2*x**5)
```

Mathematica [A] time = 0.0901555, size = 99, normalized size = 1.25

$$\frac{\sqrt{x^2(b+cx^2)}\left(-\left(b-2cx^2\right)\sqrt{b+cx^2}+3\sqrt{b}cx^2\log(x)-3\sqrt{b}cx^2\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)\right)}{2x^3\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-((b - 2*c*x^2)*Sqrt[b + c*x^2]) + 3*Sqrt
[b]*c*x^2*Log[x] - 3*Sqrt[b]*c*x^2*Log[b + Sqrt[b]*Sqrt[b + c*x^2
]]))/(2*x^3*Sqrt[b + c*x^2])
```

Maple [A] time = 0.01, size = 102, normalized size = 1.3

$$-\frac{1}{2\,bx^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b^{3/2}x^2c-\left(cx^2+b\right)^{\frac{3}{2}}x^2c+\left(cx^2+b\right)^{\frac{5}{2}}-3\,\sqrt{cx^2+b}x^2bc\right)\left(cx^2+b\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/2*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b
^(3/2)*x^2*c-(c*x^2+b)^(3/2)*x^2*c+(c*x^2+b)^(5/2)-3*(c*x^2+b)^(1
/2)*x^2*b*c)/x^5/(c*x^2+b)^(3/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276789, size = 1, normalized size = 0.01

$$\left[\frac{3\,\sqrt{b}cx^{3}\log\left(-\frac{cx^{3}+2\,bx-2\,\sqrt{cx^{4}+bx^{2}}\sqrt{b}}{x^{3}}\right)+2\,\sqrt{cx^{4}+bx^{2}}\left(2\,cx^{2}-b\right)}{4\,x^{3}},\right.$$
$$\left.-\frac{3\,\sqrt{-b}cx^{3}\arctan\left(\frac{bx}{\sqrt{cx^{4}+bx^{2}}\sqrt{-b}}\right)-\sqrt{cx^{4}+bx^{2}}\left(2\,cx^{2}-b\right)}{2\,x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/4*(3*sqrt(b)*c*x^3*log(-(c*x^3 + 2*b*x - 2*sqrt(c*x^4 + b*x^2)
*sqrt(b))/x^3) + 2*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3, -1/2*(
3*sqrt(-b)*c*x^3*arctan(b*x/(sqrt(c*x^4 + b*x^2)*sqrt(-b))) - sqr
t(c*x^4 + b*x^2)*(2*c*x^2 - b))/x^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**6, x)
```

GIAC/XCAS [A] time = 0.29601, size = 80, normalized size = 1.01

$$\frac{1}{2}\left(\frac{3\,b\arctan\left(\frac{\sqrt{cx^{2}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\,\sqrt{cx^{2}+b}-\frac{\sqrt{cx^{2}+b}b}{cx^{2}}\right)c\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2*(3*b*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(c*x^2
 + b) - sqrt(c*x^2 + b)*b/(c*x^2))*c*sign(x)
```

3.256 $\int \frac{(bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=81

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}$$

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7
) - (3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rubi [A] time = 0.169131, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}-\frac{(bx^2+cx^4)^{3/2}}{4x^7}-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(8*x^3) - (b*x^2 + c*x^4)^(3/2)/(4*x^7
) - (3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*Sqrt[b])
```

Rubi in Sympy [A] time = 20.5003, size = 75, normalized size = 0.93

$$-\frac{3c\sqrt{bx^2+cx^4}}{8x^3}-\frac{(bx^2+cx^4)^{\frac{3}{2}}}{4x^7}-\frac{3c^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] -3*c*sqrt(b*x**2 + c*x**4)/(8*x**3) - (b*x**2 + c*x**4)**(3/2)/(4
*x**7) - 3*c**2*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(8*sqrt(b)
)
```

Mathematica [A] time = 0.0948606, size = 105, normalized size = 1.3

$$\frac{\sqrt{x^2(b+cx^2)}\left(-3c^2x^4\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)-\sqrt{b}\sqrt{b+cx^2}\left(2b+5cx^2\right)+3c^2x^4\log(x)\right)}{8\sqrt{b}x^5\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(2*b + 5*c*x^2)
) + 3*c^2*x^4*Log[x] - 3*c^2*x^4*Log[b + Sqrt[b]*Sqrt[b + c*x^2]]
))/(8*Sqrt[b]*x^5*Sqrt[b + c*x^2])
```

Maple [A] time = 0.01, size = 125, normalized size = 1.5

$$-\frac{1}{8b^2x^7}\left(cx^4+bx^2\right)^{\frac{3}{2}}\Bigg(3\ln\left(2\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b^{3/2}x^4c^2-\left(cx^2+b\right)^{\frac{3}{2}}x^4c^2+\left(cx^2+b\right)^{\frac{5}{2}}x^2c-3\sqrt{cx^2+b}x^4bc^2+2\ (cx^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^8,x)
```

```
[Out] -1/8*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b
^(3/2)*x^4*c^2-(c*x^2+b)^(3/2)*x^4*c^2+(c*x^2+b)^(5/2)*x^2*c-3*(c
*x^2+b)^(1/2)*x^4*b*c^2+2*(c*x^2+b)^(5/2)*b)/x^7/(c*x^2+b)^(3/2)/
b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279968, size = 1, normalized size = 0.01

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{(cx^3+2bx)\sqrt{b}-2\sqrt{cx^4+bx^2}b}{x^3}\right)-2\sqrt{cx^4+bx^2}(5bcx^2+2b^2)}{16bx^5}, \frac{3\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)-\sqrt{cx^4+bx^2}(5bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c
*x^4 + b*x^2)*b)/x^3) - 2*sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2)
)/(b*x^5), 1/8*(3*sqrt(-b)*c^2*x^5*arctan(sqrt(-b)*x/sqrt(c*x^4 +
 b*x^2)) - sqrt(c*x^4 + b*x^2)*(5*b*c*x^2 + 2*b^2))/(b*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**8, x)
```

GIAC/XCAS [A] time = 0.297139, size = 85, normalized size = 1.05

$$\frac{1}{8}c^2\left(\frac{3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\frac{5\left(cx^2+b\right)^{\frac{3}{2}}-3\sqrt{cx^2+b}b}{c^2x^4}\right)\operatorname{sign}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/8*c^2*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/sqrt(-b) - (5*(c*x^2
+ b)^(3/2) - 3*sqrt(c*x^2 + b)*b)/(c^2*x^4))*sign(x)
```

3.257 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{10}}\,dx$

Optimal. Leaf size=109

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}$$

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (c^2*Sqrt[b*x^2 + c*x^4])/(16*
b*x^3) - (b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rubi [A] time = 0.252975, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{c^3\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{3/2}}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}-\frac{(bx^2+cx^4)^{3/2}}{6x^9}-\frac{c\sqrt{bx^2+cx^4}}{8x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(8*x^5) - (c^2*Sqrt[b*x^2 + c*x^4])/(16*
b*x^3) - (b*x^2 + c*x^4)^(3/2)/(6*x^9) + (c^3*ArcTanh[(Sqrt[b]*x)
/Sqrt[b*x^2 + c*x^4]])/(16*b^(3/2))
```

Rubi in Sympy [A] time = 29.4524, size = 94, normalized size = 0.86

$$-\frac{c\sqrt{bx^2+cx^4}}{8x^5}-\frac{(bx^2+cx^4)^{\frac{3}{2}}}{6x^9}-\frac{c^2\sqrt{bx^2+cx^4}}{16bx^3}+\frac{c^3\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{16b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] -c*sqrt(b*x**2 + c*x**4)/(8*x**5) - (b*x**2 + c*x**4)**(3/2)/(6*x
**9) - c**2*sqrt(b*x**2 + c*x**4)/(16*b*x**3) + c**3*atanh(sqrt(b
)*x/sqrt(b*x**2 + c*x**4))/(16*b**(3/2))
```

Mathematica [A] time = 0.113159, size = 115, normalized size = 1.06

$$-\frac{\sqrt{x^2 (b + cx^2)} \left(\sqrt{b}\sqrt{b + cx^2} (8b^2 + 14bcx^2 + 3c^2x^4) - 3c^3x^6 \log \left(\sqrt{b}\sqrt{b + cx^2} + b\right) + 3c^3x^6 \log(x)\right)}{48b^{3/2}x^7\sqrt{b + cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^10,x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(Sqrt[b]*Sqrt[b + c*x^2]*(8*b^2 + 14*b*c*
x^2 + 3*c^2*x^4) + 3*c^3*x^6*Log[x] - 3*c^3*x^6*Log[b + Sqrt[b]*S
qrt[b + c*x^2]]))/(48*b^(3/2)*x^7*Sqrt[b + c*x^2])
```

Maple [A] time = 0.014, size = 145, normalized size = 1.3

$$\frac{1}{48\,x^9b^3} \left(cx^4 + bx^2\right)^{\frac{3}{2}} \left(3 \ln \left(2\,\frac{\sqrt{b}\sqrt{cx^2 + b} + b}{x}\right) b^{3/2}x^6c^3 - \left(cx^2 + b\right)^{\frac{3}{2}} x^6c^3 + \left(cx^2 + b\right)^{\frac{5}{2}} x^4c^2 - 3\,\sqrt{cx^2 + b}x^6bc^3 + 2\,\left(cx^2 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^10,x)
```

```
[Out] 1/48*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b
^(3/2)*x^6*c^3-(c*x^2+b)^(3/2)*x^6*c^3+(c*x^2+b)^(5/2)*x^4*c^2-3*
(c*x^2+b)^(1/2)*x^6*b*c^3+2*(c*x^2+b)^(5/2)*x^2*b*c-8*(c*x^2+b)^(
5/2)*b^2)/x^9/(c*x^2+b)^(3/2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285315, size = 1, normalized size = 0.01

$$\left[\frac{3\sqrt{b}c^3x^7\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}+2\,\sqrt{cx^4+bx^2}b}{x^3}\right)-2\left(3\,bc^2x^4+14\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{96\,b^2x^7},\right.$$
$$\left.-\frac{3\sqrt{-b}c^3x^7\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\left(3\,bc^2x^4+14\,b^2cx^2+8\,b^3\right)\sqrt{cx^4+bx^2}}{48\,b^2x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] [1/96*(3*sqrt(b)*c^3*x^7*log(-((c*x^3 + 2*b*x)*sqrt(b) + 2*sqrt(c
*x^4 + b*x^2)*b)/x^3) - 2*(3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b^3)*sq
rt(c*x^4 + b*x^2))/(b^2*x^7), -1/48*(3*sqrt(-b)*c^3*x^7*arctan(sq
rt(-b)*x/sqrt(c*x^4 + b*x^2)) + (3*b*c^2*x^4 + 14*b^2*c*x^2 + 8*b
^3)*sqrt(c*x^4 + b*x^2))/(b^2*x^7)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**10,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**10, x)
```

GIAC/XCAS [A] time = 0.303543, size = 111, normalized size = 1.02

$$-\frac{1}{48}\,c^3\left(\frac{3\,\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}+\frac{3\left(cx^2+b\right)^{\frac{5}{2}}+8\left(cx^2+b\right)^{\frac{3}{2}}b-3\sqrt{cx^2+b}b^2}{bc^3x^6}\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/48*c^3*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(
c*x^2 + b)^(5/2) + 8*(c*x^2 + b)^(3/2)*b - 3*sqrt(c*x^2 + b)*b^2)
/(b*c^3*x^6))*sign(x)
```

3.258 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=137

$$-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}$$

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(16*x^7) - (c^2*Sqrt[b*x^2 + c*x^4])/(64
*b*x^5) + (3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*
x^4)^(3/2)/(8*x^11) - (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(128*b^(5/2))
```

Rubi [A] time = 0.342378, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{3c^4\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{5/2}}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}-\frac{(bx^2+cx^4)^{3/2}}{8x^{11}}-\frac{c\sqrt{bx^2+cx^4}}{16x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] -(c*Sqrt[b*x^2 + c*x^4])/(16*x^7) - (c^2*Sqrt[b*x^2 + c*x^4])/(64
*b*x^5) + (3*c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^3) - (b*x^2 + c*
x^4)^(3/2)/(8*x^11) - (3*c^4*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x
^4]])/(128*b^(5/2))
```

Rubi in Sympy [A] time = 38.2706, size = 122, normalized size = 0.89

$$-\frac{c\sqrt{bx^2+cx^4}}{16x^7}-\frac{(bx^2+cx^4)^{\frac{3}{2}}}{8x^{11}}-\frac{c^2\sqrt{bx^2+cx^4}}{64bx^5}+\frac{3c^3\sqrt{bx^2+cx^4}}{128b^2x^3}-\frac{3c^4\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{128b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] -c*sqrt(b*x**2 + c*x**4)/(16*x**7) - (b*x**2 + c*x**4)**(3/2)/(8*
x**11) - c**2*sqrt(b*x**2 + c*x**4)/(64*b*x**5) + 3*c**3*sqrt(b*x
**2 + c*x**4)/(128*b**2*x**3) - 3*c**4*atanh(sqrt(b)*x/sqrt(b*x**
```

```
2 + c*x**4))/(128*b**(5/2))
```

Mathematica [A] time = 0.143223, size = 127, normalized size = 0.93

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(-\sqrt{b}\sqrt{b+cx^2}\left(16b^3+24b^2cx^2+2bc^2x^4-3c^3x^6\right)-3c^4x^8\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+3c^4x^8\log(x)\right)}{128b^{5/2}x^9\sqrt{b+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^12,x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-(Sqrt[b]*Sqrt[b + c*x^2]*(16*b^3 + 24*b^
2*c*x^2 + 2*b*c^2*x^4 - 3*c^3*x^6)) + 3*c^4*x^8*Log[x] - 3*c^4*x^
8*Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(128*b^(5/2)*x^9*Sqrt[b + c*
x^2])
```

Maple [A] time = 0.024, size = 165, normalized size = 1.2

$$-\frac{1}{128\,b^4x^{11}}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b^{3/2}x^8c^4-\left(cx^2+b\right)^{\frac{3}{2}}x^8c^4+\left(cx^2+b\right)^{\frac{5}{2}}x^6c^3-3\,\sqrt{cx^2+b}x^8bc^4+2\,(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^12,x)
```

```
[Out] -1/128*(c*x^4+b*x^2)^(3/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)
*b^(3/2)*x^8*c^4-(c*x^2+b)^(3/2)*x^8*c^4+(c*x^2+b)^(5/2)*x^6*c^3-
3*(c*x^2+b)^(1/2)*x^8*b*c^4+2*(c*x^2+b)^(5/2)*x^4*b*c^2-8*(c*x^2+
b)^(5/2)*x^2*b^2*c+16*(c*x^2+b)^(5/2)*b^3)/x^11/(c*x^2+b)^(3/2)/b
^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298518, size = 1, normalized size = 0.01

$$\left[\frac{3\sqrt{b}c^4x^9\log\left(-\frac{(cx^3+2bx)\sqrt{b}-2\sqrt{cx^4+bx^2}b}{x^3}\right)+2\left(3bc^3x^6-2b^2c^2x^4-24b^3cx^2-16b^4\right)\sqrt{cx^4+bx^2}}{256b^3x^9}, \frac{3\sqrt{-b}c^4x^9\arctan\left(\frac{\sqrt{-}}{\sqrt{cx^4}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] [1/256*(3*sqrt(b)*c^4*x^9*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(
c*x^4 + b*x^2)*b)/x^3) + 2*(3*b*c^3*x^6 - 2*b^2*c^2*x^4 - 24*b^3*
c*x^2 - 16*b^4)*sqrt(c*x^4 + b*x^2))/(b^3*x^9), 1/128*(3*sqrt(-b)
*c^4*x^9*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2)) + (3*b*c^3*x^6 -
2*b^2*c^2*x^4 - 24*b^3*c*x^2 - 16*b^4)*sqrt(c*x^4 + b*x^2))/(b^3*
x^9)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**12,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**12, x)
```

GIAC/XCAS [A] time = 0.325999, size = 130, normalized size = 0.95

$$\frac{1}{128}c^4\left(\frac{3\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{3\left(cx^2+b\right)^{\frac{7}{2}}-11\left(cx^2+b\right)^{\frac{5}{2}}b-11\left(cx^2+b\right)^{\frac{3}{2}}b^2+3\sqrt{cx^2+b}b^3}{b^2c^4x^8}\right)\mathrm{sign}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] 1/128*c^4*(3*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^2) + (3
*(c*x^2 + b)^(7/2) - 11*(c*x^2 + b)^(5/2)*b - 11*(c*x^2 + b)^(3/2
)*b^2 + 3*sqrt(c*x^2 + b)*b^3)/(b^2*c^4*x^8))*sign(x)
```

3.259 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=165

$$\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}$$

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(
160*b*x^7) + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqr
t[b*x^2 + c*x^4])/(256*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13)
 + (3*c^5*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rubi [A] time = 0.43497, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{3c^5\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{7/2}}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}-\frac{(bx^2+cx^4)^{3/2}}{10x^{13}}-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] (-3*c*Sqrt[b*x^2 + c*x^4])/(80*x^9) - (c^2*Sqrt[b*x^2 + c*x^4])/(
160*b*x^7) + (c^3*Sqrt[b*x^2 + c*x^4])/(128*b^2*x^5) - (3*c^4*Sqr
t[b*x^2 + c*x^4])/(256*b^3*x^3) - (b*x^2 + c*x^4)^(3/2)/(10*x^13)
 + (3*c^5*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(256*b^(7/2))
```

Rubi in Sympy [A] time = 47.7787, size = 150, normalized size = 0.91

$$-\frac{3c\sqrt{bx^2+cx^4}}{80x^9}-\frac{(bx^2+cx^4)^{\frac{3}{2}}}{10x^{13}}-\frac{c^2\sqrt{bx^2+cx^4}}{160bx^7}+\frac{c^3\sqrt{bx^2+cx^4}}{128b^2x^5}-\frac{3c^4\sqrt{bx^2+cx^4}}{256b^3x^3}+\frac{3c^5\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{256b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] -3*c*sqrt(b*x**2 + c*x**4)/(80*x**9) - (b*x**2 + c*x**4)**(3/2)/(
10*x**13) - c**2*sqrt(b*x**2 + c*x**4)/(160*b*x**7) + c**3*sqrt(b
*x**2 + c*x**4)/(128*b**2*x**5) - 3*c**4*sqrt(b*x**2 + c*x**4)/(2
```

```
56*b**3*x**3) + 3*c**5*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(25
6*b**(7/2))
```

Mathematica [A] time = 0.270946, size = 137, normalized size = 0.83

$$-\frac{\sqrt{b+cx^2}\left(\sqrt{b}\sqrt{b+cx^2}\left(128b^4+176b^3cx^2+8b^2c^2x^4-10bc^3x^6+15c^4x^8\right)-15c^5x^{10}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+15c^5x^{10}\log(x)\right)}{1280b^{7/2}x^9\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^14,x]
```

```
[Out] -(Sqrt[b + c*x^2]*(Sqrt[b]*Sqrt[b + c*x^2]*(128*b^4 + 176*b^3*c*x
^2 + 8*b^2*c^2*x^4 - 10*b*c^3*x^6 + 15*c^4*x^8) + 15*c^5*x^10*Log
[x] - 15*c^5*x^10*Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(1280*b^(7/2
)*x^9*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.043, size = 186, normalized size = 1.1

$$\frac{1}{1280\,x^{13}b^5}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(15\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b^{3/2}x^{10}c^5-5\,\left(cx^2+b\right)^{3/2}x^{10}c^5+5\,\left(cx^2+b\right)^{5/2}x^8c^4-15\,\sqrt{cx^2+b}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^14,x)
```

```
[Out] 1/1280*(c*x^4+b*x^2)^(3/2)*(15*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x
)*b^(3/2)*x^10*c^5-5*(c*x^2+b)^(3/2)*x^10*c^5+5*(c*x^2+b)^(5/2)*x
^8*c^4-15*(c*x^2+b)^(1/2)*x^10*b*c^5+10*(c*x^2+b)^(5/2)*x^6*b*c^3
-40*(c*x^2+b)^(5/2)*x^4*b^2*c^2+80*(c*x^2+b)^(5/2)*x^2*b^3*c-128*
(c*x^2+b)^(5/2)*b^4)/x^13/(c*x^2+b)^(3/2)/b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.33097, size = 1, normalized size = 0.01

$$\left[\frac{15\sqrt{b}c^5x^{11}\log\left(-\frac{(cx^3+2bx)\sqrt{b}+2\sqrt{cx^4+bx^2}b}{x^3}\right)-2\left(15bc^4x^8-10b^2c^3x^6+8b^3c^2x^4+176b^4cx^2+128b^5\right)\sqrt{cx^4+bx^2}}{2560b^4x^{11}},\right.$$
$$\left.-\frac{15\sqrt{-b}c^5x^{11}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\left(15bc^4x^8-10b^2c^3x^6+8b^3c^2x^4+176b^4cx^2+128b^5\right)\sqrt{cx^4+bx^2}}{1280b^4x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*sqrt(b)*c^5*x^11*log(-((c*x^3 + 2*b*x)*sqrt(b) + 2*sq
rt(c*x^4 + b*x^2)*b)/x^3) - 2*(15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*
b^3*c^2*x^4 + 176*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*
x^11), -1/1280*(15*sqrt(-b)*c^5*x^11*arctan(sqrt(-b)*x/sqrt(c*x^4
 + b*x^2)) + (15*b*c^4*x^8 - 10*b^2*c^3*x^6 + 8*b^3*c^2*x^4 + 176
*b^4*c*x^2 + 128*b^5)*sqrt(c*x^4 + b*x^2))/(b^4*x^11)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**14,x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**14, x)
```

GIAC/XCAS [A] time = 0.34379, size = 149, normalized size = 0.9

$$-\frac{1}{1280}c^5\left(\frac{15\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{15\left(cx^2+b\right)^{\frac{9}{2}}-70\left(cx^2+b\right)^{\frac{7}{2}}b+128\left(cx^2+b\right)^{\frac{5}{2}}b^2+70\left(cx^2+b\right)^{\frac{3}{2}}b^3-15\sqrt{cx^2+b}b^4}{b^3c^5x^{10}}\right)\mathrm{sig}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/1280*c^5*(15*arctan(sqrt(c*x^2 + b)/sqrt(-b))/(sqrt(-b)*b^3) +
 (15*(c*x^2 + b)^(9/2) - 70*(c*x^2 + b)^(7/2)*b + 128*(c*x^2 + b)
^(5/2)*b^2 + 70*(c*x^2 + b)^(3/2)*b^3 - 15*sqrt(c*x^2 + b)*b^4)/(
b^3*c^5*x^10))*sign(x)
```

3.260 $\int \frac{x^7}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^
4])/(24*c^2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(
Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rubi [A] time = 0.270571, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{7/2}}+\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (5*b^2*Sqrt[b*x^2 + c*x^4])/(16*c^3) - (5*b*x^2*Sqrt[b*x^2 + c*x^
4])/(24*c^2) + (x^4*Sqrt[b*x^2 + c*x^4])/(6*c) - (5*b^3*ArcTanh[(
Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(16*c^(7/2))
```

Rubi in Sympy [A] time = 23.1461, size = 104, normalized size = 0.91

$$-\frac{5b^3\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{16c^{\frac{7}{2}}}+\frac{5b^2\sqrt{bx^2+cx^4}}{16c^3}-\frac{5bx^2\sqrt{bx^2+cx^4}}{24c^2}+\frac{x^4\sqrt{bx^2+cx^4}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -5*b**3*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(16*c**(7/2)) +
 5*b**2*sqrt(b*x**2 + c*x**4)/(16*c**3) - 5*b*x**2*sqrt(b*x**2 +
c*x**4)/(24*c**2) + x**4*sqrt(b*x**2 + c*x**4)/(6*c)
```

Mathematica [A] time = 0.0764267, size = 103, normalized size = 0.9

$$\frac{x\left(\sqrt{c}x\left(15b^3+5b^2cx^2-2bc^2x^4+8c^3x^6\right)-15b^3\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{48c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(15*b^3 + 5*b^2*c*x^2 - 2*b*c^2*x^4 + 8*c^3*x^6) -
15*b^3*Sqrt[b + c*x^2]*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(48*c
^(7/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.013, size = 105, normalized size = 0.9

$$-\frac{x}{48}\sqrt{cx^2+b}\left(-8\,x^5\sqrt{cx^2+b}c^{7/2}+10\,\sqrt{cx^2+b}c^{5/2}x^3b-15\,\sqrt{cx^2+b}c^{3/2}xb^2+15\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^3c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/48*x*(c*x^2+b)^(1/2)*(-8*x^5*(c*x^2+b)^(1/2)*c^(7/2)+10*(c*x^2
+b)^(1/2)*c^(5/2)*x^3*b-15*(c*x^2+b)^(1/2)*c^(3/2)*x*b^2+15*ln(x*
c^(1/2)+(c*x^2+b)^(1/2))*b^3*c)/(c*x^4+b*x^2)^(1/2)/c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283934, size = 1, normalized size = 0.01

$$\left[\frac{15\,b^3\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\,\left(8\,c^3x^4-10\,bc^2x^2+15\,b^2c\right)\sqrt{cx^4+bx^2}}{96\,c^4},\ \frac{15\,b^3\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*b^3*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4 +
 b*x^2)*c) + 2*(8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 +
 b*x^2))/c^4, 1/48*(15*b^3*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^
4 + b*x^2)) + (8*c^3*x^4 - 10*b*c^2*x^2 + 15*b^2*c)*sqrt(c*x^4 +
b*x^2))/c^4]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^7/sqrt(c*x^4 + b*x^2), x)
```

3.261 $\int \frac{x^5}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=86

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}} - \frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4
*c) + (3*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/
2))
```

Rubi [A] time = 0.20116, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{5/2}} - \frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[b*x^2 + c*x^4])/(4
*c) + (3*b^2*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(5/
2))
```

Rubi in Sympy [A] time = 18.2654, size = 76, normalized size = 0.88

$$\frac{3b^2 \operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{\frac{5}{2}}} - \frac{3b\sqrt{bx^2+cx^4}}{8c^2} + \frac{x^2\sqrt{bx^2+cx^4}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 3*b**2*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(8*c**(5/2)) - 3
*b*sqrt(b*x**2 + c*x**4)/(8*c**2) + x**2*sqrt(b*x**2 + c*x**4)/(4
*c)
```

Mathematica [A] time = 0.0655255, size = 92, normalized size = 1.07

$$\frac{x\left(\sqrt{c}x\left(-3b^2-bcx^2+2c^2x^4\right)+3b^2\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{8c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(-3*b^2 - b*c*x^2 + 2*c^2*x^4) + 3*b^2*Sqrt[b + c*x
^2]*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(8*c^(5/2)*Sqrt[x^2*(b +
 c*x^2)])
```

Maple [A] time = 0.014, size = 85, normalized size = 1.

$$\frac{x}{8}\sqrt{cx^2+b}\left(2\,x^3\sqrt{cx^2+b}c^{5/2}-3\,\sqrt{cx^2+b}c^{3/2}xb+3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)b^2c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/8*x*(c*x^2+b)^(1/2)*(2*x^3*(c*x^2+b)^(1/2)*c^(5/2)-3*(c*x^2+b)^
(1/2)*c^(3/2)*x*b+3*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*b^2*c)/(c*x^4+b
*x^2)^(1/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278198, size = 1, normalized size = 0.01

$$\left[\frac{3\,b^2\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)+2\,\sqrt{cx^4+bx^2}\left(2\,c^2x^2-3\,bc\right)}{16\,c^3},\right.$$
$$\left.-\frac{3\,b^2\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)-\sqrt{cx^4+bx^2}\left(2\,c^2x^2-3\,bc\right)}{8\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^2*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 +
b*x^2)*c) + 2*sqrt(c*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3, -1/8*
(3*b^2*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2)) - sqrt(c
*x^4 + b*x^2)*(2*c^2*x^2 - 3*b*c))/c^3]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(c*x^4 + b*x^2), x)
```

3.262 $\int \frac{x^3}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(3/2))
```

Rubi [A] time = 0.145444, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(3/2))
```

Rubi in Sympy [A] time = 13.5846, size = 48, normalized size = 0.83

$$-\frac{b\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{\frac{3}{2}}}+\frac{\sqrt{bx^2+cx^4}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -b*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(2*c**(3/2)) + sqrt(
b*x**2 + c*x**4)/(2*c)
```

Mathematica [A] time = 0.0509327, size = 76, normalized size = 1.31

$$\frac{x\left(\sqrt{c}x\left(b+cx^2\right)-b\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*Log[c*x + Sqrt[c]*S
qrt[b + c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 64, normalized size = 1.1

$$-\frac{x}{2}\sqrt{cx^2+b}\left(-x\sqrt{cx^2+b}c^{\frac{3}{2}}+b\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)^(1/2)*(-x*(c*x^2+b)^(1/2)*c^(3/2)+b*ln(x*c^(1/2)
+(c*x^2+b)^(1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276032, size = 1, normalized size = 0.02

$$\left[\frac{b\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c^2},\ \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4 + b*x^2
)*c) + 2*sqrt(c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(
-c)*x^2/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.296756, size = 80, normalized size = 1.38

$$\frac{b\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/4*b*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))
/c^(3/2) + 1/2*sqrt(c*x^4 + b*x^2)/c
```

3.263 $\int \frac{x}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi [A] time = 0.0837795, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[b*x^2 + c*x^4],x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi in Sympy [A] time = 8.293, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2)**(1/2),x)

[Out] atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/sqrt(c)
```

Mathematica [A] time = 0.0252847, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]
*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 44, normalized size = 1.4

$$x\sqrt{cx^2+b}\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1
/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268929, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 + b*x^2)*c)/sqrt(c
), -sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.290604, size = 53, normalized size = 1.71

$$-\frac{\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/
sqrt(c)
```

3.264 $\int \frac{1}{x\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rubi [A] time = 0.0689733, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[b*x^2 + c*x^4]),x]

[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rubi in Sympy [A] time = 8.16235, size = 19, normalized size = 0.83

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2)**(1/2),x)

[Out] -sqrt(b*x**2 + c*x**4)/(b*x**2)
```

Mathematica [A] time = 0.0229697, size = 23, normalized size = 1.

$$-\frac{\sqrt{x^2(b+cx^2)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[b*x^2 + c*x^4]),x]

[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Maple [A] time = 0.005, size = 26, normalized size = 1.1

$$-\frac{cx^2+b}{b}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2)^(1/2),x)

[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25907, size = 28, normalized size = 1.22

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x),x, algorithm="fricas")

[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [A] time = 0.276991, size = 19, normalized size = 0.83

$$-\frac{\sqrt{c+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x),x, algorithm="giac")
```

```
[Out] -sqrt(c + b/x^2)/b
```

3.265 $\int \frac{1}{x^3\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2
*x^2)
```

Rubi [A] time = 0.141434, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2
*x^2)
```

Rubi in Sympy [A] time = 14.284, size = 44, normalized size = 0.85

$$-\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(3*b*x**4) + 2*c*sqrt(b*x**2 + c*x**4)/(3*
b**2*x**2)
```

Mathematica [A] time = 0.0375158, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Maple [A] time = 0.006, size = 37, normalized size = 0.7

$$-\frac{(cx^2+b)(-2\,cx^2+b)}{3\,b^2x^2}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264754, size = 42, normalized size = 0.81

$$\frac{\sqrt{cx^4+bx^2}(2\,cx^2-b)}{3\,b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [A] time = 0.278295, size = 36, normalized size = 0.69

$$-\frac{\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}-3\sqrt{c+\frac{b}{x^2}}c}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^3),x, algorithm="giac")
```

```
[Out] -1/3*((c + b/x^2)^(3/2) - 3*sqrt(c + b/x^2)*c)/b^2
```

3.266 $\int \frac{1}{x^5\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(5*b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^
2*x^4) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rubi [A] time = 0.216793, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{\sqrt{bx^2+cx^4}}{5bx^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(5*b*x^6) + (4*c*Sqrt[b*x^2 + c*x^4])/(15*b^
2*x^4) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^3*x^2)
```

Rubi in Sympy [A] time = 21.3119, size = 71, normalized size = 0.89

$$-\frac{\sqrt{bx^2+cx^4}}{5bx^6}+\frac{4c\sqrt{bx^2+cx^4}}{15b^2x^4}-\frac{8c^2\sqrt{bx^2+cx^4}}{15b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(5*b*x**6) + 4*c*sqrt(b*x**2 + c*x**4)/(15
*b**2*x**4) - 8*c**2*sqrt(b*x**2 + c*x**4)/(15*b**3*x**2)
```

Mathematica [A] time = 0.0394257, size = 46, normalized size = 0.57

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}\left(3b^2-4bcx^2+8c^2x^4\right)}{15b^3x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[x^2*(b + c*x^2)]*(3*b^2 - 4*b*c*x^2 + 8*c^2*x^4))/(15*b^3*
x^6)
```

Maple [A] time = 0.007, size = 50, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(8\,c^2x^4-4\,bcx^2+3\,b^2\right)}{15\,b^3x^4}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/15*(c*x^2+b)*(8*c^2*x^4-4*b*c*x^2+3*b^2)/x^4/b^3/(c*x^4+b*x^2)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265429, size = 57, normalized size = 0.71

$$-\frac{\left(8\,c^2x^4-4\,bcx^2+3\,b^2\right)\sqrt{cx^4+bx^2}}{15\,b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^5),x, algorithm="fricas")
```

```
[Out] -1/15*(8*c^2*x^4 - 4*b*c*x^2 + 3*b^2)*sqrt(c*x^4 + b*x^2)/(b^3*x^
6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [A] time = 0.27804, size = 58, normalized size = 0.72

$$-\frac{3\left(c+\frac{b}{x^2}\right)^{\frac{5}{2}}-10\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}c+15\sqrt{c+\frac{b}{x^2}}c^2}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^5),x, algorithm="giac")
```

```
[Out] -1/15*(3*(c + b/x^2)^(5/2) - 10*(c + b/x^2)^(3/2)*c + 15*sqrt(c +
 b/x^2)*c^2)/b^3
```

3.267 $\int \frac{1}{x^7\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(7*b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^
2*x^6) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[
b*x^2 + c*x^4])/(35*b^4*x^2)
```

Rubi [A] time = 0.294481, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{\sqrt{bx^2+cx^4}}{7bx^8}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(7*b*x^8) + (6*c*Sqrt[b*x^2 + c*x^4])/(35*b^
2*x^6) - (8*c^2*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^4) + (16*c^3*Sqrt[
b*x^2 + c*x^4])/(35*b^4*x^2)
```

Rubi in Sympy [A] time = 29.2785, size = 99, normalized size = 0.92

$$-\frac{\sqrt{bx^2+cx^4}}{7bx^8}+\frac{6c\sqrt{bx^2+cx^4}}{35b^2x^6}-\frac{8c^2\sqrt{bx^2+cx^4}}{35b^3x^4}+\frac{16c^3\sqrt{bx^2+cx^4}}{35b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(7*b*x**8) + 6*c*sqrt(b*x**2 + c*x**4)/(35
*b**2*x**6) - 8*c**2*sqrt(b*x**2 + c*x**4)/(35*b**3*x**4) + 16*c*
*3*sqrt(b*x**2 + c*x**4)/(35*b**4*x**2)
```

Mathematica [A] time = 0.0414759, size = 57, normalized size = 0.53

$$\frac{\sqrt{x^2(b+cx^2)}\left(-5b^3+6b^2cx^2-8bc^2x^4+16c^3x^6\right)}{35b^4x^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-5*b^3 + 6*b^2*c*x^2 - 8*b*c^2*x^4 + 16*c
^3*x^6))/(35*b^4*x^8)
```

Maple [A] time = 0.008, size = 61, normalized size = 0.6

$$-\frac{(cx^2+b)\left(-16\,c^3x^6+8\,bc^2x^4-6\,b^2cx^2+5\,b^3\right)}{35\,b^4x^6}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-16*c^3*x^6+8*b*c^2*x^4-6*b^2*c*x^2+5*b^3)/x^6/b
^4/(c*x^4+b*x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272968, size = 72, normalized size = 0.67

$$\frac{\left(16\,c^3x^6-8\,bc^2x^4+6\,b^2cx^2-5\,b^3\right)\sqrt{cx^4+bx^2}}{35\,b^4x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^7),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^6 - 8*b*c^2*x^4 + 6*b^2*c*x^2 - 5*b^3)*sqrt(c*x^4
+ b*x^2)/(b^4*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [A] time = 0.278944, size = 77, normalized size = 0.71

$$-\frac{5\left(c+\frac{b}{x^2}\right)^{\frac{7}{2}}-21\left(c+\frac{b}{x^2}\right)^{\frac{5}{2}}c+35\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}c^2-35\sqrt{c+\frac{b}{x^2}}c^3}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^7),x, algorithm="giac")
```

```
[Out] -1/35*(5*(c + b/x^2)^(7/2) - 21*(c + b/x^2)^(5/2)*c + 35*(c + b/x
^2)^(3/2)*c^2 - 35*sqrt(c + b/x^2)*c^3)/b^4
```

3.268 $\int \frac{x^4}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3
*c)
```

Rubi [A] time = 0.0928261, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3
*c)
```

Rubi in Sympy [A] time = 15.1887, size = 41, normalized size = 0.82

$$-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*b*sqrt(b*x**2 + c*x**4)/(3*c**2*x) + x*sqrt(b*x**2 + c*x**4)/(
3*c)
```

Mathematica [A] time = 0.0273739, size = 34, normalized size = 0.68

$$\frac{\left(cx^2-2b\right)\sqrt{x^2\left(b+cx^2\right)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Maple [A] time = 0.007, size = 37, normalized size = 0.7

$$-\frac{\left(cx^{2}+b\right)\left(-cx^{2}+2\,b\right)x}{3\,c^{2}}\frac{1}{\sqrt{cx^{4}+bx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.704087, size = 46, normalized size = 0.92

$$\frac{c^{2}x^{4}-bcx^{2}-2\,b^{2}}{3\,\sqrt{cx^{2}+b}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A] time = 0.265739, size = 41, normalized size = 0.82

$$\frac{\sqrt{cx^{4}+bx^{2}}\left(cx^{2}-2\,b\right)}{3\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

3.269 $\int \frac{x^2}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rubi [A] time = 0.0143295, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[b*x^2 + c*x^4],x]

[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rubi in Sympy [A] time = 8.37126, size = 15, normalized size = 0.68

$$\frac{\sqrt{bx^2+cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2)**(1/2),x)

[Out] sqrt(b*x**2 + c*x**4)/(c*x)
```

Mathematica [A] time = 0.0104087, size = 22, normalized size = 1.

$$\frac{\sqrt{x^2\left(b+cx^2\right)}}{cx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[b*x^2 + c*x^4],x]

[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Maple [A] time = 0.004, size = 26, normalized size = 1.2

$$\frac{x\left(cx^2+b\right)}{c}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2)^(1/2),x)

[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.741918, size = 18, normalized size = 0.82

$$\frac{\sqrt{cx^2+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] sqrt(c*x^2 + b)/c
```

Fricas [A] time = 0.25933, size = 27, normalized size = 1.23

$$\frac{\sqrt{cx^4+bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.27591, size = 42, normalized size = 1.91

$$-\frac{2\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

3.270 $\int \frac{1}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rubi [A] time = 0.0236768, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[b*x^2 + c*x^4],x]

[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rubi in Sympy [A] time = 5.71561, size = 27, normalized size = 0.9

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**(1/2),x)

[Out] -atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/sqrt(b)
```

Mathematica [A] time = 0.0438278, size = 58, normalized size = 1.93

$$\frac{x\sqrt{b+cx^2}\left(\log(x)-\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Log[x] - Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(
Sqrt[b]*Sqrt[x^2*(b + c*x^2)])
```

Maple [B] time = 0.011, size = 50, normalized size = 1.7

$$-x\sqrt{cx^2+b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c
*x^2+b)^(1/2)+b)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266487, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{\left(cx^3+2\,bx\right)\sqrt{b}-2\,\sqrt{cx^4+bx^2}\,b}{x^3}\right)}{2\,\sqrt{b}}, \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c*x^4 + b*x^2)*b)/x^3
)/sqrt(b), sqrt(-b)*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.272844, size = 62, normalized size = 2.07

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sign}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sign(x)/sqrt(-b) + arctan(sqrt(c*x^2 +
b)/sqrt(-b))/(sqrt(-b)*sign(x))
```

3.271 $\int \frac{1}{x^2\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^
2 + c*x^4]])/(2*b^(3/2))
```

Rubi [A] time = 0.0965936, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^
2 + c*x^4]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 13.587, size = 49, normalized size = 0.83

$$-\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(2*b*x**3) + c*atanh(sqrt(b)*x/sqrt(b*x**2
 + c*x**4))/(2*b**(3/2))
```

Mathematica [A] time = 0.0728432, size = 97, normalized size = 1.64

$$\frac{-\sqrt{b}\left(b+cx^2\right)-cx^2\log(x)\sqrt{b+cx^2}+cx^2\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)}{2b^{3/2}x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)) - c*x^2*Sqrt[b + c*x^2]*Log[x] + c*x^2*Sq
rt[b + c*x^2]*Log[b + Sqrt[b]*Sqrt[b + c*x^2]])/(2*b^(3/2)*x*Sqrt
[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 73, normalized size = 1.2

$$\frac{1}{2x}\sqrt{cx^2+b}\left(c\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2b-\sqrt{cx^2+b}\,b^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(c*x^2+b)^(1/2)*(c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*
b-(c*x^2+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276566, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{b}cx^3\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}+2\,\sqrt{cx^4+bx^2}b}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,b^2x^3},\,-\frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-((c*x^3 + 2*b*x)*sqrt(b) + 2*sqrt(c*x^4
+ b*x^2)*b)/x^3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt
(-b)*c*x^3*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 +
b*x^2)*b)/(b^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.272 $\int \frac{1}{x^4\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2
*x^3) - (3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/
2))
```

Rubi [A] time = 0.175166, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2
*x^3) - (3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/
2))
```

Rubi in Sympy [A] time = 20.585, size = 78, normalized size = 0.9

$$-\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(4*b*x**5) + 3*c*sqrt(b*x**2 + c*x**4)/(8*
b**2*x**3) - 3*c**2*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(8*b**
(5/2))
```

Mathematica [A] time = 0.084177, size = 114, normalized size = 1.31

$$\frac{\sqrt{b}\left(-2b^2+bcx^2+3c^2x^4\right)+3c^2x^4\log(x)\sqrt{b+cx^2}-3c^2x^4\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)}{8b^{5/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4) + 3*c^2*x^4*Sqrt[b + c*x^
2]*Log[x] - 3*c^2*x^4*Sqrt[b + c*x^2]*Log[b + Sqrt[b]*Sqrt[b + c*
x^2]])/(8*b^(5/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 94, normalized size = 1.1

$$-\frac{1}{8\,x^3}\sqrt{cx^2+b}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4bc^2-3\,\sqrt{cx^2+b}b^{3/2}x^2c+2\,\sqrt{cx^2+b}b^{5/2}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8/x^3*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x
^4*b*c^2-3*(c*x^2+b)^(1/2)*b^(3/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2
))/(c*x^4+b*x^2)^(1/2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277986, size = 1, normalized size = 0.01

$$\left[\frac{3\,\sqrt{b}c^2x^5\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}-2\,\sqrt{cx^4+bx^2}b}{x^3}\right)+2\,\sqrt{cx^4+bx^2}(3\,bcx^2-2\,b^2)}{16\,b^3x^5}, \frac{3\,\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}(3\,bc}{8\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c
*x^4 + b*x^2)*b)/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2)
)/(b^3*x^5), 1/8*(3*sqrt(-b)*c^2*x^5*arctan(sqrt(-b)*x/sqrt(c*x^4
 + b*x^2)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.273 $\int \frac{x^9}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=109

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^
3) + (5*x^2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7/2))
```

Rubi [A] time = 0.257205, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{7/2}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}-\frac{x^6}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^6/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/(8*c^
3) + (5*x^2*Sqrt[b*x^2 + c*x^4])/(4*c^2) + (15*b^2*ArcTanh[(Sqrt[
c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(8*c^(7/2))
```

Rubi in Sympy [A] time = 23.4583, size = 99, normalized size = 0.91

$$\frac{15b^2\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{8c^{\frac{7}{2}}}-\frac{15b\sqrt{bx^2+cx^4}}{8c^3}-\frac{x^6}{c\sqrt{bx^2+cx^4}}+\frac{5x^2\sqrt{bx^2+cx^4}}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 15*b**2*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(8*c**(7/2)) -
15*b*sqrt(b*x**2 + c*x**4)/(8*c**3) - x**6/(c*sqrt(b*x**2 + c*x**
4)) + 5*x**2*sqrt(b*x**2 + c*x**4)/(4*c**2)
```

Mathematica [A] time = 0.0758357, size = 92, normalized size = 0.84

$$\frac{x\left(\sqrt{c}x\left(-15b^2-5bcx^2+2c^2x^4\right)+15b^2\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{8c^{7/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(-15*b^2 - 5*b*c*x^2 + 2*c^2*x^4) + 15*b^2*Sqrt[b +
 c*x^2]*Log[c*x + Sqrt[c]*Sqrt[b + c*x^2]]))/(8*c^(7/2)*Sqrt[x^2*
(b + c*x^2)])
```

Maple [A] time = 0.015, size = 87, normalized size = 0.8

$$\frac{x^3\left(cx^2+b\right)}{8}\left(2\,x^5c^{7/2}-5\,c^{5/2}x^3b-15\,c^{3/2}xb^2+15\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}b^2c\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}c^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/8*x^3*(c*x^2+b)*(2*x^5*c^(7/2)-5*c^(5/2)*x^3*b-15*c^(3/2)*x*b^2
+15*ln(x*c^(1/2)+(c*x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b^2*c)/(c*x^4+b
*x^2)^(3/2)/c^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281126, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^{2}cx^{2}+b^{3}\right)\sqrt{c}\log\left(-\left(2\,cx^{2}+b\right)\sqrt{c}-2\,\sqrt{cx^{4}+bx^{2}}c\right)+2\left(2\,c^{3}x^{4}-5\,bc^{2}x^{2}-15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{16\left(c^{5}x^{2}+bc^{4}\right)},\right.$$
$$\left.-\frac{15\left(b^{2}cx^{2}+b^{3}\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^{2}}{\sqrt{cx^{4}+bx^{2}}}\right)-\left(2\,c^{3}x^{4}-5\,bc^{2}x^{2}-15\,b^{2}c\right)\sqrt{cx^{4}+bx^{2}}}{8\left(c^{5}x^{2}+bc^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c*x^2 + b^3)*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) -
2*sqrt(c*x^4 + b*x^2)*c) + 2*(2*c^3*x^4 - 5*b*c^2*x^2 - 15*b^2*c)
*sqrt(c*x^4 + b*x^2))/(c^5*x^2 + b*c^4), -1/8*(15*(b^2*c*x^2 + b^
3)*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2)) - (2*c^3*x^4
 - 5*b*c^2*x^2 - 15*b^2*c)*sqrt(c*x^4 + b*x^2))/(c^5*x^2 + b*c^4)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{9}}{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**9/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{9}}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^9/(c*x^4 + b*x^2)^(3/2), x)
```

3.274 $\int \frac{x^7}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2)
- (3*b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rubi [A] time = 0.1983, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$-\frac{3b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{5/2}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}-\frac{x^4}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^4/(c*Sqrt[b*x^2 + c*x^4])) + (3*Sqrt[b*x^2 + c*x^4])/(2*c^2)
- (3*b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]])/(2*c^(5/2))
```

Rubi in Sympy [A] time = 18.1033, size = 71, normalized size = 0.88

$$-\frac{3b\,\mathrm{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{\frac{5}{2}}}-\frac{x^4}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -3*b*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(2*c**(5/2)) - x**
4/(c*sqrt(b*x**2 + c*x**4)) + 3*sqrt(b*x**2 + c*x**4)/(2*c**2)
```

Mathematica [A] time = 0.0570869, size = 78, normalized size = 0.96

$$\frac{x\left(\sqrt{c}x\left(3b+cx^2\right)-3b\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{2c^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[c]*x*(3*b + c*x^2) - 3*b*Sqrt[b + c*x^2]*Log[c*x + Sqrt[
c]*Sqrt[b + c*x^2]]))/(2*c^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.01, size = 74, normalized size = 0.9

$$-\frac{x^3\left(cx^2+b\right)}{2}\left(-x^3c^{\frac{5}{2}}-3\,c^{3/2}xb+3\,\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\sqrt{cx^2+b}bc\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}c^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x^3*(c*x^2+b)*(-x^3*c^(5/2)-3*c^(3/2)*x*b+3*ln(x*c^(1/2)+(c*
x^2+b)^(1/2))*(c*x^2+b)^(1/2)*b*c)/(c*x^4+b*x^2)^(3/2)/c^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291557, size = 1, normalized size = 0.01

$$\left[\frac{3\left(bcx^2+b^2\right)\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\,\sqrt{cx^4+bx^2}\left(c^2x^2+3\,bc\right)}{4\left(c^4x^2+bc^3\right)},\frac{3\left(bcx^2+b^2\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)}{2\left(c^4x^2+bc\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(b*c*x^2 + b^2)*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sq
rt(c*x^4 + b*x^2)*c) + 2*sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c))/(
c^4*x^2 + b*c^3), 1/2*(3*(b*c*x^2 + b^2)*sqrt(-c)*arctan(sqrt(-c)
*x^2/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 + b*x^2)*(c^2*x^2 + 3*b*c)
)/(c^4*x^2 + b*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**7/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(c*x^4 + b*x^2)^(3/2), x)
```

3.275 $\int \frac{x^5}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2
 + c*x^4]]/c^(3/2)
```

Rubi [A] time = 0.156033, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{3/2}}-\frac{x^2}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^2/(c*Sqrt[b*x^2 + c*x^4])) + ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2
 + c*x^4]]/c^(3/2)
```

Rubi in Sympy [A] time = 13.9388, size = 46, normalized size = 0.84

$$-\frac{x^2}{c\sqrt{bx^2+cx^4}}+\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -x**2/(c*sqrt(b*x**2 + c*x**4)) + atanh(sqrt(c)*x**2/sqrt(b*x**2
+ c*x**4))/c**(3/2)
```

Mathematica [A] time = 0.0464279, size = 65, normalized size = 1.18

$$\frac{x\left(\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)-\sqrt{c}x\right)}{c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(-(Sqrt[c]*x) + Sqrt[b + c*x^2]*Log[c*x + Sqrt[c]*Sqrt[b + c*x
^2]]))/(c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.011, size = 62, normalized size = 1.1

$$x^3\left(cx^2+b\right)\left(-xc^{\frac{3}{2}}+\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)c\sqrt{cx^2+b}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] x^3*(c*x^2+b)*(-x*c^(3/2)+ln(x*c^(1/2)+(c*x^2+b)^(1/2))*c*(c*x^2+
b)^(1/2))/(c*x^4+b*x^2)^(3/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275846, size = 1, normalized size = 0.02

$$\left[\frac{\left(cx^2+b\right)\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)-2\,\sqrt{cx^4+bx^2}c}{2\,(c^3x^2+bc^2)},\; -\frac{\left(cx^2+b\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}c}{c^3x^2+bc^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^2 + b)*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x
^4 + b*x^2)*c) - 2*sqrt(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2), -((c
*x^2 + b)*sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2)) + sqr
t(c*x^4 + b*x^2)*c)/(c^3*x^2 + b*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.284797, size = 55, normalized size = 1.

$$-\frac{\arctan\left(\frac{\sqrt{c+\frac{b}{x^2}}}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{1}{\sqrt{c+\frac{b}{x^2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(c + b/x^2)/sqrt(-c))/(sqrt(-c)*c) - 1/(sqrt(c + b/x^
2)*c)
```

3.276 $\int \frac{x^3}{(bx^2+cx^4)^{3/2}} dx$

Optimal. Leaf size=22

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

```
[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.0763867, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(b*x^2 + c*x^4)^(3/2),x]

[Out] x^2/(b*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 8.03696, size = 17, normalized size = 0.77

$$\frac{x^2}{b\sqrt{bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2)**(3/2),x)

[Out] x**2/(b*sqrt(b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0196495, size = 22, normalized size = 1.

$$\frac{x^2}{b\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^2/(b*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.004, size = 28, normalized size = 1.3

$$\frac{\left(c x^2+b\right) x^4}{b}\left(c x^4+b x^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] (c*x^2+b)/b*x^4/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.694344, size = 27, normalized size = 1.23

$$\frac{x^2}{\sqrt{c x^4+b x^2} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A] time = 0.25827, size = 35, normalized size = 1.59

$$\frac{\sqrt{c x^4+b x^2}}{b c x^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)/(b*c*x^2 + b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x**3/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.293738, size = 47, normalized size = 2.14

$$\frac{1}{\left(\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}+b\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")

[Out] 1/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) + b)*sqrt(c))
```

3.277 $\int \frac{x}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

```
[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rubi [A] time = 0.0780723, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118

$$-\frac{b+2cx^2}{b^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/(b^2*Sqrt[b*x^2 + c*x^4]))
```

Rubi in Sympy [A] time = 7.32352, size = 29, normalized size = 1.04

$$-\frac{2b+4cx^2}{2b^2\sqrt{bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -(2*b + 4*c*x**2)/(2*b**2*sqrt(b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0288852, size = 29, normalized size = 1.04

$$\frac{-b-2cx^2}{b^2\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(b*x^2 + c*x^4)^(3/2),x]

[Out] (-b - 2*c*x^2)/(b^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 37, normalized size = 1.3

$$-\frac{x^2\left(cx^2+b\right)\left(2\,cx^2+b\right)}{b^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2)^(3/2),x)

[Out] -x^2*(c*x^2+b)*(2*c*x^2+b)/b^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.693805, size = 55, normalized size = 1.96

$$-\frac{2\,cx^2}{\sqrt{cx^4+bx^2}b^2}-\frac{1}{\sqrt{cx^4+bx^2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")

[Out] -2*c*x^2/(sqrt(c*x^4 + b*x^2)*b^2) - 1/(sqrt(c*x^4 + b*x^2)*b)
```

Fricas [A] time = 0.261684, size = 55, normalized size = 1.96

$$-\frac{\sqrt{cx^4+bx^2}\left(2\,cx^2+b\right)}{b^2cx^4+b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")

[Out] -sqrt(c*x^4 + b*x^2)*(2*c*x^2 + b)/(b^2*c*x^4 + b^3*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.291577, size = 38, normalized size = 1.36

$$-\frac{\frac{2\,cx^2}{b^2}+\frac{1}{b}}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")

[Out] -(2*c*x^2/b^2 + 1/b)/sqrt(c*x^4 + b*x^2)
```

3.278 $\int \frac{1}{x(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=74

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^
4) + (8*c*Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rubi [A] time = 0.215646, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{1}{bx^2\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^2*Sqrt[b*x^2 + c*x^4]) - (4*Sqrt[b*x^2 + c*x^4])/(3*b^2*x^
4) + (8*c*Sqrt[b*x^2 + c*x^4])/(3*b^3*x^2)
```

Rubi in Sympy [A] time = 21.7421, size = 68, normalized size = 0.92

$$\frac{1}{bx^2\sqrt{bx^2+cx^4}}-\frac{4\sqrt{bx^2+cx^4}}{3b^2x^4}+\frac{8c\sqrt{bx^2+cx^4}}{3b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x**2*sqrt(b*x**2 + c*x**4)) - 4*sqrt(b*x**2 + c*x**4)/(3*b**
2*x**4) + 8*c*sqrt(b*x**2 + c*x**4)/(3*b**3*x**2)
```

Mathematica [A] time = 0.0337585, size = 46, normalized size = 0.62

$$\frac{-b^2+4bcx^2+8c^2x^4}{3b^3x^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(b*x^2 + c*x^4)^(3/2)),x]

[Out] (-b^2 + 4*b*c*x^2 + 8*c^2*x^4)/(3*b^3*x^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 45, normalized size = 0.6

$$-\frac{\left(cx^{2}+b\right)\left(-8\,c^{2}x^{4}-4\,bcx^{2}+b^{2}\right)}{3\,b^{3}}\left(cx^{4}+bx^{2}\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2)^(3/2),x)

[Out] -1/3*(c*x^2+b)*(-8*c^2*x^4-4*b*c*x^2+b^2)/b^3/(c*x^4+b*x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268223, size = 73, normalized size = 0.99

$$\frac{\left(8\,c^{2}x^{4}+4\,bcx^{2}-b^{2}\right)\sqrt{cx^{4}+bx^{2}}}{3\,(b^{3}cx^{6}+b^{4}x^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x),x, algorithm="fricas")

[Out] 1/3*(8*c^2*x^4 + 4*b*c*x^2 - b^2)*sqrt(c*x^4 + b*x^2)/(b^3*c*x^6
+ b^4*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x), x)
```

3.279 $\int \frac{1}{x^3(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=102

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^
6) + (8*c*Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 +
 c*x^4])/(5*b^4*x^2)
```

Rubi [A] time = 0.295929, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{1}{bx^4\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^4*Sqrt[b*x^2 + c*x^4]) - (6*Sqrt[b*x^2 + c*x^4])/(5*b^2*x^
6) + (8*c*Sqrt[b*x^2 + c*x^4])/(5*b^3*x^4) - (16*c^2*Sqrt[b*x^2 +
 c*x^4])/(5*b^4*x^2)
```

Rubi in Sympy [A] time = 29.9222, size = 95, normalized size = 0.93

$$\frac{1}{bx^4\sqrt{bx^2+cx^4}}-\frac{6\sqrt{bx^2+cx^4}}{5b^2x^6}+\frac{8c\sqrt{bx^2+cx^4}}{5b^3x^4}-\frac{16c^2\sqrt{bx^2+cx^4}}{5b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x**4*sqrt(b*x**2 + c*x**4)) - 6*sqrt(b*x**2 + c*x**4)/(5*b**
2*x**6) + 8*c*sqrt(b*x**2 + c*x**4)/(5*b**3*x**4) - 16*c**2*sqrt(
b*x**2 + c*x**4)/(5*b**4*x**2)
```

Mathematica [A] time = 0.0408813, size = 57, normalized size = 0.56

$$\frac{-b^3+2b^2cx^2-8bc^2x^4-16c^3x^6}{5b^4x^4\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-b^3 + 2*b^2*c*x^2 - 8*b*c^2*x^4 - 16*c^3*x^6)/(5*b^4*x^4*Sqrt[x
^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 59, normalized size = 0.6

$$-\frac{\left(cx^2+b\right)\left(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3\right)}{5\,b^4x^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/5*(c*x^2+b)*(16*c^3*x^6+8*b*c^2*x^4-2*b^2*c*x^2+b^3)/x^2/b^4/(
c*x^4+b*x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270514, size = 85, normalized size = 0.83

$$-\frac{\left(16\,c^3x^6+8\,bc^2x^4-2\,b^2cx^2+b^3\right)\sqrt{cx^4+bx^2}}{5\left(b^4cx^8+b^5x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/5*(16*c^3*x^6 + 8*b*c^2*x^4 - 2*b^2*c*x^2 + b^3)*sqrt(c*x^4 +
b*x^2)/(b^4*c*x^8 + b^5*x^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^3), x)
```

3.280 $\int \frac{1}{x^5(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=130

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^
8) + (48*c*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2
 + c*x^4])/(35*b^4*x^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x
^2)
```

Rubi [A] time = 0.380131, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{1}{bx^6\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^6*Sqrt[b*x^2 + c*x^4]) - (8*Sqrt[b*x^2 + c*x^4])/(7*b^2*x^
8) + (48*c*Sqrt[b*x^2 + c*x^4])/(35*b^3*x^6) - (64*c^2*Sqrt[b*x^2
 + c*x^4])/(35*b^4*x^4) + (128*c^3*Sqrt[b*x^2 + c*x^4])/(35*b^5*x
^2)
```

Rubi in Sympy [A] time = 38.3023, size = 122, normalized size = 0.94

$$\frac{1}{bx^6\sqrt{bx^2+cx^4}}-\frac{8\sqrt{bx^2+cx^4}}{7b^2x^8}+\frac{48c\sqrt{bx^2+cx^4}}{35b^3x^6}-\frac{64c^2\sqrt{bx^2+cx^4}}{35b^4x^4}+\frac{128c^3\sqrt{bx^2+cx^4}}{35b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x**6*sqrt(b*x**2 + c*x**4)) - 8*sqrt(b*x**2 + c*x**4)/(7*b**
2*x**8) + 48*c*sqrt(b*x**2 + c*x**4)/(35*b**3*x**6) - 64*c**2*sqr
t(b*x**2 + c*x**4)/(35*b**4*x**4) + 128*c**3*sqrt(b*x**2 + c*x**4
)/(35*b**5*x**2)
```

Mathematica [A] time = 0.049327, size = 68, normalized size = 0.52

$$\frac{-5b^4+8b^3cx^2-16b^2c^2x^4+64bc^3x^6+128c^4x^8}{35b^5x^6\sqrt{x^2\,(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-5*b^4 + 8*b^3*c*x^2 - 16*b^2*c^2*x^4 + 64*b*c^3*x^6 + 128*c^4*x
^8)/(35*b^5*x^6*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 72, normalized size = 0.6

$$-\frac{(cx^2+b)\,(-128\,c^4x^8-64\,c^3x^6b+16\,c^2x^4b^2-8\,cx^2b^3+5\,b^4)}{35\,x^4b^5}\,(cx^4+bx^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/35*(c*x^2+b)*(-128*c^4*x^8-64*b*c^3*x^6+16*b^2*c^2*x^4-8*b^3*c
*x^2+5*b^4)/x^4/b^5/(c*x^4+b*x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285215, size = 103, normalized size = 0.79

$$\frac{(128\,c^4x^8+64\,bc^3x^6-16\,b^2c^2x^4+8\,b^3cx^2-5\,b^4)\,\sqrt{cx^4+bx^2}}{35\,(b^5cx^{10}+b^6x^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5),x, algorithm="fricas")
```

```
[Out] 1/35*(128*c^4*x^8 + 64*b*c^3*x^6 - 16*b^2*c^2*x^4 + 8*b^3*c*x^2 -
 5*b^4)*sqrt(c*x^4 + b*x^2)/(b^5*c*x^10 + b^6*x^8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2\right)^{\frac{3}{2}} x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^5), x)
```

3.281 $\int \frac{x^6}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rubi [A] time = 0.0940801, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2\sqrt{bx^2+cx^4}}{c^2x}-\frac{x^3}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(b*x^2 + c*x^4)^(3/2),x]

[Out] -(x^3/(c*Sqrt[b*x^2 + c*x^4])) + (2*Sqrt[b*x^2 + c*x^4])/(c^2*x)
```

Rubi in Sympy [A] time = 15.662, size = 37, normalized size = 0.79

$$-\frac{x^3}{c\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2)**(3/2),x)

[Out] -x**3/(c*sqrt(b*x**2 + c*x**4)) + 2*sqrt(b*x**2 + c*x**4)/(c**2*x
)
```

Mathematica [A] time = 0.0260927, size = 29, normalized size = 0.62

$$\frac{x\left(2b+cx^2\right)}{c^2\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*b + c*x^2))/(c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 37, normalized size = 0.8

$$\frac{\left(cx^2+b\right)\left(cx^2+2b\right)x^3}{c^2}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] (c*x^2+b)*(c*x^2+2*b)*x^3/c^2/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.735932, size = 30, normalized size = 0.64

$$\frac{cx^2+2b}{\sqrt{cx^2+b}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (c*x^2 + 2*b)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A] time = 0.26623, size = 53, normalized size = 1.13

$$\frac{\sqrt{cx^4+bx^2}\left(cx^2+2b\right)}{c^3x^3+bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2)*(c*x^2 + 2*b)/(c^3*x^3 + b*c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.282159, size = 70, normalized size = 1.49

$$-\frac{2\sqrt{b}}{\left(\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c\right)c}+\frac{b}{\sqrt{c+\frac{b}{x^2}}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/(((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)*c) + b/(sqrt(c
+ b/x^2)*c^2*x)
```

3.282 $\int \frac{x^4}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rubi [A] time = 0.0148875, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(b*x^2 + c*x^4)^(3/2),x]

[Out] -(x/(c*Sqrt[b*x^2 + c*x^4]))
```

Rubi in Sympy [A] time = 8.40508, size = 17, normalized size = 0.81

$$-\frac{x}{c\sqrt{bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2)**(3/2),x)

[Out] -x/(c*sqrt(b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0116586, size = 21, normalized size = 1.

$$-\frac{x}{c\sqrt{x^2(b+cx^2)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(b*x^2 + c*x^4)^(3/2),x]

[Out] -(x/(c*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] time = 0.004, size = 29, normalized size = 1.4

$$-\frac{x^3\left(cx^2+b\right)}{c}\left(cx^4+bx^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2)^(3/2),x)

[Out] -(c*x^2+b)/c*x^3/(c*x^4+b*x^2)^(3/2)
```

Maxima [A] time = 0.735953, size = 19, normalized size = 0.9

$$-\frac{1}{\sqrt{cx^2+b}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")

[Out] -1/(sqrt(c*x^2 + b)*c)
```

Fricas [A] time = 0.259737, size = 39, normalized size = 1.86

$$-\frac{\sqrt{cx^4+bx^2}}{c^2x^3+bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")

[Out] -sqrt(c*x^4 + b*x^2)/(c^2*x^3 + b*c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [A] time = 0.282877, size = 23, normalized size = 1.1

$$-\frac{1}{\sqrt{c+\frac{b}{x^2}}\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(c + b/x^2)*c*x)
```

3.283 $\int \frac{x^2}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^
4]]/b^(3/2)
```

Rubi [A] time = 0.1007, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x/(b*Sqrt[b*x^2 + c*x^4]) - ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^
4]]/b^(3/2)
```

Rubi in Sympy [A] time = 13.8143, size = 42, normalized size = 0.82

$$\frac{x}{b\sqrt{bx^2+cx^4}}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] x/(b*sqrt(b*x**2 + c*x**4)) - atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**
4))/b**(3/2)
```

Mathematica [A] time = 0.0590084, size = 75, normalized size = 1.47

$$\frac{x\left(\log(x)\sqrt{b+cx^2}-\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)+\sqrt{b}\right)}{b^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(Sqrt[b] + Sqrt[b + c*x^2]*Log[x] - Sqrt[b + c*x^2]*Log[b + Sq
rt[b]*Sqrt[b + c*x^2]]))/(b^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.008, size = 67, normalized size = 1.3

$$-x^3\left(cx^2+b\right)\left(\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)b\sqrt{cx^2+b}-b^{\frac{3}{2}}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -x^3*(c*x^2+b)*(ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*b*(c*x^2+b)^(
1/2)-b^(3/2))/(c*x^4+b*x^2)^(3/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275976, size = 1, normalized size = 0.02

$$\left[\frac{\left(cx^3+bx\right)\sqrt{b}\log\left(-\frac{\left(cx^3+2\,bx\right)\sqrt{b}-2\,\sqrt{cx^4+bx^2}b}{x^3}\right)+2\,\sqrt{cx^4+bx^2}b}{2\left(b^2cx^3+b^3x\right)},\ \frac{\left(cx^3+bx\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}b}{b^2cx^3+b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((c*x^3 + b*x)*sqrt(b)*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqr
t(c*x^4 + b*x^2)*b)/x^3) + 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 +
b^3*x), ((c*x^3 + b*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*
x^2)) + sqrt(c*x^4 + b*x^2)*b)/(b^2*c*x^3 + b^3*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**2/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.284 $\int \frac{1}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=81

$$\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3)
 + (3*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rubi [A] time = 0.119543, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$\frac{3c\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{5/2}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{1}{bx\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] 1/(b*x*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2 + c*x^4])/(2*b^2*x^3)
 + (3*c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(2*b^(5/2))
```

Rubi in Sympy [A] time = 18.8115, size = 73, normalized size = 0.9

$$\frac{1}{bx\sqrt{bx^2+cx^4}}-\frac{3\sqrt{bx^2+cx^4}}{2b^2x^3}+\frac{3c\,\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+cx^4}}\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x*sqrt(b*x**2 + c*x**4)) - 3*sqrt(b*x**2 + c*x**4)/(2*b**2*x
**3) + 3*c*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(2*b**(5/2))
```

Mathematica [A] time = 0.0693013, size = 99, normalized size = 1.22

$$\frac{-\sqrt{b}\left(b+3cx^2\right)-3cx^2\log(x)\sqrt{b+cx^2}+3cx^2\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)}{2b^{5/2}x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] (-(Sqrt[b]*(b + 3*c*x^2)) - 3*c*x^2*Sqrt[b + c*x^2]*Log[x] + 3*c*
x^2*Sqrt[b + c*x^2]*Log[b + Sqrt[b]*Sqrt[b + c*x^2]])/(2*b^(5/2)*
x*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.01, size = 79, normalized size = 1.

$$\frac{x\left(cx^2+b\right)}{2}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{cx^2+b}x^2bc-3\,b^{3/2}x^2c-b^{\frac{5}{2}}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2*x*(c*x^2+b)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c*x^2+b)^
(1/2)*x^2*b*c-3*b^(3/2)*x^2*c-b^(5/2))/(c*x^4+b*x^2)^(3/2)/b^(7/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(-3/2), x)
```

Fricas [A] time = 0.278149, size = 1, normalized size = 0.01

$$\left[\frac{3\left(c^{2}x^{5}+bcx^{3}\right)\sqrt{b}\log\left(-\frac{\left(cx^{3}+2\,bx\right)\sqrt{b}+2\,\sqrt{cx^{4}+bx^{2}}b}{x^{3}}\right)-2\,\sqrt{cx^{4}+bx^{2}}\left(3\,bcx^{2}+b^{2}\right)}{4\left(b^{3}cx^{5}+b^{4}x^{3}\right)},\right.$$
$$\left.-\frac{3\left(c^{2}x^{5}+bcx^{3}\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^{4}+bx^{2}}}\right)+\sqrt{cx^{4}+bx^{2}}\left(3\,bcx^{2}+b^{2}\right)}{2\left(b^{3}cx^{5}+b^{4}x^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(c^2*x^5 + b*c*x^3)*sqrt(b)*log(-((c*x^3 + 2*b*x)*sqrt(b)
 + 2*sqrt(c*x^4 + b*x^2)*b)/x^3) - 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x
^2 + b^2))/(b^3*c*x^5 + b^4*x^3), -1/2*(3*(c^2*x^5 + b*c*x^3)*sqr
t(-b)*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 + b*x^2
)*(3*b*c*x^2 + b^2))/(b^3*c*x^5 + b^4*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral((b*x**2 + c*x**4)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.640204, size = 4, normalized size = 0.05

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(-3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.285 $\int \frac{1}{x^2(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=109

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^
5) + (15*c*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sq
rt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rubi [A] time = 0.260075, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{15c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{7/2}}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{1}{bx^3\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^3*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(4*b^2*x^
5) + (15*c*Sqrt[b*x^2 + c*x^4])/(8*b^3*x^3) - (15*c^2*ArcTanh[(Sq
rt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(7/2))
```

Rubi in Sympy [A] time = 29.9249, size = 102, normalized size = 0.94

$$\frac{1}{bx^3\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{4b^2x^5}+\frac{15c\sqrt{bx^2+cx^4}}{8b^3x^3}-\frac{15c^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x**3*sqrt(b*x**2 + c*x**4)) - 5*sqrt(b*x**2 + c*x**4)/(4*b**
2*x**5) + 15*c*sqrt(b*x**2 + c*x**4)/(8*b**3*x**3) - 15*c**2*atan
h(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(8*b**(7/2))
```

Mathematica [A] time = 0.100422, size = 115, normalized size = 1.06

$$\frac{\sqrt{b}\left(-2b^2+5bcx^2+15c^2x^4\right)+15c^2x^4\log(x)\sqrt{b+cx^2}-15c^2x^4\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)}{8b^{7/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + 5*b*c*x^2 + 15*c^2*x^4) + 15*c^2*x^4*Sqrt[b +
c*x^2]*Log[x] - 15*c^2*x^4*Sqrt[b + c*x^2]*Log[b + Sqrt[b]*Sqrt[b
 + c*x^2]])/(8*b^(7/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.012, size = 94, normalized size = 0.9

$$-\frac{cx^2+b}{8\,x}\left(15\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\sqrt{cx^2+b}x^4bc^2-15\,b^{3/2}x^4c^2-5\,b^{5/2}x^2c+2\,b^{7/2}\right)\left(cx^4+bx^2\right)^{-\frac{3}{2}}b^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/8/x*(c*x^2+b)*(15*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*(c*x^2+b
)^(1/2)*x^4*b*c^2-15*b^(3/2)*x^4*c^2-5*b^(5/2)*x^2*c+2*b^(7/2))/(
c*x^4+b*x^2)^(3/2)/b^(9/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

Fricas [A] time = 0.280649, size = 1, normalized size = 0.01

$$\left[\frac{15\left(c^{3}x^{7}+bc^{2}x^{5}\right)\sqrt{b}\log\left(-\frac{\left(cx^{3}+2\,bx\right)\sqrt{b}-2\,\sqrt{cx^{4}+bx^{2}}b}{x^{3}}\right)+2\left(15\,bc^{2}x^{4}+5\,b^{2}cx^{2}-2\,b^{3}\right)\sqrt{cx^{4}+bx^{2}}}{16\left(b^{4}cx^{7}+b^{5}x^{5}\right)},\frac{15\left(c^{3}x^{7}+bc^{2}x^{5}\right)\sqrt{-b}\,\text{ar}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(c^3*x^7 + b*c^2*x^5)*sqrt(b)*log(-((c*x^3 + 2*b*x)*sqr
t(b) - 2*sqrt(c*x^4 + b*x^2)*b)/x^3) + 2*(15*b*c^2*x^4 + 5*b^2*c*
x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/(b^4*c*x^7 + b^5*x^5), 1/8*(15*
(c^3*x^7 + b*c^2*x^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x
^2)) + (15*b*c^2*x^4 + 5*b^2*c*x^2 - 2*b^3)*sqrt(c*x^4 + b*x^2))/
(b^4*c*x^7 + b^5*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^2), x)
```

3.286 $\int \frac{x^3}{\sqrt{3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)-\frac{1}{8}\sqrt{3x^2-4x^4}$$

```
[Out] -Sqrt[3*x^2 - 4*x^4]/8 - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rubi [A] time = 0.0989784, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$-\frac{3}{32}\sin^{-1}\left(1-\frac{8x^2}{3}\right)-\frac{1}{8}\sqrt{3x^2-4x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] -Sqrt[3*x^2 - 4*x^4]/8 - (3*ArcSin[1 - (8*x^2)/3])/32
```

Rubi in Sympy [A] time = 11.0833, size = 27, normalized size = 0.79

$$-\frac{\sqrt{-4x^4+3x^2}}{8}+\frac{3\,\mathrm{asin}\left(\frac{8x^2}{3}-1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

```
[Out] -sqrt(-4*x**4 + 3*x**2)/8 + 3*asin(8*x**2/3 - 1)/32
```

Mathematica [A] time = 0.0332946, size = 58, normalized size = 1.71

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\log\left(\sqrt{4x^2-3}+2x\right)-6x\right)}{16\sqrt{3x^2-4x^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[3*x^2 - 4*x^4],x]
```

```
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*Log[2*x + Sqrt[-3 + 4*x^2]]
))/(16*Sqrt[3*x^2 - 4*x^4])
```

Maple [A] time = 0.009, size = 48, normalized size = 1.4

$$\frac{x}{16}\sqrt{-4\,x^2+3}\left(-2\,x\sqrt{-4\,x^2+3}+3\,\arcsin\left(2/3\,x\sqrt{3}\right)\right)\frac{1}{\sqrt{-4\,x^4+3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-4*x^4+3*x^2)^(1/2),x)
```

```
[Out] 1/16*x*(-4*x^2+3)^(1/2)*(-2*x*(-4*x^2+3)^(1/2)+3*arcsin(2/3*x*3^(
1/2)))/(-4*x^4+3*x^2)^(1/2)
```

Maxima [A] time = 0.770841, size = 35, normalized size = 1.03

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{32}\,\arcsin\left(-\frac{8}{3}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-4*x^4 + 3*x^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/32*arcsin(-8/3*x^2 + 1)
```

Fricas [A] time = 0.265215, size = 50, normalized size = 1.47

$$-\frac{1}{8}\sqrt{-4\,x^4+3\,x^2}-\frac{3}{16}\,\arctan\left(\frac{\sqrt{-4\,x^4+3\,x^2}}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-4*x^4 + 3*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) - 3/16*arctan(1/2*sqrt(-4*x^4 + 3*x^2)/
x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-x^2\,(4x^2-3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 - 3)), x)
```

GIAC/XCAS [A] time = 0.280773, size = 35, normalized size = 1.03

$$-\frac{1}{8}\,\sqrt{-4\,x^4+3\,x^2}+\frac{3}{32}\,\arcsin\left(\frac{8}{3}\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-4*x^4 + 3*x^2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^4 + 3*x^2) + 3/32*arcsin(8/3*x^2 - 1)
```

3.287 $\int \frac{x^3}{\sqrt{-3x^2-4x^4}}\,dx$

Optimal. Leaf size=34

$$-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)-\frac{1}{8}\sqrt{-4x^4-3x^2}$$

```
[Out] -Sqrt[-3*x^2 - 4*x^4]/8 - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rubi [A] time = 0.103323, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$-\frac{3}{32}\sin^{-1}\left(\frac{8x^2}{3}+1\right)-\frac{1}{8}\sqrt{-4x^4-3x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[-3*x^2 - 4*x^4],x]

[Out] -Sqrt[-3*x^2 - 4*x^4]/8 - (3*ArcSin[1 + (8*x^2)/3])/32
```

Rubi in Sympy [A] time = 10.7705, size = 31, normalized size = 0.91

$$-\frac{\sqrt{-4x^4-3x^2}}{8}-\frac{3\,\mathrm{asin}\left(\frac{8x^2}{3}+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)

[Out] -sqrt(-4*x**4 - 3*x**2)/8 - 3*asin(8*x**2/3 + 1)/32
```

Mathematica [A] time = 0.0337851, size = 52, normalized size = 1.53

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{-x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[-3*x^2 - 4*x^4],x]
```

```
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*
Sqrt[-(x^2*(3 + 4*x^2))])
```

Maple [B] time = 0.013, size = 54, normalized size = 1.6

$$-\frac{x}{16}\sqrt{-4\,x^2-3}\left(2\,x\sqrt{-4\,x^2-3}+3\,\arctan\left(2\,\frac{x}{\sqrt{-4\,x^2-3}}\right)\right)\frac{1}{\sqrt{-4\,x^4-3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-4*x^4-3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(-4*x^2-3)^(1/2)*(2*x*(-4*x^2-3)^(1/2)+3*arctan(2*x/(-4*x
^2-3)^(1/2)))/(-4*x^4-3*x^2)^(1/2)
```

Maxima [A] time = 0.766011, size = 35, normalized size = 1.03

$$-\frac{1}{8}\sqrt{-4\,x^4-3\,x^2}+\frac{3}{32}\,\arcsin\left(-\frac{8}{3}\,x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-4*x^4 - 3*x^2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^4 - 3*x^2) + 3/32*arcsin(-8/3*x^2 - 1)
```

Fricas [A] time = 0.263826, size = 80, normalized size = 2.35

$$-\frac{1}{8}\sqrt{-4\,x^2-3}x-\frac{3}{32}i\,\log\left(-\frac{8\,x+4i\,\sqrt{-4\,x^2-3}}{x}\right)+\frac{3}{32}i\,\log\left(-\frac{8\,x-4i\,\sqrt{-4\,x^2-3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-4*x^4 - 3*x^2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 - 3)*x - 3/32*I*log(-(8*x + 4*I*sqrt(-4*x^2 - 3)
)/x) + 3/32*I*log(-(8*x - 4*I*sqrt(-4*x^2 - 3))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-x^2\left(4x^2+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(4*x**2 + 3)), x)
```

GIAC/XCAS [A] time = 0.287021, size = 36, normalized size = 1.06

$$-\frac{1}{8}\,\sqrt{4\,x^4+3\,x^2}i-\frac{3}{32}\,\arcsin\left(\frac{8}{3}\,x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-4*x^4 - 3*x^2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(4*x^4 + 3*x^2)*i - 3/32*arcsin(8/3*x^2 + 1)
```

3.288 $\int \frac{x^3}{\sqrt{3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/
16
```

Rubi [A] time = 0.104432, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{1}{8}\sqrt{4x^4+3x^2}-\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[3*x^2 + 4*x^4]/8 - (3*ArcTanh[(2*x^2)/Sqrt[3*x^2 + 4*x^4]])/
16
```

Rubi in Sympy [A] time = 10.5405, size = 37, normalized size = 0.82

$$\frac{\sqrt{4x^4+3x^2}}{8}-\frac{3\,\mathrm{atanh}\left(\frac{2x^2}{\sqrt{4x^4+3x^2}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

```
[Out] sqrt(4*x**4 + 3*x**2)/8 - 3*atanh(2*x**2/sqrt(4*x**4 + 3*x**2))/1
6
```

Mathematica [A] time = 0.0240704, size = 51, normalized size = 1.13

$$\frac{x\left(8x^3-3\sqrt{4x^2+3}\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)+6x\right)}{16\sqrt{x^2\left(4x^2+3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[3*x^2 + 4*x^4],x]
```

```
[Out] (x*(6*x + 8*x^3 - 3*Sqrt[3 + 4*x^2]*ArcSinh[(2*x)/Sqrt[3]]))/(16*
Sqrt[x^2*(3 + 4*x^2)])
```

Maple [A] time = 0.009, size = 48, normalized size = 1.1

$$-\frac{x}{16}\sqrt{4\,x^2+3}\left(-2\,x\sqrt{4\,x^2+3}+3\,Arcsinh\left(2/3\,x\sqrt{3}\right)\right)\frac{1}{\sqrt{4\,x^4+3\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(4*x^4+3*x^2)^(1/2),x)
```

```
[Out] -1/16*x*(4*x^2+3)^(1/2)*(-2*x*(4*x^2+3)^(1/2)+3*arcsinh(2/3*x*3^(
1/2)))/(4*x^4+3*x^2)^(1/2)
```

Maxima [A] time = 0.759455, size = 55, normalized size = 1.22

$$\frac{1}{8}\sqrt{4\,x^4+3\,x^2}-\frac{3}{32}\log\left(8\,x^2+4\sqrt{4\,x^4+3\,x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(4*x^4 + 3*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) - 3/32*log(8*x^2 + 4*sqrt(4*x^4 + 3*x^2)
+ 3)
```

Fricas [A] time = 0.259486, size = 61, normalized size = 1.36

$$\frac{1}{8}\sqrt{4\,x^4+3\,x^2}+\frac{3}{16}\log\left(-\frac{2\,x^2-\sqrt{4\,x^4+3\,x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(4*x^4 + 3*x^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/16*log(-(2*x^2 - sqrt(4*x^4 + 3*x^2))
/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\,(4x^2+3)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(4*x**4+3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 + 3)), x)
```

GIAC/XCAS [A] time = 0.279171, size = 55, normalized size = 1.22

$$\frac{1}{8}\,\sqrt{4\,x^4+3\,x^2}+\frac{3}{32}\,\ln\left(8\,x^2-4\,\sqrt{4\,x^4+3\,x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(4*x^4 + 3*x^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 + 3*x^2) + 3/32*ln(8*x^2 - 4*sqrt(4*x^4 + 3*x^2) +
 3)
```

3.289 $\int \frac{x^3}{\sqrt{-3x^2+4x^4}}\,dx$

Optimal. Leaf size=45

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]]
)/16
```

Rubi [A] time = 0.101116, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{1}{8}\sqrt{4x^4-3x^2}+\frac{3}{16}\tanh^{-1}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] Sqrt[-3*x^2 + 4*x^4]/8 + (3*ArcTanh[(2*x^2)/Sqrt[-3*x^2 + 4*x^4]]
)/16
```

Rubi in Sympy [A] time = 10.7373, size = 37, normalized size = 0.82

$$\frac{\sqrt{4x^4-3x^2}}{8}+\frac{3\operatorname{atanh}\left(\frac{2x^2}{\sqrt{4x^4-3x^2}}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(4*x**4-3*x**2)**(1/2),x)
```

```
[Out] sqrt(4*x**4 - 3*x**2)/8 + 3*atanh(2*x**2/sqrt(4*x**4 - 3*x**2))/1
6
```

Mathematica [A] time = 0.0214401, size = 58, normalized size = 1.29

$$\frac{x\left(8x^3+3\sqrt{4x^2-3}\log\left(\sqrt{4x^2-3}+2x\right)-6x\right)}{16\sqrt{x^2\left(4x^2-3\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[-3*x^2 + 4*x^4],x]
```

```
[Out] (x*(-6*x + 8*x^3 + 3*Sqrt[-3 + 4*x^2]*Log[2*x + Sqrt[-3 + 4*x^2]]
))/(16*Sqrt[x^2*(-3 + 4*x^2)])
```

Maple [A]   time = 0.01, size = 60, normalized size = 1.3

$$\frac{x}{32}\sqrt{4\,x^{2}-3}\left(3\,\ln\left(x\sqrt{4}+\sqrt{4\,x^{2}-3}\right)\sqrt{4}+4\,x\sqrt{4\,x^{2}-3}\right)\frac{1}{\sqrt{4\,x^{4}-3\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(4*x^4-3*x^2)^(1/2),x)
```

```
[Out] 1/32*x*(4*x^2-3)^(1/2)*(3*ln(x*4^(1/2)+(4*x^2-3)^(1/2))*4^(1/2)+4
*x*(4*x^2-3)^(1/2))/(4*x^4-3*x^2)^(1/2)
```

Maxima [A]   time = 0.764865, size = 55, normalized size = 1.22

$$\frac{1}{8}\,\sqrt{4\,x^{4}-3\,x^{2}}+\frac{3}{32}\,\log\left(8\,x^{2}+4\,\sqrt{4\,x^{4}-3\,x^{2}}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(4*x^4 - 3*x^2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(4*x^4 - 3*x^2) + 3/32*log(8*x^2 + 4*sqrt(4*x^4 - 3*x^2)
- 3)
```

Fricas [A]   time = 0.261799, size = 61, normalized size = 1.36

$$\frac{1}{8}\,\sqrt{4\,x^{4}-3\,x^{2}}-\frac{3}{16}\,\log\left(-\frac{2\,x^{2}-\sqrt{4\,x^{4}-3\,x^{2}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(4*x^4 - 3*x^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/16*log(-(2*x^2 - sqrt(4*x^4 - 3*x^2))
/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2 (4x^2 - 3)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(4*x**4-3*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(4*x**2 - 3)), x)
```

GIAC/XCAS [A] time = 0.282338, size = 57, normalized size = 1.27

$$\frac{1}{8}\sqrt{4\,x^4 - 3\,x^2} - \frac{3}{32}\ln\left(\left|-8\,x^2 + 4\,\sqrt{4\,x^4 - 3\,x^2} + 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(4*x^4 - 3*x^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(4*x^4 - 3*x^2) - 3/32*ln(abs(-8*x^2 + 4*sqrt(4*x^4 - 3*x
^2) + 3))
```

3.290 $\int \frac{x^3}{\sqrt{ax^2+bx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 +
 b*x^4]])/(2*b^(3/2))
```

Rubi [A] time = 0.153705, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21

$$\frac{\sqrt{ax^2+bx^4}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] Sqrt[a*x^2 + b*x^4]/(2*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^2 +
 b*x^4]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 13.4634, size = 48, normalized size = 0.83

$$-\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2+bx^4}}\right)}{2b^{\frac{3}{2}}}+\frac{\sqrt{ax^2+bx^4}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

```
[Out] -a*atanh(sqrt(b)*x**2/sqrt(a*x**2 + b*x**4))/(2*b**(3/2)) + sqrt(
a*x**2 + b*x**4)/(2*b)
```

Mathematica [A] time = 0.0618191, size = 76, normalized size = 1.31

$$\frac{x\left(\sqrt{b}x\left(a+bx^2\right)-a\sqrt{a+bx^2}\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)\right)}{2b^{3/2}\sqrt{x^2\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a*x^2 + b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(a + b*x^2) - a*Sqrt[a + b*x^2]*Log[b*x + Sqrt[b]*S
qrt[a + b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a + b*x^2)])
```

Maple [A] time = 0.01, size = 64, normalized size = 1.1

$$-\frac{x}{2}\sqrt{bx^2+a}\left(-x\sqrt{bx^2+a}b^{\frac{3}{2}}+a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b\right)\frac{1}{\sqrt{bx^4+ax^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^4+a*x^2)^(1/2),x)
```

```
[Out] -1/2*x*(b*x^2+a)^(1/2)*(-x*(b*x^2+a)^(1/2)*b^(3/2)+a*ln(x*b^(1/2)
+(b*x^2+a)^(1/2))*b)/(b*x^4+a*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(b*x^4 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276668, size = 1, normalized size = 0.02

$$\left[\frac{a\sqrt{b}\log\left(-\left(2\,bx^2+a\right)\sqrt{b}+2\,\sqrt{bx^4+ax^2}b\right)+2\,\sqrt{bx^4+ax^2}b}{4\,b^2},\ \frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x^2}{\sqrt{bx^4+ax^2}}\right)+\sqrt{bx^4+ax^2}b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(b*x^4 + a*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(a*sqrt(b)*log(-(2*b*x^2 + a)*sqrt(b) + 2*sqrt(b*x^4 + a*x^2
)*b) + 2*sqrt(b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(-b)*arctan(sqrt(
-b)*x^2/sqrt(b*x^4 + a*x^2)) + sqrt(b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\,(a + bx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.296976, size = 80, normalized size = 1.38

$$\frac{a\ln\left(\left|-2\left(\sqrt{b}x^2 - \sqrt{bx^4 + ax^2}\right)\sqrt{b} - a\right|\right)}{4\,b^{\frac{3}{2}}} + \frac{\sqrt{bx^4 + ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(b*x^4 + a*x^2),x, algorithm="giac")
```

```
[Out] 1/4*a*ln(abs(-2*(sqrt(b)*x^2 - sqrt(b*x^4 + a*x^2))*sqrt(b) - a))
/b^(3/2) + 1/2*sqrt(b*x^4 + a*x^2)/b
```

3.291 $\int \frac{x^3}{\sqrt{ax^2-bx^4}}\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

```
[Out] -Sqrt[a*x^2 - b*x^4]/(2*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 -
 b*x^4]])/(2*b^(3/2))
```

Rubi [A] time = 0.157947, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] -Sqrt[a*x^2 - b*x^4]/(2*b) + (a*ArcTan[(Sqrt[b]*x^2)/Sqrt[a*x^2 -
 b*x^4]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 13.7813, size = 48, normalized size = 0.8

$$\frac{a\,\mathrm{atan}\left(\frac{\sqrt{b}x^2}{\sqrt{ax^2-bx^4}}\right)}{2b^{\frac{3}{2}}}-\frac{\sqrt{ax^2-bx^4}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

```
[Out] a*atan(sqrt(b)*x**2/sqrt(a*x**2 - b*x**4))/(2*b**(3/2)) - sqrt(a*
x**2 - b*x**4)/(2*b)
```

Mathematica [A] time = 0.0838189, size = 77, normalized size = 1.28

$$\frac{x\left(\sqrt{b}x\left(bx^2-a\right)+a\sqrt{a-bx^2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a-bx^2}}\right)\right)}{2b^{3/2}\sqrt{x^2\left(a-bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a*x^2 - b*x^4],x]
```

```
[Out] (x*(Sqrt[b]*x*(-a + b*x^2) + a*Sqrt[a - b*x^2]*ArcTan[(Sqrt[b]*x)
/Sqrt[a - b*x^2]]))/(2*b^(3/2)*Sqrt[x^2*(a - b*x^2)])
```

Maple [A] time = 0.014, size = 67, normalized size = 1.1

$$\frac{x}{2}\sqrt{-bx^2+a}\left(-x\sqrt{-bx^2+a}b^{\frac{3}{2}}+a\arctan\left(x\sqrt{b}\frac{1}{\sqrt{-bx^2+a}}\right)b\right)\frac{1}{\sqrt{-bx^4+ax^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-b*x^4+a*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(-b*x^2+a)^(1/2)*(-x*(-b*x^2+a)^(1/2)*b^(3/2)+a*arctan(b^(1
/2)*x/(-b*x^2+a)^(1/2))*b)/(-b*x^4+a*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-b*x^4 + a*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274385, size = 1, normalized size = 0.02

$$\left[-\frac{a\sqrt{-b}\log\left(\left(2\,bx^2-a\right)\sqrt{-b}+2\,\sqrt{-bx^4+ax^2}b\right)+2\,\sqrt{-bx^4+ax^2}b}{4\,b^2},\frac{a\sqrt{b}\arctan\left(\frac{\sqrt{b}x^2}{\sqrt{-bx^4+ax^2}}\right)-\sqrt{-bx^4+ax^2}b}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-b*x^4 + a*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(a*sqrt(-b)*log((2*b*x^2 - a)*sqrt(-b) + 2*sqrt(-b*x^4 + a*
x^2)*b) + 2*sqrt(-b*x^4 + a*x^2)*b)/b^2, 1/2*(a*sqrt(b)*arctan(sq
rt(b)*x^2/sqrt(-b*x^4 + a*x^2)) - sqrt(-b*x^4 + a*x^2)*b)/b^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-x^2\left(-a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-b*x**4+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(-x**2*(-a + b*x**2)), x)
```

GIAC/XCAS [A] time = 0.298044, size = 92, normalized size = 1.53

$$-\frac{a\ln\left(\left|2\left(\sqrt{-b}x^2-\sqrt{-bx^4+ax^2}\right)\sqrt{-b}+a\right|\right)}{4\sqrt{-b}b}-\frac{\sqrt{-bx^4+ax^2}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-b*x^4 + a*x^2),x, algorithm="giac")
```

```
[Out] -1/4*a*ln(abs(2*(sqrt(-b)*x^2 - sqrt(-b*x^4 + a*x^2))*sqrt(-b) +
a))/(sqrt(-b)*b) - 1/2*sqrt(-b*x^4 + a*x^2)/b
```

3.292 $\int x^{7/2} \left(bx^2+cx^4\right)\,dx$

Optimal. Leaf size=21

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Rubi [A] time = 0.0140501, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)*(b*x^2 + c*x^4),x]

[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Rubi in Sympy [A] time = 4.08332, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{13}{2}}}{13}+\frac{2cx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(7/2)*(c*x**4+b*x**2),x)

[Out] 2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

Mathematica [A] time = 0.0087109, size = 21, normalized size = 1.

$$\frac{2}{13}bx^{13/2}+\frac{2}{17}cx^{17/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(13/2))/13 + (2*c*x^(17/2))/17
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$\frac{26\,cx^2+34\,b}{221}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/221*x^(13/2)*(13*c*x^2+17*b)
```

Maxima [A] time = 0.678305, size = 18, normalized size = 0.86

$$\frac{2}{17}\,cx^{\frac{17}{2}}+\frac{2}{13}\,bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

Fricas [A] time = 0.256989, size = 24, normalized size = 1.14

$$\frac{2}{221}\left(13\,cx^8+17\,bx^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/221*(13*c*x^8 + 17*b*x^6)*sqrt(x)
```

Sympy [A] time = 34.9266, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{13}{2}}}{13}+\frac{2cx^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(13/2)/13 + 2*c*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.269443, size = 18, normalized size = 0.86

$$\frac{2}{17}cx^{\frac{17}{2}} + \frac{2}{13}bx^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/17*c*x^(17/2) + 2/13*b*x^(13/2)
```

3.293 $\int x^{5/2} \left(bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rubi [A] time = 0.0139778, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rubi in Sympy [A] time = 4.1525, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{11}{2}}}{11} + \frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(5/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Mathematica [A] time = 0.00774103, size = 21, normalized size = 1.

$$\frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$\frac{22\,cx^{2}+30\,b}{165}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/165*x^(11/2)*(11*c*x^2+15*b)
```

Maxima [A] time = 0.684087, size = 18, normalized size = 0.86

$$\frac{2}{15}\,cx^{\frac{15}{2}}+\frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

Fricas [A] time = 0.260159, size = 24, normalized size = 1.14

$$\frac{2}{165}\left(11\,cx^{7}+15\,bx^{5}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/165*(11*c*x^7 + 15*b*x^5)*sqrt(x)
```

Sympy [A] time = 17.0236, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.26639, size = 18, normalized size = 0.86

$$\frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{11}\,bx^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2)
```

3.294 $\int x^{3/2} \left(bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rubi [A] time = 0.0140933, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rubi in Sympy [A] time = 4.07805, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(3/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Mathematica [A] time = 0.00731289, size = 21, normalized size = 1.

$$\frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4),x]
```

```
[Out] (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$\frac{18\,cx^2+26\,b}{117}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/117*x^(9/2)*(9*c*x^2+13*b)
```

Maxima [A] time = 0.69624, size = 18, normalized size = 0.86

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

Fricas [A] time = 0.257142, size = 24, normalized size = 1.14

$$\frac{2}{117}\left(9\,cx^6+13\,bx^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/117*(9*c*x^6 + 13*b*x^4)*sqrt(x)
```

Sympy [A] time = 8.13698, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.267721, size = 18, normalized size = 0.86

$$\frac{2}{13}\,cx^{\frac{13}{2}} + \frac{2}{9}\,bx^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2)
```

3.295 $\int \sqrt{x} \left(bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=21

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11

Rubi [A] time = 0.0138933, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(b*x^2 + c*x^4),x]

[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11

Rubi in Sympy [A] time = 4.1029, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{7}{2}}}{7} + \frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(x**(1/2)*(c*x**4+b*x**2),x)

[Out] 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11

Mathematica [A] time = 0.00761752, size = 21, normalized size = 1.

$$\frac{2}{7}bx^{7/2} + \frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(b*x^2 + c*x^4),x]

```
[Out] (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$\frac{14\,c x^2+22\,b}{77} x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(c*x^4+b*x^2),x)
```

```
[Out] 2/77*x^(7/2)*(7*c*x^2+11*b)
```

Maxima [A] time = 0.734461, size = 18, normalized size = 0.86

$$\frac{2}{11}\,c x^{\frac{11}{2}}+\frac{2}{7}\,b x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

Fricas [A] time = 0.25727, size = 24, normalized size = 1.14

$$\frac{2}{77}\left(7\,c x^5+11\,b x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/77*(7*c*x^5 + 11*b*x^3)*sqrt(x)
```

Sympy [A] time = 2.50547, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2),x)
```

```
[Out] 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.268264, size = 18, normalized size = 0.86

$$\frac{2}{11}\,cx^{\frac{11}{2}} + \frac{2}{7}\,bx^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2)
```

3.296 $\int \frac{bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rubi [A] time = 0.0138588, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/Sqrt[x],x]

[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rubi in Sympy [A] time = 4.09157, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**(1/2),x)

[Out] 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Mathematica [A] time = 0.00751608, size = 21, normalized size = 1.

$$\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{10\,cx^2+18\,b}{45}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] 2/45*x^(5/2)*(5*c*x^2+9*b)
```

Maxima [A] time = 0.684498, size = 18, normalized size = 0.86

$$\frac{2}{9}\,cx^{\frac{9}{2}}+\frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

Fricas [A] time = 0.256675, size = 24, normalized size = 1.14

$$\frac{2}{45}\left(5\,cx^4+9\,bx^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/45*(5*c*x^4 + 9*b*x^2)*sqrt(x)
```

Sympy [A] time = 2.04216, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

GIAC/XCAS [A] time = 0.267206, size = 18, normalized size = 0.86

$$\frac{2}{9}\,cx^{\frac{9}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2)
```

3.297 $\int \frac{bx^2+cx^4}{x^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rubi [A] time = 0.0138706, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rubi in Sympy [A] time = 4.12468, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2)/x**(3/2),x)
```

```
[Out] 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Mathematica [A] time = 0.00717274, size = 21, normalized size = 1.

$$\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$\frac{6\,cx^2+14\,b}{21}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2)/x^(3/2),x)
```

```
[Out] 2/21*x^(3/2)*(3*c*x^2+7*b)
```

Maxima [A] time = 0.687804, size = 18, normalized size = 0.86

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

Fricas [A] time = 0.257905, size = 22, normalized size = 1.05

$$\frac{2}{21}\left(3\,cx^3+7\,bx\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*c*x^3 + 7*b*x)*sqrt(x)
```

Sympy [A] time = 2.38784, size = 19, normalized size = 0.9

$$\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**(3/2),x)

[Out] 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

GIAC/XCAS [A] time = 0.266973, size = 18, normalized size = 0.86

$$\frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(3/2),x, algorithm="giac")

[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2)
```

3.298 $\int \frac{bx^2+cx^4}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rubi [A] time = 0.013934, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rubi in Sympy [A] time = 4.02418, size = 17, normalized size = 0.89

$$2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**(5/2),x)
```

```
[Out] 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Mathematica [A] time = 0.00660381, size = 19, normalized size = 1.

$$2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$\frac{2\,c x^{2}+10\,b}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^(5/2),x)
```

```
[Out] 2/5*x^(1/2)*(c*x^2+5*b)
```

Maxima [A] time = 0.686513, size = 18, normalized size = 0.95

$$\frac{2}{5}\,c x^{\frac{5}{2}}+2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

Fricas [A] time = 0.256567, size = 19, normalized size = 1.

$$\frac{2}{5}\left(c x^{2}+5\,b\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/5*(c*x^2 + 5*b)*sqrt(x)
```

Sympy [A] time = 3.23358, size = 17, normalized size = 0.89

$$2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**(5/2),x)

[Out] 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

GIAC/XCAS [A] time = 0.268962, size = 18, normalized size = 0.95

$$\frac{2}{5}\,cx^{\frac{5}{2}} + 2\,b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(5/2),x, algorithm="giac")

[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x)
```

3.299 $\int \frac{bx^2+cx^4}{x^{7/2}} \, dx$

Optimal. Leaf size=19

$$\frac{2}{3}cx^{3/2} - \frac{2b}{\sqrt{x}}$$

```
[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rubi [A] time = 0.0136511, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{2}{3}cx^{3/2} - \frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)/x^(7/2),x]

[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rubi in Sympy [A] time = 4.1188, size = 17, normalized size = 0.89

$$-\frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)/x**(7/2),x)

[Out] -2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Mathematica [A] time = 0.0081906, size = 19, normalized size = 1.

$$\frac{2}{3}cx^{3/2} - \frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)/x^(7/2),x]

[Out] (-2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$-\frac{-2\,cx^{2}+6\,b}{3}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)/x^(7/2),x)

[Out] -2/3/x^(1/2)*(-c*x^2+3*b)
```

Maxima [A] time = 0.684489, size = 18, normalized size = 0.95

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(7/2),x, algorithm="maxima")

[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

Fricas [A] time = 0.257435, size = 19, normalized size = 1.

$$\frac{2\,\left(cx^{2}-3\,b\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(7/2),x, algorithm="fricas")

[Out] 2/3*(c*x^2 - 3*b)/sqrt(x)
```

Sympy [A] time = 6.14591, size = 17, normalized size = 0.89

$$-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)/x**(7/2),x)
```

```
[Out] -2*b/sqrt(x) + 2*c*x**(3/2)/3
```

GIAC/XCAS [A] time = 0.269237, size = 18, normalized size = 0.95

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*c*x^(3/2) - 2*b/sqrt(x)
```

3.300 $\int x^{7/2} \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{17}b^2x^{17/2} + \frac{4}{21}bcx^{21/2} + \frac{2}{25}c^2x^{25/2}$$

```
[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rubi [A] time = 0.0347012, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{17}b^2x^{17/2} + \frac{4}{21}bcx^{21/2} + \frac{2}{25}c^2x^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(b*x^2 + c*x^4)^2,x]

[Out] (2*b^2*x^(17/2))/17 + (4*b*c*x^(21/2))/21 + (2*c^2*x^(25/2))/25
```

Rubi in Sympy [A] time = 6.7435, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{17}{2}}}{17} + \frac{4bcx^{\frac{21}{2}}}{21} + \frac{2c^2x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)

[Out] 2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25
```

Mathematica [A] time = 0.0126077, size = 30, normalized size = 0.83

$$\frac{2x^{17/2}\left(525b^2 + 850bcx^2 + 357c^2x^4\right)}{8925}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(17/2)*(525*b^2 + 850*b*c*x^2 + 357*c^2*x^4))/8925
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$\frac{714\,c^{2}x^{4}+1700\,bc x^{2}+1050\,b^{2}}{8925}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/8925*x^(17/2)*(357*c^2*x^4+850*b*c*x^2+525*b^2)
```

Maxima [A] time = 0.684728, size = 32, normalized size = 0.89

$$\frac{2}{25}\,c^{2}x^{\frac{25}{2}}+\frac{4}{21}\,bcx^{\frac{21}{2}}+\frac{2}{17}\,b^{2}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

Fricas [A] time = 0.258653, size = 39, normalized size = 1.08

$$\frac{2}{8925}\left(357\,c^{2}x^{12}+850\,bcx^{10}+525\,b^{2}x^{8}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/8925*(357*c^2*x^12 + 850*b*c*x^10 + 525*b^2*x^8)*sqrt(x)
```

Sympy [A] time = 113.024, size = 34, normalized size = 0.94

$$\frac{2b^{2}x^{\frac{17}{2}}}{17}+\frac{4bcx^{\frac{21}{2}}}{21}+\frac{2c^{2}x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(17/2)/17 + 4*b*c*x**(21/2)/21 + 2*c**2*x**(25/2)/25
```

GIAC/XCAS [A] time = 0.267766, size = 32, normalized size = 0.89

$$\frac{2}{25}\,c^2x^{\frac{25}{2}} + \frac{4}{21}\,bcx^{\frac{21}{2}} + \frac{2}{17}\,b^2x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/25*c^2*x^(25/2) + 4/21*b*c*x^(21/2) + 2/17*b^2*x^(17/2)
```

3.301 $\int x^{5/2} \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{15}b^2x^{15/2} + \frac{4}{19}bcx^{19/2} + \frac{2}{23}c^2x^{23/2}$$

```
[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rubi [A] time = 0.0344196, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{15}b^2x^{15/2} + \frac{4}{19}bcx^{19/2} + \frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(b*x^2 + c*x^4)^2,x]

[Out] (2*b^2*x^(15/2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rubi in Sympy [A] time = 6.76346, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{15}{2}}}{15} + \frac{4bcx^{\frac{19}{2}}}{19} + \frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)

[Out] 2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

Mathematica [A] time = 0.0125462, size = 30, normalized size = 0.83

$$\frac{2x^{15/2}\left(437b^2 + 690bcx^2 + 285c^2x^4\right)}{6555}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(15/2)*(437*b^2 + 690*b*c*x^2 + 285*c^2*x^4))/6555
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$\frac{570\,c^{2}x^{4}+1380\,bc x^{2}+874\,b^{2}}{6555}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/6555*x^(15/2)*(285*c^2*x^4+690*b*c*x^2+437*b^2)
```

Maxima [A] time = 0.684798, size = 32, normalized size = 0.89

$$\frac{2}{23}\,c^{2}x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^{2}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

Fricas [A] time = 0.267866, size = 39, normalized size = 1.08

$$\frac{2}{6555}\left(285\,c^{2}x^{11}+690\,bcx^{9}+437\,b^{2}x^{7}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/6555*(285*c^2*x^11 + 690*b*c*x^9 + 437*b^2*x^7)*sqrt(x)
```

Sympy [A] time = 65.7329, size = 34, normalized size = 0.94

$$\frac{2b^{2}x^{\frac{15}{2}}}{15}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^{2}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.267947, size = 32, normalized size = 0.89

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,b^2x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2)
```

3.302 $\int x^{3/2} \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{13}b^2x^{13/2} + \frac{4}{17}bcx^{17/2} + \frac{2}{21}c^2x^{21/2}$$

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rubi [A] time = 0.0358954, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{13}b^2x^{13/2} + \frac{4}{17}bcx^{17/2} + \frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(13/2))/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rubi in Sympy [A] time = 6.74751, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13} + \frac{4bcx^{\frac{17}{2}}}{17} + \frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

Mathematica [A] time = 0.0119907, size = 30, normalized size = 0.83

$$\frac{2x^{13/2}\left(357b^2 + 546bcx^2 + 221c^2x^4\right)}{4641}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(13/2)*(357*b^2 + 546*b*c*x^2 + 221*c^2*x^4))/4641
```

Maple [A] time = 0.008, size = 27, normalized size = 0.8

$$\frac{442\,c^2x^4+1092\,bcx^2+714\,b^2}{4641}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/4641*x^(13/2)*(221*c^2*x^4+546*b*c*x^2+357*b^2)
```

Maxima [A] time = 0.680109, size = 32, normalized size = 0.89

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

Fricas [A] time = 0.263985, size = 39, normalized size = 1.08

$$\frac{2}{4641}\left(221\,c^2x^{10}+546\,bcx^8+357\,b^2x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/4641*(221*c^2*x^10 + 546*b*c*x^8 + 357*b^2*x^6)*sqrt(x)
```

Sympy [A] time = 35.2437, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.26673, size = 32, normalized size = 0.89

$$\frac{2}{21}\,c^2x^{\frac{21}{2}} + \frac{4}{17}\,bcx^{\frac{17}{2}} + \frac{2}{13}\,b^2x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2)
```

3.303 $\int \sqrt{x} \left(bx^2 + cx^4\right)^2 \, dx$

Optimal. Leaf size=36

$$\frac{2}{11}b^2x^{11/2} + \frac{4}{15}bcx^{15/2} + \frac{2}{19}c^2x^{19/2}$$

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rubi [A] time = 0.034386, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{11}b^2x^{11/2} + \frac{4}{15}bcx^{15/2} + \frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*b^2*x^(11/2))/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rubi in Sympy [A] time = 6.73658, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{11}{2}}}{11} + \frac{4bcx^{\frac{15}{2}}}{15} + \frac{2c^2x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19
```

Mathematica [A] time = 0.0116157, size = 30, normalized size = 0.83

$$\frac{2x^{11/2}\left(285b^2 + 418bcx^2 + 165c^2x^4\right)}{3135}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(11/2)*(285*b^2 + 418*b*c*x^2 + 165*c^2*x^4))/3135
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$\frac{330\,c^{2}x^{4}+836\,bc\,x^{2}+570\,b^{2}}{3135}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3135*x^(11/2)*(165*c^2*x^4+418*b*c*x^2+285*b^2)
```

Maxima [A] time = 0.687893, size = 32, normalized size = 0.89

$$\frac{2}{19}\,c^{2}x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^{2}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

Fricas [A] time = 0.266094, size = 39, normalized size = 1.08

$$\frac{2}{3135}\left(165\,c^{2}x^{9}+418\,bcx^{7}+285\,b^{2}x^{5}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2)^2*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/3135*(165*c^2*x^9 + 418*b*c*x^7 + 285*b^2*x^5)*sqrt(x)
```

Sympy [A] time = 7.83137, size = 34, normalized size = 0.94

$$\frac{2b^{2}x^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^{2}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2)**2,x)
```

```
[Out] 2*b**2*x**(11/2)/11 + 4*b*c*x**(15/2)/15 + 2*c**2*x**(19/2)/19
```

GIAC/XCAS [A] time = 0.267039, size = 32, normalized size = 0.89

$$\frac{2}{19}\,c^2x^{\frac{19}{2}} + \frac{4}{15}\,bcx^{\frac{15}{2}} + \frac{2}{11}\,b^2x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2)
```

3.304 $\int \frac{(bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rubi [A] time = 0.0334001, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{9}b^2x^{9/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/Sqrt[x],x]

[Out] (2*b^2*x^(9/2))/9 + (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rubi in Sympy [A] time = 6.72134, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**(1/2),x)

[Out] 2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

Mathematica [A] time = 0.0116563, size = 30, normalized size = 0.83

$$\frac{2x^{9/2}\left(221b^2+306bcx^2+117c^2x^4\right)}{1989}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/Sqrt[x],x]

[Out] (2*x^(9/2)*(221*b^2 + 306*b*c*x^2 + 117*c^2*x^4))/1989
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$\frac{234\,c^{2}x^{4}+612\,bcx^{2}+442\,b^{2}}{1989}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^(1/2),x)

[Out] 2/1989*x^(9/2)*(117*c^2*x^4+306*b*c*x^2+221*b^2)
```

Maxima [A] time = 0.671416, size = 32, normalized size = 0.89

$$\frac{2}{17}\,c^{2}x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^{2}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/sqrt(x),x, algorithm="maxima")

[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

Fricas [A] time = 0.265671, size = 39, normalized size = 1.08

$$\frac{2}{1989}\left(117\,c^{2}x^{8}+306\,bcx^{6}+221\,b^{2}x^{4}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/sqrt(x),x, algorithm="fricas")

[Out] 2/1989*(117*c^2*x^8 + 306*b*c*x^6 + 221*b^2*x^4)*sqrt(x)
```

Sympy [A] time = 14.2896, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bcx^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 2*b**2*x**(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.267471, size = 32, normalized size = 0.89

$$\frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2)
```

3.305 $\int \frac{(bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rubi [A] time = 0.0342171, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{7}b^2x^{7/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*b^2*x^(7/2))/7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rubi in Sympy [A] time = 6.75102, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

Mathematica [A] time = 0.0118806, size = 30, normalized size = 0.83

$$\frac{2x^{7/2}\left(165b^2+210bcx^2+77c^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^(3/2),x]

[Out] (2*x^(7/2)*(165*b^2 + 210*b*c*x^2 + 77*c^2*x^4))/1155
```

Maple [A] time = 0.007, size = 27, normalized size = 0.8

$$\frac{154\,c^2x^4 + 420\,bcx^2 + 330\,b^2}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^(3/2),x)

[Out] 2/1155*x^(7/2)*(77*c^2*x^4+210*b*c*x^2+165*b^2)
```

Maxima [A] time = 0.688772, size = 32, normalized size = 0.89

$$\frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(3/2),x, algorithm="maxima")

[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

Fricas [A] time = 0.265793, size = 39, normalized size = 1.08

$$\frac{2}{1155}\left(77\,c^2x^7 + 210\,bcx^5 + 165\,b^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(3/2),x, algorithm="fricas")

[Out] 2/1155*(77*c^2*x^7 + 210*b*c*x^5 + 165*b^2*x^3)*sqrt(x)
```

Sympy [A] time = 15.8135, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{7}{2}}}{7} + \frac{4bcx^{\frac{11}{2}}}{11} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**(3/2),x)
```

```
[Out] 2*b**2*x**(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.268018, size = 32, normalized size = 0.89

$$\frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2)
```

3.306 $\int \frac{(bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rubi [A] time = 0.0348967, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{5}b^2x^{5/2}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^(5/2),x]

[Out] (2*b^2*x^(5/2))/5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rubi in Sympy [A] time = 6.67276, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**(5/2),x)

[Out] 2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

Mathematica [A] time = 0.0115613, size = 30, normalized size = 0.83

$$\frac{2}{585}x^{5/2}\left(117b^2+130bcx^2+45c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^(5/2),x]

[Out] (2*x^(5/2)*(117*b^2 + 130*b*c*x^2 + 45*c^2*x^4))/585
```

Maple [A] time = 0.008, size = 27, normalized size = 0.8

$$\frac{90\,c^{2}x^{4}+260\,bc\,x^{2}+234\,b^{2}}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^(5/2),x)

[Out] 2/585*x^(5/2)*(45*c^2*x^4+130*b*c*x^2+117*b^2)
```

Maxima [A] time = 0.687731, size = 32, normalized size = 0.89

$$\frac{2}{13}\,c^{2}x^{\frac{13}{2}}+\frac{4}{9}\,bc\,x^{\frac{9}{2}}+\frac{2}{5}\,b^{2}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(5/2),x, algorithm="maxima")

[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

Fricas [A] time = 0.271015, size = 39, normalized size = 1.08

$$\frac{2}{585}\left(45\,c^{2}x^{6}+130\,bc\,x^{4}+117\,b^{2}x^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(5/2),x, algorithm="fricas")

[Out] 2/585*(45*c^2*x^6 + 130*b*c*x^4 + 117*b^2*x^2)*sqrt(x)
```

Sympy [A] time = 18.6524, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**(5/2),x)
```

```
[Out] 2*b**2*x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.268494, size = 32, normalized size = 0.89

$$\frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2)
```

3.307 $\int \frac{(bx^2+cx^4)^2}{x^{7/2}} dx$

Optimal. Leaf size=36

$$\frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rubi [A] time = 0.0341358, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{3}b^2x^{3/2} + \frac{4}{7}bcx^{7/2} + \frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*b^2*x^(3/2))/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rubi in Sympy [A] time = 6.69232, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

Mathematica [A] time = 0.0116151, size = 30, normalized size = 0.83

$$\frac{2}{231}x^{3/2}\left(77b^2 + 66bcx^2 + 21c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^2/x^(7/2),x]

[Out] (2*x^(3/2)*(77*b^2 + 66*b*c*x^2 + 21*c^2*x^4))/231
```

Maple [A] time = 0.008, size = 27, normalized size = 0.8

$$\frac{42\,c^{2}x^{4}+132\,bc x^{2}+154\,b^{2}}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^2/x^(7/2),x)

[Out] 2/231*x^(3/2)*(21*c^2*x^4+66*b*c*x^2+77*b^2)
```

Maxima [A] time = 0.720246, size = 32, normalized size = 0.89

$$\frac{2}{11}\,c^{2}x^{\frac{11}{2}}+\frac{4}{7}\,bc x^{\frac{7}{2}}+\frac{2}{3}\,b^{2}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(7/2),x, algorithm="maxima")

[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

Fricas [A] time = 0.26874, size = 36, normalized size = 1.

$$\frac{2}{231}\left(21\,c^{2}x^{5}+66\,bc x^{3}+77\,b^{2}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(7/2),x, algorithm="fricas")

[Out] 2/231*(21*c^2*x^5 + 66*b*c*x^3 + 77*b^2*x)*sqrt(x)
```

Sympy [A] time = 26.2046, size = 34, normalized size = 0.94

$$\frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**2/x**(7/2),x)
```

```
[Out] 2*b**2*x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.267801, size = 32, normalized size = 0.89

$$\frac{2}{11}c^2x^{\frac{11}{2}} + \frac{4}{7}bcx^{\frac{7}{2}} + \frac{2}{3}b^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2)
```

3.308 $\int x^{7/2} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/
29 + (2*c^3*x^(33/2))/33
```

Rubi [A] time = 0.0474317, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/
29 + (2*c^3*x^(33/2))/33
```

Rubi in Sympy [A] time = 8.19082, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{21}{2}}}{21}+\frac{6b^2cx^{\frac{25}{2}}}{25}+\frac{6bc^2x^{\frac{29}{2}}}{29}+\frac{2c^3x^{\frac{33}{2}}}{33}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(21/2)/21 + 6*b**2*c*x**(25/2)/25 + 6*b*c**2*x**(29/2)/
29 + 2*c**3*x**(33/2)/33
```

Mathematica [A] time = 0.0172375, size = 51, normalized size = 1.

$$\frac{2}{21}b^3x^{21/2}+\frac{6}{25}b^2cx^{25/2}+\frac{6}{29}bc^2x^{29/2}+\frac{2}{33}c^3x^{33/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(21/2))/21 + (6*b^2*c*x^(25/2))/25 + (6*b*c^2*x^(29/2))/
29 + (2*c^3*x^(33/2))/33
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$\frac{10150\,c^3x^6+34650\,bc^2x^4+40194\,b^2cx^2+15950\,b^3}{167475}x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/167475*x^(21/2)*(5075*c^3*x^6+17325*b*c^2*x^4+20097*b^2*c*x^2+7
975*b^3)
```

Maxima [A] time = 0.709979, size = 47, normalized size = 0.92

$$\frac{2}{33}\,c^3x^{\frac{33}{2}}+\frac{6}{29}\,bc^2x^{\frac{29}{2}}+\frac{6}{25}\,b^2cx^{\frac{25}{2}}+\frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2
/21*b^3*x^(21/2)
```

Fricas [A] time = 0.270673, size = 54, normalized size = 1.06

$$\frac{2}{167475}\left(5075\,c^3x^{16}+17325\,bc^2x^{14}+20097\,b^2cx^{12}+7975\,b^3x^{10}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/167475*(5075*c^3*x^16 + 17325*b*c^2*x^14 + 20097*b^2*c*x^12 + 7
975*b^3*x^10)*sqrt(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26906, size = 47, normalized size = 0.92

$$\frac{2}{33}\,c^3x^{\frac{33}{2}} + \frac{6}{29}\,bc^2x^{\frac{29}{2}} + \frac{6}{25}\,b^2cx^{\frac{25}{2}} + \frac{2}{21}\,b^3x^{\frac{21}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(7/2),x, algorithm="giac")
```

```
[Out] 2/33*c^3*x^(33/2) + 6/29*b*c^2*x^(29/2) + 6/25*b^2*c*x^(25/2) + 2
/21*b^3*x^(21/2)
```

3.309 $\int x^{5/2} \left(bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{19}b^3x^{19/2} + \frac{6}{23}b^2cx^{23/2} + \frac{2}{9}bc^2x^{27/2} + \frac{2}{31}c^3x^{31/2}$$

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/
9 + (2*c^3*x^(31/2))/31
```

Rubi [A] time = 0.0447688, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{19}b^3x^{19/2} + \frac{6}{23}b^2cx^{23/2} + \frac{2}{9}bc^2x^{27/2} + \frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/
9 + (2*c^3*x^(31/2))/31
```

Rubi in Sympy [A] time = 8.02933, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{19}{2}}}{19} + \frac{6b^2cx^{\frac{23}{2}}}{23} + \frac{2bc^2x^{\frac{27}{2}}}{9} + \frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/
9 + 2*c**3*x**(31/2)/31
```

Mathematica [A] time = 0.0166241, size = 51, normalized size = 1.

$$\frac{2}{19}b^3x^{19/2} + \frac{6}{23}b^2cx^{23/2} + \frac{2}{9}bc^2x^{27/2} + \frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(19/2))/19 + (6*b^2*c*x^(23/2))/23 + (2*b*c^2*x^(27/2))/
9 + (2*c^3*x^(31/2))/31
```

Maple [A] time = 0.008, size = 38, normalized size = 0.8

$$\frac{7866\,c^3x^6+27094\,bc^2x^4+31806\,b^2cx^2+12834\,b^3}{121923}x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/121923*x^(19/2)*(3933*c^3*x^6+13547*b*c^2*x^4+15903*b^2*c*x^2+6
417*b^3)
```

Maxima [A] time = 0.681329, size = 47, normalized size = 0.92

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/
19*b^3*x^(19/2)
```

Fricas [A] time = 0.272167, size = 54, normalized size = 1.06

$$\frac{2}{121923}\left(3933\,c^3x^{15}+13547\,bc^2x^{13}+15903\,b^2cx^{11}+6417\,b^3x^9\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/121923*(3933*c^3*x^15 + 13547*b*c^2*x^13 + 15903*b^2*c*x^11 + 6
417*b^3*x^9)*sqrt(x)
```

Sympy [A] time = 175.417, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{19}{2}}}{19}+\frac{6b^2cx^{\frac{23}{2}}}{23}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x**(27/2)/
9 + 2*c**3*x**(31/2)/31
```

GIAC/XCAS [A] time = 0.26822, size = 47, normalized size = 0.92

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\,b^2cx^{\frac{23}{2}}+\frac{2}{19}\,b^3x^{\frac{19}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 2/
19*b^3*x^(19/2)
```

3.310 $\int x^{3/2} \left(bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{17}b^3x^{17/2} + \frac{2}{7}b^2cx^{21/2} + \frac{6}{25}bc^2x^{25/2} + \frac{2}{29}c^3x^{29/2}$$

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/2
5 + (2*c^3*x^(29/2))/29
```

Rubi [A] time = 0.0446754, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{17}b^3x^{17/2} + \frac{2}{7}b^2cx^{21/2} + \frac{6}{25}bc^2x^{25/2} + \frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/2
5 + (2*c^3*x^(29/2))/29
```

Rubi in Sympy [A] time = 8.02041, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{17}{2}}}{17} + \frac{2b^2cx^{\frac{21}{2}}}{7} + \frac{6bc^2x^{\frac{25}{2}}}{25} + \frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/2
5 + 2*c**3*x**(29/2)/29
```

Mathematica [A] time = 0.0156824, size = 51, normalized size = 1.

$$\frac{2}{17}b^3x^{17/2} + \frac{2}{7}b^2cx^{21/2} + \frac{6}{25}bc^2x^{25/2} + \frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(17/2))/17 + (2*b^2*c*x^(21/2))/7 + (6*b*c^2*x^(25/2))/2
5 + (2*c^3*x^(29/2))/29
```

Maple [A] time = 0.008, size = 38, normalized size = 0.8

$$\frac{5950\,c^{3}x^{6}+20706\,bc^{2}x^{4}+24650\,b^{2}cx^{2}+10150\,b^{3}}{86275}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/86275*x^(17/2)*(2975*c^3*x^6+10353*b*c^2*x^4+12325*b^2*c*x^2+50
75*b^3)
```

Maxima [A] time = 0.679481, size = 47, normalized size = 0.92

$$\frac{2}{29}\,c^{3}x^{\frac{29}{2}}+\frac{6}{25}\,bc^{2}x^{\frac{25}{2}}+\frac{2}{7}\,b^{2}cx^{\frac{21}{2}}+\frac{2}{17}\,b^{3}x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/
17*b^3*x^(17/2)
```

Fricas [A] time = 0.269464, size = 54, normalized size = 1.06

$$\frac{2}{86275}\left(2975\,c^{3}x^{14}+10353\,bc^{2}x^{12}+12325\,b^{2}cx^{10}+5075\,b^{3}x^{8}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/86275*(2975*c^3*x^14 + 10353*b*c^2*x^12 + 12325*b^2*c*x^10 + 50
75*b^3*x^8)*sqrt(x)
```

Sympy [A] time = 113.632, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{17}{2}}}{17} + \frac{2b^2cx^{\frac{21}{2}}}{7} + \frac{6bc^2x^{\frac{25}{2}}}{25} + \frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(25/2)/2
5 + 2*c**3*x**(29/2)/29
```

GIAC/XCAS [A] time = 0.268399, size = 47, normalized size = 0.92

$$\frac{2}{29}\,c^3x^{\frac{29}{2}} + \frac{6}{25}\,bc^2x^{\frac{25}{2}} + \frac{2}{7}\,b^2cx^{\frac{21}{2}} + \frac{2}{17}\,b^3x^{\frac{17}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/
17*b^3*x^(17/2)
```

3.311 $\int \sqrt{x} \left(bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/
23 + (2*c^3*x^(27/2))/27
```

Rubi [A] time = 0.0439077, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{15}b^3x^{15/2}+\frac{6}{19}b^2cx^{19/2}+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*b^3*x^(15/2))/15 + (6*b^2*c*x^(19/2))/19 + (6*b*c^2*x^(23/2))/
23 + (2*c^3*x^(27/2))/27
```

Rubi in Sympy [A] time = 8.04849, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{15}{2}}}{15}+\frac{6b^2cx^{\frac{19}{2}}}{19}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2c^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/
23 + 2*c**3*x**(27/2)/27
```

Mathematica [A] time = 0.014284, size = 41, normalized size = 0.8

$$\frac{2x^{15/2}\left(3933b^3+9315b^2cx^2+7695bc^2x^4+2185c^3x^6\right)}{58995}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*x^(15/2)*(3933*b^3 + 9315*b^2*c*x^2 + 7695*b*c^2*x^4 + 2185*c^
3*x^6))/58995
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$\frac{4370\,c^3x^6+15390\,bc^2x^4+18630\,b^2cx^2+7866\,b^3}{58995}x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(c*x^4+b*x^2)^3,x)
```

```
[Out] 2/58995*x^(15/2)*(2185*c^3*x^6+7695*b*c^2*x^4+9315*b^2*c*x^2+3933
*b^3)
```

Maxima [A] time = 0.685767, size = 47, normalized size = 0.92

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\,b^2cx^{\frac{19}{2}}+\frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2
/15*b^3*x^(15/2)
```

Fricas [A] time = 0.268871, size = 54, normalized size = 1.06

$$\frac{2}{58995}\left(2185\,c^3x^{13}+7695\,bc^2x^{11}+9315\,b^2cx^9+3933\,b^3x^7\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/58995*(2185*c^3*x^13 + 7695*b*c^2*x^11 + 9315*b^2*c*x^9 + 3933*
b^3*x^7)*sqrt(x)
```

Sympy [A] time = 24.5034, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{15}{2}}}{15} + \frac{6b^2cx^{\frac{19}{2}}}{19} + \frac{6bc^2x^{\frac{23}{2}}}{23} + \frac{2c^3x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2)**3,x)
```

```
[Out] 2*b**3*x**(15/2)/15 + 6*b**2*c*x**(19/2)/19 + 6*b*c**2*x**(23/2)/
23 + 2*c**3*x**(27/2)/27
```

GIAC/XCAS [A] time = 0.269346, size = 47, normalized size = 0.92

$$\frac{2}{27}\,c^3x^{\frac{27}{2}} + \frac{6}{23}\,bc^2x^{\frac{23}{2}} + \frac{6}{19}\,b^2cx^{\frac{19}{2}} + \frac{2}{15}\,b^3x^{\frac{15}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 2
/15*b^3*x^(15/2)
```

3.312 $\int \frac{(bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/
7 + (2*c^3*x^(25/2))/25
```

Rubi [A] time = 0.0452104, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{13}b^3x^{13/2}+\frac{6}{17}b^2cx^{17/2}+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*b^3*x^(13/2))/13 + (6*b^2*c*x^(17/2))/17 + (2*b*c^2*x^(21/2))/
7 + (2*c^3*x^(25/2))/25
```

Rubi in Sympy [A] time = 7.95514, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{13}{2}}}{13}+\frac{6b^2cx^{\frac{17}{2}}}{17}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/
7 + 2*c**3*x**(25/2)/25
```

Mathematica [A] time = 0.0144536, size = 41, normalized size = 0.8

$$\frac{2x^{13/2}\left(2975b^3+6825b^2cx^2+5525bc^2x^4+1547c^3x^6\right)}{38675}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] (2*x^(13/2)*(2975*b^3 + 6825*b^2*c*x^2 + 5525*b*c^2*x^4 + 1547*c^
3*x^6))/38675
```

Maple [A] time = 0.008, size = 38, normalized size = 0.8

$$\frac{3094\,c^3x^6 + 11050\,bc^2x^4 + 13650\,b^2cx^2 + 5950\,b^3}{38675}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] 2/38675*x^(13/2)*(1547*c^3*x^6+5525*b*c^2*x^4+6825*b^2*c*x^2+2975
*b^3)
```

Maxima [A] time = 0.682451, size = 47, normalized size = 0.92

$$\frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/
13*b^3*x^(13/2)
```

Fricas [A] time = 0.270909, size = 54, normalized size = 1.06

$$\frac{2}{38675}\left(1547\,c^3x^{12} + 5525\,bc^2x^{10} + 6825\,b^2cx^8 + 2975\,b^3x^6\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/38675*(1547*c^3*x^12 + 5525*b*c^2*x^10 + 6825*b^2*c*x^8 + 2975*
b^3*x^6)*sqrt(x)
```

Sympy [A] time = 57.546, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{13}{2}}}{13} + \frac{6b^2cx^{\frac{17}{2}}}{17} + \frac{2bc^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)/
7 + 2*c**3*x**(25/2)/25
```

GIAC/XCAS [A] time = 0.267997, size = 47, normalized size = 0.92

$$\frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/
13*b^3*x^(13/2)
```

3.313 $\int \frac{(bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/1
9 + (2*c^3*x^(23/2))/23
```

Rubi [A] time = 0.047409, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{11}b^3x^{11/2}+\frac{2}{5}b^2cx^{15/2}+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*b^3*x^(11/2))/11 + (2*b^2*c*x^(15/2))/5 + (6*b*c^2*x^(19/2))/1
9 + (2*c^3*x^(23/2))/23
```

Rubi in Sympy [A] time = 8.00813, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2cx^{\frac{15}{2}}}{5}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/1
9 + 2*c**3*x**(23/2)/23
```

Mathematica [A] time = 0.0149064, size = 41, normalized size = 0.8

$$\frac{2x^{11/2}\left(2185b^3+4807b^2cx^2+3795bc^2x^4+1045c^3x^6\right)}{24035}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (2*x^(11/2)*(2185*b^3 + 4807*b^2*c*x^2 + 3795*b*c^2*x^4 + 1045*c^
3*x^6))/24035
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$\frac{2090\,c^3x^6 + 7590\,bc^2x^4 + 9614\,b^2cx^2 + 4370\,b^3}{24035}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^(3/2),x)
```

```
[Out] 2/24035*x^(11/2)*(1045*c^3*x^6+3795*b*c^2*x^4+4807*b^2*c*x^2+2185
*b^3)
```

Maxima [A] time = 0.683687, size = 47, normalized size = 0.92

$$\frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/
11*b^3*x^(11/2)
```

Fricas [A] time = 0.275405, size = 54, normalized size = 1.06

$$\frac{2}{24035}\left(1045\,c^3x^{11} + 3795\,bc^2x^9 + 4807\,b^2cx^7 + 2185\,b^3x^5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/24035*(1045*c^3*x^11 + 3795*b*c^2*x^9 + 4807*b^2*c*x^7 + 2185*b
^3*x^5)*sqrt(x)
```

Sympy [A] time = 62.838, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{11}{2}}}{11}+\frac{2b^2cx^{\frac{15}{2}}}{5}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**(3/2),x)
```

```
[Out] 2*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/1
9 + 2*c**3*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.267754, size = 47, normalized size = 0.92

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\,b^2cx^{\frac{15}{2}}+\frac{2}{11}\,b^3x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/
11*b^3*x^(11/2)
```

3.314 $\int \frac{(bx^2+cx^4)^3}{x^{5/2}} \, dx$

Optimal. Leaf size=51

$$\frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17
 + (2*c^3*x^(21/2))/21
```

Rubi [A] time = 0.0464129, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2}{9}b^3x^{9/2} + \frac{6}{13}b^2cx^{13/2} + \frac{6}{17}bc^2x^{17/2} + \frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*b^3*x^(9/2))/9 + (6*b^2*c*x^(13/2))/13 + (6*b*c^2*x^(17/2))/17
 + (2*c^3*x^(21/2))/21
```

Rubi in Sympy [A] time = 7.98225, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{9}{2}}}{9} + \frac{6b^2cx^{\frac{13}{2}}}{13} + \frac{6bc^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17
 + 2*c**3*x**(21/2)/21
```

Mathematica [A] time = 0.0137426, size = 41, normalized size = 0.8

$$\frac{2x^{9/2}\left(1547b^3 + 3213b^2cx^2 + 2457bc^2x^4 + 663c^3x^6\right)}{13923}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (2*x^(9/2)*(1547*b^3 + 3213*b^2*c*x^2 + 2457*b*c^2*x^4 + 663*c^3*
x^6))/13923
```

Maple [A] time = 0.008, size = 38, normalized size = 0.8

$$\frac{1326\,c^3x^6+4914\,bc^2x^4+6426\,b^2cx^2+3094\,b^3}{13923}x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^(5/2),x)
```

```
[Out] 2/13923*x^(9/2)*(663*c^3*x^6+2457*b*c^2*x^4+3213*b^2*c*x^2+1547*b
^3)
```

Maxima [A] time = 0.685531, size = 47, normalized size = 0.92

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2
/9*b^3*x^(9/2)
```

Fricas [A] time = 0.261001, size = 54, normalized size = 1.06

$$\frac{2}{13923}\left(663\,c^3x^{10}+2457\,bc^2x^8+3213\,b^2cx^6+1547\,b^3x^4\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*c^3*x^10 + 2457*b*c^2*x^8 + 3213*b^2*c*x^6 + 1547*b^
3*x^4)*sqrt(x)
```

Sympy [A] time = 72.2183, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{9}{2}}}{9} + \frac{6b^2cx^{\frac{13}{2}}}{13} + \frac{6bc^2x^{\frac{17}{2}}}{17} + \frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**(5/2),x)
```

```
[Out] 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/17
 + 2*c**3*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.2677, size = 47, normalized size = 0.92

$$\frac{2}{21}\,c^3x^{\frac{21}{2}} + \frac{6}{17}\,bc^2x^{\frac{17}{2}} + \frac{6}{13}\,b^2cx^{\frac{13}{2}} + \frac{2}{9}\,b^3x^{\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 2
/9*b^3*x^(9/2)
```

3.315 $\int \frac{(bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5
+ (2*c^3*x^(19/2))/19
```

Rubi [A] time = 0.0462983, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105

$$\frac{2}{7}b^3x^{7/2}+\frac{6}{11}b^2cx^{11/2}+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*b^3*x^(7/2))/7 + (6*b^2*c*x^(11/2))/11 + (2*b*c^2*x^(15/2))/5
+ (2*c^3*x^(19/2))/19
```

Rubi in Sympy [A] time = 8.07675, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5
+ 2*c**3*x**(19/2)/19
```

Mathematica [A] time = 0.0135756, size = 41, normalized size = 0.8

$$\frac{2x^{7/2}\left(1045b^3+1995b^2cx^2+1463bc^2x^4+385c^3x^6\right)}{7315}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (2*x^(7/2)*(1045*b^3 + 1995*b^2*c*x^2 + 1463*b*c^2*x^4 + 385*c^3*
x^6))/7315
```

Maple [A] time = 0.008, size = 38, normalized size = 0.8

$$\frac{770\,c^3x^6 + 2926\,bc^2x^4 + 3990\,b^2cx^2 + 2090\,b^3}{7315}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^3/x^(7/2),x)
```

```
[Out] 2/7315*x^(7/2)*(385*c^3*x^6+1463*b*c^2*x^4+1995*b^2*c*x^2+1045*b^
3)
```

Maxima [A] time = 0.676428, size = 47, normalized size = 0.92

$$\frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{2}{5}\,bc^2x^{\frac{15}{2}} + \frac{6}{11}\,b^2cx^{\frac{11}{2}} + \frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/
7*b^3*x^(7/2)
```

Fricas [A] time = 0.261942, size = 54, normalized size = 1.06

$$\frac{2}{7315}\left(385\,c^3x^9 + 1463\,bc^2x^7 + 1995\,b^2cx^5 + 1045\,b^3x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*c^3*x^9 + 1463*b*c^2*x^7 + 1995*b^2*c*x^5 + 1045*b^3*
x^3)*sqrt(x)
```

Sympy [A] time = 90.6278, size = 49, normalized size = 0.96

$$\frac{2b^3x^{\frac{7}{2}}}{7} + \frac{6b^2cx^{\frac{11}{2}}}{11} + \frac{2bc^2x^{\frac{15}{2}}}{5} + \frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**3/x**(7/2),x)
```

```
[Out] 2*b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5
+ 2*c**3*x**(19/2)/19
```

GIAC/XCAS [A] time = 0.266617, size = 47, normalized size = 0.92

$$\frac{2}{19}\,c^3x^{\frac{19}{2}} + \frac{2}{5}\,bc^2x^{\frac{15}{2}} + \frac{6}{11}\,b^2cx^{\frac{11}{2}} + \frac{2}{7}\,b^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 2/
7*b^3*x^(7/2)
```

3.316 $\int \frac{x^{13/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=217

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{11/4}}-\frac{2bx^{3/2}}{3c^2}+\frac{2x^{7/2}}{7c}$$

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)
) + (b^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

Rubi [A] time = 0.449725, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{11/4}}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{11/4}}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{11/4}}-\frac{2bx^{3/2}}{3c^2}+\frac{2x^{7/2}}{7c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*x^(3/2))/(3*c^2) + (2*x^(7/2))/(7*c) - (b^(7/4)*ArcTan[1 -
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)) + (b^(7/4)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(11/4)
) + (b^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt
[c]*x])/(2*Sqrt[2]*c^(11/4)) - (b^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(11/4))
```

Rubi in Sympy [A] time = 71.3276, size = 206, normalized size = 0.95

$$\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{11}{4}}}-\frac{\sqrt{2}b^{\frac{7}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{11}{4}}}-\frac{\sqrt{2}b^{\frac{7}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{11}{4}}}+\frac{\sqrt{2}b^{\frac{7}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{11}{4}}}-\frac{2bx^{\frac{3}{2}}}{3c^{2}}+\frac{2x^{\frac{7}{2}}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(2)*b**(7/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b)
 + sqrt(c)*x)/(4*c**(11/4)) - sqrt(2)*b**(7/4)*log(sqrt(2)*b**(1/
4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*c**(11/4)) - sqrt(2
)*b**(7/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*c**(11/
4)) + sqrt(2)*b**(7/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4)
)/(2*c**(11/4)) - 2*b*x**(3/2)/(3*c**2) + 2*x**(7/2)/(7*c)
```

Mathematica [A] time = 0.118326, size = 203, normalized size = 0.94

$$\frac{21\sqrt{2}b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-21\sqrt{2}b^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-42\sqrt{2}b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+42}{84c^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-56*b*c^(3/4)*x^(3/2) + 24*c^(7/4)*x^(7/2) - 42*Sqrt[2]*b^(7/4)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 42*Sqrt[2]*b^(7/4
)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 21*Sqrt[2]*b^(7
/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] -
21*Sqrt[2]*b^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x])/(84*c^(11/4))
```

Maple [A] time = 0.039, size = 158, normalized size = 0.7

$$\frac{2}{7c}x^{\frac{7}{2}}-\frac{2b}{3c^2}x^{\frac{3}{2}}+\frac{b^2\sqrt{2}}{4c^3}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$+\frac{b^2\sqrt{2}}{2c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{b^2\sqrt{2}}{2c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/7*x^(7/2)/c-2/3*b*x^(3/2)/c^2+1/4*b^2/c^3/(b/c)^(1/4)*2^(1/2)*l
n((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2)))+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*b^2/c^3/(b/c)^(1/4)*2^(1/2)*ar
ctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282712, size = 223, normalized size = 1.03

$$\frac{84\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}}{b^5\sqrt{x}+\sqrt{-b^7c^5\sqrt{-\frac{b^7}{c^{11}}}+b^{10}x}}\right)+21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}+b^5\sqrt{x}\right)-21\,c^2\left(-\frac{b^7}{c^{11}}\right)^{\frac{1}{4}}\log\left(-c^8\left(-\frac{b^7}{c^{11}}\right)^{\frac{3}{4}}\right.}{42\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/42*(84*c^2*(-b^7/c^11)^(1/4)*arctan(c^8*(-b^7/c^11)^(3/4)/(b^5*
sqrt(x) + sqrt(-b^7*c^5*sqrt(-b^7/c^11) + b^10*x))) + 21*c^2*(-b^
7/c^11)^(1/4)*log(c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) - 21*c^2*(
-b^7/c^11)^(1/4)*log(-c^8*(-b^7/c^11)^(3/4) + b^5*sqrt(x)) + 4*(3
*c*x^3 - 7*b*x)*sqrt(x))/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278023, size = 266, normalized size = 1.23

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{5}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{5}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} b \ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4c^{5}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}} b \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}} + x + \sqrt{\frac{b}{c}}\right)}{4c^{5}} + \frac{2\left(3c^{6}x^{\frac{7}{2}} - 7bc^{5}x^{\frac{3}{2}}\right)}{21c^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/c^5 + 1/2*sqrt(2)*(b*c^3)^(3/4)*b*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
c^5 - 1/4*sqrt(2)*(b*c^3)^(3/4)*b*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/c^5 + 1/4*sqrt(2)*(b*c^3)^(3/4)*b*ln(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^5 + 2/21*(3*c^6*x^(7/2) - 7*
b*c^5*x^(3/2))/c^7
```

3.317 $\int \frac{x^{11/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=215

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{9/4}}-\frac{2b\sqrt{x}}{c^2}+\frac{2x^{5/2}}{5c}$$

```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b
^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rubi [A] time = 0.40537, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}+\frac{b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{9/4}}-\frac{b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{9/4}}+\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{9/4}}-\frac{2b\sqrt{x}}{c^2}+\frac{2x^{5/2}}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*b*Sqrt[x])/c^2 + (2*x^(5/2))/(5*c) - (b^(5/4)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) + (b^(5/4)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(9/4)) - (b
^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(2*Sqrt[2]*c^(9/4)) + (b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(9/4))
```

Rubi in Sympy [A] time = 72.0698, size = 204, normalized size = 0.95

$$-\frac{\sqrt{2}b^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{9}{4}}}+\frac{\sqrt{2}b^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{9}{4}}}-\frac{\sqrt{2}b^{\frac{5}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{9}{4}}}+\frac{\sqrt{2}b^{\frac{5}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{9}{4}}}-\frac{2b\sqrt{x}}{c^2}+\frac{2x^{\frac{5}{2}}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(2)*b**(5/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b
) + sqrt(c)*x)/(4*c**(9/4)) + sqrt(2)*b**(5/4)*log(sqrt(2)*b**(1/
4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*c**(9/4)) - sqrt(2)
*b**(5/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*c**(9/4)
) + sqrt(2)*b**(5/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/
(2*c**(9/4)) - 2*b*sqrt(x)/c**2 + 2*x**(5/2)/(5*c)
```

Mathematica [A] time = 0.0731065, size = 203, normalized size = 0.94

$$\frac{-5\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+5\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-10\sqrt{2}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+10\cdots}{20c^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-40*b*c^(1/4)*Sqrt[x] + 8*c^(5/4)*x^(5/2) - 10*Sqrt[2]*b^(5/4)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 10*Sqrt[2]*b^(5/4)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 5*Sqrt[2]*b^(5/4
)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 5*
Sqrt[2]*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x])/(20*c^(9/4))
```

Maple [A] time = 0.01, size = 152, normalized size = 0.7

$$\frac{2}{5\,c}x^{\frac{5}{2}} - 2\,\frac{b\sqrt{x}}{c^{2}} + \frac{b\sqrt{2}}{4\,c^{2}}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right) + \frac{b\sqrt{2}}{2\,c^{2}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right) + \frac{b\sqrt{2}}{2\,c^{2}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/5*x^(5/2)/c-2*b*x^(1/2)/c^2+1/4*b/c^2*(b/c)^(1/4)*2^(1/2)*ln((x
+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*
2^(1/2)+(b/c)^(1/2)))+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(b/c)^(1/4)*x^(1/2)+1)+1/2*b/c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280556, size = 207, normalized size = 0.96

$$-\frac{20\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}}{b\sqrt{x}+\sqrt{c^{4}\sqrt{-\frac{b^{5}}{c^{9}}}+b^{2}x}}\right) - 5\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\log\left(c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}+b\sqrt{x}\right) + 5\,c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}\log\left(-c^{2}\left(-\frac{b^{5}}{c^{9}}\right)^{\frac{1}{4}}+b\sqrt{x}\right) -}{10\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*c^2*(-b^5/c^9)^(1/4)*arctan(c^2*(-b^5/c^9)^(1/4)/(b*sqr
t(x) + sqrt(c^4*sqrt(-b^5/c^9) + b^2*x))) - 5*c^2*(-b^5/c^9)^(1/4
)*log(c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) + 5*c^2*(-b^5/c^9)^(1/4)*
log(-c^2*(-b^5/c^9)^(1/4) + b*sqrt(x)) - 4*(c*x^2 - 5*b)*sqrt(x))
/c^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276417, size = 265, normalized size = 1.23

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} b \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} b \ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} b \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^3}+\frac{2\left(c^4x^{\frac{5}{2}}-5\,bc^3\sqrt{x}\right)}{5c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/c^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*b*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
c^3 + 1/4*sqrt(2)*(b*c^3)^(1/4)*b*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/c^3 - 1/4*sqrt(2)*(b*c^3)^(1/4)*b*ln(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 2/5*(c^4*x^(5/2) - 5*b*c
^3*sqrt(x))/c^5
```

3.318 $\int \frac{x^{9/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{7/4}}+\frac{2x^{3/2}}{3c}$$

```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)])/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
 + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(7/4))
```

Rubi [A] time = 0.351756, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{7/4}}+\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{7/4}}-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{7/4}}+\frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(b*x^2 + c*x^4),x]
```


```
[Out] (2*x^(3/2))/(3*c) + (b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)])/(Sqrt[2]*c^(7/4)) - (b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*c^(7/4)) - (b^(3/4)*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(7/4))
 + (b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[
c]*x])/(2*Sqrt[2]*c^(7/4))
```

Rubi in Sympy [A] time = 66.2651, size = 192, normalized size = 0.94

$$-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{7}{4}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{7}{4}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{7}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{7}{4}}}+\frac{2x^{\frac{3}{2}}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(2)*b**(3/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b
) + sqrt(c)*x)/(4*c**(7/4)) + sqrt(2)*b**(3/4)*log(sqrt(2)*b**(1/
4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*c**(7/4)) + sqrt(2)
*b**(3/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*c**(7/4)
) - sqrt(2)*b**(3/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/
(2*c**(7/4)) + 2*x**(3/2)/(3*c)
```

Mathematica [A] time = 0.0630479, size = 190, normalized size = 0.93

$$\frac{-3\sqrt{2}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+3\sqrt{2}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+6\sqrt{2}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-6\sqrt{2}}{12c^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (8*c^(3/4)*x^(3/2) + 6*Sqrt[2]*b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)] - 6*Sqrt[2]*b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)] - 3*Sqrt[2]*b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 3*Sqrt[2]*b^(3/4)*Log[Sqrt[b
] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(12*c^(7/4))
```

Maple [A] time = 0.009, size = 143, normalized size = 0.7

$$\frac{2}{3c}x^{\frac{3}{2}} - \frac{b\sqrt{2}}{4c^2}\ln\left(1\left(x - \sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2} + \sqrt{\frac{b}{c}}\right)\left(x + \sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2} + \sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} - \frac{b\sqrt{2}}{2c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}} + 1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} - \frac{b\sqrt{2}}{2c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}} - 1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2/3*x^(3/2)/c-1/4*b/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1
/2)))-1/2*b/c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)+1)-1/2*b/c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)
*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282619, size = 200, normalized size = 0.98

$$-\frac{12\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\arctan\left(\frac{c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}}}{b^2\sqrt{x}+\sqrt{-b^3c^3\sqrt{-\frac{b^3}{c^7}}+b^4x}}\right) + 3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}} + b^2\sqrt{x}\right) - 3\,c\left(-\frac{b^3}{c^7}\right)^{\frac{1}{4}}\log\left(-c^5\left(-\frac{b^3}{c^7}\right)^{\frac{3}{4}} + b^2\sqrt{x}\right.}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] -1/6*(12*c*(-b^3/c^7)^(1/4)*arctan(c^5*(-b^3/c^7)^(3/4)/(b^2*sqrt
(x) + sqrt(-b^3*c^3*sqrt(-b^3/c^7) + b^4*x))) + 3*c*(-b^3/c^7)^(1
/4)*log(c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) - 3*c*(-b^3/c^7)^(1/4
)*log(-c^5*(-b^3/c^7)^(3/4) + b^2*sqrt(x)) - 4*x^(3/2))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277103, size = 240, normalized size = 1.18

$$\frac{2\,x^{\frac{3}{2}}}{3\,c} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^{4}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,c^{4}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^{4}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)/c - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 1/2*sqrt(2)*(b*c
^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(
b/c)^(1/4))/c^4 + 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(-sq
rt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4
```

3.319 $\int \frac{x^{7/2}}{bx^2+cx^4} \, dx$

Optimal. Leaf size=202

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{5/4}}+\frac{2\sqrt{x}}{c}$$

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/(Sqrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (
b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(2*Sqrt[2]*c^(5/4))
```

Rubi [A] time = 0.347886, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}c^{5/4}}+\frac{\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}c^{5/4}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}c^{5/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + (b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(Sqrt[2]*c^(5/4)) - (b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/(Sqrt[2]*c^(5/4)) + (b^(1/4)*Log[Sqrt[b] - Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*c^(5/4)) - (
b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x
])/(2*Sqrt[2]*c^(5/4))
```

Rubi in Sympy [A] time = 65.4467, size = 190, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{5}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4c^{\frac{5}{4}}}+\frac{\sqrt{2}\sqrt[4]{b}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{5}{4}}}-\frac{\sqrt{2}\sqrt[4]{b}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2c^{\frac{5}{4}}}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(2)*b**(1/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b)
 + sqrt(c)*x)/(4*c**(5/4)) - sqrt(2)*b**(1/4)*log(sqrt(2)*b**(1/4
)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*c**(5/4)) + sqrt(2)*
b**(1/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*c**(5/4))
 - sqrt(2)*b**(1/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(
2*c**(5/4)) + 2*sqrt(x)/c
```

Mathematica [A] time = 0.0474746, size = 189, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+2\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-2\sqrt{2}\sqrt[4]{b}\tan}{4c^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (8*c^(1/4)*Sqrt[x] + 2*Sqrt[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)] - 2*Sqrt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)] + Sqrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Sqrt[2]*b^(1/4)*Log[Sqrt[b] +
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(4*c^(5/4))
```

Maple [A] time = 0.01, size = 140, normalized size = 0.7

$$2\frac{\sqrt{x}}{c}-\frac{\sqrt{2}}{4c}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{\sqrt{2}}{2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{\sqrt{2}}{2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] 2*x^(1/2)/c-1/4/c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2
^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-
1/2/c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1
/2/c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280655, size = 144, normalized size = 0.71

$$\frac{4c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}}{\sqrt{c^2\sqrt{-\frac{b}{c^5}}+x}+\sqrt{x}}\right)-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}\log\left(-c\left(-\frac{b}{c^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+4\sqrt{x}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*(-b/c^5)^(1/4)*arctan(c*(-b/c^5)^(1/4)/(sqrt(c^2*sqrt(-b
/c^5) + x) + sqrt(x))) - c*(-b/c^5)^(1/4)*log(c*(-b/c^5)^(1/4) +
```

```
sqrt(x)) + c*(-b/c^5)^(1/4)*log(-c*(-b/c^5)^(1/4) + sqrt(x)) + 4*
sqrt(x))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275789, size = 240, normalized size = 1.19

$$-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{2}}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2c^{2}}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{2}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4c^{2}}+\frac{2\sqrt{x}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/c^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^2
 - 1/4*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/c^2 + 1/4*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)*sqrt(x)*(
b/c)^(1/4) + x + sqrt(b/c))/c^2 + 2*sqrt(x)/c
```

3.320 $\int \frac{x^{5/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*
c^(3/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2
]*b^(1/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(3/4)
)
```

Rubi [A] time = 0.293387, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(1/4)*
c^(3/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2
]*b^(1/4)*c^(3/4)) + Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(3/4)) - Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(1/4)*c^(3/4)
)
```

Rubi in Sympy [A] time = 60.1542, size = 182, normalized size = 0.95

$$\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4\sqrt[4]{b}c^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4\sqrt[4]{b}c^{\frac{3}{4}}}-\frac{\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2\sqrt[4]{b}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c
)*x)/(4*b**(1/4)*c**(3/4)) - sqrt(2)*log(sqrt(2)*b**(1/4)*c**(1/4
)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(1/4)*c**(3/4)) - sqrt(2)*
atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(1/4)*c**(3/4))
 + sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(1/4
)*c**(3/4))
```

Mathematica [A] time = 0.0433887, size = 146, normalized size = 0.76

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}\sqrt[4]{b}c^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(1/4)*c^(3/4))
```

Maple [A] time = 0.009, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4c}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2/c/(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2/c/(b/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281669, size = 159, normalized size = 0.83

$$2\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\arctan\left(\frac{bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}}{\sqrt{-bc\sqrt{-\frac{1}{bc^3}}+x}+\sqrt{x}}\right)+\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{bc^3}\right)^{\frac{1}{4}}\log\left(-bc^2\left(-\frac{1}{bc^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 2*(-1/(b*c^3))^(1/4)*arctan(b*c^2*(-1/(b*c^3))^(3/4)/(sqrt(-b*c*s
qrt(-1/(b*c^3)) + x) + sqrt(x))) + 1/2*(-1/(b*c^3))^(1/4)*log(b*c
^2*(-1/(b*c^3))^(3/4) + sqrt(x)) - 1/2*(-1/(b*c^3))^(1/4)*log(-b*
c^2*(-1/(b*c^3))^(3/4) + sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278451, size = 246, normalized size = 1.28

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{3}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc^{3}}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^{3}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 1/2*sqrt(2)*(b*c^3)^(3/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
(b*c^3) - 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4
) + x + sqrt(b/c))/(b*c^3) + 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3)
```

3.321 $\int \frac{x^{3/2}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=192

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*
c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2
]*b^(3/4)*c^(1/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3/4)*c^(1/4)
)
```

Rubi [A] time = 0.289167, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] -(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2]*b^(3/4)*
c^(1/4))) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]/(Sqrt[2
]*b^(3/4)*c^(1/4)) - Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x
] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3/4)*c^(1/4)) + Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(2*Sqrt[2]*b^(3/4)*c^(1/4)
)
```

Rubi in Sympy [A] time = 58.7277, size = 182, normalized size = 0.95

$$-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{3}{4}}\sqrt[4]{c}}-\frac{\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{3}{4}}\sqrt[4]{c}}+\frac{\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{3}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] -sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(
c)*x)/(4*b**(3/4)*c**(1/4)) + sqrt(2)*log(sqrt(2)*b**(1/4)*c**(1/
4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(3/4)*c**(1/4)) - sqrt(2)
*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(3/4)*c**(1/4)
) + sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(3/
4)*c**(1/4))
```

Mathematica [A] time = 0.0365801, size = 146, normalized size = 0.76

$$\frac{-\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{2\sqrt{2}b^{3/4}\sqrt[4]{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(b*x^2 + c*x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - Log[Sqrt[b] - Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(3/4)*c^(1/4))
```

Maple [A] time = 0.008, size = 132, normalized size = 0.7

$$\frac{\sqrt{2}}{4b}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{2b}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{\sqrt{2}}{2b}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)
^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*(b/c)^(1
/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*(b/c)^(1/
4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281383, size = 147, normalized size = 0.77

$$-2\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\arctan\left(\frac{b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}}{\sqrt{b^2\sqrt{-\frac{1}{b^3c}}+x}+\sqrt{x}}\right)+\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)-\frac{1}{2}\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}\log\left(-b\left(-\frac{1}{b^3c}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] -2*(-1/(b^3*c))^(1/4)*arctan(b*(-1/(b^3*c))^(1/4)/(sqrt(b^2*sqrt(
-1/(b^3*c)) + x) + sqrt(x))) + 1/2*(-1/(b^3*c))^(1/4)*log(b*(-1/(
b^3*c))^(1/4) + sqrt(x)) - 1/2*(-1/(b^3*c))^(1/4)*log(-b*(-1/(b^3
*c))^(1/4) + sqrt(x))
```

Sympy [A] time = 153.089, size = 170, normalized size = 0.89

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{3cx^{\frac{3}{2}}} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } c = 0 \\ -\frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}c^{9}\left(\frac{1}{c}\right)^{\frac{35}{4}}} + \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{3}{4}}c^{9}\left(\frac{1}{c}\right)^{\frac{35}{4}}} - \frac{\sqrt[4]{-1}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{3}{4}}c^{9}\left(\frac{1}{c}\right)^{\frac{35}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(b, 0) & Eq(c, 0)), (-2/(3*c*x**(3/2))
, Eq(b, 0)), (2*sqrt(x)/b, Eq(c, 0)), (-(-1)**(1/4)*log(-(-1)**(1
/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(3/4)*c**9*(1/c)**(35/
4)) + (-1)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x)
)/(2*b**(3/4)*c**9*(1/c)**(35/4)) - (-1)**(1/4)*atan((-1)**(3/4)*
sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(3/4)*c**9*(1/c)**(35/4)), T
rue))
```

GIAC/XCAS [A] time = 0.278133, size = 246, normalized size = 1.28

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,bc} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 + 2*sqrt(x))/(b/c)^(1/4))/(b*c) + 1/2*sqrt(2)*(b*c^3)^(1/4)*arct
an(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b
*c) + 1/4*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/(b*c) - 1/4*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)*sqrt
(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c)
```

3.322 $\int \frac{\sqrt{x}}{bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{5/4}}-\frac{2}{b\sqrt{x}}$$

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) +
(c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(5/4))
```

Rubi [A] time = 0.338436, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{5/4}}+\frac{\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{5/4}}-\frac{\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{5/4}}-\frac{2}{b\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] -2/(b*Sqrt[x]) + (c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/(Sqrt[2]*b^(5/4)) - (c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*
Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(5/4)) - (c^(1/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(5/4)) +
(c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(2*Sqrt[2]*b^(5/4))
```

Rubi in Sympy [A] time = 65.8307, size = 190, normalized size = 0.94

$$-\frac{2}{b\sqrt{x}} - \frac{\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{4b^{\frac{5}{4}}} + \frac{\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right)}{4b^{\frac{5}{4}}} + \frac{\sqrt{2}\sqrt[4]{c}\,\mathrm{atan}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{5}{4}}} - \frac{\sqrt{2}\sqrt[4]{c}\,\mathrm{atan}\left(1 + \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] -2/(b*sqrt(x)) - sqrt(2)*c**(1/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*
sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(5/4)) + sqrt(2)*c**(1/4)*lo
g(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**
(5/4)) + sqrt(2)*c**(1/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1
/4))/(2*b**(5/4)) - sqrt(2)*c**(1/4)*atan(1 + sqrt(2)*c**(1/4)*sq
rt(x)/b**(1/4))/(2*b**(5/4))
```

Mathematica [A] time = 0.118246, size = 189, normalized size = 0.94

$$\frac{-\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) + \sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x} + \sqrt{b} + \sqrt{c}x\right) + 2\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right) - 2\sqrt{2}\sqrt[4]{c}\tan}{4b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(b*x^2 + c*x^4),x]
```

```
[Out] ((-8*b^(1/4))/Sqrt[x] + 2*Sqrt[2]*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)] - 2*Sqrt[2]*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*Sqrt[x])/b^(1/4)] - Sqrt[2]*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + Sqrt[2]*c^(1/4)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(4*b^(5/4))
```

Maple [A] time = 0.012, size = 140, normalized size = 0.7

$$-\frac{\sqrt{2}}{4\,b}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$-\frac{\sqrt{2}}{2\,b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{\sqrt{2}}{2\,b}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-2\,\frac{1}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/4/b/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c
)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2/b/(b/c)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2/b/(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/b/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283527, size = 180, normalized size = 0.89

$$-\frac{4\,b\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}}{c\sqrt{x}+\sqrt{-b^3c\sqrt{-\frac{c}{b^5}}+c^2x}}\right)+b\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{x}\right)-b\sqrt{x}\left(-\frac{c}{b^5}\right)^{\frac{1}{4}}\log\left(-b^4\left(-\frac{c}{b^5}\right)^{\frac{3}{4}}+c\sqrt{\right.}\right.}{2\,b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*sqrt(x)*(-c/b^5)^(1/4)*arctan(b^4*(-c/b^5)^(3/4)/(c*sqr
t(x) + sqrt(-b^3*c*sqrt(-c/b^5) + c^2*x))) + b*sqrt(x)*(-c/b^5)^(
1/4)*log(b^4*(-c/b^5)^(3/4) + c*sqrt(x)) - b*sqrt(x)*(-c/b^5)^(1/
4)*log(-b^4*(-c/b^5)^(3/4) + c*sqrt(x)) + 4)/(b*sqrt(x))
```

Sympy [A] time = 86.7168, size = 175, normalized size = 0.87

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\ b = 0 \wedge c = 0 \\ -\frac{2}{5cx^{\frac{5}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for}\ c = 0 \\ -\frac{2}{b\sqrt{x}} + \frac{(-1)^{\frac{3}{4}} \log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{5}{4}}c\left(\frac{1}{c}\right)^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}} \log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{5}{4}}c\left(\frac{1}{c}\right)^{\frac{5}{4}}} - \frac{(-1)^{\frac{3}{4}} \operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{5}{4}}c\left(\frac{1}{c}\right)^{\frac{5}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(b, 0) & Eq(c, 0)), (-2/(5*c*x**(5/2))
, Eq(b, 0)), (-2/(b*sqrt(x)), Eq(c, 0)), (-2/(b*sqrt(x)) + (-1)**
(3/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(5/
4)*c*(1/c)**(5/4)) - (-1)**(3/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**
(1/4) + sqrt(x))/(2*b**(5/4)*c*(1/c)**(5/4)) - (-1)**(3/4)*atan((
-1)**(3/4)*sqrt(x)/(b**(1/4)*(1/c)**(1/4)))/(b**(5/4)*c*(1/c)**(5
/4)), True))
```

GIAC/XCAS [A] time = 0.277707, size = 257, normalized size = 1.27

$$-\frac{2}{b\sqrt{x}} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^2c^2} + \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2} - \frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}} \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -2/(b*sqrt(x)) - 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sq
rt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) - 1/2*sqrt(
2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqr
t(x))/(b/c)^(1/4))/(b^2*c^2) + 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(
2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 1/4*sqrt(2)*(
b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^
2*c^2)
```

3.323 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=204

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{7/4}}-\frac{2}{3bx^{3/2}}$$

```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))
- (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(2*Sqrt[2]*b^(7/4))
```

Rubi [A] time = 0.340984, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\frac{c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}-\frac{c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{7/4}}+\frac{c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{7/4}}-\frac{c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{7/4}}-\frac{2}{3bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*b*x^(3/2)) + (c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)])/(Sqrt[2]*b^(7/4)) - (c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(7/4)) + (c^(3/4)*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(7/4))
- (c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(2*Sqrt[2]*b^(7/4))
```

Rubi in Sympy [A] time = 64.6819, size = 192, normalized size = 0.94

$$-\frac{2}{3bx^{\frac{3}{2}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{7}{4}}}-\frac{\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{7}{4}}}+\frac{\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{7}{4}}}-\frac{\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] -2/(3*b*x**(3/2)) + sqrt(2)*c**(3/4)*log(-sqrt(2)*b**(1/4)*c**(1/
4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(7/4)) - sqrt(2)*c**(3/4)
*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*
b**(7/4)) + sqrt(2)*c**(3/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b*
*(1/4))/(2*b**(7/4)) - sqrt(2)*c**(3/4)*atan(1 + sqrt(2)*c**(1/4)
*sqrt(x)/b**(1/4))/(2*b**(7/4))
```

Mathematica [A] time = 0.116057, size = 190, normalized size = 0.93

$$\frac{-\frac{8b^{3/4}}{x^{3/2}}+3\sqrt{2}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-3\sqrt{2}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+6\sqrt{2}c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{12b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)),x]
```

```
[Out] ((-8*b^(3/4))/x^(3/2) + 6*Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)] - 6*Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*Sqrt[x])/b^(1/4)] + 3*Sqrt[2]*c^(3/4)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 3*Sqrt[2]*c^(3/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(12*b^(7/4))
```

Maple [A] time = 0.012, size = 143, normalized size = 0.7

$$-\frac{c\sqrt{2}}{4\,b^2}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right) - \frac{c\sqrt{2}}{2\,b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right) - \frac{c\sqrt{2}}{2\,b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right) - \frac{2}{3\,b}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)/x^(1/2),x)
```

```
[Out] -1/4*c/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+
(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*c/b
^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-1/2*
c/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2
/3/b/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282166, size = 197, normalized size = 0.97

$$\frac{12\,bx^{\frac{3}{2}}\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}}{c\sqrt{x}+\sqrt{b^4\sqrt{-\frac{c^3}{b^7}}+c^2x}}\right) - 3\,bx^{\frac{3}{2}}\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{x}\right) + 3\,bx^{\frac{3}{2}}\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}\log\left(-b^2\left(-\frac{c^3}{b^7}\right)^{\frac{1}{4}}+c\sqrt{\right.}}{6\,bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(12*b*x^(3/2)*(-c^3/b^7)^(1/4)*arctan(b^2*(-c^3/b^7)^(1/4)/(c
*sqrt(x) + sqrt(b^4*sqrt(-c^3/b^7) + c^2*x))) - 3*b*x^(3/2)*(-c^3
/b^7)^(1/4)*log(b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) + 3*b*x^(3/2)*(
-c^3/b^7)^(1/4)*log(-b^2*(-c^3/b^7)^(1/4) + c*sqrt(x)) - 4)/(b*x^
(3/2))
```

Sympy [A] time = 140.023, size = 184, normalized size = 0.9

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } b = 0 \wedge c = 0 \\ -\frac{2}{7cx^{\frac{7}{2}}} & \text{for } b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } c = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} - \frac{\sqrt[4]{-1}\log\left(\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}c^{3}\left(\frac{1}{c}\right)^{\frac{15}{4}}} + \frac{\sqrt[4]{-1}\operatorname{atan}\left(\frac{(-1)^{\frac{3}{4}}\sqrt{x}}{\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}}\right)}{b^{\frac{7}{4}}c^{3}\left(\frac{1}{c}\right)^{\frac{15}{4}}} + \frac{\sqrt[4]{-1}\log\left(-\sqrt[4]{-1}\sqrt[4]{b}\sqrt[4]{\frac{1}{c}}+\sqrt{x}\right)}{2b^{\frac{7}{4}}c^{41}\left(\frac{1}{c}\right)^{\frac{167}{4}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(b, 0) & Eq(c, 0)), (-2/(7*c*x**(7/2))
, Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(c, 0)), (-2/(3*b*x**(3/2)) -
(-1)**(1/4)*log((-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b
**(7/4)*c**3*(1/c)**(15/4)) + (-1)**(1/4)*atan((-1)**(3/4)*sqrt(x
)/(b**(1/4)*(1/c)**(1/4)))/(b**(7/4)*c**3*(1/c)**(15/4)) + (-1)**
(1/4)*log(-(-1)**(1/4)*b**(1/4)*(1/c)**(1/4) + sqrt(x))/(2*b**(7/
4)*c**41*(1/c)**(167/4)), True))
```

GIAC/XCAS [A] time = 0.273638, size = 240, normalized size = 1.18

$$-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{2}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{2}} - \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^{2}} + \frac{\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^{2}} - \frac{2}{3bx^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*sqrt(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/b^2 - 1/2*sqrt(2)*(b*c^3)^(1/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^2
 - 1/4*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/b^2 + 1/4*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)*sqrt(x)*(
b/c)^(1/4) + x + sqrt(b/c))/b^2 - 2/3/(b*x^(3/2))
```

3.324 $\int \frac{1}{x^{3/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=215

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}+\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{9/4}}+\frac{2c}{b^2\sqrt{x}}-\frac{2}{5bx^{5/2}}$$

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c
^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.392114, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\frac{c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{9/4}}-\frac{c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{9/4}}+\frac{c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{9/4}}+\frac{2c}{b^2\sqrt{x}}-\frac{2}{5bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*b*x^(5/2)) + (2*c)/(b^2*Sqrt[x]) - (c^(5/4)*ArcTan[1 - (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c^(5/4)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(9/4)) + (c
^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(2*Sqrt[2]*b^(9/4)) - (c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(9/4))
```

Rubi in Sympy [A] time = 71.7935, size = 204, normalized size = 0.95

$$-\frac{2}{5bx^{\frac{5}{2}}}+\frac{2c}{b^2\sqrt{x}}+\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{9}{4}}}-\frac{\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{9}{4}}}-\frac{\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{9}{4}}}+\frac{\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] -2/(5*b*x**(5/2)) + 2*c/(b**2*sqrt(x)) + sqrt(2)*c**(5/4)*log(-sq
rt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(9/4
)) - sqrt(2)*c**(5/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqr
t(b) + sqrt(c)*x)/(4*b**(9/4)) - sqrt(2)*c**(5/4)*atan(1 - sqrt(2
)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(9/4)) + sqrt(2)*c**(5/4)*atan
(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(9/4))
```

Mathematica [A] time = 0.173897, size = 203, normalized size = 0.94

$$\frac{-\frac{8b^{5/4}}{x^{5/2}}+5\sqrt{2}c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-5\sqrt{2}c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-10\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{20b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] ((-8*b^(5/4))/x^(5/2) + (40*b^(1/4)*c)/Sqrt[x] - 10*Sqrt[2]*c^(5/
4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 10*Sqrt[2]*c^(
5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 5*Sqrt[2]*c^
(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
- 5*Sqrt[2]*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x])/(20*b^(9/4))
```

Maple [A] time = 0.015, size = 152, normalized size = 0.7

$$\frac{c\sqrt{2}}{4\,b^{2}}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} + \frac{c\sqrt{2}}{2\,b^{2}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{c\sqrt{2}}{2\,b^{2}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{2}{5\,b}x^{-\frac{5}{2}}+2\,\frac{c}{b^{2}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*c/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(
b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*c/b^
2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1/2*c
/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/
5/b/x^(5/2)+2*c/b^2/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285196, size = 230, normalized size = 1.07

$$\frac{20\,b^{2}x^{\frac{5}{2}}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{b^{7}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{3}{4}}}{c^{4}\sqrt{x}+\sqrt{-b^{5}c^{5}\sqrt{-\frac{c^{5}}{b^{9}}}+c^{8}x}}\right)+5\,b^{2}x^{\frac{5}{2}}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(b^{7}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{3}{4}}+c^{4}\sqrt{x}\right)-5\,b^{2}x^{\frac{5}{2}}\left(-\frac{c^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(-b^{7}\left(-\frac{c^{5}}{b^{9}}\right.\right.}{10\,b^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(20*b^2*x^(5/2)*(-c^5/b^9)^(1/4)*arctan(b^7*(-c^5/b^9)^(3/4)
/(c^4*sqrt(x) + sqrt(-b^5*c^5*sqrt(-c^5/b^9) + c^8*x))) + 5*b^2*x
^(5/2)*(-c^5/b^9)^(1/4)*log(b^7*(-c^5/b^9)^(3/4) + c^4*sqrt(x)) -
 5*b^2*x^(5/2)*(-c^5/b^9)^(1/4)*log(-b^7*(-c^5/b^9)^(3/4) + c^4*s
qrt(x)) + 20*c*x^2 - 4*b)/(b^2*x^(5/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275866, size = 270, normalized size = 1.26

$$\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}c}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^{3}c}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{3}c}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^{3}c}+\frac{2\left(5\,cx^{2}-b\right)}{5\,b^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(3/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 1/2*sqrt(2)*(b*c^3)^(3/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
(b^3*c) - 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4
) + x + sqrt(b/c))/(b^3*c) + 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 2/5*(5*c*x^2 - b
)/(b^2*x^(5/2))
```

3.325 $\int \frac{1}{x^{5/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=217

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{11/4}}+\frac{2c}{3b^2x^{3/2}}-\frac{2}{7bx^{7/2}}$$

```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4))
- (c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.387007, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$-\frac{c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}+\frac{c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{11/4}}-\frac{c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{11/4}}+\frac{c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{11/4}}+\frac{2c}{3b^2x^{3/2}}-\frac{2}{7bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(7*b*x^(7/2)) + (2*c)/(3*b^2*x^(3/2)) - (c^(7/4)*ArcTan[1 - (S
qrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4)) + (c^(7/4)*A
rcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2]*b^(11/4))
- (c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(2*Sqrt[2]*b^(11/4)) + (c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(11/4))
```

Rubi in Sympy [A] time = 71.9281, size = 206, normalized size = 0.95

$$-\frac{2}{7bx^{\frac{7}{2}}}+\frac{2c}{3b^2x^{\frac{3}{2}}}-\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{11}{4}}}+\frac{\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{11}{4}}}-\frac{\sqrt{2}c^{\frac{7}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{11}{4}}}+\frac{\sqrt{2}c^{\frac{7}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] -2/(7*b*x**(7/2)) + 2*c/(3*b**2*x**(3/2)) - sqrt(2)*c**(7/4)*log(
-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(
11/4)) + sqrt(2)*c**(7/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) +
 sqrt(b) + sqrt(c)*x)/(4*b**(11/4)) - sqrt(2)*c**(7/4)*atan(1 - s
qrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(11/4)) + sqrt(2)*c**(7/4
)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(11/4))
```

Mathematica [A] time = 0.0965798, size = 221, normalized size = 1.02

$$\frac{56b^{3/4}cx^2-24b^{7/4}-21\sqrt{2}c^{7/4}x^{7/2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+21\sqrt{2}c^{7/4}x^{7/2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-42\sqrt{2}}{84b^{11/4}x^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-24*b^(7/4) + 56*b^(3/4)*c*x^2 - 42*Sqrt[2]*c^(7/4)*x^(7/2)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 42*Sqrt[2]*c^(7/4)*x^
(7/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 21*Sqrt[2]*
c^(7/4)*x^(7/2)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + S
qrt[c]*x] + 21*Sqrt[2]*c^(7/4)*x^(7/2)*Log[Sqrt[b] + Sqrt[2]*b^(1
/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(84*b^(11/4)*x^(7/2))
```

Maple [A] time = 0.014, size = 158, normalized size = 0.7

$$\frac{c^2\sqrt{2}}{4\,b^3}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)$$
$$+\frac{c^2\sqrt{2}}{2\,b^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{c^2\sqrt{2}}{2\,b^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{2}{7\,b}x^{-\frac{7}{2}}+\frac{2\,c}{3\,b^2}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(c*x^4+b*x^2),x)
```

```
[Out] 1/4*c^2/b^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1/2*c^
2/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+1
/2*c^2/b^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)
-1)-2/7/b/x^(7/2)+2/3*c/b^2/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293846, size = 224, normalized size = 1.03

$$-\frac{84\,b^2x^{\frac{7}{2}}\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}}{c^2\sqrt{x}+\sqrt{b^6\sqrt{-\frac{c^7}{b^{11}}}+c^4x}}\right)-21\,b^2x^{\frac{7}{2}}\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}+c^2\sqrt{x}\right)+21\,b^2x^{\frac{7}{2}}\left(-\frac{c^7}{b^{11}}\right)^{\frac{1}{4}}\log\left(-b^3\left(\right.\right.}{42\,b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/42*(84*b^2*x^(7/2)*(-c^7/b^11)^(1/4)*arctan(b^3*(-c^7/b^11)^(1
/4)/(c^2*sqrt(x) + sqrt(b^6*sqrt(-c^7/b^11) + c^4*x))) - 21*b^2*x
^(7/2)*(-c^7/b^11)^(1/4)*log(b^3*(-c^7/b^11)^(1/4) + c^2*sqrt(x))
 + 21*b^2*x^(7/2)*(-c^7/b^11)^(1/4)*log(-b^3*(-c^7/b^11)^(1/4) +
c^2*sqrt(x)) - 28*c*x^2 + 12*b)/(b^2*x^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274811, size = 259, normalized size = 1.19

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} c \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} c \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2\,b^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} c \ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}} c \ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4\,b^3}+\frac{2\left(7\,cx^2-3\,b\right)}{21\,b^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2)*x^(5/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/b^3 + 1/2*sqrt(2)*(b*c^3)^(1/4)*c*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
b^3 + 1/4*sqrt(2)*(b*c^3)^(1/4)*c*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/b^3 - 1/4*sqrt(2)*(b*c^3)^(1/4)*c*ln(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^3 + 2/21*(7*c*x^2 - 3*b)/(b^
2*x^(7/2))
```

3.326 $\int \frac{1}{x^{7/2}(bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}-\frac{c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{13/4}}-\frac{2c^2}{b^3\sqrt{x}}+\frac{2c}{5b^2x^{5/2}}-\frac{2}{9bx^{9/2}}$$

```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x])
+ (c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2
]*b^(13/4)) - (c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(Sqrt[2]*b^(13/4)) - (c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) + (c^(9/4)*Log
[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[
2]*b^(13/4))
```

Rubi [A] time = 0.430817, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$-\frac{c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{2\sqrt{2}b^{13/4}}+\frac{c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt{2}b^{13/4}}-\frac{c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt{2}b^{13/4}}-\frac{2c^2}{b^3\sqrt{x}}+\frac{2c}{5b^2x^{5/2}}-\frac{2}{9bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] -2/(9*b*x^(9/2)) + (2*c)/(5*b^2*x^(5/2)) - (2*c^2)/(b^3*Sqrt[x])
+ (c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(Sqrt[2
]*b^(13/4)) - (c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(Sqrt[2]*b^(13/4)) - (c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[2]*b^(13/4)) + (c^(9/4)*Log
[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(2*Sqrt[
2]*b^(13/4))
```

Rubi in Sympy [A] time = 80.3726, size = 219, normalized size = 0.95

$$-\frac{2}{9bx^{\frac{9}{2}}}+\frac{2c}{5b^2x^{\frac{5}{2}}}-\frac{2c^2}{b^3\sqrt{x}}-\frac{\sqrt{2}c^{\frac{9}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{13}{4}}}+\frac{\sqrt{2}c^{\frac{9}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{4b^{\frac{13}{4}}}+\frac{\sqrt{2}c^{\frac{9}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{13}{4}}}-\frac{\sqrt{2}c^{\frac{9}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{2b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] -2/(9*b*x**(9/2)) + 2*c/(5*b**2*x**(5/2)) - 2*c**2/(b**3*sqrt(x))
 - sqrt(2)*c**(9/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt
(b) + sqrt(c)*x)/(4*b**(13/4)) + sqrt(2)*c**(9/4)*log(sqrt(2)*b**
(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(4*b**(13/4)) + sqr
t(2)*c**(9/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(2*b**(
13/4)) - sqrt(2)*c**(9/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1
/4))/(2*b**(13/4))
```

Mathematica [A] time = 0.104406, size = 234, normalized size = 1.02

$$\frac{72b^{5/4}cx^2-40b^{9/4}-45\sqrt{2}c^{9/4}x^{9/2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+45\sqrt{2}c^{9/4}x^{9/2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+90\sqrt{2}c}{180b^{13/4}x^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(b*x^2 + c*x^4)),x]
```

```
[Out] (-40*b^(9/4) + 72*b^(5/4)*c*x^2 - 360*b^(1/4)*c^2*x^4 + 90*Sqrt[2
]*c^(9/4)*x^(9/2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] -
 90*Sqrt[2]*c^(9/4)*x^(9/2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)] - 45*Sqrt[2]*c^(9/4)*x^(9/2)*Log[Sqrt[b] - Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*c^(9/4)*x^(9/2)*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(180*b^(1
3/4)*x^(9/2))
```

Maple [A] time = 0.016, size = 169, normalized size = 0.7

$$-\frac{c^{2}\sqrt{2}}{4\,b^{3}}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$-\frac{c^{2}\sqrt{2}}{2\,b^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$-\frac{c^{2}\sqrt{2}}{2\,b^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{2}{9\,b}x^{-\frac{9}{2}}-2\,\frac{c^{2}}{b^{3}\sqrt{x}}+\frac{2\,c}{5\,b^{2}}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(c*x^4+b*x^2),x)
```

```
[Out] -1/4*c^2/b^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-1/2*c
^2/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-
1/2*c^2/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)-1)-2/9/b/x^(9/2)-2*c^2/b^3/x^(1/2)+2/5*c/b^2/x^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28539, size = 244, normalized size = 1.06

$$-\frac{180\,b^{3}x^{\frac{9}{2}}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{b^{10}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{3}{4}}}{c^{7}\sqrt{x}+\sqrt{-b^{7}c^{9}\sqrt{-\frac{c^{9}}{b^{13}}}+c^{14}x}}\right)+45\,b^{3}x^{\frac{9}{2}}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}\log\left(b^{10}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{3}{4}}+c^{7}\sqrt{x}\right)-45\,b^{3}x^{\frac{9}{2}}\left(-\frac{c^{9}}{b^{13}}\right)^{\frac{1}{4}}\log\Big(}{90\,b^{3}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] -1/90*(180*b^3*x^(9/2)*(-c^9/b^13)^(1/4)*arctan(b^10*(-c^9/b^13)^
(3/4)/(c^7*sqrt(x) + sqrt(-b^7*c^9*sqrt(-c^9/b^13) + c^14*x))) +
45*b^3*x^(9/2)*(-c^9/b^13)^(1/4)*log(b^10*(-c^9/b^13)^(3/4) + c^7
*sqrt(x)) - 45*b^3*x^(9/2)*(-c^9/b^13)^(1/4)*log(-b^10*(-c^9/b^13
)^(3/4) + c^7*sqrt(x)) + 180*c^2*x^4 - 36*b*c*x^2 + 20*b^2)/(b^3*
x^(9/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(c*x**4+b*x**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276245, size = 269, normalized size = 1.17

$$-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{4}}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{2b^{4}}+\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^{4}}-\frac{\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{4b^{4}}-\frac{2\left(45c^{2}x^{4}-9bcx^{2}+5b^{2}\right)}{45b^{3}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)*x^(7/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 1/2*sqrt(2)*(b*c^3)^(3/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^4
```

```
 + 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/b^4 - 1/4*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(
b/c)^(1/4) + x + sqrt(b/c))/b^4 - 2/45*(45*c^2*x^4 - 9*b*c*x^2 +
5*b^2)/(b^3*x^(9/2))
```

3.327 $\int \frac{x^{19/2}}{(bx^2+cx^4)^2} dx$

Optimal. Leaf size=243

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{13/4}}-\frac{9b\sqrt{x}}{2c^3}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}+\frac{9x^{5/2}}{10c^2}$$

```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b +
 c*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rubi [A] time = 0.440643, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{13/4}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{13/4}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{13/4}}-\frac{9b\sqrt{x}}{2c^3}-\frac{x^{9/2}}{2c\left(b+cx^2\right)}+\frac{9x^{5/2}}{10c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```



```
[Out] (-9*b*Sqrt[x])/(2*c^3) + (9*x^(5/2))/(10*c^2) - x^(9/2)/(2*c*(b +
 c*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*c^(13/4)) + (9*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(13/4)) - (9*b^(5/4)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(13
/4)) + (9*b^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*c^(13/4))
```

Rubi in Sympy [A] time = 79.4076, size = 230, normalized size = 0.95

$$-\frac{9\sqrt{2}b^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16c^{\frac{13}{4}}}+\frac{9\sqrt{2}b^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16c^{\frac{13}{4}}}-\frac{9\sqrt{2}b^{\frac{5}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8c^{\frac{13}{4}}}+\frac{9\sqrt{2}b^{\frac{5}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8c^{\frac{13}{4}}}-\frac{9b\sqrt{x}}{2c^3}-\frac{x^{\frac{9}{2}}}{2c\left(b+cx^2\right)}+\frac{9x^{\frac{5}{2}}}{10c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -9*sqrt(2)*b**(5/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt
(b) + sqrt(c)*x)/(16*c**(13/4)) + 9*sqrt(2)*b**(5/4)*log(sqrt(2)*
b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*c**(13/4)) -
 9*sqrt(2)*b**(5/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(
8*c**(13/4)) + 9*sqrt(2)*b**(5/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(
x)/b**(1/4))/(8*c**(13/4)) - 9*b*sqrt(x)/(2*c**3) - x**(9/2)/(2*c
*(b + c*x**2)) + 9*x**(5/2)/(10*c**2)
```

Mathematica [A] time = 0.376528, size = 227, normalized size = 0.93

$$\frac{-45\sqrt{2}b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+45\sqrt{2}b^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-90\sqrt{2}b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)+9}{80c^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(19/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (-320*b*c^(1/4)*Sqrt[x] + 32*c^(5/4)*x^(5/2) - (40*b^2*c^(1/4)*Sq
rt[x])/(b + c*x^2) - 90*Sqrt[2]*b^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*b^(5/4)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*b^(5/4)*Log[Sqrt[b] - Sqrt[2]
*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[2]*b^(5/4)*Log[Sq
rt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*c^(13/4
))
```

Maple [A] time = 0.018, size = 172, normalized size = 0.7

$$\frac{2}{5\,c^2}x^{\frac{5}{2}}-4\,\frac{b\sqrt{x}}{c^3}-\frac{b^2}{2\,c^3\,(cx^2+b)}\sqrt{x}+\frac{9\,b\sqrt{2}}{16\,c^3}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{9\,b\sqrt{2}}{8\,c^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{9\,b\sqrt{2}}{8\,c^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(19/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/5*x^(5/2)/c^2-4*b*x^(1/2)/c^3-1/2/c^3*b^2*x^(1/2)/(c*x^2+b)+9/1
6/c^3*b*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/
c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+9/8/c^3*b*
(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/c^3
*b*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(19/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285359, size = 284, normalized size = 1.17

$$-\frac{180\,(c^4x^2+bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}}{b\sqrt{x}+\sqrt{c^6\sqrt{-\frac{b^5}{c^{13}}}+b^2x}}\right)-45\,(c^4x^2+bc^3)\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}\log\left(9\,c^3\left(-\frac{b^5}{c^{13}}\right)^{\frac{1}{4}}+9\,b\sqrt{x}\right)+45\,(c^4x^2+}{40\,(c^4x^2+bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(19/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/40*(180*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*arctan(c^3*(-b^5/c
^13)^(1/4)/(b*sqrt(x) + sqrt(c^6*sqrt(-b^5/c^13) + b^2*x))) - 45*
(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(9*c^3*(-b^5/c^13)^(1/4) +
 9*b*sqrt(x)) + 45*(c^4*x^2 + b*c^3)*(-b^5/c^13)^(1/4)*log(-9*c^3
*(-b^5/c^13)^(1/4) + 9*b*sqrt(x)) - 4*(4*c^2*x^4 - 36*b*c*x^2 - 4
5*b^2)*sqrt(x))/(c^4*x^2 + b*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(19/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278981, size = 292, normalized size = 1.2

$$\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{4}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8c^{4}}+\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{4}}-\frac{9\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}b\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16c^{4}}-\frac{b^{2}\sqrt{x}}{2\left(cx^{2}+b\right)c^{3}}+\frac{2\left(c^{8}x^{\frac{5}{2}}-10\,bc^{7}\sqrt{x}\right)}{5c^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(19/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 9/8*sqrt(2)*(b*c^3)^(1/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 + 9/8*sqrt(2)*(b*c^3)^(1/4)*b*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
c^4 + 9/16*sqrt(2)*(b*c^3)^(1/4)*b*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4)
 + x + sqrt(b/c))/c^4 - 9/16*sqrt(2)*(b*c^3)^(1/4)*b*ln(-sqrt(2)*
```

```
sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 - 1/2*b^2*sqrt(x)/((c*x^
2 + b)*c^3) + 2/5*(c^8*x^(5/2) - 10*b*c^7*sqrt(x))/c^10
```

3.328 $\int \frac{x^{17/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{11/4}}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}+\frac{7x^{3/2}}{6c^2}$$

```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) -
 (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*c^(11/4)) - (7*b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/4)) + (7*b^(3/4)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^
(11/4))
```

Rubi [A] time = 0.393876, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{11/4}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{11/4}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{11/4}}-\frac{x^{7/2}}{2c\left(b+cx^2\right)}+\frac{7x^{3/2}}{6c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```


```
[Out] (7*x^(3/2))/(6*c^2) - x^(7/2)/(2*c*(b + c*x^2)) + (7*b^(3/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(11/4)) -
 (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*c^(11/4)) - (7*b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(11/4)) + (7*b^(3/4)*Log[Sqrt
[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^
(11/4))
```

Rubi in Sympy [A] time = 72.9672, size = 216, normalized size = 0.94

$$-\frac{7\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16c^{\frac{11}{4}}}+\frac{7\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16c^{\frac{11}{4}}}+\frac{7\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8c^{\frac{11}{4}}}-\frac{7\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8c^{\frac{11}{4}}}-\frac{x^{\frac{7}{2}}}{2c\left(b+cx^{2}\right)}+\frac{7x^{\frac{3}{2}}}{6c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -7*sqrt(2)*b**(3/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt
(b) + sqrt(c)*x)/(16*c**(11/4)) + 7*sqrt(2)*b**(3/4)*log(sqrt(2)*
b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*c**(11/4)) +
 7*sqrt(2)*b**(3/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(
8*c**(11/4)) - 7*sqrt(2)*b**(3/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(
x)/b**(1/4))/(8*c**(11/4)) - x**(7/2)/(2*c*(b + c*x**2)) + 7*x**(
3/2)/(6*c**2)
```

Mathematica [A] time = 0.28352, size = 212, normalized size = 0.92

$$\frac{-21\sqrt{2}b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+21\sqrt{2}b^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+42\sqrt{2}b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)-4}{48c^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(17/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (32*c^(3/4)*x^(3/2) + (24*b*c^(3/4)*x^(3/2))/(b + c*x^2) + 42*Sqr
t[2]*b^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 42*S
qrt[2]*b^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 21
*Sqrt[2]*b^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
Sqrt[c]*x] + 21*Sqrt[2]*b^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(48*c^(11/4))
```

Maple [A] time = 0.018, size = 161, normalized size = 0.7

$$\frac{2}{3\,c^{2}}x^{\frac{3}{2}}+\frac{b}{2\,c^{2}\,(cx^{2}+b)}x^{\frac{3}{2}}-\frac{7\,b\sqrt{2}}{16\,c^{3}}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{7\,b\sqrt{2}}{8\,c^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{7\,b\sqrt{2}}{8\,c^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(17/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2/3*x^(3/2)/c^2+1/2*b/c^2*x^(3/2)/(c*x^2+b)-7/16*b/c^3/(b/c)^(1/4
)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-7/8*b/c^3/(b/c)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-7/8*b/c^3/(b/c)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28522, size = 288, normalized size = 1.25

$$-\frac{84\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{b^{3}}{c^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{343\,c^{8}\left(-\frac{b^{3}}{c^{11}}\right)^{\frac{3}{4}}}{343\,b^{2}\sqrt{x}+\sqrt{-117649\,b^{3}c^{5}\sqrt{-\frac{b^{3}}{c^{11}}}+117649\,b^{4}x}}\right)+21\left(c^{3}x^{2}+bc^{2}\right)\left(-\frac{b^{3}}{c^{11}}\right)^{\frac{1}{4}}\log\left(343\,c^{8}\left(-\frac{b^{3}}{c^{11}}\right)^{\frac{3}{4}}+34\right.}{24\left(c^{3}x^{2}+bc^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(84*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*arctan(343*c^8*(-b^
3/c^11)^(3/4)/(343*b^2*sqrt(x) + sqrt(-117649*b^3*c^5*sqrt(-b^3/c
^11) + 117649*b^4*x))) + 21*(c^3*x^2 + b*c^2)*(-b^3/c^11)^(1/4)*l
og(343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*sqrt(x)) - 21*(c^3*x^2 + b
*c^2)*(-b^3/c^11)^(1/4)*log(-343*c^8*(-b^3/c^11)^(3/4) + 343*b^2*
sqrt(x)) - 4*(4*c*x^3 + 7*b*x)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(17/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279171, size = 265, normalized size = 1.15

$$\frac{bx^{\frac{3}{2}}}{2\,(cx^2+b)c^2}+\frac{2\,x^{\frac{3}{2}}}{3\,c^2}-\frac{7\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}-\frac{7\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^5}+\frac{7\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}-\frac{7\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^(3/2)/((c*x^2 + b)*c^2) + 2/3*x^(3/2)/c^2 - 7/8*sqrt(2)*(
b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))
/(b/c)^(1/4))/c^5 - 7/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^5 + 7/16*sqrt(2
)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c
^5 - 7/16*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) +
```

```
x + sqrt(b/c))/c^5
```

3.329 $\int \frac{x^{15/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{9/4}}-\frac{x^{5/2}}{2c\,(b+cx^2)}+\frac{5\sqrt{x}}{2c^2}$$

```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) -
(5*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt
[2]*c^(9/4)) + (5*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*S
qrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/
4))
```

Rubi [A] time = 0.384232, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}c^{9/4}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}c^{9/4}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}c^{9/4}}-\frac{x^{5/2}}{2c\,(b+cx^2)}+\frac{5\sqrt{x}}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (5*Sqrt[x])/(2*c^2) - x^(5/2)/(2*c*(b + c*x^2)) + (5*b^(1/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*c^(9/4)) -
(5*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt
[2]*c^(9/4)) + (5*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*S
qrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/4)) - (5*b^(1/4)*Log[Sqrt[b]
 + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*c^(9/
4))
```

Rubi in Sympy [A] time = 71.6924, size = 216, normalized size = 0.94

$$\frac{5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16c^{\frac{9}{4}}}-\frac{5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16c^{\frac{9}{4}}}+\frac{5\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8c^{\frac{9}{4}}}-\frac{5\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8c^{\frac{9}{4}}}-\frac{x^{\frac{5}{2}}}{2c\left(b+cx^{2}\right)}+\frac{5\sqrt{x}}{2c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] 5*sqrt(2)*b**(1/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(
b) + sqrt(c)*x)/(16*c**(9/4)) - 5*sqrt(2)*b**(1/4)*log(sqrt(2)*b*
*(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*c**(9/4)) + 5*
sqrt(2)*b**(1/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*c
**(9/4)) - 5*sqrt(2)*b**(1/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b
**(1/4))/(8*c**(9/4)) - x**(5/2)/(2*c*(b + c*x**2)) + 5*sqrt(x)/(
2*c**2)
```

Mathematica [A] time = 0.269557, size = 212, normalized size = 0.92

$$\frac{\frac{8b\sqrt[4]{c}\sqrt{x}}{b+cx^2}+5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+10\sqrt{2}\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{16c^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(15/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] (32*c^(1/4)*Sqrt[x] + (8*b*c^(1/4)*Sqrt[x])/(b + c*x^2) + 10*Sqrt
[2]*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 10*Sq
rt[2]*b^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 5*S
qrt[2]*b^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x] - 5*Sqrt[2]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(16*c^(9/4))
```

Maple [A] time = 0.019, size = 158, normalized size = 0.7

$$2\,{\frac {\sqrt {x}}{{c}^{2}}}+{\frac {b}{2\,{c}^{2} \left(c{x}^{2}+b \right) }\sqrt {x}}-{\frac {5\,\sqrt {2}}{16\,{c}^{2}}\sqrt [4]{{\frac {b}{c}}}\ln \left(1 \left(x+\sqrt [4]{{\frac {b}{c}}}\sqrt {x}\sqrt {2}+\sqrt {{\frac {b}{c}}} \right) \left(x-\sqrt [4]{{\frac {b}{c}}}\sqrt {x}\sqrt {2}+\sqrt {{\frac {b}{c}}} \right) ^{-1} \right) }-{\frac {5\,\sqrt {2}}{8\,{c}^{2}}\sqrt [4]{{\frac {b}{c}}}\arctan \left(\sqrt {2}\sqrt {x}{\frac {1}{\sqrt [4]{{\frac {b}{c}}}}}+1 \right) }-{\frac {5\,\sqrt {2}}{8\,{c}^{2}}\sqrt [4]{{\frac {b}{c}}}\arctan \left(\sqrt {2}\sqrt {x}{\frac {1}{\sqrt [4]{{\frac {b}{c}}}}}-1 \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(15/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 2*x^(1/2)/c^2+1/2*b/c^2*x^(1/2)/(c*x^2+b)-5/16/c^2*(b/c)^(1/4)*2^
(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/
4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arct
an(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-5/8/c^2*(b/c)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281724, size = 239, normalized size = 1.04

$$\frac{20\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\arctan\left(\frac{c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}}{\sqrt{c^4\sqrt{-\frac{b}{c^9}}+x}+\sqrt{x}}\right)-5\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}\log\left(5\,c^2\left(-\frac{b}{c^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)+5\,(c^3x^2+bc^2)\left(-\frac{b}{c^9}\right)}{8\,(c^3x^2+bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(20*(c^3*x^2 + b*c^2)*(-b/c^9)^(1/4)*arctan(c^2*(-b/c^9)^(1/4
)/(sqrt(c^4*sqrt(-b/c^9) + x) + sqrt(x))) - 5*(c^3*x^2 + b*c^2)*(
-b/c^9)^(1/4)*log(5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) + 5*(c^3*x^2
+ b*c^2)*(-b/c^9)^(1/4)*log(-5*c^2*(-b/c^9)^(1/4) + 5*sqrt(x)) +
4*(4*c*x^2 + 5*b)*sqrt(x))/(c^3*x^2 + b*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278131, size = 265, normalized size = 1.15

$$-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{3}}-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,c^{3}}-\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{3}}+\frac{5\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,c^{3}}+\frac{b\sqrt{x}}{2\,(cx^{2}+b)c^{2}}+\frac{2\sqrt{x}}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -5/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/c^3 - 5/8*sqrt(2)*(b*c^3)^(1/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/c^3
 - 5/16*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
+ sqrt(b/c))/c^3 + 5/16*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)*sqrt(x)
*(b/c)^(1/4) + x + sqrt(b/c))/c^3 + 1/2*b*sqrt(x)/((c*x^2 + b)*c^
2) + 2*sqrt(x)/c^2
```

3.330 $\int \frac{x^{13/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

```
[Out] -x^(3/2)/(2*c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2
]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[
2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rubi [A] time = 0.347405, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}\sqrt[4]{b}c^{7/4}}-\frac{x^{3/2}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -x^(3/2)/(2*c*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2
]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(1/4)*c^(7/4)) + (3*Log
[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[
2]*b^(1/4)*c^(7/4)) - (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sq
rt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(1/4)*c^(7/4))
```

Rubi in Sympy [A] time = 66.2433, size = 204, normalized size = 0.94

$$-\frac{x^{\frac{3}{2}}}{2c\left(b+cx^{2}\right)}+\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16\sqrt[4]{b}c^{\frac{7}{4}}}-\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16\sqrt[4]{b}c^{\frac{7}{4}}}-\frac{3\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8\sqrt[4]{b}c^{\frac{7}{4}}}+\frac{3\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8\sqrt[4]{b}c^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -x**(3/2)/(2*c*(b + c*x**2)) + 3*sqrt(2)*log(-sqrt(2)*b**(1/4)*c*
*(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(1/4)*c**(7/4)) - 3*
sqrt(2)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)
*x)/(16*b**(1/4)*c**(7/4)) - 3*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*
sqrt(x)/b**(1/4))/(8*b**(1/4)*c**(7/4)) + 3*sqrt(2)*atan(1 + sqrt
(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b**(1/4)*c**(7/4))
```

Mathematica [A] time = 0.218743, size = 199, normalized size = 0.91

$$\frac{-\frac{8c^{3/4}x^{3/2}}{b+cx^{2}}+\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{b}}-\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{b}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{b}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{b}}}{16c^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*x^(3/2))/(b + c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2
]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(1/4) + (3*Sqrt[2]*Log[Sqrt[b]
- Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(1/4) - (3*Sqrt
[2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b
^(1/4))/(16*c^(7/4))
```

Maple [A] time = 0.016, size = 149, normalized size = 0.7

$$-\frac{1}{2\,c\,(cx^2+b)}x^{\frac{3}{2}}+\frac{3\sqrt{2}}{16\,c^2}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\sqrt{2}}{8\,c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\sqrt{2}}{8\,c^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(3/2)/c/(c*x^2+b)+3/16/c^2/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))+3/8/c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)+1)+3/8/c^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277807, size = 234, normalized size = 1.07

$$\frac{12\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\arctan\left(\frac{bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}}{\sqrt{-bc^3\sqrt{-\frac{1}{bc^7}}+x}+\sqrt{x}}\right)+3\left(c^2x^2+bc\right)\left(-\frac{1}{bc^7}\right)^{\frac{1}{4}}\log\left(bc^5\left(-\frac{1}{bc^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\left(c^2x^2+bc\right)\left(-\frac{1}{bc}\right.}{8\left(c^2x^2+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(12*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*arctan(b*c^5*(-1/(b*c^
7))^(3/4)/(sqrt(-b*c^3*sqrt(-1/(b*c^7)) + x) + sqrt(x))) + 3*(c^2
*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(b*c^5*(-1/(b*c^7))^(3/4) + sqr
t(x)) - 3*(c^2*x^2 + b*c)*(-1/(b*c^7))^(1/4)*log(-b*c^5*(-1/(b*c^
7))^(3/4) + sqrt(x)) - 4*x^(3/2))/(c^2*x^2 + b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(13/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280278, size = 269, normalized size = 1.23

$$-\frac{x^{\frac{3}{2}}}{2\,(cx^2+b)c}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^4}-\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^4}+\frac{3\,\sqrt{2}\,(bc^3)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^(3/2)/((c*x^2 + b)*c) + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1
/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^4)
 + 3/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c^4) - 3/16*sqrt(2)*(b*c^3)^(3/
4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^4) + 3/16
*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt
(b/c))/(b*c^4)
```

3.331 $\int \frac{x^{11/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{3/4}c^{5/4}}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

```
[Out] -Sqrt[x]/(2*c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c
^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rubi [A] time = 0.348331, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}+\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{3/4}c^{5/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{3/4}c^{5/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{3/4}c^{5/4}}-\frac{\sqrt{x}}{2c\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -Sqrt[x]/(2*c*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])
/b^(1/4)]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) + ArcTan[1 + (Sqrt[2]*c^(1/
4)*Sqrt[x])/b^(1/4)]/(4*Sqrt[2]*b^(3/4)*c^(5/4)) - Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(3/4)*c
^(5/4)) + Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x]/(8*Sqrt[2]*b^(3/4)*c^(5/4))
```

Rubi in Sympy [A] time = 64.9957, size = 197, normalized size = 0.9

$$-\frac{\sqrt{x}}{2c\left(b+cx^{2}\right)}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{3}{4}}c^{\frac{5}{4}}}-\frac{\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{3}{4}}c^{\frac{5}{4}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{3}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] -sqrt(x)/(2*c*(b + c*x**2)) - sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1
/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(3/4)*c**(5/4)) + sqrt(
2)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(
16*b**(3/4)*c**(5/4)) - sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)
/b**(1/4))/(8*b**(3/4)*c**(5/4)) + sqrt(2)*atan(1 + sqrt(2)*c**(1
/4)*sqrt(x)/b**(1/4))/(8*b**(3/4)*c**(5/4))
```

Mathematica [A] time = 0.227221, size = 198, normalized size = 0.91

$$\frac{-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}-\frac{8\sqrt[4]{c}\sqrt{x}}{b+cx^{2}}}{16c^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-8*c^(1/4)*Sqrt[x])/(b + c*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2
]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/b^(3/4) - (Sqrt[2]*Log[Sqrt[b] -
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (Sqrt[2]*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/
4))/(16*c^(5/4))
```

Maple [A] time = 0.017, size = 158, normalized size = 0.7

$$-\frac{1}{2\,c\left(c x^{2}+b\right)}\sqrt{x}+\frac{\sqrt{2}}{16\,bc}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{\sqrt{2}}{8\,bc}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{\sqrt{2}}{8\,bc}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2*x^(1/2)/c/(c*x^2+b)+1/16/c*(b/c)^(1/4)/b*2^(1/2)*ln((x+(b/c)
^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2)))+1/8/c*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)+1)+1/8/c*(b/c)^(1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285606, size = 225, normalized size = 1.03

$$-\frac{4\left(c^{2}x^{2}+bc\right)\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}\arctan\left(\frac{bc\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}}{\sqrt{b^{2}c^{2}\sqrt{-\frac{1}{b^{3}c^{5}}}+x+\sqrt{x}}}\right)-\left(c^{2}x^{2}+bc\right)\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}\log\left(bc\left(-\frac{1}{b^{3}c^{5}}\right)^{\frac{1}{4}}+\sqrt{x}\right)+\left(c^{2}x^{2}+bc\right)\left(-\frac{1}{b^{3}c^{5}}\right.}{8\left(c^{2}x^{2}+bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*(c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*arctan(b*c*(-1/(b^3*
c^5))^(1/4)/(sqrt(b^2*c^2*sqrt(-1/(b^3*c^5)) + x) + sqrt(x))) - (
c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(b*c*(-1/(b^3*c^5))^(1/4)
+ sqrt(x)) + (c^2*x^2 + b*c)*(-1/(b^3*c^5))^(1/4)*log(-b*c*(-1/(b
^3*c^5))^(1/4) + sqrt(x)) + 4*sqrt(x))/(c^2*x^2 + b*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279098, size = 269, normalized size = 1.23

$$\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,bc^2}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,bc^2}-\frac{\sqrt{x}}{2\left(cx^2+b\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 + 2*sqrt(x))/(b/c)^(1/4))/(b*c^2) + 1/8*sqrt(2)*(b*c^3)^(1/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
(b*c^2) + 1/16*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b*c^2) - 1/16*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^2) - 1/2*sqrt(x)/((
c*x^2 + b)*c)
```

3.332 $\int \frac{x^{9/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)]/(4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)]/(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^
(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rubi [A] time = 0.341862, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{5/4}c^{3/4}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{5/4}c^{3/4}}+\frac{x^{3/2}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```


```
[Out] x^(3/2)/(2*b*(b + c*x^2)) - ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/
b^(1/4)]/(4*Sqrt[2]*b^(5/4)*c^(3/4)) + ArcTan[1 + (Sqrt[2]*c^(1/4
)*Sqrt[x])/b^(1/4)]/(4*Sqrt[2]*b^(5/4)*c^(3/4)) + Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]/(8*Sqrt[2]*b^(5/4)*c^
(3/4)) - Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x]/(8*Sqrt[2]*b^(5/4)*c^(3/4))
```

Rubi in Sympy [A] time = 65.8037, size = 197, normalized size = 0.9

$$\frac{x^{\frac{3}{2}}}{2b(b+cx^2)}+\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{5}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{5}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{5}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{5}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] x**(3/2)/(2*b*(b + c*x**2)) + sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1
/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(5/4)*c**(3/4)) - sqrt(
2)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(
16*b**(5/4)*c**(3/4)) - sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)
/b**(1/4))/(8*b**(5/4)*c**(3/4)) + sqrt(2)*atan(1 + sqrt(2)*c**(1
/4)*sqrt(x)/b**(1/4))/(8*b**(5/4)*c**(3/4))
```

Mathematica [A] time = 0.264392, size = 198, normalized size = 0.91

$$\frac{\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{c^{3/4}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{c^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{c^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{c^{3/4}}+\frac{8\sqrt[4]{b}x^{3/2}}{b+cx^2}}{16b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*b^(1/4)*x^(3/2))/(b + c*x^2) - (2*Sqrt[2]*ArcTan[1 - (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(3/4) + (2*Sqrt[2]*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(3/4) + (Sqrt[2]*Log[Sqrt[b] - S
qrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(3/4) - (Sqrt[2]*L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(3/4
))/(16*b^(5/4))
```

Maple [A] time = 0.012, size = 158, normalized size = 0.7

$$\frac{1}{2\,b\,(cx^2+b)}x^{\frac{3}{2}}+\frac{\sqrt{2}}{16\,bc}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{8\,bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{\sqrt{2}}{8\,bc}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(3/2)/b/(c*x^2+b)+1/16/b/c/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2)))+1/8/b/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)+1)+1/8/b/c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284939, size = 235, normalized size = 1.08

$$\frac{4\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\arctan\left(\frac{b^4c^2\left(-\frac{1}{b^5c^3}\right)^{\frac{3}{4}}}{\sqrt{-b^3c\sqrt{-\frac{1}{b^5c^3}}+x+\sqrt{x}}}\right)+\left(bcx^2+b^2\right)\left(-\frac{1}{b^5c^3}\right)^{\frac{1}{4}}\log\left(b^4c^2\left(-\frac{1}{b^5c^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\left(bcx^2+b^2\right)\left(-\frac{1}{b^5}\cdots\right.}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*(b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*arctan(b^4*c^2*(-1/(b
^5*c^3))^(3/4)/(sqrt(-b^3*c*sqrt(-1/(b^5*c^3)) + x) + sqrt(x))) +
 (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(b^4*c^2*(-1/(b^5*c^3))^
(3/4) + sqrt(x)) - (b*c*x^2 + b^2)*(-1/(b^5*c^3))^(1/4)*log(-b^4*
c^2*(-1/(b^5*c^3))^(3/4) + sqrt(x)) + 4*x^(3/2))/(b*c*x^2 + b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280132, size = 269, normalized size = 1.23

$$\frac{x^{\frac{3}{2}}}{2\,(cx^2+b)b}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^2c^3}-\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^3}+\frac{\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^(3/2)/((c*x^2 + b)*b) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(1/
2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3
) + 1/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^3) - 1/16*sqrt(2)*(b*c^3)^
(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^3) +
 1/16*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
 sqrt(b/c))/(b^2*c^3)
```

3.333 $\int \frac{x^{7/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=218

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2
]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rubi [A] time = 0.330145, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{7/4}\sqrt[4]{c}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{7/4}\sqrt[4]{c}}+\frac{\sqrt{x}}{2b\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] Sqrt[x]/(2*b*(b + c*x^2)) - (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) + (3*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(7/4)*c^(1/4)) - (3*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2
]*b^(7/4)*c^(1/4)) + (3*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(7/4)*c^(1/4))
```

Rubi in Sympy [A] time = 63.7103, size = 204, normalized size = 0.94

$$\frac{\sqrt{x}}{2b\left(b+cx^2\right)}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{7}{4}}\sqrt[4]{c}}-\frac{3\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{7}{4}}\sqrt[4]{c}}+\frac{3\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{7}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] sqrt(x)/(2*b*(b + c*x**2)) - 3*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(
1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(7/4)*c**(1/4)) + 3*sq
rt(2)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x
)/(16*b**(7/4)*c**(1/4)) - 3*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sq
rt(x)/b**(1/4))/(8*b**(7/4)*c**(1/4)) + 3*sqrt(2)*atan(1 + sqrt(2
)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b**(7/4)*c**(1/4))
```

Mathematica [A] time = 0.225975, size = 199, normalized size = 0.91

$$\frac{\frac{8b^{3/4}\sqrt{x}}{b+cx^2}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{c}}}{16b^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((8*b^(3/4)*Sqrt[x])/(b + c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/c^(1/4) - (3*Sqrt[2]*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + (3*Sqrt[
2]*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^
(1/4))/(16*b^(7/4))
```

Maple [A] time = 0.012, size = 149, normalized size = 0.7

$$\frac{1}{2b(cx^2+b)}\sqrt{x}+\frac{3\sqrt{2}}{16b^2}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)$$
$$+\frac{3\sqrt{2}}{8b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{3\sqrt{2}}{8b^2}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2*x^(1/2)/b/(c*x^2+b)+3/16/b^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^
(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)
+(b/c)^(1/2)))+3/8/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(
1/4)*x^(1/2)+1)+3/8/b^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283612, size = 221, normalized size = 1.01

$$-\frac{12\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}}{\sqrt{b^4\sqrt{-\frac{1}{b^7c}}+x+\sqrt{x}}}\right)-3\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{1}{b^7c}\right)^{\frac{1}{4}}+\sqrt{x}\right)+3\left(bcx^2+b^2\right)\left(-\frac{1}{b^7c}\right.}{8\left(bcx^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(12*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*arctan(b^2*(-1/(b^7*c
))^(1/4)/(sqrt(b^4*sqrt(-1/(b^7*c)) + x) + sqrt(x))) - 3*(b*c*x^2
 + b^2)*(-1/(b^7*c))^(1/4)*log(b^2*(-1/(b^7*c))^(1/4) + sqrt(x))
+ 3*(b*c*x^2 + b^2)*(-1/(b^7*c))^(1/4)*log(-b^2*(-1/(b^7*c))^(1/4
) + sqrt(x)) - 4*sqrt(x))/(b*c*x^2 + b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278, size = 269, normalized size = 1.23

$$\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{2}c}+\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{2}c}+\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{2}c}-\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{2}c}+\frac{\sqrt{x}}{2\,(cx^{2}+b)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4)
 + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c) + 3/8*sqrt(2)*(b*c^3)^(1/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/
(b^2*c) + 3/16*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/
4) + x + sqrt(b/c))/(b^2*c) - 3/16*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c) + 1/2*sqrt(x)/((
c*x^2 + b)*b)
```

3.334 $\int \frac{x^{5/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{5\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{9/4}}-\frac{5}{2b^2\sqrt{x}}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) -
 (5*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(9/4)) - (5*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)) + (5*c^(1/4)*Log[Sqrt[b
] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9
/4))
```

Rubi [A] time = 0.386203, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526

$$-\frac{5\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{9/4}}-\frac{5}{2b^2\sqrt{x}}+\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -5/(2*b^2*Sqrt[x]) + 1/(2*b*Sqrt[x]*(b + c*x^2)) + (5*c^(1/4)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(9/4)) -
 (5*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqr
t[2]*b^(9/4)) - (5*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9/4)) + (5*c^(1/4)*Log[Sqrt[b
] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(9
/4))
```

Rubi in Sympy [A] time = 71.9633, size = 218, normalized size = 0.95

$$\frac{1}{2b\sqrt{x}\left(b+cx^2\right)}-\frac{5}{2b^2\sqrt{x}}-\frac{5\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{9}{4}}}+\frac{5\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{9}{4}}}+\frac{5\sqrt{2}\sqrt[4]{c}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{9}{4}}}-\frac{5\sqrt{2}\sqrt[4]{c}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*sqrt(x)*(b + c*x**2)) - 5/(2*b**2*sqrt(x)) - 5*sqrt(2)*c**
(1/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*
x)/(16*b**(9/4)) + 5*sqrt(2)*c**(1/4)*log(sqrt(2)*b**(1/4)*c**(1/
4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(9/4)) + 5*sqrt(2)*c**(1
/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b**(9/4)) - 5*
sqrt(2)*c**(1/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b
**(9/4))
```

Mathematica [A] time = 0.415363, size = 212, normalized size = 0.92

$$\frac{-\frac{8\sqrt[4]{b}cx^{3/2}}{b+cx^2}-5\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+5\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+10\sqrt{2}\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-32*b^(1/4))/Sqrt[x] - (8*b^(1/4)*c*x^(3/2))/(b + c*x^2) + 10*S
qrt[2]*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 10
*Sqrt[2]*c^(1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] -
5*Sqrt[2]*c^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x] + 5*Sqrt[2]*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(16*b^(9/4))
```

Maple [A] time = 0.02, size = 158, normalized size = 0.7

$$-\frac{c}{2\,b^2\,(cx^2+b)}x^{\frac{3}{2}}-\frac{5\sqrt{2}}{16\,b^2}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{5\sqrt{2}}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{5\sqrt{2}}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-2\,\frac{1}{b^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x^(3/2)/(c*x^2+b)-5/16/b^2/(b/c)^(1/4)*2^(1/2)*ln((x-(
b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2)))-5/8/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b
/c)^(1/4)*x^(1/2)+1)-5/8/b^2/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
b/c)^(1/4)*x^(1/2)-1)-2/b^2/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292784, size = 259, normalized size = 1.13

$$-\frac{20\,cx^2+20\left(b^2cx^2+b^3\right)\sqrt{x}\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{125\,b^7\left(-\frac{c}{b^9}\right)^{\frac{3}{4}}}{125\,c\sqrt{x}+\sqrt{-15625\,b^5c\sqrt{-\frac{c}{b^9}}+15625\,c^2x}}\right)+5\left(b^2cx^2+b^3\right)\sqrt{x}\left(-\frac{c}{b^9}\right)^{\frac{1}{4}}\log\left(125\,b^7\left(-\cdots\right.\right.}{8\left(b^2cx^2+b^3\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*c*x^2 + 20*(b^2*c*x^2 + b^3)*sqrt(x)*(-c/b^9)^(1/4)*arct
an(125*b^7*(-c/b^9)^(3/4)/(125*c*sqrt(x) + sqrt(-15625*b^5*c*sqrt
(-c/b^9) + 15625*c^2*x))) + 5*(b^2*c*x^2 + b^3)*sqrt(x)*(-c/b^9)^
(1/4)*log(125*b^7*(-c/b^9)^(3/4) + 125*c*sqrt(x)) - 5*(b^2*c*x^2
+ b^3)*sqrt(x)*(-c/b^9)^(1/4)*log(-125*b^7*(-c/b^9)^(3/4) + 125*c
*sqrt(x)) + 16*b)/((b^2*c*x^2 + b^3)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280915, size = 284, normalized size = 1.23

$$-\frac{5\,cx^2+4\,b}{2\left(cx^{\frac{5}{2}}+b\sqrt{x}\right)b^2}-\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}-\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3c^2}+\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^2}-\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*c*x^2 + 4*b)/((c*x^(5/2) + b*sqrt(x))*b^2) - 5/8*sqrt(2)*
(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x)
)/(b/c)^(1/4))/(b^3*c^2) - 5/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*
sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c^2)
+ 5/16*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
```

```
 sqrt(b/c))/(b^3*c^2) - 5/16*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sq
rt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^2)
```

3.335 $\int \frac{x^{3/2}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=230

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{11/4}}-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}(b+cx^2)}$$

```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4))
- (7*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sq
rt[2]*b^(11/4)) + (7*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b
^(11/4))
```

Rubi [A] time = 0.378972, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{7c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{11/4}}+\frac{7c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{11/4}}-\frac{7}{6b^2x^{3/2}}+\frac{1}{2bx^{3/2}(b+cx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```


```
[Out] -7/(6*b^2*x^(3/2)) + 1/(2*b*x^(3/2)*(b + c*x^2)) + (7*c^(3/4)*Arc
Tan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(11/4))
- (7*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sq
rt[2]*b^(11/4)) + (7*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(11/4)) - (7*c^(3/4)*Log[Sqr
t[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b
^(11/4))
```

Rubi in Sympy [A] time = 70.957, size = 218, normalized size = 0.95

$$\frac{1}{2bx^{\frac{3}{2}}(b+cx^2)}-\frac{7}{6b^2x^{\frac{3}{2}}}+\frac{7\sqrt{2}c^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{11}{4}}}-\frac{7\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{11}{4}}}+\frac{7\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{11}{4}}}-\frac{7\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*x**(3/2)*(b + c*x**2)) - 7/(6*b**2*x**(3/2)) + 7*sqrt(2)*c
**(3/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c
)*x)/(16*b**(11/4)) - 7*sqrt(2)*c**(3/4)*log(sqrt(2)*b**(1/4)*c**
(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(11/4)) + 7*sqrt(2)*c
**(3/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b**(11/4))
 - 7*sqrt(2)*c**(3/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))
/(8*b**(11/4))
```

Mathematica [A] time = 0.382629, size = 212, normalized size = 0.92

$$\frac{-\frac{24b^{3/4}c\sqrt{x}}{b+cx^2}-\frac{32b^{3/4}}{x^{3/2}}+21\sqrt{2}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-21\sqrt{2}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+42\sqrt{2}c^{3/4}\tan^{-}}{48b^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-32*b^(3/4))/x^(3/2) - (24*b^(3/4)*c*Sqrt[x])/(b + c*x^2) + 42*
Sqrt[2]*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 4
2*Sqrt[2]*c^(3/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] +
 21*Sqrt[2]*c^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
 + Sqrt[c]*x] - 21*Sqrt[2]*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(48*b^(11/4))
```

Maple [A] time = 0.019, size = 161, normalized size = 0.7

$$-\frac{c}{2\,b^{2}\,(cx^{2}+b)}\sqrt{x}-\frac{7\,c\sqrt{2}}{16\,b^{3}}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)$$
$$-\frac{7\,c\sqrt{2}}{8\,b^{3}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{7\,c\sqrt{2}}{8\,b^{3}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{2}{3\,b^{2}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^2*c*x^(1/2)/(c*x^2+b)-7/16/b^3*c*(b/c)^(1/4)*2^(1/2)*ln((x
+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*
2^(1/2)+(b/c)^(1/2)))-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(b/c)^(1/4)*x^(1/2)+1)-7/8/b^3*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/3/b^2/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286773, size = 282, normalized size = 1.23

$$-\frac{28\,cx^{2}-84\,\left(b^{2}cx^{3}+b^{3}x\right)\sqrt{x}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{b^{3}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}}{c\sqrt{x}+\sqrt{b^{6}\sqrt{-\frac{c^{3}}{b^{11}}}+c^{2}x}}\right)+21\,\left(b^{2}cx^{3}+b^{3}x\right)\sqrt{x}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}\log\left(7\,b^{3}\left(-\frac{c^{3}}{b^{11}}\right)^{\frac{1}{4}}+7\,c\right.}{24\,(b^{2}cx^{3}+b^{3}x)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/24*(28*c*x^2 - 84*(b^2*c*x^3 + b^3*x)*sqrt(x)*(-c^3/b^11)^(1/4
)*arctan(b^3*(-c^3/b^11)^(1/4)/(c*sqrt(x) + sqrt(b^6*sqrt(-c^3/b^
11) + c^2*x))) + 21*(b^2*c*x^3 + b^3*x)*sqrt(x)*(-c^3/b^11)^(1/4)
*log(7*b^3*(-c^3/b^11)^(1/4) + 7*c*sqrt(x)) - 21*(b^2*c*x^3 + b^3
*x)*sqrt(x)*(-c^3/b^11)^(1/4)*log(-7*b^3*(-c^3/b^11)^(1/4) + 7*c*
sqrt(x)) + 16*b)/((b^2*c*x^3 + b^3*x)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280238, size = 265, normalized size = 1.15

$$-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^3}-\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3}+\frac{7\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^3}-\frac{c\sqrt{x}}{2\left(cx^2+b\right)b^2}-\frac{2}{3\,b^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] -7/8*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/b^3 - 7/8*sqrt(2)*(b*c^3)^(1/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^3
 - 7/16*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x
+ sqrt(b/c))/b^3 + 7/16*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)*sqrt(x)
*(b/c)^(1/4) + x + sqrt(b/c))/b^3 - 1/2*c*sqrt(x)/((c*x^2 + b)*b^
2) - 2/3/(b^2*x^(3/2))
```

3.336 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=243

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{13/4}}+\frac{9c}{2b^3\sqrt{x}}-\frac{9}{10b^2x^{5/2}}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b +
 c*x^2)) - (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*b^(13/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13
/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.430819, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{9c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{13/4}}-\frac{9c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{13/4}}+\frac{9c}{2b^3\sqrt{x}}-\frac{9}{10b^2x^{5/2}}+\frac{1}{2bx^{5/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -9/(10*b^2*x^(5/2)) + (9*c)/(2*b^3*Sqrt[x]) + 1/(2*b*x^(5/2)*(b +
 c*x^2)) - (9*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(4*Sqrt[2]*b^(13/4)) + (9*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(13/4)) + (9*c^(5/4)*Log[Sqrt[b]
 - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(13
/4)) - (9*c^(5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] +
 Sqrt[c]*x])/(8*Sqrt[2]*b^(13/4))
```

Rubi in Sympy [A] time = 79.7024, size = 231, normalized size = 0.95

$$\frac{1}{2bx^{\frac{5}{2}}\left(b+cx^{2}\right)}-\frac{9}{10b^{2}x^{\frac{5}{2}}}+\frac{9c}{2b^{3}\sqrt{x}}+\frac{9\sqrt{2}c^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{13}{4}}}-\frac{9\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{13}{4}}}-\frac{9\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{13}{4}}}+\frac{9\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*x**(5/2)*(b + c*x**2)) - 9/(10*b**2*x**(5/2)) + 9*c/(2*b**
3*sqrt(x)) + 9*sqrt(2)*c**(5/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sq
rt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(13/4)) - 9*sqrt(2)*c**(5/4)*
log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*
b**(13/4)) - 9*sqrt(2)*c**(5/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)
/b**(1/4))/(8*b**(13/4)) + 9*sqrt(2)*c**(5/4)*atan(1 + sqrt(2)*c*
*(1/4)*sqrt(x)/b**(1/4))/(8*b**(13/4))
```

Mathematica [A] time = 0.405078, size = 227, normalized size = 0.93

$$\frac{-\frac{32b^{5/4}}{x^{5/2}}+45\sqrt{2}c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-45\sqrt{2}c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-90\sqrt{2}c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{b}}\right.}{80b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-32*b^(5/4))/x^(5/2) + (320*b^(1/4)*c)/Sqrt[x] + (40*b^(1/4)*c^
2*x^(3/2))/(b + c*x^2) - 90*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*c
^(1/4)*Sqrt[x])/b^(1/4)] + 90*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)] + 45*Sqrt[2]*c^(5/4)*Log[Sqrt[b] - Sqr
t[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 45*Sqrt[2]*c^(5/4)*Lo
g[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(80*b^(
13/4))
```

Maple [A] time = 0.025, size = 172, normalized size = 0.7

$$\frac{c^{2}}{2\,b^{3}\left(cx^{2}+b\right)}x^{\frac{3}{2}}+\frac{9\,c\sqrt{2}}{16\,b^{3}}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$+\frac{9\,c\sqrt{2}}{8\,b^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{9\,c\sqrt{2}}{8\,b^{3}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{2}{5\,b^{2}}x^{-\frac{5}{2}}+4\,\frac{c}{b^{3}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] 1/2/b^3*c^2*x^(3/2)/(c*x^2+b)+9/16/b^3*c/(b/c)^(1/4)*2^(1/2)*ln((
x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2)))+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(b/c)^(1/4)*x^(1/2)+1)+9/8/b^3*c/(b/c)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/5/b^2/x^(5/2)+4*c/b^3/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284435, size = 325, normalized size = 1.34

$$\frac{180\,c^{2}x^{4}+144\,bcx^{2}+180\left(b^{3}cx^{4}+b^{4}x^{2}\right)\sqrt{x}\left(-\frac{c^{5}}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{729\,b^{10}\left(-\frac{c^{5}}{b^{13}}\right)^{\frac{3}{4}}}{729\,c^{4}\sqrt{x}+\sqrt{-531441\,b^{7}c^{5}\sqrt{-\frac{c^{5}}{b^{13}}}+531441\,c^{8}x}}\right)+45\left(b^{3}cx^{4}+b^{4}x^{2}\right)\sqrt{x}}{40\left(b^{3}cx^{4}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/40*(180*c^2*x^4 + 144*b*c*x^2 + 180*(b^3*c*x^4 + b^4*x^2)*sqrt(
x)*(-c^5/b^13)^(1/4)*arctan(729*b^10*(-c^5/b^13)^(3/4)/(729*c^4*s
qrt(x) + sqrt(-531441*b^7*c^5*sqrt(-c^5/b^13) + 531441*c^8*x))) +
 45*(b^3*c*x^4 + b^4*x^2)*sqrt(x)*(-c^5/b^13)^(1/4)*log(729*b^10*
(-c^5/b^13)^(3/4) + 729*c^4*sqrt(x)) - 45*(b^3*c*x^4 + b^4*x^2)*s
qrt(x)*(-c^5/b^13)^(1/4)*log(-729*b^10*(-c^5/b^13)^(3/4) + 729*c^
4*sqrt(x)) - 16*b^2)/((b^3*c*x^4 + b^4*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280156, size = 297, normalized size = 1.22

$$\frac{c^2x^{\frac{3}{2}}}{2\,(cx^2+b)b^3}+\frac{9\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c}+\frac{9\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^4c}-\frac{9\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c}+\frac{9\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^4c}+\frac{2\left(10\,cx^2-b\right)}{5\,b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^(3/2)/((c*x^2 + b)*b^3) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arc
tan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b
^4*c) + 9/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b
/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^4*c) - 9/16*sqrt(2)*(b*c^3
)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c) +
```

```
9/16*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x +
sqrt(b/c))/(b^4*c) + 2/5*(10*c*x^2 - b)/(b^3*x^(5/2))
```

3.337 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=243

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}+\frac{11c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{15/4}}+\frac{11c}{6b^3x^{3/2}}-\frac{11}{14b^2x^{7/2}}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b
 + c*x^2)) - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(4*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (11*c^(7/4)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*
b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rubi [A] time = 0.433978, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{11c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}+\frac{11c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{15/4}}-\frac{11c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{15/4}}+\frac{11c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{15/4}}+\frac{11c}{6b^3x^{3/2}}-\frac{11}{14b^2x^{7/2}}+\frac{1}{2bx^{7/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -11/(14*b^2*x^(7/2)) + (11*c)/(6*b^3*x^(3/2)) + 1/(2*b*x^(7/2)*(b
 + c*x^2)) - (11*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(
1/4)])/(4*Sqrt[2]*b^(15/4)) + (11*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(
1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(15/4)) - (11*c^(7/4)*Log[Sq
rt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*
b^(15/4)) + (11*c^(7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqr
t[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(15/4))
```

Rubi in Sympy [A] time = 78.5541, size = 231, normalized size = 0.95

$$\frac{1}{2bx^{\frac{7}{2}}\left(b+cx^2\right)}-\frac{11}{14b^2x^{\frac{7}{2}}}+\frac{11c}{6b^3x^{\frac{3}{2}}}-\frac{11\sqrt{2}c^{\frac{7}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{15}{4}}}+\frac{11\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{15}{4}}}-\frac{11\sqrt{2}c^{\frac{7}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{15}{4}}}+\frac{11\sqrt{2}c^{\frac{7}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] 1/(2*b*x**(7/2)*(b + c*x**2)) - 11/(14*b**2*x**(7/2)) + 11*c/(6*b
**3*x**(3/2)) - 11*sqrt(2)*c**(7/4)*log(-sqrt(2)*b**(1/4)*c**(1/4
)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(15/4)) + 11*sqrt(2)*c**(
7/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)
/(16*b**(15/4)) - 11*sqrt(2)*c**(7/4)*atan(1 - sqrt(2)*c**(1/4)*s
qrt(x)/b**(1/4))/(8*b**(15/4)) + 11*sqrt(2)*c**(7/4)*atan(1 + sqr
t(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b**(15/4))
```

Mathematica [A] time = 0.429004, size = 227, normalized size = 0.93

$$\frac{\frac{168b^{3/4}c^2\sqrt{x}}{b+cx^2}+\frac{448b^{3/4}c}{x^{3/2}}-\frac{96b^{7/4}}{x^{7/2}}-231\sqrt{2}c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+231\sqrt{2}c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-4}{336b^{15/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-96*b^(7/4))/x^(7/2) + (448*b^(3/4)*c)/x^(3/2) + (168*b^(3/4)*c
^2*Sqrt[x])/(b + c*x^2) - 462*Sqrt[2]*c^(7/4)*ArcTan[1 - (Sqrt[2]
*c^(1/4)*Sqrt[x])/b^(1/4)] + 462*Sqrt[2]*c^(7/4)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 231*Sqrt[2]*c^(7/4)*Log[Sqrt[b] -
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 231*Sqrt[2]*c^(7/
4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(3
36*b^(15/4))
```

Maple [A] time = 0.022, size = 178, normalized size = 0.7

$$\frac{c^2}{2\,b^3\,(cx^2+b)}\sqrt{x}+\frac{11\,c^2\sqrt{2}}{16\,b^4}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)$$
$$+\frac{11\,c^2\sqrt{2}}{8\,b^4}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{11\,c^2\sqrt{2}}{8\,b^4}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{2}{7\,b^2}x^{-\frac{7}{2}}+\frac{4\,c}{3\,b^3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^2/x^(1/2),x)
```

```
[Out] 1/2/b^3*c^2*x^(1/2)/(c*x^2+b)+11/16/b^4*c^2*(b/c)^(1/4)*2^(1/2)*l
n((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1
/2)*2^(1/2)+(b/c)^(1/2)))+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*arctan
(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+11/8/b^4*c^2*(b/c)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/7/b^2/x^(7/2)+4/3*c/b^3/x
^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289265, size = 316, normalized size = 1.3

$$\frac{308\,c^2x^4+176\,bcx^2-924\left(b^3cx^5+b^4x^3\right)\sqrt{x}\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{b^4\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}}{c^2\sqrt{x}+\sqrt{b^8\sqrt{-\frac{c^7}{b^{15}}}+c^4x}}\right)+231\left(b^3cx^5+b^4x^3\right)\sqrt{x}\left(-\frac{c^7}{b^{15}}\right)^{\frac{1}{4}}\log\left(1\right.}{168\left(b^3cx^5+b^4x^3\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/168*(308*c^2*x^4 + 176*b*c*x^2 - 924*(b^3*c*x^5 + b^4*x^3)*sqrt
(x)*(-c^7/b^15)^(1/4)*arctan(b^4*(-c^7/b^15)^(1/4)/(c^2*sqrt(x) +
 sqrt(b^8*sqrt(-c^7/b^15) + c^4*x))) + 231*(b^3*c*x^5 + b^4*x^3)*
sqrt(x)*(-c^7/b^15)^(1/4)*log(11*b^4*(-c^7/b^15)^(1/4) + 11*c^2*s
qrt(x)) - 231*(b^3*c*x^5 + b^4*x^3)*sqrt(x)*(-c^7/b^15)^(1/4)*log
(-11*b^4*(-c^7/b^15)^(1/4) + 11*c^2*sqrt(x)) - 48*b^2)/((b^3*c*x^
5 + b^4*x^3)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**2/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276142, size = 286, normalized size = 1.18

$$\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}}+\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{4}}+\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{4}}-\frac{11\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{4}}+\frac{c^{2}\sqrt{x}}{2\left(cx^{2}+b\right)b^{3}}+\frac{2\left(14\,cx^{2}-3\,b\right)}{21\,b^{3}x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*sqrt(x)),x, algorithm="giac")
```

```
[Out] 11/8*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 + 11/8*sqrt(2)*(b*c^3)^(1/4)*c*
arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/b^4 + 11/16*sqrt(2)*(b*c^3)^(1/4)*c*ln(sqrt(2)*sqrt(x)*(b/c)^(1
/4) + x + sqrt(b/c))/b^4 - 11/16*sqrt(2)*(b*c^3)^(1/4)*c*ln(-sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 + 1/2*c^2*sqrt(x)/((
c*x^2 + b)*b^3) + 2/21*(14*c*x^2 - 3*b)/(b^3*x^(7/2))
```

3.338 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=258

$$-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}-\frac{13c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{17/4}}-\frac{13c^2}{2b^4\sqrt{x}}+\frac{13c}{10b^3x^{5/2}}-\frac{13}{18b^2x^{9/2}}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*
Sqrt[x]) + 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(
9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4)*Log[Sqrt[b] +
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4
))
```

Rubi [A] time = 0.488015, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526

$$-\frac{13c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{8\sqrt{2}b^{17/4}}+\frac{13c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{4\sqrt{2}b^{17/4}}-\frac{13c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{4\sqrt{2}b^{17/4}}-\frac{13c^2}{2b^4\sqrt{x}}+\frac{13c}{10b^3x^{5/2}}-\frac{13}{18b^2x^{9/2}}+\frac{1}{2bx^{9/2}\left(b+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] -13/(18*b^2*x^(9/2)) + (13*c)/(10*b^3*x^(5/2)) - (13*c^2)/(2*b^4*
Sqrt[x]) + 1/(2*b*x^(9/2)*(b + c*x^2)) + (13*c^(9/4)*ArcTan[1 - (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^(17/4)) - (13*c^(
9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(4*Sqrt[2]*b^
(17/4)) - (13*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4)) + (13*c^(9/4)*Log[Sqrt[b] +
```

```
 Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(8*Sqrt[2]*b^(17/4
))
```

Rubi in Sympy [A] time = 86.9627, size = 246, normalized size = 0.95

$$\frac{1}{2bx^{\frac{9}{2}}(b+cx^2)} - \frac{13}{18b^2x^{\frac{9}{2}}} + \frac{13c}{10b^3x^{\frac{5}{2}}} - \frac{13c^2}{2b^4\sqrt{x}} - \frac{13\sqrt{2}c^{\frac{9}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{17}{4}}} + \frac{13\sqrt{2}c^{\frac{9}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{16b^{\frac{17}{4}}} + \frac{13\sqrt{2}c^{\frac{9}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{17}{4}}} - \frac{13\sqrt{2}c^{\frac{9}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{8b^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] 1/(2*b*x**(9/2)*(b + c*x**2)) - 13/(18*b**2*x**(9/2)) + 13*c/(10*
b**3*x**(5/2)) - 13*c**2/(2*b**4*sqrt(x)) - 13*sqrt(2)*c**(9/4)*l
og(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(16*
b**(17/4)) + 13*sqrt(2)*c**(9/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sq
rt(x) + sqrt(b) + sqrt(c)*x)/(16*b**(17/4)) + 13*sqrt(2)*c**(9/4)
*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b**(17/4)) - 13*s
qrt(2)*c**(9/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(8*b*
*(17/4))
```

Mathematica [A] time = 0.487296, size = 242, normalized size = 0.94

$$\frac{\frac{576b^{5/4}c}{x^{5/2}} - \frac{160b^{9/4}}{x^{9/2}} - 585\sqrt{2}c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right) + 585\sqrt{2}c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right) + 1170\sqrt{2}c^{9/4}\text{ta}}{720b^{17/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-160*b^(9/4))/x^(9/2) + (576*b^(5/4)*c)/x^(5/2) - (4320*b^(1/4)
*c^2)/Sqrt[x] - (360*b^(1/4)*c^3*x^(3/2))/(b + c*x^2) + 1170*Sqrt
[2]*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 1170*
Sqrt[2]*c^(9/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 5
85*Sqrt[2]*c^(9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x]
+ Sqrt[c]*x] + 585*Sqrt[2]*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(720*b^(17/4))
```

Maple [A] time = 0.024, size = 189, normalized size = 0.7

$$-\frac{c^3}{2\,b^4\,(cx^2+b)}x^{\frac{3}{2}}-\frac{13\,c^2\sqrt{2}}{16\,b^4}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$-\frac{13\,c^2\sqrt{2}}{8\,b^4}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$
$$-\frac{13\,c^2\sqrt{2}}{8\,b^4}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{2}{9\,b^2}x^{-\frac{9}{2}}-6\,\frac{c^2}{b^4\sqrt{x}}+\frac{4\,c}{5\,b^3}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^4+b*x^2)^2,x)
```

```
[Out] -1/2/b^4*c^3*x^(3/2)/(c*x^2+b)-13/16/b^4*c^2/(b/c)^(1/4)*2^(1/2)*
ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(
1/2)*2^(1/2)+(b/c)^(1/2)))-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-13/8/b^4*c^2/(b/c)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)-2/9/b^2/x^(9/2)-6*c^2/b^4/
x^(1/2)+4/5*c/b^3/x^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288778, size = 340, normalized size = 1.32

$$-\frac{2340\,c^3x^6+1872\,bc^2x^4-208\,b^2cx^2+80\,b^3+2340\,\left(b^4cx^6+b^5x^4\right)\sqrt{x}\left(-\frac{c^9}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{2197\,b^{13}\left(-\frac{c^9}{b^{17}}\right)^{\frac{3}{4}}}{2197\,c^7\sqrt{x}+\sqrt{-4826809\,b^9c^9\sqrt{-\frac{c^9}{b^{17}}}+4826809}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/360*(2340*c^3*x^6 + 1872*b*c^2*x^4 - 208*b^2*c*x^2 + 80*b^3 +
2340*(b^4*c*x^6 + b^5*x^4)*sqrt(x)*(-c^9/b^17)^(1/4)*arctan(2197*
b^13*(-c^9/b^17)^(3/4)/(2197*c^7*sqrt(x) + sqrt(-4826809*b^9*c^9*
sqrt(-c^9/b^17) + 4826809*c^14*x))) + 585*(b^4*c*x^6 + b^5*x^4)*s
qrt(x)*(-c^9/b^17)^(1/4)*log(2197*b^13*(-c^9/b^17)^(3/4) + 2197*c
^7*sqrt(x)) - 585*(b^4*c*x^6 + b^5*x^4)*sqrt(x)*(-c^9/b^17)^(1/4)
*log(-2197*b^13*(-c^9/b^17)^(3/4) + 2197*c^7*sqrt(x)))/((b^4*c*x^
6 + b^5*x^4)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(c*x**4+b*x**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278379, size = 296, normalized size = 1.15

$$-\frac{c^{3}x^{\frac{3}{2}}}{2\,(cx^{2}+b)b^{4}}-\frac{13\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{5}}-\frac{13\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{8\,b^{5}}+\frac{13\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{5}}-\frac{13\,\sqrt{2}\,\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{16\,b^{5}}-\frac{2\,\left(135\,c^{2}x^{4}-18\,bcx^{2}+5\,b^{2}\right)}{45\,b^{4}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^2*x^(3/2)),x, algorithm="giac")
```

```
[Out] -1/2*c^3*x^(3/2)/((c*x^2 + b)*b^4) - 13/8*sqrt(2)*(b*c^3)^(3/4)*a
rctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/
b^5 - 13/8*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/
c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/b^5 + 13/16*sqrt(2)*(b*c^3)^(3
/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 - 13/16*s
qrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b
/c))/b^5 - 2/45*(135*c^2*x^4 - 18*b*c*x^2 + 5*b^2)/(b^4*x^(9/2))
```

3.339 $\int \frac{x^{23/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}c^{13/4}}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}+\frac{45\sqrt{x}}{16c^3}$$

```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))
/(16*c^2*(b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b
^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```

Rubi [A] time = 0.438835, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526

$$\frac{45\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}c^{13/4}}+\frac{45\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}c^{13/4}}-\frac{45\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}c^{13/4}}-\frac{9x^{5/2}}{16c^2\left(b+cx^2\right)}-\frac{x^{9/2}}{4c\left(b+cx^2\right)^2}+\frac{45\sqrt{x}}{16c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (45*Sqrt[x])/(16*c^3) - x^(9/2)/(4*c*(b + c*x^2)^2) - (9*x^(5/2))
/(16*c^2*(b + c*x^2)) + (45*b^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*c^(13/4)) + (45*b
^(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(64*Sqrt[2]*c^(13/4)) - (45*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*c^(13/4))
```

Rubi in Sympy [A] time = 79.5493, size = 236, normalized size = 0.94

$$\frac{45\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128c^{\frac{13}{4}}}-\frac{45\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128c^{\frac{13}{4}}}+\frac{45\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64c^{\frac{13}{4}}}-\frac{45\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64c^{\frac{13}{4}}}-\frac{x^{\frac{9}{2}}}{4c(b+cx^2)^2}-\frac{9x^{\frac{5}{2}}}{16c^2(b+cx^2)}+\frac{45\sqrt{x}}{16c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 45*sqrt(2)*b**(1/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt
(b) + sqrt(c)*x)/(128*c**(13/4)) - 45*sqrt(2)*b**(1/4)*log(sqrt(2
)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*c**(13/4)
) + 45*sqrt(2)*b**(1/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4
))/(64*c**(13/4)) - 45*sqrt(2)*b**(1/4)*atan(1 + sqrt(2)*c**(1/4)
*sqrt(x)/b**(1/4))/(64*c**(13/4)) - x**(9/2)/(4*c*(b + c*x**2)**2
) - 9*x**(5/2)/(16*c**2*(b + c*x**2)) + 45*sqrt(x)/(16*c**3)
```

Mathematica [A] time = 0.19042, size = 236, normalized size = 0.94

$$\frac{-\frac{32b^2\sqrt[4]{c}\sqrt{x}}{(b+cx^2)^2}+\frac{136b\sqrt[4]{c}\sqrt{x}}{b+cx^2}+45\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-45\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+90\sqrt{2}\sqrt[4]{b}\tan}{128c^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(23/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] (256*c^(1/4)*Sqrt[x] - (32*b^2*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (
136*b*c^(1/4)*Sqrt[x])/(b + c*x^2) + 90*Sqrt[2]*b^(1/4)*ArcTan[1
- (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 90*Sqrt[2]*b^(1/4)*ArcTan[
1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 45*Sqrt[2]*b^(1/4)*Log[S
qrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 45*Sqrt[2
]*b^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]
*x])/(128*c^(13/4))
```

Maple [A] time = 0.024, size = 178, normalized size = 0.7

$$2\frac{\sqrt{x}}{c^3}+\frac{17\,b}{16\,c^2\left(cx^2+b\right)^2}x^{\frac{5}{2}}+\frac{13\,b^2}{16\,c^3\left(cx^2+b\right)^2}\sqrt{x}-\frac{45\sqrt{2}}{128\,c^3}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{45\sqrt{2}}{64\,c^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{45\sqrt{2}}{64\,c^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(23/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*x^(1/2)/c^3+17/16/c^2*b/(c*x^2+b)^2*x^(5/2)+13/16/c^3*b^2/(c*x^
2+b)^2*x^(1/2)-45/128/c^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x
^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^
(1/2)))-45/64/c^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)+1)-45/64/c^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/
4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(23/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285824, size = 313, normalized size = 1.25

$$\frac{180\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{c^3\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}}{\sqrt{c^6\sqrt{-\frac{b}{c^{13}}}+x}+\sqrt{x}}\right)-45\left(c^5x^4+2\,bc^4x^2+b^2c^3\right)\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}\log\left(45\,c^3\left(-\frac{b}{c^{13}}\right)^{\frac{1}{4}}+45\right.}{64\left(c^5x^4+2\,bc^4x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(23/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(180*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*arcta
n(c^3*(-b/c^13)^(1/4)/(sqrt(c^6*sqrt(-b/c^13) + x) + sqrt(x))) -
45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^3)*(-b/c^13)^(1/4)*log(45*c^3*(
-b/c^13)^(1/4) + 45*sqrt(x)) + 45*(c^5*x^4 + 2*b*c^4*x^2 + b^2*c^
3)*(-b/c^13)^(1/4)*log(-45*c^3*(-b/c^13)^(1/4) + 45*sqrt(x)) + 4*
(32*c^2*x^4 + 81*b*c*x^2 + 45*b^2)*sqrt(x))/(c^5*x^4 + 2*b*c^4*x^
2 + b^2*c^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(23/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279218, size = 281, normalized size = 1.12

$$-\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^{4}}-\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,c^{4}}-\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^{4}}+\frac{45\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,c^{4}}+\frac{2\sqrt{x}}{c^{3}}+\frac{17\,bcx^{\frac{5}{2}}+13\,b^{2}\sqrt{x}}{16\left(cx^{2}+b\right)^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(23/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -45/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/c^4 - 45/64*sqrt(2)*(b*c^3)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))
/c^4 - 45/128*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4
```

```
) + x + sqrt(b/c))/c^4 + 45/128*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/c^4 + 2*sqrt(x)/c^3 + 1/16*
(17*b*c*x^(5/2) + 13*b^2*sqrt(x))/((c*x^2 + b)^2*c^3)
```

3.340 $\int \frac{x^{21/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

```
[Out] -x^(7/2)/(4*c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) -
 (21*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b
^(1/4)*c^(11/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
 - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rubi [A] time = 0.391989, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{21\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{21\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}+\frac{21\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}\sqrt[4]{b}c^{11/4}}-\frac{7x^{3/2}}{16c^2\left(b+cx^2\right)}-\frac{x^{7/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(7/2)/(4*c*(b + c*x^2)^2) - (7*x^(3/2))/(16*c^2*(b + c*x^2)) -
 (21*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b
^(1/4)*c^(11/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/(32*Sqrt[2]*b^(1/4)*c^(11/4)) + (21*Log[Sqrt[b] - Sqrt[2]*b^
(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(1/4)*c^(11/4))
 - (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(64*Sqrt[2]*b^(1/4)*c^(11/4))
```

Rubi in Sympy [A] time = 72.8846, size = 224, normalized size = 0.94

$$-\frac{x^{\frac{7}{2}}}{4c\left(b+cx^2\right)^2}-\frac{7x^{\frac{3}{2}}}{16c^2\left(b+cx^2\right)}+\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128\sqrt[4]{b}c^{\frac{11}{4}}}-\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128\sqrt[4]{b}c^{\frac{11}{4}}}-\frac{21\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64\sqrt[4]{b}c^{\frac{11}{4}}}+\frac{21\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64\sqrt[4]{b}c^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -x**(7/2)/(4*c*(b + c*x**2)**2) - 7*x**(3/2)/(16*c**2*(b + c*x**2
)) + 21*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b)
+ sqrt(c)*x)/(128*b**(1/4)*c**(11/4)) - 21*sqrt(2)*log(sqrt(2)*b*
*(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(1/4)*c**(
11/4)) - 21*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(
64*b**(1/4)*c**(11/4)) + 21*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqr
t(x)/b**(1/4))/(64*b**(1/4)*c**(11/4))
```

Mathematica [A] time = 0.207736, size = 221, normalized size = 0.92

$$\frac{-\frac{88c^{3/4}x^{3/2}}{b+cx^2}+\frac{32bc^{3/4}x^{3/2}}{(b+cx^2)^2}+\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{b}}-\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{b}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{b}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{\sqrt[4]{b}}}{128c^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(21/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b*c^(3/4)*x^(3/2))/(b + c*x^2)^2 - (88*c^(3/4)*x^(3/2))/(b +
 c*x^2) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/b^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/b^(1/4) + (21*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/b^(1/4) - (21*Sqrt[2]*Log[Sqrt[b] + Sqrt
[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(1/4))/(128*c^(11/4))
```

Maple [A] time = 0.02, size = 161, normalized size = 0.7

$$2\frac{1}{(cx^2+b)^2}\left(-\frac{11\,x^{7/2}}{32\,c}-\frac{7\,bx^{3/2}}{32\,c^2}\right)+\frac{21\sqrt{2}}{128\,c^3}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{21\sqrt{2}}{64\,c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{21\sqrt{2}}{64\,c^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(21/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-11/32*x^(7/2)/c-7/32*b*x^(3/2)/c^2)/(c*x^2+b)^2+21/128/c^3/(b
/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/
(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+21/64/c^3/(b/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/c^3/(b/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(21/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284646, size = 319, normalized size = 1.33

$$\frac{84\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}}}{\sqrt{-bc^5\sqrt{-\frac{1}{bc^{11}}}+x}+\sqrt{x}}\right)+21\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)\left(-\frac{1}{bc^{11}}\right)^{\frac{1}{4}}\log\left(bc^8\left(-\frac{1}{bc^{11}}\right)^{\frac{3}{4}}+\cdots\right.}{64\left(c^4x^4+2\,bc^3x^2+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(21/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(84*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*ar
ctan(b*c^8*(-1/(b*c^11))^(3/4)/(sqrt(-b*c^5*sqrt(-1/(b*c^11)) + x
) + sqrt(x))) + 21*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11)
)^(1/4)*log(b*c^8*(-1/(b*c^11))^(3/4) + sqrt(x)) - 21*(c^4*x^4 +
2*b*c^3*x^2 + b^2*c^2)*(-1/(b*c^11))^(1/4)*log(-b*c^8*(-1/(b*c^11
))^(3/4) + sqrt(x)) - 4*(11*c*x^3 + 7*b*x)*sqrt(x))/(c^4*x^4 + 2*
b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(21/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281711, size = 282, normalized size = 1.18

$$-\frac{11\,cx^{\frac{7}{2}}+7\,bx^{\frac{3}{2}}}{16\,(cx^2+b)^2c^2}+\frac{21\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5}+\frac{21\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^5}-\frac{21\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^5}+\frac{21\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(21/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/16*(11*c*x^(7/2) + 7*b*x^(3/2))/((c*x^2 + b)^2*c^2) + 21/64*sq
rt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*s
```

```
qrt(x))/(b/c)^(1/4))/(b*c^5) + 21/64*sqrt(2)*(b*c^3)^(3/4)*arctan
(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b*c
^5) - 21/128*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4)
 + x + sqrt(b/c))/(b*c^5) + 21/128*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^5)
```

3.341 $\int \frac{x^{19/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{3/4}c^{9/4}}-\frac{5\sqrt{x}}{16c^2(b+cx^2)}-\frac{x^{5/2}}{4c(b+cx^2)^2}$$

```
[Out] -x^(5/2)/(4*c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) -
 (5*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(3/4)*c^(9/4)) + (5*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(3/4)*c^(9/4)) - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4)) + (5*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(3/4)*c^(9/4))
```

Rubi [A] time = 0.391573, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$-\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{3/4}c^{9/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{3/4}c^{9/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{3/4}c^{9/4}}-\frac{5\sqrt{x}}{16c^2(b+cx^2)}-\frac{x^{5/2}}{4c(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -x^(5/2)/(4*c*(b + c*x^2)^2) - (5*Sqrt[x])/(16*c^2*(b + c*x^2)) -
 (5*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(3/4)*c^(9/4)) + (5*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(3/4)*c^(9/4)) - (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(3/4)*c^(9/4)) + (5*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(3/4)*c^(9/4))
```

Rubi in Sympy [A] time = 72.7818, size = 224, normalized size = 0.94

$$-\frac{x^{\frac{5}{2}}}{4c\left(b+cx^{2}\right)^{2}}-\frac{5\sqrt{x}}{16c^{2}\left(b+cx^{2}\right)}-\frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{3}{4}}c^{\frac{9}{4}}}+\frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{3}{4}}c^{\frac{9}{4}}}-\frac{5\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{3}{4}}c^{\frac{9}{4}}}+\frac{5\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{3}{4}}c^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -x**(5/2)/(4*c*(b + c*x**2)**2) - 5*sqrt(x)/(16*c**2*(b + c*x**2)
) - 5*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) +
sqrt(c)*x)/(128*b**(3/4)*c**(9/4)) + 5*sqrt(2)*log(sqrt(2)*b**(1/
4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(3/4)*c**(9/4)
) - 5*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**
(3/4)*c**(9/4)) + 5*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**
(1/4))/(64*b**(3/4)*c**(9/4))
```

Mathematica [A] time = 0.201094, size = 221, normalized size = 0.92

$$\frac{-\frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}+\frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{3/4}}-\frac{10\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{3/4}}+\frac{10\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{3/4}}+\frac{32b\sqrt[4]{c}\sqrt{x}}{\left(b+cx^{2}\right)^{2}}-72}{128c^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(19/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 - (72*c^(1/4)*Sqrt[x])/(b +
 c*x^2) - (10*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/b^(3/4) + (10*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/b^(3/4) - (5*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/b^(3/4) + (5*Sqrt[2]*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(3/4))/(128*c^(9/4))
```

Maple [A] time = 0.022, size = 170, normalized size = 0.7

$$
\begin{aligned}
&2\,\frac{1}{\left(cx^{2}+b\right)^{2}}\left(-\frac{9\,x^{5/2}}{32\,c}-\frac{5\,b\sqrt{x}}{32\,c^{2}}\right)\\
&+\frac{5\,\sqrt{2}}{128\,bc^{2}}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\\
&+\frac{5\,\sqrt{2}}{64\,bc^{2}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{5\,\sqrt{2}}{64\,bc^{2}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(19/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(-9/32*x^(5/2)/c-5/32*b*x^(1/2)/c^2)/(c*x^2+b)^2+5/128/c^2*(b/c
)^(1/4)/b*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/
(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+5/64/c^2*(b/c)^(1/4)
/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+5/64/c^2*(b/c)^(
1/4)/b*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(19/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285217, size = 319, normalized size = 1.33

$$
-\frac{20\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{bc^{2}\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}}{\sqrt{b^{2}c^{4}\sqrt{-\frac{1}{b^{3}c^{9}}}+x}+\sqrt{x}}\right)-5\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}\log\left(bc^{2}\left(-\frac{1}{b^{3}c^{9}}\right)^{\frac{1}{4}}\right.}{64\left(c^{4}x^{4}+2\,bc^{3}x^{2}+b^{2}c^{2}\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(19/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(20*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*
arctan(b*c^2*(-1/(b^3*c^9))^(1/4)/(sqrt(b^2*c^4*sqrt(-1/(b^3*c^9)
) + x) + sqrt(x))) - 5*(c^4*x^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3
*c^9))^(1/4)*log(b*c^2*(-1/(b^3*c^9))^(1/4) + sqrt(x)) + 5*(c^4*x
^4 + 2*b*c^3*x^2 + b^2*c^2)*(-1/(b^3*c^9))^(1/4)*log(-b*c^2*(-1/(
b^3*c^9))^(1/4) + sqrt(x)) + 4*(9*c*x^2 + 5*b)*sqrt(x))/(c^4*x^4
+ 2*b*c^3*x^2 + b^2*c^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(19/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278622, size = 282, normalized size = 1.18

$$\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3}+\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,bc^3}+\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^3}-\frac{5\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,bc^3}-\frac{9\,cx^{\frac{5}{2}}+5\,b\sqrt{x}}{16\left(cx^2+b\right)^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(19/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 5/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/(b*c^3) + 5/64*sqrt(2)*(b*c^3)^(1/4)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4)
)/(b*c^3) + 5/128*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^
```

```
(1/4) + x + sqrt(b/c))/(b*c^3) - 5/128*sqrt(2)*(b*c^3)^(1/4)*ln(-
sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b*c^3) - 1/16*(9*c*
x^(5/2) + 5*b*sqrt(x))/((c*x^2 + b)^2*c^2)
```

3.342 $\int \frac{x^{17/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=242

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

```
[Out] -x^(3/2)/(4*c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) -
 (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(5/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4)) - (3*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(5/4)*c^(7/4))
```

Rubi [A] time = 0.391432, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{5/4}c^{7/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{5/4}c^{7/4}}+\frac{3x^{3/2}}{16bc\left(b+cx^2\right)}-\frac{x^{3/2}}{4c\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -x^(3/2)/(4*c*(b + c*x^2)^2) + (3*x^(3/2))/(16*b*c*(b + c*x^2)) -
 (3*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(5/4)*c^(7/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/(32*Sqrt[2]*b^(5/4)*c^(7/4)) + (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)
*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(5/4)*c^(7/4)) - (3*
Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*S
qrt[2]*b^(5/4)*c^(7/4))
```

Rubi in Sympy [A] time = 72.7242, size = 224, normalized size = 0.93

$$-\frac{x^{\frac{3}{2}}}{4c\left(b+cx^{2}\right)^{2}}+\frac{3x^{\frac{3}{2}}}{16bc\left(b+cx^{2}\right)}+\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{5}{4}}c^{\frac{7}{4}}}-\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{5}{4}}c^{\frac{7}{4}}}-\frac{3\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{5}{4}}c^{\frac{7}{4}}}+\frac{3\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{5}{4}}c^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -x**(3/2)/(4*c*(b + c*x**2)**2) + 3*x**(3/2)/(16*b*c*(b + c*x**2)
) + 3*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) +
sqrt(c)*x)/(128*b**(5/4)*c**(7/4)) - 3*sqrt(2)*log(sqrt(2)*b**(1/
4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(5/4)*c**(7/4)
) - 3*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**
(5/4)*c**(7/4)) + 3*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**
(1/4))/(64*b**(5/4)*c**(7/4))
```

Mathematica [A] time = 0.21586, size = 223, normalized size = 0.92

$$\frac{\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{5/4}}-\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{5/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{5/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{5/4}}+\frac{24c^{3/4}x^{3/2}}{b^{2}+bcx^{2}}-\frac{32c^{3/4}x^{3/2}}{\left(b+cx^{2}\right)^{2}}}{128c^{7/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(17/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-32*c^(3/4)*x^(3/2))/(b + c*x^2)^2 + (24*c^(3/4)*x^(3/2))/(b^2
+ b*c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/b^(5/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)])/b^(5/4) + (3*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/b^(5/4) - (3*Sqrt[2]*Log[Sqrt[b] + Sqrt[
2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(5/4))/(128*c^(7/4))
```

Maple [A] time = 0.02, size = 169, normalized size = 0.7

$$2\frac{1}{(cx^2+b)^2}\left(\frac{3x^{7/2}}{32b}-1/32\frac{x^{3/2}}{c}\right)+\frac{3\sqrt{2}}{128bc^2}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\sqrt{2}}{64bc^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{3\sqrt{2}}{64bc^2}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(17/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(3/32/b*x^(7/2)-1/32*x^(3/2)/c)/(c*x^2+b)^2+3/128/c^2/b/(b/c)^(
1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x+(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/64/c^2/b/(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/64/c^2/b/(b/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282984, size = 335, normalized size = 1.38

$$\frac{12\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\arctan\left(\frac{b^4c^5\left(-\frac{1}{b^5c^7}\right)^{\frac{3}{4}}}{\sqrt{-b^3c^3\sqrt{-\frac{1}{b^5c^7}}+x+\sqrt{x}}}\right)+3\left(bc^3x^4+2b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^5c^7}\right)^{\frac{1}{4}}\log\left(b^4c^5\left(-\frac{1}{b^5c^7}\right.\right.}{64\left(bc^3x^4+2b^2c^2x^2+b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)
*arctan(b^4*c^5*(-1/(b^5*c^7))^(3/4)/(sqrt(-b^3*c^3*sqrt(-1/(b^5*
c^7)) + x) + sqrt(x))) + 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-
1/(b^5*c^7))^(1/4)*log(b^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) -
3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^5*c^7))^(1/4)*log(-b
^4*c^5*(-1/(b^5*c^7))^(3/4) + sqrt(x)) + 4*(3*c*x^3 - b*x)*sqrt(x
))/(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(17/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280892, size = 286, normalized size = 1.18

$$\frac{3\,cx^{\frac{7}{2}}-bx^{\frac{3}{2}}}{16\,(cx^2+b)^2bc}+\frac{3\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4}+\frac{3\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^2c^4}-\frac{3\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}+\frac{3\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(3*c*x^(7/2) - b*x^(3/2))/((c*x^2 + b)^2*b*c) + 3/64*sqrt(2)
*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x
))/(b/c)^(1/4))/(b^2*c^4) + 3/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^4
) - 3/128*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) +
x + sqrt(b/c))/(b^2*c^4) + 3/128*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2
)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^4)
```

3.343 $\int \frac{x^{15/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{\sqrt{x}}{16bc\,(b+cx^2)}-\frac{\sqrt{x}}{4c\,(b+cx^2)^2}$$

```
[Out] -Sqrt[x]/(4*c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4
)*c^(5/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(7/4)*c^(5/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(7/4)*c^(5/4))
```

Rubi [A] time = 0.397999, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{7/4}c^{5/4}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{7/4}c^{5/4}}+\frac{\sqrt{x}}{16bc\,(b+cx^2)}-\frac{\sqrt{x}}{4c\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -Sqrt[x]/(4*c*(b + c*x^2)^2) + Sqrt[x]/(16*b*c*(b + c*x^2)) - (3*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(7/4
)*c^(5/4)) + (3*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(3
2*Sqrt[2]*b^(7/4)*c^(5/4)) - (3*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(
1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(7/4)*c^(5/4)) + (3*Log[
Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[
2]*b^(7/4)*c^(5/4))
```

Rubi in Sympy [A] time = 71.7558, size = 223, normalized size = 0.92

$$-\frac{\sqrt{x}}{4c\left(b+cx^2\right)^2}+\frac{\sqrt{x}}{16bc\left(b+cx^2\right)}-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{7}{4}}c^{\frac{5}{4}}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{7}{4}}c^{\frac{5}{4}}}-\frac{3\sqrt{2}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{7}{4}}c^{\frac{5}{4}}}+\frac{3\sqrt{2}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{7}{4}}c^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] -sqrt(x)/(4*c*(b + c*x**2)**2) + sqrt(x)/(16*b*c*(b + c*x**2)) -
3*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt
(c)*x)/(128*b**(7/4)*c**(5/4)) + 3*sqrt(2)*log(sqrt(2)*b**(1/4)*c
**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(7/4)*c**(5/4)) -
3*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(7/4
)*c**(5/4)) + 3*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4
))/(64*b**(7/4)*c**(5/4))
```

Mathematica [A] time = 0.223664, size = 223, normalized size = 0.92

$$\frac{-\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}+\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{b^{7/4}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{b^{7/4}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{b^{7/4}}+\frac{8\sqrt[4]{c}\sqrt{x}}{b^2+bcx^2}-\frac{32\sqrt[4]{c}\sqrt{x}}{\left(b+cx^2\right)^2}}{128c^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(15/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-32*c^(1/4)*Sqrt[x])/(b + c*x^2)^2 + (8*c^(1/4)*Sqrt[x])/(b^2 +
 b*c*x^2) - (6*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/
4)])/b^(7/4) + (6*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)])/b^(7/4) - (3*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4
)*Sqrt[x] + Sqrt[c]*x])/b^(7/4) + (3*Sqrt[2]*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/b^(7/4))/(128*c^(5/4))
```

Maple [A] time = 0.022, size = 169, normalized size = 0.7

$$2\,\frac{1}{(cx^2+b)^2}\left(1/32\,\frac{x^{5/2}}{b}-\frac{3\sqrt{x}}{32\,c}\right)+\frac{3\sqrt{2}}{128\,b^2c}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{3\sqrt{2}}{64\,b^2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{3\sqrt{2}}{64\,b^2c}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(15/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 2*(1/32/b*x^(5/2)-3/32*x^(1/2)/c)/(c*x^2+b)^2+3/128/c/b^2*(b/c)^(
1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))/(x-(b
/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+3/64/c/b^2*(b/c)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+3/64/c/b^2*(b/c)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284425, size = 323, normalized size = 1.33

$$-\frac{12\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\arctan\left(\frac{b^2c\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}}{\sqrt{b^4c^2\sqrt{-\frac{1}{b^7c^5}}+x}+\sqrt{x}}\right)-3\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)\left(-\frac{1}{b^7c^5}\right)^{\frac{1}{4}}\log\left(b^2c\left(-\frac{1}{b^7c^5}\right)\right.}{64\left(bc^3x^4+2\,b^2c^2x^2+b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4
)*arctan(b^2*c*(-1/(b^7*c^5))^(1/4)/(sqrt(b^4*c^2*sqrt(-1/(b^7*c^
5)) + x) + sqrt(x))) - 3*(b*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/
(b^7*c^5))^(1/4)*log(b^2*c*(-1/(b^7*c^5))^(1/4) + sqrt(x)) + 3*(b
*c^3*x^4 + 2*b^2*c^2*x^2 + b^3*c)*(-1/(b^7*c^5))^(1/4)*log(-b^2*c
*(-1/(b^7*c^5))^(1/4) + sqrt(x)) - 4*(c*x^2 - 3*b)*sqrt(x))/(b*c^
3*x^4 + 2*b^2*c^2*x^2 + b^3*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(15/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278388, size = 285, normalized size = 1.18

$$\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{2}c^{2}}+\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{2}c^{2}}+\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{2}c^{2}}-\frac{3\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{2}c^{2}}+\frac{cx^{\frac{5}{2}}-3\,b\sqrt{x}}{16\,\left(cx^{2}+b\right)^{2}bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4
) + 2*sqrt(x))/(b/c)^(1/4))/(b^2*c^2) + 3/64*sqrt(2)*(b*c^3)^(1/4
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/
4))/(b^2*c^2) + 3/128*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b
```

```
/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) - 3/128*sqrt(2)*(b*c^3)^(1/4
)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^2*c^2) + 1/
16*(c*x^(5/2) - 3*b*sqrt(x))/((c*x^2 + b)^2*b*c)
```

3.344 $\int \frac{x^{13/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=239

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{9/4}c^{3/4}}+\frac{5x^{3/2}}{16b^2\left(b+cx^2\right)}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) -
(5*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(
9/4)*c^(3/4)) + (5*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4)) - (5*L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sq
rt[2]*b^(9/4)*c^(3/4))
```

Rubi [A] time = 0.394296, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$\frac{5\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{9/4}c^{3/4}}-\frac{5\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{9/4}c^{3/4}}+\frac{5\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{9/4}c^{3/4}}+\frac{5x^{3/2}}{16b^2\left(b+cx^2\right)}+\frac{x^{3/2}}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] x^(3/2)/(4*b*(b + c*x^2)^2) + (5*x^(3/2))/(16*b^2*(b + c*x^2)) -
(5*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(
9/4)*c^(3/4)) + (5*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(32*Sqrt[2]*b^(9/4)*c^(3/4)) + (5*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(9/4)*c^(3/4)) - (5*L
og[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sq
rt[2]*b^(9/4)*c^(3/4))
```

Rubi in Sympy [A] time = 72.1363, size = 224, normalized size = 0.94

$$\frac{x^{\frac{3}{2}}}{4b\left(b+cx^2\right)^2}+\frac{5x^{\frac{3}{2}}}{16b^2\left(b+cx^2\right)}+\frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{9}{4}}c^{\frac{3}{4}}}-\frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{9}{4}}c^{\frac{3}{4}}}-\frac{5\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{9}{4}}c^{\frac{3}{4}}}+\frac{5\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{9}{4}}c^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] x**(3/2)/(4*b*(b + c*x**2)**2) + 5*x**(3/2)/(16*b**2*(b + c*x**2)
) + 5*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) +
sqrt(c)*x)/(128*b**(9/4)*c**(3/4)) - 5*sqrt(2)*log(sqrt(2)*b**(1/
4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(9/4)*c**(3/4)
) - 5*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**
(9/4)*c**(3/4)) + 5*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**
(1/4))/(64*b**(9/4)*c**(3/4))
```

Mathematica [A] time = 0.18733, size = 220, normalized size = 0.92

$$\frac{\frac{32b^{5/4}x^{3/2}}{\left(b+cx^2\right)^2}+\frac{5\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{c^{3/4}}-\frac{5\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{c^{3/4}}-\frac{10\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{c^{3/4}}+\frac{10\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{c^{3/4}}+\frac{40\sqrt[4]{b}}{b+c}}{128b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b^(5/4)*x^(3/2))/(b + c*x^2)^2 + (40*b^(1/4)*x^(3/2))/(b + c
*x^2) - (10*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/c^(3/4) + (10*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/c^(3/4) + (5*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*
Sqrt[x] + Sqrt[c]*x])/c^(3/4) - (5*Sqrt[2]*Log[Sqrt[b] + Sqrt[2]*
b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(3/4))/(128*b^(9/4))
```

Maple [A] time = 0.013, size = 175, normalized size = 0.7

$$\frac{1}{4\,b\,(cx^2+b)^2}x^{\frac{3}{2}}+\frac{5}{16\,b^2\,(cx^2+b)}x^{\frac{3}{2}}+\frac{5\sqrt{2}}{128\,b^2c}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{5\sqrt{2}}{64\,b^2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{5\sqrt{2}}{64\,b^2c}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(3/2)/b/(c*x^2+b)^2+5/16*x^(3/2)/b^2/(c*x^2+b)+5/128/b^2/c/
(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)
)/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+5/64/b^2/c/(b/c)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+5/64/b^2/c/(b/
c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285562, size = 321, normalized size = 1.34

$$\frac{20\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\arctan\left(\frac{b^7c^2\left(-\frac{1}{b^9c^3}\right)^{\frac{3}{4}}}{\sqrt{-b^5c\sqrt{-\frac{1}{b^9c^3}}+x}+\sqrt{x}}\right)+5\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^9c^3}\right)^{\frac{1}{4}}\log\left(b^7c^2\left(-\frac{1}{b^9c^3}\right)^{\frac{3}{4}}\right.}{64\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(20*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*a
rctan(b^7*c^2*(-1/(b^9*c^3))^(3/4)/(sqrt(-b^5*c*sqrt(-1/(b^9*c^3)
) + x) + sqrt(x))) + 5*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9
*c^3))^(1/4)*log(b^7*c^2*(-1/(b^9*c^3))^(3/4) + sqrt(x)) - 5*(b^2
*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^9*c^3))^(1/4)*log(-b^7*c^2*(
-1/(b^9*c^3))^(3/4) + sqrt(x)) + 4*(5*c*x^3 + 9*b*x)*sqrt(x))/(b^
2*c^2*x^4 + 2*b^3*c*x^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(13/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279883, size = 282, normalized size = 1.18

$$\frac{5\,cx^{\frac{7}{2}}+9\,bx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^2}+\frac{5\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^3}+\frac{5\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^3c^3}-\frac{5\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^3}+\frac{5\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 1/16*(5*c*x^(7/2) + 9*b*x^(3/2))/((c*x^2 + b)^2*b^2) + 5/64*sqrt(
2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) + 2*sqrt
(x))/(b/c)^(1/4))/(b^3*c^3) + 5/64*sqrt(2)*(b*c^3)^(3/4)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))/(b^3*c
^3) - 5/128*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4)
+ x + sqrt(b/c))/(b^3*c^3) + 5/128*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt
(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c^3)
```

3.345 $\int \frac{x^{11/2}}{(bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=239

$$-\frac{21 \log \left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{21 \log \left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{21 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{21 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) -
(21*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
+ (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rubi [A] time = 0.381699, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474

$$-\frac{21 \log \left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{21 \log \left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{11/4}\sqrt[4]{c}}-\frac{21 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{21 \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{11/4}\sqrt[4]{c}}+\frac{7\sqrt{x}}{16b^2\left(b+cx^2\right)}+\frac{\sqrt{x}}{4b\left(b+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] Sqrt[x]/(4*b*(b + c*x^2)^2) + (7*Sqrt[x])/(16*b^2*(b + c*x^2)) -
(21*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^
(11/4)*c^(1/4)) + (21*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4
)])/(32*Sqrt[2]*b^(11/4)*c^(1/4)) - (21*Log[Sqrt[b] - Sqrt[2]*b^(
1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(11/4)*c^(1/4))
+ (21*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(11/4)*c^(1/4))
```

Rubi in Sympy [A] time = 71.215, size = 224, normalized size = 0.94

$$\frac{\sqrt{x}}{4b\left(b+cx^{2}\right)^{2}}+\frac{7\sqrt{x}}{16b^{2}\left(b+cx^{2}\right)}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{11}{4}}\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{11}{4}}\sqrt[4]{c}}-\frac{21\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{11}{4}}\sqrt[4]{c}}+\frac{21\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{11}{4}}\sqrt[4]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] sqrt(x)/(4*b*(b + c*x**2)**2) + 7*sqrt(x)/(16*b**2*(b + c*x**2))
- 21*sqrt(2)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + s
qrt(c)*x)/(128*b**(11/4)*c**(1/4)) + 21*sqrt(2)*log(sqrt(2)*b**(1
/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(11/4)*c**(1/
4)) - 21*sqrt(2)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*
b**(11/4)*c**(1/4)) + 21*sqrt(2)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x
)/b**(1/4))/(64*b**(11/4)*c**(1/4))
```

Mathematica [A] time = 0.179051, size = 220, normalized size = 0.92

$$\frac{\frac{32b^{7/4}\sqrt{x}}{\left(b+cx^{2}\right)^{2}}+\frac{56b^{3/4}\sqrt{x}}{b+cx^{2}}-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{\sqrt[4]{c}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{\sqrt[4]{c}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right.}{\sqrt[4]{c}}}{128b^{11/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((32*b^(7/4)*Sqrt[x])/(b + c*x^2)^2 + (56*b^(3/4)*Sqrt[x])/(b + c
*x^2) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)]
)/c^(1/4) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1
/4)])/c^(1/4) - (21*Sqrt[2]*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)
*Sqrt[x] + Sqrt[c]*x])/c^(1/4) + (21*Sqrt[2]*Log[Sqrt[b] + Sqrt[2
]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/c^(1/4))/(128*b^(11/4))
```

Maple [A] time = 0.012, size = 166, normalized size = 0.7

$$\frac{1}{4\,b\,(cx^2+b)^2}\sqrt{x}+\frac{7}{16\,b^2\,(cx^2+b)}\sqrt{x}+\frac{21\sqrt{2}}{128\,b^3}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{21\sqrt{2}}{64\,b^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{21\sqrt{2}}{64\,b^3}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 1/4*x^(1/2)/b/(c*x^2+b)^2+7/16*x^(1/2)/b^2/(c*x^2+b)+21/128/b^3*(
b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+21/64/b^3*(b/c)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)+21/64/b^3*(b/c)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285464, size = 305, normalized size = 1.28

$$-\frac{84\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\arctan\left(\frac{b^3\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}}{\sqrt{b^6\sqrt{-\frac{1}{b^{11}c}}+x}+\sqrt{x}}\right)-21\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}\log\left(b^3\left(-\frac{1}{b^{11}c}\right)^{\frac{1}{4}}+\right.}{64\left(b^2c^2x^4+2\,b^3cx^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(84*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*a
rctan(b^3*(-1/(b^11*c))^(1/4)/(sqrt(b^6*sqrt(-1/(b^11*c)) + x) +
sqrt(x))) - 21*(b^2*c^2*x^4 + 2*b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1
/4)*log(b^3*(-1/(b^11*c))^(1/4) + sqrt(x)) + 21*(b^2*c^2*x^4 + 2*
b^3*c*x^2 + b^4)*(-1/(b^11*c))^(1/4)*log(-b^3*(-1/(b^11*c))^(1/4)
 + sqrt(x)) - 4*(7*c*x^2 + 11*b)*sqrt(x))/(b^2*c^2*x^4 + 2*b^3*c*
x^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281359, size = 282, normalized size = 1.18

$$\frac{21\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c}+\frac{21\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{3}c}+\frac{21\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c}-\frac{21\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{3}c}+\frac{7\,cx^{\frac{5}{2}}+11\,b\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 21/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/
4) + 2*sqrt(x))/(b/c)^(1/4))/(b^3*c) + 21/64*sqrt(2)*(b*c^3)^(1/4
)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/
4))/(b^3*c) + 21/128*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/
```

```
c)^(1/4) + x + sqrt(b/c))/(b^3*c) - 21/128*sqrt(2)*(b*c^3)^(1/4)*
ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^3*c) + 1/16*(
7*c*x^(5/2) + 11*b*sqrt(x))/((c*x^2 + b)^2*b^2)
```

3.346 $\int \frac{x^{9/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$-\frac{45\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}}-\frac{45\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{13/4}}-\frac{45}{16b^3\sqrt{x}}+\frac{9}{16b^2\sqrt{x}\,(b+cx^2)}+\frac{1}{4b\sqrt{x}\,(b+cx^2)^2}$$

```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*
Sqrt[x]*(b + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sq
rt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^
(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.437017, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526

$$-\frac{45\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{13/4}}+\frac{45\sqrt[4]{c}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{13/4}}-\frac{45\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{13/4}}-\frac{45}{16b^3\sqrt{x}}+\frac{9}{16b^2\sqrt{x}\,(b+cx^2)}+\frac{1}{4b\sqrt{x}\,(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -45/(16*b^3*Sqrt[x]) + 1/(4*b*Sqrt[x]*(b + c*x^2)^2) + 9/(16*b^2*
Sqrt[x]*(b + c*x^2)) + (45*c^(1/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sq
rt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^(1/4)*ArcTan[1 + (
Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(13/4)) - (45*c^
(1/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])
/(64*Sqrt[2]*b^(13/4)) + (45*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4
)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(13/4))
```

Rubi in Sympy [A] time = 80.4502, size = 238, normalized size = 0.95

$$\frac{1}{4b\sqrt{x}\left(b+cx^2\right)^2}+\frac{9}{16b^2\sqrt{x}\left(b+cx^2\right)}-\frac{45}{16b^3\sqrt{x}}-\frac{45\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{13}{4}}}+\frac{45\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{13}{4}}}+\frac{45\sqrt{2}\sqrt[4]{c}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{13}{4}}}-\frac{45\sqrt{2}\sqrt[4]{c}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*sqrt(x)*(b + c*x**2)**2) + 9/(16*b**2*sqrt(x)*(b + c*x**2)
) - 45/(16*b**3*sqrt(x)) - 45*sqrt(2)*c**(1/4)*log(-sqrt(2)*b**(1
/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(13/4)) + 45*
sqrt(2)*c**(1/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b)
+ sqrt(c)*x)/(128*b**(13/4)) + 45*sqrt(2)*c**(1/4)*atan(1 - sqrt(
2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(13/4)) - 45*sqrt(2)*c**(1/4
)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(13/4))
```

Mathematica [A] time = 0.220509, size = 234, normalized size = 0.93

$$\frac{-\frac{32b^{5/4}cx^{3/2}}{\left(b+cx^2\right)^2}-\frac{104\sqrt[4]{b}cx^{3/2}}{b+cx^2}-45\sqrt{2}\sqrt[4]{c}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+45\sqrt{2}\sqrt[4]{c}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+90\sqrt{2}\sqrt[4]{c}\ \mathrm{ta}}{128b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-256*b^(1/4))/Sqrt[x] - (32*b^(5/4)*c*x^(3/2))/(b + c*x^2)^2 -
(104*b^(1/4)*c*x^(3/2))/(b + c*x^2) + 90*Sqrt[2]*c^(1/4)*ArcTan[1
 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 90*Sqrt[2]*c^(1/4)*ArcTan
[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 45*Sqrt[2]*c^(1/4)*Log[
Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 45*Sqrt[
2]*c^(1/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
]*x])/(128*b^(13/4))
```

Maple [A] time = 0.025, size = 178, normalized size = 0.7

$$-\frac{13\,c^2}{16\,b^3\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{17\,c}{16\,b^2\left(cx^2+b\right)^2}x^{\frac{3}{2}}-\frac{45\sqrt{2}}{128\,b^3}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{45\sqrt{2}}{64\,b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{45\sqrt{2}}{64\,b^3}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-2\,\frac{1}{b^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -13/16*c^2/b^3/(c*x^2+b)^2*x^(7/2)-17/16*c/b^2/(c*x^2+b)^2*x^(3/2
)-45/128/b^3/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^(1/2
)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-45/64
/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1)-45
/64/b^3/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)-1)
-2/b^3/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289779, size = 333, normalized size = 1.33

$$-\frac{180\,c^2x^4+324\,bcx^2+180\left(b^3c^2x^4+2\,b^4cx^2+b^5\right)\sqrt{x}\left(-\frac{c}{b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{91125\,b^{10}\left(-\frac{c}{b^{13}}\right)^{\frac{3}{4}}}{91125\,c\sqrt{x}+\sqrt{-8303765625\,b^7c\sqrt{-\frac{c}{b^{13}}}+8303765625\,c^2x}}\right)+45\,(b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(180*c^2*x^4 + 324*b*c*x^2 + 180*(b^3*c^2*x^4 + 2*b^4*c*x^2
 + b^5)*sqrt(x)*(-c/b^13)^(1/4)*arctan(91125*b^10*(-c/b^13)^(3/4)
/(91125*c*sqrt(x) + sqrt(-8303765625*b^7*c*sqrt(-c/b^13) + 830376
5625*c^2*x))) + 45*(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)*sqrt(x)*(-c/
b^13)^(1/4)*log(91125*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) - 4
5*(b^3*c^2*x^4 + 2*b^4*c*x^2 + b^5)*sqrt(x)*(-c/b^13)^(1/4)*log(-
91125*b^10*(-c/b^13)^(3/4) + 91125*c*sqrt(x)) + 128*b^2)/((b^3*c^
2*x^4 + 2*b^4*c*x^2 + b^5)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280329, size = 297, normalized size = 1.18

$$-\frac{2}{b^3\sqrt{x}} - \frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^2} - \frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4c^2} + \frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c^2} - \frac{45\,\sqrt{2}\,\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4c^2} - \frac{13\,c^2x^{\frac{7}{2}}+17\,bcx^{\frac{3}{2}}}{16\,(cx^2+b)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -2/(b^3*sqrt(x)) - 45/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(b/c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) - 45/64
```

```
*sqrt(2)*(b*c^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) -
 2*sqrt(x))/(b/c)^(1/4))/(b^4*c^2) + 45/128*sqrt(2)*(b*c^3)^(3/4)
*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^4*c^2) - 45/1
28*sqrt(2)*(b*c^3)^(3/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sq
rt(b/c))/(b^4*c^2) - 1/16*(13*c^2*x^(7/2) + 17*b*c*x^(3/2))/((c*x
^2 + b)^2*b^3)
```

3.347 $\int \frac{x^{7/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=251

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{15/4}}-\frac{77}{48b^3x^{3/2}}+\frac{11}{16b^2x^{3/2}(b+cx^2)}+\frac{1}{4bx^{3/2}(b+cx^2)^2}$$

```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2
*x^(3/2)*(b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c
^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```

Rubi [A] time = 0.430924, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526

$$\frac{77c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{15/4}}+\frac{77c^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{15/4}}-\frac{77c^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{15/4}}-\frac{77}{48b^3x^{3/2}}+\frac{11}{16b^2x^{3/2}(b+cx^2)}+\frac{1}{4bx^{3/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -77/(48*b^3*x^(3/2)) + 1/(4*b*x^(3/2)*(b + c*x^2)^2) + 11/(16*b^2
*x^(3/2)*(b + c*x^2)) + (77*c^(3/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*S
qrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(15/4)) + (77*c
^(3/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x]
)/(64*Sqrt[2]*b^(15/4)) - (77*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(15/4))
```

Rubi in Sympy [A] time = 81.4421, size = 238, normalized size = 0.95

$$\frac{1}{4bx^{\frac{3}{2}}\left(b+cx^{2}\right)^{2}}+\frac{11}{16b^{2}x^{\frac{3}{2}}\left(b+cx^{2}\right)}-\frac{77}{48b^{3}x^{\frac{3}{2}}}+\frac{77\sqrt{2}c^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{15}{4}}}-\frac{77\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{15}{4}}}+\frac{77\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{15}{4}}}-\frac{77\sqrt{2}c^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x**(3/2)*(b + c*x**2)**2) + 11/(16*b**2*x**(3/2)*(b + c*x*
*2)) - 77/(48*b**3*x**(3/2)) + 77*sqrt(2)*c**(3/4)*log(-sqrt(2)*b
**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(15/4)) -
 77*sqrt(2)*c**(3/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt
(b) + sqrt(c)*x)/(128*b**(15/4)) + 77*sqrt(2)*c**(3/4)*atan(1 - s
qrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(15/4)) - 77*sqrt(2)*c**
(3/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(15/4))
```

Mathematica [A] time = 0.209879, size = 234, normalized size = 0.93

$$\frac{-\frac{96b^{7/4}c\sqrt{x}}{\left(b+cx^{2}\right)^{2}}-\frac{360b^{3/4}c\sqrt{x}}{b+cx^{2}}-\frac{256b^{3/4}}{x^{3/2}}+231\sqrt{2}c^{3/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-231\sqrt{2}c^{3/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{384b^{15/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-256*b^(3/4))/x^(3/2) - (96*b^(7/4)*c*Sqrt[x])/(b + c*x^2)^2 -
(360*b^(3/4)*c*Sqrt[x])/(b + c*x^2) + 462*Sqrt[2]*c^(3/4)*ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 462*Sqrt[2]*c^(3/4)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 231*Sqrt[2]*c^(3/4)*L
og[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 231*S
qrt[2]*c^(3/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sq
rt[c]*x])/(384*b^(15/4))
```

Maple [A] time = 0.025, size = 181, normalized size = 0.7

$$-\frac{15\,c^{2}}{16\,b^{3}\left(cx^{2}+b\right)^{2}}x^{\frac{5}{2}}-\frac{19\,c}{16\,b^{2}\left(cx^{2}+b\right)^{2}}\sqrt{x}$$
$$-\frac{77\,c\sqrt{2}}{128\,b^{4}}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)$$
$$-\frac{77\,c\sqrt{2}}{64\,b^{4}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{77\,c\sqrt{2}}{64\,b^{4}}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{2}{3\,b^{3}}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -15/16/b^3*c^2/(c*x^2+b)^2*x^(5/2)-19/16/b^2*c/(c*x^2+b)^2*x^(1/2
)-77/128/b^4*c*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*2^(1
/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))-77/
64/b^4*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2)+1
)-77/64/b^4*c*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1
/2)-1)-2/3/b^3/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290843, size = 356, normalized size = 1.42

$$-\frac{308\,c^{2}x^{4}+484\,bcx^{2}-924\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right)\sqrt{x}\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{b^{4}\left(-\frac{c^{3}}{b^{15}}\right)^{\frac{1}{4}}}{c\sqrt{x}+\sqrt{b^{8}\sqrt{-\frac{c^{3}}{b^{15}}}+c^{2}x}}\right)+231\left(b^{3}c^{2}x^{5}+2\,b^{4}cx^{3}+b^{5}x\right.}{192\left(b^{3}c^{2}x^{5}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/192*(308*c^2*x^4 + 484*b*c*x^2 - 924*(b^3*c^2*x^5 + 2*b^4*c*x^
3 + b^5*x)*sqrt(x)*(-c^3/b^15)^(1/4)*arctan(b^4*(-c^3/b^15)^(1/4)
/(c*sqrt(x) + sqrt(b^8*sqrt(-c^3/b^15) + c^2*x))) + 231*(b^3*c^2*
x^5 + 2*b^4*c*x^3 + b^5*x)*sqrt(x)*(-c^3/b^15)^(1/4)*log(77*b^4*(
-c^3/b^15)^(1/4) + 77*c*sqrt(x)) - 231*(b^3*c^2*x^5 + 2*b^4*c*x^3
 + b^5*x)*sqrt(x)*(-c^3/b^15)^(1/4)*log(-77*b^4*(-c^3/b^15)^(1/4)
 + 77*c*sqrt(x)) + 128*b^2)/((b^3*c^2*x^5 + 2*b^4*c*x^3 + b^5*x)*
sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280037, size = 281, normalized size = 1.12

$$-\frac{77\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4}-\frac{77\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^4}-\frac{77\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4}+\frac{77\,\sqrt{2}\left(bc^3\right)^{\frac{1}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^4}-\frac{15\,c^2x^{\frac{5}{2}}+19\,bc\sqrt{x}}{16\left(cx^2+b\right)^2b^3}-\frac{2}{3\,b^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -77/64*sqrt(2)*(b*c^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/b^4 - 77/64*sqrt(2)*(b*c^3)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4))
```

```
/b^4 - 77/128*sqrt(2)*(b*c^3)^(1/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(1/4
) + x + sqrt(b/c))/b^4 + 77/128*sqrt(2)*(b*c^3)^(1/4)*ln(-sqrt(2)
*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^4 - 1/16*(15*c^2*x^(5/2)
+ 19*b*c*sqrt(x))/((c*x^2 + b)^2*b^3) - 2/3/(b^3*x^(3/2))
```

3.348 $\int \frac{x^{5/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=264

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{17/4}}+\frac{117c}{16b^4\sqrt{x}}-\frac{117}{80b^3x^{5/2}}+\frac{13}{16b^2x^{5/2}(b+cx^2)}+\frac{1}{4bx^{5/2}(b+cx^2)^2}$$

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)
*(b + c*x^2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/
4)) + (117*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) - (117*c^(
5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(64*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.498042, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{117c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{17/4}}-\frac{117c^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{17/4}}+\frac{117c^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{17/4}}+\frac{117c}{16b^4\sqrt{x}}-\frac{117}{80b^3x^{5/2}}+\frac{13}{16b^2x^{5/2}(b+cx^2)}+\frac{1}{4bx^{5/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -117/(80*b^3*x^(5/2)) + (117*c)/(16*b^4*Sqrt[x]) + 1/(4*b*x^(5/2)
*(b + c*x^2)^2) + 13/(16*b^2*x^(5/2)*(b + c*x^2)) - (117*c^(5/4)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(17/
4)) + (117*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(32*Sqrt[2]*b^(17/4)) + (117*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(17/4)) - (117*c^(
```

```
5/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(64*Sqrt[2]*b^(17/4))
```

Rubi in Sympy [A] time = 97.4506, size = 252, normalized size = 0.95

$$\frac{1}{4bx^{\frac{5}{2}}\left(b+cx^2\right)^2}+\frac{13}{16b^2x^{\frac{5}{2}}\left(b+cx^2\right)}-\frac{117}{80b^3x^{\frac{5}{2}}}+\frac{117c}{16b^4\sqrt{x}}+\frac{117\sqrt{2}c^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{17}{4}}}-\frac{117\sqrt{2}c^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{17}{4}}}-\frac{117\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{17}{4}}}+\frac{117\sqrt{2}c^{\frac{5}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{17}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x**(5/2)*(b + c*x**2)**2) + 13/(16*b**2*x**(5/2)*(b + c*x*
*2)) - 117/(80*b**3*x**(5/2)) + 117*c/(16*b**4*sqrt(x)) + 117*sqr
t(2)*c**(5/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) +
sqrt(c)*x)/(128*b**(17/4)) - 117*sqrt(2)*c**(5/4)*log(sqrt(2)*b**
(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(17/4)) - 1
17*sqrt(2)*c**(5/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(
64*b**(17/4)) + 117*sqrt(2)*c**(5/4)*atan(1 + sqrt(2)*c**(1/4)*sq
rt(x)/b**(1/4))/(64*b**(17/4))
```

Mathematica [A] time = 0.238205, size = 251, normalized size = 0.95

$$\frac{\frac{160b^{5/4}c^2x^{3/2}}{\left(b+cx^2\right)^2}-\frac{256b^{5/4}}{x^{5/2}}+585\sqrt{2}c^{5/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-585\sqrt{2}c^{5/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-1170\sqrt{2}c^5}{640b^{17/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-256*b^(5/4))/x^(5/2) + (3840*b^(1/4)*c)/Sqrt[x] + (160*b^(5/4)
*c^2*x^(3/2))/(b + c*x^2)^2 + (840*b^(1/4)*c^2*x^(3/2))/(b + c*x^
2) - 1170*Sqrt[2]*c^(5/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^
(1/4)] + 1170*Sqrt[2]*c^(5/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)] + 585*Sqrt[2]*c^(5/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^
(1/4)*Sqrt[x] + Sqrt[c]*x] - 585*Sqrt[2]*c^(5/4)*Log[Sqrt[b] + Sq
```

```
rt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(640*b^(17/4))
```

Maple [A] time = 0.027, size = 192, normalized size = 0.7

$$\frac{21\,c^{3}}{16\,b^{4}\left(cx^{2}+b\right)^{2}}x^{\frac{7}{2}}+\frac{25\,c^{2}}{16\,b^{3}\left(cx^{2}+b\right)^{2}}x^{\frac{3}{2}}+\frac{117\,c\sqrt{2}}{128\,b^{4}}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{117\,c\sqrt{2}}{64\,b^{4}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}+\frac{117\,c\sqrt{2}}{64\,b^{4}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{2}{5\,b^{3}}x^{-\frac{5}{2}}+6\,\frac{c}{b^{4}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 21/16*c^3/b^4/(c*x^2+b)^2*x^(7/2)+25/16*c^2/b^3/(c*x^2+b)^2*x^(3/
2)+117/128*c/b^4/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)*2^
(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))+1
17/64*c/b^4/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^(1/2
)+1)+117/64*c/b^4/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*
x^(1/2)-1)-2/5/b^3/x^(5/2)+6*c/b^4/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289045, size = 400, normalized size = 1.52

$$\frac{2340\,c^3x^6+4212\,bc^2x^4+1664\,b^2cx^2-128\,b^3+2340\left(b^4c^2x^6+2\,b^5cx^4+b^6x^2\right)\sqrt{x}\left(-\frac{c^5}{b^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{1601613}{1601613\,c^4\sqrt{x}+\sqrt{-2565164201}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(2340*c^3*x^6 + 4212*b*c^2*x^4 + 1664*b^2*c*x^2 - 128*b^3 +
 2340*(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)*sqrt(x)*(-c^5/b^17)^(
1/4)*arctan(1601613*b^13*(-c^5/b^17)^(3/4)/(1601613*c^4*sqrt(x) +
 sqrt(-2565164201769*b^9*c^5*sqrt(-c^5/b^17) + 2565164201769*c^8*
x))) + 585*(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)*sqrt(x)*(-c^5/b^
17)^(1/4)*log(1601613*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x
)) - 585*(b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)*sqrt(x)*(-c^5/b^17
)^(1/4)*log(-1601613*b^13*(-c^5/b^17)^(3/4) + 1601613*c^4*sqrt(x)
))/((b^4*c^2*x^6 + 2*b^5*c*x^4 + b^6*x^2)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282072, size = 313, normalized size = 1.19

$$\frac{117\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c}+\frac{117\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^5c}-\frac{117\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c}+\frac{117\,\sqrt{2}\left(bc^3\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5c}+\frac{21\,c^3x^{\frac{7}{2}}+25\,bc^2x^{\frac{3}{2}}}{16\left(cx^2+b\right)^2b^4}+\frac{2\left(15\,cx^2-b\right)}{5\,b^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 117/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(1
/4) + 2*sqrt(x))/(b/c)^(1/4))/(b^5*c) + 117/64*sqrt(2)*(b*c^3)^(3
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/(b^5*c) - 117/128*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*
(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 117/128*sqrt(2)*(b*c^3)^(3
/4)*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/(b^5*c) + 1/
16*(21*c^3*x^(7/2) + 25*b*c^2*x^(3/2))/((c*x^2 + b)^2*b^4) + 2/5*
(15*c*x^2 - b)/(b^4*x^(5/2))
```

3.349 $\int \frac{x^{3/2}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=264

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{19/4}}+\frac{55c}{16b^4x^{3/2}}-\frac{165}{112b^3x^{7/2}}+\frac{15}{16b^2x^{7/2}(b+cx^2)}+\frac{1}{4bx^{7/2}(b+cx^2)^2}$$

```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)
*(b + c*x^2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/
4)) + (165*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) + (165*c^(
7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(64*Sqrt[2]*b^(19/4))
```

Rubi [A] time = 0.491455, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{165c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{19/4}}-\frac{165c^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{19/4}}+\frac{165c^{7/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{19/4}}+\frac{55c}{16b^4x^{3/2}}-\frac{165}{112b^3x^{7/2}}+\frac{15}{16b^2x^{7/2}(b+cx^2)}+\frac{1}{4bx^{7/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -165/(112*b^3*x^(7/2)) + (55*c)/(16*b^4*x^(3/2)) + 1/(4*b*x^(7/2)
*(b + c*x^2)^2) + 15/(16*b^2*x^(7/2)*(b + c*x^2)) - (165*c^(7/4)*
ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(19/
4)) + (165*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])
/(32*Sqrt[2]*b^(19/4)) - (165*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/
4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(19/4)) + (165*c^(
```

```
7/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/
(64*Sqrt[2]*b^(19/4))
```

Rubi in Sympy [A] time = 99.5096, size = 252, normalized size = 0.95

$$\frac{1}{4bx^{\frac{7}{2}}(b+cx^2)^2}+\frac{15}{16b^2x^{\frac{7}{2}}(b+cx^2)}-\frac{165}{112b^3x^{\frac{7}{2}}}+\frac{55c}{16b^4x^{\frac{3}{2}}}-\frac{165\sqrt{2}c^{\frac{7}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{19}{4}}}+\frac{165\sqrt{2}c^{\frac{7}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{19}{4}}}-\frac{165\sqrt{2}c^{\frac{7}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{19}{4}}}+\frac{165\sqrt{2}c^{\frac{7}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{19}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x**(7/2)*(b + c*x**2)**2) + 15/(16*b**2*x**(7/2)*(b + c*x*
*2)) - 165/(112*b**3*x**(7/2)) + 55*c/(16*b**4*x**(3/2)) - 165*sq
rt(2)*c**(7/4)*log(-sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) +
 sqrt(c)*x)/(128*b**(19/4)) + 165*sqrt(2)*c**(7/4)*log(sqrt(2)*b*
*(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(19/4)) -
165*sqrt(2)*c**(7/4)*atan(1 - sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/
(64*b**(19/4)) + 165*sqrt(2)*c**(7/4)*atan(1 + sqrt(2)*c**(1/4)*s
qrt(x)/b**(1/4))/(64*b**(19/4))
```

Mathematica [A] time = 0.236725, size = 251, normalized size = 0.95

$$\frac{\frac{1288b^{3/4}c^2\sqrt{x}}{b+cx^2}+\frac{224b^{7/4}c^2\sqrt{x}}{(b+cx^2)^2}+\frac{1792b^{3/4}c}{x^{3/2}}-\frac{256b^{7/4}}{x^{7/2}}-1155\sqrt{2}c^{7/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+1155\sqrt{2}c^{7/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}\right.}{896b^{19/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-256*b^(7/4))/x^(7/2) + (1792*b^(3/4)*c)/x^(3/2) + (224*b^(7/4)
*c^2*Sqrt[x])/(b + c*x^2)^2 + (1288*b^(3/4)*c^2*Sqrt[x])/(b + c*x
^2) - 2310*Sqrt[2]*c^(7/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b
^(1/4)] + 2310*Sqrt[2]*c^(7/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x
])/b^(1/4)] - 1155*Sqrt[2]*c^(7/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 1155*Sqrt[2]*c^(7/4)*Log[Sqrt[b] +
```

```
Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(896*b^(19/4))
```

Maple [A] time = 0.027, size = 198, normalized size = 0.8

$$\frac{23\,c^3}{16\,b^4\,(cx^2+b)^2}x^{\frac{5}{2}}+\frac{27\,c^2}{16\,b^3\,(cx^2+b)^2}\sqrt{x}+\frac{165\,c^2\sqrt{2}}{128\,b^5}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)+\frac{165\,c^2\sqrt{2}}{64\,b^5}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)+\frac{165\,c^2\sqrt{2}}{64\,b^5}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{2}{7\,b^3}x^{-\frac{7}{2}}+2\,\frac{c}{b^4x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] 23/16/b^4*c^3/(c*x^2+b)^2*x^(5/2)+27/16/b^3*c^2/(c*x^2+b)^2*x^(1/
2)+165/128/b^5*c^2*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)*
2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2)))
+165/64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x^
(1/2)+1)+165/64/b^5*c^2*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^
(1/4)*x^(1/2)-1)-2/7/b^3/x^(7/2)+2*c/b^4/x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292187, size = 390, normalized size = 1.48

$$\frac{1540\,c^3x^6+2420\,bc^2x^4+640\,b^2cx^2-128\,b^3-4620\left(b^4c^2x^7+2\,b^5cx^5+b^6x^3\right)\sqrt{x}\left(-\frac{c^7}{b^{19}}\right)^{\frac14}\arctan\left(\frac{b^5\left(-\frac{c^7}{b^{19}}\right)^{\frac14}}{c^2\sqrt{x}+\sqrt{b^{10}\sqrt{-\frac{c^7}{b^{19}}}+c^4x}}\right)+1\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/448*(1540*c^3*x^6 + 2420*b*c^2*x^4 + 640*b^2*c*x^2 - 128*b^3 -
4620*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*sqrt(x)*(-c^7/b^19)^(1
/4)*arctan(b^5*(-c^7/b^19)^(1/4)/(c^2*sqrt(x) + sqrt(b^10*sqrt(-c
^7/b^19) + c^4*x))) + 1155*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*
sqrt(x)*(-c^7/b^19)^(1/4)*log(165*b^5*(-c^7/b^19)^(1/4) + 165*c^2
*sqrt(x)) - 1155*(b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*sqrt(x)*(-
c^7/b^19)^(1/4)*log(-165*b^5*(-c^7/b^19)^(1/4) + 165*c^2*sqrt(x))
)/((b^4*c^2*x^7 + 2*b^5*c*x^5 + b^6*x^3)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280856, size = 302, normalized size = 1.14

$$\frac{165\sqrt{2}\left(bc^3\right)^{\frac14}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}+2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,b^5}+\frac{165\sqrt{2}\left(bc^3\right)^{\frac14}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac14}-2\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac14}}\right)}{64\,b^5}$$
$$+\frac{165\sqrt{2}\left(bc^3\right)^{\frac14}c\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}$$
$$-\frac{165\sqrt{2}\left(bc^3\right)^{\frac14}c\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac14}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^5}+\frac{23\,c^3x^{\frac52}+27\,bc^2\sqrt{x}}{16\left(cx^2+b\right)^2b^4}+\frac{2\left(7\,cx^2-b\right)}{7\,b^4x^{\frac72}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] 165/64*sqrt(2)*(b*c^3)^(1/4)*c*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^
(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^5 + 165/64*sqrt(2)*(b*c^3)^(1/4
)*c*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(
1/4))/b^5 + 165/128*sqrt(2)*(b*c^3)^(1/4)*c*ln(sqrt(2)*sqrt(x)*(b
/c)^(1/4) + x + sqrt(b/c))/b^5 - 165/128*sqrt(2)*(b*c^3)^(1/4)*c*
ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^5 + 1/16*(23*c
^3*x^(5/2) + 27*b*c^2*sqrt(x))/((c*x^2 + b)^2*b^4) + 2/7*(7*c*x^2
 - b)/(b^4*x^(7/2))
```

3.350 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=279

$$-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}}-\frac{221c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{21/4}}-\frac{221c^2}{16b^5\sqrt{x}}+\frac{221c}{80b^4x^{5/2}}-\frac{221}{144b^3x^{9/2}}+\frac{17}{16b^2x^{9/2}(b+cx^2)}+\frac{1}{4bx^{9/2}(b+cx^2)^2}$$

```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16
*b^5*Sqrt[x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2
)*(b + c*x^2)) + (221*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) - (221*c^(9/
4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(6
4*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rubi [A] time = 0.548161, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$-\frac{221c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{21/4}}+\frac{221c^{9/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{21/4}}-\frac{221c^{9/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{21/4}}-\frac{221c^2}{16b^5\sqrt{x}}+\frac{221c}{80b^4x^{5/2}}-\frac{221}{144b^3x^{9/2}}+\frac{17}{16b^2x^{9/2}(b+cx^2)}+\frac{1}{4bx^{9/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```


```
[Out] -221/(144*b^3*x^(9/2)) + (221*c)/(80*b^4*x^(5/2)) - (221*c^2)/(16
*b^5*Sqrt[x]) + 1/(4*b*x^(9/2)*(b + c*x^2)^2) + 17/(16*b^2*x^(9/2
)*(b + c*x^2)) + (221*c^(9/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x]
)/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) - (221*c^(9/4)*ArcTan[1 + (Sqrt
[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(21/4)) - (221*c^(9/
4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(6
```

```
4*Sqrt[2]*b^(21/4)) + (221*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*
c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(21/4))
```

Rubi in Sympy [A] time = 104.615, size = 267, normalized size = 0.96

$$\frac{1}{4bx^{\frac{9}{2}}\left(b+cx^2\right)^2}+\frac{17}{16b^2x^{\frac{9}{2}}\left(b+cx^2\right)}-\frac{221}{144b^3x^{\frac{9}{2}}}+\frac{221c}{80b^4x^{\frac{5}{2}}}-\frac{221c^2}{16b^5\sqrt{x}}-\frac{221\sqrt{2}c^{\frac{9}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{21}{4}}}+\frac{221\sqrt{2}c^{\frac{9}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{21}{4}}}+\frac{221\sqrt{2}c^{\frac{9}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{21}{4}}}-\frac{221\sqrt{2}c^{\frac{9}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{21}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] 1/(4*b*x**(9/2)*(b + c*x**2)**2) + 17/(16*b**2*x**(9/2)*(b + c*x*
*2)) - 221/(144*b**3*x**(9/2)) + 221*c/(80*b**4*x**(5/2)) - 221*c
**2/(16*b**5*sqrt(x)) - 221*sqrt(2)*c**(9/4)*log(-sqrt(2)*b**(1/4
)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(21/4)) + 221*s
qrt(2)*c**(9/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqrt(b) +
 sqrt(c)*x)/(128*b**(21/4)) + 221*sqrt(2)*c**(9/4)*atan(1 - sqrt(
2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(21/4)) - 221*sqrt(2)*c**(9/
4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(21/4))
```

Mathematica [A] time = 0.281639, size = 266, normalized size = 0.95

$$\frac{-\frac{1440b^{5/4}c^3x^{3/2}}{\left(b+cx^2\right)^2}+\frac{6912b^{5/4}c}{x^{5/2}}-\frac{1280b^{9/4}}{x^{9/2}}-9945\sqrt{2}c^{9/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)+9945\sqrt{2}c^{9/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{}\right.}{5760b^{21/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-1280*b^(9/4))/x^(9/2) + (6912*b^(5/4)*c)/x^(5/2) - (69120*b^(1
/4)*c^2)/Sqrt[x] - (1440*b^(5/4)*c^3*x^(3/2))/(b + c*x^2)^2 - (10
440*b^(1/4)*c^3*x^(3/2))/(b + c*x^2) + 19890*Sqrt[2]*c^(9/4)*ArcT
an[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 19890*Sqrt[2]*c^(9/4)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 9945*Sqrt[2]*c^(
9/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x] +
```

```
 9945*Sqrt[2]*c^(9/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[
x] + Sqrt[c]*x])/(5760*b^(21/4))
```

Maple [A] time = 0.029, size = 209, normalized size = 0.8

$$-\frac{29\,c^4}{16\,b^5\left(cx^2+b\right)^2}x^{\frac{7}{2}}-\frac{33\,c^3}{16\,b^4\left(cx^2+b\right)^2}x^{\frac{3}{2}} -\frac{221\,c^2\sqrt{2}}{128\,b^5}\ln\left(1\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} -\frac{221\,c^2\sqrt{2}}{64\,b^5}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}} -\frac{221\,c^2\sqrt{2}}{64\,b^5}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)\frac{1}{\sqrt[4]{\frac{b}{c}}}-\frac{2}{9\,b^3}x^{-\frac{9}{2}}-12\,\frac{c^2}{b^5\sqrt{x}}+\frac{6\,c}{5\,b^4}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2)^3,x)
```

```
[Out] -29/16*c^4/b^5/(c*x^2+b)^2*x^(7/2)-33/16*c^3/b^4/(c*x^2+b)^2*x^(3
/2)-221/128*c^2/b^5/(b/c)^(1/4)*2^(1/2)*ln((x-(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2))/(x+(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
)-221/64*c^2/b^5/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)-221/64*c^2/b^5/(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)-1)-2/9/b^3/x^(9/2)-12*c^2/b^5/x^(1/2)+6/5*c/b^4/x^
(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290753, size = 414, normalized size = 1.48

$$-\frac{39780\, c^4x^8 + 71604\, bc^3x^6 + 28288\, b^2c^2x^4 - 2176\, b^3cx^2 + 640\, b^4 + 39780\, \left(b^5c^2x^8 + 2\, b^6cx^6 + b^7x^4\right)\sqrt{x}\left(-\frac{c^9}{b^{21}}\right)^{\frac{1}{4}} \arctan\left(\frac{\quad}{107}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/2880*(39780*c^4*x^8 + 71604*b*c^3*x^6 + 28288*b^2*c^2*x^4 - 21
76*b^3*c*x^2 + 640*b^4 + 39780*(b^5*c^2*x^8 + 2*b^6*c*x^6 + b^7*x
^4)*sqrt(x)*(-c^9/b^21)^(1/4)*arctan(10793861*b^16*(-c^9/b^21)^(3
/4)/(10793861*c^7*sqrt(x) + sqrt(-116507435287321*b^11*c^9*sqrt(-
c^9/b^21) + 116507435287321*c^14*x))) + 9945*(b^5*c^2*x^8 + 2*b^6
*c*x^6 + b^7*x^4)*sqrt(x)*(-c^9/b^21)^(1/4)*log(10793861*b^16*(-c
^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)) - 9945*(b^5*c^2*x^8 + 2*b^
6*c*x^6 + b^7*x^4)*sqrt(x)*(-c^9/b^21)^(1/4)*log(-10793861*b^16*(
-c^9/b^21)^(3/4) + 10793861*c^7*sqrt(x)))/((b^5*c^2*x^8 + 2*b^6*c
*x^6 + b^7*x^4)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282112, size = 312, normalized size = 1.12

$$-\frac{221\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}-\frac{221\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}+\frac{221\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}-\frac{221\,\sqrt{2}\left(bc^{3}\right)^{\frac{3}{4}}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}-\frac{29\,c^{4}x^{\frac{7}{2}}+33\,bc^{3}x^{\frac{3}{2}}}{16\left(cx^{2}+b\right)^{2}b^{5}}-\frac{2\left(270\,c^{2}x^{4}-27\,bcx^{2}+5\,b^{2}\right)}{45\,b^{5}x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] -221/64*sqrt(2)*(b*c^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(b/c)^(
1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 - 221/64*sqrt(2)*(b*c^3)^(3/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b/c)^(1/4
))/b^6 + 221/128*sqrt(2)*(b*c^3)^(3/4)*ln(sqrt(2)*sqrt(x)*(b/c)^(
1/4) + x + sqrt(b/c))/b^6 - 221/128*sqrt(2)*(b*c^3)^(3/4)*ln(-sqr
t(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 1/16*(29*c^4*x^(7
/2) + 33*b*c^3*x^(3/2))/((c*x^2 + b)^2*b^5) - 2/45*(270*c^2*x^4 -
 27*b*c*x^2 + 5*b^2)/(b^5*x^(9/2))
```

3.351 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)^3} dx$

Optimal. Leaf size=279

$$\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}}-\frac{285c^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{23/4}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{285}{176b^3x^{11/2}}+\frac{19}{16b^2x^{11/2}(b+cx^2)}+\frac{1}{4bx^{11/2}(b+cx^2)^2}$$

```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(1
6*b^5*x^(3/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(1
1/2)*(b + c*x^2)) + (285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) + (285*
c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt[2]*b
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rubi [A] time = 0.539037, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$

$$\frac{285c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}-\frac{285c^{11/4}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{64\sqrt{2}b^{23/4}}+\frac{285c^{11/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{32\sqrt{2}b^{23/4}}-\frac{285c^{11/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}+1\right)}{32\sqrt{2}b^{23/4}}-\frac{95c^2}{16b^5x^{3/2}}+\frac{285c}{112b^4x^{7/2}}-\frac{285}{176b^3x^{11/2}}+\frac{19}{16b^2x^{11/2}(b+cx^2)}+\frac{1}{4bx^{11/2}(b+cx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] -285/(176*b^3*x^(11/2)) + (285*c)/(112*b^4*x^(7/2)) - (95*c^2)/(1
6*b^5*x^(3/2)) + 1/(4*b*x^(11/2)*(b + c*x^2)^2) + 19/(16*b^2*x^(1
1/2)*(b + c*x^2)) + (285*c^(11/4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqr
t[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*ArcTan[1 +
(Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)])/(32*Sqrt[2]*b^(23/4)) + (285*
c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*
x])/(64*Sqrt[2]*b^(23/4)) - (285*c^(11/4)*Log[Sqrt[b] + Sqrt[2]*b
```

```
^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c]*x])/(64*Sqrt[2]*b^(23/4))
```

Rubi in Sympy [A] time = 102.195, size = 267, normalized size = 0.96

$$\frac{1}{4bx^{\frac{11}{2}}(b+cx^2)^2}+\frac{19}{16b^2x^{\frac{11}{2}}(b+cx^2)}-\frac{285}{176b^3x^{\frac{11}{2}}}+\frac{285c}{112b^4x^{\frac{7}{2}}}-\frac{95c^2}{16b^5x^{\frac{3}{2}}}+\frac{285\sqrt{2}c^{\frac{11}{4}}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{23}{4}}}-\frac{285\sqrt{2}c^{\frac{11}{4}}\log\left(\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)}{128b^{\frac{23}{4}}}+\frac{285\sqrt{2}c^{\frac{11}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{23}{4}}}-\frac{285\sqrt{2}c^{\frac{11}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)}{64b^{\frac{23}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] 1/(4*b*x**(11/2)*(b + c*x**2)**2) + 19/(16*b**2*x**(11/2)*(b + c*
x**2)) - 285/(176*b**3*x**(11/2)) + 285*c/(112*b**4*x**(7/2)) - 9
5*c**2/(16*b**5*x**(3/2)) + 285*sqrt(2)*c**(11/4)*log(-sqrt(2)*b*
*(1/4)*c**(1/4)*sqrt(x) + sqrt(b) + sqrt(c)*x)/(128*b**(23/4)) -
285*sqrt(2)*c**(11/4)*log(sqrt(2)*b**(1/4)*c**(1/4)*sqrt(x) + sqr
t(b) + sqrt(c)*x)/(128*b**(23/4)) + 285*sqrt(2)*c**(11/4)*atan(1
- sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(23/4)) - 285*sqrt(2)
*c**(11/4)*atan(1 + sqrt(2)*c**(1/4)*sqrt(x)/b**(1/4))/(64*b**(23
/4))
```

Mathematica [A] time = 0.263038, size = 266, normalized size = 0.95

$$\frac{-\frac{19096b^{3/4}c^3\sqrt{x}}{b+cx^2}-\frac{2464b^{7/4}c^3\sqrt{x}}{(b+cx^2)^2}-\frac{39424b^{3/4}c^2}{x^{3/2}}+\frac{8448b^{7/4}c}{x^{7/2}}-\frac{1792b^{11/4}}{x^{11/2}}+21945\sqrt{2}c^{11/4}\log\left(-\sqrt{2}\sqrt[4]{b}\sqrt[4]{c}\sqrt{x}+\sqrt{b}+\sqrt{c}x\right)-21945\sqrt{2}}{9856b^{23/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-1792*b^(11/4))/x^(11/2) + (8448*b^(7/4)*c)/x^(7/2) - (39424*b^
(3/4)*c^2)/x^(3/2) - (2464*b^(7/4)*c^3*Sqrt[x])/(b + c*x^2)^2 - (
19096*b^(3/4)*c^3*Sqrt[x])/(b + c*x^2) + 43890*Sqrt[2]*c^(11/4)*A
rcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] - 43890*Sqrt[2]*c^(1
1/4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[x])/b^(1/4)] + 21945*Sqrt[2
]*c^(11/4)*Log[Sqrt[b] - Sqrt[2]*b^(1/4)*c^(1/4)*Sqrt[x] + Sqrt[c
```

```
]*x] - 21945*Sqrt[2]*c^(11/4)*Log[Sqrt[b] + Sqrt[2]*b^(1/4)*c^(1/
4)*Sqrt[x] + Sqrt[c]*x])/(9856*b^(23/4))
```

Maple [A] time = 0.028, size = 209, normalized size = 0.8

$$-\frac{31\,c^4}{16\,b^5\,(cx^2+b)^2}x^{\frac{5}{2}}-\frac{35\,c^3}{16\,b^4\,(cx^2+b)^2}\sqrt{x}-\frac{285\,c^3\sqrt{2}}{128\,b^6}\sqrt[4]{\frac{b}{c}}\ln\left(1\left(x+\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)\left(x-\sqrt[4]{\frac{b}{c}}\sqrt{x}\sqrt{2}+\sqrt{\frac{b}{c}}\right)^{-1}\right)-\frac{285\,c^3\sqrt{2}}{64\,b^6}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}+1\right)-\frac{285\,c^3\sqrt{2}}{64\,b^6}\sqrt[4]{\frac{b}{c}}\arctan\left(\sqrt{2}\sqrt{x}\frac{1}{\sqrt[4]{\frac{b}{c}}}-1\right)-\frac{2}{11\,b^3}x^{-\frac{11}{2}}-4\,\frac{c^2}{b^5x^{3/2}}+\frac{6\,c}{7\,b^4}x^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^3/x^(1/2),x)
```

```
[Out] -31/16/b^5*c^4/(c*x^2+b)^2*x^(5/2)-35/16/b^4*c^3/(c*x^2+b)^2*x^(1
/2)-285/128/b^6*c^3*(b/c)^(1/4)*2^(1/2)*ln((x+(b/c)^(1/4)*x^(1/2)
*2^(1/2)+(b/c)^(1/2))/(x-(b/c)^(1/4)*x^(1/2)*2^(1/2)+(b/c)^(1/2))
)-285/64/b^6*c^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)^(1/4)*x
^(1/2)+1)-285/64/b^6*c^3*(b/c)^(1/4)*2^(1/2)*arctan(2^(1/2)/(b/c)
^(1/4)*x^(1/2)-1)-2/11/b^3/x^(11/2)-4*c^2/b^5/x^(3/2)+6/7*c/b^4/x
^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^3*sqrt(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29314, size = 405, normalized size = 1.45

$$-\frac{29260\,c^4x^8 + 45980\,bc^3x^6 + 12160\,b^2c^2x^4 - 2432\,b^3cx^2 + 896\,b^4 - 87780\,\left(b^5c^2x^9 + 2\,b^6cx^7 + b^7x^5\right)\sqrt{x}\left(-\frac{c^{11}}{b^{23}}\right)^{\frac{1}{4}}\arctan\left(\frac{\ }{c^{3}\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^3*sqrt(x)),x, algorithm="fricas")
```

```
[Out] -1/4928*(29260*c^4*x^8 + 45980*b*c^3*x^6 + 12160*b^2*c^2*x^4 - 24
32*b^3*c*x^2 + 896*b^4 - 87780*(b^5*c^2*x^9 + 2*b^6*c*x^7 + b^7*x
^5)*sqrt(x)*(-c^11/b^23)^(1/4)*arctan(b^6*(-c^11/b^23)^(1/4)/(c^3
*sqrt(x) + sqrt(b^12*sqrt(-c^11/b^23) + c^6*x))) + 21945*(b^5*c^2
*x^9 + 2*b^6*c*x^7 + b^7*x^5)*sqrt(x)*(-c^11/b^23)^(1/4)*log(285*
b^6*(-c^11/b^23)^(1/4) + 285*c^3*sqrt(x)) - 21945*(b^5*c^2*x^9 +
2*b^6*c*x^7 + b^7*x^5)*sqrt(x)*(-c^11/b^23)^(1/4)*log(-285*b^6*(-
c^11/b^23)^(1/4) + 285*c^3*sqrt(x)))/((b^5*c^2*x^9 + 2*b^6*c*x^7
+ b^7*x^5)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**3/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276693, size = 328, normalized size = 1.18

$$-\frac{285\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}-\frac{285\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{b}{c}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{b}{c}\right)^{\frac{1}{4}}}\right)}{64\,b^{6}}-\frac{285\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\ln\left(\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}+\frac{285\,\sqrt{2}\left(bc^{3}\right)^{\frac{1}{4}}c^{2}\ln\left(-\sqrt{2}\sqrt{x}\left(\frac{b}{c}\right)^{\frac{1}{4}}+x+\sqrt{\frac{b}{c}}\right)}{128\,b^{6}}-\frac{31\,c^{4}x^{\frac{5}{2}}+35\,bc^{3}\sqrt{x}}{16\left(cx^{2}+b\right)^{2}b^{5}}-\frac{2\left(154\,c^{2}x^{4}-33\,bcx^{2}+7\,b^{2}\right)}{77\,b^{5}x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^3*sqrt(x)),x, algorithm="giac")
```

```
[Out] -285/64*sqrt(2)*(b*c^3)^(1/4)*c^2*arctan(1/2*sqrt(2)*(sqrt(2)*(b/
c)^(1/4) + 2*sqrt(x))/(b/c)^(1/4))/b^6 - 285/64*sqrt(2)*(b*c^3)^(
1/4)*c^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(b/c)^(1/4) - 2*sqrt(x))/(b
/c)^(1/4))/b^6 - 285/128*sqrt(2)*(b*c^3)^(1/4)*c^2*ln(sqrt(2)*sqr
t(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 + 285/128*sqrt(2)*(b*c^3)^(
1/4)*c^2*ln(-sqrt(2)*sqrt(x)*(b/c)^(1/4) + x + sqrt(b/c))/b^6 - 1
/16*(31*c^4*x^(5/2) + 35*b*c^3*sqrt(x))/((c*x^2 + b)^2*b^5) - 2/7
7*(154*c^2*x^4 - 33*b*c*x^2 + 7*b^2)/(b^5*x^(11/2))
```

3.352 $\int x^{7/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=323

$$\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{28b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

```
[Out] (28*b^3*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (28*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(585*c
^2) + (4*b*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) + (2*x^(9/2)*Sqrt
[b*x^2 + c*x^4])/13 - (28*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (14*
b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1
95*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.744765, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{14b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}-\frac{28b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{195c^{11/4}\sqrt{bx^2+cx^4}}+\frac{28b^3x^{3/2}\left(b+cx^2\right)}{195c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{2}{13}x^{9/2}\sqrt{bx^2+cx^4}+\frac{4bx^{5/2}\sqrt{bx^2+cx^4}}{117c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (28*b^3*x^(3/2)*(b + c*x^2))/(195*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*S
qrt[b*x^2 + c*x^4]) - (28*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(585*c
```

```
^2) + (4*b*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(117*c) + (2*x^(9/2)*Sqrt
[b*x^2 + c*x^4])/13 - (28*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(195*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (14*
b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1
95*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 67.7961, size = 304, normalized size = 0.94

$$-\frac{28b^{\frac{13}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{195c^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{14b^{\frac{13}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{195c^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{28b^3\sqrt{bx^2+cx^4}}{195c^{\frac{5}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{28b^2\sqrt{x}\sqrt{bx^2+cx^4}}{585c^2}+\frac{4bx^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{117c}+\frac{2x^{\frac{9}{2}}\sqrt{bx^2+cx^4}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -28*b**(13/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b
) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*s
qrt(x)/b**(1/4)), 1/2)/(195*c**(11/4)*x*(b + c*x**2)) + 14*b**(13
/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c
)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**
(1/4)), 1/2)/(195*c**(11/4)*x*(b + c*x**2)) + 28*b**3*sqrt(b*x**2
 + c*x**4)/(195*c**(5/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 28*b**2
*sqrt(x)*sqrt(b*x**2 + c*x**4)/(585*c**2) + 4*b*x**(5/2)*sqrt(b*x
**2 + c*x**4)/(117*c) + 2*x**(9/2)*sqrt(b*x**2 + c*x**4)/13
```

Mathematica [C] time = 0.341723, size = 201, normalized size = 0.62

$$\frac{2x^{3/2}\left(-42b^{7/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+42b^{7/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-14b^3-4\right.\right.}{585c^{5/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-14*b^3 - 4*b^
2*c*x^2 + 55*b*c^2*x^4 + 45*c^3*x^6) + 42*b^(7/2)*Sqrt[1 + (c*x^2
)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] - 42*b
^(7/2)*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)
/Sqrt[b]]], -1]))/(585*c^(5/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x
^2*(b + c*x^2)])
```

Maple [A] time = 0.095, size = 237, normalized size = 0.7

$$\frac{2}{(585\,cx^2+585\,b)\,c^3}\sqrt{cx^4+bx^2}\left(45\,x^8c^4+55\,x^6bc^3+42\,b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/585*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^3*(45*x^8*c^4+55*x^
6*b*c^3+42*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((
-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*
EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21
*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*x^4*b^2*c^
2-14*x^2*b^3*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2}\,x^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2}\,x^{\frac{7}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2} x^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(7/2), x)
```

3.353 $\int x^{5/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=176

$$\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}} - \frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}$$

```
[Out] (-20*b^2*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (4*b*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(77*c) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 +
(10*b^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.459476, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{10b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}} - \frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{2}{11}x^{7/2}\sqrt{bx^2+cx^4} + \frac{4bx^{3/2}\sqrt{bx^2+cx^4}}{77c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-20*b^2*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (4*b*x^(3/2)*Sq
rt[b*x^2 + c*x^4])/(77*c) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/11 +
(10*b^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(231*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 42.2362, size = 168, normalized size = 0.95

$$\frac{10b^{\frac{11}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{231c^{\frac{9}{4}}x\left(b+cx^2\right)} - \frac{20b^2\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{4bx^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{77c} + \frac{2x^{\frac{7}{2}}\sqrt{bx^2+cx^4}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 10*b**(11/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
 + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sq
rt(x)/b**(1/4)), 1/2)/(231*c**(9/4)*x*(b + c*x**2)) - 20*b**2*sqr
t(b*x**2 + c*x**4)/(231*c**2*sqrt(x)) + 4*b*x**(3/2)*sqrt(b*x**2
+ c*x**4)/(77*c) + 2*x**(7/2)*sqrt(b*x**2 + c*x**4)/11
```

Mathematica [C] time = 0.601087, size = 133, normalized size = 0.76

$$\frac{1}{231}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{2\left(-10b^2+6bcx^2+21c^2x^4\right)}{c^2\sqrt{x}} + \frac{20ib^3\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{c^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*((2*(-10*b^2 + 6*b*c*x^2 + 21*c^2*x^4))/(c
^2*Sqrt[x]) + ((20*I)*b^3*Sqrt[1 + b/(c*x^2)]*EllipticF[I*ArcSinh
[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[
c]]*c^2*(b + c*x^2))))/231
```

Maple [A] time = 0.033, size = 157, normalized size = 0.9

$$\frac{2}{\left(231\,cx^2+231\,b\right)c^3}\sqrt{cx^4+bx^2}\left(21\,x^7c^4+5\,b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(21*x^7*c^4+5*b^3*(-b
*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+27*x^5*
b*c^3-4*x^3*b^2*c^2-10*x*b^3*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{cx^4 + bx^2}x^{\frac{5}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{5}{2}}\sqrt{x^2\left(b + cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2}x^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(5/2), x)
```

3.354 $\int x^{3/2}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=293

$$-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}-\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

```
[Out] (-4*b^2*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c) + (
2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/9 + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4
]) - (2*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.621729, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{2b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}+\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{7/4}\sqrt{bx^2+cx^4}}-\frac{4b^2x^{3/2}\left(b+cx^2\right)}{15c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c}+\frac{2}{9}x^{5/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-4*b^2*x^(3/2)*(b + c*x^2))/(15*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c) + (
```

```
2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/9 + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c
]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan
[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4
]) - (2*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(15*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 56.5116, size = 275, normalized size = 0.94

$$\frac{4b^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{7}{4}}x\left(b+cx^2\right)} - \frac{2b^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{7}{4}}x\left(b+cx^2\right)} - \frac{4b^2\sqrt{bx^2+cx^4}}{15c^{\frac{3}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)} + \frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{45c} + \frac{2x^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 4*b**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) +
 sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqrt
(x)/b**(1/4)), 1/2)/(15*c**(7/4)*x*(b + c*x**2)) - 2*b**(9/4)*sqr
t((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sq
rt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)),
 1/2)/(15*c**(7/4)*x*(b + c*x**2)) - 4*b**2*sqrt(b*x**2 + c*x**4)
/(15*c**(3/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 4*b*sqrt(x)*sqrt(b
*x**2 + c*x**4)/(45*c) + 2*x**(5/2)*sqrt(b*x**2 + c*x**4)/9
```

Mathematica [C] time = 0.314554, size = 190, normalized size = 0.65

$$\frac{2x^{3/2}\left(6b^{5/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-6b^{5/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(2b^2+7bcx^2+\right.}{45c^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(2*b^2 + 7*b*c*
x^2 + 5*c^2*x^4) - 6*b^(5/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcS
inh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 6*b^(5/2)*Sqrt[1 + (c*x^2
)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(45*
c^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.033, size = 226, normalized size = 0.8

$$-\frac{2}{(45\,cx^2+45\,b)\,c^2}\sqrt{cx^4+bx^2}\left(-5\,c^3x^6+6\,b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c^2*(-5*c^3*x^6+6*b^3
*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-3*b^3*((c*x+(-b*
c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-7*b*c^2*x^4-2*b^2*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4+bx^2}x^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2}\,x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)*x^(3/2), x)
```

3.355 $\int \sqrt{x}\sqrt{bx^2+cx^4}\,dx$

Optimal. Leaf size=146

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2
+ c*x^4])/7 - (2*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.355041, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{2b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{5/4}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2}{7}x^{3/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(21*c*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*x^2
+ c*x^4])/7 - (2*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(21*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 32.4169, size = 139, normalized size = 0.95

$$-\frac{2b^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{21c^{\frac{5}{4}}x\left(b+cx^2\right)}+\frac{4b\sqrt{bx^2+cx^4}}{21c\sqrt{x}}+\frac{2x^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*b**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqr
```

```
t(x)/b**(1/4)), 1/2)/(21*c**(5/4)*x*(b + c*x**2)) + 4*b*sqrt(b*x*
*2 + c*x**4)/(21*c*sqrt(x)) + 2*x**(3/2)*sqrt(b*x**2 + c*x**4)/7
```

Mathematica [C] time = 0.427096, size = 120, normalized size = 0.82

$$\frac{1}{21}\sqrt{x^2\left(b+cx^2\right)}\left(-\frac{4ib^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{c\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}+\frac{4b}{c\sqrt{x}}+6x^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*((4*b)/(c*Sqrt[x]) + 6*x^(3/2) - ((4*I)*b^
2*Sqrt[1 + b/(c*x^2)]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c
]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[c]]*c*(b + c*x^2))))/21
```

Maple [A] time = 0.032, size = 145, normalized size = 1.

$$-\frac{2}{\left(21\,cx^2+21\,b\right)c^2}\sqrt{cx^4+bx^2}\left(b^2\sqrt{-bc}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}EllipticF\left(\sqrt{1\left(cx\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b^2*(-b*c)^(1/2)*((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-3*c^3*x^5-5*b*c^2*x^
3-2*b^2*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+bx^2}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*sqrt(x),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{cx^4+bx^2}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*sqrt(x),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(sqrt(x)*sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+bx^2}\sqrt{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)*sqrt(x),x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

3.356 $\int \frac{\sqrt{bx^2+cx^4}}{\sqrt{x}}\,dx$

Optimal. Leaf size=263

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} - \frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4} + \frac{4bx^{3/2}\left(b+cx^2\right)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

```
[Out] (4*b*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (4*b^(5/4)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*S
qrt[b*x^2 + c*x^4]) + (2*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.505995, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{2b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} - \frac{4b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{3/4}\sqrt{bx^2+cx^4}} + \frac{2}{5}\sqrt{x}\sqrt{bx^2+cx^4} + \frac{4bx^{3/2}\left(b+cx^2\right)}{5\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (4*b*x^(3/2)*(b + c*x^2))/(5*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (4*b^(5/4)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
```

```
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(3/4)*S
qrt[b*x^2 + c*x^4]) + (2*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(5*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 45.2928, size = 248, normalized size = 0.94

$$-\frac{4b^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{2b^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{4b\sqrt{bx^2+cx^4}}{5\sqrt{c}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] -4*b**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(5*c**(3/4)*x*(b + c*x**2)) + 2*b**(5/4)*sqr
t((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sq
rt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)),
 1/2)/(5*c**(3/4)*x*(b + c*x**2)) + 4*b*sqrt(b*x**2 + c*x**4)/(5*
sqrt(c)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 2*sqrt(x)*sqrt(b*x**2 +
c*x**4)/5
```

Mathematica [C] time = 0.269764, size = 176, normalized size = 0.67

$$\frac{2x^{3/2}\left(-2b^{3/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+2b^{3/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(b+cx^2\right)\right)}{5\sqrt{c}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/Sqrt[x],x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(b + c*x^2) + 2
*b^(3/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*
x)/Sqrt[b]]], -1] - 2*b^(3/2)*Sqrt[1 + (c*x^2)/b]*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(5*Sqrt[c]*Sqrt[(I*Sqrt[
c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.032, size = 213, normalized size = 0.8

$$\frac{2}{(5\,cx^2+5\,b)\,c}\sqrt{cx^4+bx^2}\left(2\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)-b^2\sqrt{1\left(cx+\sqrt{-\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(1/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)/c*(2*b^2*((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-b^2*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x
*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2),1/2*2^(1/2))+c^2*x^4+b*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/sqrt(x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/sqrt(x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\,(b+cx^2)}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(1/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/sqrt(x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/sqrt(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/sqrt(x), x)
```

3.357 $\int \frac{\sqrt{bx^2+cx^4}}{x^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x
^4])
```

Rubi [A] time = 0.265855, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{2b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) + (2*b^(3/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(1/4)*Sqrt[b*x^2 + c*x
^4])
```

Rubi in Sympy [A] time = 24.6002, size = 114, normalized size = 0.97

$$\frac{2b^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{c}x\left(b+cx^2\right)}+\frac{2\sqrt{bx^2+cx^4}}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] 2*b**(3/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) +
 sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt
(x)/b**(1/4)), 1/2)/(3*c**(1/4)*x*(b + c*x**2)) + 2*sqrt(b*x**2 +
 c*x**4)/(3*sqrt(x))
```

Mathematica [C] time = 0.228649, size = 102, normalized size = 0.86

$$\frac{2}{3}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{1}{\sqrt{x}}+\frac{2ib\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(3/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(1/Sqrt[x] + ((2*I)*b*Sqrt[1 + b/(c*x^2)
]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(S
qrt[(I*Sqrt[b])/Sqrt[c]]*(b + c*x^2))))/3
```

Maple [A] time = 0.031, size = 130, normalized size = 1.1

$$\frac{2}{\left(3\,cx^2+3\,b\right)c}\sqrt{cx^4+bx^2}\left(b\sqrt{-bc}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}EllipticF\left(\sqrt{1\left(cx+\sqrt{-\right.}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(3/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(b*(-b*c)^(1/2)*((c*x+(
-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^2*x^3+b*c*x)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 (b + cx^2)}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(3/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(3/2), x)
```

3.358 $\int \frac{\sqrt{bx^2+cx^4}}{x^{5/2}}\,dx$

Optimal. Leaf size=254

$$\frac{2\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}} - \frac{4\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}} + \frac{4\sqrt{c}x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

```
[Out] (4*Sqrt[c]*x^(3/2)*(b + c*x^2))/((Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/x^(3/2) - (4*b^(1/4)*c^(1/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2
 + c*x^4] + (2*b^(1/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4]
```

Rubi [A] time = 0.507384, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{2\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}} - \frac{4\sqrt[4]{b}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{bx^2+cx^4}} + \frac{4\sqrt{c}x^{3/2}\left(b+cx^2\right)}{\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\sqrt{bx^2+cx^4}}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (4*Sqrt[c]*x^(3/2)*(b + c*x^2))/((Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/x^(3/2) - (4*b^(1/4)*c^(1/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
```

```
]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/Sqrt[b*x^2
 + c*x^4] + (2*b^(1/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/Sqrt[b*x^2 + c*x^4]
```

Rubi in Sympy [A] time = 47.6613, size = 240, normalized size = 0.94

$$-\frac{4\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{x\left(b+cx^2\right)}+\frac{2\sqrt[4]{b}\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{x\left(b+cx^2\right)}+\frac{4\sqrt{c}\sqrt{bx^2+cx^4}}{\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{2\sqrt{bx^2+cx^4}}{x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

```
[Out] -4*b**(1/4)*c**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*
(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**
(1/4)*sqrt(x)/b**(1/4)), 1/2)/(x*(b + c*x**2)) + 2*b**(1/4)*c**(1
/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c
)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**
(1/4)), 1/2)/(x*(b + c*x**2)) + 4*sqrt(c)*sqrt(b*x**2 + c*x**4)/(
sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 2*sqrt(b*x**2 + c*x**4)/x**(3/2)
```

Mathematica [C] time = 0.257217, size = 175, normalized size = 0.69

$$-\frac{2\sqrt{x}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(b+cx^2\right)+2\sqrt{b}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-2\sqrt{b}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)\right)}{\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(5/2),x]
```

```
[Out] (-2*Sqrt[x]*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(b + c*x^2) - 2*Sqrt[b]*
Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]
*x)/Sqrt[b]]], -1] + 2*Sqrt[b]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*Elli
pticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(Sqrt[(I*Sqrt
```

```
[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.039, size = 202, normalized size = 0.8

$$2\,{\frac{\sqrt{cx^4+bx^2}}{x^{3/2}\left(cx^2+b\right)}\left(2\,\sqrt{{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}}\sqrt{{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}}\sqrt{-{\frac{cx}{\sqrt{-bc}}}}EllipticE\left(\sqrt{{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}},1/2\,\sqrt{2}\right)\sqrt{2}b-\sqrt{{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}}\sqrt{{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(5/2),x)
```

```
[Out] 2*(c*x^4+b*x^2)^(1/2)/x^(3/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*b-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(
1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2
))*2^(1/2)*b-c*x^2-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\left(b+cx^2\right)}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(5/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(5/2), x)
```

3.359 $\int \frac{\sqrt{bx^2+cx^4}}{x^{7/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*
x^4])
```

Rubi [A] time = 0.266757, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{2c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*x^(5/2)) + (2*c^(3/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(1/4)*Sqrt[b*x^2 + c*
x^4])
```

Rubi in Sympy [A] time = 24.8866, size = 114, normalized size = 0.97

$$-\frac{2\sqrt{bx^2+cx^4}}{3x^{\frac{5}{2}}}+\frac{2c^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{3\sqrt[4]{b}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(3*x**(5/2)) + 2*c**(3/4)*sqrt((b + c*x*
*2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 +
 c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(3*b*
*(1/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.316624, size = 104, normalized size = 0.88

$$\frac{2}{3}\sqrt{x^2\left(b+cx^2\right)}\left(-\frac{1}{x^{5/2}}+\frac{2ic\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(7/2),x]
```

```
[Out] (2*Sqrt[x^2*(b + c*x^2)]*(-x^(-5/2) + ((2*I)*c*Sqrt[1 + b/(c*x^2)
]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(S
qrt[(I*Sqrt[b])/Sqrt[c]]*(b + c*x^2))))/3
```

Maple [A] time = 0.036, size = 125, normalized size = 1.1

$$\frac{2}{3\,cx^2+3\,b}\sqrt{cx^4+bx^2}\left(EllipticF\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(7/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(1/2)/x^(5/2)/(c*x^2+b)*(EllipticF(((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*(-b*c)^(1/2)*2^(1/2)*x-c*x^2-b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2\,(b + cx^2)}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(7/2),x)
```

```
[Out] Integral(sqrt(x**2*(b + c*x**2))/x**(7/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(7/2), x)
```

3.360 $\int \frac{\sqrt{bx^2+cx^4}}{x^{9/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}+\frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}}-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

```
[Out] (4*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) - (4*c*Sqrt[
b*x^2 + c*x^4])/(5*b*x^(3/2)) - (4*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) +
 (2*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.6187, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{2c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{bx^2+cx^4}}+\frac{4c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{5bx^{3/2}}-\frac{2\sqrt{bx^2+cx^4}}{5x^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (4*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b
*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*x^(7/2)) - (4*c*Sqrt[
```

```
b*x^2 + c*x^4])/(5*b*x^(3/2)) - (4*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x
)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c
^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4]) +
 (2*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(5*b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 57.3716, size = 274, normalized size = 0.94

$$-\frac{2\sqrt{bx^2+cx^4}}{5x^{\frac{7}{2}}}+\frac{4c^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{5b\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{4c\sqrt{bx^2+cx^4}}{5bx^{\frac{3}{2}}}-\frac{4c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{2c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{3}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(5*x**(7/2)) + 4*c**(3/2)*sqrt(b*x**2 +
c*x**4)/(5*b*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 4*c*sqrt(b*x**2 + c
*x**4)/(5*b*x**(3/2)) - 4*c**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + s
qrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*ellipti
c_e(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*b**(3/4)*x*(b + c*
x**2)) + 2*c**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(
sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(
1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*b**(3/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.364164, size = 196, normalized size = 0.67

$$-\frac{2\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(b^2+3bcx^2+2c^2x^4\right)+2\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-2\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{\cdots}}{\sqrt{\cdots}}}\cdots\right.\right.\right.}{5bx^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(9/2),x]
```

```
[Out] (-2*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(b^2 + 3*b*c*x^2 + 2*c^2*x^4) -
2*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqr
t[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 2*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 + (
c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))
/(5*b*x^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.042, size = 224, normalized size = 0.8

$$\frac{2}{(5\,cx^2+5\,b)\,b}\sqrt{cx^4+bx^2}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^2bc-\sqrt{1\left(cx+\sqrt{\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(1/2)/x^(7/2)/(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b*c-((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*2^(1/2)*x^2*b*c-2*c^2*x^4-3*b*c*x^2-b^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(9/2), x)
```

3.361 $\int \frac{\sqrt{bx^2+cx^4}}{x^{11/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/
(21*b*x^(5/2)) - (2*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.359456, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{2c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7x^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*x^(9/2)) - (4*c*Sqrt[b*x^2 + c*x^4])/
(21*b*x^(5/2)) - (2*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 32.5988, size = 141, normalized size = 0.97

$$-\frac{2\sqrt{bx^2+cx^4}}{7x^{\frac{9}{2}}}-\frac{4c\sqrt{bx^2+cx^4}}{21bx^{\frac{5}{2}}}-\frac{2c^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{21b^{\frac{5}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(7*x**(9/2)) - 4*c*sqrt(b*x**2 + c*x**4)
/(21*b*x**(5/2)) - 2*c**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c
)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2
*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(21*b**(5/4)*x*(b + c*x**2
))
```

Mathematica [C] time = 0.429399, size = 122, normalized size = 0.84

$$\frac{1}{21}\sqrt{x^2\left(b+cx^2\right)}\left(-\frac{2\left(3b+2cx^2\right)}{bx^{9/2}}-\frac{4ic^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{b\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(11/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*((-2*(3*b + 2*c*x^2))/(b*x^(9/2)) - ((4*I)
*c^2*Sqrt[1 + b/(c*x^2)]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqr
t[c]]/Sqrt[x]], -1])/(b*Sqrt[(I*Sqrt[b])/Sqrt[c]]*(b + c*x^2))))/
21
```

Maple [A] time = 0.039, size = 142, normalized size = 1.

$$-\frac{2}{\left(21\,cx^2+21\,b\right)b}\sqrt{cx^4+bx^2}\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}EllipticF\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{}{\sqrt{}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(11/2),x)
```

```
[Out] -2/21*(c*x^4+b*x^2)^(1/2)/x^(9/2)/(c*x^2+b)*(((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^3*c+2*c^2*x^4+5*b*c*x^
2+3*b^2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(11/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(11/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(11/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(11/2),x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(11/2), x)
```

3.362 $\int \frac{\sqrt{bx^2+cx^4}}{x^{13/2}}\,dx$

Optimal. Leaf size=323

$$-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

```
[Out] (-4*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) - (4*c*
Sqrt[b*x^2 + c*x^4])/(45*b*x^(7/2)) + (4*c^2*Sqrt[b*x^2 + c*x^4])
/(15*b^2*x^(3/2)) + (4*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (2*c^(9/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4
)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.739989, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{2c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}+\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{bx^2+cx^4}}-\frac{4c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{3/2}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{7/2}}-\frac{2\sqrt{bx^2+cx^4}}{9x^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

```
[Out] (-4*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^2*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*x^(11/2)) - (4*c*
```

```
Sqrt[b*x^2 + c*x^4])/(45*b*x^(7/2)) + (4*c^2*Sqrt[b*x^2 + c*x^4])
/(15*b^2*x^(3/2)) + (4*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b +
c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[
x])/b^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (2*c^(9/4)
*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(7/4
)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 68.1986, size = 304, normalized size = 0.94

$$-\frac{2\sqrt{bx^2+cx^4}}{9x^{\frac{11}{2}}}-\frac{4c\sqrt{bx^2+cx^4}}{45bx^{\frac{7}{2}}}-\frac{4c^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{15b^2\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{4c^2\sqrt{bx^2+cx^4}}{15b^2x^{\frac{3}{2}}}+\frac{4c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}x\left(b+cx^2\right)}-\frac{2c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{7}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(9*x**(11/2)) - 4*c*sqrt(b*x**2 + c*x**4
)/(45*b*x**(7/2)) - 4*c**(5/2)*sqrt(b*x**2 + c*x**4)/(15*b**2*sqr
t(x)*(sqrt(b) + sqrt(c)*x)) + 4*c**2*sqrt(b*x**2 + c*x**4)/(15*b*
*2*x**(3/2)) + 4*c**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)
**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*ata
n(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(15*b**(7/4)*x*(b + c*x**2)) -
 2*c**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(15*b**(7/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.312162, size = 209, normalized size = 0.65

$$\frac{2\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-5b^3-7b^2cx^2+4bc^2x^4+6c^3x^6\right)+6\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-6\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}E\left(i\,\mathrm{si}\right.\right.}{45b^2x^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(13/2),x]
```

```
[Out] (2*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-5*b^3 - 7*b^2*c*x^2 + 4*b*c^2*x
^4 + 6*c^3*x^6) - 6*Sqrt[b]*c^(5/2)*x^5*Sqrt[1 + (c*x^2)/b]*Ellip
ticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 6*Sqrt[b]*c^(5
/2)*x^5*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x
)/Sqrt[b]]], -1]))/(45*b^2*x^(7/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sq
rt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.042, size = 239, normalized size = 0.7

$$-\frac{2}{(45\,cx^2+45\,b)\,b^2}\sqrt{cx^4+bx^2}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^4bc^2-3\sqrt{\frac{cx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(13/2),x)
```

```
[Out] -2/45*(c*x^4+b*x^2)^(1/2)/x^(11/2)/(c*x^2+b)*(6*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-3*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/
(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-6*c^3*x^6-4*b*c^2*x^4+7*b^2*c
*x^2+5*b^3)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{13}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(13/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(13/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{13}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(13/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(13/2), x)
```

3.363 $\int \frac{\sqrt{bx^2+cx^4}}{x^{15/2}}\,dx$

Optimal. Leaf size=176

$$\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) - (4*c*Sqrt[b*x^2 + c*x^4]
)/(77*b*x^(9/2)) + (20*c^2*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2))
 + (10*c^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.453419, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{10c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11x^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*x^(13/2)) - (4*c*Sqrt[b*x^2 + c*x^4]
)/(77*b*x^(9/2)) + (20*c^2*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5/2))
 + (10*c^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
 + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1
/2])/(231*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 41.2058, size = 168, normalized size = 0.95

$$-\frac{2\sqrt{bx^2+cx^4}}{11x^{\frac{13}{2}}}-\frac{4c\sqrt{bx^2+cx^4}}{77bx^{\frac{9}{2}}}+\frac{20c^2\sqrt{bx^2+cx^4}}{231b^2x^{\frac{5}{2}}}+\frac{10c^{\frac{11}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{231b^{\frac{9}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(11*x**(13/2)) - 4*c*sqrt(b*x**2 + c*x**
4)/(77*b*x**(9/2)) + 20*c**2*sqrt(b*x**2 + c*x**4)/(231*b**2*x**(
5/2)) + 10*c**(11/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*
(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**
(1/4)*sqrt(x)/b**(1/4)), 1/2)/(231*b**(9/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.515758, size = 133, normalized size = 0.76

$$\frac{1}{231}\sqrt{x^2\left(b+cx^2\right)}\left(\frac{2\left(-21b^2-6bcx^2+10c^2x^4\right)}{b^2x^{13/2}}+\frac{20ic^3\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(b+cx^2\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[b*x^2 + c*x^4]/x^(15/2),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*((2*(-21*b^2 - 6*b*c*x^2 + 10*c^2*x^4))/(b
^2*x^(13/2)) + ((20*I)*c^3*Sqrt[1 + b/(c*x^2)]*EllipticF[I*ArcSin
h[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(b^2*Sqrt[(I*Sqrt[b])/
Sqrt[c]]*(b + c*x^2))))/231
```

Maple [A] time = 0.04, size = 156, normalized size = 0.9

$$\frac{2}{(231\,cx^2+231\,b)\,b^2}\sqrt{cx^4+bx^2}\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^5c^2+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(1/2)/x^(15/2),x)
```

```
[Out] 2/231*(c*x^4+b*x^2)^(1/2)/x^(13/2)/(c*x^2+b)*(5*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^5*c^2+10*c^3*x^6+4*
```

```
b*c^2*x^4-27*b^2*c*x^2-21*b^3)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(1/2)/x**(15/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2}}{x^{\frac{15}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2)/x^(15/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2)/x^(15/2), x)
```

3.364 $\int x^{3/2} \left(bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=350

$$\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{56b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

```
[Out] (56*b^4*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (56*b^3*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(3315
*c^2) + (8*b^2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663*c) + (12*b*x^(9/
2)*Sqrt[b*x^2 + c*x^4])/221 + (2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/1
7 - (56*b^(17/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (28*b^(17/4)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt
[b*x^2 + c*x^4])
```

Rubi [A] time = 0.865078, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{28b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}-\frac{56b^{17/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105c^{11/4}\sqrt{bx^2+cx^4}}+\frac{56b^4x^{3/2}\left(b+cx^2\right)}{1105c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{5/2}\sqrt{bx^2+cx^4}}{663c}+\frac{12}{221}bx^{9/2}\sqrt{bx^2+cx^4}+\frac{2}{17}x^{5/2}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (56*b^4*x^(3/2)*(b + c*x^2))/(1105*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*
Sqrt[b*x^2 + c*x^4]) - (56*b^3*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(3315
```

```
*c^2) + (8*b^2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(663*c) + (12*b*x^(9/
2)*Sqrt[b*x^2 + c*x^4])/221 + (2*x^(5/2)*(b*x^2 + c*x^4)^(3/2))/1
7 - (56*b^(17/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b
] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
1/2])/(1105*c^(11/4)*Sqrt[b*x^2 + c*x^4]) + (28*b^(17/4)*x*(Sqrt[
b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1105*c^(11/4)*Sqrt
[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 81.9083, size = 330, normalized size = 0.94

$$-\frac{56b^{\frac{17}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{1105c^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{28b^{\frac{17}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{1105c^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{56b^4\sqrt{bx^2+cx^4}}{1105c^{\frac{5}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{56b^3\sqrt{x}\sqrt{bx^2+cx^4}}{3315c^2}+\frac{8b^2x^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{663c}+\frac{12bx^{\frac{9}{2}}\sqrt{bx^2+cx^4}}{221}+\frac{2x^{\frac{5}{2}}\left(bx^2+cx^4\right)^{\frac{3}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -56*b**(17/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b
) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*s
qrt(x)/b**(1/4)), 1/2)/(1105*c**(11/4)*x*(b + c*x**2)) + 28*b**(1
7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(
c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b*
*(1/4)), 1/2)/(1105*c**(11/4)*x*(b + c*x**2)) + 56*b**4*sqrt(b*x*
*2 + c*x**4)/(1105*c**(5/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 56*b
**3*sqrt(x)*sqrt(b*x**2 + c*x**4)/(3315*c**2) + 8*b**2*x**(5/2)*s
qrt(b*x**2 + c*x**4)/(663*c) + 12*b*x**(9/2)*sqrt(b*x**2 + c*x**4
)/221 + 2*x**(5/2)*(b*x**2 + c*x**4)**(3/2)/17
```

Mathematica [C] time = 0.4362, size = 212, normalized size = 0.61

$$\frac{2x^{3/2}\left(-84b^{9/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+84b^{9/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-28b^4-8\right.}{3315c^{5/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-28*b^4 - 8*b^
3*c*x^2 + 305*b^2*c^2*x^4 + 480*b*c^3*x^6 + 195*c^4*x^8) + 84*b^(
9/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/S
qrt[b]]], -1] - 84*b^(9/2)*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSin
h[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(3315*c^(5/2)*Sqrt[(I*Sqrt[
c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.034, size = 248, normalized size = 0.7

$$\frac{2}{3315\,(cx^2+b)^2\,c^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(195\,x^{10}c^5+480\,x^8bc^4+305\,x^6b^2c^3+84\,b^5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}Ellipt\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 2/3315*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^3*(195*x^10*c^5+
480*x^8*b*c^4+305*x^6*b^2*c^3+84*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))-42*b^5*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(
1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2
*2^(1/2))-8*x^4*b^3*c^2-28*x^2*b^4*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^5+bx^3\right)\sqrt{cx^4+bx^2}\sqrt{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2),x, algorithm="fricas")

[Out] integral((c*x^5 + b*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} x^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)*x^(3/2), x)
```

3.365 $\int \sqrt{x} \left(bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=203

$$\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

```
[Out] (-8*b^3*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (8*b^2*x^(3/2)*S
qrt[b*x^2 + c*x^4])/(385*c) + (4*b*x^(7/2)*Sqrt[b*x^2 + c*x^4])/5
5 + (2*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/15 + (4*b^(15/4)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x
^2 + c*x^4])
```

Rubi [A] time = 0.55031, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{4b^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231c^{9/4}\sqrt{bx^2+cx^4}}-\frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}}+\frac{8b^2x^{3/2}\sqrt{bx^2+cx^4}}{385c}+\frac{4}{55}bx^{7/2}\sqrt{bx^2+cx^4}+\frac{2}{15}x^{3/2}\left(bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^2 + c*x^4])/(231*c^2*Sqrt[x]) + (8*b^2*x^(3/2)*S
qrt[b*x^2 + c*x^4])/(385*c) + (4*b*x^(7/2)*Sqrt[b*x^2 + c*x^4])/5
5 + (2*x^(3/2)*(b*x^2 + c*x^4)^(3/2))/15 + (4*b^(15/4)*x*(Sqrt[b]
 + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF
[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*c^(9/4)*Sqrt[b*x
^2 + c*x^4])
```

Rubi in Sympy [A] time = 52.0726, size = 194, normalized size = 0.96

$$\frac{4b^{\frac{15}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{231c^{\frac{9}{4}}x\left(b+cx^2\right)} - \frac{8b^3\sqrt{bx^2+cx^4}}{231c^2\sqrt{x}} + \frac{8b^2x^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{385c} + \frac{4bx^{\frac{7}{2}}\sqrt{bx^2+cx^4}}{55} + \frac{2x^{\frac{3}{2}}\left(bx^2+cx^4\right)^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] 4*b**(15/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(231*c**(9/4)*x*(b + c*x**2)) - 8*b**3*sqrt(
b*x**2 + c*x**4)/(231*c**2*sqrt(x)) + 8*b**2*x**(3/2)*sqrt(b*x**2
 + c*x**4)/(385*c) + 4*b*x**(7/2)*sqrt(b*x**2 + c*x**4)/55 + 2*x*
*(3/2)*(b*x**2 + c*x**4)**(3/2)/15
```

Mathematica [C] time = 0.32106, size = 164, normalized size = 0.81

$$\frac{40ib^4x^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)+2x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-20b^4-8b^3cx^2+131b^2c^2x^4+196bc^3x^6+77c^4x^8\right)}{1155c^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2)*(-20*b^4 - 8*b^3*c*x^2 + 131
*b^2*c^2*x^4 + 196*b*c^3*x^6 + 77*c^4*x^8) + (40*I)*b^4*Sqrt[1 +
b/(c*x^2)]*x^2*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt
[x]], -1])/(1155*Sqrt[(I*Sqrt[b])/Sqrt[c]]*c^2*Sqrt[x^2*(b + c*x^
2)])
```

Maple [A] time = 0.036, size = 168, normalized size = 0.8

$$\frac{2}{1155\left(cx^2+b\right)^2c^3}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(77x^9c^5+196x^7bc^4+10b^4\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)*x^(1/2),x)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(77*x^9*c^5+196*x^
7*b*c^4+10*b^4*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))+131*x^5*b^2*c^3-8*x^3*b^3*c^2-20*x*b^4*c)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2\right)^{\frac{3}{2}} \sqrt{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x} \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)*x**(1/2),x)
```

```
[Out] Integral(sqrt(x)*(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2\right)^{\frac{3}{2}} \sqrt{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)*sqrt(x), x)
```

3.366 $\int \frac{(bx^2+cx^4)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=320

$$-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}-\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}$$

```
[Out] (-8*b^3*x^(3/2)*(b + c*x^2))/(65*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) + (8*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(195*c)
+ (4*b*x^(5/2)*Sqrt[b*x^2 + c*x^4])/39 + (2*Sqrt[x]*(b*x^2 + c*x^
4)^(3/2))/13 + (8*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*b^(13/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*S
qrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.722549, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{4b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}+\frac{8b^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65c^{7/4}\sqrt{bx^2+cx^4}}-\frac{8b^3x^{3/2}\left(b+cx^2\right)}{65c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c}+\frac{2}{13}\sqrt{x}\left(bx^2+cx^4\right)^{3/2}+\frac{4}{39}bx^{5/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```



```
[Out] (-8*b^3*x^(3/2)*(b + c*x^2))/(65*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) + (8*b^2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(195*c)
```

```
+ (4*b*x^(5/2)*Sqrt[b*x^2 + c*x^4])/39 + (2*Sqrt[x]*(b*x^2 + c*x^
4)^(3/2))/13 + (8*b^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^
2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/
b^(1/4)], 1/2])/(65*c^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*b^(13/4)*x*
(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*E
llipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*c^(7/4)*S
qrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 67.419, size = 301, normalized size = 0.94

$$\frac{8b^{\frac{13}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{65c^{\frac{7}{4}}x\left(b+cx^2\right)} - \frac{4b^{\frac{13}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{65c^{\frac{7}{4}}x\left(b+cx^2\right)} - \frac{8b^3\sqrt{bx^2+cx^4}}{65c^{\frac{3}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)} + \frac{8b^2\sqrt{x}\sqrt{bx^2+cx^4}}{195c} + \frac{4bx^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{39} + \frac{2\sqrt{x}\left(bx^2+cx^4\right)^{\frac{3}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] 8*b**(13/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(65*c**(7/4)*x*(b + c*x**2)) - 4*b**(13/4)*s
qrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*
sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)
), 1/2)/(65*c**(7/4)*x*(b + c*x**2)) - 8*b**3*sqrt(b*x**2 + c*x**
4)/(65*c**(3/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 8*b**2*sqrt(x)*s
qrt(b*x**2 + c*x**4)/(195*c) + 4*b*x**(5/2)*sqrt(b*x**2 + c*x**4)
/39 + 2*sqrt(x)*(b*x**2 + c*x**4)**(3/2)/13
```

Mathematica [C] time = 0.325854, size = 201, normalized size = 0.63

$$\frac{2x^{3/2}\left(12b^{7/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-12b^{7/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(4b^3+29b^2c\right.\right.}{195c^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(4*b^3 + 29*b^2
*c*x^2 + 40*b*c^2*x^4 + 15*c^3*x^6) - 12*b^(7/2)*Sqrt[1 + (c*x^2)
/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 12*b^
(7/2)*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/
Sqrt[b]]], -1]))/(195*c^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^
2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 237, normalized size = 0.7

$$-\frac{2}{195\,(cx^2+b)^2\,c^2}\,(cx^4+bx^2)^{\frac{3}{2}}\left(-15\,x^8c^4-40\,x^6bc^3+12\,b^4\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathit{EllipticE}\left(\sqrt{\frac{cx+\sqrt{}}{\sqrt{-b}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c^2*(-15*x^8*c^4-4
0*x^6*b*c^3+12*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2
)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1
/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2)
)-6*b^4*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-
b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellipti
cF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-29*x^4*b^
2*c^2-4*x^2*b^3*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/sqrt(x), x)
```

3.367 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}$$

```
[Out] (8*b^2*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) + (12*b*x^(3/2)*Sqrt[b
*x^2 + c*x^4])/77 + (2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) - (4*b
^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77
*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.447064, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{4b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{5/4}\sqrt{bx^2+cx^4}}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{11\sqrt{x}}+\frac{12}{77}bx^{3/2}\sqrt{bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (8*b^2*Sqrt[b*x^2 + c*x^4])/(77*c*Sqrt[x]) + (12*b*x^(3/2)*Sqrt[b
*x^2 + c*x^4])/77 + (2*(b*x^2 + c*x^4)^(3/2))/(11*Sqrt[x]) - (4*b
^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[
c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(77
*c^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 39.9226, size = 165, normalized size = 0.95

$$-\frac{4b^{\frac{11}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{77c^{\frac{5}{4}}x\left(b+cx^2\right)}+\frac{8b^2\sqrt{bx^2+cx^4}}{77c\sqrt{x}}+\frac{12bx^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{77}+\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{11\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] -4*b**(11/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
 + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sq
rt(x)/b**(1/4)), 1/2)/(77*c**(5/4)*x*(b + c*x**2)) + 8*b**2*sqrt(
b*x**2 + c*x**4)/(77*c*sqrt(x)) + 12*b*x**(3/2)*sqrt(b*x**2 + c*x
**4)/77 + 2*(b*x**2 + c*x**4)**(3/2)/(11*sqrt(x))
```

Mathematica [C] time = 0.298389, size = 153, normalized size = 0.88

$$\frac{2x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(4b^3+17b^2cx^2+20bc^2x^4+7c^3x^6\right)-8ib^3x^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{77c\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(3/2),x]
```

```
[Out] (2*Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2)*(4*b^3 + 17*b^2*c*x^2 + 20*b
*c^2*x^4 + 7*c^3*x^6) - (8*I)*b^3*Sqrt[1 + b/(c*x^2)]*x^2*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(77*Sqrt[(I
*Sqrt[b])/Sqrt[c]]*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 157, normalized size = 0.9

$$-\frac{2}{77\left(cx^2+b\right)^2c^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(-7x^7c^4+2b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(3/2),x)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(-7*x^7*c^4+2*b^3*(
-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ell
ipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-20*x^
5*b*c^3-17*x^3*b^2*c^2-4*x*b^3*c)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^3 + bx\right)}{\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^3 + b*x)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(3/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(3/2), x)
```

3.368 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=290

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}} - \frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}} + \frac{8b^2x^{3/2}\left(b+cx^2\right)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

```
[Out] (8*b^2*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[b*x^2 + c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2*(b*
x^2 + c*x^4)^(3/2))/(9*x^(3/2)) - (8*b^(9/4)*x*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]
) + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.609721, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{4b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}} - \frac{8b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{3/4}\sqrt{bx^2+cx^4}} + \frac{8b^2x^{3/2}\left(b+cx^2\right)}{15\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} + \frac{4}{15}b\sqrt{x}\sqrt{bx^2+cx^4} + \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (8*b^2*x^(3/2)*(b + c*x^2))/(15*Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[b*x^2 + c*x^4]) + (4*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/15 + (2*(b*
```

```
x^2 + c*x^4)^(3/2))/(9*x^(3/2)) - (8*b^(9/4)*x*(Sqrt[b] + Sqrt[c]
*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[
(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4]
) + (4*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(15*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 55.3901, size = 274, normalized size = 0.94

$$-\frac{8b^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{4b^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{8b^2\sqrt{bx^2+cx^4}}{15\sqrt{c}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{4b\sqrt{x}\sqrt{bx^2+cx^4}}{15}+\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{9x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(5/2),x)
```

```
[Out] -8*b**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(15*c**(3/4)*x*(b + c*x**2)) + 4*b**(9/4)*sq
rt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*s
qrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4))
, 1/2)/(15*c**(3/4)*x*(b + c*x**2)) + 8*b**2*sqrt(b*x**2 + c*x**4
)/(15*sqrt(c)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 4*b*sqrt(x)*sqrt(b
*x**2 + c*x**4)/15 + 2*(b*x**2 + c*x**4)**(3/2)/(9*x**(3/2))
```

Mathematica [C] time = 0.306533, size = 190, normalized size = 0.66

$$\frac{2x^{3/2}\left(-12b^{5/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+12b^{5/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(11b^2+16b\right.\right.}{45\sqrt{c}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(5/2),x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(11*b^2 + 16*b*
c*x^2 + 5*c^2*x^4) + 12*b^(5/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*A
rcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] - 12*b^(5/2)*Sqrt[1 + (c
*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/
(45*Sqrt[c]*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 226, normalized size = 0.8

$$\frac{2}{45\left(cx^2+b\right)^2 c}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(5c^3x^6+12b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(5/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2/c*(5*c^3*x^6+12*b^3*
((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*b^3*((c*x+(-b*c
)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+16*b*c^2*x^4+11*b^2*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/sqrt(x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^{2}\left(b+cx^{2}\right)\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(5/2),x)

[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(5/2), x)
```

3.369 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{7/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(7*Sqrt[x]) + (2*(b*x^2 + c*x^4)^(3/2))
/(7*x^(5/2)) + (4*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(7*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.348573, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{4b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (4*b*Sqrt[b*x^2 + c*x^4])/(7*Sqrt[x]) + (2*(b*x^2 + c*x^4)^(3/2))
/(7*x^(5/2)) + (4*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(7*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 31.5862, size = 138, normalized size = 0.97

$$\frac{4b^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{7\sqrt[4]{c}x\left(b+cx^2\right)}+\frac{4b\sqrt{bx^2+cx^4}}{7\sqrt{x}}+\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{7x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] 4*b**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) +
 sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt
(x)/b**(1/4)), 1/2)/(7*c**(1/4)*x*(b + c*x**2)) + 4*b*sqrt(b*x**2
 + c*x**4)/(7*sqrt(x)) + 2*(b*x**2 + c*x**4)**(3/2)/(7*x**(5/2))
```

Mathematica [C] time = 0.38754, size = 119, normalized size = 0.83

$$\frac{2x^{3/2}\left(\frac{4ib^2\sqrt{x}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}+3b^2+4bcx^2+c^2x^4\right)}{7\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(7/2),x]
```

```
[Out] (2*x^(3/2)*(3*b^2 + 4*b*c*x^2 + c^2*x^4 + ((4*I)*b^2*Sqrt[1 + b/(
c*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqr
t[x]], -1])/Sqrt[(I*Sqrt[b])/Sqrt[c]]))/(7*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 145, normalized size = 1.

$$\frac{2}{7\,\left(cx^2+b\right)^2c}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\,b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)+c^3x^5+\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(7/2),x)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(2*b^2*(-b*c)^(1/2)*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+c^3*x^5+4*b*c^2*x^
3+3*b^2*c*x)/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2\left(b + cx^2\right)\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(7/2),x)
```

```
[Out] Integral((x**2*(b + c*x**2))**(3/2)/x**(7/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(7/2), x)
```

3.370 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{9/2}}\,dx$

Optimal. Leaf size=286

$$\frac{12b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} - \frac{24b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} + \frac{24b\sqrt{c}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

```
[Out] (24*b*Sqrt[c]*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (2*(b*x^
2 + c*x^4)^(3/2))/x^(7/2) - (24*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (1
2*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.604087, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{12b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} - \frac{24b^{5/4}\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{12}{5}c\sqrt{x}\sqrt{bx^2+cx^4} + \frac{24b\sqrt{c}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{x^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (24*b*Sqrt[c]*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (12*c*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/5 - (2*(b*x^
```

```
2 + c*x^4)^(3/2))/x^(7/2) - (24*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4]) + (1
2*b^(5/4)*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 56.2901, size = 270, normalized size = 0.94

$$-\frac{24b^{\frac{5}{4}}\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5x\left(b+cx^2\right)}+\frac{12b^{\frac{5}{4}}\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5x\left(b+cx^2\right)}+\frac{24b\sqrt{c}\sqrt{bx^2+cx^4}}{5\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{12c\sqrt{x}\sqrt{bx^2+cx^4}}{5}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] -24*b**(5/4)*c**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)
*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c*
*(1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*x*(b + c*x**2)) + 12*b**(5/4)*c
**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sq
rt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)
/b**(1/4)), 1/2)/(5*x*(b + c*x**2)) + 24*b*sqrt(c)*sqrt(b*x**2 +
c*x**4)/(5*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 12*c*sqrt(x)*sqrt(b*x
**2 + c*x**4)/5 - 2*(b*x**2 + c*x**4)**(3/2)/x**(7/2)
```

Mathematica [C] time = 0.272446, size = 190, normalized size = 0.66

$$\frac{2\sqrt{x}\left(-12b^{3/2}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+12b^{3/2}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-5b^2-\right.\right.}{5\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(9/2),x]
```

```
[Out] (2*Sqrt[x]*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-5*b^2 - 4*b*c*x^2 + c^2
*x^4) + 12*b^(3/2)*Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcS
inh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] - 12*b^(3/2)*Sqrt[c]*x*Sqrt
[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]],
 -1]))/(5*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.024, size = 216, normalized size = 0.8

$$\frac{2}{5\left(cx^{2}+b\right)^{2}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(12\,b^{2}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathit{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-6\,b^{2}\sqrt{\frac{cx+\sqrt{}}{\sqrt{-b}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(9/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(7/2)/(c*x^2+b)^2*(12*b^2*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-6*b^2*((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))+c^2*x^4-4*b*c*x^2-5*b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}}\left(cx^{2}+b\right)}{x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(5/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(9/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(9/2), x)
```

3.371 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{11/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

```
[Out] (4*c*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) - (2*(b*x^2 + c*x^4)^(3/2))
/(3*x^(9/2)) + (4*b^(3/4)*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(3*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.3531, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{4b^{3/4}c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3\sqrt{bx^2+cx^4}}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{3x^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (4*c*Sqrt[b*x^2 + c*x^4])/(3*Sqrt[x]) - (2*(b*x^2 + c*x^4)^(3/2))
/(3*x^(9/2)) + (4*b^(3/4)*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
 + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sq
rt[x])/b^(1/4)], 1/2])/(3*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 36.2326, size = 138, normalized size = 0.97

$$\frac{4b^{\frac{3}{4}}c^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{3x\left(b+cx^2\right)}+\frac{4c\sqrt{bx^2+cx^4}}{3\sqrt{x}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{3x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(11/2),x)
```

```
[Out] 4*b**(3/4)*c**(3/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(
sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(
1/4)*sqrt(x)/b**(1/4)), 1/2)/(3*x*(b + c*x**2)) + 4*c*sqrt(b*x**2
 + c*x**4)/(3*sqrt(x)) - 2*(b*x**2 + c*x**4)**(3/2)/(3*x**(9/2))
```

Mathematica [C] time = 0.337041, size = 111, normalized size = 0.78

$$\frac{2\left(-b^2+\frac{4ibcx^{5/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}+c^2x^4\right)}{3\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(11/2),x]
```

```
[Out] (2*(-b^2 + c^2*x^4 + ((4*I)*b*c*Sqrt[1 + b/(c*x^2)]*x^(5/2)*Ellip
ticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/Sqrt[(I*S
qrt[b])/Sqrt[c]]))/(3*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.021, size = 130, normalized size = 0.9

$$\frac{2}{3\left(cx^2+b\right)^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}xb+c^2x^4-b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(11/2),x)
```

```
[Out] 2/3*(c*x^4+b*x^2)^(3/2)/x^(9/2)/(c*x^2+b)^2*(2*(-b*c)^(1/2)*((c*x
+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x*b+c^2*x^4-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(7/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(11/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(11/2), x)
```

3.372 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{13/2}} dx$

Optimal. Leaf size=287

$$\frac{12\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} - \frac{24\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{24c^{3/2}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

```
[Out] (24*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*
x^2 + c*x^4]) - (12*c*Sqrt[b*x^2 + c*x^4])/(5*x^(3/2)) - (2*(b*x^
2 + c*x^4)^(3/2))/(5*x^(11/2)) - (24*b^(1/4)*c^(5/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
 + (12*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.603707, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{12\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} - \frac{24\sqrt[4]{b}c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5\sqrt{bx^2+cx^4}} + \frac{24c^{3/2}x^{3/2}\left(b+cx^2\right)}{5\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{12c\sqrt{bx^2+cx^4}}{5x^{3/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{5x^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (24*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*
x^2 + c*x^4]) - (12*c*Sqrt[b*x^2 + c*x^4])/(5*x^(3/2)) - (2*(b*x^
```

```
2 + c*x^4)^(3/2))/(5*x^(11/2)) - (24*b^(1/4)*c^(5/4)*x*(Sqrt[b] +
 Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2
*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
 + (12*b^(1/4)*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(
Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1
/4)], 1/2])/(5*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 54.452, size = 270, normalized size = 0.94

$$-\frac{24\sqrt[4]{b}c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5x\left(b+cx^2\right)}+\frac{12\sqrt[4]{b}c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5x\left(b+cx^2\right)}+\frac{24c^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{5\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{12c\sqrt{bx^2+cx^4}}{5x^{\frac{3}{2}}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{5x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(13/2),x)
```

```
[Out] -24*b**(1/4)*c**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)
*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c*
*(1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*x*(b + c*x**2)) + 12*b**(1/4)*c
**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sq
rt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)
/b**(1/4)), 1/2)/(5*x*(b + c*x**2)) + 24*c**(3/2)*sqrt(b*x**2 + c
*x**4)/(5*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 12*c*sqrt(b*x**2 + c*x
**4)/(5*x**(3/2)) - 2*(b*x**2 + c*x**4)**(3/2)/(5*x**(11/2))
```

Mathematica [C] time = 0.344032, size = 193, normalized size = 0.67

$$-\frac{2\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(b^2+8bcx^2+7c^2x^4\right)+12\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-12\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\right.\right.\right.}{5x^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(13/2),x]
```

```
[Out] (-2*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(b^2 + 8*b*c*x^2 + 7*c^2*x^4) -
12*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sq
rt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 12*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 +
 (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]
))/(5*x^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.023, size = 221, normalized size = 0.8

$$\frac{2}{5\left(cx^{2}+b\right)^{2}}\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^{2}bc-6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(13/2),x)
```

```
[Out] 2/5*(c*x^4+b*x^2)^(3/2)/x^(11/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*
(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b*c-6*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*x^2*b*c-7*c^2*x^4-8*b*c*x^2-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^{4}+bx^{2}\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^{4}+bx^{2}}\left(cx^{2}+b\right)}{x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(9/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(13/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{13}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(13/2), x)
```

3.373 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{15/2}}\,dx$

Optimal. Leaf size=143

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(7*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2)
)/(7*x^(13/2)) + (4*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.350265, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$\frac{4c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{bx^2+cx^4}}-\frac{4c\sqrt{bx^2+cx^4}}{7x^{5/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{7x^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(7*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2)
)/(7*x^(13/2)) + (4*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 31.5493, size = 138, normalized size = 0.97

$$-\frac{4c\sqrt{bx^2+cx^4}}{7x^{\frac{5}{2}}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{7x^{\frac{13}{2}}}+\frac{4c^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{7\sqrt[4]{b}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] -4*c*sqrt(b*x**2 + c*x**4)/(7*x**(5/2)) - 2*(b*x**2 + c*x**4)**(3
/2)/(7*x**(13/2)) + 4*c**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(
c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(
2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(7*b**(1/4)*x*(b + c*x**2
))
```

Mathematica [C] time = 0.475801, size = 120, normalized size = 0.84

$$\frac{2\left(-b^2+\frac{4ic^2x^{9/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}-4bcx^2-3c^2x^4\right)}{7x^{5/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(15/2),x]
```

```
[Out] (2*(-b^2 - 4*b*c*x^2 - 3*c^2*x^4 + ((4*I)*c^2*Sqrt[1 + b/(c*x^2)]
*x^(9/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]],
-1])/Sqrt[(I*Sqrt[b])/Sqrt[c]]))/(7*x^(5/2)*Sqrt[x^2*(b + c*x^2)]
)
```

Maple [A] time = 0.021, size = 140, normalized size = 1.

$$\frac{2}{7\left(cx^2+b\right)^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^3c-3c^2x^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(15/2),x)
```

```
[Out] 2/7*(c*x^4+b*x^2)^(3/2)/x^(13/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^3*c-3*c^2*x^4-4*b*c
*x^2-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2}\,(cx^2 + b)}{x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(11/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(15/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2)^{\frac{3}{2}}}{x^{\frac{15}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2),x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(15/2), x)
```

3.374 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{17/2}}\,dx$

Optimal. Leaf size=320

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} - \frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} + \frac{8c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

```
[Out] (8*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(15*x^(7/2)) - (8*c^2
*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/
(9*x^(15/2)) - (8*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(9/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqr
t[b*x^2 + c*x^4])
```

Rubi [A] time = 0.730754, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{4c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} - \frac{8c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{bx^2+cx^4}} + \frac{8c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{3/2}} - \frac{4c\sqrt{bx^2+cx^4}}{15x^{7/2}} - \frac{2\left(bx^2+cx^4\right)^{3/2}}{9x^{15/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

```
[Out] (8*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(15*x^(7/2)) - (8*c^2
```

```
*Sqrt[b*x^2 + c*x^4])/(15*b*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/
(9*x^(15/2)) - (8*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2
)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b
^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[b*x^2 + c*x^4]) + (4*c^(9/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(3/4)*Sqr
t[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 65.8946, size = 299, normalized size = 0.93

$$-\frac{4c\sqrt{bx^2+cx^4}}{15x^{\frac{7}{2}}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{9x^{\frac{15}{2}}}+\frac{8c^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{15b\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{8c^2\sqrt{bx^2+cx^4}}{15bx^{\frac{3}{2}}}$$
$$-\frac{8c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}x\left(b+cx^2\right)}$$
$$+\frac{4c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{3}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] -4*c*sqrt(b*x**2 + c*x**4)/(15*x**(7/2)) - 2*(b*x**2 + c*x**4)**(
3/2)/(9*x**(15/2)) + 8*c**(5/2)*sqrt(b*x**2 + c*x**4)/(15*b*sqrt(
x)*(sqrt(b) + sqrt(c)*x)) - 8*c**2*sqrt(b*x**2 + c*x**4)/(15*b*x*
*(3/2)) - 8*c**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*
(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**
(1/4)*sqrt(x)/b**(1/4)), 1/2)/(15*b**(3/4)*x*(b + c*x**2)) + 4*c*
*(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqr
t(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/
b**(1/4)), 1/2)/(15*b**(3/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.476275, size = 209, normalized size = 0.65

$$-\frac{2\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(5b^3+16b^2cx^2+23bc^2x^4+12c^3x^6\right)+12\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-12\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}\right.}{45bx^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(17/2),x]
```

```
[Out] (-2*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(5*b^3 + 16*b^2*c*x^2 + 23*b*c^2
*x^4 + 12*c^3*x^6) - 12*Sqrt[b]*c^(5/2)*x^5*Sqrt[1 + (c*x^2)/b]*E
llipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 12*Sqrt[b]
*c^(5/2)*x^5*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt
[c]*x)/Sqrt[b]]], -1]))/(45*b*x^(7/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]
*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.024, size = 239, normalized size = 0.8

$$\frac{2}{45\,(cx^2+b)^2\,b}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^4bc^2-6\sqrt{\frac{cx}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(17/2),x)
```

```
[Out] 2/45*(c*x^4+b*x^2)^(3/2)/x^(15/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-6*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-12*c^3*x^6-23*b*c^2*x^4-16*
b^2*c*x^2-5*b^3)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\,(cx^2+b)}{x^{\frac{13}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(13/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(17/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{17}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(17/2), x)
```

3.375 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{19/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

```
[Out] (-12*c*Sqrt[b*x^2 + c*x^4])/(77*x^(9/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(77*b*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) -
(4*c^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(77*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.446944, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{4c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{bx^2+cx^4}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{5/2}}-\frac{12c\sqrt{bx^2+cx^4}}{77x^{9/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{11x^{17/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

```
[Out] (-12*c*Sqrt[b*x^2 + c*x^4])/(77*x^(9/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(77*b*x^(5/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(11*x^(17/2)) -
(4*c^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(77*b^(5/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 42.6877, size = 167, normalized size = 0.97

$$-\frac{12c\sqrt{bx^2+cx^4}}{77x^{\frac{9}{2}}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{11x^{\frac{17}{2}}}-\frac{8c^2\sqrt{bx^2+cx^4}}{77bx^{\frac{5}{2}}}-\frac{4c^{\frac{11}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{77b^{\frac{5}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(19/2),x)
```

```
[Out] -12*c*sqrt(b*x**2 + c*x**4)/(77*x**(9/2)) - 2*(b*x**2 + c*x**4)**
(3/2)/(11*x**(17/2)) - 8*c**2*sqrt(b*x**2 + c*x**4)/(77*b*x**(5/2
)) - 4*c**(11/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqr
t(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4
)*sqrt(x)/b**(1/4)), 1/2)/(77*b**(5/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.321682, size = 154, normalized size = 0.89

$$-\frac{2\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(7b^3+20b^2cx^2+17bc^2x^4+4c^3x^6\right)+4ic^3x^{13/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{77bx^{9/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(19/2),x]
```

```
[Out] (-2*(Sqrt[(I*Sqrt[b])/Sqrt[c]]*(7*b^3 + 20*b^2*c*x^2 + 17*b*c^2*x
^4 + 4*c^3*x^6) + (4*I)*c^3*Sqrt[1 + b/(c*x^2)]*x^(13/2)*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1]))/(77*b*Sqrt[
(I*Sqrt[b])/Sqrt[c]]*x^(9/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.021, size = 156, normalized size = 0.9

$$-\frac{2}{77\left(cx^2+b\right)^2b}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^5c^2+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(19/2),x)
```

```
[Out] -2/77*(c*x^4+b*x^2)^(3/2)/x^(17/2)/(c*x^2+b)^2*(2*((c*x+(-b*c)^(1
/2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^5*c^2+4*c^3*x^6+1
7*b*c^2*x^4+20*b^2*c*x^2+7*b^3)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^4 + bx^2}\left(cx^2 + b\right)}{x^{\frac{15}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(15/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(19/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{19}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(19/2), x)
```

3.376 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{21/2}}\,dx$

Optimal. Leaf size=350

$$-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}+\frac{8c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}-\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}-\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

```
[Out] (-8*c^(7/2)*x^(3/2)*(b + c*x^2))/(65*b^2*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(39*x^(11/2)) - (8
*c^2*Sqrt[b*x^2 + c*x^4])/(195*b*x^(7/2)) + (8*c^3*Sqrt[b*x^2 + c
*x^4])/(65*b^2*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(13*x^(19/2))
 + (8*c^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*c^(13/4)*x*(Sqrt[b] + S
qrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 +
c*x^4])
```

Rubi [A] time = 0.864534, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{4c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}+\frac{8c^{13/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{bx^2+cx^4}}-\frac{8c^{7/2}x^{3/2}\left(b+cx^2\right)}{65b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{3/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{7/2}}-\frac{4c\sqrt{bx^2+cx^4}}{39x^{11/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{13x^{19/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (-8*c^(7/2)*x^(3/2)*(b + c*x^2))/(65*b^2*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (4*c*Sqrt[b*x^2 + c*x^4])/(39*x^(11/2)) - (8
*c^2*Sqrt[b*x^2 + c*x^4])/(195*b*x^(7/2)) + (8*c^3*Sqrt[b*x^2 + c
*x^4])/(65*b^2*x^(3/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(13*x^(19/2))
 + (8*c^(13/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(65*b^(7/4)*Sqrt[b*x^2 + c*x^4]) - (4*c^(13/4)*x*(Sqrt[b] + S
qrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*A
rcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[b*x^2 +
c*x^4])
```

Rubi in Sympy [A] time = 83.4985, size = 330, normalized size = 0.94

$$-\frac{4c\sqrt{bx^2+cx^4}}{39x^{\frac{11}{2}}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{13x^{\frac{19}{2}}}-\frac{8c^2\sqrt{bx^2+cx^4}}{195bx^{\frac{7}{2}}}-\frac{8c^{\frac{7}{2}}\sqrt{bx^2+cx^4}}{65b^2\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{8c^3\sqrt{bx^2+cx^4}}{65b^2x^{\frac{3}{2}}}+\frac{8c^{\frac{13}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{65b^{\frac{7}{4}}x\left(b+cx^2\right)}-\frac{4c^{\frac{13}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{65b^{\frac{7}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(21/2),x)
```

```
[Out] -4*c*sqrt(b*x**2 + c*x**4)/(39*x**(11/2)) - 2*(b*x**2 + c*x**4)**
(3/2)/(13*x**(19/2)) - 8*c**2*sqrt(b*x**2 + c*x**4)/(195*b*x**(7/
2)) - 8*c**(7/2)*sqrt(b*x**2 + c*x**4)/(65*b**2*sqrt(x)*(sqrt(b)
+ sqrt(c)*x)) + 8*c**3*sqrt(b*x**2 + c*x**4)/(65*b**2*x**(3/2)) +
 8*c**(13/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
 + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sq
rt(x)/b**(1/4)), 1/2)/(65*b**(7/4)*x*(b + c*x**2)) - 4*c**(13/4)*
sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)
*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4
)), 1/2)/(65*b**(7/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.448136, size = 220, normalized size = 0.63

$$\frac{2\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-15b^4-40b^3cx^2-29b^2c^2x^4+8bc^3x^6+12c^4x^8\right)+12\sqrt{b}c^{7/2}x^7\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-12\sqrt{b}c^{7/2}}{195b^2x^{11/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(21/2),x]
```

```
[Out] (2*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-15*b^4 - 40*b^3*c*x^2 - 29*b^2*
c^2*x^4 + 8*b*c^3*x^6 + 12*c^4*x^8) - 12*Sqrt[b]*c^(7/2)*x^7*Sqrt
[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]],
 -1] + 12*Sqrt[b]*c^(7/2)*x^7*Sqrt[1 + (c*x^2)/b]*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(195*b^2*x^(11/2)*Sqrt[(
I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.048, size = 250, normalized size = 0.7

$$-\frac{2}{195\,(cx^2+b)^2 b^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(12\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\mathit{EllipticE}\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^6bc^3-6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(21/2),x)
```

```
[Out] -2/195*(c*x^4+b*x^2)^(3/2)/x^(19/2)/(c*x^2+b)^2*(12*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^6*b*c^3-6*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-
x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))*2^(1/2)*x^6*b*c^3-12*x^8*c^4-8*x^6*b*c^3+29
*x^4*b^2*c^2+40*x^2*b^3*c+15*b^4)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}(cx^2+b)}{x^{\frac{17}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(17/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(21/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2)^{\frac{3}{2}}}{x^{\frac{21}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(21/2), x)
```

3.377 $\int \frac{(bx^2+cx^4)^{3/2}}{x^{23/2}}\,dx$

Optimal. Leaf size=203

$$\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(55*x^(13/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(385*b*x^(9/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5
/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(15*x^(21/2)) + (4*c^(15/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*b^(9/4)*Sq
rt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.551647, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{4c^{15/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{bx^2+cx^4}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{5/2}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{9/2}}-\frac{4c\sqrt{bx^2+cx^4}}{55x^{13/2}}-\frac{2\left(bx^2+cx^4\right)^{3/2}}{15x^{21/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (-4*c*Sqrt[b*x^2 + c*x^4])/(55*x^(13/2)) - (8*c^2*Sqrt[b*x^2 + c*
x^4])/(385*b*x^(9/2)) + (8*c^3*Sqrt[b*x^2 + c*x^4])/(231*b^2*x^(5
/2)) - (2*(b*x^2 + c*x^4)^(3/2))/(15*x^(21/2)) + (4*c^(15/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(231*b^(9/4)*Sq
rt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 59.3392, size = 194, normalized size = 0.96

$$-\frac{4c\sqrt{bx^2+cx^4}}{55x^{\frac{13}{2}}}-\frac{2\left(bx^2+cx^4\right)^{\frac{3}{2}}}{15x^{\frac{21}{2}}}-\frac{8c^2\sqrt{bx^2+cx^4}}{385bx^{\frac{9}{2}}}+\frac{8c^3\sqrt{bx^2+cx^4}}{231b^2x^{\frac{5}{2}}}+\frac{4c^{\frac{15}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{231b^{\frac{9}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2)**(3/2)/x**(23/2),x)
```

```
[Out] -4*c*sqrt(b*x**2 + c*x**4)/(55*x**(13/2)) - 2*(b*x**2 + c*x**4)**
(3/2)/(15*x**(21/2)) - 8*c**2*sqrt(b*x**2 + c*x**4)/(385*b*x**(9/
2)) + 8*c**3*sqrt(b*x**2 + c*x**4)/(231*b**2*x**(5/2)) + 4*c**(15
/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c
)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**
(1/4)), 1/2)/(231*b**(9/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.350614, size = 165, normalized size = 0.81

$$\frac{2\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-77b^4-196b^3cx^2-131b^2c^2x^4+8bc^3x^6+20c^4x^8\right)+20ic^4x^{17/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)\right)}{1155b^2x^{13/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x^2 + c*x^4)^(3/2)/x^(23/2),x]
```

```
[Out] (2*(Sqrt[(I*Sqrt[b])/Sqrt[c]]*(-77*b^4 - 196*b^3*c*x^2 - 131*b^2*
c^2*x^4 + 8*b*c^3*x^6 + 20*c^4*x^8) + (20*I)*c^4*Sqrt[1 + b/(c*x^
2)]*x^(17/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x
]], -1]))/(1155*b^2*Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(13/2)*Sqrt[x^2*(
b + c*x^2)])
```

Maple [A] time = 0.045, size = 167, normalized size = 0.8

$$\frac{2}{1155\left(cx^2+b\right)^2b^2}\left(cx^4+bx^2\right)^{\frac{3}{2}}\left(10\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^7c^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2)^(3/2)/x^(23/2),x)
```

```
[Out] 2/1155*(c*x^4+b*x^2)^(3/2)/x^(21/2)/(c*x^2+b)^2*(10*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^7*c^3+20*x^8*c^
4+8*x^6*b*c^3-131*x^4*b^2*c^2-196*x^2*b^3*c-77*b^4)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2\right)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4+bx^2}\left(cx^2+b\right)}{x^{\frac{19}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)/x^(19/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2)**(3/2)/x**(23/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}{x^{\frac{23}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2)^(3/2)/x^(23/2), x)
```

3.378 $\int \frac{x^{13/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}+\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}$$

```
[Out] (30*b^2*Sqrt[b*x^2 + c*x^4])/(77*c^3*Sqrt[x]) - (18*b*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^2) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*
c) - (15*b^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.462804, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{15b^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77c^{13/4}\sqrt{bx^2+cx^4}}+\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{3/2}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{7/2}\sqrt{bx^2+cx^4}}{11c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (30*b^2*Sqrt[b*x^2 + c*x^4])/(77*c^3*Sqrt[x]) - (18*b*x^(3/2)*Sqr
t[b*x^2 + c*x^4])/(77*c^2) + (2*x^(7/2)*Sqrt[b*x^2 + c*x^4])/(11*
c) - (15*b^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[
b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)],
 1/2])/(77*c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 45.1031, size = 172, normalized size = 0.96

$$-\frac{15b^{\frac{11}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{77c^{\frac{13}{4}}x\left(b+cx^2\right)}+\frac{30b^2\sqrt{bx^2+cx^4}}{77c^3\sqrt{x}}-\frac{18bx^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{77c^2}+\frac{2x^{\frac{7}{2}}\sqrt{bx^2+cx^4}}{11c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -15*b**(11/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b
) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*s
qrt(x)/b**(1/4)), 1/2)/(77*c**(13/4)*x*(b + c*x**2)) + 30*b**2*sq
rt(b*x**2 + c*x**4)/(77*c**3*sqrt(x)) - 18*b*x**(3/2)*sqrt(b*x**2
 + c*x**4)/(77*c**2) + 2*x**(7/2)*sqrt(b*x**2 + c*x**4)/(11*c)
```

Mathematica [C] time = 0.186801, size = 153, normalized size = 0.85

$$\frac{2x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(15b^3+6b^2cx^2-2bc^2x^4+7c^3x^6\right)-30ib^3x^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{77c^3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2)*(15*b^3 + 6*b^2*c*x^2 - 2*b*
c^2*x^4 + 7*c^3*x^6) - (30*I)*b^3*Sqrt[1 + b/(c*x^2)]*x^2*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(77*Sqrt[(I
*Sqrt[b])/Sqrt[c]]*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.036, size = 148, normalized size = 0.8

$$-\frac{1}{77\,c^4}\sqrt{x}\left(-14\,x^7c^4+15\,b^3\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)+4\,x^5bc^3-12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-14*x^7*c^4+15*b^3*(-b*c)^(1/2
)*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c
*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+4*x^5*b*c^3-12*
x^3*b^2*c^2-30*x*b^3*c)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(13/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(c*x^4 + b*x^2), x)
```

3.379 $\int \frac{x^{11/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} - \frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} + \frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}$$

```
[Out] (14*b^2*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (14*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^2)
+ (2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (14*b^(9/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^
2 + c*x^4]) + (7*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.629464, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{7b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} - \frac{14b^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15c^{11/4}\sqrt{bx^2+cx^4}} + \frac{14b^2x^{3/2}\left(b+cx^2\right)}{15c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2} + \frac{2x^{5/2}\sqrt{bx^2+cx^4}}{9c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (14*b^2*x^(3/2)*(b + c*x^2))/(15*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (14*b*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(45*c^2)
```

```
+ (2*x^(5/2)*Sqrt[b*x^2 + c*x^4])/(9*c) - (14*b^(9/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^
2 + c*x^4]) + (7*b^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(15*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 60.4571, size = 279, normalized size = 0.94

$$-\frac{14b^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{7b^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15c^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{14b^2\sqrt{bx^2+cx^4}}{15c^{\frac{5}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{14b\sqrt{x}\sqrt{bx^2+cx^4}}{45c^2}+\frac{2x^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{9c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -14*b**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
 + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sq
rt(x)/b**(1/4)), 1/2)/(15*c**(11/4)*x*(b + c*x**2)) + 7*b**(9/4)*
sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)
*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4
)), 1/2)/(15*c**(11/4)*x*(b + c*x**2)) + 14*b**2*sqrt(b*x**2 + c*
x**4)/(15*c**(5/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 14*b*sqrt(x)*
sqrt(b*x**2 + c*x**4)/(45*c**2) + 2*x**(5/2)*sqrt(b*x**2 + c*x**4
)/(9*c)
```

Mathematica [C] time = 0.253082, size = 190, normalized size = 0.64

$$\frac{2x^{3/2}\left(-21b^{5/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+21b^{5/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-7b^2-2b\right.\right.}{45c^{5/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-7*b^2 - 2*b*c
*x^2 + 5*c^2*x^4) + 21*b^(5/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*Ar
cSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] - 21*b^(5/2)*Sqrt[1 + (c*
x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(
45*c^(5/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.036, size = 217, normalized size = 0.7

$$\frac{1}{45\,c^3}\sqrt{x}\left(10\,c^3x^6+42\,b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-21\,b^3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/45/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^3*(10*c^3*x^6+42*b^3*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21*b^3*((c*x+(-b*c)^(1/2))
/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*
c)^(1/2))^(1/2),1/2*2^(1/2))-4*b*c^2*x^4-14*b^2*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] integral(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")

[Out] integrate(x^(11/2)/sqrt(c*x^4 + b*x^2), x)
```

3.380 $\int \frac{x^{9/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

```
[Out] (-10*b*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*
x^2 + c*x^4])/(7*c) + (5*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.363642, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{5b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21c^{9/4}\sqrt{bx^2+cx^4}}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{3/2}\sqrt{bx^2+cx^4}}{7c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-10*b*Sqrt[b*x^2 + c*x^4])/(21*c^2*Sqrt[x]) + (2*x^(3/2)*Sqrt[b*
x^2 + c*x^4])/(7*c) + (5*b^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(21*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 36.5821, size = 143, normalized size = 0.96

$$\frac{5b^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{21c^{\frac{9}{4}}x\left(b+cx^2\right)}-\frac{10b\sqrt{bx^2+cx^4}}{21c^2\sqrt{x}}+\frac{2x^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 5*b**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) +
 sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt
(x)/b**(1/4)), 1/2)/(21*c**(9/4)*x*(b + c*x**2)) - 10*b*sqrt(b*x*
*2 + c*x**4)/(21*c**2*sqrt(x)) + 2*x**(3/2)*sqrt(b*x**2 + c*x**4)
/(7*c)
```

Mathematica [C] time = 0.161149, size = 144, normalized size = 0.97

$$\frac{x\left(b+cx^2\right)\left(\frac{2x^{5/2}}{7c}-\frac{10b\sqrt{x}}{21c^2}\right)}{\sqrt{x^2\left(b+cx^2\right)}}+\frac{10ib^2x^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{21c^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(b + c*x^2)*((-10*b*Sqrt[x])/(21*c^2) + (2*x^(5/2))/(7*c)))/Sq
rt[x^2*(b + c*x^2)] + (((10*I)/21)*b^2*Sqrt[1 + b/(c*x^2)]*x^2*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(Sqrt[
(I*Sqrt[b])/Sqrt[c]]*c^2*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 137, normalized size = 0.9

$$\frac{1}{21\,c^3}\sqrt{x}\left(5\,b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)+6\,c^3x^5-4\,bc^2x^3-10\,b^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(5*b^2*(-b*c)^(1/2)*((c*x+(-b*c)
^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+6*c^3*x^5-4*b*c^2*x^3-10*b^2*
c*x)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{9}{2}}}{\sqrt{cx^4 + bx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")

[Out] integrate(x^(9/2)/sqrt(c*x^4 + b*x^2), x)
```

3.381 $\int \frac{x^{7/2}}{\sqrt{bx^2+cx^4}} dx$

Optimal. Leaf size=266

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-\frac{6bx^{3/2}\left(b+cx^2\right)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$

```
[Out] (-6*b*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (6*b^(5
/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7
/4)*Sqrt[b*x^2 + c*x^4]) - (3*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.509562, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{3b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}+\frac{6b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{7/4}\sqrt{bx^2+cx^4}}-\frac{6bx^{3/2}\left(b+cx^2\right)}{5c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (-6*b*x^(3/2)*(b + c*x^2))/(5*c^(3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[
b*x^2 + c*x^4]) + (2*Sqrt[x]*Sqrt[b*x^2 + c*x^4])/(5*c) + (6*b^(5
/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
```

```
)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7
/4)*Sqrt[b*x^2 + c*x^4]) - (3*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 54.3762, size = 250, normalized size = 0.94

$$\frac{6b^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{4}}x\left(b+cx^2\right)} - \frac{3b^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{7}{4}}x\left(b+cx^2\right)} - \frac{6b\sqrt{bx^2+cx^4}}{5c^{\frac{3}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)} + \frac{2\sqrt{x}\sqrt{bx^2+cx^4}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 6*b**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) +
 sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqrt
(x)/b**(1/4)), 1/2)/(5*c**(7/4)*x*(b + c*x**2)) - 3*b**(5/4)*sqrt
((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqr
t(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)),
1/2)/(5*c**(7/4)*x*(b + c*x**2)) - 6*b*sqrt(b*x**2 + c*x**4)/(5*c
**(3/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 2*sqrt(x)*sqrt(b*x**2 +
c*x**4)/(5*c)
```

Mathematica [C] time = 0.244677, size = 176, normalized size = 0.66

$$\frac{2x^{3/2}\left(3b^{3/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-3b^{3/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(b+cx^2\right)\right)}{5c^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(b + c*x^2) - 3
*b^(3/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*
x)/Sqrt[b]]], -1] + 3*b^(3/2)*Sqrt[1 + (c*x^2)/b]*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(5*c^(3/2)*Sqrt[(I*Sqrt[
c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.019, size = 206, normalized size = 0.8

$$-\frac{1}{5\,c^2}\sqrt{x}\left(6\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)-3\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)*x^(1/2)/c^2*(6*b^2*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)
^(1/2))^(1/2),1/2*2^(1/2))-3*b^2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-
b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/
2),1/2*2^(1/2))-2*c^2*x^4-2*b*c*x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/sqrt(c*x^4 + b*x^2), x)
```

3.382 $\int \frac{x^{5/2}}{\sqrt{bx^2+cx^4}} \, dx$

Optimal. Leaf size=121

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x
^4])
```

Rubi [A] time = 0.271154, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}-\frac{b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3c^{5/4}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[b*x^2 + c*x^4])/(3*c*Sqrt[x]) - (b^(3/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*c^(5/4)*Sqrt[b*x^2 + c*x
^4])
```

Rubi in Sympy [A] time = 27.4183, size = 114, normalized size = 0.94

$$-\frac{b^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{3c^{\frac{5}{4}}x\left(b+cx^2\right)}+\frac{2\sqrt{bx^2+cx^4}}{3c\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -b**(3/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) +
sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(
x)/b**(1/4)), 1/2)/(3*c**(5/4)*x*(b + c*x**2)) + 2*sqrt(b*x**2 +
c*x**4)/(3*c*sqrt(x))
```

Mathematica [C] time = 0.103233, size = 126, normalized size = 1.04

$$\frac{2x^{3/2}\left(b+cx^2\right)}{3c\sqrt{x^2\left(b+cx^2\right)}}-\frac{2ibx^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{3c\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(b + c*x^2))/(3*c*Sqrt[x^2*(b + c*x^2)]) - (((2*I)/3)*
b*Sqrt[1 + b/(c*x^2)]*x^2*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sq
rt[c]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[c]]*c*Sqrt[x^2*(b +
c*x^2)])
```

Maple [A] time = 0.016, size = 123, normalized size = 1.

$$-\frac{1}{3\,c^2}\sqrt{x}\left(b\sqrt{-bc}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\mathit{EllipticF}\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(b*(-b*c)^(1/2)*((c*x+(-b*c)^(1/
2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c^2*x^3-2*b*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{5}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2 (b + cx^2)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/sqrt(c*x^4 + b*x^2), x)
```

3.383 $\int \frac{x^{3/2}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=231

$$\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2x^{3/2}\left(b+cx^2\right)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

```
[Out] (2*x^(3/2)*(b + c*x^2))/(Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (2*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (b^(1/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4)*Sqrt[b*x^2 +
c*x^4])
```

Rubi [A] time = 0.401544, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{bx^2+cx^4}}+\frac{2x^{3/2}\left(b+cx^2\right)}{\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (2*x^(3/2)*(b + c*x^2))/(Sqrt[c]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2
 + c*x^4]) - (2*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(c^(3/4)*Sqrt[b*x^2 + c*x^4]) + (b^(1/4)*x*(Sqrt[b]
+ Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[
2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(3/4)*Sqrt[b*x^2 +
c*x^4])
```

Rubi in Sympy [A] time = 38.0441, size = 218, normalized size = 0.94

$$-\frac{2\sqrt[4]{b}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{\sqrt[4]{b}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{2\sqrt{bx^2+cx^4}}{\sqrt{c}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*b**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(c**(3/4)*x*(b + c*x**2)) + b**(1/4)*sqrt((b
 + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b
*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2
)/(c**(3/4)*x*(b + c*x**2)) + 2*sqrt(b*x**2 + c*x**4)/(sqrt(c)*sq
rt(x)*(sqrt(b) + sqrt(c)*x))
```

Mathematica [C] time = 0.0882347, size = 112, normalized size = 0.48

$$\frac{2ix^{5/2}\sqrt{\frac{cx^2}{b}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{c}x}{\sqrt{b}}\right)^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((2*I)*x^(5/2)*Sqrt[1 + (c*x^2)/b]*(EllipticE[I*ArcSinh[Sqrt[(I*S
qrt[c]*x)/Sqrt[b]]], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)
/Sqrt[b]]], -1]))/(((I*Sqrt[c]*x)/Sqrt[b])^(3/2)*Sqrt[x^2*(b + c*
x^2)])
```

Maple [A] time = 0.018, size = 131, normalized size = 0.6

$$\frac{b\sqrt{2}}{c}\sqrt{x}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\left(2\,EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-EllipticF\left(\sqrt{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*b/c*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*(2*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))-EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(
1/2),1/2*2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] integral(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(x**(3/2)/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")

[Out] integrate(x^(3/2)/sqrt(c*x^4 + b*x^2), x)
```

3.384 $\int \frac{\sqrt{x}}{\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

```
[Out] (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c
^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.181139, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2
]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(1/4)*c
^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 16.9803, size = 88, normalized size = 0.98

$$\frac{\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt[4]{c}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)
*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4
```

```
)), 1/2)/(b**(1/4)*c**(1/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.0447842, size = 85, normalized size = 0.94

$$\frac{2ix^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/Sqrt[b*x^2 + c*x^4],x]
```

```
[Out] ((2*I)*Sqrt[1 + b/(c*x^2)]*x^2*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b
])/Sqrt[c]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[b])/Sqrt[c]]*Sqrt[x^2*(b
 + c*x^2)])
```

Maple [A] time = 0.022, size = 106, normalized size = 1.2

$$\frac{\sqrt{2}}{c}\sqrt{x}\sqrt{-bc}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}EllipticF\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\frac{1}{\sqrt{cx^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(-b*c)^(1/2)*((c*x+(-b*c)^(1/2))/(-
b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2
)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2),1/2*2^(1/2))/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^4 + bx^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] integral(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x^2\,(b + cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(sqrt(x)/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{cx^4 + bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/sqrt(c*x^4 + b*x^2),x, algorithm="giac")

[Out] integrate(sqrt(x)/sqrt(c*x^4 + b*x^2), x)
```

3.385 $\int \frac{1}{\sqrt{x}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=259

$$\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

```
[Out] (2*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x
^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*c^(1/4)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqr
t[b*x^2 + c*x^4]) + (c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.501985, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{bx^2+cx^4}}+\frac{2\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[c]*x^(3/2)*(b + c*x^2))/(b*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x
^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(b*x^(3/2)) - (2*c^(1/4)*x
*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*
EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*Sqr
t[b*x^2 + c*x^4]) + (c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*
x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x]
)/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 49.3426, size = 241, normalized size = 0.93

$$\frac{2\sqrt{c}\sqrt{bx^2+cx^4}}{b\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{2\sqrt{bx^2+cx^4}}{bx^{\frac{3}{2}}}-\frac{2\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}x\left(b+cx^2\right)}+\frac{\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] 2*sqrt(c)*sqrt(b*x**2 + c*x**4)/(b*sqrt(x)*(sqrt(b) + sqrt(c)*x))
 - 2*sqrt(b*x**2 + c*x**4)/(b*x**(3/2)) - 2*c**(1/4)*sqrt((b + c*
x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2
 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(b*
*(3/4)*x*(b + c*x**2)) + c**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sq
rt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic
_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(b**(3/4)*x*(b + c*x**
2))
```

Mathematica [C] time = 0.189927, size = 177, normalized size = 0.68

$$-\frac{2\sqrt{x}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(b+cx^2\right)+\sqrt{b}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-\sqrt{b}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)\right)}{b\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[x]*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(b + c*x^2) - Sqrt[b]*Sq
rt[c]*x*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x
)/Sqrt[b]]], -1] + Sqrt[b]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*Elliptic
F[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(b*Sqrt[(I*Sqrt[c
]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.024, size = 195, normalized size = 0.8

$$\frac{1}{b}\sqrt{x}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}b-\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(-cx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x^(1/2)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2)
)^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/
2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2
^(1/2))*2^(1/2)*b-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*((-c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*Ellip
ticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)
*b-2*c*x^2-2*b)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2}\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(1/(sqrt(x)*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

3.386 $\int \frac{1}{x^{3/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*
x^4])
```

Rubi [A] time = 0.277644, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{bx^2+cx^4}}-\frac{2\sqrt{bx^2+cx^4}}{3bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(3*b*x^(5/2)) - (c^(3/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(3*b^(5/4)*Sqrt[b*x^2 + c*
x^4])
```

Rubi in Sympy [A] time = 26.0402, size = 116, normalized size = 0.96

$$-\frac{2\sqrt{bx^2+cx^4}}{3bx^{\frac{5}{2}}}-\frac{c^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{3b^{\frac{5}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(3*b*x**(5/2)) - c**(3/4)*sqrt((b + c*x*
*2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 +
 c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(3*b*
*(5/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.273239, size = 110, normalized size = 0.91

$$\frac{2\left(-\frac{icx^{5/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}-b-cx^2\right)}{3b\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*(-b - c*x^2 - (I*c*Sqrt[1 + b/(c*x^2)]*x^(5/2)*EllipticF[I*Arc
Sinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[b])/Sq
rt[c]]))/(3*b*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.022, size = 119, normalized size = 1.

$$-\frac{1}{3\,b}\left(EllipticF\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}\sqrt{-bc}\sqrt{2}x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3/(c*x^4+b*x^2)^(1/2)/x^(1/2)*(EllipticF(((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*(-b*c)^(1/2)*2^(1/2)*x+2*c*x^2+2*b)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)),x, algorithm="fricas")

[Out] integral(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(1/(x**(3/2)*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(3/2)), x)
```

3.387 $\int \frac{1}{x^{5/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=296

$$-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

```
[Out] (-6*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[b*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + (6*c*
Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) + (6*c^(5/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*
x^4]) - (3*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.625011, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{3c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}+\frac{6c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{bx^2+cx^4}}-\frac{6c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{3/2}}-\frac{2\sqrt{bx^2+cx^4}}{5bx^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-6*c^(3/2)*x^(3/2)*(b + c*x^2))/(5*b^2*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[b*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(5*b*x^(7/2)) + (6*c*
```

```
Sqrt[b*x^2 + c*x^4])/(5*b^2*x^(3/2)) + (6*c^(5/4)*x*(Sqrt[b] + Sq
rt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*Ar
cTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*
x^4]) - (3*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt
[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)]
, 1/2])/(5*b^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 58.5464, size = 279, normalized size = 0.94

$$-\frac{2\sqrt{bx^2+cx^4}}{5bx^{\frac{7}{2}}}-\frac{6c^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{5b^2\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{6c\sqrt{bx^2+cx^4}}{5b^2x^{\frac{3}{2}}}+\frac{6c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}x\left(b+cx^2\right)}-\frac{3c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{7}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(5*b*x**(7/2)) - 6*c**(3/2)*sqrt(b*x**2
+ c*x**4)/(5*b**2*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + 6*c*sqrt(b*x**
2 + c*x**4)/(5*b**2*x**(3/2)) + 6*c**(5/4)*sqrt((b + c*x**2)/(sqr
t(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)
*elliptic_e(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*b**(7/4)*x
*(b + c*x**2)) - 3*c**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*
x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*a
tan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*b**(7/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.240444, size = 199, normalized size = 0.67

$$\frac{2\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-b^2+2bcx^2+3c^2x^4\right)+6\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-6\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right.\right.}{5b^2x^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-b^2 + 2*b*c*x^2 + 3*c^2*x^4) - 6
*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt
[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 6*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 + (c
*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1])/(
5*b^2*x^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.024, size = 215, normalized size = 0.7

$$-\frac{1}{5\,b^2}\left(6\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)\sqrt{2}x^2bc-3\,\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/5/(c*x^4+b*x^2)^(1/2)/x^(3/2)*(6*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*2^(1/2)*x^2*b*c-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^
(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/
2))*2^(1/2)*x^2*b*c-6*c^2*x^4-4*b*c*x^2+2*b^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)),x, algorithm="fricas")

[Out] integral(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(5/2)/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(1/(x**(5/2)*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(5/2)), x)
```

3.388 $\int \frac{1}{x^{7/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=149

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + (10*c*Sqrt[b*x^2 + c*x^4
])/(21*b^2*x^(5/2)) + (5*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.366148, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{5c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{bx^2+cx^4}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{5/2}}-\frac{2\sqrt{bx^2+cx^4}}{7bx^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(7*b*x^(9/2)) + (10*c*Sqrt[b*x^2 + c*x^4
])/(21*b^2*x^(5/2)) + (5*c^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b
+ c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqr
t[x])/b^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 34.905, size = 143, normalized size = 0.96

$$-\frac{2\sqrt{bx^2+cx^4}}{7bx^{\frac{9}{2}}}+\frac{10c\sqrt{bx^2+cx^4}}{21b^2x^{\frac{5}{2}}}+\frac{5c^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{21b^{\frac{9}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(7*b*x**(9/2)) + 10*c*sqrt(b*x**2 + c*x*
*4)/(21*b**2*x**(5/2)) + 5*c**(7/4)*sqrt((b + c*x**2)/(sqrt(b) +
sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*ellipt
ic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(21*b**(9/4)*x*(b +
c*x**2))
```

Mathematica [C] time = 0.161682, size = 144, normalized size = 0.97

$$\frac{x\left(\frac{10c}{21b^2x^{3/2}}-\frac{2}{7bx^{7/2}}\right)\left(b+cx^2\right)}{\sqrt{x^2\left(b+cx^2\right)}}+\frac{10ic^2x^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{21b^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] ((-2/(7*b*x^(7/2)) + (10*c)/(21*b^2*x^(3/2)))*x*(b + c*x^2))/Sqrt
[x^2*(b + c*x^2)] + (((10*I)/21)*c^2*Sqrt[1 + b/(c*x^2)]*x^2*Elli
pticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(b^2*Sqr
t[(I*Sqrt[b])/Sqrt[c]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.022, size = 134, normalized size = 0.9

$$\frac{1}{21\,b^2}\left(5\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^3c+10\,c^2x^4+4\,bcx^2-6\,b^2\right)x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/21/(c*x^4+b*x^2)^(1/2)/x^(5/2)*(5*((c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^
(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/
2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^3*c+10*c^2*x^4+4*b*c*x^2-6*b^2)
/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\left(b + cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(7/2)), x)
```

3.389 $\int \frac{1}{x^{9/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=326

$$\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} - \frac{14c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} + \frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

```
[Out] (14*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + (14
*c*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) - (14*c^2*Sqrt[b*x^2 + c
*x^4])/(15*b^3*x^(3/2)) - (14*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (7
*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1
5*b^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.746663, antiderivative size = 326, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{7c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} - \frac{14c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{bx^2+cx^4}} + \frac{14c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{3/2}} + \frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{7/2}} - \frac{2\sqrt{bx^2+cx^4}}{9bx^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (14*c^(5/2)*x^(3/2)*(b + c*x^2))/(15*b^3*(Sqrt[b] + Sqrt[c]*x)*Sq
rt[b*x^2 + c*x^4]) - (2*Sqrt[b*x^2 + c*x^4])/(9*b*x^(11/2)) + (14
```

```
*c*Sqrt[b*x^2 + c*x^4])/(45*b^2*x^(7/2)) - (14*c^2*Sqrt[b*x^2 + c
*x^4])/(15*b^3*x^(3/2)) - (14*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(15*b^(11/4)*Sqrt[b*x^2 + c*x^4]) + (7
*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt
[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(1
5*b^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 76.4918, size = 308, normalized size = 0.94

$$-\frac{2\sqrt{bx^2+cx^4}}{9bx^{\frac{11}{2}}}+\frac{14c\sqrt{bx^2+cx^4}}{45b^2x^{\frac{7}{2}}}+\frac{14c^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{15b^3\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{14c^2\sqrt{bx^2+cx^4}}{15b^3x^{\frac{3}{2}}}-\frac{14c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{11}{4}}x\left(b+cx^2\right)}+\frac{7c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{11}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(9*b*x**(11/2)) + 14*c*sqrt(b*x**2 + c*x
**4)/(45*b**2*x**(7/2)) + 14*c**(5/2)*sqrt(b*x**2 + c*x**4)/(15*b
**3*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 14*c**2*sqrt(b*x**2 + c*x**4
)/(15*b**3*x**(3/2)) - 14*c**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + s
qrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*ellipti
c_e(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(15*b**(11/4)*x*(b +
c*x**2)) + 7*c**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)
*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c*
*(1/4)*sqrt(x)/b**(1/4)), 1/2)/(15*b**(11/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.260137, size = 210, normalized size = 0.64

$$\frac{-2\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(5b^3-2b^2cx^2+14bc^2x^4+21c^3x^6\right)-42\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+42\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}E\left(\right.}{45b^3x^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(9/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(5*b^3 - 2*b^2*c*x^2 + 14*b*c^2*x
^4 + 21*c^3*x^6) + 42*Sqrt[b]*c^(5/2)*x^5*Sqrt[1 + (c*x^2)/b]*Ell
ipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] - 42*Sqrt[b]*c
^(5/2)*x^5*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c
]*x)/Sqrt[b]]], -1])/(45*b^3*x^(7/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*
Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.025, size = 230, normalized size = 0.7

$$-\frac{1}{45\,b^3}\left(21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^4bc^2-42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(9/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/45/(c*x^4+b*x^2)^(1/2)/x^(7/2)*(21*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-42*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1
/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*
2^(1/2))*2^(1/2)*x^4*b*c^2+42*c^3*x^6+28*b*c^2*x^4-4*b^2*c*x^2+10
*b^3)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(9/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2}x^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(9/2)), x)
```

3.390 $\int \frac{1}{x^{11/2}\sqrt{bx^2+cx^4}}\,dx$

Optimal. Leaf size=179

$$-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + (18*c*Sqrt[b*x^2 + c*x
^4])/(77*b^2*x^(9/2)) - (30*c^2*Sqrt[b*x^2 + c*x^4])/(77*b^3*x^(5
/2)) - (15*c^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(77*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.4544, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$-\frac{15c^{11/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{13/4}\sqrt{bx^2+cx^4}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{5/2}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{9/2}}-\frac{2\sqrt{bx^2+cx^4}}{11bx^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[b*x^2 + c*x^4])/(11*b*x^(13/2)) + (18*c*Sqrt[b*x^2 + c*x
^4])/(77*b^2*x^(9/2)) - (30*c^2*Sqrt[b*x^2 + c*x^4])/(77*b^3*x^(5
/2)) - (15*c^(11/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqr
t[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)
], 1/2])/(77*b^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 45.6709, size = 172, normalized size = 0.96

$$-\frac{2\sqrt{bx^2+cx^4}}{11bx^{\frac{13}{2}}}+\frac{18c\sqrt{bx^2+cx^4}}{77b^2x^{\frac{9}{2}}}-\frac{30c^2\sqrt{bx^2+cx^4}}{77b^3x^{\frac{5}{2}}}-\frac{15c^{\frac{11}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{77b^{\frac{13}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*sqrt(b*x**2 + c*x**4)/(11*b*x**(13/2)) + 18*c*sqrt(b*x**2 + c*
x**4)/(77*b**2*x**(9/2)) - 30*c**2*sqrt(b*x**2 + c*x**4)/(77*b**3
*x**(5/2)) - 15*c**(11/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)
**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*ata
n(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(77*b**(13/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.336098, size = 134, normalized size = 0.75

$$\frac{2\left(-7b^3+2b^2cx^2-\frac{15ic^3x^{13/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}-6bc^2x^4-15c^3x^6\right)}{77b^3x^{9/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(11/2)*Sqrt[b*x^2 + c*x^4]),x]
```

```
[Out] (2*(-7*b^3 + 2*b^2*c*x^2 - 6*b*c^2*x^4 - 15*c^3*x^6 - ((15*I)*c^3
*Sqrt[1 + b/(c*x^2)]*x^(13/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b]
)/Sqrt[c]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[b])/Sqrt[c]]))/(77*b^3*x^(
9/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.022, size = 147, normalized size = 0.8

$$-\frac{1}{77\,b^3}\left(15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^5c^2+30\,c^3x^6+12\,bc^2x^4-4\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(11/2)/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/77/(c*x^4+b*x^2)^(1/2)/x^(9/2)*(15*((c*x+(-b*c)^(1/2))/(-b*c)^
(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c
)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),
1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^5*c^2+30*c^3*x^6+12*b*c^2*x^4
```

```
-4*b^2*c*x^2+14*b^3)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{11}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(11/2)/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2}x^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c*x^4 + b*x^2)*x^(11/2)), x)
```

3.391 $\int \frac{x^{17/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=174

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/
(7*c^3*Sqrt[x]) + (9*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c^2) + (15*b
^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*
c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.457264, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{15b^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14c^{13/4}\sqrt{bx^2+cx^4}}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}+\frac{9x^{3/2}\sqrt{bx^2+cx^4}}{7c^2}-\frac{x^{11/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(11/2)/(c*Sqrt[b*x^2 + c*x^4])) - (15*b*Sqrt[b*x^2 + c*x^4])/
(7*c^3*Sqrt[x]) + (9*x^(3/2)*Sqrt[b*x^2 + c*x^4])/(7*c^2) + (15*b
^(7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c
]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*
c^(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 46.2103, size = 165, normalized size = 0.95

$$\frac{15b^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{14c^{\frac{13}{4}}x\left(b+cx^2\right)}-\frac{15b\sqrt{bx^2+cx^4}}{7c^3\sqrt{x}}-\frac{x^{\frac{11}{2}}}{c\sqrt{bx^2+cx^4}}+\frac{9x^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{7c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(17/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 15*b**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(14*c**(13/4)*x*(b + c*x**2)) - 15*b*sqrt(b*
x**2 + c*x**4)/(7*c**3*sqrt(x)) - x**(11/2)/(c*sqrt(b*x**2 + c*x*
*4)) + 9*x**(3/2)*sqrt(b*x**2 + c*x**4)/(7*c**2)
```

Mathematica [C] time = 0.149513, size = 141, normalized size = 0.81

$$\frac{x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-15b^2-6bcx^2+2c^2x^4\right)+15ib^2x^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{7c^3\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(17/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2)*(-15*b^2 - 6*b*c*x^2 + 2*c^2*x
^4) + (15*I)*b^2*Sqrt[1 + b/(c*x^2)]*x^2*EllipticF[I*ArcSinh[Sqrt
[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(7*Sqrt[(I*Sqrt[b])/Sqrt[c]]
*c^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.051, size = 144, normalized size = 0.8

$$\frac{cx^2+b}{14\,c^4}x^{\frac{5}{2}}\left(15\,b^2\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)+4\,c^3x^5-12\,bc^2x^3-30\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(17/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(15*b^2*(-b*c)^(1/2)*(
(c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+
(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+4*c^3*x^5-12*b*c^2
*x^3-30*b^2*c*x)/c^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{17}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^{\frac{13}{2}}}{\sqrt{cx^4 + bx^2}(cx^2 + b)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")

[Out] integral(x^(13/2)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(17/2)/(c*x**4+b*x**2)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{17}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(x^(17/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.392 $\int \frac{x^{15/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=291

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}-\frac{21bx^{3/2}\left(b+cx^2\right)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) - (21*b*x^(3/2)*(b + c*x^2))/(
5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (7*Sqrt[x]
*Sqrt[b*x^2 + c*x^4])/(5*c^2) + (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
 - (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.624546, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10c^{11/4}\sqrt{bx^2+cx^4}}+\frac{21b^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5c^{11/4}\sqrt{bx^2+cx^4}}-\frac{21bx^{3/2}\left(b+cx^2\right)}{5c^{5/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}-\frac{x^{9/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(9/2)/(c*Sqrt[b*x^2 + c*x^4])) - (21*b*x^(3/2)*(b + c*x^2))/(
5*c^(5/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (7*Sqrt[x]
```

```
*Sqrt[b*x^2 + c*x^4])/(5*c^2) + (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*
x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(
c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(5*c^(11/4)*Sqrt[b*x^2 + c*x^4])
 - (21*b^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b]
+ Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/
2])/(10*c^(11/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 59.285, size = 272, normalized size = 0.93

$$\frac{21b^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5c^{\frac{11}{4}}x\left(b+cx^2\right)} - \frac{21b^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{10c^{\frac{11}{4}}x\left(b+cx^2\right)} - \frac{21b\sqrt{bx^2+cx^4}}{5c^{\frac{5}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)} - \frac{x^{\frac{9}{2}}}{c\sqrt{bx^2+cx^4}} + \frac{7\sqrt{x}\sqrt{bx^2+cx^4}}{5c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(15/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 21*b**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(5*c**(11/4)*x*(b + c*x**2)) - 21*b**(5/4)*s
qrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*
sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)
), 1/2)/(10*c**(11/4)*x*(b + c*x**2)) - 21*b*sqrt(b*x**2 + c*x**4
)/(5*c**(5/2)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - x**(9/2)/(c*sqrt(b
*x**2 + c*x**4)) + 7*sqrt(x)*sqrt(b*x**2 + c*x**4)/(5*c**2)
```

Mathematica [C] time = 0.209121, size = 179, normalized size = 0.62

$$\frac{x^{3/2}\left(21b^{3/2}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-21b^{3/2}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(7b+2cx^2\right)\right)}{5c^{5/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(15/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(7*b + 2*c*x^2) -
 21*b^(3/2)*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[
c]*x)/Sqrt[b]]], -1] + 21*b^(3/2)*Sqrt[1 + (c*x^2)/b]*EllipticF[I
*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(5*c^(5/2)*Sqrt[(I*S
qrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.046, size = 213, normalized size = 0.7

$$-\frac{cx^2+b}{10\,c^3}x^{\frac{5}{2}}\left(42\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)-21\,b^2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(15/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(42*b^2*((c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1
/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-21*b^2*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(
1/2))^(1/2),1/2*2^(1/2))-4*c^2*x^4-14*b*c*x^2)/c^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{11}{2}}}{\sqrt{cx^4+bx^2}(cx^2+b)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^(11/2)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(15/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.393 $\int \frac{x^{13/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*Sqrt[b*x^2 + c*x^4])/(3*c
^2*Sqrt[x]) - (5*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(6*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.355145, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{5b^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6c^{9/4}\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}-\frac{x^{7/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(7/2)/(c*Sqrt[b*x^2 + c*x^4])) + (5*Sqrt[b*x^2 + c*x^4])/(3*c
^2*Sqrt[x]) - (5*b^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(6*c^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 33.8411, size = 138, normalized size = 0.95

$$-\frac{5b^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{6c^{\frac{9}{4}}x\left(b+cx^2\right)}-\frac{x^{\frac{7}{2}}}{c\sqrt{bx^2+cx^4}}+\frac{5\sqrt{bx^2+cx^4}}{3c^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -5*b**(3/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(6*c**(9/4)*x*(b + c*x**2)) - x**(7/2)/(c*sq
rt(b*x**2 + c*x**4)) + 5*sqrt(b*x**2 + c*x**4)/(3*c**2*sqrt(x))
```

Mathematica [C] time = 0.136805, size = 128, normalized size = 0.88

$$\frac{x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(5b+2cx^2\right)-5ibx^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{3c^2\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2)*(5*b + 2*c*x^2) - (5*I)*b*Sqrt
[1 + b/(c*x^2)]*x^2*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]
/Sqrt[x]], -1])/(3*Sqrt[(I*Sqrt[b])/Sqrt[c]]*c^2*Sqrt[x^2*(b + c*
x^2)])
```

Maple [A] time = 0.023, size = 131, normalized size = 0.9

$$-\frac{cx^2+b}{6\,c^3}x^{\frac{5}{2}}\left(5\,b\sqrt{-bc}\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{2}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)-4\,c^2x^3-10\,bcx\right)\left(cx^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(5*b*(-b*c)^(1/2)*((c*
x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-4*c^2*x^3-10*b*c*x)/c
^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{9}{2}}}{\sqrt{cx^4+bx^2}(cx^2+b)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")

[Out] integral(x^(9/2)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(13/2)/(c*x**4+b*x**2)**(3/2),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(x^(13/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.394 $\int \frac{x^{11/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{bx^2+cx^4}} + \frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x^(3/2)*(b + c*x^2))/(c^(
3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*b^(1/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(7/4)*Sqrt[b
*x^2 + c*x^4]) + (3*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.50391, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2c^{7/4}\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{b}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{c^{7/4}\sqrt{bx^2+cx^4}} + \frac{3x^{3/2}\left(b+cx^2\right)}{c^{3/2}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{x^{5/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(5/2)/(c*Sqrt[b*x^2 + c*x^4])) + (3*x^(3/2)*(b + c*x^2))/(c^(
3/2)*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*b^(1/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
```

```
ipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(c^(7/4)*Sqrt[b
*x^2 + c*x^4]) + (3*b^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x
^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])
/b^(1/4)], 1/2])/(2*c^(7/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 47.2224, size = 241, normalized size = 0.93

$$-\frac{3\sqrt[4]{b}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{7}{4}}x\left(b+cx^2\right)}+\frac{3\sqrt[4]{b}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2c^{\frac{7}{4}}x\left(b+cx^2\right)}-\frac{x^{\frac{5}{2}}}{c\sqrt{bx^2+cx^4}}+\frac{3\sqrt{bx^2+cx^4}}{c^{\frac{3}{2}}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -3*b**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(c**(7/4)*x*(b + c*x**2)) + 3*b**(1/4)*sqrt(
(b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt
(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1
/2)/(2*c**(7/4)*x*(b + c*x**2)) - x**(5/2)/(c*sqrt(b*x**2 + c*x**
4)) + 3*sqrt(b*x**2 + c*x**4)/(c**(3/2)*sqrt(x)*(sqrt(b) + sqrt(c
)*x))
```

Mathematica [C] time = 0.177834, size = 167, normalized size = 0.64

$$-\frac{x^{3/2}\left(3\sqrt{b}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-3\sqrt{b}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)}{c^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x^(3/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]] - 3*Sqrt[b]*Sqr
t[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]]
, -1] + 3*Sqrt[b]*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I
```

```
*Sqrt[c]*x)/Sqrt[b]]], -1]))/(c^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]
*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] time = 0.025, size = 200, normalized size = 0.8

$$\frac{cx^2+b}{2\,c^2}x^{\frac{5}{2}}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}b-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*b-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*2^(1/2)*b-2*c*x^2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{7}{2}}}{\sqrt{cx^4+bx^2}(cx^2+b)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^(7/2)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.395 $\int \frac{x^{9/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=119

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

```
[Out] -(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4]
)
```

Rubi [A] time = 0.269094, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{b}c^{5/4}\sqrt{bx^2+cx^4}}-\frac{x^{3/2}}{c\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -(x^(3/2)/(c*Sqrt[b*x^2 + c*x^4])) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqr
t[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4
)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(1/4)*c^(5/4)*Sqrt[b*x^2 + c*x^4]
)
```

Rubi in Sympy [A] time = 25.0596, size = 110, normalized size = 0.92

$$-\frac{x^{\frac{3}{2}}}{c\sqrt{bx^2+cx^4}}+\frac{\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{b}c^{\frac{5}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] -x**(3/2)/(c*sqrt(b*x**2 + c*x**4)) + sqrt((b + c*x**2)/(sqrt(b)
+ sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elli
ptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(2*b**(1/4)*c**(5/
4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.0925922, size = 115, normalized size = 0.97

$$\frac{ix^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)-x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{c\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2)) + I*Sqrt[1 + b/(c*x^2)]*x^2
*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(Sq
rt[(I*Sqrt[b])/Sqrt[c]]*c*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.018, size = 120, normalized size = 1.

$$\frac{cx^2+b}{2\,c^2}x^{\frac{5}{2}}\left(\sqrt{-bc}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}EllipticF\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))-2*c*x)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{9}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{5}{2}}}{\sqrt{cx^4 + bx^2}(cx^2 + b)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^(5/2)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{(cx^4 + bx^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.396 $\int \frac{x^{7/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=260

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x^{3/2}\left(b+cx^2\right)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

```
[Out] x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x^(3/2)*(b + c*x^2))/(b*Sqrt[c
]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4]) - (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(2*b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.510587, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$-\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}c^{3/4}\sqrt{bx^2+cx^4}}+\frac{x^{5/2}}{b\sqrt{bx^2+cx^4}}-\frac{x^{3/2}\left(b+cx^2\right)}{b\sqrt{c}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(5/2)/(b*Sqrt[b*x^2 + c*x^4]) - (x^(3/2)*(b + c*x^2))/(b*Sqrt[c
]*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt
[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcT
an[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(b^(3/4)*c^(3/4)*Sqrt[b*x^2
+ c*x^4]) - (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] +
Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2]
)/(2*b^(3/4)*c^(3/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 48.6595, size = 238, normalized size = 0.92

$$\frac{x^{\frac{5}{2}}}{b\sqrt{bx^2+cx^4}}-\frac{\sqrt{bx^2+cx^4}}{b\sqrt{c}\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}+\frac{\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{3}{4}}c^{\frac{3}{4}}x\left(b+cx^2\right)}-\frac{\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2b^{\frac{3}{4}}c^{\frac{3}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] x**(5/2)/(b*sqrt(b*x**2 + c*x**4)) - sqrt(b*x**2 + c*x**4)/(b*sqr
t(c)*sqrt(x)*(sqrt(b) + sqrt(c)*x)) + sqrt((b + c*x**2)/(sqrt(b)
+ sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elli
ptic_e(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(b**(3/4)*c**(3/4)
*x*(b + c*x**2)) - sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(s
qrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1
/4)*sqrt(x)/b**(1/4)), 1/2)/(2*b**(3/4)*c**(3/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.17029, size = 168, normalized size = 0.65

$$\frac{ix^{5/2}\left(\sqrt{b}\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-\sqrt{b}\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+\sqrt{c}x\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)}{b^{3/2}\left(\frac{i\sqrt{c}x}{\sqrt{b}}\right)^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (I*x^(5/2)*(Sqrt[c]*x*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]] - Sqrt[b]*Sqrt[
1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]],
-1] + Sqrt[b]*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqr
t[c]*x)/Sqrt[b]]], -1]))/(b^(3/2)*((I*Sqrt[c]*x)/Sqrt[b])^(3/2)*S
qrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.019, size = 203, normalized size = 0.8

$$-\frac{cx^2+b}{2\,bc}x^{\frac{5}{2}}\left(2\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)\sqrt{2}b-\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{1\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/2*x^(5/2)*(c*x^2+b)*(2*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)
*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/
2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))
*2^(1/2)*b-((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^
(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((
c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*2^(1/2)*b-2*c*
x^2)/(c*x^4+b*x^2)^(3/2)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{7}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^{\frac{3}{2}}}{\sqrt{cx^4+bx^2}(cx^2+b)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^(3/2)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{\left(cx^4 + bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.397 $\int \frac{x^{5/2}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

```
[Out] x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.269729, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{5/4}\sqrt[4]{c}\sqrt{bx^2+cx^4}}+\frac{x^{3/2}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] x^(3/2)/(b*Sqrt[b*x^2 + c*x^4]) + (x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(
b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*S
qrt[x])/b^(1/4)], 1/2])/(2*b^(5/4)*c^(1/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 24.9486, size = 110, normalized size = 0.93

$$\frac{x^{\frac{3}{2}}}{b\sqrt{bx^2+cx^4}}+\frac{\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2b^{\frac{5}{4}}\sqrt[4]{c}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] x**(3/2)/(b*sqrt(b*x**2 + c*x**4)) + sqrt((b + c*x**2)/(sqrt(b) +
 sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*ellip
```

```
tic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(2*b**(5/4)*c**(1/4
)*x*(b + c*x**2))
```

Mathematica [C] time = 0.0850134, size = 114, normalized size = 0.97

$$\frac{x^{3/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}+ix^2\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{b\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[c]]*x^(3/2) + I*Sqrt[1 + b/(c*x^2)]*x^2*El
lipticF[I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(b*Sqr
t[(I*Sqrt[b])/Sqrt[c]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.026, size = 123, normalized size = 1.

$$\frac{cx^2+b}{2\,bc}x^{\frac{5}{2}}\left(\sqrt{-bc}\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{2}\sqrt{1\left(-cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}}\sqrt{-cx\frac{1}{\sqrt{-bc}}}EllipticF\left(\sqrt{1\left(cx+\sqrt{-bc}\right)\frac{1}{\sqrt{-bc}}},\frac{\sqrt{2}}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*((-b*c)^(1/2)*((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2)*2^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c
)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(
1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))+2*c*x)/c/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}(cx^2+b)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(x)/(sqrt(c*x^4 + b*x^2)*(c*x^2 + b)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{5}{2}}}{\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2)**(3/2),x)

[Out] Integral(x**(5/2)/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{5}{2}}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")

[Out] integrate(x^(5/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.398 $\int \frac{x^{3/2}}{(bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=286

$$\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

```
[Out] Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*x^(3/2)*(b + c*x^2))
/(b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2
+ c*x^4])/(b^2*x^(3/2)) - (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1
/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7
/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.62361, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{bx^2+cx^4}} - \frac{3\sqrt[4]{c}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{bx^2+cx^4}} + \frac{3\sqrt{c}x^{3/2}\left(b+cx^2\right)}{b^2\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{3\sqrt{bx^2+cx^4}}{b^2x^{3/2}} + \frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] Sqrt[x]/(b*Sqrt[b*x^2 + c*x^4]) + (3*Sqrt[c]*x^(3/2)*(b + c*x^2))
/(b^2*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (3*Sqrt[b*x^2
```

```
+ c*x^4])/(b^2*x^(3/2)) - (3*c^(1/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt
[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)
*Sqrt[x])/b^(1/4)], 1/2])/(b^(7/4)*Sqrt[b*x^2 + c*x^4]) + (3*c^(1
/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x
)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(2*b^(7
/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 58.8549, size = 269, normalized size = 0.94

$$\frac{\sqrt{x}}{b\sqrt{bx^2+cx^4}}+\frac{3\sqrt{c}\sqrt{bx^2+cx^4}}{b^2\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)}-\frac{3\sqrt{bx^2+cx^4}}{b^2x^{\frac{3}{2}}}-\frac{3\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{b^{\frac{7}{4}}x\left(b+cx^2\right)}+\frac{3\sqrt[4]{c}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{2b^{\frac{7}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] sqrt(x)/(b*sqrt(b*x**2 + c*x**4)) + 3*sqrt(c)*sqrt(b*x**2 + c*x**
4)/(b**2*sqrt(x)*(sqrt(b) + sqrt(c)*x)) - 3*sqrt(b*x**2 + c*x**4)
/(b**2*x**(3/2)) - 3*c**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c
)*x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2
*atan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(b**(7/4)*x*(b + c*x**2))
+ 3*c**(1/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
 + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sq
rt(x)/b**(1/4)), 1/2)/(2*b**(7/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.203656, size = 181, normalized size = 0.63

$$-\frac{\sqrt{x}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(2b+3cx^2\right)+3\sqrt{b}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-3\sqrt{b}\sqrt{c}x\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)\right)}{b^2\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((Sqrt[x]*(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(2*b + 3*c*x^2) - 3*Sqrt[
b]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt
[c]*x)/Sqrt[b]]], -1] + 3*Sqrt[b]*Sqrt[c]*x*Sqrt[1 + (c*x^2)/b]*E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1]))/(b^2*Sqrt[
(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2)]))
```

Maple [A] time = 0.028, size = 203, normalized size = 0.7

$$\frac{cx^2+b}{2\,b^2}x^{\frac{5}{2}}\left(6\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}b-3\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/2/(c*x^4+b*x^2)^(3/2)*x^(5/2)*(c*x^2+b)*(6*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*b-3*((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(-b*c)^(1/2
))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^
(1/2))*2^(1/2)*b-6*c*x^2-4*b)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{cx^4+bx^2}(cx^3+bx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/(sqrt(c*x^4 + b*x^2)*(c*x^3 + b*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(x^2\,(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(x**(3/2)/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2)^(3/2), x)
```

3.399 $\int \frac{\sqrt{x}}{(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(3*b^
2*x^(5/2)) - (5*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(6*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.357011, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{5c^{3/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{5/2}}+\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] 1/(b*Sqrt[x]*Sqrt[b*x^2 + c*x^4]) - (5*Sqrt[b*x^2 + c*x^4])/(3*b^
2*x^(5/2)) - (5*c^(3/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/
(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(
1/4)], 1/2])/(6*b^(9/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 33.2621, size = 139, normalized size = 0.96

$$\frac{1}{b\sqrt{x}\sqrt{bx^2+cx^4}}-\frac{5\sqrt{bx^2+cx^4}}{3b^2x^{\frac{5}{2}}}-\frac{5c^{\frac{3}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{6b^{\frac{9}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*sqrt(x)*sqrt(b*x**2 + c*x**4)) - 5*sqrt(b*x**2 + c*x**4)/(3*
b**2*x**(5/2)) - 5*c**(3/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*
x)**2)*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*a
tan(c**(1/4)*sqrt(x)/b**(1/4)), 1/2)/(6*b**(9/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.239834, size = 110, normalized size = 0.76

$$\frac{-\frac{5icx^{5/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}-2b-5cx^2}{3b^2\sqrt{x}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b - 5*c*x^2 - ((5*I)*c*Sqrt[1 + b/(c*x^2)]*x^(5/2)*EllipticF[
I*ArcSinh[Sqrt[(I*Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/Sqrt[(I*Sqrt[b
])/Sqrt[c]])/(3*b^2*Sqrt[x]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.029, size = 127, normalized size = 0.9

$$-\frac{cx^2+b}{6\,b^2}x^{\frac{3}{2}}\left(5\,EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\,\sqrt{2}\right)\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}\sqrt{-bc}\sqrt{2}x+10\,cx^2+4\,b\right)\left(cx^4+bx^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] -1/6/(c*x^4+b*x^2)^(3/2)*x^(3/2)*(c*x^2+b)*(5*EllipticF(((c*x+(-b
*c)^(1/2))/(-b*c)^(1/2))^(1/2),1/2*2^(1/2))*((c*x+(-b*c)^(1/2))/(
-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c
/(-b*c)^(1/2))^(1/2)*(-b*c)^(1/2)*2^(1/2)*x+10*c*x^2+4*b)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\left(cx^4+bx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(x^2(b+cx^2))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(x**2*(b + c*x**2))**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(cx^4+bx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2)^(3/2), x)
```

3.400 $\int \frac{1}{\sqrt{x}(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=320

$$-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) - (21*c^(3/2)*x^(3/2)*(b + c*x^
2))/(5*b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*Sqrt[b
*x^2 + c*x^4])/(5*b^2*x^(7/2)) + (21*c*Sqrt[b*x^2 + c*x^4])/(5*b^
3*x^(3/2)) + (21*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (21*c^(5/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sq
rt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.727257, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{bx^2+cx^4}}+\frac{21c^{5/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{bx^2+cx^4}}-\frac{21c^{3/2}x^{3/2}\left(b+cx^2\right)}{5b^3\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}}+\frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{3/2}}-\frac{7\sqrt{bx^2+cx^4}}{5b^2x^{7/2}}+\frac{1}{bx^{3/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(3/2)*Sqrt[b*x^2 + c*x^4]) - (21*c^(3/2)*x^(3/2)*(b + c*x^
2))/(5*b^3*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (7*Sqrt[b
```

```
*x^2 + c*x^4])/(5*b^2*x^(7/2)) + (21*c*Sqrt[b*x^2 + c*x^4])/(5*b^
3*x^(3/2)) + (21*c^(5/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)
/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^
(1/4)], 1/2])/(5*b^(11/4)*Sqrt[b*x^2 + c*x^4]) - (21*c^(5/4)*x*(S
qrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*Ell
ipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(10*b^(11/4)*Sq
rt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 70.8602, size = 303, normalized size = 0.95

$$\frac{1}{bx^{\frac{3}{2}}\sqrt{bx^2+cx^4}} - \frac{7\sqrt{bx^2+cx^4}}{5b^2x^{\frac{7}{2}}} - \frac{21c^{\frac{3}{2}}\sqrt{bx^2+cx^4}}{5b^3\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)} + \frac{21c\sqrt{bx^2+cx^4}}{5b^3x^{\frac{3}{2}}} + \frac{21c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{5b^{\frac{11}{4}}x\left(b+cx^2\right)} - \frac{21c^{\frac{5}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{10b^{\frac{11}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] 1/(b*x**(3/2)*sqrt(b*x**2 + c*x**4)) - 7*sqrt(b*x**2 + c*x**4)/(5
*b**2*x**(7/2)) - 21*c**(3/2)*sqrt(b*x**2 + c*x**4)/(5*b**3*sqrt(
x)*(sqrt(b) + sqrt(c)*x)) + 21*c*sqrt(b*x**2 + c*x**4)/(5*b**3*x*
*(3/2)) + 21*c**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)
*(sqrt(b) + sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c*
*(1/4)*sqrt(x)/b**(1/4)), 1/2)/(5*b**(11/4)*x*(b + c*x**2)) - 21*
c**(5/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + s
qrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x
)/b**(1/4)), 1/2)/(10*b**(11/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.207228, size = 198, normalized size = 0.62

$$\frac{\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(-2b^2+14bcx^2+21c^2x^4\right)+21\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)-21\sqrt{b}c^{3/2}x^3\sqrt{\frac{cx^2}{b}+1}E\left(i\sinh^{-1}\left(\sqrt{\right.\right.}{5b^3x^{3/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(-2*b^2 + 14*b*c*x^2 + 21*c^2*x^4) -
 21*Sqrt[b]*c^(3/2)*x^3*Sqrt[1 + (c*x^2)/b]*EllipticE[I*ArcSinh[S
qrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] + 21*Sqrt[b]*c^(3/2)*x^3*Sqrt[1
+ (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1
])/(5*b^3*x^(3/2)*Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*Sqrt[x^2*(b + c*x^2
)])
```

Maple [A] time = 0.028, size = 222, normalized size = 0.7

$$-\frac{cx^2+b}{10\,b^3}\sqrt{x}\left(42\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^2bc-21\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx}{\sqrt{}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^(3/2)/x^(1/2),x)
```

```
[Out] -1/10/(c*x^4+b*x^2)^(3/2)*x^(1/2)*(c*x^2+b)*(42*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^2*b*c-21*((c*x+(-b*c)^(1/2))/(-b
*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*c/(
-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1
/2),1/2*2^(1/2))*2^(1/2)*x^2*b*c-42*c^2*x^4-28*b*c*x^2+4*b^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^4+bx^2)^{\frac{3}{2}}\sqrt{x}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**(3/2)/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*sqrt(x)), x)
```

3.401 $\int \frac{1}{x^{3/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=173

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) - (9*Sqrt[b*x^2 + c*x^4])/(7*b^
2*x^(9/2)) + (15*c*Sqrt[b*x^2 + c*x^4])/(7*b^3*x^(5/2)) + (15*c^(
7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*b^
(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi [A] time = 0.461891, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$\frac{15c^{7/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{bx^2+cx^4}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{5/2}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{9/2}}+\frac{1}{bx^{5/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(5/2)*Sqrt[b*x^2 + c*x^4]) - (9*Sqrt[b*x^2 + c*x^4])/(7*b^
2*x^(9/2)) + (15*c*Sqrt[b*x^2 + c*x^4])/(7*b^3*x^(5/2)) + (15*c^(
7/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*
x)^2]*EllipticF[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(14*b^
(13/4)*Sqrt[b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 43.3747, size = 167, normalized size = 0.97

$$\frac{1}{bx^{\frac{5}{2}}\sqrt{bx^2+cx^4}}-\frac{9\sqrt{bx^2+cx^4}}{7b^2x^{\frac{9}{2}}}+\frac{15c\sqrt{bx^2+cx^4}}{7b^3x^{\frac{5}{2}}}+\frac{15c^{\frac{7}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{14b^{\frac{13}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x**(5/2)*sqrt(b*x**2 + c*x**4)) - 9*sqrt(b*x**2 + c*x**4)/(7
*b**2*x**(9/2)) + 15*c*sqrt(b*x**2 + c*x**4)/(7*b**3*x**(5/2)) +
15*c**(7/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(14*b**(13/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.139517, size = 143, normalized size = 0.83

$$\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\left(-2b^2+6bcx^2+15c^2x^4\right)+15ic^2x^{9/2}\sqrt{\frac{b}{cx^2}+1}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}}{\sqrt{x}}\right)\middle|-1\right)}{7b^3x^{5/2}\sqrt{\frac{i\sqrt{b}}{\sqrt{c}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[(I*Sqrt[b])/Sqrt[c]]*(-2*b^2 + 6*b*c*x^2 + 15*c^2*x^4) + (1
5*I)*c^2*Sqrt[1 + b/(c*x^2)]*x^(9/2)*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[b])/Sqrt[c]]/Sqrt[x]], -1])/(7*b^3*Sqrt[(I*Sqrt[b])/Sqrt[c]]
*x^(5/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.027, size = 141, normalized size = 0.8

$$\frac{cx^2+b}{14\,b^3}\left(15\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticF\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{-bc}\sqrt{2}x^3c+30\,c^2x^4+12\,bcx^2-4\,b^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/14/(c*x^4+b*x^2)^(3/2)/x^(1/2)*(c*x^2+b)*(15*((c*x+(-b*c)^(1/2)
)/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-
x*c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2
))^(1/2),1/2*2^(1/2))*(-b*c)^(1/2)*2^(1/2)*x^3*c+30*c^2*x^4+12*b*
c*x^2-4*b^2)/b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}} x^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^5 + bx^3)\sqrt{cx^4 + bx^2}\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^5 + b*x^3)*sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{3}{2}} \left(x^2 \left(b + cx^2\right)\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2)^{\frac{3}{2}} x^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(3/2)), x)
```

3.402 $\int \frac{1}{x^{5/2}(bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=350

$$\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}} - \frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}} + \frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

```
[Out] 1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (77*c^(5/2)*x^(3/2)*(b + c*x^
2))/(15*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (11*Sqrt
[b*x^2 + c*x^4])/(9*b^2*x^(11/2)) + (77*c*Sqrt[b*x^2 + c*x^4])/(4
5*b^3*x^(7/2)) - (77*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^4*x^(3/2)) -
(77*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) + (77*c^(9/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2 + c
*x^4])
```

Rubi [A] time = 0.860347, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{30b^{15/4}\sqrt{bx^2+cx^4}} - \frac{77c^{9/4}x\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{15b^{15/4}\sqrt{bx^2+cx^4}} + \frac{77c^{5/2}x^{3/2}\left(b+cx^2\right)}{15b^4\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{3/2}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{7/2}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{11/2}} + \frac{1}{bx^{7/2}\sqrt{bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] 1/(b*x^(7/2)*Sqrt[b*x^2 + c*x^4]) + (77*c^(5/2)*x^(3/2)*(b + c*x^
2))/(15*b^4*(Sqrt[b] + Sqrt[c]*x)*Sqrt[b*x^2 + c*x^4]) - (11*Sqrt
[b*x^2 + c*x^4])/(9*b^2*x^(11/2)) + (77*c*Sqrt[b*x^2 + c*x^4])/(4
5*b^3*x^(7/2)) - (77*c^2*Sqrt[b*x^2 + c*x^4])/(15*b^4*x^(3/2)) -
(77*c^(9/4)*x*(Sqrt[b] + Sqrt[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + S
qrt[c]*x)^2]*EllipticE[2*ArcTan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])
/(15*b^(15/4)*Sqrt[b*x^2 + c*x^4]) + (77*c^(9/4)*x*(Sqrt[b] + Sqr
t[c]*x)*Sqrt[(b + c*x^2)/(Sqrt[b] + Sqrt[c]*x)^2]*EllipticF[2*Arc
Tan[(c^(1/4)*Sqrt[x])/b^(1/4)], 1/2])/(30*b^(15/4)*Sqrt[b*x^2 + c
*x^4])
```

Rubi in Sympy [A] time = 83.225, size = 332, normalized size = 0.95

$$\frac{1}{bx^{\frac{7}{2}}\sqrt{bx^2+cx^4}} - \frac{11\sqrt{bx^2+cx^4}}{9b^2x^{\frac{11}{2}}} + \frac{77c\sqrt{bx^2+cx^4}}{45b^3x^{\frac{7}{2}}} + \frac{77c^{\frac{5}{2}}\sqrt{bx^2+cx^4}}{15b^4\sqrt{x}\left(\sqrt{b}+\sqrt{c}x\right)} - \frac{77c^2\sqrt{bx^2+cx^4}}{15b^4x^{\frac{3}{2}}} - \frac{77c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{15b^{\frac{15}{4}}x\left(b+cx^2\right)} + \frac{77c^{\frac{9}{4}}\sqrt{\frac{b+cx^2}{\left(\sqrt{b}+\sqrt{c}x\right)^2}}\left(\sqrt{b}+\sqrt{c}x\right)\sqrt{bx^2+cx^4}F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{b}}\right)\middle|\frac{1}{2}\right)}{30b^{\frac{15}{4}}x\left(b+cx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] 1/(b*x**(7/2)*sqrt(b*x**2 + c*x**4)) - 11*sqrt(b*x**2 + c*x**4)/(
9*b**2*x**(11/2)) + 77*c*sqrt(b*x**2 + c*x**4)/(45*b**3*x**(7/2))
 + 77*c**(5/2)*sqrt(b*x**2 + c*x**4)/(15*b**4*sqrt(x)*(sqrt(b) +
sqrt(c)*x)) - 77*c**2*sqrt(b*x**2 + c*x**4)/(15*b**4*x**(3/2)) -
77*c**(9/4)*sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b)
+ sqrt(c)*x)*sqrt(b*x**2 + c*x**4)*elliptic_e(2*atan(c**(1/4)*sqr
t(x)/b**(1/4)), 1/2)/(15*b**(15/4)*x*(b + c*x**2)) + 77*c**(9/4)*
sqrt((b + c*x**2)/(sqrt(b) + sqrt(c)*x)**2)*(sqrt(b) + sqrt(c)*x)
*sqrt(b*x**2 + c*x**4)*elliptic_f(2*atan(c**(1/4)*sqrt(x)/b**(1/4
)), 1/2)/(30*b**(15/4)*x*(b + c*x**2))
```

Mathematica [C] time = 0.23819, size = 210, normalized size = 0.6

$$\frac{-\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\left(10b^3-22b^2cx^2+154bc^2x^4+231c^3x^6\right)-231\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\right)\middle|-1\right)+231\sqrt{b}c^{5/2}x^5\sqrt{\frac{cx^2}{b}}}{45b^4x^{7/2}\sqrt{\frac{i\sqrt{c}x}{\sqrt{b}}}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]*(10*b^3 - 22*b^2*c*x^2 + 154*b*c^2
*x^4 + 231*c^3*x^6)) + 231*Sqrt[b]*c^(5/2)*x^5*Sqrt[1 + (c*x^2)/b
]*EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c]*x)/Sqrt[b]]], -1] - 231*Sqr
t[b]*c^(5/2)*x^5*Sqrt[1 + (c*x^2)/b]*EllipticF[I*ArcSinh[Sqrt[(I*
Sqrt[c]*x)/Sqrt[b]]], -1])/(45*b^4*x^(7/2)*Sqrt[(I*Sqrt[c]*x)/Sqr
t[b]]*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.03, size = 237, normalized size = 0.7

$$\frac{cx^2+b}{90\,b^4}\left(462\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{-\frac{cx}{\sqrt{-bc}}}EllipticE\left(\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}},1/2\sqrt{2}\right)\sqrt{2}x^4bc^2-231\sqrt{\frac{cx+\sqrt{-bc}}{\sqrt{-bc}}}\sqrt{\frac{-cx+}{\sqrt{-}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(c*x^4+b*x^2)^(3/2),x)
```

```
[Out] 1/90/(c*x^4+b*x^2)^(3/2)/x^(3/2)*(c*x^2+b)*(462*((c*x+(-b*c)^(1/2
))/(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(
-x*c/(-b*c)^(1/2))^(1/2)*EllipticE(((c*x+(-b*c)^(1/2))/(-b*c)^(1/
2))^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-231*((c*x+(-b*c)^(1/2))/
(-b*c)^(1/2))^(1/2)*((-c*x+(-b*c)^(1/2))/(-b*c)^(1/2))^(1/2)*(-x*
c/(-b*c)^(1/2))^(1/2)*EllipticF(((c*x+(-b*c)^(1/2))/(-b*c)^(1/2))
^(1/2),1/2*2^(1/2))*2^(1/2)*x^4*b*c^2-462*c^3*x^6-308*b*c^2*x^4+4
4*b^2*c*x^2-20*b^3)/b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4+bx^2)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^6+bx^4)\sqrt{cx^4+bx^2}\sqrt{x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^6 + b*x^4)*sqrt(c*x^4 + b*x^2)*sqrt(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{\frac{5}{2}}\left(x^2\left(b+cx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(5/2)/(c*x**4+b*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**(5/2)*(x**2*(b + c*x**2))**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2)^(3/2)*x^(5/2)), x)
```

3.403 $\int (cx)^m \left(bx^2 + cx^4\right)^3 \, dx$

Optimal. Leaf size=74

$$\frac{b^3(cx)^{m+7}}{c^7(m+7)} + \frac{3b^2(cx)^{m+9}}{c^8(m+9)} + \frac{3b(cx)^{m+11}}{c^9(m+11)} + \frac{(cx)^{m+13}}{c^{10}(m+13)}$$

```
[Out] (b^3*(c*x)^(7 + m))/(c^7*(7 + m)) + (3*b^2*(c*x)^(9 + m))/(c^8*(9
 + m)) + (3*b*(c*x)^(11 + m))/(c^9*(11 + m)) + (c*x)^(13 + m)/(c^
10*(13 + m))
```

Rubi [A] time = 0.127727, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$\frac{b^3(cx)^{m+7}}{c^7(m+7)} + \frac{3b^2(cx)^{m+9}}{c^8(m+9)} + \frac{3b(cx)^{m+11}}{c^9(m+11)} + \frac{(cx)^{m+13}}{c^{10}(m+13)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (b^3*(c*x)^(7 + m))/(c^7*(7 + m)) + (3*b^2*(c*x)^(9 + m))/(c^8*(9
 + m)) + (3*b*(c*x)^(11 + m))/(c^9*(11 + m)) + (c*x)^(13 + m)/(c^
10*(13 + m))
```

Rubi in Sympy [A] time = 24.3572, size = 65, normalized size = 0.88

$$\frac{b^3\,(cx)^{m+7}}{c^7\,(m+7)} + \frac{3b^2\,(cx)^{m+9}}{c^8\,(m+9)} + \frac{3b\,(cx)^{m+11}}{c^9\,(m+11)} + \frac{(cx)^{m+13}}{c^{10}\,(m+13)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(c*x**4+b*x**2)**3,x)
```

```
[Out] b**3*(c*x)**(m + 7)/(c**7*(m + 7)) + 3*b**2*(c*x)**(m + 9)/(c**8*
(m + 9)) + 3*b*(c*x)**(m + 11)/(c**9*(m + 11)) + (c*x)**(m + 13)/
(c**10*(m + 13))
```

Mathematica [A] time = 0.0565439, size = 59, normalized size = 0.8

$$(cx)^m\left(\frac{b^3x^7}{m+7}+\frac{3b^2cx^9}{m+9}+\frac{3bc^2x^{11}}{m+11}+\frac{c^3x^{13}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m*(b*x^2 + c*x^4)^3,x]
```

```
[Out] (c*x)^m*((b^3*x^7)/(7 + m) + (3*b^2*c*x^9)/(9 + m) + (3*b*c^2*x^1
1)/(11 + m) + (c^3*x^13)/(13 + m))
```

Maple [B] time = 0.008, size = 181, normalized size = 2.5

$$\frac{(cx)^m\left(c^3m^3x^6+27\,c^3m^2x^6+3\,bc^2m^3x^4+239\,c^3mx^6+87\,bc^2m^2x^4+693\,c^3x^6+3\,b^2cm^3x^2+813\,bc^2mx^4+93\,b^2cm^2x^2+24\right.}{(13+m)(11+m)(9+m)(7+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(c*x^4+b*x^2)^3,x)
```

```
[Out] (c*x)^m*(c^3*m^3*x^6+27*c^3*m^2*x^6+3*b*c^2*m^3*x^4+239*c^3*m*x^6
+87*b*c^2*m^2*x^4+693*c^3*x^6+3*b^2*c*m^3*x^2+813*b*c^2*m*x^4+93*
b^2*c*m^2*x^2+2457*b*c^2*x^4+b^3*m^3+933*b^2*c*m*x^2+33*b^3*m^2+3
003*b^2*c*x^2+359*b^3*m+1287*b^3)*x^7/(13+m)/(11+m)/(9+m)/(7+m)
```

Maxima [A] time = 0.699747, size = 103, normalized size = 1.39

$$\frac{c^{m+3}x^{13}x^m}{m+13}+\frac{3\,bc^{m+2}x^{11}x^m}{m+11}+\frac{3\,b^2c^{m+1}x^9x^m}{m+9}+\frac{b^3c^mx^7x^m}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*(c*x)^m,x, algorithm="maxima")
```

```
[Out] c^(m + 3)*x^13*x^m/(m + 13) + 3*b*c^(m + 2)*x^11*x^m/(m + 11) + 3
*b^2*c^(m + 1)*x^9*x^m/(m + 9) + b^3*c^m*x^7*x^m/(m + 7)
```

Fricas [A] time = 0.272449, size = 217, normalized size = 2.93

$$\frac{\left(\left(c^3m^3+27\,c^3m^2+239\,c^3m+693\,c^3\right)x^{13}+3\left(bc^2m^3+29\,bc^2m^2+271\,bc^2m+819\,bc^2\right)x^{11}+3\left(b^2cm^3+31\,b^2cm^2+311\,b^2c\right.\right.}{m^4+40\,m^3+590\,m^2+3800\,m+9009}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*(c*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^3*m^3 + 27*c^3*m^2 + 239*c^3*m + 693*c^3)*x^13 + 3*(b*c^2*m^3
 + 29*b*c^2*m^2 + 271*b*c^2*m + 819*b*c^2)*x^11 + 3*(b^2*c*m^3 +
31*b^2*c*m^2 + 311*b^2*c*m + 1001*b^2*c)*x^9 + (b^3*m^3 + 33*b^3*
m^2 + 359*b^3*m + 1287*b^3)*x^7)*(c*x)^m/(m^4 + 40*m^3 + 590*m^2
+ 3800*m + 9009)
```

Sympy [A] time = 16.1669, size = 758, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(c*x**4+b*x**2)**3,x)
```

```
[Out] Piecewise(((-b**3/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2
) + c**3*log(x))/c**13, Eq(m, -13)), ((-b**3/(4*x**4) - 3*b**2*c/
(2*x**2) + 3*b*c**2*log(x) + c**3*x**2/2)/c**11, Eq(m, -11)), ((-
b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*x**4/4)/
c**9, Eq(m, -9)), ((b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4
/4 + c**3*x**6/6)/c**7, Eq(m, -7)), (b**3*c**m*m**3*x**7*x**m/(m*
*4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 33*b**3*c**m*m**2*x**7
*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 359*b**3*c**m
*m*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 1287*b
**3*c**m*x**7*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) +
3*b**2*c*c**m*m**3*x**9*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m
+ 9009) + 93*b**2*c*c**m*m**2*x**9*x**m/(m**4 + 40*m**3 + 590*m**
2 + 3800*m + 9009) + 933*b**2*c*c**m*m*x**9*x**m/(m**4 + 40*m**3
+ 590*m**2 + 3800*m + 9009) + 3003*b**2*c*c**m*x**9*x**m/(m**4 +
40*m**3 + 590*m**2 + 3800*m + 9009) + 3*b*c**2*c**m*m**3*x**11*x*
*m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 87*b*c**2*c**m*m
**2*x**11*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009) + 813*
b*c**2*c**m*m*x**11*x**m/(m**4 + 40*m**3 + 590*m**2 + 3800*m + 90
09) + 2457*b*c**2*c**m*x**11*x**m/(m**4 + 40*m**3 + 590*m**2 + 38
00*m + 9009) + c**3*c**m*m**3*x**13*x**m/(m**4 + 40*m**3 + 590*m*
*2 + 3800*m + 9009) + 27*c**3*c**m*m**2*x**13*x**m/(m**4 + 40*m**
3 + 590*m**2 + 3800*m + 9009) + 239*c**3*c**m*m*x**13*x**m/(m**4
+ 40*m**3 + 590*m**2 + 3800*m + 9009) + 693*c**3*c**m*x**13*x**m/
(m**4 + 40*m**3 + 590*m**2 + 3800*m + 9009), True))
```

GIAC/XCAS [A] time = 0.277416, size = 400, normalized size = 5.41

$$\frac{c^3m^3x^{13}e^{(m\ln(cx))} + 27\,c^3m^2x^{13}e^{(m\ln(cx))} + 3\,bc^2m^3x^{11}e^{(m\ln(cx))} + 239\,c^3mx^{13}e^{(m\ln(cx))} + 87\,bc^2m^2x^{11}e^{(m\ln(cx))} + 693\,c^3x^{13}e^{(ml}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^3*(c*x)^m,x, algorithm="giac")
```

```
[Out] (c^3*m^3*x^13*e^(m*ln(c*x)) + 27*c^3*m^2*x^13*e^(m*ln(c*x)) + 3*b
*c^2*m^3*x^11*e^(m*ln(c*x)) + 239*c^3*m*x^13*e^(m*ln(c*x)) + 87*b
*c^2*m^2*x^11*e^(m*ln(c*x)) + 693*c^3*x^13*e^(m*ln(c*x)) + 3*b^2*
c*m^3*x^9*e^(m*ln(c*x)) + 813*b*c^2*m*x^11*e^(m*ln(c*x)) + 93*b^2
*c*m^2*x^9*e^(m*ln(c*x)) + 2457*b*c^2*x^11*e^(m*ln(c*x)) + b^3*m^
3*x^7*e^(m*ln(c*x)) + 933*b^2*c*m*x^9*e^(m*ln(c*x)) + 33*b^3*m^2*
x^7*e^(m*ln(c*x)) + 3003*b^2*c*x^9*e^(m*ln(c*x)) + 359*b^3*m*x^7*
e^(m*ln(c*x)) + 1287*b^3*x^7*e^(m*ln(c*x)))/(m^4 + 40*m^3 + 590*m
^2 + 3800*m + 9009)
```

3.404 $\int (cx)^m \left(bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{b^2(cx)^{m+5}}{c^5(m+5)}+\frac{2b(cx)^{m+7}}{c^6(m+7)}+\frac{(cx)^{m+9}}{c^7(m+9)}$$

```
[Out] (b^2*(c*x)^(5 + m))/(c^5*(5 + m)) + (2*b*(c*x)^(7 + m))/(c^6*(7 +
 m)) + (c*x)^(9 + m)/(c^7*(9 + m))
```

Rubi [A] time = 0.0916882, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{b^2(cx)^{m+5}}{c^5(m+5)}+\frac{2b(cx)^{m+7}}{c^6(m+7)}+\frac{(cx)^{m+9}}{c^7(m+9)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*(c*x)^(5 + m))/(c^5*(5 + m)) + (2*b*(c*x)^(7 + m))/(c^6*(7 +
 m)) + (c*x)^(9 + m)/(c^7*(9 + m))
```

Rubi in Sympy [A] time = 18.2137, size = 46, normalized size = 0.85

$$\frac{b^2\left(cx\right)^{m+5}}{c^5\left(m+5\right)}+\frac{2b\left(cx\right)^{m+7}}{c^6\left(m+7\right)}+\frac{\left(cx\right)^{m+9}}{c^7\left(m+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(c*x**4+b*x**2)**2,x)
```

```
[Out] b**2*(c*x)**(m + 5)/(c**5*(m + 5)) + 2*b*(c*x)**(m + 7)/(c**6*(m
+ 7)) + (c*x)**(m + 9)/(c**7*(m + 9))
```

Mathematica [A] time = 0.0333006, size = 43, normalized size = 0.8

$$(cx)^m\left(\frac{b^2x^5}{m+5}+\frac{2bcx^7}{m+7}+\frac{c^2x^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m*(b*x^2 + c*x^4)^2,x]
```

```
[Out] (c*x)^m*((b^2*x^5)/(5 + m) + (2*b*c*x^7)/(7 + m) + (c^2*x^9)/(9 +
 m))
```

Maple [A] time = 0.008, size = 96, normalized size = 1.8

$$\frac{(cx)^m \left(c^2m^2x^4 + 12\,c^2mx^4 + 2\,bcm^2x^2 + 35\,c^2x^4 + 28\,bcmx^2 + b^2m^2 + 90\,bcx^2 + 16\,b^2m + 63\,b^2\right)\,x^5}{(9+m)\,(7+m)\,(5+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(c*x^4+b*x^2)^2,x)
```

```
[Out] (c*x)^m*(c^2*m^2*x^4+12*c^2*m*x^4+2*b*c*m^2*x^2+35*c^2*x^4+28*b*c
*m*x^2+b^2*m^2+90*b*c*x^2+16*b^2*m+63*b^2)*x^5/(9+m)/(7+m)/(5+m)
```

Maxima [A] time = 0.700437, size = 74, normalized size = 1.37

$$\frac{c^{m+2}x^9x^m}{m+9} + \frac{2\,bc^{m+1}x^7x^m}{m+7} + \frac{b^2c^mx^5x^m}{m+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*(c*x)^m,x, algorithm="maxima")
```

```
[Out] c^(m + 2)*x^9*x^m/(m + 9) + 2*b*c^(m + 1)*x^7*x^m/(m + 7) + b^2*c
^m*x^5*x^m/(m + 5)
```

Fricas [A] time = 0.271168, size = 120, normalized size = 2.22

$$\frac{\left(\left(c^2m^2 + 12\,c^2m + 35\,c^2\right)x^9 + 2\,\left(bcm^2 + 14\,bcm + 45\,bc\right)x^7 + \left(b^2m^2 + 16\,b^2m + 63\,b^2\right)x^5\right)\,(cx)^m}{m^3 + 21\,m^2 + 143\,m + 315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*(c*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^2*m^2 + 12*c^2*m + 35*c^2)*x^9 + 2*(b*c*m^2 + 14*b*c*m + 45*b
*c)*x^7 + (b^2*m^2 + 16*b^2*m + 63*b^2)*x^5)*(c*x)^m/(m^3 + 21*m^
2 + 143*m + 315)
```

Sympy [A] time = 7.01663, size = 352, normalized size = 6.52

$$\begin{cases} \frac{-\frac{b^2}{4x^4} - \frac{bc}{x^2} + c^2 \log(x)}{c^9} \\ \frac{-\frac{b^2}{2x^2} + 2bc\log(x) + \frac{c^2x^2}{2}}{c^7} \\ \frac{b^2\log(x) + bcx^2 + \frac{c^2x^4}{4}}{c^5} \\ \frac{b^2c^m m^2x^5x^m}{m^3+21m^2+143m+315} + \frac{16b^2c^m mx^5x^m}{m^3+21m^2+143m+315} + \frac{63b^2c^m x^5x^m}{m^3+21m^2+143m+315} + \frac{2bcc^m m^2x^7x^m}{m^3+21m^2+143m+315} + \frac{28bcc^m mx^7x^m}{m^3+21m^2+143m+315} + \frac{90bcc^m x^7x^m}{m^3+21m^2+143m+315} + \frac{c}{m^3+} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(c*x**4+b*x**2)**2,x)
```

```
[Out] Piecewise(((-b**2/(4*x**4) - b*c/x**2 + c**2*log(x))/c**9, Eq(m,
-9)), ((-b**2/(2*x**2) + 2*b*c*log(x) + c**2*x**2/2)/c**7, Eq(m,
-7)), ((b**2*log(x) + b*c*x**2 + c**2*x**4/4)/c**5, Eq(m, -5)), (
b**2*c**m*m**2*x**5*x**m/(m**3 + 21*m**2 + 143*m + 315) + 16*b**2
*c**m*m*x**5*x**m/(m**3 + 21*m**2 + 143*m + 315) + 63*b**2*c**m*x
**5*x**m/(m**3 + 21*m**2 + 143*m + 315) + 2*b*c*c**m*m**2*x**7*x*
*m/(m**3 + 21*m**2 + 143*m + 315) + 28*b*c*c**m*m*x**7*x**m/(m**3
 + 21*m**2 + 143*m + 315) + 90*b*c*c**m*x**7*x**m/(m**3 + 21*m**2
 + 143*m + 315) + c**2*c**m*m**2*x**9*x**m/(m**3 + 21*m**2 + 143*
m + 315) + 12*c**2*c**m*m*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315
) + 35*c**2*c**m*x**9*x**m/(m**3 + 21*m**2 + 143*m + 315), True))
```

GIAC/XCAS [A] time = 0.27412, size = 215, normalized size = 3.98

$$\frac{c^2m^2x^9e^{(m\ln(cx))} + 12\,c^2mx^9e^{(m\ln(cx))} + 2\,bcm^2x^7e^{(m\ln(cx))} + 35\,c^2x^9e^{(m\ln(cx))} + 28\,bcmx^7e^{(m\ln(cx))} + b^2m^2x^5e^{(m\ln(cx))} + 90\,bcx}{m^3 + 21\,m^2 + 143\,m + 315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)^2*(c*x)^m,x, algorithm="giac")
```

```
[Out] (c^2*m^2*x^9*e^(m*ln(c*x)) + 12*c^2*m*x^9*e^(m*ln(c*x)) + 2*b*c*m
^2*x^7*e^(m*ln(c*x)) + 35*c^2*x^9*e^(m*ln(c*x)) + 28*b*c*m*x^7*e^
(m*ln(c*x)) + b^2*m^2*x^5*e^(m*ln(c*x)) + 90*b*c*x^7*e^(m*ln(c*x)
) + 16*b^2*m*x^5*e^(m*ln(c*x)) + 63*b^2*x^5*e^(m*ln(c*x)))/(m^3 +
 21*m^2 + 143*m + 315)
```

3.405 $\int (cx)^m \left(bx^2+cx^4\right) dx$

Optimal. Leaf size=34

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

```
[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rubi [A] time = 0.0361338, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059

$$\frac{b(cx)^{m+3}}{c^3(m+3)}+\frac{(cx)^{m+5}}{c^4(m+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m*(b*x^2 + c*x^4),x]

[Out] (b*(c*x)^(3 + m))/(c^3*(3 + m)) + (c*x)^(5 + m)/(c^4*(5 + m))
```

Rubi in Sympy [A] time = 10.1091, size = 27, normalized size = 0.79

$$\frac{b\,(cx)^{m+3}}{c^3\,(m+3)}+\frac{(cx)^{m+5}}{c^4\,(m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m*(c*x**4+b*x**2),x)

[Out] b*(c*x)**(m + 3)/(c**3*(m + 3)) + (c*x)**(m + 5)/(c**4*(m + 5))
```

Mathematica [A] time = 0.0275057, size = 27, normalized size = 0.79

$$(cx)^m\left(\frac{bx^3}{m+3}+\frac{cx^5}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m*(b*x^2 + c*x^4),x]
```

```
[Out] (c*x)^m*((b*x^3)/(3 + m) + (c*x^5)/(5 + m))
```

Maple [A] time = 0.003, size = 39, normalized size = 1.2

$$\frac{(cx)^m\left(cmx^2+3\,cx^2+bm+5\,b\right)x^3}{(5+m)(3+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m*(c*x^4+b*x^2),x)
```

```
[Out] (c*x)^m*(c*m*x^2+3*c*x^2+b*m+5*b)*x^3/(5+m)/(3+m)
```

Maxima [A] time = 0.69625, size = 46, normalized size = 1.35

$$\frac{c^{m+1}x^5x^m}{m+5}+\frac{bc^mx^3x^m}{m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*(c*x)^m,x, algorithm="maxima")
```

```
[Out] c^(m + 1)*x^5*x^m/(m + 5) + b*c^m*x^3*x^m/(m + 3)
```

Fricas [A] time = 0.271867, size = 53, normalized size = 1.56

$$\frac{\left((cm+3\,c)x^5+(bm+5\,b)x^3\right)(cx)^m}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*(c*x)^m,x, algorithm="fricas")
```

```
[Out] ((c*m + 3*c)*x^5 + (b*m + 5*b)*x^3)*(c*x)^m/(m^2 + 8*m + 15)
```

Sympy [A] time = 1.9643, size = 119, normalized size = 3.5

$$\begin{cases} \frac{-\frac{b}{2x^2}+c\log{(x)}}{c^5} & \text{for } m=-5 \\ \frac{b\log{(x)}+\frac{cx^2}{2}}{c^3} & \text{for } m=-3 \\ \frac{bc^m mx^3x^m}{m^2+8m+15}+\frac{5bc^m x^3x^m}{m^2+8m+15}+\frac{cc^m mx^5x^m}{m^2+8m+15}+\frac{3cc^m x^5x^m}{m^2+8m+15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m*(c*x**4+b*x**2),x)
```

```
[Out] Piecewise((((-b/(2*x**2) + c*log(x))/c**5, Eq(m, -5)), ((b*log(x)
+ c*x**2/2)/c**3, Eq(m, -3)), (b*c**m*m*x**3*x**m/(m**2 + 8*m + 1
5) + 5*b*c**m*x**3*x**m/(m**2 + 8*m + 15) + c*c**m*m*x**5*x**m/(m
**2 + 8*m + 15) + 3*c*c**m*x**5*x**m/(m**2 + 8*m + 15), True))
```

GIAC/XCAS [A] time = 0.271009, size = 86, normalized size = 2.53

$$\frac{cmx^5e^{(m\ln(cx))}+3\,cx^5e^{(m\ln(cx))}+bmx^3e^{(m\ln(cx))}+5\,bx^3e^{(m\ln(cx))}}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2)*(c*x)^m,x, algorithm="giac")
```

```
[Out] (c*m*x^5*e^(m*ln(c*x)) + 3*c*x^5*e^(m*ln(c*x)) + b*m*x^3*e^(m*ln(
c*x)) + 5*b*x^3*e^(m*ln(c*x)))/(m^2 + 8*m + 15)
```

3.406 $\int \frac{(cx)^m}{bx^2+cx^4} \, dx$

Optimal. Leaf size=45

$$-\frac{c(cx)^{m-1} {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)}$$

```
[Out] -((c*(c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -
((c*x^2)/b)])/(b*(1 - m)))
```

Rubi [A] time = 0.0633662, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{c(cx)^{m-1} {}_2F_1\left(1,\frac{m-1}{2};\frac{m+1}{2};-\frac{cx^2}{b}\right)}{b(1-m)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] -((c*(c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -
((c*x^2)/b)])/(b*(1 - m)))
```

Rubi in Sympy [A] time = 8.41014, size = 34, normalized size = 0.76

$$-\frac{c\,(cx)^{m-1} {}_2F_1\left(\begin{matrix} 1, \frac{m}{2}-\frac{1}{2} \\ \frac{m}{2}+\frac{1}{2} \end{matrix}\middle| -\frac{cx^2}{b}\right)}{b\,(-m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m/(c*x**4+b*x**2),x)
```

```
[Out] -c*(c*x)**(m - 1)*hyper((1, m/2 - 1/2), (m/2 + 1/2,), -c*x**2/b)/
(b*(-m + 1))
```

Mathematica [A] time = 0.0712378, size = 56, normalized size = 1.24

$$\frac{(cx)^m\left(\frac{b}{m-1}-\frac{cx^2\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{cx^2}{b}\right)}{m+1}\right)}{b^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m/(b*x^2 + c*x^4),x]
```

```
[Out] ((c*x)^m*(b/(-1 + m) - (c*x^2*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((c*x^2)/b)])/(1 + m)))/(b^2*x)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m/(c*x^4+b*x^2),x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{cx^4+bx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c*x^4 + b*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{x^2\,(b+cx^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m/(c*x**4+b*x**2),x)
```

```
[Out] Integral((c*x)**m/(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{cx^4+bx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2), x)
```

3.407 $\int \frac{(cx)^m}{(bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{c^3(cx)^{m-3}\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)}$$

```
[Out] -((c^3*(c*x)^(-3 + m)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2
, -((c*x^2)/b)])/(b^2*(3 - m)))
```

Rubi [A] time = 0.05965, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{c^3(cx)^{m-3}\,{}_2F_1\left(2,\frac{m-3}{2};\frac{m-1}{2};-\frac{cx^2}{b}\right)}{b^2(3-m)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```

```
[Out] -((c^3*(c*x)^(-3 + m)*Hypergeometric2F1[2, (-3 + m)/2, (-1 + m)/2
, -((c*x^2)/b)])/(b^2*(3 - m)))
```

Rubi in Sympy [A] time = 8.53209, size = 37, normalized size = 0.79

$$-\frac{c^3\,(cx)^{m-3}\,{}_2F_1\left(\begin{matrix}2,\frac{m}{2}-\frac{3}{2}\\ \frac{m}{2}-\frac{1}{2}\end{matrix}\middle|-\frac{cx^2}{b}\right)}{b^2\,(-m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m/(c*x**4+b*x**2)**2,x)
```

```
[Out] -c**3*(c*x)**(m - 3)*hyper((2, m/2 - 3/2), (m/2 - 1/2,), -c*x**2/
b)/(b**2*(-m + 3))
```

Mathematica [B] time = 0.131121, size = 109, normalized size = 2.32

$$\frac{(cx)^m\left(\frac{2c^2x^4\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{cx^2}{b}\right)}{m+1}+\frac{c^2x^4\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{cx^2}{b}\right)}{m+1}+b\left(\frac{b}{m-3}-\frac{2cx^2}{m-1}\right)\right)}{b^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m/(b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*x)^m*(b*(b/(-3 + m) - (2*c*x^2)/(-1 + m)) + (2*c^2*x^4*Hyperg
eometric2F1[1, (1 + m)/2, (3 + m)/2, -((c*x^2)/b)])/(1 + m) + (c^
2*x^4*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((c*x^2)/b)])/(
1 + m)))/(b^4*x^3)
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m/(c*x^4+b*x^2)^2,x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{c^2x^8+2\,bcx^6+b^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c^2*x^8 + 2*b*c*x^6 + b^2*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{x^4\,(b+cx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m/(c*x**4+b*x**2)**2,x)
```

```
[Out] Integral((c*x)**m/(x**4*(b + c*x**2)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^2, x)
```

3.408 $\int \frac{(cx)^m}{(bx^2+cx^4)^3} dx$

Optimal. Leaf size=47

$$-\frac{c^5(cx)^{m-5} {}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)}$$

```
[Out] -((c^5*(c*x)^(-5 + m)*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2
, -((c*x^2)/b)])/(b^3*(5 - m)))
```

Rubi [A] time = 0.0594378, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{c^5(cx)^{m-5} {}_2F_1\left(3,\frac{m-5}{2};\frac{m-3}{2};-\frac{cx^2}{b}\right)}{b^3(5-m)}$$

Antiderivative was successfully verified.

```
[In]  Int[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] -((c^5*(c*x)^(-5 + m)*Hypergeometric2F1[3, (-5 + m)/2, (-3 + m)/2
, -((c*x^2)/b)])/(b^3*(5 - m)))
```

Rubi in Sympy [A] time = 8.48306, size = 37, normalized size = 0.79

$$-\frac{c^5\,(cx)^{m-5}\,{}_2F_1\left(\begin{matrix} 3, \frac{m}{2}-\frac{5}{2} \\ \frac{m}{2}-\frac{3}{2} \end{matrix}\middle| -\frac{cx^2}{b}\right)}{b^3\,(-m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x)**m/(c*x**4+b*x**2)**3,x)
```

```
[Out] -c**5*(c*x)**(m - 5)*hyper((3, m/2 - 5/2), (m/2 - 3/2,), -c*x**2/
b)/(b**3*(-m + 5))
```

Mathematica [B] time = 0.208958, size = 164, normalized size = 3.49

$$\frac{(cx)^m\left(\frac{b^3}{m-5}-\frac{3b^2cx^2}{m-3}-\frac{6c^3x^6\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{cx^2}{b}\right)}{m+1}-\frac{3c^3x^6\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{cx^2}{b}\right)}{m+1}-\frac{c^3x^6\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{cx^2}{b}\right)}{m+1}+\frac{6bc^2x^4}{m-1}\right)}{b^6x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(c*x)^m/(b*x^2 + c*x^4)^3,x]
```

```
[Out] ((c*x)^m*(b^3/(-5 + m) - (3*b^2*c*x^2)/(-3 + m) + (6*b*c^2*x^4)/(
-1 + m) - (6*c^3*x^6*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((c*x^2)/b)])/(1 + m) - (3*c^3*x^6*Hypergeometric2F1[2, (1 + m)/2
, (3 + m)/2, -((c*x^2)/b)])/(1 + m) - (c^3*x^6*Hypergeometric2F1[
3, (1 + m)/2, (3 + m)/2, -((c*x^2)/b)])/(1 + m)))/(b^6*x^5)
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4+bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

```
[Out] int((c*x)^m/(c*x^4+b*x^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4+bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx)^m}{c^3x^{12} + 3\,bc^2x^{10} + 3\,b^2cx^8 + b^3x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x)^m/(c^3*x^12 + 3*b*c^2*x^10 + 3*b^2*c*x^8 + b^3*x^6
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)**m/(c*x**4+b*x**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx)^m}{(cx^4 + bx^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x)^m/(c*x^4 + b*x^2)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(c*x^4 + b*x^2)^3, x)
```

3.409 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

```
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi [A] time = 0.0267797, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi in Sympy [A] time = 14.1419, size = 27, normalized size = 0.9

$$-\frac{a\left(a+bx^2\right)^3}{6b^2}+\frac{\left(a+bx^2\right)^4}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)

[Out] -a*(a + b*x**2)**3/(6*b**2) + (a + b*x**2)**4/(8*b**2)
```

Mathematica [A] time = 0.00253875, size = 30, normalized size = 1.

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^4+2*a*b*x^2+a^2),x)

[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Maxima [A] time = 0.69491, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x^3,x, algorithm="maxima")

[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Fricas [A] time = 0.237872, size = 1, normalized size = 0.03

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x^3,x, algorithm="fricas")

[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

Sympy [A] time = 0.076611, size = 24, normalized size = 0.8

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

GIAC/XCAS [A] time = 0.268267, size = 32, normalized size = 1.07

$$\frac{1}{8}\,b^2x^8 + \frac{1}{3}\,abx^6 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

3.410 $\int x^2 \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi [A] time = 0.0253667, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi in Sympy [A] time = 12.0253, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)

[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Mathematica [A] time = 0.00170071, size = 30, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{2\,abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Maxima [A] time = 0.683371, size = 32, normalized size = 1.07

$$\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Fricas [A] time = 0.240613, size = 1, normalized size = 0.03

$$\frac{1}{7}x^7b^2+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.071444, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

GIAC/XCAS [A] time = 0.267457, size = 32, normalized size = 1.07

$$\frac{1}{7}b^2x^7 + \frac{2}{5}abx^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

3.411 $\int x\left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rubi [A] time = 0.0245305, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{a^2x^2}{2}+\frac{1}{2}abx^4+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + (b^2*x^6)/6
```

Rubi in Sympy [A] time = 5.91427, size = 10, normalized size = 0.33

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)

[Out] (a + b*x**2)**3/(6*b)
```

Mathematica [A] time = 0.00339342, size = 16, normalized size = 0.53

$$\frac{\left(a+bx^2\right)^3}{6b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Maple [A] time = 0.001, size = 25, normalized size = 0.8

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*b^2*x^6
```

Maxima [A] time = 0.677782, size = 32, normalized size = 1.07

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Fricas [A] time = 0.24067, size = 1, normalized size = 0.03

$$\frac{1}{6}x^6b^2 + \frac{1}{2}x^4ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 1/2*x^4*b*a + 1/2*x^2*a^2
```

Sympy [A] time = 0.074715, size = 24, normalized size = 0.8

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**4+2*a*b*x**2+a**2),x)

[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

GIAC/XCAS [A] time = 0.267847, size = 32, normalized size = 1.07

$$\frac{1}{6}\,b^2x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*x,x, algorithm="giac")

[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

3.412 $\int \left(a^2+2abx^2+b^2x^4\right)\, dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] time = 0.0159134, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a^2 + 2*a*b*x^2 + b^2*x^4,x]

[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2abx^3}{3}+\frac{b^2x^5}{5}+\int a^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(b**2*x**4+2*a*b*x**2+a**2,x)

[Out] 2*a*b*x**3/3 + b**2*x**5/5 + Integral(a**2, x)
```

Mathematica [A] time = 0.0000905552, size = 25, normalized size = 1.

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^2 + 2*a*b*x^2 + b^2*x^4,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Maple [A] time = 0.001, size = 22, normalized size = 0.9

$$a^2x + \frac{2\,abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(b^2*x^4+2*a*b*x^2+a^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Maxima [A] time = 0.692516, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b^2*x^4 + 2*a*b*x^2 + a^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Fricas [A] time = 0.239779, size = 1, normalized size = 0.04

$$\frac{1}{5}x^5b^2 + \frac{2}{3}x^3ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b^2*x^4 + 2*a*b*x^2 + a^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 2/3*x^3*b*a + x*a^2
```

Sympy [A] time = 0.070895, size = 22, normalized size = 0.88

$$a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b**2*x**4+2*a*b*x**2+a**2,x)

[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

GIAC/XCAS [A] time = 0.268589, size = 28, normalized size = 1.12

$$\frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(b^2*x^4 + 2*a*b*x^2 + a^2,x, algorithm="giac")

[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

3.413 $\int \frac{a^2+2abx^2+b^2x^4}{x} dx$

Optimal. Leaf size=23

$$a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi [A] time = 0.020732, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]

[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\log\left(x^2\right)}{2} + abx^2 + \frac{b^2\int^{x^2} x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)

[Out] a**2*log(x**2)/2 + a*b*x**2 + b**2*Integral(x, (x, x**2))/2
```

Mathematica [A] time = 0.00234004, size = 23, normalized size = 1.

$$a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x,x]
```

```
[Out] a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Maple [A] time = 0.004, size = 22, normalized size = 1.

$$abx^2 + \frac{b^2x^4}{4} + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/x,x)
```

```
[Out] a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Maxima [A] time = 0.682215, size = 32, normalized size = 1.39

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Fricas [A] time = 0.260831, size = 28, normalized size = 1.22

$$\frac{1}{4}b^2x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] time = 0.952875, size = 20, normalized size = 0.87

$$a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

GIAC/XCAS [A] time = 0.269034, size = 32, normalized size = 1.39

$$\frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*ln(x^2)
```

3.414 $\int \frac{a^2+2abx^2+b^2x^4}{x^2}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] time = 0.0231274, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi in Sympy [A] time = 11.2286, size = 19, normalized size = 0.79

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

Mathematica [A] time = 0.00153688, size = 24, normalized size = 1.

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Maple [A] time = 0.005, size = 23, normalized size = 1.

$$-\frac{a^2}{x} + 2\,abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/x^2,x)
```

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Maxima [A] time = 0.696324, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Fricas [A] time = 0.253241, size = 34, normalized size = 1.42

$$\frac{b^2x^4 + 6\,abx^2 - 3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] time = 0.951372, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**2,x)

[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

GIAC/XCAS [A] time = 0.268216, size = 30, normalized size = 1.25

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^2,x, algorithm="giac")

[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

3.415 $\int \frac{a^2+2abx^2+b^2x^4}{x^3} \, dx$

Optimal. Leaf size=27

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

```
[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] time = 0.0248118, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]

[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{2x^2} + ab\log\left(x^2\right) + \frac{\int^{x^2} b^4\,dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)

[Out] -a**2/(2*x**2) + a*b*log(x**2) + Integral(b**4, (x, x**2))/(2*b**
2)
```

Mathematica [A] time = 0.00203893, size = 27, normalized size = 1.

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^3,x]

[Out] -a^2/(2*x^2) + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Maple [A] time = 0.008, size = 24, normalized size = 0.9

$$-\frac{a^2}{2\,x^2}+\frac{b^2x^2}{2}+2\,ab\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/x^3,x)

[Out] -1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Maxima [A] time = 0.691801, size = 32, normalized size = 1.19

$$\frac{1}{2}\,b^2x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^3,x, algorithm="maxima")

[Out] 1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Fricas [A] time = 0.264417, size = 36, normalized size = 1.33

$$\frac{b^2x^4+4\,abx^2\log\left(x\right)-a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^3,x, algorithm="fricas")

[Out] 1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A] time = 1.05771, size = 24, normalized size = 0.89

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

GIAC/XCAS [A] time = 0.269987, size = 43, normalized size = 1.59

$$\frac{1}{2}b^2x^2 + ab\ln\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b^2*x^2 + a*b*ln(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

3.416 $\int \frac{a^2+2abx^2+b^2x^4}{x^4}\,dx$

Optimal. Leaf size=23

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rubi [A] time = 0.0236637, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]

[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+\frac{\int b^4\,dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)

[Out] -a**2/(3*x**3) - 2*a*b/x + Integral(b**4, x)/b**2
```

Mathematica [A] time = 0.00181302, size = 23, normalized size = 1.

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + b^2*x
```

Maple [A] time = 0.008, size = 22, normalized size = 1.

$$-\frac{a^2}{3\,x^3}-2\,\frac{ab}{x}+b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/x^4,x)
```

```
[Out] -1/3*a^2/x^3-2*a*b/x+b^2*x
```

Maxima [A] time = 0.686534, size = 30, normalized size = 1.3

$$b^2x-\frac{6\,abx^2+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^4,x, algorithm="maxima")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Fricas [A] time = 0.256052, size = 35, normalized size = 1.52

$$\frac{3\,b^2x^4-6\,abx^2-a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A] time = 1.07796, size = 20, normalized size = 0.87

$$b^2x - \frac{a^2 + 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**4,x)
```

```
[Out] b**2*x - (a**2 + 6*a*b*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.267951, size = 30, normalized size = 1.3

$$b^2x - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^4,x, algorithm="giac")
```

```
[Out] b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

3.417 $\int \frac{a^2+2abx^2+b^2x^4}{x^5}\,dx$

Optimal. Leaf size=24

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rubi [A] time = 0.0234157, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Rubi in Sympy [A] time = 13.9288, size = 24, normalized size = 1.

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+\frac{b^2\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
```

```
[Out] -a**2/(4*x**4) - a*b/x**2 + b**2*log(x**2)/2
```

Mathematica [A] time = 0.00152344, size = 24, normalized size = 1.

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^5,x]

[Out] -a^2/(4*x^4) - (a*b)/x^2 + b^2*Log[x]
```

Maple [A] time = 0.008, size = 23, normalized size = 1.

$$-\frac{a^2}{4\,x^4}-\frac{ab}{x^2}+b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/x^5,x)

[Out] -1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Maxima [A] time = 0.690988, size = 35, normalized size = 1.46

$$\frac{1}{2}\,b^2\log\left(x^2\right)-\frac{4\,abx^2+a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^5,x, algorithm="maxima")

[Out] 1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Fricas [A] time = 0.26627, size = 38, normalized size = 1.58

$$\frac{4\,b^2x^4\log(x)-4\,abx^2-a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^5,x, algorithm="fricas")

[Out] 1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A] time = 1.22327, size = 22, normalized size = 0.92

$$b^2\log(x) - \frac{a^2 + 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**5,x)
```

```
[Out] b**2*log(x) - (a**2 + 4*a*b*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.270177, size = 46, normalized size = 1.92

$$\frac{1}{2}\,b^2\ln\left(x^2\right) - \frac{3\,b^2x^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^2*ln(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

3.418 $\int \frac{a^2+2abx^2+b^2x^4}{x^6} \, dx$

Optimal. Leaf size=28

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rubi [A] time = 0.0243424, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]

[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Rubi in Sympy [A] time = 11.9154, size = 24, normalized size = 0.86

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)

[Out] -a**2/(5*x**5) - 2*a*b/(3*x**3) - b**2/x
```

Mathematica [A] time = 0.00160599, size = 28, normalized size = 1.

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - b^2/x
```

Maple [A] time = 0.007, size = 25, normalized size = 0.9

$$-\frac{a^2}{5\,x^5}-\frac{2\,ab}{3\,x^3}-\frac{b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^6,x)
```

```
[Out] -1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Maxima [A] time = 0.681319, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Fricas [A] time = 0.257628, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^6,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A] time = 1.23249, size = 27, normalized size = 0.96

$$-\frac{3a^2+10abx^2+15b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**6,x)
```

```
[Out] -(3*a**2 + 10*a*b*x**2 + 15*b**2*x**4)/(15*x**5)
```

GIAC/XCAS [A] time = 0.268836, size = 35, normalized size = 1.25

$$-\frac{15\,b^2x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

3.419 $\int \frac{a^2+2abx^2+b^2x^4}{x^7} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rubi [A] time = 0.0244774, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]

[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rubi in Sympy [A] time = 7.70336, size = 15, normalized size = 0.5

$$-\frac{\left(a + bx^2\right)^3}{6ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)

[Out] -(a + b*x**2)**3/(6*a*x**6)
```

Mathematica [A] time = 0.00177303, size = 30, normalized size = 1.

$$-\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Maple [A] time = 0.007, size = 25, normalized size = 0.8

$$-\frac{a^2}{6\,x^6}-\frac{ab}{2\,x^4}-\frac{b^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/x^7,x)
```

```
[Out] -1/6*a^2/x^6-1/2*a*b/x^4-1/2*b^2/x^2
```

Maxima [A] time = 0.68296, size = 32, normalized size = 1.07

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Fricas [A] time = 0.253928, size = 32, normalized size = 1.07

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A] time = 1.25853, size = 26, normalized size = 0.87

$$-\frac{a^2+3abx^2+3b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**7,x)
```

```
[Out] -(a**2 + 3*a*b*x**2 + 3*b**2*x**4)/(6*x**6)
```

GIAC/XCAS [A] time = 0.26704, size = 32, normalized size = 1.07

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

3.420 $\int \frac{a^2+2abx^2+b^2x^4}{x^8} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rubi [A] time = 0.0243965, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]

[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rubi in Sympy [A] time = 11.6315, size = 27, normalized size = 0.9

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)

[Out] -a**2/(7*x**7) - 2*a*b/(5*x**5) - b**2/(3*x**3)
```

Mathematica [A] time = 0.00187926, size = 30, normalized size = 1.

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Maple [A] time = 0.007, size = 25, normalized size = 0.8

$$-\frac{a^2}{7\,x^7}-\frac{2\,ab}{5\,x^5}-\frac{b^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^2*x^4+2*a*b*x^2+a^2)/x^8,x)
```

```
[Out] -1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Maxima [A] time = 0.690185, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Fricas [A] time = 0.258226, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A] time = 1.29084, size = 27, normalized size = 0.9

$$-\frac{15a^2+42abx^2+35b^2x^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/x**8,x)
```

```
[Out] -(15*a**2 + 42*a*b*x**2 + 35*b**2*x**4)/(105*x**7)
```

GIAC/XCAS [A] time = 0.267973, size = 35, normalized size = 1.17

$$-\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

3.421 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^
13)/13 + (b^4*x^15)/15
```

Rubi [A] time = 0.0794108, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^
13)/13 + (b^4*x^15)/15
```

Rubi in Sympy [A] time = 17.5146, size = 53, normalized size = 0.95

$$\frac{a^4x^7}{7}+\frac{4a^3bx^9}{9}+\frac{6a^2b^2x^{11}}{11}+\frac{4ab^3x^{13}}{13}+\frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x
**13/13 + b**4*x**15/15
```

Mathematica [A] time = 0.00448488, size = 56, normalized size = 1.

$$\frac{a^4x^7}{7}+\frac{4}{9}a^3bx^9+\frac{6}{11}a^2b^2x^{11}+\frac{4}{13}ab^3x^{13}+\frac{b^4x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^7)/7 + (4*a^3*b*x^9)/9 + (6*a^2*b^2*x^11)/11 + (4*a*b^3*x^
13)/13 + (b^4*x^15)/15
```

Maple [A] time = 0.001, size = 47, normalized size = 0.8

$$\frac{a^4x^7}{7}+\frac{4\,a^3bx^9}{9}+\frac{6\,a^2b^2x^{11}}{11}+\frac{4\,ab^3x^{13}}{13}+\frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/7*a^4*x^7+4/9*a^3*b*x^9+6/11*a^2*b^2*x^11+4/13*a*b^3*x^13+1/15*
b^4*x^15
```

Maxima [A] time = 0.687941, size = 62, normalized size = 1.11

$$\frac{1}{15}\,b^4x^{15}+\frac{4}{13}\,ab^3x^{13}+\frac{6}{11}\,a^2b^2x^{11}+\frac{4}{9}\,a^3bx^9+\frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^6,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x
^9 + 1/7*a^4*x^7
```

Fricas [A] time = 0.241513, size = 1, normalized size = 0.02

$$\frac{1}{15}x^{15}b^4+\frac{4}{13}x^{13}b^3a+\frac{6}{11}x^{11}b^2a^2+\frac{4}{9}x^9ba^3+\frac{1}{7}x^7a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^6,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^4 + 4/13*x^13*b^3*a + 6/11*x^11*b^2*a^2 + 4/9*x^9*b*a
^3 + 1/7*x^7*a^4
```

Sympy [A] time = 0.112466, size = 53, normalized size = 0.95

$$\frac{a^4x^7}{7} + \frac{4a^3bx^9}{9} + \frac{6a^2b^2x^{11}}{11} + \frac{4ab^3x^{13}}{13} + \frac{b^4x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**7/7 + 4*a**3*b*x**9/9 + 6*a**2*b**2*x**11/11 + 4*a*b**3*x
**13/13 + b**4*x**15/15
```

GIAC/XCAS [A] time = 0.268693, size = 62, normalized size = 1.11

$$\frac{1}{15}\,b^4x^{15} + \frac{4}{13}\,ab^3x^{13} + \frac{6}{11}\,a^2b^2x^{11} + \frac{4}{9}\,a^3bx^9 + \frac{1}{7}\,a^4x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^6,x, algorithm="giac")
```

```
[Out] 1/15*b^4*x^15 + 4/13*a*b^3*x^13 + 6/11*a^2*b^2*x^11 + 4/9*a^3*b*x
^9 + 1/7*a^4*x^7
```

3.422 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}$$

```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b
*x^2)^7/(14*b^3)
```

Rubi [A] time = 0.16599, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^2*(a + b*x^2)^5)/(10*b^3) - (a*(a + b*x^2)^6)/(6*b^3) + (a + b
*x^2)^7/(14*b^3)
```

Rubi in Sympy [A] time = 20.0877, size = 44, normalized size = 0.83

$$\frac{a^2\left(a+bx^2\right)^5}{10b^3}-\frac{a\left(a+bx^2\right)^6}{6b^3}+\frac{\left(a+bx^2\right)^7}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**2*(a + b*x**2)**5/(10*b**3) - a*(a + b*x**2)**6/(6*b**3) + (a
+ b*x**2)**7/(14*b**3)
```

Mathematica [A] time = 0.00347822, size = 56, normalized size = 1.06

$$\frac{a^4x^6}{6}+\frac{1}{2}a^3bx^8+\frac{3}{5}a^2b^2x^{10}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^6)/6 + (a^3*b*x^8)/2 + (3*a^2*b^2*x^10)/5 + (a*b^3*x^12)/3
 + (b^4*x^14)/14
```

Maple [A] time = 0.001, size = 47, normalized size = 0.9

$$\frac{b^4x^{14}}{14}+\frac{ab^3x^{12}}{3}+\frac{3\,a^2b^2x^{10}}{5}+\frac{a^3bx^8}{2}+\frac{a^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/14*b^4*x^14+1/3*a*b^3*x^12+3/5*a^2*b^2*x^10+1/2*a^3*b*x^8+1/6*a
^4*x^6
```

Maxima [A] time = 0.685764, size = 62, normalized size = 1.17

$$\frac{1}{14}\,b^4x^{14}+\frac{1}{3}\,ab^3x^{12}+\frac{3}{5}\,a^2b^2x^{10}+\frac{1}{2}\,a^3bx^8+\frac{1}{6}\,a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^5,x, algorithm="maxima")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8
 + 1/6*a^4*x^6
```

Fricas [A] time = 0.24218, size = 1, normalized size = 0.02

$$\frac{1}{14}x^{14}b^4+\frac{1}{3}x^{12}b^3a+\frac{3}{5}x^{10}b^2a^2+\frac{1}{2}x^8ba^3+\frac{1}{6}x^6a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^5,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^4 + 1/3*x^12*b^3*a + 3/5*x^10*b^2*a^2 + 1/2*x^8*b*a^3
 + 1/6*x^6*a^4
```

Sympy [A] time = 0.105423, size = 49, normalized size = 0.92

$$\frac{a^4x^6}{6} + \frac{a^3bx^8}{2} + \frac{3a^2b^2x^{10}}{5} + \frac{ab^3x^{12}}{3} + \frac{b^4x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**6/6 + a**3*b*x**8/2 + 3*a**2*b**2*x**10/5 + a*b**3*x**12/
3 + b**4*x**14/14
```

GIAC/XCAS [A] time = 0.26679, size = 62, normalized size = 1.17

$$\frac{1}{14}\, b^4x^{14} + \frac{1}{3}\, ab^3x^{12} + \frac{3}{5}\, a^2b^2x^{10} + \frac{1}{2}\, a^3bx^8 + \frac{1}{6}\, a^4x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^5,x, algorithm="giac")
```

```
[Out] 1/14*b^4*x^14 + 1/3*a*b^3*x^12 + 3/5*a^2*b^2*x^10 + 1/2*a^3*b*x^8
 + 1/6*a^4*x^6
```

3.423 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=56

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11
)/11 + (b^4*x^13)/13
```

Rubi [A] time = 0.0738783, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11
)/11 + (b^4*x^13)/13
```

Rubi in Sympy [A] time = 17.5397, size = 53, normalized size = 0.95

$$\frac{a^4x^5}{5}+\frac{4a^3bx^7}{7}+\frac{2a^2b^2x^9}{3}+\frac{4ab^3x^{11}}{11}+\frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**
11/11 + b**4*x**13/13
```

Mathematica [A] time = 0.0036894, size = 56, normalized size = 1.

$$\frac{a^4x^5}{5}+\frac{4}{7}a^3bx^7+\frac{2}{3}a^2b^2x^9+\frac{4}{11}ab^3x^{11}+\frac{b^4x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^5)/5 + (4*a^3*b*x^7)/7 + (2*a^2*b^2*x^9)/3 + (4*a*b^3*x^11
)/11 + (b^4*x^13)/13
```

Maple [A] time = 0.002, size = 47, normalized size = 0.8

$$\frac{a^4x^5}{5} + \frac{4\,a^3bx^7}{7} + \frac{2\,a^2b^2x^9}{3} + \frac{4\,ab^3x^{11}}{11} + \frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*a^4*x^5+4/7*a^3*b*x^7+2/3*a^2*b^2*x^9+4/11*a*b^3*x^11+1/13*b^
4*x^13
```

Maxima [A] time = 0.682261, size = 62, normalized size = 1.11

$$\frac{1}{13}\,b^4x^{13} + \frac{4}{11}\,ab^3x^{11} + \frac{2}{3}\,a^2b^2x^9 + \frac{4}{7}\,a^3bx^7 + \frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^4,x, algorithm="maxima")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7
 + 1/5*a^4*x^5
```

Fricas [A] time = 0.241833, size = 1, normalized size = 0.02

$$\frac{1}{13}x^{13}b^4 + \frac{4}{11}x^{11}b^3a + \frac{2}{3}x^9b^2a^2 + \frac{4}{7}x^7ba^3 + \frac{1}{5}x^5a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^4,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^4 + 4/11*x^11*b^3*a + 2/3*x^9*b^2*a^2 + 4/7*x^7*b*a^3
 + 1/5*x^5*a^4
```

Sympy [A] time = 0.105919, size = 53, normalized size = 0.95

$$\frac{a^4x^5}{5} + \frac{4a^3bx^7}{7} + \frac{2a^2b^2x^9}{3} + \frac{4ab^3x^{11}}{11} + \frac{b^4x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**5/5 + 4*a**3*b*x**7/7 + 2*a**2*b**2*x**9/3 + 4*a*b**3*x**
11/11 + b**4*x**13/13
```

GIAC/XCAS [A] time = 0.267534, size = 62, normalized size = 1.11

$$\frac{1}{13}\,b^4x^{13} + \frac{4}{11}\,ab^3x^{11} + \frac{2}{3}\,a^2b^2x^9 + \frac{4}{7}\,a^3bx^7 + \frac{1}{5}\,a^4x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^4,x, algorithm="giac")
```

```
[Out] 1/13*b^4*x^13 + 4/11*a*b^3*x^11 + 2/3*a^2*b^2*x^9 + 4/7*a^3*b*x^7
 + 1/5*a^4*x^5
```

3.424 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

```
[Out] -(a*(a + b*x^2)^5)/(10*b^2) + (a + b*x^2)^6/(12*b^2)
```

Rubi [A] time = 0.1025, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(a+bx^2\right)^6}{12b^2}-\frac{a\left(a+bx^2\right)^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a*(a + b*x^2)^5)/(10*b^2) + (a + b*x^2)^6/(12*b^2)
```

Rubi in Sympy [A] time = 16.5193, size = 27, normalized size = 0.79

$$-\frac{a\left(a+bx^2\right)^5}{10b^2}+\frac{\left(a+bx^2\right)^6}{12b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -a*(a + b*x**2)**5/(10*b**2) + (a + b*x**2)**6/(12*b**2)
```

Mathematica [A] time = 0.00362381, size = 56, normalized size = 1.65

$$\frac{a^4x^4}{4}+\frac{2}{3}a^3bx^6+\frac{3}{4}a^2b^2x^8+\frac{2}{5}ab^3x^{10}+\frac{b^4x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^4)/4 + (2*a^3*b*x^6)/3 + (3*a^2*b^2*x^8)/4 + (2*a*b^3*x^10
)/5 + (b^4*x^12)/12
```

Maple [A] time = 0.001, size = 47, normalized size = 1.4

$$\frac{b^4x^{12}}{12}+\frac{2\,ab^3x^{10}}{5}+\frac{3\,a^2b^2x^8}{4}+\frac{2\,a^3bx^6}{3}+\frac{a^4x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/12*b^4*x^12+2/5*a*b^3*x^10+3/4*a^2*b^2*x^8+2/3*a^3*b*x^6+1/4*a^
4*x^4
```

Maxima [A] time = 0.68829, size = 62, normalized size = 1.82

$$\frac{1}{12}\,b^4x^{12}+\frac{2}{5}\,ab^3x^{10}+\frac{3}{4}\,a^2b^2x^8+\frac{2}{3}\,a^3bx^6+\frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^3,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6
+ 1/4*a^4*x^4
```

Fricas [A] time = 0.242047, size = 1, normalized size = 0.03

$$\frac{1}{12}x^{12}b^4+\frac{2}{5}x^{10}b^3a+\frac{3}{4}x^8b^2a^2+\frac{2}{3}x^6ba^3+\frac{1}{4}x^4a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^4 + 2/5*x^10*b^3*a + 3/4*x^8*b^2*a^2 + 2/3*x^6*b*a^3
+ 1/4*x^4*a^4
```

Sympy [A] time = 0.10781, size = 53, normalized size = 1.56

$$\frac{a^4x^4}{4} + \frac{2a^3bx^6}{3} + \frac{3a^2b^2x^8}{4} + \frac{2ab^3x^{10}}{5} + \frac{b^4x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**4/4 + 2*a**3*b*x**6/3 + 3*a**2*b**2*x**8/4 + 2*a*b**3*x**
10/5 + b**4*x**12/12
```

GIAC/XCAS [A] time = 0.267913, size = 62, normalized size = 1.82

$$\frac{1}{12}\,b^4x^{12} + \frac{2}{5}\,ab^3x^{10} + \frac{3}{4}\,a^2b^2x^8 + \frac{2}{3}\,a^3bx^6 + \frac{1}{4}\,a^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^3,x, algorithm="giac")
```

```
[Out] 1/12*b^4*x^12 + 2/5*a*b^3*x^10 + 3/4*a^2*b^2*x^8 + 2/3*a^3*b*x^6
+ 1/4*a^4*x^4
```

3.425 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=56

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)
/9 + (b^4*x^11)/11
```

Rubi [A] time = 0.0737234, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)
/9 + (b^4*x^11)/11
```

Rubi in Sympy [A] time = 17.8801, size = 53, normalized size = 0.95

$$\frac{a^4x^3}{3}+\frac{4a^3bx^5}{5}+\frac{6a^2b^2x^7}{7}+\frac{4ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**
9/9 + b**4*x**11/11
```

Mathematica [A] time = 0.00335182, size = 56, normalized size = 1.

$$\frac{a^4x^3}{3}+\frac{4}{5}a^3bx^5+\frac{6}{7}a^2b^2x^7+\frac{4}{9}ab^3x^9+\frac{b^4x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*x^3)/3 + (4*a^3*b*x^5)/5 + (6*a^2*b^2*x^7)/7 + (4*a*b^3*x^9)
/9 + (b^4*x^11)/11
```

Maple [A] time = 0.002, size = 47, normalized size = 0.8

$$\frac{a^4x^3}{3} + \frac{4\,a^3bx^5}{5} + \frac{6\,a^2b^2x^7}{7} + \frac{4\,ab^3x^9}{9} + \frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*a^4*x^3+4/5*a^3*b*x^5+6/7*a^2*b^2*x^7+4/9*a*b^3*x^9+1/11*b^4*
x^11
```

Maxima [A] time = 0.686685, size = 62, normalized size = 1.11

$$\frac{1}{11}\,b^4x^{11} + \frac{4}{9}\,ab^3x^9 + \frac{6}{7}\,a^2b^2x^7 + \frac{4}{5}\,a^3bx^5 + \frac{1}{3}\,a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 +
 1/3*a^4*x^3
```

Fricas [A] time = 0.241732, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}b^4 + \frac{4}{9}x^9b^3a + \frac{6}{7}x^7b^2a^2 + \frac{4}{5}x^5ba^3 + \frac{1}{3}x^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^4 + 4/9*x^9*b^3*a + 6/7*x^7*b^2*a^2 + 4/5*x^5*b*a^3 +
 1/3*x^3*a^4
```

Sympy [A] time = 0.105999, size = 53, normalized size = 0.95

$$\frac{a^4x^3}{3}+\frac{4a^3bx^5}{5}+\frac{6a^2b^2x^7}{7}+\frac{4ab^3x^9}{9}+\frac{b^4x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**3/3 + 4*a**3*b*x**5/5 + 6*a**2*b**2*x**7/7 + 4*a*b**3*x**
9/9 + b**4*x**11/11
```

GIAC/XCAS [A] time = 0.266417, size = 62, normalized size = 1.11

$$\frac{1}{11}b^4x^{11}+\frac{4}{9}ab^3x^9+\frac{6}{7}a^2b^2x^7+\frac{4}{5}a^3bx^5+\frac{1}{3}a^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^4*x^11 + 4/9*a*b^3*x^9 + 6/7*a^2*b^2*x^7 + 4/5*a^3*b*x^5 +
 1/3*a^4*x^3
```

3.426 $\int x \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^5}{10b}$$

```
[Out] (a + b*x^2)^5/(10*b)
```

Rubi [A] time = 0.017018, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]

[Out] (a + b*x^2)^5/(10*b)
```

Rubi in Sympy [A] time = 6.66098, size = 10, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)

[Out] (a + b*x**2)**5/(10*b)
```

Mathematica [A] time = 0.0042337, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^5}{10b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a + b*x^2)^5/(10*b)
```

Maple [B] time = 0.001, size = 45, normalized size = 2.8

$$\frac{b^4x^{10}}{10}+\frac{ab^3x^8}{2}+a^2b^2x^6+a^3bx^4+\frac{a^4x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/10*b^4*x^10+1/2*a*b^3*x^8+a^2*b^2*x^6+a^3*b*x^4+1/2*a^4*x^2
```

Maxima [A] time = 0.702543, size = 59, normalized size = 3.69

$$\frac{1}{10}\,b^4x^{10}+\frac{1}{2}\,ab^3x^8+a^2b^2x^6+a^3bx^4+\frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x,x, algorithm="maxima")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4
*x^2
```

Fricas [A] time = 0.241912, size = 1, normalized size = 0.06

$$\frac{1}{10}x^{10}b^4+\frac{1}{2}x^8b^3a+x^6b^2a^2+x^4ba^3+\frac{1}{2}x^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^4 + 1/2*x^8*b^3*a + x^6*b^2*a^2 + x^4*b*a^3 + 1/2*x^2
*a^4
```

Sympy [A] time = 0.108162, size = 44, normalized size = 2.75

$$\frac{a^4x^2}{2} + a^3bx^4 + a^2b^2x^6 + \frac{ab^3x^8}{2} + \frac{b^4x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x**2/2 + a**3*b*x**4 + a**2*b**2*x**6 + a*b**3*x**8/2 + b**4
*x**10/10
```

GIAC/XCAS [A] time = 0.267687, size = 59, normalized size = 3.69

$$\frac{1}{10}\,b^4x^{10} + \frac{1}{2}\,ab^3x^8 + a^2b^2x^6 + a^3bx^4 + \frac{1}{2}\,a^4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x,x, algorithm="giac")
```

```
[Out] 1/10*b^4*x^10 + 1/2*a*b^3*x^8 + a^2*b^2*x^6 + a^3*b*x^4 + 1/2*a^4
*x^2
```

3.427 $\int \left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=51

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (
b^4*x^9)/9
```

Rubi [A] time = 0.0548659, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (
b^4*x^9)/9
```

Rubi in Sympy [A] time = 25.1308, size = 49, normalized size = 0.96

$$a^4x+\frac{4a^3bx^3}{3}+\frac{6a^2b^2x^5}{5}+\frac{4ab^3x^7}{7}+\frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 +
 b**4*x**9/9
```

Mathematica [A] time = 0.00186486, size = 51, normalized size = 1.

$$a^4x+\frac{4}{3}a^3bx^3+\frac{6}{5}a^2b^2x^5+\frac{4}{7}ab^3x^7+\frac{b^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^4*x + (4*a^3*b*x^3)/3 + (6*a^2*b^2*x^5)/5 + (4*a*b^3*x^7)/7 + (
b^4*x^9)/9
```

Maple [A] time = 0.001, size = 44, normalized size = 0.9

$$a^4x + \frac{4\,a^3bx^3}{3} + \frac{6\,a^2b^2x^5}{5} + \frac{4\,ab^3x^7}{7} + \frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] a^4*x+4/3*a^3*b*x^3+6/5*a^2*b^2*x^5+4/7*a*b^3*x^7+1/9*b^4*x^9
```

Maxima [A] time = 0.704154, size = 74, normalized size = 1.45

$$\frac{1}{9}\,b^4x^9 + \frac{4}{7}\,ab^3x^7 + \frac{4}{5}\,a^2b^2x^5 + a^4x + \frac{2}{15}\left(3\,b^2x^5 + 10\,abx^3\right)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 4/5*a^2*b^2*x^5 + a^4*x + 2/15*(3*b
^2*x^5 + 10*a*b*x^3)*a^2
```

Fricas [A] time = 0.242125, size = 1, normalized size = 0.02

$$\frac{1}{9}x^9b^4 + \frac{4}{7}x^7b^3a + \frac{6}{5}x^5b^2a^2 + \frac{4}{3}x^3ba^3 + xa^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^4 + 4/7*x^7*b^3*a + 6/5*x^5*b^2*a^2 + 4/3*x^3*b*a^3 + x
*a^4
```

Sympy [A] time = 0.111936, size = 49, normalized size = 0.96

$$a^4x + \frac{4a^3bx^3}{3} + \frac{6a^2b^2x^5}{5} + \frac{4ab^3x^7}{7} + \frac{b^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*x + 4*a**3*b*x**3/3 + 6*a**2*b**2*x**5/5 + 4*a*b**3*x**7/7 +
 b**4*x**9/9
```

GIAC/XCAS [A] time = 0.268331, size = 58, normalized size = 1.14

$$\frac{1}{9}\,b^4x^9 + \frac{4}{7}\,ab^3x^7 + \frac{6}{5}\,a^2b^2x^5 + \frac{4}{3}\,a^3bx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^4*x^9 + 4/7*a*b^3*x^7 + 6/5*a^2*b^2*x^5 + 4/3*a^3*b*x^3 + a
^4*x
```

3.428 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x}\,dx$

Optimal. Leaf size=50

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 +
 a^4*Log[x]
```

Rubi [A] time = 0.0819044, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 +
 a^4*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^4\log(x^2)}{2}+2a^3bx^2+3a^2b^2\int^{x^2}x\,dx+\frac{2ab^3x^6}{3}+\frac{b^4x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

```
[Out] a**4*log(x**2)/2 + 2*a**3*b*x**2 + 3*a**2*b**2*Integral(x, (x, x*
*2)) + 2*a*b**3*x**6/3 + b**4*x**8/8
```

Mathematica [A] time = 0.00748856, size = 50, normalized size = 1.

$$a^4\log(x)+2a^3bx^2+\frac{3}{2}a^2b^2x^4+\frac{2}{3}ab^3x^6+\frac{b^4x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x,x]
```

```
[Out] 2*a^3*b*x^2 + (3*a^2*b^2*x^4)/2 + (2*a*b^3*x^6)/3 + (b^4*x^8)/8 +
 a^4*Log[x]
```

Maple [A] time = 0.003, size = 45, normalized size = 0.9

$$2\,a^{3}bx^{2}+\frac{3\,a^{2}b^{2}x^{4}}{2}+\frac{2\,ab^{3}x^{6}}{3}+\frac{b^{4}x^{8}}{8}+a^{4}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x,x)
```

```
[Out] 2*a^3*b*x^2+3/2*a^2*b^2*x^4+2/3*a*b^3*x^6+1/8*b^4*x^8+a^4*ln(x)
```

Maxima [A] time = 0.707176, size = 63, normalized size = 1.26

$$\frac{1}{8}\,b^{4}x^{8}+\frac{2}{3}\,ab^{3}x^{6}+\frac{3}{2}\,a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+\frac{1}{2}\,a^{4}\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2
*a^4*log(x^2)
```

Fricas [A] time = 0.26777, size = 59, normalized size = 1.18

$$\frac{1}{8}\,b^{4}x^{8}+\frac{2}{3}\,ab^{3}x^{6}+\frac{3}{2}\,a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4
*log(x)
```

Sympy [A] time = 1.11365, size = 49, normalized size = 0.98

$$a^4 \log(x) + 2a^3bx^2 + \frac{3a^2b^2x^4}{2} + \frac{2ab^3x^6}{3} + \frac{b^4x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x,x)
```

```
[Out] a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/
3 + b**4*x**8/8
```

GIAC/XCAS [A] time = 0.269664, size = 63, normalized size = 1.26

$$\frac{1}{8}\,b^4x^8 + \frac{2}{3}\,ab^3x^6 + \frac{3}{2}\,a^2b^2x^4 + 2\,a^3bx^2 + \frac{1}{2}\,a^4\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*b^4*x^8 + 2/3*a*b^3*x^6 + 3/2*a^2*b^2*x^4 + 2*a^3*b*x^2 + 1/2
*a^4*ln(x^2)
```

3.429 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7
)/7
```

Rubi [A] time = 0.0639931, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7
)/7
```

Rubi in Sympy [A] time = 17.4721, size = 44, normalized size = 0.92

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4ab^3x^5}{5}+\frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
```

```
[Out] -a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*
x**7/7
```

Mathematica [A] time = 0.0130249, size = 48, normalized size = 1.

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4}{5}ab^3x^5+\frac{b^4x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^2,x]
```

```
[Out] -(a^4/x) + 4*a^3*b*x + 2*a^2*b^2*x^3 + (4*a*b^3*x^5)/5 + (b^4*x^7
)/7
```

Maple [A] time = 0.005, size = 45, normalized size = 0.9

$$-\frac{a^4}{x}+4\,a^3bx+2\,a^2b^2x^3+\frac{4\,ab^3x^5}{5}+\frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^2,x)
```

```
[Out] -a^4/x+4*a^3*b*x+2*a^2*b^2*x^3+4/5*a*b^3*x^5+1/7*b^4*x^7
```

Maxima [A] time = 0.694275, size = 59, normalized size = 1.23

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

Fricas [A] time = 0.258635, size = 65, normalized size = 1.35

$$\frac{5\,b^4x^8+28\,ab^3x^6+70\,a^2b^2x^4+140\,a^3bx^2-35\,a^4}{35\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^4*x^8 + 28*a*b^3*x^6 + 70*a^2*b^2*x^4 + 140*a^3*b*x^2 -
 35*a^4)/x
```

Sympy [A] time = 1.05458, size = 44, normalized size = 0.92

$$-\frac{a^4}{x}+4a^3bx+2a^2b^2x^3+\frac{4ab^3x^5}{5}+\frac{b^4x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**2,x)
```

```
[Out] -a**4/x + 4*a**3*b*x + 2*a**2*b**2*x**3 + 4*a*b**3*x**5/5 + b**4*
x**7/7
```

GIAC/XCAS [A] time = 0.267815, size = 59, normalized size = 1.23

$$\frac{1}{7}\,b^4x^7+\frac{4}{5}\,ab^3x^5+2\,a^2b^2x^3+4\,a^3bx-\frac{a^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*x^7 + 4/5*a*b^3*x^5 + 2*a^2*b^2*x^3 + 4*a^3*b*x - a^4/x
```

3.430 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^3}\,dx$

Optimal. Leaf size=48

$$-\frac{a^4}{2x^2}+4a^3b\log(x)+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*
Log[x]
```

Rubi [A] time = 0.0916041, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^4}{2x^2}+4a^3b\log(x)+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*
Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{2x^2}+2a^3b\log\left(x^2\right)+3a^2b^2x^2+2ab^3\int^{x^2}x\,dx+\frac{b^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)
```

```
[Out] -a**4/(2*x**2) + 2*a**3*b*log(x**2) + 3*a**2*b**2*x**2 + 2*a*b**3
*Integral(x, (x, x**2)) + b**4*x**6/6
```

Mathematica [A] time = 0.00769079, size = 48, normalized size = 1.

$$-\frac{a^4}{2x^2}+4a^3b\log(x)+3a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^3,x]
```

```
[Out] -a^4/(2*x^2) + 3*a^2*b^2*x^2 + a*b^3*x^4 + (b^4*x^6)/6 + 4*a^3*b*
Log[x]
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$-\frac{a^4}{2\,x^2}+3\,a^2b^2x^2+ab^3x^4+\frac{b^4x^6}{6}+4\,a^3b\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^3,x)
```

```
[Out] -1/2*a^4/x^2+3*a^2*b^2*x^2+a*b^3*x^4+1/6*b^4*x^6+4*a^3*b*ln(x)
```

Maxima [A] time = 0.703458, size = 62, normalized size = 1.29

$$\frac{1}{6}\,b^4x^6+ab^3x^4+3\,a^2b^2x^2+2\,a^3b\log\left(x^2\right)-\frac{a^4}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*log(x^2) - 1/2*
a^4/x^2
```

Fricas [A] time = 0.264049, size = 66, normalized size = 1.38

$$\frac{b^4x^8+6\,ab^3x^6+18\,a^2b^2x^4+24\,a^3bx^2\log\left(x\right)-3\,a^4}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 + 6*a*b^3*x^6 + 18*a^2*b^2*x^4 + 24*a^3*b*x^2*log(x)
 - 3*a^4)/x^2
```

Sympy [A] time = 1.15638, size = 46, normalized size = 0.96

$$-\frac{a^4}{2x^2} + 4a^3b\log(x) + 3a^2b^2x^2 + ab^3x^4 + \frac{b^4x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**3,x)
```

```
[Out] -a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4
 + b**4*x**6/6
```

GIAC/XCAS [A] time = 0.270609, size = 76, normalized size = 1.58

$$\frac{1}{6}b^4x^6 + ab^3x^4 + 3a^2b^2x^2 + 2a^3b\ln\left(x^2\right) - \frac{4a^3bx^2 + a^4}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*b^4*x^6 + a*b^3*x^4 + 3*a^2*b^2*x^2 + 2*a^3*b*ln(x^2) - 1/2*(
4*a^3*b*x^2 + a^4)/x^2
```

3.431 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^4}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4
*x^5)/5
```

Rubi [A] time = 0.0663837, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4
*x^5)/5
```

Rubi in Sympy [A] time = 17.3557, size = 46, normalized size = 0.92

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

```
[Out] -a**4/(3*x**3) - 4*a**3*b/x + 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b
**4*x**5/5
```

Mathematica [A] time = 0.0105393, size = 50, normalized size = 1.

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4
*x^5)/5
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$-\frac{a^4}{3x^3} - 4\frac{a^3b}{x} + 6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^4,x)
```

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Maxima [A] time = 0.703725, size = 61, normalized size = 1.22

$$\frac{1}{5}b^4x^5 + \frac{4}{3}ab^3x^3 + 6a^2b^2x - \frac{12a^3bx^2 + a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a
^4)/x^3
```

Fricas [A] time = 0.256249, size = 65, normalized size = 1.3

$$\frac{3b^4x^8 + 20ab^3x^6 + 90a^2b^2x^4 - 60a^3bx^2 - 5a^4}{15x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 -
5*a^4)/x^3
```

Sympy [A] time = 1.17604, size = 48, normalized size = 0.96

$$6a^2b^2x + \frac{4ab^3x^3}{3} + \frac{b^4x^5}{5} - \frac{a^4 + 12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 - (a**4 + 12*a**3*b
*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.268362, size = 61, normalized size = 1.22

$$\frac{1}{5}\,b^4x^5 + \frac{4}{3}\,ab^3x^3 + 6\,a^2b^2x - \frac{12\,a^3bx^2 + a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a
^4)/x^3
```

3.432 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^5}\,dx$

Optimal. Leaf size=49

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*
b^2*Log[x]
```

Rubi [A] time = 0.090487, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*
b^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+3a^2b^2\log\left(x^2\right)+2ab^3x^2+\frac{b^4\int^{x^2}x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

```
[Out] -a**4/(4*x**4) - 2*a**3*b/x**2 + 3*a**2*b**2*log(x**2) + 2*a*b**3
*x**2 + b**4*Integral(x, (x, x**2))/2
```

Mathematica [A] time = 0.00765687, size = 49, normalized size = 1.

$$-\frac{a^4}{4x^4}-\frac{2a^3b}{x^2}+6a^2b^2\log(x)+2ab^3x^2+\frac{b^4x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^5,x]
```

```
[Out] -a^4/(4*x^4) - (2*a^3*b)/x^2 + 2*a*b^3*x^2 + (b^4*x^4)/4 + 6*a^2*
b^2*Log[x]
```

Maple [A] time = 0.009, size = 46, normalized size = 0.9

$$-\frac{a^4}{4\,x^4}-2\,\frac{a^3b}{x^2}+2\,ab^3x^2+\frac{b^4x^4}{4}+6\,a^2b^2\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^5,x)
```

```
[Out] -1/4*a^4/x^4-2*a^3*b/x^2+2*a*b^3*x^2+1/4*b^4*x^4+6*a^2*b^2*ln(x)
```

Maxima [A] time = 0.6975, size = 65, normalized size = 1.33

$$\frac{1}{4}\,b^4x^4+2\,ab^3x^2+3\,a^2b^2\log\left(x^2\right)-\frac{8\,a^3bx^2+a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*log(x^2) - 1/4*(8*a^3*b*x^2
 + a^4)/x^4
```

Fricas [A] time = 0.254174, size = 66, normalized size = 1.35

$$\frac{b^4x^8+8\,ab^3x^6+24\,a^2b^2x^4\log\left(x\right)-8\,a^3bx^2-a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*x^8 + 8*a*b^3*x^6 + 24*a^2*b^2*x^4*log(x) - 8*a^3*b*x^2
- a^4)/x^4
```

Sympy [A] time = 1.33115, size = 48, normalized size = 0.98

$$6a^2b^2\log(x) + 2ab^3x^2 + \frac{b^4x^4}{4} - \frac{a^4 + 8a^3bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**5,x)
```

```
[Out] 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4 - (a**4 + 8*a**3
*b*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.26953, size = 80, normalized size = 1.63

$$\frac{1}{4}\,b^4x^4 + 2\,ab^3x^2 + 3\,a^2b^2\ln\left(x^2\right) - \frac{18\,a^2b^2x^4 + 8\,a^3bx^2 + a^4}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*b^4*x^4 + 2*a*b^3*x^2 + 3*a^2*b^2*ln(x^2) - 1/4*(18*a^2*b^2*x
^4 + 8*a^3*b*x^2 + a^4)/x^4
```

3.433 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^6} dx$

Optimal. Leaf size=50

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b
^4*x^3)/3
```

Rubi [A] time = 0.066992, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b
^4*x^3)/3
```

Rubi in Sympy [A] time = 17.142, size = 46, normalized size = 0.92

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

```
[Out] -a**4/(5*x**5) - 4*a**3*b/(3*x**3) - 6*a**2*b**2/x + 4*a*b**3*x +
 b**4*x**3/3
```

Mathematica [A] time = 0.0136454, size = 50, normalized size = 1.

$$-\frac{a^4}{5x^5}-\frac{4a^3b}{3x^3}-\frac{6a^2b^2}{x}+4ab^3x+\frac{b^4x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^6,x]
```

```
[Out] -a^4/(5*x^5) - (4*a^3*b)/(3*x^3) - (6*a^2*b^2)/x + 4*a*b^3*x + (b
^4*x^3)/3
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$-\frac{a^4}{5\,x^5}-\frac{4\,a^3b}{3\,x^3}-6\,\frac{a^2b^2}{x}+4\,ab^3x+\frac{b^4x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^6,x)
```

```
[Out] -1/5*a^4/x^5-4/3*a^3*b/x^3-6*a^2*b^2/x+4*a*b^3*x+1/3*b^4*x^3
```

Maxima [A] time = 0.696255, size = 63, normalized size = 1.26

$$\frac{1}{3}\,b^4x^3+4\,ab^3x-\frac{90\,a^2b^2x^4+20\,a^3bx^2+3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3
*a^4)/x^5
```

Fricas [A] time = 0.247926, size = 65, normalized size = 1.3

$$\frac{5\,b^4x^8+60\,ab^3x^6-90\,a^2b^2x^4-20\,a^3bx^2-3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*b^4*x^8 + 60*a*b^3*x^6 - 90*a^2*b^2*x^4 - 20*a^3*b*x^2 -
3*a^4)/x^5
```

Sympy [A] time = 1.35551, size = 48, normalized size = 0.96

$$4ab^3x + \frac{b^4x^3}{3} - \frac{3a^4 + 20a^3bx^2 + 90a^2b^2x^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**6,x)
```

```
[Out] 4*a*b**3*x + b**4*x**3/3 - (3*a**4 + 20*a**3*b*x**2 + 90*a**2*b**
2*x**4)/(15*x**5)
```

GIAC/XCAS [A] time = 0.268019, size = 63, normalized size = 1.26

$$\frac{1}{3}\,b^4x^3 + 4\,ab^3x - \frac{90\,a^2b^2x^4 + 20\,a^3bx^2 + 3\,a^4}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*b^4*x^3 + 4*a*b^3*x - 1/15*(90*a^2*b^2*x^4 + 20*a^3*b*x^2 + 3
*a^4)/x^5
```

3.434 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^7}\,dx$

Optimal. Leaf size=49

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*
b^3*Log[x]
```

Rubi [A] time = 0.087116, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*
b^3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+2ab^3\log\left(x^2\right)+\frac{\int^{x^2} b^8\,dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

```
[Out] -a**4/(6*x**6) - a**3*b/x**4 - 3*a**2*b**2/x**2 + 2*a*b**3*log(x*
*2) + Integral(b**8, (x, x**2))/(2*b**4)
```

Mathematica [A] time = 0.00800309, size = 49, normalized size = 1.

$$-\frac{a^4}{6x^6}-\frac{a^3b}{x^4}-\frac{3a^2b^2}{x^2}+4ab^3\log(x)+\frac{b^4x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^7,x]
```

```
[Out] -a^4/(6*x^6) - (a^3*b)/x^4 - (3*a^2*b^2)/x^2 + (b^4*x^2)/2 + 4*a*
b^3*Log[x]
```

Maple [A] time = 0.009, size = 46, normalized size = 0.9

$$-\frac{a^4}{6\,x^6}-\frac{a^3b}{x^4}-3\,\frac{a^2b^2}{x^2}+\frac{b^4x^2}{2}+4\,ab^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^7,x)
```

```
[Out] -1/6*a^4/x^6-a^3*b/x^4-3*a^2*b^2/x^2+1/2*b^4*x^2+4*a*b^3*ln(x)
```

Maxima [A] time = 0.709288, size = 65, normalized size = 1.33

$$\frac{1}{2}\,b^4x^2+2\,ab^3\log\left(x^2\right)-\frac{18\,a^2b^2x^4+6\,a^3bx^2+a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*log(x^2) - 1/6*(18*a^2*b^2*x^4 + 6*a^3*b*x^
2 + a^4)/x^6
```

Fricas [A] time = 0.257205, size = 68, normalized size = 1.39

$$\frac{3\,b^4x^8+24\,ab^3x^6\log(x)-18\,a^2b^2x^4-6\,a^3bx^2-a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 24*a*b^3*x^6*log(x) - 18*a^2*b^2*x^4 - 6*a^3*b*x
^2 - a^4)/x^6
```

Sympy [A] time = 1.51942, size = 48, normalized size = 0.98

$$4ab^3 \log(x) + \frac{b^4x^2}{2} - \frac{a^4 + 6a^3bx^2 + 18a^2b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**7,x)
```

```
[Out] 4*a*b**3*log(x) + b**4*x**2/2 - (a**4 + 6*a**3*b*x**2 + 18*a**2*b
**2*x**4)/(6*x**6)
```

GIAC/XCAS [A] time = 0.268514, size = 77, normalized size = 1.57

$$\frac{1}{2}\,b^4x^2 + 2\,ab^3\ln\left(x^2\right) - \frac{22\,ab^3x^6 + 18\,a^2b^2x^4 + 6\,a^3bx^2 + a^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^4*x^2 + 2*a*b^3*ln(x^2) - 1/6*(22*a*b^3*x^6 + 18*a^2*b^2*x^
4 + 6*a^3*b*x^2 + a^4)/x^6
```

3.435 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x
+ b^4*x
```

Rubi [A] time = 0.0677234, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x
+ b^4*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+\frac{\int b^8\,dx}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

```
[Out] -a**4/(7*x**7) - 4*a**3*b/(5*x**5) - 2*a**2*b**2/x**3 - 4*a*b**3/
x + Integral(b**8, x)/b**4
```

Mathematica [A] time = 0.00943438, size = 47, normalized size = 1.

$$-\frac{a^4}{7x^7}-\frac{4a^3b}{5x^5}-\frac{2a^2b^2}{x^3}-\frac{4ab^3}{x}+b^4x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^8,x]
```

```
[Out] -a^4/(7*x^7) - (4*a^3*b)/(5*x^5) - (2*a^2*b^2)/x^3 - (4*a*b^3)/x
+ b^4*x
```

Maple [A] time = 0.009, size = 44, normalized size = 0.9

$$-\frac{a^4}{7\,x^7}-\frac{4\,a^3b}{5\,x^5}-2\,\frac{a^2b^2}{x^3}-4\,\frac{ab^3}{x}+b^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^8,x)
```

```
[Out] -1/7*a^4/x^7-4/5*a^3*b/x^5-2*a^2*b^2/x^3-4*a*b^3/x+b^4*x
```

Maxima [A] time = 0.698434, size = 62, normalized size = 1.32

$$b^4x-\frac{140\,ab^3x^6+70\,a^2b^2x^4+28\,a^3bx^2+5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^8,x, algorithm="maxima")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a
^4)/x^7
```

Fricas [A] time = 0.248177, size = 65, normalized size = 1.38

$$\frac{35\,b^4x^8-140\,ab^3x^6-70\,a^2b^2x^4-28\,a^3bx^2-5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 - 140*a*b^3*x^6 - 70*a^2*b^2*x^4 - 28*a^3*b*x^2
- 5*a^4)/x^7
```

Sympy [A] time = 1.56148, size = 46, normalized size = 0.98

$$b^4x - \frac{5a^4 + 28a^3bx^2 + 70a^2b^2x^4 + 140ab^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**8,x)
```

```
[Out] b**4*x - (5*a**4 + 28*a**3*b*x**2 + 70*a**2*b**2*x**4 + 140*a*b**
3*x**6)/(35*x**7)
```

GIAC/XCAS [A] time = 0.267272, size = 62, normalized size = 1.32

$$b^4x - \frac{140\,ab^3x^6 + 70\,a^2b^2x^4 + 28\,a^3bx^2 + 5\,a^4}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^8,x, algorithm="giac")
```

```
[Out] b^4*x - 1/35*(140*a*b^3*x^6 + 70*a^2*b^2*x^4 + 28*a^3*b*x^2 + 5*a
^4)/x^7
```

3.436 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{x^9}\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3
)/x^2 + b^4*Log[x]
```

Rubi [A] time = 0.0835066, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3
)/x^2 + b^4*Log[x]
```

Rubi in Sympy [A] time = 20.1615, size = 53, normalized size = 1.06

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+\frac{b^4\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

```
[Out] -a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a*b
**3/x**2 + b**4*log(x**2)/2
```

Mathematica [A] time = 0.00791222, size = 50, normalized size = 1.

$$-\frac{a^4}{8x^8}-\frac{2a^3b}{3x^6}-\frac{3a^2b^2}{2x^4}-\frac{2ab^3}{x^2}+b^4\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^9,x]
```

```
[Out] -a^4/(8*x^8) - (2*a^3*b)/(3*x^6) - (3*a^2*b^2)/(2*x^4) - (2*a*b^3
)/x^2 + b^4*Log[x]
```

Maple [A] time = 0.009, size = 45, normalized size = 0.9

$$-\frac{a^4}{8\,x^8}-\frac{2\,a^3b}{3\,x^6}-\frac{3\,a^2b^2}{2\,x^4}-2\,\frac{ab^3}{x^2}+b^4\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^9,x)
```

```
[Out] -1/8*a^4/x^8-2/3*a^3*b/x^6-3/2*a^2*b^2/x^4-2*a*b^3/x^2+b^4*ln(x)
```

Maxima [A] time = 0.694109, size = 68, normalized size = 1.36

$$\frac{1}{2}\,b^4\log\left(x^2\right)-\frac{48\,ab^3x^6+36\,a^2b^2x^4+16\,a^3bx^2+3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^4*log(x^2) - 1/24*(48*a*b^3*x^6 + 36*a^2*b^2*x^4 + 16*a^3*b
*x^2 + 3*a^4)/x^8
```

Fricas [A] time = 0.255078, size = 68, normalized size = 1.36

$$\frac{24\,b^4x^8\log\left(x\right)-48\,ab^3x^6-36\,a^2b^2x^4-16\,a^3bx^2-3\,a^4}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*b^4*x^8*log(x) - 48*a*b^3*x^6 - 36*a^2*b^2*x^4 - 16*a^3*
b*x^2 - 3*a^4)/x^8
```

Sympy [A] time = 1.74038, size = 48, normalized size = 0.96

$$b^4 \log(x) - \frac{3a^4 + 16a^3bx^2 + 36a^2b^2x^4 + 48ab^3x^6}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**9,x)
```

```
[Out] b**4*log(x) - (3*a**4 + 16*a**3*b*x**2 + 36*a**2*b**2*x**4 + 48*a
*b**3*x**6)/(24*x**8)
```

GIAC/XCAS [A] time = 0.269035, size = 78, normalized size = 1.56

$$\frac{1}{2}\, b^4 \ln\left(x^2\right) - \frac{25\, b^4 x^8 + 48\, ab^3 x^6 + 36\, a^2 b^2 x^4 + 16\, a^3 b x^2 + 3\, a^4}{24\, x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^4*ln(x^2) - 1/24*(25*b^4*x^8 + 48*a*b^3*x^6 + 36*a^2*b^2*x^
4 + 16*a^3*b*x^2 + 3*a^4)/x^8
```

3.437 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3
)/(3*x^3) - b^4/x
```

Rubi [A] time = 0.0699256, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3
)/(3*x^3) - b^4/x
```

Rubi in Sympy [A] time = 17.491, size = 51, normalized size = 0.94

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

```
[Out] -a**4/(9*x**9) - 4*a**3*b/(7*x**7) - 6*a**2*b**2/(5*x**5) - 4*a*b
**3/(3*x**3) - b**4/x
```

Mathematica [A] time = 0.0138565, size = 54, normalized size = 1.

$$-\frac{a^4}{9x^9}-\frac{4a^3b}{7x^7}-\frac{6a^2b^2}{5x^5}-\frac{4ab^3}{3x^3}-\frac{b^4}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (4*a^3*b)/(7*x^7) - (6*a^2*b^2)/(5*x^5) - (4*a*b^3
)/(3*x^3) - b^4/x
```

Maple [A] time = 0.008, size = 47, normalized size = 0.9

$$-\frac{a^4}{9\,x^9}-\frac{4\,a^3b}{7\,x^7}-\frac{6\,a^2b^2}{5\,x^5}-\frac{4\,ab^3}{3\,x^3}-\frac{b^4}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^10,x)
```

```
[Out] -1/9*a^4/x^9-4/7*a^3*b/x^7-6/5*a^2*b^2/x^5-4/3*a*b^3/x^3-b^4/x
```

Maxima [A] time = 0.703666, size = 65, normalized size = 1.2

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b
*x^2 + 35*a^4)/x^9
```

Fricas [A] time = 0.25246, size = 65, normalized size = 1.2

$$-\frac{315\,b^4x^8+420\,ab^3x^6+378\,a^2b^2x^4+180\,a^3bx^2+35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b
*x^2 + 35*a^4)/x^9
```

Sympy [A] time = 1.7507, size = 51, normalized size = 0.94

$$-\frac{35a^4 + 180a^3bx^2 + 378a^2b^2x^4 + 420ab^3x^6 + 315b^4x^8}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**10,x)
```

```
[Out] -(35*a**4 + 180*a**3*b*x**2 + 378*a**2*b**2*x**4 + 420*a*b**3*x**
6 + 315*b**4*x**8)/(315*x**9)
```

GIAC/XCAS [A] time = 0.267739, size = 65, normalized size = 1.2

$$-\frac{315\,b^4x^8 + 420\,ab^3x^6 + 378\,a^2b^2x^4 + 180\,a^3bx^2 + 35\,a^4}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*b^4*x^8 + 420*a*b^3*x^6 + 378*a^2*b^2*x^4 + 180*a^3*b
*x^2 + 35*a^4)/x^9
```

3.438 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{11}} dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

```
[Out] -(a + b*x^2)^5/(10*a*x^10)
```

Rubi [A] time = 0.0244275, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]

[Out] -(a + b*x^2)^5/(10*a*x^10)
```

Rubi in Sympy [A] time = 8.25463, size = 15, normalized size = 0.79

$$-\frac{(a+bx^2)^5}{10ax^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)

[Out] -(a + b*x**2)**5/(10*a*x**10)
```

Mathematica [B] time = 0.00727545, size = 52, normalized size = 2.74

$$-\frac{a^4}{10x^{10}}-\frac{a^3b}{2x^8}-\frac{a^2b^2}{x^6}-\frac{ab^3}{x^4}-\frac{b^4}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^11,x]
```

```
[Out] -a^4/(10*x^10) - (a^3*b)/(2*x^8) - (a^2*b^2)/x^6 - (a*b^3)/x^4 -
b^4/(2*x^2)
```

Maple [B] time = 0.008, size = 47, normalized size = 2.5

$$-\frac{a^{3}b}{2\,x^{8}}-\frac{a^{2}b^{2}}{x^{6}}-\frac{b^{4}}{2\,x^{2}}-\frac{a^{4}}{10\,x^{10}}-\frac{ab^{3}}{x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^11,x)
```

```
[Out] -1/2*a^3*b/x^8-a^2*b^2/x^6-1/2*b^4/x^2-1/10*a^4/x^10-a*b^3/x^4
```

Maxima [A] time = 0.699252, size = 62, normalized size = 3.26

$$-\frac{5\,b^{4}x^{8}+10\,ab^{3}x^{6}+10\,a^{2}b^{2}x^{4}+5\,a^{3}bx^{2}+a^{4}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 +
a^4)/x^10
```

Fricas [A] time = 0.251983, size = 62, normalized size = 3.26

$$-\frac{5\,b^{4}x^{8}+10\,ab^{3}x^{6}+10\,a^{2}b^{2}x^{4}+5\,a^{3}bx^{2}+a^{4}}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 +
a^4)/x^10
```

Sympy [A] time = 1.8601, size = 49, normalized size = 2.58

$$-\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**11,x)
```

```
[Out] -(a**4 + 5*a**3*b*x**2 + 10*a**2*b**2*x**4 + 10*a*b**3*x**6 + 5*b
**4*x**8)/(10*x**10)
```

GIAC/XCAS [A] time = 0.267656, size = 62, normalized size = 3.26

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 +
a^4)/x^10
```

3.439 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b
^3)/(5*x^5) - b^4/(3*x^3)
```

Rubi [A] time = 0.0676182, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b
^3)/(5*x^5) - b^4/(3*x^3)
```

Rubi in Sympy [A] time = 18.0701, size = 54, normalized size = 0.96

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

```
[Out] -a**4/(11*x**11) - 4*a**3*b/(9*x**9) - 6*a**2*b**2/(7*x**7) - 4*a
*b**3/(5*x**5) - b**4/(3*x**3)
```

Mathematica [A] time = 0.0119373, size = 56, normalized size = 1.

$$-\frac{a^4}{11x^{11}}-\frac{4a^3b}{9x^9}-\frac{6a^2b^2}{7x^7}-\frac{4ab^3}{5x^5}-\frac{b^4}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^12,x]
```

```
[Out] -a^4/(11*x^11) - (4*a^3*b)/(9*x^9) - (6*a^2*b^2)/(7*x^7) - (4*a*b
^3)/(5*x^5) - b^4/(3*x^3)
```

Maple [A] time = 0.008, size = 47, normalized size = 0.8

$$-\frac{a^4}{11\,x^{11}}-\frac{4\,a^3b}{9\,x^9}-\frac{6\,a^2b^2}{7\,x^7}-\frac{4\,ab^3}{5\,x^5}-\frac{b^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^12,x)
```

```
[Out] -1/11*a^4/x^11-4/9*a^3*b/x^9-6/7*a^2*b^2/x^7-4/5*a*b^3/x^5-1/3*b^
4/x^3
```

Maxima [A] time = 0.706476, size = 65, normalized size = 1.16

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*
a^3*b*x^2 + 315*a^4)/x^11
```

Fricas [A] time = 0.251334, size = 65, normalized size = 1.16

$$-\frac{1155\,b^4x^8+2772\,ab^3x^6+2970\,a^2b^2x^4+1540\,a^3bx^2+315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*
a^3*b*x^2 + 315*a^4)/x^11
```

Sympy [A] time = 1.8903, size = 51, normalized size = 0.91

$$-\frac{315a^4 + 1540a^3bx^2 + 2970a^2b^2x^4 + 2772ab^3x^6 + 1155b^4x^8}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**12,x)
```

```
[Out] -(315*a**4 + 1540*a**3*b*x**2 + 2970*a**2*b**2*x**4 + 2772*a*b**3
*x**6 + 1155*b**4*x**8)/(3465*x**11)
```

GIAC/XCAS [A] time = 0.268765, size = 65, normalized size = 1.16

$$-\frac{1155\,b^4x^8 + 2772\,ab^3x^6 + 2970\,a^2b^2x^4 + 1540\,a^3bx^2 + 315\,a^4}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*b^4*x^8 + 2772*a*b^3*x^6 + 2970*a^2*b^2*x^4 + 1540*
a^3*b*x^2 + 315*a^4)/x^11
```

3.440 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{13}}\,dx$

Optimal. Leaf size=40

$$\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}-\frac{\left(a+bx^2\right)^5}{12ax^{12}}$$

```
[Out] -(a + b*x^2)^5/(12*a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rubi [A] time = 0.0724243, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{b\left(a+bx^2\right)^5}{60a^2x^{10}}-\frac{\left(a+bx^2\right)^5}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -(a + b*x^2)^5/(12*a*x^12) + (b*(a + b*x^2)^5)/(60*a^2*x^10)
```

Rubi in Sympy [A] time = 21.0023, size = 54, normalized size = 1.35

$$-\frac{a^4}{12x^{12}}-\frac{2a^3b}{5x^{10}}-\frac{3a^2b^2}{4x^8}-\frac{2ab^3}{3x^6}-\frac{b^4}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)
```

```
[Out] -a**4/(12*x**12) - 2*a**3*b/(5*x**10) - 3*a**2*b**2/(4*x**8) - 2*
a*b**3/(3*x**6) - b**4/(4*x**4)
```

Mathematica [A] time = 0.00694587, size = 56, normalized size = 1.4

$$-\frac{a^4}{12x^{12}}-\frac{2a^3b}{5x^{10}}-\frac{3a^2b^2}{4x^8}-\frac{2ab^3}{3x^6}-\frac{b^4}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^13,x]
```

```
[Out] -a^4/(12*x^12) - (2*a^3*b)/(5*x^10) - (3*a^2*b^2)/(4*x^8) - (2*a*
b^3)/(3*x^6) - b^4/(4*x^4)
```

Maple [A] time = 0.008, size = 47, normalized size = 1.2

$$-\frac{3\,a^{2}b^{2}}{4\,x^{8}}-\frac{a^{4}}{12\,x^{12}}-\frac{2\,ab^{3}}{3\,x^{6}}-\frac{2\,a^{3}b}{5\,x^{10}}-\frac{b^{4}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^13,x)
```

```
[Out] -3/4*a^2*b^2/x^8-1/12*a^4/x^12-2/3*a*b^3/x^6-2/5*a^3*b/x^10-1/4*b
^4/x^4
```

Maxima [A] time = 0.702922, size = 65, normalized size = 1.62

$$-\frac{15\,b^{4}x^{8}+40\,ab^{3}x^{6}+45\,a^{2}b^{2}x^{4}+24\,a^{3}bx^{2}+5\,a^{4}}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2
+ 5*a^4)/x^12
```

Fricas [A] time = 0.254856, size = 65, normalized size = 1.62

$$-\frac{15\,b^{4}x^{8}+40\,ab^{3}x^{6}+45\,a^{2}b^{2}x^{4}+24\,a^{3}bx^{2}+5\,a^{4}}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2
+ 5*a^4)/x^12
```

Sympy [A] time = 1.98, size = 51, normalized size = 1.27

$$-\frac{5a^4 + 24a^3bx^2 + 45a^2b^2x^4 + 40ab^3x^6 + 15b^4x^8}{60x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**13,x)
```

```
[Out] -(5*a**4 + 24*a**3*b*x**2 + 45*a**2*b**2*x**4 + 40*a*b**3*x**6 +
15*b**4*x**8)/(60*x**12)
```

GIAC/XCAS [A] time = 0.269104, size = 65, normalized size = 1.62

$$-\frac{15\,b^4x^8 + 40\,ab^3x^6 + 45\,a^2b^2x^4 + 24\,a^3bx^2 + 5\,a^4}{60\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^4*x^8 + 40*a*b^3*x^6 + 45*a^2*b^2*x^4 + 24*a^3*b*x^2
+ 5*a^4)/x^12
```

3.441 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{14}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a
*b^3)/(7*x^7) - b^4/(5*x^5)
```

Rubi [A] time = 0.0687819, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a
*b^3)/(7*x^7) - b^4/(5*x^5)
```

Rubi in Sympy [A] time = 17.6988, size = 54, normalized size = 0.96

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)
```

```
[Out] -a**4/(13*x**13) - 4*a**3*b/(11*x**11) - 2*a**2*b**2/(3*x**9) - 4
*a*b**3/(7*x**7) - b**4/(5*x**5)
```

Mathematica [A] time = 0.0144789, size = 56, normalized size = 1.

$$-\frac{a^4}{13x^{13}}-\frac{4a^3b}{11x^{11}}-\frac{2a^2b^2}{3x^9}-\frac{4ab^3}{7x^7}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^14,x]
```

```
[Out] -a^4/(13*x^13) - (4*a^3*b)/(11*x^11) - (2*a^2*b^2)/(3*x^9) - (4*a
*b^3)/(7*x^7) - b^4/(5*x^5)
```

Maple [A] time = 0.011, size = 47, normalized size = 0.8

$$-\frac{a^4}{13\,x^{13}}-\frac{4\,a^3b}{11\,x^{11}}-\frac{2\,a^2b^2}{3\,x^9}-\frac{4\,ab^3}{7\,x^7}-\frac{b^4}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^14,x)
```

```
[Out] -1/13*a^4/x^13-4/11*a^3*b/x^11-2/3*a^2*b^2/x^9-4/7*a*b^3/x^7-1/5*
b^4/x^5
```

Maxima [A] time = 0.696032, size = 65, normalized size = 1.16

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^14,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 546
0*a^3*b*x^2 + 1155*a^4)/x^13
```

Fricas [A] time = 0.255381, size = 65, normalized size = 1.16

$$-\frac{3003\,b^4x^8+8580\,ab^3x^6+10010\,a^2b^2x^4+5460\,a^3bx^2+1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^14,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 546
0*a^3*b*x^2 + 1155*a^4)/x^13
```

Sympy [A] time = 1.96819, size = 51, normalized size = 0.91

$$-\frac{1155a^4 + 5460a^3bx^2 + 10010a^2b^2x^4 + 8580ab^3x^6 + 3003b^4x^8}{15015x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**14,x)
```

```
[Out] -(1155*a**4 + 5460*a**3*b*x**2 + 10010*a**2*b**2*x**4 + 8580*a*b*
*3*x**6 + 3003*b**4*x**8)/(15015*x**13)
```

GIAC/XCAS [A] time = 0.26604, size = 65, normalized size = 1.16

$$-\frac{3003\,b^4x^8 + 8580\,ab^3x^6 + 10010\,a^2b^2x^4 + 5460\,a^3bx^2 + 1155\,a^4}{15015\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^14,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^4*x^8 + 8580*a*b^3*x^6 + 10010*a^2*b^2*x^4 + 546
0*a^3*b*x^2 + 1155*a^4)/x^13
```

3.442 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{15}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3
)/(2*x^8) - b^4/(6*x^6)
```

Rubi [A] time = 0.0877397, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3
)/(2*x^8) - b^4/(6*x^6)
```

Rubi in Sympy [A] time = 20.8273, size = 51, normalized size = 0.91

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

```
[Out] -a**4/(14*x**14) - a**3*b/(3*x**12) - 3*a**2*b**2/(5*x**10) - a*b
**3/(2*x**8) - b**4/(6*x**6)
```

Mathematica [A] time = 0.00711226, size = 56, normalized size = 1.

$$-\frac{a^4}{14x^{14}}-\frac{a^3b}{3x^{12}}-\frac{3a^2b^2}{5x^{10}}-\frac{ab^3}{2x^8}-\frac{b^4}{6x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^15,x]
```

```
[Out] -a^4/(14*x^14) - (a^3*b)/(3*x^12) - (3*a^2*b^2)/(5*x^10) - (a*b^3
)/(2*x^8) - b^4/(6*x^6)
```

Maple [A] time = 0.008, size = 47, normalized size = 0.8

$$-\frac{a^4}{14\,x^{14}}-\frac{a^3b}{3\,x^{12}}-\frac{3\,a^2b^2}{5\,x^{10}}-\frac{ab^3}{2\,x^8}-\frac{b^4}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^15,x)
```

```
[Out] -1/14*a^4/x^14-1/3*a^3*b/x^12-3/5*a^2*b^2/x^10-1/2*a*b^3/x^8-1/6*
b^4/x^6
```

Maxima [A] time = 0.694763, size = 65, normalized size = 1.16

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^15,x, algorithm="maxima")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x
^2 + 15*a^4)/x^14
```

Fricas [A] time = 0.257721, size = 65, normalized size = 1.16

$$-\frac{35\,b^4x^8+105\,ab^3x^6+126\,a^2b^2x^4+70\,a^3bx^2+15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^15,x, algorithm="fricas")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x
^2 + 15*a^4)/x^14
```

Sympy [A] time = 2.07676, size = 51, normalized size = 0.91

$$-\frac{15a^4 + 70a^3bx^2 + 126a^2b^2x^4 + 105ab^3x^6 + 35b^4x^8}{210x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**15,x)
```

```
[Out] -(15*a**4 + 70*a**3*b*x**2 + 126*a**2*b**2*x**4 + 105*a*b**3*x**6
 + 35*b**4*x**8)/(210*x**14)
```

GIAC/XCAS [A] time = 0.268165, size = 65, normalized size = 1.16

$$-\frac{35\,b^4x^8 + 105\,ab^3x^6 + 126\,a^2b^2x^4 + 70\,a^3bx^2 + 15\,a^4}{210\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^15,x, algorithm="giac")
```

```
[Out] -1/210*(35*b^4*x^8 + 105*a*b^3*x^6 + 126*a^2*b^2*x^4 + 70*a^3*b*x
^2 + 15*a^4)/x^14
```

3.443 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{x^{16}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4
*a*b^3)/(9*x^9) - b^4/(7*x^7)
```

Rubi [A] time = 0.0673062, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4
*a*b^3)/(9*x^9) - b^4/(7*x^7)
```

Rubi in Sympy [A] time = 17.6708, size = 54, normalized size = 0.96

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**16,x)
```

```
[Out] -a**4/(15*x**15) - 4*a**3*b/(13*x**13) - 6*a**2*b**2/(11*x**11) -
 4*a*b**3/(9*x**9) - b**4/(7*x**7)
```

Mathematica [A] time = 0.0117933, size = 56, normalized size = 1.

$$-\frac{a^4}{15x^{15}}-\frac{4a^3b}{13x^{13}}-\frac{6a^2b^2}{11x^{11}}-\frac{4ab^3}{9x^9}-\frac{b^4}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/x^16,x]
```

```
[Out] -a^4/(15*x^15) - (4*a^3*b)/(13*x^13) - (6*a^2*b^2)/(11*x^11) - (4
*a*b^3)/(9*x^9) - b^4/(7*x^7)
```

Maple [A] time = 0.008, size = 47, normalized size = 0.8

$$-\frac{a^4}{15\,x^{15}}-\frac{4\,a^3b}{13\,x^{13}}-\frac{6\,a^2b^2}{11\,x^{11}}-\frac{4\,ab^3}{9\,x^9}-\frac{b^4}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/x^16,x)
```

```
[Out] -1/15*a^4/x^15-4/13*a^3*b/x^13-6/11*a^2*b^2/x^11-4/9*a*b^3/x^9-1/
7*b^4/x^7
```

Maxima [A] time = 0.695141, size = 65, normalized size = 1.16

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13
860*a^3*b*x^2 + 3003*a^4)/x^15
```

Fricas [A] time = 0.251345, size = 65, normalized size = 1.16

$$-\frac{6435\,b^4x^8+20020\,ab^3x^6+24570\,a^2b^2x^4+13860\,a^3bx^2+3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13
860*a^3*b*x^2 + 3003*a^4)/x^15
```

Sympy [A] time = 2.09507, size = 51, normalized size = 0.91

$$-\frac{3003a^4 + 13860a^3bx^2 + 24570a^2b^2x^4 + 20020ab^3x^6 + 6435b^4x^8}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/x**16,x)
```

```
[Out] -(3003*a**4 + 13860*a**3*b*x**2 + 24570*a**2*b**2*x**4 + 20020*a*
b**3*x**6 + 6435*b**4*x**8)/(45045*x**15)
```

GIAC/XCAS [A] time = 0.268397, size = 65, normalized size = 1.16

$$-\frac{6435\,b^4x^8 + 20020\,ab^3x^6 + 24570\,a^2b^2x^4 + 13860\,a^3bx^2 + 3003\,a^4}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(6435*b^4*x^8 + 20020*a*b^3*x^6 + 24570*a^2*b^2*x^4 + 13
860*a^3*b*x^2 + 3003*a^4)/x^15
```

3.444 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b
^3*x^15)/3 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21
)/21
```

Rubi [A] time = 0.116048, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b
^3*x^15)/3 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21
)/21
```

Rubi in Sympy [A] time = 23.2947, size = 80, normalized size = 0.98

$$\frac{a^6x^9}{9}+\frac{6a^5bx^{11}}{11}+\frac{15a^4b^2x^{13}}{13}+\frac{4a^3b^3x^{15}}{3}+\frac{15a^2b^4x^{17}}{17}+\frac{6ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*
b**3*x**15/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x
**21/21
```

Mathematica [A] time = 0.00525348, size = 82, normalized size = 1.

$$\frac{a^6x^9}{9}+\frac{6}{11}a^5bx^{11}+\frac{15}{13}a^4b^2x^{13}+\frac{4}{3}a^3b^3x^{15}+\frac{15}{17}a^2b^4x^{17}+\frac{6}{19}ab^5x^{19}+\frac{b^6x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^9)/9 + (6*a^5*b*x^11)/11 + (15*a^4*b^2*x^13)/13 + (4*a^3*b
^3*x^15)/3 + (15*a^2*b^4*x^17)/17 + (6*a*b^5*x^19)/19 + (b^6*x^21
)/21
```

Maple [A] time = 0.001, size = 69, normalized size = 0.8

$$\frac{a^6x^9}{9}+\frac{6\,a^5bx^{11}}{11}+\frac{15\,a^4b^2x^{13}}{13}+\frac{4\,a^3b^3x^{15}}{3}+\frac{15\,a^2b^4x^{17}}{17}+\frac{6\,ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/9*a^6*x^9+6/11*a^5*b*x^11+15/13*a^4*b^2*x^13+4/3*a^3*b^3*x^15+1
5/17*a^2*b^4*x^17+6/19*a*b^5*x^19+1/21*b^6*x^21
```

Maxima [A] time = 0.703172, size = 92, normalized size = 1.12

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^8,x, algorithm="maxima")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^
3*x^15 + 15/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

Fricas [A] time = 0.235543, size = 1, normalized size = 0.01

$$\frac{1}{21}x^{21}b^6+\frac{6}{19}x^{19}b^5a+\frac{15}{17}x^{17}b^4a^2+\frac{4}{3}x^{15}b^3a^3+\frac{15}{13}x^{13}b^2a^4+\frac{6}{11}x^{11}ba^5+\frac{1}{9}x^9a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^8,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^6 + 6/19*x^19*b^5*a + 15/17*x^17*b^4*a^2 + 4/3*x^15*b
^3*a^3 + 15/13*x^13*b^2*a^4 + 6/11*x^11*b*a^5 + 1/9*x^9*a^6
```

Sympy [A] time = 0.131709, size = 80, normalized size = 0.98

$$\frac{a^6x^9}{9}+\frac{6a^5bx^{11}}{11}+\frac{15a^4b^2x^{13}}{13}+\frac{4a^3b^3x^{15}}{3}+\frac{15a^2b^4x^{17}}{17}+\frac{6ab^5x^{19}}{19}+\frac{b^6x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**9/9 + 6*a**5*b*x**11/11 + 15*a**4*b**2*x**13/13 + 4*a**3*
b**3*x**15/3 + 15*a**2*b**4*x**17/17 + 6*a*b**5*x**19/19 + b**6*x
**21/21
```

GIAC/XCAS [A] time = 0.267199, size = 92, normalized size = 1.12

$$\frac{1}{21}\,b^6x^{21}+\frac{6}{19}\,ab^5x^{19}+\frac{15}{17}\,a^2b^4x^{17}+\frac{4}{3}\,a^3b^3x^{15}+\frac{15}{13}\,a^4b^2x^{13}+\frac{6}{11}\,a^5bx^{11}+\frac{1}{9}\,a^6x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^8,x, algorithm="giac")
```

```
[Out] 1/21*b^6*x^21 + 6/19*a*b^5*x^19 + 15/17*a^2*b^4*x^17 + 4/3*a^3*b^
3*x^15 + 15/13*a^4*b^2*x^13 + 6/11*a^5*b*x^11 + 1/9*a^6*x^9
```

3.445 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=72

$$-\frac{a^3 \left(a+bx^2\right)^7}{14b^4}+\frac{3a^2 \left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a \left(a+bx^2\right)^9}{6b^4}$$

```
[Out] -(a^3*(a + b*x^2)^7)/(14*b^4) + (3*a^2*(a + b*x^2)^8)/(16*b^4) -
(a*(a + b*x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rubi [A] time = 0.255441, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{a^3 \left(a+bx^2\right)^7}{14b^4}+\frac{3a^2 \left(a+bx^2\right)^8}{16b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}-\frac{a \left(a+bx^2\right)^9}{6b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^3*(a + b*x^2)^7)/(14*b^4) + (3*a^2*(a + b*x^2)^8)/(16*b^4) -
(a*(a + b*x^2)^9)/(6*b^4) + (a + b*x^2)^10/(20*b^4)
```

Rubi in Sympy [A] time = 27.5963, size = 63, normalized size = 0.88

$$-\frac{a^3 \left(a+bx^2\right)^7}{14b^4}+\frac{3a^2 \left(a+bx^2\right)^8}{16b^4}-\frac{a \left(a+bx^2\right)^9}{6b^4}+\frac{\left(a+bx^2\right)^{10}}{20b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -a**3*(a + b*x**2)**7/(14*b**4) + 3*a**2*(a + b*x**2)**8/(16*b**4
) - a*(a + b*x**2)**9/(6*b**4) + (a + b*x**2)**10/(20*b**4)
```

Mathematica [A] time = 0.00438665, size = 82, normalized size = 1.14

$$\frac{a^6x^8}{8}+\frac{3}{5}a^5bx^{10}+\frac{5}{4}a^4b^2x^{12}+\frac{10}{7}a^3b^3x^{14}+\frac{15}{16}a^2b^4x^{16}+\frac{1}{3}ab^5x^{18}+\frac{b^6x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^8)/8 + (3*a^5*b*x^10)/5 + (5*a^4*b^2*x^12)/4 + (10*a^3*b^3
*x^14)/7 + (15*a^2*b^4*x^16)/16 + (a*b^5*x^18)/3 + (b^6*x^20)/20
```

Maple [A] time = 0.002, size = 69, normalized size = 1.

$$\frac{b^6x^{20}}{20}+\frac{ab^5x^{18}}{3}+\frac{15\,a^2b^4x^{16}}{16}+\frac{10\,a^3b^3x^{14}}{7}+\frac{5\,a^4b^2x^{12}}{4}+\frac{3\,a^5bx^{10}}{5}+\frac{a^6x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/20*b^6*x^20+1/3*a*b^5*x^18+15/16*a^2*b^4*x^16+10/7*a^3*b^3*x^14
+5/4*a^4*b^2*x^12+3/5*a^5*b*x^10+1/8*a^6*x^8
```

Maxima [A] time = 0.704549, size = 92, normalized size = 1.28

$$\frac{1}{20}\,b^6x^{20}+\frac{1}{3}\,ab^5x^{18}+\frac{15}{16}\,a^2b^4x^{16}+\frac{10}{7}\,a^3b^3x^{14}+\frac{5}{4}\,a^4b^2x^{12}+\frac{3}{5}\,a^5bx^{10}+\frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^7,x, algorithm="maxima")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^
3*x^14 + 5/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

Fricas [A] time = 0.236289, size = 1, normalized size = 0.01

$$\frac{1}{20}x^{20}b^6+\frac{1}{3}x^{18}b^5a+\frac{15}{16}x^{16}b^4a^2+\frac{10}{7}x^{14}b^3a^3+\frac{5}{4}x^{12}b^2a^4+\frac{3}{5}x^{10}ba^5+\frac{1}{8}x^8a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^7,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^6 + 1/3*x^18*b^5*a + 15/16*x^16*b^4*a^2 + 10/7*x^14*b
^3*a^3 + 5/4*x^12*b^2*a^4 + 3/5*x^10*b*a^5 + 1/8*x^8*a^6
```

Sympy [A] time = 0.13885, size = 78, normalized size = 1.08

$$\frac{a^6x^8}{8} + \frac{3a^5bx^{10}}{5} + \frac{5a^4b^2x^{12}}{4} + \frac{10a^3b^3x^{14}}{7} + \frac{15a^2b^4x^{16}}{16} + \frac{ab^5x^{18}}{3} + \frac{b^6x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**8/8 + 3*a**5*b*x**10/5 + 5*a**4*b**2*x**12/4 + 10*a**3*b*
*3*x**14/7 + 15*a**2*b**4*x**16/16 + a*b**5*x**18/3 + b**6*x**20/
20
```

GIAC/XCAS [A] time = 0.268042, size = 92, normalized size = 1.28

$$\frac{1}{20}\,b^6x^{20} + \frac{1}{3}\,ab^5x^{18} + \frac{15}{16}\,a^2b^4x^{16} + \frac{10}{7}\,a^3b^3x^{14} + \frac{5}{4}\,a^4b^2x^{12} + \frac{3}{5}\,a^5bx^{10} + \frac{1}{8}\,a^6x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^7,x, algorithm="giac")
```

```
[Out] 1/20*b^6*x^20 + 1/3*a*b^5*x^18 + 15/16*a^2*b^4*x^16 + 10/7*a^3*b^
3*x^14 + 5/4*a^4*b^2*x^12 + 3/5*a^5*b*x^10 + 1/8*a^6*x^8
```

3.446 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=79

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^
3*x^13)/13 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rubi [A] time = 0.104694, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^
3*x^13)/13 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Rubi in Sympy [A] time = 23.1571, size = 76, normalized size = 0.96

$$\frac{a^6x^7}{7}+\frac{2a^5bx^9}{3}+\frac{15a^4b^2x^{11}}{11}+\frac{20a^3b^3x^{13}}{13}+a^2b^4x^{15}+\frac{6ab^5x^{17}}{17}+\frac{b^6x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b
**3*x**13/13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/1
9
```

Mathematica [A] time = 0.00407754, size = 79, normalized size = 1.

$$\frac{a^6x^7}{7}+\frac{2}{3}a^5bx^9+\frac{15}{11}a^4b^2x^{11}+\frac{20}{13}a^3b^3x^{13}+a^2b^4x^{15}+\frac{6}{17}ab^5x^{17}+\frac{b^6x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^7)/7 + (2*a^5*b*x^9)/3 + (15*a^4*b^2*x^11)/11 + (20*a^3*b^
3*x^13)/13 + a^2*b^4*x^15 + (6*a*b^5*x^17)/17 + (b^6*x^19)/19
```

Maple [A] time = 0.001, size = 68, normalized size = 0.9

$$\frac{a^{6}x^{7}}{7}+\frac{2\,a^{5}bx^{9}}{3}+\frac{15\,a^{4}b^{2}x^{11}}{11}+\frac{20\,a^{3}b^{3}x^{13}}{13}+a^{2}b^{4}x^{15}+\frac{6\,ab^{5}x^{17}}{17}+\frac{b^{6}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/7*a^6*x^7+2/3*a^5*b*x^9+15/11*a^4*b^2*x^11+20/13*a^3*b^3*x^13+a
^2*b^4*x^15+6/17*a*b^5*x^17+1/19*b^6*x^19
```

Maxima [A] time = 0.695062, size = 90, normalized size = 1.14

$$\frac{1}{19}\,b^{6}x^{19}+\frac{6}{17}\,ab^{5}x^{17}+a^{2}b^{4}x^{15}+\frac{20}{13}\,a^{3}b^{3}x^{13}+\frac{15}{11}\,a^{4}b^{2}x^{11}+\frac{2}{3}\,a^{5}bx^{9}+\frac{1}{7}\,a^{6}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^6,x, algorithm="maxima")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^
13 + 15/11*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

Fricas [A] time = 0.236235, size = 1, normalized size = 0.01

$$\frac{1}{19}x^{19}b^{6}+\frac{6}{17}x^{17}b^{5}a+x^{15}b^{4}a^{2}+\frac{20}{13}x^{13}b^{3}a^{3}+\frac{15}{11}x^{11}b^{2}a^{4}+\frac{2}{3}x^{9}ba^{5}+\frac{1}{7}x^{7}a^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^6,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^6 + 6/17*x^17*b^5*a + x^15*b^4*a^2 + 20/13*x^13*b^3*a
^3 + 15/11*x^11*b^2*a^4 + 2/3*x^9*b*a^5 + 1/7*x^7*a^6
```

Sympy [A] time = 0.134051, size = 76, normalized size = 0.96

$$\frac{a^6x^7}{7}+\frac{2a^5bx^9}{3}+\frac{15a^4b^2x^{11}}{11}+\frac{20a^3b^3x^{13}}{13}+a^2b^4x^{15}+\frac{6ab^5x^{17}}{17}+\frac{b^6x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**7/7 + 2*a**5*b*x**9/3 + 15*a**4*b**2*x**11/11 + 20*a**3*b
**3*x**13/13 + a**2*b**4*x**15 + 6*a*b**5*x**17/17 + b**6*x**19/1
9
```

GIAC/XCAS [A] time = 0.267808, size = 90, normalized size = 1.14

$$\frac{1}{19}\,b^6x^{19}+\frac{6}{17}\,ab^5x^{17}+a^2b^4x^{15}+\frac{20}{13}\,a^3b^3x^{13}+\frac{15}{11}\,a^4b^2x^{11}+\frac{2}{3}\,a^5bx^9+\frac{1}{7}\,a^6x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^6,x, algorithm="giac")
```

```
[Out] 1/19*b^6*x^19 + 6/17*a*b^5*x^17 + a^2*b^4*x^15 + 20/13*a^3*b^3*x^
13 + 15/11*a^4*b^2*x^11 + 2/3*a^5*b*x^9 + 1/7*a^6*x^7
```

3.447 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=53

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}$$

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b
*x^2)^9/(18*b^3)
```

Rubi [A] time = 0.190359, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^2*(a + b*x^2)^7)/(14*b^3) - (a*(a + b*x^2)^8)/(8*b^3) + (a + b
*x^2)^9/(18*b^3)
```

Rubi in Sympy [A] time = 22.3288, size = 44, normalized size = 0.83

$$\frac{a^2\left(a+bx^2\right)^7}{14b^3}-\frac{a\left(a+bx^2\right)^8}{8b^3}+\frac{\left(a+bx^2\right)^9}{18b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**2*(a + b*x**2)**7/(14*b**3) - a*(a + b*x**2)**8/(8*b**3) + (a
+ b*x**2)**9/(18*b**3)
```

Mathematica [A] time = 0.00425449, size = 82, normalized size = 1.55

$$\frac{a^6x^6}{6}+\frac{3}{4}a^5bx^8+\frac{3}{2}a^4b^2x^{10}+\frac{5}{3}a^3b^3x^{12}+\frac{15}{14}a^2b^4x^{14}+\frac{3}{8}ab^5x^{16}+\frac{b^6x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^6)/6 + (3*a^5*b*x^8)/4 + (3*a^4*b^2*x^10)/2 + (5*a^3*b^3*x
^12)/3 + (15*a^2*b^4*x^14)/14 + (3*a*b^5*x^16)/8 + (b^6*x^18)/18
```

Maple [A] time = 0.001, size = 69, normalized size = 1.3

$$\frac{b^6x^{18}}{18}+\frac{3\,ab^5x^{16}}{8}+\frac{15\,a^2b^4x^{14}}{14}+\frac{5\,a^3b^3x^{12}}{3}+\frac{3\,a^4b^2x^{10}}{2}+\frac{3\,a^5bx^8}{4}+\frac{a^6x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/18*b^6*x^18+3/8*a*b^5*x^16+15/14*a^2*b^4*x^14+5/3*a^3*b^3*x^12+
3/2*a^4*b^2*x^10+3/4*a^5*b*x^8+1/6*a^6*x^6
```

Maxima [A] time = 0.704358, size = 92, normalized size = 1.74

$$\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^5,x, algorithm="maxima")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3
*x^12 + 3/2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

Fricas [A] time = 0.236853, size = 1, normalized size = 0.02

$$\frac{1}{18}x^{18}b^6+\frac{3}{8}x^{16}b^5a+\frac{15}{14}x^{14}b^4a^2+\frac{5}{3}x^{12}b^3a^3+\frac{3}{2}x^{10}b^2a^4+\frac{3}{4}x^8ba^5+\frac{1}{6}x^6a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^5,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^6 + 3/8*x^16*b^5*a + 15/14*x^14*b^4*a^2 + 5/3*x^12*b^
3*a^3 + 3/2*x^10*b^2*a^4 + 3/4*x^8*b*a^5 + 1/6*x^6*a^6
```

Sympy [A] time = 0.134084, size = 80, normalized size = 1.51

$$\frac{a^6x^6}{6}+\frac{3a^5bx^8}{4}+\frac{3a^4b^2x^{10}}{2}+\frac{5a^3b^3x^{12}}{3}+\frac{15a^2b^4x^{14}}{14}+\frac{3ab^5x^{16}}{8}+\frac{b^6x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**6/6 + 3*a**5*b*x**8/4 + 3*a**4*b**2*x**10/2 + 5*a**3*b**3
*x**12/3 + 15*a**2*b**4*x**14/14 + 3*a*b**5*x**16/8 + b**6*x**18/
18
```

GIAC/XCAS [A] time = 0.268115, size = 92, normalized size = 1.74

$$\frac{1}{18}\,b^6x^{18}+\frac{3}{8}\,ab^5x^{16}+\frac{15}{14}\,a^2b^4x^{14}+\frac{5}{3}\,a^3b^3x^{12}+\frac{3}{2}\,a^4b^2x^{10}+\frac{3}{4}\,a^5bx^8+\frac{1}{6}\,a^6x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^5,x, algorithm="giac")
```

```
[Out] 1/18*b^6*x^18 + 3/8*a*b^5*x^16 + 15/14*a^2*b^4*x^14 + 5/3*a^3*b^3
*x^12 + 3/2*a^4*b^2*x^10 + 3/4*a^5*b*x^8 + 1/6*a^6*x^6
```

3.448 $\int x^4 \left(a^2 + 2abx^2 + b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^5}{5} + \frac{6}{7}a^5bx^7 + \frac{5}{3}a^4b^2x^9 + \frac{20}{11}a^3b^3x^{11} + \frac{15}{13}a^2b^4x^{13} + \frac{2}{5}ab^5x^{15} + \frac{b^6x^{17}}{17}$$

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x
^11)/11 + (15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rubi [A] time = 0.103931, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{a^6x^5}{5} + \frac{6}{7}a^5bx^7 + \frac{5}{3}a^4b^2x^9 + \frac{20}{11}a^3b^3x^{11} + \frac{15}{13}a^2b^4x^{13} + \frac{2}{5}ab^5x^{15} + \frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x
^11)/11 + (15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Rubi in Sympy [A] time = 24.5998, size = 80, normalized size = 0.98

$$\frac{a^6x^5}{5} + \frac{6a^5bx^7}{7} + \frac{5a^4b^2x^9}{3} + \frac{20a^3b^3x^{11}}{11} + \frac{15a^2b^4x^{13}}{13} + \frac{2ab^5x^{15}}{5} + \frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3
*x**11/11 + 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17
/17
```

Mathematica [A] time = 0.00411498, size = 82, normalized size = 1.

$$\frac{a^6x^5}{5} + \frac{6}{7}a^5bx^7 + \frac{5}{3}a^4b^2x^9 + \frac{20}{11}a^3b^3x^{11} + \frac{15}{13}a^2b^4x^{13} + \frac{2}{5}ab^5x^{15} + \frac{b^6x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^5)/5 + (6*a^5*b*x^7)/7 + (5*a^4*b^2*x^9)/3 + (20*a^3*b^3*x
^11)/11 + (15*a^2*b^4*x^13)/13 + (2*a*b^5*x^15)/5 + (b^6*x^17)/17
```

Maple [A] time = 0.001, size = 69, normalized size = 0.8

$$\frac{a^6x^5}{5}+\frac{6\,a^5bx^7}{7}+\frac{5\,a^4b^2x^9}{3}+\frac{20\,a^3b^3x^{11}}{11}+\frac{15\,a^2b^4x^{13}}{13}+\frac{2\,ab^5x^{15}}{5}+\frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*a^6*x^5+6/7*a^5*b*x^7+5/3*a^4*b^2*x^9+20/11*a^3*b^3*x^11+15/1
3*a^2*b^4*x^13+2/5*a*b^5*x^15+1/17*b^6*x^17
```

Maxima [A] time = 0.678619, size = 92, normalized size = 1.12

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^4,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b
^3*x^11 + 5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

Fricas [A] time = 0.234654, size = 1, normalized size = 0.01

$$\frac{1}{17}x^{17}b^6+\frac{2}{5}x^{15}b^5a+\frac{15}{13}x^{13}b^4a^2+\frac{20}{11}x^{11}b^3a^3+\frac{5}{3}x^9b^2a^4+\frac{6}{7}x^7ba^5+\frac{1}{5}x^5a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^4,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^6 + 2/5*x^15*b^5*a + 15/13*x^13*b^4*a^2 + 20/11*x^11*
b^3*a^3 + 5/3*x^9*b^2*a^4 + 6/7*x^7*b*a^5 + 1/5*x^5*a^6
```

Sympy [A] time = 0.128291, size = 80, normalized size = 0.98

$$\frac{a^6x^5}{5}+\frac{6a^5bx^7}{7}+\frac{5a^4b^2x^9}{3}+\frac{20a^3b^3x^{11}}{11}+\frac{15a^2b^4x^{13}}{13}+\frac{2ab^5x^{15}}{5}+\frac{b^6x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**5/5 + 6*a**5*b*x**7/7 + 5*a**4*b**2*x**9/3 + 20*a**3*b**3
*x**11/11 + 15*a**2*b**4*x**13/13 + 2*a*b**5*x**15/5 + b**6*x**17
/17
```

GIAC/XCAS [A] time = 0.268755, size = 92, normalized size = 1.12

$$\frac{1}{17}\,b^6x^{17}+\frac{2}{5}\,ab^5x^{15}+\frac{15}{13}\,a^2b^4x^{13}+\frac{20}{11}\,a^3b^3x^{11}+\frac{5}{3}\,a^4b^2x^9+\frac{6}{7}\,a^5bx^7+\frac{1}{5}\,a^6x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^4,x, algorithm="giac")
```

```
[Out] 1/17*b^6*x^17 + 2/5*a*b^5*x^15 + 15/13*a^2*b^4*x^13 + 20/11*a^3*b
^3*x^11 + 5/3*a^4*b^2*x^9 + 6/7*a^5*b*x^7 + 1/5*a^6*x^5
```

3.449 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=34

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

```
[Out] -(a*(a + b*x^2)^7)/(14*b^2) + (a + b*x^2)^8/(16*b^2)
```

Rubi [A] time = 0.118149, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(a+bx^2\right)^8}{16b^2}-\frac{a\left(a+bx^2\right)^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] -(a*(a + b*x^2)^7)/(14*b^2) + (a + b*x^2)^8/(16*b^2)
```

Rubi in Sympy [A] time = 17.9423, size = 27, normalized size = 0.79

$$-\frac{a\left(a+bx^2\right)^7}{14b^2}+\frac{\left(a+bx^2\right)^8}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] -a*(a + b*x**2)**7/(14*b**2) + (a + b*x**2)**8/(16*b**2)
```

Mathematica [B] time = 0.00453736, size = 77, normalized size = 2.26

$$\frac{a^6x^4}{4}+a^5bx^6+\frac{15}{8}a^4b^2x^8+2a^3b^3x^{10}+\frac{5}{4}a^2b^4x^{12}+\frac{3}{7}ab^5x^{14}+\frac{b^6x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^4)/4 + a^5*b*x^6 + (15*a^4*b^2*x^8)/8 + 2*a^3*b^3*x^10 + (
5*a^2*b^4*x^12)/4 + (3*a*b^5*x^14)/7 + (b^6*x^16)/16
```

Maple [B] time = 0.002, size = 68, normalized size = 2.

$$\frac{b^6x^{16}}{16}+\frac{3\,ab^5x^{14}}{7}+\frac{5\,a^2b^4x^{12}}{4}+2\,a^3b^3x^{10}+\frac{15\,a^4b^2x^8}{8}+a^5bx^6+\frac{a^6x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/16*b^6*x^16+3/7*a*b^5*x^14+5/4*a^2*b^4*x^12+2*a^3*b^3*x^10+15/8
*a^4*b^2*x^8+a^5*b*x^6+1/4*a^6*x^4
```

Maxima [A] time = 0.692653, size = 90, normalized size = 2.65

$$\frac{1}{16}\,b^6x^{16}+\frac{3}{7}\,ab^5x^{14}+\frac{5}{4}\,a^2b^4x^{12}+2\,a^3b^3x^{10}+\frac{15}{8}\,a^4b^2x^8+a^5bx^6+\frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^1
0 + 15/8*a^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

Fricas [A] time = 0.235161, size = 1, normalized size = 0.03

$$\frac{1}{16}x^{16}b^6+\frac{3}{7}x^{14}b^5a+\frac{5}{4}x^{12}b^4a^2+2x^{10}b^3a^3+\frac{15}{8}x^8b^2a^4+x^6ba^5+\frac{1}{4}x^4a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^6 + 3/7*x^14*b^5*a + 5/4*x^12*b^4*a^2 + 2*x^10*b^3*a^
3 + 15/8*x^8*b^2*a^4 + x^6*b*a^5 + 1/4*x^4*a^6
```

Sympy [A] time = 0.127591, size = 75, normalized size = 2.21

$$\frac{a^6x^4}{4} + a^5bx^6 + \frac{15a^4b^2x^8}{8} + 2a^3b^3x^{10} + \frac{5a^2b^4x^{12}}{4} + \frac{3ab^5x^{14}}{7} + \frac{b^6x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**4/4 + a**5*b*x**6 + 15*a**4*b**2*x**8/8 + 2*a**3*b**3*x**
10 + 5*a**2*b**4*x**12/4 + 3*a*b**5*x**14/7 + b**6*x**16/16
```

GIAC/XCAS [A] time = 0.269357, size = 90, normalized size = 2.65

$$\frac{1}{16}\,b^6x^{16} + \frac{3}{7}\,ab^5x^{14} + \frac{5}{4}\,a^2b^4x^{12} + 2\,a^3b^3x^{10} + \frac{15}{8}\,a^4b^2x^8 + a^5bx^6 + \frac{1}{4}\,a^6x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^3,x, algorithm="giac")
```

```
[Out] 1/16*b^6*x^16 + 3/7*a*b^5*x^14 + 5/4*a^2*b^4*x^12 + 2*a^3*b^3*x^1
0 + 15/8*a^4*b^2*x^8 + a^5*b*x^6 + 1/4*a^6*x^4
```

3.450 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=82

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*
x^9)/9 + (15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rubi [A] time = 0.103393, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*
x^9)/9 + (15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Rubi in Sympy [A] time = 23.4822, size = 80, normalized size = 0.98

$$\frac{a^6x^3}{3}+\frac{6a^5bx^5}{5}+\frac{15a^4b^2x^7}{7}+\frac{20a^3b^3x^9}{9}+\frac{15a^2b^4x^{11}}{11}+\frac{6ab^5x^{13}}{13}+\frac{b^6x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**
3*x**9/9 + 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15
/15
```

Mathematica [A] time = 0.0041169, size = 82, normalized size = 1.

$$\frac{a^6x^3}{3}+\frac{6}{5}a^5bx^5+\frac{15}{7}a^4b^2x^7+\frac{20}{9}a^3b^3x^9+\frac{15}{11}a^2b^4x^{11}+\frac{6}{13}ab^5x^{13}+\frac{b^6x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*x^3)/3 + (6*a^5*b*x^5)/5 + (15*a^4*b^2*x^7)/7 + (20*a^3*b^3*
x^9)/9 + (15*a^2*b^4*x^11)/11 + (6*a*b^5*x^13)/13 + (b^6*x^15)/15
```

Maple [A] time = 0.002, size = 69, normalized size = 0.8

$$\frac{a^6x^3}{3}+\frac{6\,a^5bx^5}{5}+\frac{15\,a^4b^2x^7}{7}+\frac{20\,a^3b^3x^9}{9}+\frac{15\,a^2b^4x^{11}}{11}+\frac{6\,ab^5x^{13}}{13}+\frac{b^6x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*a^6*x^3+6/5*a^5*b*x^5+15/7*a^4*b^2*x^7+20/9*a^3*b^3*x^9+15/11
*a^2*b^4*x^11+6/13*a*b^5*x^13+1/15*b^6*x^15
```

Maxima [A] time = 0.688571, size = 92, normalized size = 1.12

$$\frac{1}{15}\,b^6x^{15}+\frac{6}{13}\,ab^5x^{13}+\frac{15}{11}\,a^2b^4x^{11}+\frac{20}{9}\,a^3b^3x^9+\frac{15}{7}\,a^4b^2x^7+\frac{6}{5}\,a^5bx^5+\frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b
^3*x^9 + 15/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

Fricas [A] time = 0.232105, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}b^6+\frac{6}{13}x^{13}b^5a+\frac{15}{11}x^{11}b^4a^2+\frac{20}{9}x^9b^3a^3+\frac{15}{7}x^7b^2a^4+\frac{6}{5}x^5ba^5+\frac{1}{3}x^3a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^6 + 6/13*x^13*b^5*a + 15/11*x^11*b^4*a^2 + 20/9*x^9*b
^3*a^3 + 15/7*x^7*b^2*a^4 + 6/5*x^5*b*a^5 + 1/3*x^3*a^6
```

Sympy [A] time = 0.129586, size = 80, normalized size = 0.98

$$\frac{a^6x^3}{3} + \frac{6a^5bx^5}{5} + \frac{15a^4b^2x^7}{7} + \frac{20a^3b^3x^9}{9} + \frac{15a^2b^4x^{11}}{11} + \frac{6ab^5x^{13}}{13} + \frac{b^6x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**3/3 + 6*a**5*b*x**5/5 + 15*a**4*b**2*x**7/7 + 20*a**3*b**
3*x**9/9 + 15*a**2*b**4*x**11/11 + 6*a*b**5*x**13/13 + b**6*x**15
/15
```

GIAC/XCAS [A] time = 0.26884, size = 92, normalized size = 1.12

$$\frac{1}{15}\,b^6x^{15} + \frac{6}{13}\,ab^5x^{13} + \frac{15}{11}\,a^2b^4x^{11} + \frac{20}{9}\,a^3b^3x^9 + \frac{15}{7}\,a^4b^2x^7 + \frac{6}{5}\,a^5bx^5 + \frac{1}{3}\,a^6x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^2,x, algorithm="giac")
```

```
[Out] 1/15*b^6*x^15 + 6/13*a*b^5*x^13 + 15/11*a^2*b^4*x^11 + 20/9*a^3*b
^3*x^9 + 15/7*a^4*b^2*x^7 + 6/5*a^5*b*x^5 + 1/3*a^6*x^3
```

3.451 $\int x\left(a^2+2abx^2+b^2x^4\right)^3\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^2\right)^7}{14b}$$

```
[Out] (a + b*x^2)^7/(14*b)
```

Rubi [A] time = 0.0180342, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] (a + b*x^2)^7/(14*b)
```

Rubi in Sympy [A] time = 6.49307, size = 10, normalized size = 0.62

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] (a + b*x**2)**7/(14*b)
```

Mathematica [A] time = 0.00344942, size = 16, normalized size = 1.

$$\frac{\left(a+bx^2\right)^7}{14b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a + b*x^2)^7/(14*b)
```

Maple [B] time = 0.001, size = 69, normalized size = 4.3

$$\frac{b^6x^{14}}{14} + \frac{ab^5x^{12}}{2} + \frac{3\,a^2b^4x^{10}}{2} + \frac{5\,a^3b^3x^8}{2} + \frac{5\,a^4b^2x^6}{2} + \frac{3\,a^5bx^4}{2} + \frac{a^6x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/14*b^6*x^14+1/2*a*b^5*x^12+3/2*a^2*b^4*x^10+5/2*a^3*b^3*x^8+5/2
*a^4*b^2*x^6+3/2*a^5*b*x^4+1/2*a^6*x^2
```

Maxima [A] time = 0.684204, size = 92, normalized size = 5.75

$$\frac{1}{14}\,b^6x^{14} + \frac{1}{2}\,ab^5x^{12} + \frac{3}{2}\,a^2b^4x^{10} + \frac{5}{2}\,a^3b^3x^8 + \frac{5}{2}\,a^4b^2x^6 + \frac{3}{2}\,a^5bx^4 + \frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x
^8 + 5/2*a^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

Fricas [A] time = 0.231797, size = 1, normalized size = 0.06

$$\frac{1}{14}x^{14}b^6 + \frac{1}{2}x^{12}b^5a + \frac{3}{2}x^{10}b^4a^2 + \frac{5}{2}x^8b^3a^3 + \frac{5}{2}x^6b^2a^4 + \frac{3}{2}x^4ba^5 + \frac{1}{2}x^2a^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^6 + 1/2*x^12*b^5*a + 3/2*x^10*b^4*a^2 + 5/2*x^8*b^3*a
^3 + 5/2*x^6*b^2*a^4 + 3/2*x^4*b*a^5 + 1/2*x^2*a^6
```

Sympy [A] time = 0.127134, size = 78, normalized size = 4.88

$$\frac{a^6x^2}{2}+\frac{3a^5bx^4}{2}+\frac{5a^4b^2x^6}{2}+\frac{5a^3b^3x^8}{2}+\frac{3a^2b^4x^{10}}{2}+\frac{ab^5x^{12}}{2}+\frac{b^6x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x**2/2 + 3*a**5*b*x**4/2 + 5*a**4*b**2*x**6/2 + 5*a**3*b**3*
x**8/2 + 3*a**2*b**4*x**10/2 + a*b**5*x**12/2 + b**6*x**14/14
```

GIAC/XCAS [A] time = 0.266826, size = 92, normalized size = 5.75

$$\frac{1}{14}\,b^6x^{14}+\frac{1}{2}\,ab^5x^{12}+\frac{3}{2}\,a^2b^4x^{10}+\frac{5}{2}\,a^3b^3x^8+\frac{5}{2}\,a^4b^2x^6+\frac{3}{2}\,a^5bx^4+\frac{1}{2}\,a^6x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x,x, algorithm="giac")
```

```
[Out] 1/14*b^6*x^14 + 1/2*a*b^5*x^12 + 3/2*a^2*b^4*x^10 + 5/2*a^3*b^3*x
^8 + 5/2*a^4*b^2*x^6 + 3/2*a^5*b*x^4 + 1/2*a^6*x^2
```

3.452 $\int \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=73

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2
*b^4*x^9)/3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rubi [A] time = 0.0839687, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2
*b^4*x^9)/3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Rubi in Sympy [A] time = 32.6733, size = 73, normalized size = 1.

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20a^3b^3x^7}{7}+\frac{5a^2b^4x^9}{3}+\frac{6ab^5x^{11}}{11}+\frac{b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 +
 5*a**2*b**4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

Mathematica [A] time = 0.00211349, size = 73, normalized size = 1.

$$a^6x+2a^5bx^3+3a^4b^2x^5+\frac{20}{7}a^3b^3x^7+\frac{5}{3}a^2b^4x^9+\frac{6}{11}ab^5x^{11}+\frac{b^6x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^6*x + 2*a^5*b*x^3 + 3*a^4*b^2*x^5 + (20*a^3*b^3*x^7)/7 + (5*a^2
*b^4*x^9)/3 + (6*a*b^5*x^11)/11 + (b^6*x^13)/13
```

Maple [A] time = 0.001, size = 66, normalized size = 0.9

$$a^{6}x+2\,a^{5}bx^{3}+3\,a^{4}b^{2}x^{5}+\frac{20\,a^{3}b^{3}x^{7}}{7}+\frac{5\,a^{2}b^{4}x^{9}}{3}+\frac{6\,ab^{5}x^{11}}{11}+\frac{b^{6}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] a^6*x+2*a^5*b*x^3+3*a^4*b^2*x^5+20/7*a^3*b^3*x^7+5/3*a^2*b^4*x^9+
6/11*a*b^5*x^11+1/13*b^6*x^13
```

Maxima [A] time = 0.689783, size = 135, normalized size = 1.85

$$\frac{1}{13}\,b^{6}x^{13}+\frac{6}{11}\,ab^{5}x^{11}+\frac{4}{3}\,a^{2}b^{4}x^{9}+\frac{8}{7}\,a^{3}b^{3}x^{7}+a^{6}x+\frac{1}{5}\left(3\,b^{2}x^{5}+10\,abx^{3}\right)a^{4}+\frac{1}{105}\left(35\,b^{4}x^{9}+180\,ab^{3}x^{7}+252\,a^{2}b^{2}x^{5}\right)a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 4/3*a^2*b^4*x^9 + 8/7*a^3*b^3*x
^7 + a^6*x + 1/5*(3*b^2*x^5 + 10*a*b*x^3)*a^4 + 1/105*(35*b^4*x^9
 + 180*a*b^3*x^7 + 252*a^2*b^2*x^5)*a^2
```

Fricas [A] time = 0.231526, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}b^{6}+\frac{6}{11}x^{11}b^{5}a+\frac{5}{3}x^{9}b^{4}a^{2}+\frac{20}{7}x^{7}b^{3}a^{3}+3x^{5}b^{2}a^{4}+2x^{3}ba^{5}+xa^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^6 + 6/11*x^11*b^5*a + 5/3*x^9*b^4*a^2 + 20/7*x^7*b^3*
a^3 + 3*x^5*b^2*a^4 + 2*x^3*b*a^5 + x*a^6
```

Sympy [A] time = 0.121299, size = 73, normalized size = 1.

$$a^6x + 2a^5bx^3 + 3a^4b^2x^5 + \frac{20a^3b^3x^7}{7} + \frac{5a^2b^4x^9}{3} + \frac{6ab^5x^{11}}{11} + \frac{b^6x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*x + 2*a**5*b*x**3 + 3*a**4*b**2*x**5 + 20*a**3*b**3*x**7/7 +
 5*a**2*b**4*x**9/3 + 6*a*b**5*x**11/11 + b**6*x**13/13
```

GIAC/XCAS [A] time = 0.26861, size = 88, normalized size = 1.21

$$\frac{1}{13}\,b^6x^{13} + \frac{6}{11}\,ab^5x^{11} + \frac{5}{3}\,a^2b^4x^9 + \frac{20}{7}\,a^3b^3x^7 + 3\,a^4b^2x^5 + 2\,a^5bx^3 + a^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/13*b^6*x^13 + 6/11*a*b^5*x^11 + 5/3*a^2*b^4*x^9 + 20/7*a^3*b^3*
x^7 + 3*a^4*b^2*x^5 + 2*a^5*b*x^3 + a^6*x
```

3.453 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x} dx$

Optimal. Leaf size=76

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b
^4*x^8)/8 + (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rubi [A] time = 0.116476, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b
^4*x^8)/8 + (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^6\log(x^2)}{2}+3a^5bx^2+\frac{15a^4b^2\int^{x^2}x\,dx}{2}+\frac{10a^3b^3x^6}{3}+\frac{15a^2b^4x^8}{8}+\frac{3ab^5x^{10}}{5}+\frac{b^6x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

```
[Out] a**6*log(x**2)/2 + 3*a**5*b*x**2 + 15*a**4*b**2*Integral(x, (x, x
**2))/2 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x*
*10/5 + b**6*x**12/12
```

Mathematica [A] time = 0.00745496, size = 76, normalized size = 1.

$$a^6\log(x)+3a^5bx^2+\frac{15}{4}a^4b^2x^4+\frac{10}{3}a^3b^3x^6+\frac{15}{8}a^2b^4x^8+\frac{3}{5}ab^5x^{10}+\frac{b^6x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x,x]
```

```
[Out] 3*a^5*b*x^2 + (15*a^4*b^2*x^4)/4 + (10*a^3*b^3*x^6)/3 + (15*a^2*b
^4*x^8)/8 + (3*a*b^5*x^10)/5 + (b^6*x^12)/12 + a^6*Log[x]
```

Maple [A] time = 0.004, size = 67, normalized size = 0.9

$$3 a^{5}b x^{2}+\frac{15 a^{4}b^{2}x^{4}}{4}+\frac{10 a^{3}b^{3}x^{6}}{3}+\frac{15 a^{2}b^{4}x^{8}}{8}+\frac{3 a b^{5}x^{10}}{5}+\frac{b^{6}x^{12}}{12}+a^{6}\ln \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x,x)
```

```
[Out] 3*a^5*b*x^2+15/4*a^4*b^2*x^4+10/3*a^3*b^3*x^6+15/8*a^2*b^4*x^8+3/
5*a*b^5*x^10+1/12*b^6*x^12+a^6*ln(x)
```

Maxima [A] time = 0.683994, size = 93, normalized size = 1.22

$$\frac{1}{12}\, b^{6}x^{12}+\frac{3}{5}\, a b^{5}x^{10}+\frac{15}{8}\, a^{2}b^{4}x^{8}+\frac{10}{3}\, a^{3}b^{3}x^{6}+\frac{15}{4}\, a^{4}b^{2}x^{4}+3\, a^{5}b x^{2}+\frac{1}{2}\, a^{6}\log \left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*
x^6 + 15/4*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*log(x^2)
```

Fricas [A] time = 0.2551, size = 89, normalized size = 1.17

$$\frac{1}{12}\, b^{6}x^{12}+\frac{3}{5}\, a b^{5}x^{10}+\frac{15}{8}\, a^{2}b^{4}x^{8}+\frac{10}{3}\, a^{3}b^{3}x^{6}+\frac{15}{4}\, a^{4}b^{2}x^{4}+3\, a^{5}b x^{2}+a^{6}\log \left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*
x^6 + 15/4*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6*log(x)
```

Sympy [A] time = 1.15884, size = 76, normalized size = 1.

$$a^6 \log(x) + 3a^5bx^2 + \frac{15a^4b^2x^4}{4} + \frac{10a^3b^3x^6}{3} + \frac{15a^2b^4x^8}{8} + \frac{3ab^5x^{10}}{5} + \frac{b^6x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x,x)
```

```
[Out] a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*
x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12
```

GIAC/XCAS [A] time = 0.270104, size = 93, normalized size = 1.22

$$\frac{1}{12}\,b^6x^{12} + \frac{3}{5}\,ab^5x^{10} + \frac{15}{8}\,a^2b^4x^8 + \frac{10}{3}\,a^3b^3x^6 + \frac{15}{4}\,a^4b^2x^4 + 3\,a^5bx^2 + \frac{1}{2}\,a^6\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*b^6*x^12 + 3/5*a*b^5*x^10 + 15/8*a^2*b^4*x^8 + 10/3*a^3*b^3*
x^6 + 15/4*a^4*b^2*x^4 + 3*a^5*b*x^2 + 1/2*a^6*ln(x^2)
```

3.454 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^2} dx$

Optimal. Leaf size=72

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^
4*x^7)/7 + (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rubi [A] time = 0.0932312, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^
4*x^7)/7 + (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Rubi in Sympy [A] time = 23.5268, size = 70, normalized size = 0.97

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15a^2b^4x^7}{7}+\frac{2ab^5x^9}{3}+\frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

```
[Out] -a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a
**2*b**4*x**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

Mathematica [A] time = 0.0149605, size = 72, normalized size = 1.

$$-\frac{a^6}{x}+6a^5bx+5a^4b^2x^3+4a^3b^3x^5+\frac{15}{7}a^2b^4x^7+\frac{2}{3}ab^5x^9+\frac{b^6x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^2,x]
```

```
[Out] -(a^6/x) + 6*a^5*b*x + 5*a^4*b^2*x^3 + 4*a^3*b^3*x^5 + (15*a^2*b^
4*x^7)/7 + (2*a*b^5*x^9)/3 + (b^6*x^11)/11
```

Maple [A] time = 0.005, size = 67, normalized size = 0.9

$$-\frac{a^{6}}{x}+6\,a^{5}bx+5\,a^{4}b^{2}x^{3}+4\,a^{3}b^{3}x^{5}+\frac{15\,a^{2}b^{4}x^{7}}{7}+\frac{2\,ab^{5}x^{9}}{3}+\frac{b^{6}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^2,x)
```

```
[Out] -a^6/x+6*a^5*b*x+5*a^4*b^2*x^3+4*a^3*b^3*x^5+15/7*a^2*b^4*x^7+2/3
*a*b^5*x^9+1/11*b^6*x^11
```

Maxima [A] time = 0.688664, size = 89, normalized size = 1.24

$$\frac{1}{11}\,b^{6}x^{11}+\frac{2}{3}\,ab^{5}x^{9}+\frac{15}{7}\,a^{2}b^{4}x^{7}+4\,a^{3}b^{3}x^{5}+5\,a^{4}b^{2}x^{3}+6\,a^{5}bx-\frac{a^{6}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5
+ 5*a^4*b^2*x^3 + 6*a^5*b*x - a^6/x
```

Fricas [A] time = 0.248366, size = 95, normalized size = 1.32

$$\frac{21\,b^{6}x^{12}+154\,ab^{5}x^{10}+495\,a^{2}b^{4}x^{8}+924\,a^{3}b^{3}x^{6}+1155\,a^{4}b^{2}x^{4}+1386\,a^{5}bx^{2}-231\,a^{6}}{231\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/231*(21*b^6*x^12 + 154*a*b^5*x^10 + 495*a^2*b^4*x^8 + 924*a^3*b
^3*x^6 + 1155*a^4*b^2*x^4 + 1386*a^5*b*x^2 - 231*a^6)/x
```

Sympy [A] time = 1.13537, size = 70, normalized size = 0.97

$$-\frac{a^6}{x} + 6a^5bx + 5a^4b^2x^3 + 4a^3b^3x^5 + \frac{15a^2b^4x^7}{7} + \frac{2ab^5x^9}{3} + \frac{b^6x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**2,x)
```

```
[Out] -a**6/x + 6*a**5*b*x + 5*a**4*b**2*x**3 + 4*a**3*b**3*x**5 + 15*a
**2*b**4*x**7/7 + 2*a*b**5*x**9/3 + b**6*x**11/11
```

GIAC/XCAS [A] time = 0.267064, size = 89, normalized size = 1.24

$$\frac{1}{11}\,b^6x^{11} + \frac{2}{3}\,ab^5x^9 + \frac{15}{7}\,a^2b^4x^7 + 4\,a^3b^3x^5 + 5\,a^4b^2x^3 + 6\,a^5bx - \frac{a^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*b^6*x^11 + 2/3*a*b^5*x^9 + 15/7*a^2*b^4*x^7 + 4*a^3*b^3*x^5
+ 5*a^4*b^2*x^3 + 6*a^5*b*x - a^6/x
```

3.455 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^3}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^
6)/2 + (3*a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rubi [A] time = 0.135607, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^
6)/2 + (3*a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{2x^2}+3a^5b\log\left(x^2\right)+\frac{15a^4b^2x^2}{2}+10a^3b^3\int^{x^2}x\,dx+\frac{5a^2b^4x^6}{2}+\frac{3ab^5x^8}{4}+\frac{b^6x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

```
[Out] -a**6/(2*x**2) + 3*a**5*b*log(x**2) + 15*a**4*b**2*x**2/2 + 10*a*
*3*b**3*Integral(x, (x, x**2)) + 5*a**2*b**4*x**6/2 + 3*a*b**5*x*
*8/4 + b**6*x**10/10
```

Mathematica [A] time = 0.0133999, size = 77, normalized size = 1.

$$-\frac{a^6}{2x^2}+6a^5b\log(x)+\frac{15}{2}a^4b^2x^2+5a^3b^3x^4+\frac{5}{2}a^2b^4x^6+\frac{3}{4}ab^5x^8+\frac{b^6x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^3,x]
```

```
[Out] -a^6/(2*x^2) + (15*a^4*b^2*x^2)/2 + 5*a^3*b^3*x^4 + (5*a^2*b^4*x^
6)/2 + (3*a*b^5*x^8)/4 + (b^6*x^10)/10 + 6*a^5*b*Log[x]
```

Maple [A] time = 0.008, size = 68, normalized size = 0.9

$$-\frac{a^6}{2\,x^2}+\frac{15\,a^4b^2x^2}{2}+5\,a^3b^3x^4+\frac{5\,a^2b^4x^6}{2}+\frac{3\,ab^5x^8}{4}+\frac{b^6x^{10}}{10}+6\,a^5b\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^3,x)
```

```
[Out] -1/2*a^6/x^2+15/2*a^4*b^2*x^2+5*a^3*b^3*x^4+5/2*a^2*b^4*x^6+3/4*a
*b^5*x^8+1/10*b^6*x^10+6*a^5*b*ln(x)
```

Maxima [A] time = 0.68812, size = 93, normalized size = 1.21

$$\frac{1}{10}\,b^6x^{10}+\frac{3}{4}\,ab^5x^8+\frac{5}{2}\,a^2b^4x^6+5\,a^3b^3x^4+\frac{15}{2}\,a^4b^2x^2+3\,a^5b\log\left(x^2\right)-\frac{a^6}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 +
 15/2*a^4*b^2*x^2 + 3*a^5*b*log(x^2) - 1/2*a^6/x^2
```

Fricas [A] time = 0.25718, size = 97, normalized size = 1.26

$$\frac{2\,b^6x^{12}+15\,ab^5x^{10}+50\,a^2b^4x^8+100\,a^3b^3x^6+150\,a^4b^2x^4+120\,a^5bx^2\log(x)-10\,a^6}{20\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^6*x^12 + 15*a*b^5*x^10 + 50*a^2*b^4*x^8 + 100*a^3*b^3*x
^6 + 150*a^4*b^2*x^4 + 120*a^5*b*x^2*log(x) - 10*a^6)/x^2
```

Sympy [A] time = 1.26079, size = 76, normalized size = 0.99

$$-\frac{a^6}{2x^2} + 6a^5b\log(x) + \frac{15a^4b^2x^2}{2} + 5a^3b^3x^4 + \frac{5a^2b^4x^6}{2} + \frac{3ab^5x^8}{4} + \frac{b^6x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**3,x)
```

```
[Out] -a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b
**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10
```

GIAC/XCAS [A] time = 0.271257, size = 107, normalized size = 1.39

$$\frac{1}{10}b^6x^{10} + \frac{3}{4}ab^5x^8 + \frac{5}{2}a^2b^4x^6 + 5a^3b^3x^4 + \frac{15}{2}a^4b^2x^2 + 3a^5b\ln\left(x^2\right) - \frac{6a^5bx^2 + a^6}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*b^6*x^10 + 3/4*a*b^5*x^8 + 5/2*a^2*b^4*x^6 + 5*a^3*b^3*x^4 +
 15/2*a^4*b^2*x^2 + 3*a^5*b*ln(x^2) - 1/2*(6*a^5*b*x^2 + a^6)/x^2
```

3.456 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 +
3*a^2*b^4*x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rubi [A] time = 0.0941614, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 +
3*a^2*b^4*x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Rubi in Sympy [A] time = 23.0684, size = 71, normalized size = 0.96

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20a^3b^3x^3}{3}+3a^2b^4x^5+\frac{6ab^5x^7}{7}+\frac{b^6x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

```
[Out] -a**6/(3*x**3) - 6*a**5*b/x + 15*a**4*b**2*x + 20*a**3*b**3*x**3/
3 + 3*a**2*b**4*x**5 + 6*a*b**5*x**7/7 + b**6*x**9/9
```

Mathematica [A] time = 0.0151336, size = 74, normalized size = 1.

$$-\frac{a^6}{3x^3}-\frac{6a^5b}{x}+15a^4b^2x+\frac{20}{3}a^3b^3x^3+3a^2b^4x^5+\frac{6}{7}ab^5x^7+\frac{b^6x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^4,x]
```

```
[Out] -a^6/(3*x^3) - (6*a^5*b)/x + 15*a^4*b^2*x + (20*a^3*b^3*x^3)/3 +
3*a^2*b^4*x^5 + (6*a*b^5*x^7)/7 + (b^6*x^9)/9
```

Maple [A] time = 0.008, size = 67, normalized size = 0.9

$$-\frac{a^6}{3\,x^3}-6\,\frac{a^5b}{x}+15\,a^4b^2x+\frac{20\,a^3b^3x^3}{3}+3\,a^2b^4x^5+\frac{6\,ab^5x^7}{7}+\frac{b^6x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^4,x)
```

```
[Out] -1/3*a^6/x^3-6*a^5*b/x+15*a^4*b^2*x+20/3*a^3*b^3*x^3+3*a^2*b^4*x^
5+6/7*a*b^5*x^7+1/9*b^6*x^9
```

Maxima [A] time = 0.688714, size = 90, normalized size = 1.22

$$\frac{1}{9}\,b^6x^9+\frac{6}{7}\,ab^5x^7+3\,a^2b^4x^5+\frac{20}{3}\,a^3b^3x^3+15\,a^4b^2x-\frac{18\,a^5bx^2+a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 +
15*a^4*b^2*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

Fricas [A] time = 0.254882, size = 95, normalized size = 1.28

$$\frac{7\,b^6x^{12}+54\,ab^5x^{10}+189\,a^2b^4x^8+420\,a^3b^3x^6+945\,a^4b^2x^4-378\,a^5bx^2-21\,a^6}{63\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^6*x^12 + 54*a*b^5*x^10 + 189*a^2*b^4*x^8 + 420*a^3*b^3*
x^6 + 945*a^4*b^2*x^4 - 378*a^5*b*x^2 - 21*a^6)/x^3
```

Sympy [A] time = 1.27317, size = 73, normalized size = 0.99

$$15a^4b^2x + \frac{20a^3b^3x^3}{3} + 3a^2b^4x^5 + \frac{6ab^5x^7}{7} + \frac{b^6x^9}{9} - \frac{a^6 + 18a^5bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**4,x)
```

```
[Out] 15*a**4*b**2*x + 20*a**3*b**3*x**3/3 + 3*a**2*b**4*x**5 + 6*a*b**
5*x**7/7 + b**6*x**9/9 - (a**6 + 18*a**5*b*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.26776, size = 90, normalized size = 1.22

$$\frac{1}{9}\,b^6x^9 + \frac{6}{7}\,ab^5x^7 + 3\,a^2b^4x^5 + \frac{20}{3}\,a^3b^3x^3 + 15\,a^4b^2x - \frac{18\,a^5bx^2 + a^6}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*b^6*x^9 + 6/7*a*b^5*x^7 + 3*a^2*b^4*x^5 + 20/3*a^3*b^3*x^3 +
15*a^4*b^2*x - 1/3*(18*a^5*b*x^2 + a^6)/x^3
```

3.457 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^5}\,dx$

Optimal. Leaf size=72

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/
4 + a*b^5*x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rubi [A] time = 0.129092, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/
4 + a*b^5*x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+\frac{15a^4b^2\log\left(x^2\right)}{2}+10a^3b^3x^2+\frac{15a^2b^4\int^{x^2}x\,dx}{2}+ab^5x^6+\frac{b^6x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

```
[Out] -a**6/(4*x**4) - 3*a**5*b/x**2 + 15*a**4*b**2*log(x**2)/2 + 10*a*
*3*b**3*x**2 + 15*a**2*b**4*Integral(x, (x, x**2))/2 + a*b**5*x**
6 + b**6*x**8/8
```

Mathematica [A] time = 0.00866674, size = 72, normalized size = 1.

$$-\frac{a^6}{4x^4}-\frac{3a^5b}{x^2}+15a^4b^2\log(x)+10a^3b^3x^2+\frac{15}{4}a^2b^4x^4+ab^5x^6+\frac{b^6x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^5,x]
```

```
[Out] -a^6/(4*x^4) - (3*a^5*b)/x^2 + 10*a^3*b^3*x^2 + (15*a^2*b^4*x^4)/
4 + a*b^5*x^6 + (b^6*x^8)/8 + 15*a^4*b^2*Log[x]
```

Maple [A] time = 0.009, size = 67, normalized size = 0.9

$$-\frac{a^6}{4\,x^4}-3\,\frac{a^5b}{x^2}+10\,a^3b^3x^2+\frac{15\,a^2b^4x^4}{4}+ab^5x^6+\frac{b^6x^8}{8}+15\,a^4b^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^5,x)
```

```
[Out] -1/4*a^6/x^4-3*a^5*b/x^2+10*a^3*b^3*x^2+15/4*a^2*b^4*x^4+a*b^5*x^
6+1/8*b^6*x^8+15*a^4*b^2*ln(x)
```

Maxima [A] time = 0.683651, size = 93, normalized size = 1.29

$$\frac{1}{8}\,b^6x^8+ab^5x^6+\frac{15}{4}\,a^2b^4x^4+10\,a^3b^3x^2+\frac{15}{2}\,a^4b^2\log\left(x^2\right)-\frac{12\,a^5bx^2+a^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/
2*a^4*b^2*log(x^2) - 1/4*(12*a^5*b*x^2 + a^6)/x^4
```

Fricas [A] time = 0.259409, size = 96, normalized size = 1.33

$$\frac{b^6x^{12}+8\,ab^5x^{10}+30\,a^2b^4x^8+80\,a^3b^3x^6+120\,a^4b^2x^4\log(x)-24\,a^5bx^2-2\,a^6}{8\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(b^6*x^12 + 8*a*b^5*x^10 + 30*a^2*b^4*x^8 + 80*a^3*b^3*x^6 +
120*a^4*b^2*x^4*log(x) - 24*a^5*b*x^2 - 2*a^6)/x^4
```

Sympy [A] time = 1.453, size = 71, normalized size = 0.99

$$15a^4b^2\log(x) + 10a^3b^3x^2 + \frac{15a^2b^4x^4}{4} + ab^5x^6 + \frac{b^6x^8}{8} - \frac{a^6 + 12a^5bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**5,x)
```

```
[Out] 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a
*b**5*x**6 + b**6*x**8/8 - (a**6 + 12*a**5*b*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.270276, size = 108, normalized size = 1.5

$$\frac{1}{8}b^6x^8 + ab^5x^6 + \frac{15}{4}a^2b^4x^4 + 10a^3b^3x^2 + \frac{15}{2}a^4b^2\ln\left(x^2\right) - \frac{45a^4b^2x^4 + 12a^5bx^2 + a^6}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^5,x, algorithm="giac")
```

```
[Out] 1/8*b^6*x^8 + a*b^5*x^6 + 15/4*a^2*b^4*x^4 + 10*a^3*b^3*x^2 + 15/
2*a^4*b^2*ln(x^2) - 1/4*(45*a^4*b^2*x^4 + 12*a^5*b*x^2 + a^6)/x^4
```

3.458 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^6} dx$

Optimal. Leaf size=72

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*
a^2*b^4*x^3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rubi [A] time = 0.095626, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*
a^2*b^4*x^3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Rubi in Sympy [A] time = 22.1085, size = 70, normalized size = 0.97

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6ab^5x^5}{5}+\frac{b^6x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

```
[Out] -a**6/(5*x**5) - 2*a**5*b/x**3 - 15*a**4*b**2/x + 20*a**3*b**3*x
+ 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7
```

Mathematica [A] time = 0.0105662, size = 72, normalized size = 1.

$$-\frac{a^6}{5x^5}-\frac{2a^5b}{x^3}-\frac{15a^4b^2}{x}+20a^3b^3x+5a^2b^4x^3+\frac{6}{5}ab^5x^5+\frac{b^6x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^6,x]
```

```
[Out] -a^6/(5*x^5) - (2*a^5*b)/x^3 - (15*a^4*b^2)/x + 20*a^3*b^3*x + 5*
a^2*b^4*x^3 + (6*a*b^5*x^5)/5 + (b^6*x^7)/7
```

Maple [A] time = 0.008, size = 67, normalized size = 0.9

$$-\frac{a^{6}}{5\,x^{5}}-2\,\frac{a^{5}b}{x^{3}}-15\,\frac{a^{4}b^{2}}{x}+20\,a^{3}b^{3}x+5\,a^{2}b^{4}x^{3}+\frac{6\,ab^{5}x^{5}}{5}+\frac{b^{6}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^6,x)
```

```
[Out] -1/5*a^6/x^5-2*a^5*b/x^3-15*a^4*b^2/x+20*a^3*b^3*x+5*a^2*b^4*x^3+
6/5*a*b^5*x^5+1/7*b^6*x^7
```

Maxima [A] time = 0.692387, size = 90, normalized size = 1.25

$$\frac{1}{7}\,b^{6}x^{7}+\frac{6}{5}\,ab^{5}x^{5}+5\,a^{2}b^{4}x^{3}+20\,a^{3}b^{3}x-\frac{75\,a^{4}b^{2}x^{4}+10\,a^{5}bx^{2}+a^{6}}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^6,x, algorithm="maxima")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*
(75*a^4*b^2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

Fricas [A] time = 0.250349, size = 95, normalized size = 1.32

$$\frac{5\,b^{6}x^{12}+42\,ab^{5}x^{10}+175\,a^{2}b^{4}x^{8}+700\,a^{3}b^{3}x^{6}-525\,a^{4}b^{2}x^{4}-70\,a^{5}bx^{2}-7\,a^{6}}{35\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^6,x, algorithm="fricas")
```

```
[Out] 1/35*(5*b^6*x^12 + 42*a*b^5*x^10 + 175*a^2*b^4*x^8 + 700*a^3*b^3*
x^6 - 525*a^4*b^2*x^4 - 70*a^5*b*x^2 - 7*a^6)/x^5
```

Sympy [A] time = 1.53811, size = 71, normalized size = 0.99

$$20a^3b^3x + 5a^2b^4x^3 + \frac{6ab^5x^5}{5} + \frac{b^6x^7}{7} - \frac{a^6 + 10a^5bx^2 + 75a^4b^2x^4}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**6,x)
```

```
[Out] 20*a**3*b**3*x + 5*a**2*b**4*x**3 + 6*a*b**5*x**5/5 + b**6*x**7/7
 - (a**6 + 10*a**5*b*x**2 + 75*a**4*b**2*x**4)/(5*x**5)
```

GIAC/XCAS [A] time = 0.267685, size = 90, normalized size = 1.25

$$\frac{1}{7}\,b^6x^7 + \frac{6}{5}\,ab^5x^5 + 5\,a^2b^4x^3 + 20\,a^3b^3x - \frac{75\,a^4b^2x^4 + 10\,a^5bx^2 + a^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^6,x, algorithm="giac")
```

```
[Out] 1/7*b^6*x^7 + 6/5*a*b^5*x^5 + 5*a^2*b^4*x^3 + 20*a^3*b^3*x - 1/5*
(75*a^4*b^2*x^4 + 10*a^5*b*x^2 + a^6)/x^5
```

3.459 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^7}\,dx$

Optimal. Leaf size=79

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2
*b^4*x^2)/2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rubi [A] time = 0.129644, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2
*b^4*x^2)/2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+10a^3b^3\log\left(x^2\right)+\frac{15a^2b^4x^2}{2}+3ab^5\int^{x^2}x\,dx+\frac{b^6x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

```
[Out] -a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15*a**4*b**2/(2*x**2) + 10*a
**3*b**3*log(x**2) + 15*a**2*b**4*x**2/2 + 3*a*b**5*Integral(x, (
x, x**2)) + b**6*x**6/6
```

Mathematica [A] time = 0.00852243, size = 79, normalized size = 1.

$$-\frac{a^6}{6x^6}-\frac{3a^5b}{2x^4}-\frac{15a^4b^2}{2x^2}+20a^3b^3\log(x)+\frac{15}{2}a^2b^4x^2+\frac{3}{2}ab^5x^4+\frac{b^6x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^7,x]
```

```
[Out] -a^6/(6*x^6) - (3*a^5*b)/(2*x^4) - (15*a^4*b^2)/(2*x^2) + (15*a^2
*b^4*x^2)/2 + (3*a*b^5*x^4)/2 + (b^6*x^6)/6 + 20*a^3*b^3*Log[x]
```

Maple [A] time = 0.01, size = 68, normalized size = 0.9

$$-\frac{a^6}{6\,x^6}-\frac{3\,a^5b}{2\,x^4}-\frac{15\,a^4b^2}{2\,x^2}+\frac{15\,a^2b^4x^2}{2}+\frac{3\,ab^5x^4}{2}+\frac{b^6x^6}{6}+20\,a^3b^3\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^7,x)
```

```
[Out] -1/6*a^6/x^6-3/2*a^5*b/x^4-15/2*a^4*b^2/x^2+15/2*a^2*b^4*x^2+3/2*
a*b^5*x^4+1/6*b^6*x^6+20*a^3*b^3*ln(x)
```

Maxima [A] time = 0.688635, size = 95, normalized size = 1.2

$$\frac{1}{6}\,b^6x^6+\frac{3}{2}\,ab^5x^4+\frac{15}{2}\,a^2b^4x^2+10\,a^3b^3\log\left(x^2\right)-\frac{45\,a^4b^2x^4+9\,a^5bx^2+a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^7,x, algorithm="maxima")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*log(x
^2) - 1/6*(45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x^6
```

Fricas [A] time = 0.260625, size = 96, normalized size = 1.22

$$\frac{b^6x^{12}+9\,ab^5x^{10}+45\,a^2b^4x^8+120\,a^3b^3x^6\log\left(x\right)-45\,a^4b^2x^4-9\,a^5bx^2-a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(b^6*x^12 + 9*a*b^5*x^10 + 45*a^2*b^4*x^8 + 120*a^3*b^3*x^6*l
og(x) - 45*a^4*b^2*x^4 - 9*a^5*b*x^2 - a^6)/x^6
```

Sympy [A] time = 1.65704, size = 75, normalized size = 0.95

$$20a^3b^3\log(x) + \frac{15a^2b^4x^2}{2} + \frac{3ab^5x^4}{2} + \frac{b^6x^6}{6} - \frac{a^6 + 9a^5bx^2 + 45a^4b^2x^4}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**7,x)
```

```
[Out] 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**
6*x**6/6 - (a**6 + 9*a**5*b*x**2 + 45*a**4*b**2*x**4)/(6*x**6)
```

GIAC/XCAS [A] time = 0.271227, size = 109, normalized size = 1.38

$$\frac{1}{6}\,b^6x^6 + \frac{3}{2}\,ab^5x^4 + \frac{15}{2}\,a^2b^4x^2 + 10\,a^3b^3\ln\left(x^2\right) - \frac{110\,a^3b^3x^6 + 45\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^7,x, algorithm="giac")
```

```
[Out] 1/6*b^6*x^6 + 3/2*a*b^5*x^4 + 15/2*a^2*b^4*x^2 + 10*a^3*b^3*ln(x^
2) - 1/6*(110*a^3*b^3*x^6 + 45*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/x
^6
```

3.460 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^8} dx$

Optimal. Leaf size=72

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)
/x + 15*a^2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rubi [A] time = 0.0939195, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)
/x + 15*a^2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Rubi in Sympy [A] time = 22.3804, size = 70, normalized size = 0.97

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

```
[Out] -a**6/(7*x**7) - 6*a**5*b/(5*x**5) - 5*a**4*b**2/x**3 - 20*a**3*b
**3/x + 15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5
```

Mathematica [A] time = 0.0156504, size = 72, normalized size = 1.

$$-\frac{a^6}{7x^7}-\frac{6a^5b}{5x^5}-\frac{5a^4b^2}{x^3}-\frac{20a^3b^3}{x}+15a^2b^4x+2ab^5x^3+\frac{b^6x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^8,x]
```

```
[Out] -a^6/(7*x^7) - (6*a^5*b)/(5*x^5) - (5*a^4*b^2)/x^3 - (20*a^3*b^3)
/x + 15*a^2*b^4*x + 2*a*b^5*x^3 + (b^6*x^5)/5
```

Maple [A] time = 0.009, size = 67, normalized size = 0.9

$$-\frac{a^{6}}{7\,x^{7}}-\frac{6\,a^{5}b}{5\,x^{5}}-5\,\frac{a^{4}b^{2}}{x^{3}}-20\,\frac{a^{3}b^{3}}{x}+15\,a^{2}b^{4}x+2\,ab^{5}x^{3}+\frac{b^{6}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^8,x)
```

```
[Out] -1/7*a^6/x^7-6/5*a^5*b/x^5-5*a^4*b^2/x^3-20*a^3*b^3/x+15*a^2*b^4*
x+2*a*b^5*x^3+1/5*b^6*x^5
```

Maxima [A] time = 0.691544, size = 93, normalized size = 1.29

$$\frac{1}{5}\,b^{6}x^{5}+2\,ab^{5}x^{3}+15\,a^{2}b^{4}x-\frac{700\,a^{3}b^{3}x^{6}+175\,a^{4}b^{2}x^{4}+42\,a^{5}bx^{2}+5\,a^{6}}{35\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^8,x, algorithm="maxima")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6
+ 175*a^4*b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

Fricas [A] time = 0.25375, size = 95, normalized size = 1.32

$$\frac{7\,b^{6}x^{12}+70\,ab^{5}x^{10}+525\,a^{2}b^{4}x^{8}-700\,a^{3}b^{3}x^{6}-175\,a^{4}b^{2}x^{4}-42\,a^{5}bx^{2}-5\,a^{6}}{35\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^8,x, algorithm="fricas")
```

```
[Out] 1/35*(7*b^6*x^12 + 70*a*b^5*x^10 + 525*a^2*b^4*x^8 - 700*a^3*b^3*
x^6 - 175*a^4*b^2*x^4 - 42*a^5*b*x^2 - 5*a^6)/x^7
```

Sympy [A] time = 1.67949, size = 71, normalized size = 0.99

$$15a^2b^4x + 2ab^5x^3 + \frac{b^6x^5}{5} - \frac{5a^6 + 42a^5bx^2 + 175a^4b^2x^4 + 700a^3b^3x^6}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**8,x)
```

```
[Out] 15*a**2*b**4*x + 2*a*b**5*x**3 + b**6*x**5/5 - (5*a**6 + 42*a**5*
b*x**2 + 175*a**4*b**2*x**4 + 700*a**3*b**3*x**6)/(35*x**7)
```

GIAC/XCAS [A] time = 0.268358, size = 93, normalized size = 1.29

$$\frac{1}{5}\,b^6x^5 + 2\,ab^5x^3 + 15\,a^2b^4x - \frac{700\,a^3b^3x^6 + 175\,a^4b^2x^4 + 42\,a^5bx^2 + 5\,a^6}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^8,x, algorithm="giac")
```

```
[Out] 1/5*b^6*x^5 + 2*a*b^5*x^3 + 15*a^2*b^4*x - 1/35*(700*a^3*b^3*x^6
+ 175*a^4*b^2*x^4 + 42*a^5*b*x^2 + 5*a^6)/x^7
```

3.461 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^9}\,dx$

Optimal. Leaf size=73

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/
x^2 + 3*a*b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rubi [A] time = 0.125281, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/
x^2 + 3*a*b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+\frac{15a^2b^4\log\left(x^2\right)}{2}+3ab^5x^2+\frac{b^6\int^{x^2}x\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
```

```
[Out] -a**6/(8*x**8) - a**5*b/x**6 - 15*a**4*b**2/(4*x**4) - 10*a**3*b*
*3/x**2 + 15*a**2*b**4*log(x**2)/2 + 3*a*b**5*x**2 + b**6*Integra
l(x, (x, x**2))/2
```

Mathematica [A] time = 0.013678, size = 73, normalized size = 1.

$$-\frac{a^6}{8x^8}-\frac{a^5b}{x^6}-\frac{15a^4b^2}{4x^4}-\frac{10a^3b^3}{x^2}+15a^2b^4\log(x)+3ab^5x^2+\frac{b^6x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^9,x]
```

```
[Out] -a^6/(8*x^8) - (a^5*b)/x^6 - (15*a^4*b^2)/(4*x^4) - (10*a^3*b^3)/
x^2 + 3*a*b^5*x^2 + (b^6*x^4)/4 + 15*a^2*b^4*Log[x]
```

Maple [A] time = 0.01, size = 68, normalized size = 0.9

$$-\frac{a^6}{8\,x^8}-\frac{a^5b}{x^6}-\frac{15\,a^4b^2}{4\,x^4}-10\,\frac{a^3b^3}{x^2}+3\,ab^5x^2+\frac{b^6x^4}{4}+15\,a^2b^4\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^9,x)
```

```
[Out] -1/8*a^6/x^8-a^5*b/x^6-15/4*a^4*b^2/x^4-10*a^3*b^3/x^2+3*a*b^5*x^
2+1/4*b^6*x^4+15*a^2*b^4*ln(x)
```

Maxima [A] time = 0.688746, size = 95, normalized size = 1.3

$$\frac{1}{4}\,b^6x^4+3\,ab^5x^2+\frac{15}{2}\,a^2b^4\log\left(x^2\right)-\frac{80\,a^3b^3x^6+30\,a^4b^2x^4+8\,a^5bx^2+a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^9,x, algorithm="maxima")
```

```
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*log(x^2) - 1/8*(80*a^3*b
^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

Fricas [A] time = 0.255083, size = 97, normalized size = 1.33

$$\frac{2\,b^6x^{12}+24\,ab^5x^{10}+120\,a^2b^4x^8\log\left(x\right)-80\,a^3b^3x^6-30\,a^4b^2x^4-8\,a^5bx^2-a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^9,x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^6*x^12 + 24*a*b^5*x^10 + 120*a^2*b^4*x^8*log(x) - 80*a^3
*b^3*x^6 - 30*a^4*b^2*x^4 - 8*a^5*b*x^2 - a^6)/x^8
```

Sympy [A] time = 1.92091, size = 71, normalized size = 0.97

$$15a^2b^4\log{(x)} + 3ab^5x^2 + \frac{b^6x^4}{4} - \frac{a^6 + 8a^5bx^2 + 30a^4b^2x^4 + 80a^3b^3x^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**9,x)
```

```
[Out] 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4 - (a**6 + 8*a**
5*b*x**2 + 30*a**4*b**2*x**4 + 80*a**3*b**3*x**6)/(8*x**8)
```

GIAC/XCAS [A] time = 0.269831, size = 109, normalized size = 1.49

$$\frac{1}{4}\,b^6x^4 + 3\,ab^5x^2 + \frac{15}{2}\,a^2b^4\ln\left(x^2\right) - \frac{125\,a^2b^4x^8 + 80\,a^3b^3x^6 + 30\,a^4b^2x^4 + 8\,a^5bx^2 + a^6}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^9,x, algorithm="giac")
```

```
[Out] 1/4*b^6*x^4 + 3*a*b^5*x^2 + 15/2*a^2*b^4*ln(x^2) - 1/8*(125*a^2*b
^4*x^8 + 80*a^3*b^3*x^6 + 30*a^4*b^2*x^4 + 8*a^5*b*x^2 + a^6)/x^8
```

3.462 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{10}}\,dx$

Optimal. Leaf size=74

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)
/(3*x^3) - (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rubi [A] time = 0.0937105, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)
/(3*x^3) - (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Rubi in Sympy [A] time = 22.4119, size = 71, normalized size = 0.96

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

```
[Out] -a**6/(9*x**9) - 6*a**5*b/(7*x**7) - 3*a**4*b**2/x**5 - 20*a**3*b
**3/(3*x**3) - 15*a**2*b**4/x + 6*a*b**5*x + b**6*x**3/3
```

Mathematica [A] time = 0.0154171, size = 74, normalized size = 1.

$$-\frac{a^6}{9x^9}-\frac{6a^5b}{7x^7}-\frac{3a^4b^2}{x^5}-\frac{20a^3b^3}{3x^3}-\frac{15a^2b^4}{x}+6ab^5x+\frac{b^6x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^10,x]
```

```
[Out] -a^6/(9*x^9) - (6*a^5*b)/(7*x^7) - (3*a^4*b^2)/x^5 - (20*a^3*b^3)
/(3*x^3) - (15*a^2*b^4)/x + 6*a*b^5*x + (b^6*x^3)/3
```

Maple [A] time = 0.009, size = 67, normalized size = 0.9

$$-\frac{a^6}{9\,x^9}-\frac{6\,a^5b}{7\,x^7}-3\,\frac{a^4b^2}{x^5}-\frac{20\,a^3b^3}{3\,x^3}-15\,\frac{a^2b^4}{x}+6\,ab^5x+\frac{b^6x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^10,x)
```

```
[Out] -1/9*a^6/x^9-6/7*a^5*b/x^7-3*a^4*b^2/x^5-20/3*a^3*b^3/x^3-15*a^2*
b^4/x+6*a*b^5*x+1/3*b^6*x^3
```

Maxima [A] time = 0.69522, size = 93, normalized size = 1.26

$$\frac{1}{3}\,b^6x^3+6\,ab^5x-\frac{945\,a^2b^4x^8+420\,a^3b^3x^6+189\,a^4b^2x^4+54\,a^5bx^2+7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6
 + 189*a^4*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

Fricas [A] time = 0.246904, size = 95, normalized size = 1.28

$$\frac{21\,b^6x^{12}+378\,ab^5x^{10}-945\,a^2b^4x^8-420\,a^3b^3x^6-189\,a^4b^2x^4-54\,a^5bx^2-7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(21*b^6*x^12 + 378*a*b^5*x^10 - 945*a^2*b^4*x^8 - 420*a^3*b^
3*x^6 - 189*a^4*b^2*x^4 - 54*a^5*b*x^2 - 7*a^6)/x^9
```

Sympy [A] time = 1.96051, size = 71, normalized size = 0.96

$$6ab^5x + \frac{b^6x^3}{3} - \frac{7a^6 + 54a^5bx^2 + 189a^4b^2x^4 + 420a^3b^3x^6 + 945a^2b^4x^8}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**10,x)
```

```
[Out] 6*a*b**5*x + b**6*x**3/3 - (7*a**6 + 54*a**5*b*x**2 + 189*a**4*b*
*2*x**4 + 420*a**3*b**3*x**6 + 945*a**2*b**4*x**8)/(63*x**9)
```

GIAC/XCAS [A] time = 0.269932, size = 93, normalized size = 1.26

$$\frac{1}{3}\,b^6x^3 + 6\,ab^5x - \frac{945\,a^2b^4x^8 + 420\,a^3b^3x^6 + 189\,a^4b^2x^4 + 54\,a^5bx^2 + 7\,a^6}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^6*x^3 + 6*a*b^5*x - 1/63*(945*a^2*b^4*x^8 + 420*a^3*b^3*x^6
 + 189*a^4*b^2*x^4 + 54*a^5*b*x^2 + 7*a^6)/x^9
```

3.463 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{11}}\,dx$

Optimal. Leaf size=77

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3
*b^3)/x^4 - (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rubi [A] time = 0.121594, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3
*b^3)/x^4 - (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+3ab^5\log\left(x^2\right)+\frac{\int^{x^2} b^{12}\,dx}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

```
[Out] -a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b**2/(2*x**6) - 5*a
**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 3*a*b**5*log(x**2) + Inte
gral(b**12, (x, x**2))/(2*b**6)
```

Mathematica [A] time = 0.00850227, size = 77, normalized size = 1.

$$-\frac{a^6}{10x^{10}}-\frac{3a^5b}{4x^8}-\frac{5a^4b^2}{2x^6}-\frac{5a^3b^3}{x^4}-\frac{15a^2b^4}{2x^2}+6ab^5\log(x)+\frac{b^6x^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^11,x]
```

```
[Out] -a^6/(10*x^10) - (3*a^5*b)/(4*x^8) - (5*a^4*b^2)/(2*x^6) - (5*a^3
*b^3)/x^4 - (15*a^2*b^4)/(2*x^2) + (b^6*x^2)/2 + 6*a*b^5*Log[x]
```

Maple [A] time = 0.011, size = 68, normalized size = 0.9

$$-\frac{a^{6}}{10\,x^{10}}-\frac{3\,a^{5}b}{4\,x^{8}}-\frac{5\,a^{4}b^{2}}{2\,x^{6}}-5\,\frac{a^{3}b^{3}}{x^{4}}-\frac{15\,a^{2}b^{4}}{2\,x^{2}}+\frac{b^{6}x^{2}}{2}+6\,ab^{5}\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^11,x)
```

```
[Out] -1/10*a^6/x^10-3/4*a^5*b/x^8-5/2*a^4*b^2/x^6-5*a^3*b^3/x^4-15/2*a
^2*b^4/x^2+1/2*b^6*x^2+6*a*b^5*ln(x)
```

Maxima [A] time = 0.691438, size = 97, normalized size = 1.26

$$\frac{1}{2}\,b^{6}x^{2}+3\,ab^{5}\log\left(x^{2}\right)-\frac{150\,a^{2}b^{4}x^{8}+100\,a^{3}b^{3}x^{6}+50\,a^{4}b^{2}x^{4}+15\,a^{5}bx^{2}+2\,a^{6}}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^11,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*log(x^2) - 1/20*(150*a^2*b^4*x^8 + 100*a^3*
b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/x^10
```

Fricas [A] time = 0.256822, size = 97, normalized size = 1.26

$$\frac{10\,b^{6}x^{12}+120\,ab^{5}x^{10}\log\left(x\right)-150\,a^{2}b^{4}x^{8}-100\,a^{3}b^{3}x^{6}-50\,a^{4}b^{2}x^{4}-15\,a^{5}bx^{2}-2\,a^{6}}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^11,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 120*a*b^5*x^10*log(x) - 150*a^2*b^4*x^8 - 100
*a^3*b^3*x^6 - 50*a^4*b^2*x^4 - 15*a^5*b*x^2 - 2*a^6)/x^10
```

Sympy [A] time = 2.24141, size = 73, normalized size = 0.95

$$6ab^5\log(x) + \frac{b^6x^2}{2} - \frac{2a^6 + 15a^5bx^2 + 50a^4b^2x^4 + 100a^3b^3x^6 + 150a^2b^4x^8}{20x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**11,x)
```

```
[Out] 6*a*b**5*log(x) + b**6*x**2/2 - (2*a**6 + 15*a**5*b*x**2 + 50*a**
4*b**2*x**4 + 100*a**3*b**3*x**6 + 150*a**2*b**4*x**8)/(20*x**10)
```

GIAC/XCAS [A] time = 0.270585, size = 109, normalized size = 1.42

$$\frac{1}{2}\,b^6x^2 + 3\,ab^5\ln\left(x^2\right) - \frac{137\,ab^5x^{10} + 150\,a^2b^4x^8 + 100\,a^3b^3x^6 + 50\,a^4b^2x^4 + 15\,a^5bx^2 + 2\,a^6}{20\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^6*x^2 + 3*a*b^5*ln(x^2) - 1/20*(137*a*b^5*x^10 + 150*a^2*b^
4*x^8 + 100*a^3*b^3*x^6 + 50*a^4*b^2*x^4 + 15*a^5*b*x^2 + 2*a^6)/
x^10
```

3.464 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{12}} dx$

Optimal. Leaf size=71

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^
3*b^3)/x^5 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rubi [A] time = 0.0980073, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^
3*b^3)/x^5 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+\frac{\int b^{12}\,dx}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

```
[Out] -a**6/(11*x**11) - 2*a**5*b/(3*x**9) - 15*a**4*b**2/(7*x**7) - 4*
a**3*b**3/x**5 - 5*a**2*b**4/x**3 - 6*a*b**5/x + Integral(b**12,
x)/b**6
```

Mathematica [A] time = 0.00955629, size = 71, normalized size = 1.

$$-\frac{a^6}{11x^{11}}-\frac{2a^5b}{3x^9}-\frac{15a^4b^2}{7x^7}-\frac{4a^3b^3}{x^5}-\frac{5a^2b^4}{x^3}-\frac{6ab^5}{x}+b^6x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^12,x]
```

```
[Out] -a^6/(11*x^11) - (2*a^5*b)/(3*x^9) - (15*a^4*b^2)/(7*x^7) - (4*a^
3*b^3)/x^5 - (5*a^2*b^4)/x^3 - (6*a*b^5)/x + b^6*x
```

Maple [A] time = 0.009, size = 66, normalized size = 0.9

$$-\frac{a^{6}}{11\,x^{11}}-\frac{2\,a^{5}b}{3\,x^{9}}-\frac{15\,a^{4}b^{2}}{7\,x^{7}}-4\,\frac{a^{3}b^{3}}{x^{5}}-5\,\frac{a^{2}b^{4}}{x^{3}}-6\,\frac{ab^{5}}{x}+b^{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^12,x)
```

```
[Out] -1/11*a^6/x^11-2/3*a^5*b/x^9-15/7*a^4*b^2/x^7-4*a^3*b^3/x^5-5*a^2
*b^4/x^3-6*a*b^5/x+b^6*x
```

Maxima [A] time = 0.685793, size = 92, normalized size = 1.3

$$b^{6}x-\frac{1386\,ab^{5}x^{10}+1155\,a^{2}b^{4}x^{8}+924\,a^{3}b^{3}x^{6}+495\,a^{4}b^{2}x^{4}+154\,a^{5}bx^{2}+21\,a^{6}}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^12,x, algorithm="maxima")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x
^6 + 495*a^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

Fricas [A] time = 0.249377, size = 95, normalized size = 1.34

$$\frac{231\,b^{6}x^{12}-1386\,ab^{5}x^{10}-1155\,a^{2}b^{4}x^{8}-924\,a^{3}b^{3}x^{6}-495\,a^{4}b^{2}x^{4}-154\,a^{5}bx^{2}-21\,a^{6}}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^12,x, algorithm="fricas")
```

```
[Out] 1/231*(231*b^6*x^12 - 1386*a*b^5*x^10 - 1155*a^2*b^4*x^8 - 924*a^
3*b^3*x^6 - 495*a^4*b^2*x^4 - 154*a^5*b*x^2 - 21*a^6)/x^11
```

Sympy [A] time = 2.23864, size = 70, normalized size = 0.99

$$b^6x - \frac{21a^6 + 154a^5bx^2 + 495a^4b^2x^4 + 924a^3b^3x^6 + 1155a^2b^4x^8 + 1386ab^5x^{10}}{231x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**12,x)
```

```
[Out] b**6*x - (21*a**6 + 154*a**5*b*x**2 + 495*a**4*b**2*x**4 + 924*a*
*3*b**3*x**6 + 1155*a**2*b**4*x**8 + 1386*a*b**5*x**10)/(231*x**1
1)
```

GIAC/XCAS [A] time = 0.269346, size = 92, normalized size = 1.3

$$b^6x - \frac{1386\,ab^5x^{10} + 1155\,a^2b^4x^8 + 924\,a^3b^3x^6 + 495\,a^4b^2x^4 + 154\,a^5bx^2 + 21\,a^6}{231\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^12,x, algorithm="giac")
```

```
[Out] b^6*x - 1/231*(1386*a*b^5*x^10 + 1155*a^2*b^4*x^8 + 924*a^3*b^3*x
^6 + 495*a^4*b^2*x^4 + 154*a^5*b*x^2 + 21*a^6)/x^11
```

3.465 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{13}} dx$

Optimal. Leaf size=76

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*
a^3*b^3)/(3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log
[x]
```

Rubi [A] time = 0.119613, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*
a^3*b^3)/(3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log
[x]
```

Rubi in Sympy [A] time = 27.2467, size = 80, normalized size = 1.05

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+\frac{b^6\log(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

```
[Out] -a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8) - 1
0*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b*
*6*log(x**2)/2
```

Mathematica [A] time = 0.00848403, size = 76, normalized size = 1.

$$-\frac{a^6}{12x^{12}}-\frac{3a^5b}{5x^{10}}-\frac{15a^4b^2}{8x^8}-\frac{10a^3b^3}{3x^6}-\frac{15a^2b^4}{4x^4}-\frac{3ab^5}{x^2}+b^6\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^13,x]
```

```
[Out] -a^6/(12*x^12) - (3*a^5*b)/(5*x^10) - (15*a^4*b^2)/(8*x^8) - (10*
a^3*b^3)/(3*x^6) - (15*a^2*b^4)/(4*x^4) - (3*a*b^5)/x^2 + b^6*Log
[x]
```

Maple [A] time = 0.01, size = 67, normalized size = 0.9

$$-\frac{a^6}{12\,x^{12}}-\frac{3\,a^5b}{5\,x^{10}}-\frac{15\,a^4b^2}{8\,x^8}-\frac{10\,a^3b^3}{3\,x^6}-\frac{15\,a^2b^4}{4\,x^4}-3\,\frac{ab^5}{x^2}+b^6\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^13,x)
```

```
[Out] -1/12*a^6/x^12-3/5*a^5*b/x^10-15/8*a^4*b^2/x^8-10/3*a^3*b^3/x^6-1
5/4*a^2*b^4/x^4-3*a*b^5/x^2+b^6*ln(x)
```

Maxima [A] time = 0.684928, size = 97, normalized size = 1.28

$$\frac{1}{2}\,b^6\log\left(x^2\right)-\frac{360\,ab^5x^{10}+450\,a^2b^4x^8+400\,a^3b^3x^6+225\,a^4b^2x^4+72\,a^5bx^2+10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^13,x, algorithm="maxima")
```

```
[Out] 1/2*b^6*log(x^2) - 1/120*(360*a*b^5*x^10 + 450*a^2*b^4*x^8 + 400*
a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a^6)/x^12
```

Fricas [A] time = 0.255739, size = 97, normalized size = 1.28

$$\frac{120\,b^6x^{12}\log(x)-360\,ab^5x^{10}-450\,a^2b^4x^8-400\,a^3b^3x^6-225\,a^4b^2x^4-72\,a^5bx^2-10\,a^6}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^13,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^6*x^12*log(x) - 360*a*b^5*x^10 - 450*a^2*b^4*x^8 - 4
00*a^3*b^3*x^6 - 225*a^4*b^2*x^4 - 72*a^5*b*x^2 - 10*a^6)/x^12
```

Sympy [A] time = 2.52048, size = 71, normalized size = 0.93

$$b^6 \log(x) - \frac{10a^6 + 72a^5bx^2 + 225a^4b^2x^4 + 400a^3b^3x^6 + 450a^2b^4x^8 + 360ab^5x^{10}}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**13,x)
```

```
[Out] b**6*log(x) - (10*a**6 + 72*a**5*b*x**2 + 225*a**4*b**2*x**4 + 40
0*a**3*b**3*x**6 + 450*a**2*b**4*x**8 + 360*a*b**5*x**10)/(120*x*
*12)
```

GIAC/XCAS [A] time = 0.270636, size = 108, normalized size = 1.42

$$\frac{1}{2}\, b^6 \ln\left(x^2\right) - \frac{147\, b^6 x^{12} + 360\, ab^5 x^{10} + 450\, a^2 b^4 x^8 + 400\, a^3 b^3 x^6 + 225\, a^4 b^2 x^4 + 72\, a^5 b x^2 + 10\, a^6}{120\, x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^13,x, algorithm="giac")
```

```
[Out] 1/2*b^6*ln(x^2) - 1/120*(147*b^6*x^12 + 360*a*b^5*x^10 + 450*a^2*
b^4*x^8 + 400*a^3*b^3*x^6 + 225*a^4*b^2*x^4 + 72*a^5*b*x^2 + 10*a
^6)/x^12
```

3.466 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{x^{14}}\,dx$

Optimal. Leaf size=76

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*
a^3*b^3)/(7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rubi [A] time = 0.0989531, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*
a^3*b^3)/(7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Rubi in Sympy [A] time = 23.1144, size = 75, normalized size = 0.99

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

```
[Out] -a**6/(13*x**13) - 6*a**5*b/(11*x**11) - 5*a**4*b**2/(3*x**9) - 2
0*a**3*b**3/(7*x**7) - 3*a**2*b**4/x**5 - 2*a*b**5/x**3 - b**6/x
```

Mathematica [A] time = 0.015828, size = 76, normalized size = 1.

$$-\frac{a^6}{13x^{13}}-\frac{6a^5b}{11x^{11}}-\frac{5a^4b^2}{3x^9}-\frac{20a^3b^3}{7x^7}-\frac{3a^2b^4}{x^5}-\frac{2ab^5}{x^3}-\frac{b^6}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^14,x]
```

```
[Out] -a^6/(13*x^13) - (6*a^5*b)/(11*x^11) - (5*a^4*b^2)/(3*x^9) - (20*
a^3*b^3)/(7*x^7) - (3*a^2*b^4)/x^5 - (2*a*b^5)/x^3 - b^6/x
```

Maple [A] time = 0.008, size = 69, normalized size = 0.9

$$-\frac{a^{6}}{13\,x^{13}}-\frac{6\,a^{5}b}{11\,x^{11}}-\frac{5\,a^{4}b^{2}}{3\,x^{9}}-\frac{20\,a^{3}b^{3}}{7\,x^{7}}-3\,\frac{a^{2}b^{4}}{x^{5}}-2\,\frac{ab^{5}}{x^{3}}-\frac{b^{6}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^14,x)
```

```
[Out] -1/13*a^6/x^13-6/11*a^5*b/x^11-5/3*a^4*b^2/x^9-20/7*a^3*b^3/x^7-3
*a^2*b^4/x^5-2*a*b^5/x^3-b^6/x
```

Maxima [A] time = 0.690469, size = 95, normalized size = 1.25

$$-\frac{3003\,b^{6}x^{12}+6006\,ab^{5}x^{10}+9009\,a^{2}b^{4}x^{8}+8580\,a^{3}b^{3}x^{6}+5005\,a^{4}b^{2}x^{4}+1638\,a^{5}bx^{2}+231\,a^{6}}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 858
0*a^3*b^3*x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Fricas [A] time = 0.249954, size = 95, normalized size = 1.25

$$-\frac{3003\,b^{6}x^{12}+6006\,ab^{5}x^{10}+9009\,a^{2}b^{4}x^{8}+8580\,a^{3}b^{3}x^{6}+5005\,a^{4}b^{2}x^{4}+1638\,a^{5}bx^{2}+231\,a^{6}}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 858
0*a^3*b^3*x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

Sympy [A] time = 2.49734, size = 75, normalized size = 0.99

$$-\frac{231a^6 + 1638a^5bx^2 + 5005a^4b^2x^4 + 8580a^3b^3x^6 + 9009a^2b^4x^8 + 6006ab^5x^{10} + 3003b^6x^{12}}{3003x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**14,x)
```

```
[Out] -(231*a**6 + 1638*a**5*b*x**2 + 5005*a**4*b**2*x**4 + 8580*a**3*b
**3*x**6 + 9009*a**2*b**4*x**8 + 6006*a*b**5*x**10 + 3003*b**6*x*
*12)/(3003*x**13)
```

GIAC/XCAS [A] time = 0.268149, size = 95, normalized size = 1.25

$$-\frac{3003\, b^6x^{12} + 6006\, ab^5x^{10} + 9009\, a^2b^4x^8 + 8580\, a^3b^3x^6 + 5005\, a^4b^2x^4 + 1638\, a^5bx^2 + 231\, a^6}{3003\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^14,x, algorithm="giac")
```



```
[Out] -1/3003*(3003*b^6*x^12 + 6006*a*b^5*x^10 + 9009*a^2*b^4*x^8 + 858
0*a^3*b^3*x^6 + 5005*a^4*b^2*x^4 + 1638*a^5*b*x^2 + 231*a^6)/x^13
```

3.467 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{15}}\,dx$

Optimal. Leaf size=19

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

```
[Out] -(a + b*x^2)^7/(14*a*x^14)
```

Rubi [A] time = 0.0254873, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]

[Out] -(a + b*x^2)^7/(14*a*x^14)
```

Rubi in Sympy [A] time = 8.45368, size = 15, normalized size = 0.79

$$-\frac{(a+bx^2)^7}{14ax^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)

[Out] -(a + b*x**2)**7/(14*a*x**14)
```

Mathematica [B] time = 0.0128915, size = 82, normalized size = 4.32

$$-\frac{a^6}{14x^{14}}-\frac{a^5b}{2x^{12}}-\frac{3a^4b^2}{2x^{10}}-\frac{5a^3b^3}{2x^8}-\frac{5a^2b^4}{2x^6}-\frac{3ab^5}{2x^4}-\frac{b^6}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^15,x]
```

```
[Out] -a^6/(14*x^14) - (a^5*b)/(2*x^12) - (3*a^4*b^2)/(2*x^10) - (5*a^3
*b^3)/(2*x^8) - (5*a^2*b^4)/(2*x^6) - (3*a*b^5)/(2*x^4) - b^6/(2*
x^2)
```

Maple [B] time = 0.008, size = 69, normalized size = 3.6

$$-\frac{5\,a^3b^3}{2\,x^8}-\frac{a^5b}{2\,x^{12}}-\frac{a^6}{14\,x^{14}}-\frac{5\,a^2b^4}{2\,x^6}-\frac{b^6}{2\,x^2}-\frac{3\,a^4b^2}{2\,x^{10}}-\frac{3\,ab^5}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^15,x)
```

```
[Out] -5/2*a^3*b^3/x^8-1/2*a^5*b/x^12-1/14*a^6/x^14-5/2*a^2*b^4/x^6-1/2
*b^6/x^2-3/2*a^4*b^2/x^10-3/2*a*b^5/x^4
```

Maxima [A] time = 0.685935, size = 92, normalized size = 4.84

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^15,x, algorithm="maxima")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x
^6 + 21*a^4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Fricas [A] time = 0.248838, size = 92, normalized size = 4.84

$$-\frac{7\,b^6x^{12}+21\,ab^5x^{10}+35\,a^2b^4x^8+35\,a^3b^3x^6+21\,a^4b^2x^4+7\,a^5bx^2+a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^15,x, algorithm="fricas")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x
^6 + 21*a^4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

Sympy [A] time = 2.62463, size = 73, normalized size = 3.84

$$-\frac{a^6 + 7a^5bx^2 + 21a^4b^2x^4 + 35a^3b^3x^6 + 35a^2b^4x^8 + 21ab^5x^{10} + 7b^6x^{12}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**15,x)
```

```
[Out] -(a**6 + 7*a**5*b*x**2 + 21*a**4*b**2*x**4 + 35*a**3*b**3*x**6 +
35*a**2*b**4*x**8 + 21*a*b**5*x**10 + 7*b**6*x**12)/(14*x**14)
```

GIAC/XCAS [A] time = 0.269838, size = 92, normalized size = 4.84

$$-\frac{7\,b^6x^{12} + 21\,ab^5x^{10} + 35\,a^2b^4x^8 + 35\,a^3b^3x^6 + 21\,a^4b^2x^4 + 7\,a^5bx^2 + a^6}{14\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^15,x, algorithm="giac")
```

```
[Out] -1/14*(7*b^6*x^12 + 21*a*b^5*x^10 + 35*a^2*b^4*x^8 + 35*a^3*b^3*x
^6 + 21*a^4*b^2*x^4 + 7*a^5*b*x^2 + a^6)/x^14
```

3.468 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{16}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (
20*a^3*b^3)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) -
b^6/(3*x^3)
```

Rubi [A] time = 0.0962093, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (
20*a^3*b^3)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) -
b^6/(3*x^3)
```

Rubi in Sympy [A] time = 23.4628, size = 82, normalized size = 1.

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

```
[Out] -a**6/(15*x**15) - 6*a**5*b/(13*x**13) - 15*a**4*b**2/(11*x**11)
- 20*a**3*b**3/(9*x**9) - 15*a**2*b**4/(7*x**7) - 6*a*b**5/(5*x**
5) - b**6/(3*x**3)
```

Mathematica [A] time = 0.0161643, size = 82, normalized size = 1.

$$-\frac{a^6}{15x^{15}}-\frac{6a^5b}{13x^{13}}-\frac{15a^4b^2}{11x^{11}}-\frac{20a^3b^3}{9x^9}-\frac{15a^2b^4}{7x^7}-\frac{6ab^5}{5x^5}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^16,x]
```

```
[Out] -a^6/(15*x^15) - (6*a^5*b)/(13*x^13) - (15*a^4*b^2)/(11*x^11) - (
20*a^3*b^3)/(9*x^9) - (15*a^2*b^4)/(7*x^7) - (6*a*b^5)/(5*x^5) -
b^6/(3*x^3)
```

Maple [A] time = 0.008, size = 69, normalized size = 0.8

$$-\frac{a^{6}}{15\,x^{15}}-\frac{6\,a^{5}b}{13\,x^{13}}-\frac{15\,a^{4}b^{2}}{11\,x^{11}}-\frac{20\,a^{3}b^{3}}{9\,x^{9}}-\frac{15\,a^{2}b^{4}}{7\,x^{7}}-\frac{6\,ab^{5}}{5\,x^{5}}-\frac{b^{6}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^16,x)
```

```
[Out] -1/15*a^6/x^15-6/13*a^5*b/x^13-15/11*a^4*b^2/x^11-20/9*a^3*b^3/x^
9-15/7*a^2*b^4/x^7-6/5*a*b^5/x^5-1/3*b^6/x^3
```

Maxima [A] time = 0.682474, size = 95, normalized size = 1.16

$$-\frac{15015\,b^{6}x^{12}+54054\,ab^{5}x^{10}+96525\,a^{2}b^{4}x^{8}+100100\,a^{3}b^{3}x^{6}+61425\,a^{4}b^{2}x^{4}+20790\,a^{5}bx^{2}+3003\,a^{6}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 +
 100100*a^3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*
a^6)/x^15
```

Fricas [A] time = 0.247509, size = 95, normalized size = 1.16

$$-\frac{15015\,b^{6}x^{12}+54054\,ab^{5}x^{10}+96525\,a^{2}b^{4}x^{8}+100100\,a^{3}b^{3}x^{6}+61425\,a^{4}b^{2}x^{4}+20790\,a^{5}bx^{2}+3003\,a^{6}}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 +
 100100*a^3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*
a^6)/x^15
```

Sympy [A]   time = 2.62738, size = 75, normalized size = 0.91

$$-\frac{3003a^6 + 20790a^5bx^2 + 61425a^4b^2x^4 + 100100a^3b^3x^6 + 96525a^2b^4x^8 + 54054ab^5x^{10} + 15015b^6x^{12}}{45045x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**16,x)
```

```
[Out] -(3003*a**6 + 20790*a**5*b*x**2 + 61425*a**4*b**2*x**4 + 100100*a
**3*b**3*x**6 + 96525*a**2*b**4*x**8 + 54054*a*b**5*x**10 + 15015
*b**6*x**12)/(45045*x**15)
```

GIAC/XCAS [A]   time = 0.268457, size = 95, normalized size = 1.16

$$-\frac{15015\,b^6x^{12} + 54054\,ab^5x^{10} + 96525\,a^2b^4x^8 + 100100\,a^3b^3x^6 + 61425\,a^4b^2x^4 + 20790\,a^5bx^2 + 3003\,a^6}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(15015*b^6*x^12 + 54054*a*b^5*x^10 + 96525*a^2*b^4*x^8 +
 100100*a^3*b^3*x^6 + 61425*a^4*b^2*x^4 + 20790*a^5*b*x^2 + 3003*
a^6)/x^15
```

3.469 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{17}} \, dx$

Optimal. Leaf size=40

$$\frac{b(a+bx^2)^7}{112a^2x^{14}} - \frac{(a+bx^2)^7}{16ax^{16}}$$

```
[Out] -(a + b*x^2)^7/(16*a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rubi [A] time = 0.0723366, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{b(a+bx^2)^7}{112a^2x^{14}} - \frac{(a+bx^2)^7}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -(a + b*x^2)^7/(16*a*x^16) + (b*(a + b*x^2)^7)/(112*a^2*x^14)
```

Rubi in Sympy [A] time = 13.7933, size = 32, normalized size = 0.8

$$-\frac{(a+bx^2)^7}{16ax^{16}} + \frac{b(a+bx^2)^7}{112a^2x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

```
[Out] -(a + b*x**2)**7/(16*a*x**16) + b*(a + b*x**2)**7/(112*a**2*x**14
)
```

Mathematica [A] time = 0.00806517, size = 78, normalized size = 1.95

$$-\frac{a^6}{16x^{16}} - \frac{3a^5b}{7x^{14}} - \frac{5a^4b^2}{4x^{12}} - \frac{2a^3b^3}{x^{10}} - \frac{15a^2b^4}{8x^8} - \frac{ab^5}{x^6} - \frac{b^6}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^17,x]
```

```
[Out] -a^6/(16*x^16) - (3*a^5*b)/(7*x^14) - (5*a^4*b^2)/(4*x^12) - (2*a
^3*b^3)/x^10 - (15*a^2*b^4)/(8*x^8) - (a*b^5)/x^6 - b^6/(4*x^4)
```

Maple [A] time = 0.008, size = 69, normalized size = 1.7

$$-\frac{3\,a^5b}{7\,x^{14}}-\frac{a^6}{16\,x^{16}}-\frac{15\,a^2b^4}{8\,x^8}-\frac{ab^5}{x^6}-\frac{5\,a^4b^2}{4\,x^{12}}-2\,\frac{a^3b^3}{x^{10}}-\frac{b^6}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^17,x)
```

```
[Out] -3/7*a^5*b/x^14-1/16*a^6/x^16-15/8*a^2*b^4/x^8-a*b^5/x^6-5/4*a^4*
b^2/x^12-2*a^3*b^3/x^10-1/4*b^6/x^4
```

Maxima [A] time = 0.695571, size = 95, normalized size = 2.38

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^17,x, algorithm="maxima")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*
b^3*x^6 + 140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Fricas [A] time = 0.247548, size = 95, normalized size = 2.38

$$-\frac{28\,b^6x^{12}+112\,ab^5x^{10}+210\,a^2b^4x^8+224\,a^3b^3x^6+140\,a^4b^2x^4+48\,a^5bx^2+7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^17,x, algorithm="fricas")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*
b^3*x^6 + 140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

Sympy [A] time = 2.82124, size = 75, normalized size = 1.88

$$-\frac{7a^6 + 48a^5bx^2 + 140a^4b^2x^4 + 224a^3b^3x^6 + 210a^2b^4x^8 + 112ab^5x^{10} + 28b^6x^{12}}{112x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**17,x)
```

```
[Out] -(7*a**6 + 48*a**5*b*x**2 + 140*a**4*b**2*x**4 + 224*a**3*b**3*x*
*6 + 210*a**2*b**4*x**8 + 112*a*b**5*x**10 + 28*b**6*x**12)/(112*
x**16)
```

GIAC/XCAS [A] time = 0.269336, size = 95, normalized size = 2.38

$$-\frac{28\,b^6x^{12} + 112\,ab^5x^{10} + 210\,a^2b^4x^8 + 224\,a^3b^3x^6 + 140\,a^4b^2x^4 + 48\,a^5bx^2 + 7\,a^6}{112\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^17,x, algorithm="giac")
```

```
[Out] -1/112*(28*b^6*x^12 + 112*a*b^5*x^10 + 210*a^2*b^4*x^8 + 224*a^3*
b^3*x^6 + 140*a^4*b^2*x^4 + 48*a^5*b*x^2 + 7*a^6)/x^16
```

3.470 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{18}}\,dx$

Optimal. Leaf size=82

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (2
0*a^3*b^3)/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) -
b^6/(5*x^5)
```

Rubi [A] time = 0.0968595, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (2
0*a^3*b^3)/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) -
b^6/(5*x^5)
```

Rubi in Sympy [A] time = 23.2612, size = 82, normalized size = 1.

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**18,x)
```

```
[Out] -a**6/(17*x**17) - 2*a**5*b/(5*x**15) - 15*a**4*b**2/(13*x**13) -
 20*a**3*b**3/(11*x**11) - 5*a**2*b**4/(3*x**9) - 6*a*b**5/(7*x**
7) - b**6/(5*x**5)
```

Mathematica [A] time = 0.0113776, size = 82, normalized size = 1.

$$-\frac{a^6}{17x^{17}}-\frac{2a^5b}{5x^{15}}-\frac{15a^4b^2}{13x^{13}}-\frac{20a^3b^3}{11x^{11}}-\frac{5a^2b^4}{3x^9}-\frac{6ab^5}{7x^7}-\frac{b^6}{5x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^18,x]
```

```
[Out] -a^6/(17*x^17) - (2*a^5*b)/(5*x^15) - (15*a^4*b^2)/(13*x^13) - (2
0*a^3*b^3)/(11*x^11) - (5*a^2*b^4)/(3*x^9) - (6*a*b^5)/(7*x^7) -
b^6/(5*x^5)
```

Maple [A] time = 0.008, size = 69, normalized size = 0.8

$$-\frac{a^6}{17\,x^{17}}-\frac{2\,a^5b}{5\,x^{15}}-\frac{15\,a^4b^2}{13\,x^{13}}-\frac{20\,a^3b^3}{11\,x^{11}}-\frac{5\,a^2b^4}{3\,x^9}-\frac{6\,ab^5}{7\,x^7}-\frac{b^6}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^18,x)
```

```
[Out] -1/17*a^6/x^17-2/5*a^5*b/x^15-15/13*a^4*b^2/x^13-20/11*a^3*b^3/x^
11-5/3*a^2*b^4/x^9-6/7*a*b^5/x^7-1/5*b^6/x^5
```

Maxima [A] time = 0.687985, size = 95, normalized size = 1.16

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^18,x, algorithm="maxima")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^
8 + 464100*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 +
15015*a^6)/x^17
```

Fricas [A] time = 0.248219, size = 95, normalized size = 1.16

$$-\frac{51051\,b^6x^{12}+218790\,ab^5x^{10}+425425\,a^2b^4x^8+464100\,a^3b^3x^6+294525\,a^4b^2x^4+102102\,a^5bx^2+15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^18,x, algorithm="fricas")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^
8 + 464100*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 +
15015*a^6)/x^17
```

Sympy [A] time = 2.81031, size = 75, normalized size = 0.91

$$-\frac{15015a^6 + 102102a^5bx^2 + 294525a^4b^2x^4 + 464100a^3b^3x^6 + 425425a^2b^4x^8 + 218790ab^5x^{10} + 51051b^6x^{12}}{255255x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**18,x)
```

```
[Out] -(15015*a**6 + 102102*a**5*b*x**2 + 294525*a**4*b**2*x**4 + 46410
0*a**3*b**3*x**6 + 425425*a**2*b**4*x**8 + 218790*a*b**5*x**10 +
51051*b**6*x**12)/(255255*x**17)
```

GIAC/XCAS [A] time = 0.268499, size = 95, normalized size = 1.16

$$-\frac{51051\,b^6x^{12} + 218790\,ab^5x^{10} + 425425\,a^2b^4x^8 + 464100\,a^3b^3x^6 + 294525\,a^4b^2x^4 + 102102\,a^5bx^2 + 15015\,a^6}{255255\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^18,x, algorithm="giac")
```

```
[Out] -1/255255*(51051*b^6*x^12 + 218790*a*b^5*x^10 + 425425*a^2*b^4*x^
8 + 464100*a^3*b^3*x^6 + 294525*a^4*b^2*x^4 + 102102*a^5*b*x^2 +
15015*a^6)/x^17
```

3.471 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{19}}\,dx$

Optimal. Leaf size=62

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

```
[Out] -(a + b*x^2)^7/(18*a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b
^2*(a + b*x^2)^7)/(504*a^3*x^14)
```

Rubi [A] time = 0.107072, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{(a+bx^2)^7}{18ax^{18}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -(a + b*x^2)^7/(18*a*x^18) + (b*(a + b*x^2)^7)/(72*a^2*x^16) - (b
^2*(a + b*x^2)^7)/(504*a^3*x^14)
```

Rubi in Sympy [A] time = 17.5843, size = 53, normalized size = 0.85

$$-\frac{(a+bx^2)^7}{18ax^{18}}+\frac{b(a+bx^2)^7}{72a^2x^{16}}-\frac{b^2(a+bx^2)^7}{504a^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

```
[Out] -(a + b*x**2)**7/(18*a*x**18) + b*(a + b*x**2)**7/(72*a**2*x**16)
 - b**2*(a + b*x**2)**7/(504*a**3*x**14)
```

Mathematica [A] time = 0.0075356, size = 82, normalized size = 1.32

$$-\frac{a^6}{18x^{18}}-\frac{3a^5b}{8x^{16}}-\frac{15a^4b^2}{14x^{14}}-\frac{5a^3b^3}{3x^{12}}-\frac{3a^2b^4}{2x^{10}}-\frac{3ab^5}{4x^8}-\frac{b^6}{6x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^19,x]
```

```
[Out] -a^6/(18*x^18) - (3*a^5*b)/(8*x^16) - (15*a^4*b^2)/(14*x^14) - (5
*a^3*b^3)/(3*x^12) - (3*a^2*b^4)/(2*x^10) - (3*a*b^5)/(4*x^8) - b
^6/(6*x^6)
```

Maple [A] time = 0.007, size = 69, normalized size = 1.1

$$-\frac{3\,a^5b}{8\,x^{16}}-\frac{3\,ab^5}{4\,x^8}-\frac{a^6}{18\,x^{18}}-\frac{5\,a^3b^3}{3\,x^{12}}-\frac{15\,a^4b^2}{14\,x^{14}}-\frac{b^6}{6\,x^6}-\frac{3\,a^2b^4}{2\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^19,x)
```

```
[Out] -3/8*a^5*b/x^16-3/4*a*b^5/x^8-1/18*a^6/x^18-5/3*a^3*b^3/x^12-15/1
4*a^4*b^2/x^14-1/6*b^6/x^6-3/2*a^2*b^4/x^10
```

Maxima [A] time = 0.679205, size = 95, normalized size = 1.53

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^19,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*
b^3*x^6 + 540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Fricas [A] time = 0.250805, size = 95, normalized size = 1.53

$$-\frac{84\,b^6x^{12}+378\,ab^5x^{10}+756\,a^2b^4x^8+840\,a^3b^3x^6+540\,a^4b^2x^4+189\,a^5bx^2+28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^19,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*
b^3*x^6 + 540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

Sympy [A] time = 2.95366, size = 75, normalized size = 1.21

$$-\frac{28a^6 + 189a^5bx^2 + 540a^4b^2x^4 + 840a^3b^3x^6 + 756a^2b^4x^8 + 378ab^5x^{10} + 84b^6x^{12}}{504x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**19,x)
```

```
[Out] -(28*a**6 + 189*a**5*b*x**2 + 540*a**4*b**2*x**4 + 840*a**3*b**3*
x**6 + 756*a**2*b**4*x**8 + 378*a*b**5*x**10 + 84*b**6*x**12)/(50
4*x**18)
```

GIAC/XCAS [A] time = 0.269581, size = 95, normalized size = 1.53

$$-\frac{84\,b^6x^{12} + 378\,ab^5x^{10} + 756\,a^2b^4x^8 + 840\,a^3b^3x^6 + 540\,a^4b^2x^4 + 189\,a^5bx^2 + 28\,a^6}{504\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^19,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^6*x^12 + 378*a*b^5*x^10 + 756*a^2*b^4*x^8 + 840*a^3*
b^3*x^6 + 540*a^4*b^2*x^4 + 189*a^5*b*x^2 + 28*a^6)/x^18
```

3.472 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{20}}\,dx$

Optimal. Leaf size=80

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b
^3)/(13*x^13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/
(7*x^7)
```

Rubi [A] time = 0.10016, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b
^3)/(13*x^13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/
(7*x^7)
```

Rubi in Sympy [A] time = 22.9772, size = 78, normalized size = 0.98

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**20,x)
```

```
[Out] -a**6/(19*x**19) - 6*a**5*b/(17*x**17) - a**4*b**2/x**15 - 20*a**
3*b**3/(13*x**13) - 15*a**2*b**4/(11*x**11) - 2*a*b**5/(3*x**9) -
 b**6/(7*x**7)
```

Mathematica [A] time = 0.0159598, size = 80, normalized size = 1.

$$-\frac{a^6}{19x^{19}}-\frac{6a^5b}{17x^{17}}-\frac{a^4b^2}{x^{15}}-\frac{20a^3b^3}{13x^{13}}-\frac{15a^2b^4}{11x^{11}}-\frac{2ab^5}{3x^9}-\frac{b^6}{7x^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^20,x]
```

```
[Out] -a^6/(19*x^19) - (6*a^5*b)/(17*x^17) - (a^4*b^2)/x^15 - (20*a^3*b
^3)/(13*x^13) - (15*a^2*b^4)/(11*x^11) - (2*a*b^5)/(3*x^9) - b^6/
(7*x^7)
```

Maple [A] time = 0.009, size = 69, normalized size = 0.9

$$-\frac{a^{6}}{19\,x^{19}}-\frac{6\,a^{5}b}{17\,x^{17}}-\frac{a^{4}b^{2}}{x^{15}}-\frac{20\,a^{3}b^{3}}{13\,x^{13}}-\frac{15\,a^{2}b^{4}}{11\,x^{11}}-\frac{2\,ab^{5}}{3\,x^{9}}-\frac{b^{6}}{7\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^20,x)
```

```
[Out] -1/19*a^6/x^19-6/17*a^5*b/x^17-a^4*b^2/x^15-20/13*a^3*b^3/x^13-15
/11*a^2*b^4/x^11-2/3*a*b^5/x^9-1/7*b^6/x^7
```

Maxima [A] time = 0.686543, size = 95, normalized size = 1.19

$$-\frac{138567\,b^{6}x^{12}+646646\,ab^{5}x^{10}+1322685\,a^{2}b^{4}x^{8}+1492260\,a^{3}b^{3}x^{6}+969969\,a^{4}b^{2}x^{4}+342342\,a^{5}bx^{2}+51051\,a^{6}}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^20,x, algorithm="maxima")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*
x^8 + 1492260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2
 + 51051*a^6)/x^19
```

Fricas [A] time = 0.248245, size = 95, normalized size = 1.19

$$-\frac{138567\,b^{6}x^{12}+646646\,ab^{5}x^{10}+1322685\,a^{2}b^{4}x^{8}+1492260\,a^{3}b^{3}x^{6}+969969\,a^{4}b^{2}x^{4}+342342\,a^{5}bx^{2}+51051\,a^{6}}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^20,x, algorithm="fricas")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*
x^8 + 1492260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2
 + 51051*a^6)/x^19
```

Sympy [A]   time = 2.96512, size = 75, normalized size = 0.94

$$-\frac{51051a^6 + 342342a^5bx^2 + 969969a^4b^2x^4 + 1492260a^3b^3x^6 + 1322685a^2b^4x^8 + 646646ab^5x^{10} + 138567b^6x^{12}}{969969x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**20,x)
```

```
[Out] -(51051*a**6 + 342342*a**5*b*x**2 + 969969*a**4*b**2*x**4 + 14922
60*a**3*b**3*x**6 + 1322685*a**2*b**4*x**8 + 646646*a*b**5*x**10
+ 138567*b**6*x**12)/(969969*x**19)
```

GIAC/XCAS [A]   time = 0.268685, size = 95, normalized size = 1.19

$$-\frac{138567\,b^6x^{12} + 646646\,ab^5x^{10} + 1322685\,a^2b^4x^8 + 1492260\,a^3b^3x^6 + 969969\,a^4b^2x^4 + 342342\,a^5bx^2 + 51051\,a^6}{969969\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^20,x, algorithm="giac")
```

```
[Out] -1/969969*(138567*b^6*x^12 + 646646*a*b^5*x^10 + 1322685*a^2*b^4*
x^8 + 1492260*a^3*b^3*x^6 + 969969*a^4*b^2*x^4 + 342342*a^5*b*x^2
 + 51051*a^6)/x^19
```

3.473 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{21}}\,dx$

Optimal. Leaf size=84

$$\frac{b^3(a+bx^2)^7}{1680a^4x^{14}}-\frac{b^2(a+bx^2)^7}{240a^3x^{16}}+\frac{b(a+bx^2)^7}{60a^2x^{18}}-\frac{(a+bx^2)^7}{20ax^{20}}$$

```
[Out] -(a + b*x^2)^7/(20*a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b
^2*(a + b*x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*
x^14)
```

Rubi [A] time = 0.145369, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{b^3(a+bx^2)^7}{1680a^4x^{14}}-\frac{b^2(a+bx^2)^7}{240a^3x^{16}}+\frac{b(a+bx^2)^7}{60a^2x^{18}}-\frac{(a+bx^2)^7}{20ax^{20}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -(a + b*x^2)^7/(20*a*x^20) + (b*(a + b*x^2)^7)/(60*a^2*x^18) - (b
^2*(a + b*x^2)^7)/(240*a^3*x^16) + (b^3*(a + b*x^2)^7)/(1680*a^4*
x^14)
```

Rubi in Sympy [A] time = 26.7697, size = 80, normalized size = 0.95

$$-\frac{a^6}{20x^{20}}-\frac{a^5b}{3x^{18}}-\frac{15a^4b^2}{16x^{16}}-\frac{10a^3b^3}{7x^{14}}-\frac{5a^2b^4}{4x^{12}}-\frac{3ab^5}{5x^{10}}-\frac{b^6}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

```
[Out] -a**6/(20*x**20) - a**5*b/(3*x**18) - 15*a**4*b**2/(16*x**16) - 1
0*a**3*b**3/(7*x**14) - 5*a**2*b**4/(4*x**12) - 3*a*b**5/(5*x**10
) - b**6/(8*x**8)
```

Mathematica [A] time = 0.0134185, size = 82, normalized size = 0.98

$$-\frac{a^6}{20x^{20}}-\frac{a^5b}{3x^{18}}-\frac{15a^4b^2}{16x^{16}}-\frac{10a^3b^3}{7x^{14}}-\frac{5a^2b^4}{4x^{12}}-\frac{3ab^5}{5x^{10}}-\frac{b^6}{8x^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^21,x]
```

```
[Out] -a^6/(20*x^20) - (a^5*b)/(3*x^18) - (15*a^4*b^2)/(16*x^16) - (10*
a^3*b^3)/(7*x^14) - (5*a^2*b^4)/(4*x^12) - (3*a*b^5)/(5*x^10) - b
^6/(8*x^8)
```

Maple [A] time = 0.008, size = 69, normalized size = 0.8

$$-\frac{a^5b}{3\,x^{18}}-\frac{b^6}{8\,x^8}-\frac{3\,ab^5}{5\,x^{10}}-\frac{10\,a^3b^3}{7\,x^{14}}-\frac{15\,a^4b^2}{16\,x^{16}}-\frac{5\,a^2b^4}{4\,x^{12}}-\frac{a^6}{20\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^21,x)
```

```
[Out] -1/3*a^5*b/x^18-1/8*b^6/x^8-3/5*a*b^5/x^10-10/7*a^3*b^3/x^14-15/1
6*a^4*b^2/x^16-5/4*a^2*b^4/x^12-1/20*a^6/x^20
```

Maxima [A] time = 0.685417, size = 95, normalized size = 1.13

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^21,x, algorithm="maxima")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400
*a^3*b^3*x^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Fricas [A] time = 0.247338, size = 95, normalized size = 1.13

$$-\frac{210\,b^6x^{12}+1008\,ab^5x^{10}+2100\,a^2b^4x^8+2400\,a^3b^3x^6+1575\,a^4b^2x^4+560\,a^5bx^2+84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^21,x, algorithm="fricas")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400
*a^3*b^3*x^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

Sympy [A] time = 3.16232, size = 75, normalized size = 0.89

$$-\frac{84a^6 + 560a^5bx^2 + 1575a^4b^2x^4 + 2400a^3b^3x^6 + 2100a^2b^4x^8 + 1008ab^5x^{10} + 210b^6x^{12}}{1680x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**21,x)
```

```
[Out] -(84*a**6 + 560*a**5*b*x**2 + 1575*a**4*b**2*x**4 + 2400*a**3*b**
3*x**6 + 2100*a**2*b**4*x**8 + 1008*a*b**5*x**10 + 210*b**6*x**12
)/(1680*x**20)
```

GIAC/XCAS [A] time = 0.26917, size = 95, normalized size = 1.13

$$-\frac{210\,b^6x^{12} + 1008\,ab^5x^{10} + 2100\,a^2b^4x^8 + 2400\,a^3b^3x^6 + 1575\,a^4b^2x^4 + 560\,a^5bx^2 + 84\,a^6}{1680\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^21,x, algorithm="giac")
```

```
[Out] -1/1680*(210*b^6*x^12 + 1008*a*b^5*x^10 + 2100*a^2*b^4*x^8 + 2400
*a^3*b^3*x^6 + 1575*a^4*b^2*x^4 + 560*a^5*b*x^2 + 84*a^6)/x^20
```

3.474 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{x^{22}} dx$

Optimal. Leaf size=82

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (
4*a^3*b^3)/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11
) - b^6/(9*x^9)
```

Rubi [A] time = 0.0996926, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (
4*a^3*b^3)/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11
) - b^6/(9*x^9)
```

Rubi in Sympy [A] time = 23.0091, size = 82, normalized size = 1.

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**22,x)
```

```
[Out] -a**6/(21*x**21) - 6*a**5*b/(19*x**19) - 15*a**4*b**2/(17*x**17)
- 4*a**3*b**3/(3*x**15) - 15*a**2*b**4/(13*x**13) - 6*a*b**5/(11*
x**11) - b**6/(9*x**9)
```

Mathematica [A] time = 0.0158606, size = 82, normalized size = 1.

$$-\frac{a^6}{21x^{21}}-\frac{6a^5b}{19x^{19}}-\frac{15a^4b^2}{17x^{17}}-\frac{4a^3b^3}{3x^{15}}-\frac{15a^2b^4}{13x^{13}}-\frac{6ab^5}{11x^{11}}-\frac{b^6}{9x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/x^22,x]
```

```
[Out] -a^6/(21*x^21) - (6*a^5*b)/(19*x^19) - (15*a^4*b^2)/(17*x^17) - (
4*a^3*b^3)/(3*x^15) - (15*a^2*b^4)/(13*x^13) - (6*a*b^5)/(11*x^11
) - b^6/(9*x^9)
```

Maple [A] time = 0.008, size = 69, normalized size = 0.8

$$-\frac{a^6}{21\,x^{21}}-\frac{6\,a^5b}{19\,x^{19}}-\frac{15\,a^4b^2}{17\,x^{17}}-\frac{4\,a^3b^3}{3\,x^{15}}-\frac{15\,a^2b^4}{13\,x^{13}}-\frac{6\,ab^5}{11\,x^{11}}-\frac{b^6}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/x^22,x)
```

```
[Out] -1/21*a^6/x^21-6/19*a^5*b/x^19-15/17*a^4*b^2/x^17-4/3*a^3*b^3/x^1
5-15/13*a^2*b^4/x^13-6/11*a*b^5/x^11-1/9*b^6/x^9
```

Maxima [A] time = 0.689781, size = 95, normalized size = 1.16

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^22,x, algorithm="maxima")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^
4*x^8 + 3879876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*
x^2 + 138567*a^6)/x^21
```

Fricas [A] time = 0.251884, size = 95, normalized size = 1.16

$$-\frac{323323\,b^6x^{12}+1587222\,ab^5x^{10}+3357585\,a^2b^4x^8+3879876\,a^3b^3x^6+2567565\,a^4b^2x^4+918918\,a^5bx^2+138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^22,x, algorithm="fricas")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^
4*x^8 + 3879876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*
x^2 + 138567*a^6)/x^21
```

Sympy [A] time = 3.13562, size = 75, normalized size = 0.91

$$-\frac{138567a^6 + 918918a^5bx^2 + 2567565a^4b^2x^4 + 3879876a^3b^3x^6 + 3357585a^2b^4x^8 + 1587222ab^5x^{10} + 323323b^6x^{12}}{2909907x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/x**22,x)
```

```
[Out] -(138567*a**6 + 918918*a**5*b*x**2 + 2567565*a**4*b**2*x**4 + 387
9876*a**3*b**3*x**6 + 3357585*a**2*b**4*x**8 + 1587222*a*b**5*x**
10 + 323323*b**6*x**12)/(2909907*x**21)
```

GIAC/XCAS [A] time = 0.269936, size = 95, normalized size = 1.16

$$-\frac{323323\,b^6x^{12} + 1587222\,ab^5x^{10} + 3357585\,a^2b^4x^8 + 3879876\,a^3b^3x^6 + 2567565\,a^4b^2x^4 + 918918\,a^5bx^2 + 138567\,a^6}{2909907\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/x^22,x, algorithm="giac")
```

```
[Out] -1/2909907*(323323*b^6*x^12 + 1587222*a*b^5*x^10 + 3357585*a^2*b^
4*x^8 + 3879876*a^3*b^3*x^6 + 2567565*a^4*b^2*x^4 + 918918*a^5*b*
x^2 + 138567*a^6)/x^21
```

3.475 $\int \frac{x^{11}}{a^2+2abx^2+b^2x^4} dx$

Optimal. Leaf size=83

$$\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8
*b^2) + a^5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] time = 0.177103, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-2*a^3*x^2)/b^5 + (3*a^2*x^4)/(4*b^4) - (a*x^6)/(3*b^3) + x^8/(8
*b^2) + a^5/(2*b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/(2*b^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5}{2b^6(a+bx^2)}+\frac{5a^4\log(a+bx^2)}{2b^6}-\frac{2a^3x^2}{b^5}+\frac{3a^2\int^{x^2} x\,dx}{2b^4}-\frac{ax^6}{3b^3}+\frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**5/(2*b**6*(a + b*x**2)) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*
a**3*x**2/b**5 + 3*a**2*Integral(x, (x, x**2))/(2*b**4) - a*x**6/
(3*b**3) + x**8/(8*b**2)
```

Mathematica [A] time = 0.035825, size = 72, normalized size = 0.87

$$\frac{\frac{12a^5}{a+bx^2}+60a^4\log(a+bx^2)-48a^3bx^2+18a^2b^2x^4-8ab^3x^6+3b^4x^8}{24b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a
^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Maple [A] time = 0.01, size = 74, normalized size = 0.9

$$-2\,\frac{a^3x^2}{b^5}+\frac{3\,a^2x^4}{4\,b^4}-\frac{ax^6}{3\,b^3}+\frac{x^8}{8\,b^2}+\frac{a^5}{2\,b^6\,(bx^2+a)}+\frac{5\,a^4\ln\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/
b^6/(b*x^2+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Maxima [A] time = 0.680935, size = 104, normalized size = 1.25

$$\frac{a^5}{2\,(b^7x^2+ab^6)}+\frac{5\,a^4\log\left(bx^2+a\right)}{2\,b^6}+\frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*
b^3*x^8 - 8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Fricas [A] time = 0.254832, size = 126, normalized size = 1.52

$$\frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\,\left(a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{24\,(b^7x^2+ab^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4
- 48*a^4*b*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b
^7*x^2 + a*b^6)
```

Sympy [A] time = 1.59141, size = 80, normalized size = 0.96

$$\frac{a^5}{2ab^6 + 2b^7x^2} + \frac{5a^4 \log\left(a + bx^2\right)}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) -
 2*a**3*x**2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8
/(8*b**2)
```

GIAC/XCAS [A] time = 0.270742, size = 124, normalized size = 1.49

$$\frac{5\,a^4 \ln\left(\left|bx^2 + a\right|\right)}{2\,b^6} - \frac{5\,a^4 bx^2 + 4\,a^5}{2\,(bx^2 + a)b^6} + \frac{3\,b^6 x^8 - 8\,ab^5 x^6 + 18\,a^2 b^4 x^4 - 48\,a^3 b^3 x^2}{24\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^4*ln(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^
2 + a)*b^6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48
*a^3*b^3*x^2)/b^8
```

3.476 $\int \frac{x^9}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=70

$$-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*
(a + b*x^2)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rubi [A] time = 0.144676, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*a^2*x^2)/(2*b^4) - (a*x^4)/(2*b^3) + x^6/(6*b^2) - a^4/(2*b^5*
(a + b*x^2)) - (2*a^3*Log[a + b*x^2])/b^5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{2b^5(a+bx^2)}-\frac{2a^3\log(a+bx^2)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{a\int^{x^2}x\,dx}{b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4/(2*b**5*(a + b*x**2)) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**
2*x**2/(2*b**4) - a*Integral(x, (x, x**2))/b**3 + x**6/(6*b**2)
```

Mathematica [A] time = 0.037407, size = 60, normalized size = 0.86

$$\frac{-\frac{3a^4}{a+bx^2}-12a^3\log(a+bx^2)+9a^2bx^2-3ab^2x^4+b^3x^6}{6b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a
^3*Log[a + b*x^2])/(6*b^5)
```

Maple [A] time = 0.01, size = 63, normalized size = 0.9

$$\frac{3\,a^2x^2}{2\,b^4}-\frac{ax^4}{2\,b^3}+\frac{x^6}{6\,b^2}-\frac{a^4}{2\,b^5\,(bx^2+a)}-2\,\frac{a^3\ln\left(bx^2+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2
*a^3*ln(b*x^2+a)/b^5
```

Maxima [A] time = 0.691388, size = 88, normalized size = 1.26

$$-\frac{a^4}{2\,(b^6x^2+ab^5)}-\frac{2\,a^3\log\left(bx^2+a\right)}{b^5}+\frac{b^2x^6-3\,abx^4+9\,a^2x^2}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*
x^6 - 3*a*b*x^4 + 9*a^2*x^2)/b^4
```

Fricas [A] time = 0.254108, size = 109, normalized size = 1.56

$$\frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\,\left(a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{6\,(b^6x^2+ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4
- 12*(a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] time = 1.53291, size = 66, normalized size = 0.94

$$-\frac{a^4}{2ab^5+2b^6x^2}-\frac{2a^3\log\left(a+bx^2\right)}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{ax^4}{2b^3}+\frac{x^6}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*
a**2*x**2/(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

GIAC/XCAS [A] time = 0.269975, size = 108, normalized size = 1.54

$$-\frac{2\,a^3\ln\left(\left|bx^2+a\right|\right)}{b^5}+\frac{b^4x^6-3\,ab^3x^4+9\,a^2b^2x^2}{6\,b^6}+\frac{4\,a^3bx^2+3\,a^4}{2\,(bx^2+a)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -2*a^3*ln(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^
2*b^2*x^2)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

3.477 $\int \frac{x^7}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=57

$$\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*L
og[a + b*x^2])/(2*b^4)
```

Rubi [A] time = 0.12208, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}-\frac{ax^2}{b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -((a*x^2)/b^3) + x^4/(4*b^2) + a^3/(2*b^4*(a + b*x^2)) + (3*a^2*L
og[a + b*x^2])/(2*b^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3}{2b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{2b^4}-\frac{ax^2}{b^3}+\frac{\int^{x^2} x\,dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3/(2*b**4*(a + b*x**2)) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*
x**2/b**3 + Integral(x, (x, x**2))/(2*b**2)
```

Mathematica [A] time = 0.0272952, size = 49, normalized size = 0.86

$$\frac{\frac{2a^3}{a+bx^2}+6a^2\log(a+bx^2)-4abx^2+b^2x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2
])/(4*b^4)
```

Maple [A]   time = 0.01, size = 52, normalized size = 0.9

$$-\frac{ax^2}{b^3}+\frac{x^4}{4\,b^2}+\frac{a^3}{2\,b^4\,(bx^2+a)}+\frac{3\,a^2\ln\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/
b^4
```

Maxima [A]   time = 0.682896, size = 73, normalized size = 1.28

$$\frac{a^3}{2\,(b^5x^2+ab^4)}+\frac{3\,a^2\log\left(bx^2+a\right)}{2\,b^4}+\frac{bx^4-4\,ax^2}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] 1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x
^4 - 4*a*x^2)/b^3
```

Fricas [A]   time = 0.254004, size = 95, normalized size = 1.67

$$\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\,\left(a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\,(b^5x^2+ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 +
 a^3)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] time = 1.47057, size = 53, normalized size = 0.93

$$\frac{a^3}{2ab^4 + 2b^5x^2} + \frac{3a^2 \log\left(a + bx^2\right)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) -
 a*x**2/b**3 + x**4/(4*b**2)
```

GIAC/XCAS [A] time = 0.269771, size = 90, normalized size = 1.58

$$\frac{3\,a^2 \ln\left(\left|bx^2 + a\right|\right)}{2\,b^4} + \frac{b^2x^4 - 4\,abx^2}{4\,b^4} - \frac{3\,a^2bx^2 + 2\,a^3}{2\,(bx^2 + a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 3/2*a^2*ln(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 -
1/2*(3*a^2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

3.478 $\int \frac{x^5}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=44

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rubi [A] time = 0.0949822, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x^2/(2*b^2) - a^2/(2*b^3*(a + b*x^2)) - (a*Log[a + b*x^2])/b^3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{2b^3\left(a+bx^2\right)}-\frac{a\log\left(a+bx^2\right)}{b^3}+\frac{b^2\int^{x^2}\frac{1}{b^4}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2/(2*b**3*(a + b*x**2)) - a*log(a + b*x**2)/b**3 + b**2*Integ
ral(b**(-4), (x, x**2))/2
```

Mathematica [A] time = 0.025886, size = 38, normalized size = 0.86

$$\frac{-\frac{a^2}{a+bx^2}-2a\log\left(a+bx^2\right)+bx^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Maple [A] time = 0.01, size = 41, normalized size = 0.9

$$\frac{x^2}{2\,b^2} - \frac{a^2}{2\,b^3\,(bx^2 + a)} - \frac{a\ln\left(bx^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Maxima [A] time = 0.681478, size = 58, normalized size = 1.32

$$-\frac{a^2}{2\,(b^4x^2 + ab^3)} + \frac{x^2}{2\,b^2} - \frac{a\log\left(bx^2 + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Fricas [A] time = 0.256762, size = 76, normalized size = 1.73

$$\frac{b^2x^4 + abx^2 - a^2 - 2\left(abx^2 + a^2\right)\log\left(bx^2 + a\right)}{2\,(b^4x^2 + ab^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/
(b^4*x^2 + a*b^3)
```

Sympy [A] time = 1.40509, size = 39, normalized size = 0.89

$$-\frac{a^2}{2ab^3 + 2b^4x^2} - \frac{a\log\left(a + bx^2\right)}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2
*b**2)
```

GIAC/XCAS [A] time = 0.270368, size = 66, normalized size = 1.5

$$\frac{x^2}{2\,b^2} - \frac{a\ln\left(\left|bx^2 + a\right|\right)}{b^3} + \frac{2\,abx^2 + a^2}{2\,(bx^2 + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^2 - a*ln(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((
b*x^2 + a)*b^3)
```

3.479 $\int \frac{x^3}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=33

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rubi [A] time = 0.0710244, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] a/(2*b^2*(a + b*x^2)) + Log[a + b*x^2]/(2*b^2)
```

Rubi in Sympy [A] time = 14.9052, size = 26, normalized size = 0.79

$$\frac{a}{2b^2\left(a+bx^2\right)}+\frac{\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a/(2*b**2*(a + b*x**2)) + log(a + b*x**2)/(2*b**2)
```

Mathematica [A] time = 0.0148498, size = 27, normalized size = 0.82

$$\frac{\frac{a}{a+bx^2}+\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] (a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Maple [A] time = 0.009, size = 30, normalized size = 0.9

$$\frac{a}{2\,b^{2}\left(bx^{2}+a\right)}+\frac{\ln\left(bx^{2}+a\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^4+2*a*b*x^2+a^2),x)

[Out] 1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Maxima [A] time = 0.688939, size = 43, normalized size = 1.3

$$\frac{a}{2\left(b^{3}x^{2}+ab^{2}\right)}+\frac{\log\left(bx^{2}+a\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")

[Out] 1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Fricas [A] time = 0.252199, size = 47, normalized size = 1.42

$$\frac{\left(bx^{2}+a\right)\log\left(bx^{2}+a\right)+a}{2\left(b^{3}x^{2}+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")

[Out] 1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] time = 1.26407, size = 29, normalized size = 0.88

$$\frac{a}{2ab^2 + 2b^3x^2} + \frac{\log(a + bx^2)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

GIAC/XCAS [A] time = 0.271376, size = 41, normalized size = 1.24

$$\frac{\ln(|bx^2 + a|)}{2\,b^2} + \frac{a}{2\,(bx^2 + a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(b*x^2 + a))/b^2 + 1/2*a/((b*x^2 + a)*b^2)
```

3.480 $\int \frac{x}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{2b\left(a+bx^2\right)}$$

```
[Out] -1/(2*b*(a + b*x^2))
```

Rubi [A] time = 0.0153704, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]

[Out] -1/(2*b*(a + b*x^2))
```

Rubi in Sympy [A] time = 6.74245, size = 12, normalized size = 0.75

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)

[Out] -1/(2*b*(a + b*x**2))
```

Mathematica [A] time = 0.00470919, size = 16, normalized size = 1.

$$-\frac{1}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -1/(2*b*(a + b*x^2))
```

Maple [A] time = 0.005, size = 15, normalized size = 0.9

$$-\frac{1}{2\,b\,(b x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/b/(b*x^2+a)
```

Maxima [A] time = 0.696605, size = 20, normalized size = 1.25

$$-\frac{1}{2\,(b^2 x^2 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Fricas [A] time = 0.246033, size = 20, normalized size = 1.25

$$-\frac{1}{2\,(b^2 x^2 + ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] -1/2/(b^2*x^2 + a*b)
```

Sympy [A] time = 1.15025, size = 15, normalized size = 0.94

$$-\frac{1}{2ab + 2b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -1/(2*a*b + 2*b**2*x**2)
```

GIAC/XCAS [A] time = 0.270046, size = 19, normalized size = 1.19

$$-\frac{1}{2\,(bx^2 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -1/2/((b*x^2 + a)*b)
```

3.481 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rubi [A] time = 0.0880225, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rubi in Sympy [A] time = 19.349, size = 34, normalized size = 0.89

$$\frac{1}{2a\left(a+bx^2\right)}+\frac{\log\left(x^2\right)}{2a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*(a + b*x**2)) + log(x**2)/(2*a**2) - log(a + b*x**2)/(2*a*
*2)
```

Mathematica [A] time = 0.0224913, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2}-\log\left(a+bx^2\right)+2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Maple [A] time = 0.017, size = 35, normalized size = 0.9

$$\frac{1}{2\,a\,(bx^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln\left(bx^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Maxima [A] time = 0.685047, size = 50, normalized size = 1.32

$$\frac{1}{2\,(abx^2+a^2)}-\frac{\log\left(bx^2+a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x),x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Fricas [A] time = 0.259953, size = 63, normalized size = 1.66

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log(x)-a}{2\,(a^2bx^2+a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x),x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2
*b*x^2 + a^3)
```

Sympy [A] time = 1.613, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

GIAC/XCAS [A] time = 0.272543, size = 63, normalized size = 1.66

$$\frac{\ln\left(x^2\right)}{2\,a^2} - \frac{\ln\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{bx^2 + 2\,a}{2\,(bx^2 + a)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2)/a^2 - 1/2*ln(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/
((b*x^2 + a)*a^2)
```

3.482 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Lo
g[a + b*x^2])/a^3
```

Rubi [A] time = 0.105857, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Lo
g[a + b*x^2])/a^3
```

Rubi in Sympy [A] time = 19.2768, size = 46, normalized size = 0.94

$$-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -b/(2*a**2*(a + b*x**2)) - 1/(2*a**2*x**2) - b*log(x**2)/a**3 + b
*log(a + b*x**2)/a**3
```

Mathematica [A] time = 0.0640177, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/(
2*a^3)
```

Maple [A] time = 0.018, size = 46, normalized size = 0.9

$$-\frac{1}{2\,a^{2}x^{2}}-\frac{b}{2\,a^{2}\left(bx^{2}+a\right)}-2\,\frac{b\ln\left(x\right)}{a^{3}}+\frac{b\ln\left(bx^{2}+a\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Maxima [A] time = 0.684604, size = 70, normalized size = 1.43

$$-\frac{2\,bx^{2}+a}{2\,\left(a^{2}bx^{4}+a^{3}x^{2}\right)}+\frac{b\log\left(bx^{2}+a\right)}{a^{3}}-\frac{b\log\left(x^{2}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 -
 b*log(x^2)/a^3
```

Fricas [A] time = 0.259303, size = 99, normalized size = 2.02

$$-\frac{2\,abx^{2}+a^{2}-2\,\left(b^{2}x^{4}+abx^{2}\right)\log\left(bx^{2}+a\right)+4\,\left(b^{2}x^{4}+abx^{2}\right)\log\left(x\right)}{2\,\left(a^{3}bx^{4}+a^{4}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*
(b^2*x^4 + a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] time = 1.92785, size = 49, normalized size = 1.

$$-\frac{a+2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -(a + 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 +
 b*log(a/b + x**2)/a**3
```

GIAC/XCAS [A] time = 0.269854, size = 69, normalized size = 1.41

$$-\frac{b\ln\left(x^2\right)}{a^3}+\frac{b\ln\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\,(bx^4+ax^2)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^3),x, algorithm="giac")
```

```
[Out] -b*ln(x^2)/a^3 + b*ln(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b
*x^4 + a*x^2)*a^2)
```

3.483 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=66

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*L
og[x])/a^4 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] time = 0.120071, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}+\frac{3b^2\log(x)}{a^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(4*a^2*x^4) + b/(a^3*x^2) + b^2/(2*a^3*(a + b*x^2)) + (3*b^2*L
og[x])/a^4 - (3*b^2*Log[a + b*x^2])/(2*a^4)
```

Rubi in Sympy [A] time = 22.2745, size = 66, normalized size = 1.

$$-\frac{1}{4a^2x^4}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{b}{a^3x^2}+\frac{3b^2\log\left(x^2\right)}{2a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -1/(4*a**2*x**4) + b**2/(2*a**3*(a + b*x**2)) + b/(a**3*x**2) + 3
*b**2*log(x**2)/(2*a**4) - 3*b**2*log(a + b*x**2)/(2*a**4)
```

Mathematica [A] time = 0.117276, size = 57, normalized size = 0.86

$$\frac{-6b^2\log\left(a+bx^2\right)+a\left(\frac{2b^2}{a+bx^2}-\frac{a}{x^4}+\frac{4b}{x^2}\right)+12b^2\log(x)}{4a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] -
 6*b^2*Log[a + b*x^2])/(4*a^4)
```

Maple [A] time = 0.02, size = 61, normalized size = 0.9

$$-\frac{1}{4\,a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2\,a^3\,(bx^2+a)}+3\,\frac{b^2\ln(x)}{a^4}-\frac{3\,b^2\ln\left(bx^2+a\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*
b^2*ln(b*x^2+a)/a^4
```

Maxima [A] time = 0.689011, size = 95, normalized size = 1.44

$$\frac{6\,b^2x^4+3\,abx^2-a^2}{4\,(a^3bx^6+a^4x^4)}-\frac{3\,b^2\log\left(bx^2+a\right)}{2\,a^4}+\frac{3\,b^2\log\left(x^2\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^5),x, algorithm="maxima")
```

```
[Out] 1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2
*log(b*x^2 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Fricas [A] time = 0.259477, size = 122, normalized size = 1.85

$$\frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log(x)}{4\,(a^4bx^6+a^5x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^5),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*lo
g(b*x^2 + a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*
x^4)
```

Sympy [A] time = 2.42986, size = 68, normalized size = 1.03

$$\frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6)
+ 3*b**2*log(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

GIAC/XCAS [A] time = 0.269886, size = 116, normalized size = 1.76

$$\frac{3\,b^2\ln\left(x^2\right)}{2\,a^4}-\frac{3\,b^2\ln\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{3\,b^3x^2+4\,ab^2}{2\,(bx^2+a)a^4}-\frac{9\,b^2x^4-4\,abx^2+a^2}{4\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^5),x, algorithm="giac")
```

```
[Out] 3/2*b^2*ln(x^2)/a^4 - 3/2*b^2*ln(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3
*x^2 + 4*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 +
a^2)/(a^4*x^4)
```

3.484 $\int \frac{x^{10}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=92

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\,(a+bx^2)}+\frac{9x^7}{14b^2}$$

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (
9*x^7)/(14*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] time = 0.129437, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}-\frac{9a^3x}{2b^5}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\,(a+bx^2)}+\frac{9x^7}{14b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a^3*x)/(2*b^5) + (3*a^2*x^3)/(2*b^4) - (9*a*x^5)/(10*b^3) + (
9*x^7)/(14*b^2) - x^9/(2*b*(a + b*x^2)) + (9*a^(7/2)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{9a^{\frac{7}{2}}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{11}{2}}}+\frac{3a^2x^3}{2b^4}-\frac{9ax^5}{10b^3}-\frac{x^9}{2b\,(a+bx^2)}+\frac{9x^7}{14b^2}-\frac{9\int a^3\,dx}{2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 9*a**(7/2)*atan(sqrt(b)*x/sqrt(a))/(2*b**(11/2)) + 3*a**2*x**3/(2
*b**4) - 9*a*x**5/(10*b**3) - x**9/(2*b*(a + b*x**2)) + 9*x**7/(1
4*b**2) - 9*Integral(a**3, x)/(2*b**5)
```

Mathematica [A] time = 0.0953357, size = 82, normalized size = 0.89

$$\frac{9a^{7/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}+\frac{x\left(-\frac{35a^4}{a+bx^2}-280a^3+70a^2bx^2-28ab^2x^4+10b^3x^6\right)}{70b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4
)/(a + b*x^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*b^(11/2))
```

Maple [A] time = 0.017, size = 78, normalized size = 0.9

$$\frac{x^7}{7\,b^2}-\frac{2\,ax^5}{5\,b^3}+\frac{a^2x^3}{b^4}-4\,\frac{a^3x}{b^5}-\frac{a^4x}{2\,b^5\,(bx^2+a)}+\frac{9\,a^4}{2\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/7*x^7/b^2-2/5*a*x^5/b^3+a^2*x^3/b^4-4*a^3*x/b^5-1/2/b^5*a^4*x/(
b*x^2+a)+9/2/b^5*a^4/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262158, size = 1, normalized size = 0.01

$$\left[\frac{20\,b^{4}x^{9} - 36\,ab^{3}x^{7} + 84\,a^{2}b^{2}x^{5} - 420\,a^{3}bx^{3} - 630\,a^{4}x + 315\,\left(a^{3}bx^{2} + a^{4}\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)}{140\,(b^{6}x^{2} + ab^{5})}, \frac{10\,b^{4}x^{9} - 18\,ab^{3}x^{7} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^
3 - 630*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b
*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*
x^9 - 18*a*b^3*x^7 + 42*a^2*b^2*x^5 - 210*a^3*b*x^3 - 315*a^4*x +
 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(x/sqrt(a/b)))/(b^6*x^2 +
a*b^5)]
```

Sympy [A] time = 1.71917, size = 134, normalized size = 1.46

$$-\frac{a^{4}x}{2ab^{5} + 2b^{6}x^{2}} - \frac{4a^{3}x}{b^{5}} + \frac{a^{2}x^{3}}{b^{4}} - \frac{2ax^{5}}{5b^{3}} - \frac{9\sqrt{-\frac{a^{7}}{b^{11}}}\log\left(x - \frac{b^{5}\sqrt{-\frac{a^{7}}{b^{11}}}}{a^{3}}\right)}{4} + \frac{9\sqrt{-\frac{a^{7}}{b^{11}}}\log\left(x + \frac{b^{5}\sqrt{-\frac{a^{7}}{b^{11}}}}{a^{3}}\right)}{4} + \frac{x^{7}}{7b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4
 - 2*a*x**5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**
7/b**11)/a**3)/4 + 9*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b*
*11)/a**3)/4 + x**7/(7*b**2)
```

GIAC/XCAS [A] time = 0.26944, size = 113, normalized size = 1.23

$$\frac{9\,a^{4}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^{5}} - \frac{a^{4}x}{2\,(bx^{2} + a)b^{5}} + \frac{5\,b^{12}x^{7} - 14\,ab^{11}x^{5} + 35\,a^{2}b^{10}x^{3} - 140\,a^{3}b^{9}x}{35\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2
 + a)*b^5) + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 -
 140*a^3*b^9*x)/b^14
```

3.485 $\int \frac{x^8}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2
*b*(a + b*x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9
/2))
```

Rubi [A] time = 0.110292, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}+\frac{7a^2x}{2b^4}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (7*a^2*x)/(2*b^4) - (7*a*x^3)/(6*b^3) + (7*x^5)/(10*b^2) - x^7/(2
*b*(a + b*x^2)) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9
/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{7a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{9}{2}}}-\frac{7ax^3}{6b^3}-\frac{x^7}{2b\left(a+bx^2\right)}+\frac{7x^5}{10b^2}+\frac{7\int a^2\,dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -7*a**(5/2)*atan(sqrt(b)*x/sqrt(a))/(2*b**(9/2)) - 7*a*x**3/(6*b*
*3) - x**7/(2*b*(a + b*x**2)) + 7*x**5/(10*b**2) + 7*Integral(a**
2, x)/(2*b**4)
```

Mathematica [A] time = 0.0849795, size = 71, normalized size = 0.9

$$\frac{x\left(\frac{15a^3}{a+bx^2}+90a^2-20abx^2+6b^2x^4\right)}{30b^4}-\frac{7a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*
b^4) - (7*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Maple [A] time = 0.012, size = 68, normalized size = 0.9

$$\frac{x^5}{5\,b^2}-\frac{2\,ax^3}{3\,b^3}+3\,\frac{a^2x}{b^4}+\frac{a^3x}{2\,b^4\,(bx^2+a)}-\frac{7\,a^3}{2\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/5*x^5/b^2-2/3*a*x^3/b^3+3*a^2*x/b^4+1/2/b^4*a^3*x/(b*x^2+a)-7/2
/b^4*a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263148, size = 1, normalized size = 0.01

$$\left[\frac{12\,b^3x^7-28\,ab^2x^5+140\,a^2bx^3+210\,a^3x+105\left(a^2bx^2+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{60\,(b^5x^2+ab^4)},\frac{6\,b^3x^7-14\,ab^2x^5+70\,a^2bx^3+1\cdots}{30\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 10
5*(a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)))/(b^5*x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5
+ 70*a^2*b*x^3 + 105*a^3*x - 105*(a^2*b*x^2 + a^3)*sqrt(a/b)*arct
an(x/sqrt(a/b)))/(b^5*x^2 + a*b^4)]
```

Sympy [A] time = 1.63131, size = 124, normalized size = 1.57

$$\frac{a^3x}{2ab^4+2b^5x^2}+\frac{3a^2x}{b^4}-\frac{2ax^3}{3b^3}+\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}-\frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**
3) + 7*sqrt(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7
*sqrt(-a**5/b**9)*log(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5
*b**2)
```

GIAC/XCAS [A] time = 0.270136, size = 99, normalized size = 1.25

$$-\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4}+\frac{a^3x}{2\,(bx^2+a)b^4}+\frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^
2 + a)*b^4) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

3.486 $\int \frac{x^6}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=66

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] time = 0.0927202, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}-\frac{5ax}{2b^3}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-5*a*x)/(2*b^3) + (5*x^3)/(6*b^2) - x^5/(2*b*(a + b*x^2)) + (5*a
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{5a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{7}{2}}}-\frac{x^5}{2b\left(a+bx^2\right)}+\frac{5x^3}{6b^2}-\frac{5\int a\,dx}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 5*a**(3/2)*atan(sqrt(b)*x/sqrt(a))/(2*b**(7/2)) - x**5/(2*b*(a +
b*x**2)) + 5*x**3/(6*b**2) - 5*Integral(a, x)/(2*b**3)
```

Mathematica [A] time = 0.072393, size = 60, normalized size = 0.91

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x\left(-\frac{3a^2}{a+bx^2}-12a+2bx^2\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Maple [A] time = 0.013, size = 57, normalized size = 0.9

$$\frac{x^3}{3\,b^2}-2\,\frac{ax}{b^3}-\frac{a^2x}{2\,b^3\,(bx^2+a)}+\frac{5\,a^2}{2\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/3*x^3/b^2-2*a*x/b^3-1/2/b^3*a^2*x/(b*x^2+a)+5/2/b^3*a^2/(a*b)^(
1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262571, size = 1, normalized size = 0.02

$$\left[\frac{4\,b^2x^5-20\,abx^3-30\,a^2x+15\left(abx^2+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\,(b^4x^2+ab^3)},\ \frac{2\,b^2x^5-10\,abx^3-15\,a^2x+15\left(abx^2+a^2\right)\sqrt{\frac{a}{b}}\,\mathrm{arct}}{6\,(b^4x^2+ab^3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqr
t(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2
 + a*b^3), 1/6*(2*b^2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 +
 a^2)*sqrt(a/b)*arctan(x/sqrt(a/b)))/(b^4*x^2 + a*b^3)]
```

Sympy [A] time = 1.56263, size = 107, normalized size = 1.62

$$-\frac{a^2x}{2ab^3+2b^4x^2}-\frac{2ax}{b^3}-\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7
)*log(x - b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x +
 b**3*sqrt(-a**3/b**7)/a)/4 + x**3/(3*b**2)
```

GIAC/XCAS [A] time = 0.267938, size = 82, normalized size = 1.24

$$\frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3}-\frac{a^2x}{2\,(bx^2+a)b^3}+\frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2
 + a)*b^3) + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

3.487 $\int \frac{x^4}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=55

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*b^(5/2))
```

Rubi [A] time = 0.0708196, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (3*x)/(2*b^2) - x^3/(2*b*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(2*b^(5/2))
```

Rubi in Sympy [A] time = 19.4335, size = 48, normalized size = 0.87

$$-\frac{3\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}-\frac{x^3}{2b\left(a+bx^2\right)}+\frac{3x}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -3*sqrt(a)*atan(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) - x**3/(2*b*(a +
b*x**2)) + 3*x/(2*b**2)
```

Mathematica [A] time = 0.0588881, size = 51, normalized size = 0.93

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}+\frac{ax}{2b^2\left(a+bx^2\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(2*b^(5/2))
```

Maple [A] time = 0.011, size = 43, normalized size = 0.8

$$\frac{x}{b^2}+\frac{ax}{2\,b^2\,(bx^2+a)}-\frac{3\,a}{2\,b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] x/b^2+1/2/b^2*a*x/(b*x^2+a)-3/2/b^2*a/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26314, size = 1, normalized size = 0.02

$$\left[\frac{4\,bx^3+3\left(bx^2+a\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\left(b^3x^2+ab^2\right)},\ \frac{2\,bx^3-3\left(bx^2+a\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)+3\,ax}{2\left(b^3x^2+ab^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(
-a/b) - a)/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3
- 3*(b*x^2 + a)*sqrt(a/b)*arctan(x/sqrt(a/b)) + 3*a*x)/(b^3*x^2 +
 a*b^2)]
```

Sympy [A] time = 1.45644, size = 83, normalized size = 1.51

$$\frac{ax}{2ab^2+2b^3x^2}+\frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}-\frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/
b**5) + x)/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/
b**2
```

GIAC/XCAS [A] time = 0.271422, size = 57, normalized size = 1.04

$$-\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}+\frac{ax}{2\,(bx^2+a)b^2}+\frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 +
a)*b^2) + x/b^2
```

3.488 $\int \frac{x^2}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(
3/2))
```

Rubi [A] time = 0.0480662, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(
3/2))
```

Rubi in Sympy [A] time = 13.0002, size = 36, normalized size = 0.8

$$-\frac{x}{2b\left(a+bx^2\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -x/(2*b*(a + b*x**2)) + atan(sqrt(b)*x/sqrt(a))/(2*sqrt(a)*b**(3/
2))
```

Mathematica [A] time = 0.0362842, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -x/(2*b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(
3/2))
```

Maple [A] time = 0.01, size = 36, normalized size = 0.8

$$-\frac{x}{2\,b\,(bx^2+a)}+\frac{1}{2\,b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262935, size = 1, normalized size = 0.02

$$\left[\frac{\left(bx^2+a\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\,\sqrt{-ab}x}{4\,(b^2x^2+ab)\sqrt{-ab}},\ \frac{\left(bx^2+a\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\sqrt{ab}x}{2\,(b^2x^2+ab)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*x^2 + a)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) - 2*sqrt(-a*b)*x)/((b^2*x^2 + a*b)*sqrt(-a*b)), 1/2*((b*x^2
+ a)*arctan(sqrt(a*b)*x/a) - sqrt(a*b)*x)/((b^2*x^2 + a*b)*sqrt(a
*b))]
```

Sympy [A] time = 1.28289, size = 78, normalized size = 1.73

$$-\frac{x}{2ab+2b^2x^2}-\frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*
b**3)) + x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/
4
```

GIAC/XCAS [A] time = 0.269045, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b}-\frac{x}{2\,(bx^2+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

3.489 $\int \frac{1}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[b])
```

Rubi [A] time = 0.0429897, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[b])
```

Rubi in Sympy [A] time = 19.5511, size = 36, normalized size = 0.8

$$\frac{x}{2a\left(a+bx^2\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] x/(2*a*(a + b*x**2)) + atan(sqrt(b)*x/sqrt(a))/(2*a**(3/2)*sqrt(b
))
```

Mathematica [A] time = 0.042551, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1),x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[b])
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$\frac{x}{2\,a\,(bx^2+a)}+\frac{1}{2\,a}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.260443, size = 1, normalized size = 0.02

$$\left[\frac{(bx^2+a)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\sqrt{-ab}x}{4\,(abx^2+a^2)\sqrt{-ab}},\frac{(bx^2+a)\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\sqrt{ab}x}{2\,(abx^2+a^2)\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*x^2 + a)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 +
 a)) + 2*sqrt(-a*b)*x)/((a*b*x^2 + a^2)*sqrt(-a*b)), 1/2*((b*x^2
+ a)*arctan(sqrt(a*b)*x/a) + sqrt(a*b)*x)/((a*b*x^2 + a^2)*sqrt(a
*b))]
```

Sympy [A] time = 1.34016, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a*
*3*b)) + x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)
/4
```

GIAC/XCAS [A] time = 0.268251, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2(bx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

3.490 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] time = 0.0719584, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*a^(5/2))
```

Rubi in Sympy [A] time = 19.6246, size = 48, normalized size = 0.84

$$\frac{1}{2ax\left(a+bx^2\right)}-\frac{3}{2a^2x}-\frac{3\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*x*(a + b*x**2)) - 3/(2*a**2*x) - 3*sqrt(b)*atan(sqrt(b)*x/
sqrt(a))/(2*a**(5/2))
```

Mathematica [A] time = 0.0648362, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*a^(5/2))
```

Maple [A] time = 0.014, size = 46, normalized size = 0.8

$$-\frac{bx}{2\,a^2\,(bx^2+a)}-\frac{3\,b}{2\,a^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*b/a^2*x/(b*x^2+a)-3/2*b/a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))-1/a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263742, size = 1, normalized size = 0.02

$$\left[-\frac{6\,bx^2-3\,(bx^3+ax)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+4\,a}{4\,(a^2bx^3+a^3x)},-\frac{3\,bx^2+3\,(bx^3+ax)\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+2\,a}{2\,(a^2bx^3+a^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sq
rt(-b/a) - a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*
x^2 + 3*(b*x^3 + a*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 2*a)/
(a^2*b*x^3 + a^3*x)]
```

Sympy [A] time = 1.65214, size = 90, normalized size = 1.58

$$\frac{3\sqrt{-\frac{b}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b}{a^5}}}{b}+x\right)}{4}-\frac{2a+3bx^2}{2a^3x+2a^2bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5
)*log(a**3*sqrt(-b/a**5)/b + x)/4 - (2*a + 3*b*x**2)/(2*a**3*x +
2*a**2*b*x**3)
```

GIAC/XCAS [A] time = 0.270102, size = 63, normalized size = 1.11

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}-\frac{3\,bx^2+2\,a}{2\,(bx^3+ax)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^2),x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a
)/((b*x^3 + a*x)*a^2)
```

3.491 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] time = 0.093508, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi in Sympy [A] time = 27.6985, size = 61, normalized size = 0.9

$$\frac{1}{2ax^3\left(a+bx^2\right)}-\frac{5}{6a^2x^3}+\frac{5b}{2a^3x}+\frac{5b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*x**3*(a + b*x**2)) - 5/(6*a**2*x**3) + 5*b/(2*a**3*x) + 5*
b**(3/2)*atan(sqrt(b)*x/sqrt(a))/(2*a**(7/2))
```

Mathematica [A] time = 0.0749999, size = 67, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{b^2x}{2a^3\left(a+bx^2\right)}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(3*a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5
*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Maple [A] time = 0.016, size = 59, normalized size = 0.9

$$\frac{b^2x}{2\,a^3\,(bx^2+a)}+\frac{5\,b^2}{2\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{3\,a^2x^3}+2\,\frac{b}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*b^2/a^3*x/(b*x^2+a)+5/2*b^2/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))-1/3/a^2/x^3+2*b/a^3/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267793, size = 1, normalized size = 0.01

$$\left[\frac{30\,b^2x^4+20\,abx^2+15\,\left(b^2x^5+abx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-4\,a^2}{12\,(a^3bx^5+a^4x^3)},\frac{15\,b^2x^4+10\,abx^2+15\,\left(b^2x^5+abx^3\right)\sqrt{\frac{b}{a}}\arctan\left(\right.}{6\,(a^3bx^5+a^4x^3)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^4),x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a
)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b
*x^5 + a^4*x^3), 1/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b
*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) - 2*a^2)/(a^3*b*x^5 + a
^4*x^3)]
```

Sympy [A] time = 1.95564, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sq
rt(-b**3/a**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 +
 10*a*b*x**2 + 15*b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

GIAC/XCAS [A] time = 0.27013, size = 80, normalized size = 1.18

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3}+\frac{b^2x}{2\,(bx^2+a)a^3}+\frac{6\,bx^2-a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^4),x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2
 + a)*a^3) + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

3.492 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=81

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5(a+bx^2)}$$

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*
x^5*(a + b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(
9/2))
```

Rubi [A] time = 0.11887, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{7b^2}{2a^4x}+\frac{7b}{6a^3x^3}-\frac{7}{10a^2x^5}+\frac{1}{2ax^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -7/(10*a^2*x^5) + (7*b)/(6*a^3*x^3) - (7*b^2)/(2*a^4*x) + 1/(2*a*
x^5*(a + b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(
9/2))
```

Rubi in Sympy [A] time = 33.9936, size = 75, normalized size = 0.93

$$\frac{1}{2ax^5(a+bx^2)}-\frac{7}{10a^2x^5}+\frac{7b}{6a^3x^3}-\frac{7b^2}{2a^4x}-\frac{7b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*x**5*(a + b*x**2)) - 7/(10*a**2*x**5) + 7*b/(6*a**3*x**3)
- 7*b**2/(2*a**4*x) - 7*b**(5/2)*atan(sqrt(b)*x/sqrt(a))/(2*a**(9
/2))
```

Mathematica [A] time = 0.0800348, size = 80, normalized size = 0.99

$$-\frac{7b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}-\frac{b^3x}{2a^4(a+bx^2)}-\frac{3b^2}{a^4x}+\frac{2b}{3a^3x^3}-\frac{1}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] -1/(5*a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2
*a^4*(a + b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^
(9/2))
```

Maple [A] time = 0.017, size = 70, normalized size = 0.9

$$-\frac{b^3x}{2\,a^4\,(bx^2+a)}-\frac{7\,b^3}{2\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{5\,a^2x^5}-3\,\frac{b^2}{a^4x}+\frac{2\,b}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*b^3/a^4*x/(b*x^2+a)-7/2*b^3/a^4/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))-1/5/a^2/x^5-3*b^2/a^4/x+2/3*b/a^3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2652, size = 1, normalized size = 0.01

$$\left[-\frac{210\, b^3x^6 + 140\, ab^2x^4 - 28\, a^2bx^2 + 12\, a^3 - 105\, \left(b^3x^7 + ab^2x^5\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2 - 2\, ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a}\right)}{60\, \left(a^4bx^7 + a^5x^5\right)},\right.$$
$$\left.-\frac{105\, b^3x^6 + 70\, ab^2x^4 - 14\, a^2bx^2 + 6\, a^3 + 105\, \left(b^3x^7 + ab^2x^5\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{30\, \left(a^4bx^7 + a^5x^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^6),x, algorithm="fricas")
```

```
[Out] [-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105
*(b^3*x^7 + a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)))/(a^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*
a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt
(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^4*b*x^7 + a^5*x^5)]
```

Sympy [A] time = 2.62633, size = 126, normalized size = 1.56

$$\frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3} + x\right)}{4} - \frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3} + x\right)}{4} - \frac{6a^3 - 14a^2bx^2 + 70ab^2x^4 + 105b^3x^6}{30a^5x^5 + 30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqr
t(-b**5/a**9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - (6*a**3 - 1
4*a**2*b*x**2 + 70*a*b**2*x**4 + 105*b**3*x**6)/(30*a**5*x**5 + 3
0*a**4*b*x**7)
```

GIAC/XCAS [A] time = 0.269998, size = 95, normalized size = 1.17

$$-\frac{7\, b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, \sqrt{ab}a^4} - \frac{b^3x}{2\, (bx^2 + a)a^4} - \frac{45\, b^2x^4 - 10\, abx^2 + 3\, a^2}{15\, a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*x^6),x, algorithm="giac")
```

```
[Out] -7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^
2 + a)*a^4) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

3.493 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=91

$$\frac{a^5}{6b^6(a+bx^2)^3}-\frac{5a^4}{4b^6(a+bx^2)^2}+\frac{5a^3}{b^6(a+bx^2)}+\frac{5a^2\log(a+bx^2)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4
)/(4*b^6*(a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[
a + b*x^2])/b^6
```

Rubi [A] time = 0.20761, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^5}{6b^6(a+bx^2)^3}-\frac{5a^4}{4b^6(a+bx^2)^2}+\frac{5a^3}{b^6(a+bx^2)}+\frac{5a^2\log(a+bx^2)}{b^6}-\frac{2ax^2}{b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-2*a*x^2)/b^5 + x^4/(4*b^4) + a^5/(6*b^6*(a + b*x^2)^3) - (5*a^4
)/(4*b^6*(a + b*x^2)^2) + (5*a^3)/(b^6*(a + b*x^2)) + (5*a^2*Log[
a + b*x^2])/b^6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^5}{6b^6(a+bx^2)^3}-\frac{5a^4}{4b^6(a+bx^2)^2}+\frac{5a^3}{b^6(a+bx^2)}+\frac{5a^2\log(a+bx^2)}{b^6}-\frac{2ax^2}{b^5}+\frac{\int^{x^2} x\,dx}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**5/(6*b**6*(a + b*x**2)**3) - 5*a**4/(4*b**6*(a + b*x**2)**2) +
 5*a**3/(b**6*(a + b*x**2)) + 5*a**2*log(a + b*x**2)/b**6 - 2*a*x
**2/b**5 + Integral(x, (x, x**2))/(2*b**4)
```

Mathematica [A] time = 0.0537335, size = 78, normalized size = 0.86

$$\frac{\frac{2a^5}{(a+bx^2)^3}-\frac{15a^4}{(a+bx^2)^2}+\frac{60a^3}{a+bx^2}+60a^2\log\left(a+bx^2\right)-24abx^2+3b^2x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-24*a*b*x^2 + 3*b^2*x^4 + (2*a^5)/(a + b*x^2)^3 - (15*a^4)/(a +
b*x^2)^2 + (60*a^3)/(a + b*x^2) + 60*a^2*Log[a + b*x^2])/(12*b^6)
```

Maple [A] time = 0.017, size = 86, normalized size = 1.

$$-2\,\frac{ax^2}{b^5}+\frac{x^4}{4\,b^4}+\frac{a^5}{6\,b^6\,(bx^2+a)^3}-\frac{5\,a^4}{4\,b^6\,(bx^2+a)^2}+5\,\frac{a^3}{b^6\,(bx^2+a)}+5\,\frac{a^2\ln\left(bx^2+a\right)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2*a*x^2/b^5+1/4*x^4/b^4+1/6*a^5/b^6/(b*x^2+a)^3-5/4*a^4/b^6/(b*x
^2+a)^2+5*a^3/b^6/(b*x^2+a)+5*a^2*ln(b*x^2+a)/b^6
```

Maxima [A] time = 0.683516, size = 134, normalized size = 1.47

$$\frac{60\,a^3b^2x^4+105\,a^4bx^2+47\,a^5}{12\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}+\frac{5\,a^2\log\left(bx^2+a\right)}{b^6}+\frac{bx^4-8\,ax^2}{4\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*a^3*b^2*x^4 + 105*a^4*b*x^2 + 47*a^5)/(b^9*x^6 + 3*a*b^8
*x^4 + 3*a^2*b^7*x^2 + a^3*b^6) + 5*a^2*log(b*x^2 + a)/b^6 + 1/4*
(b*x^4 - 8*a*x^2)/b^5
```

Fricas [A] time = 0.252528, size = 185, normalized size = 2.03

$$\frac{3\,b^5x^{10}-15\,ab^4x^8-63\,a^2b^3x^6-9\,a^3b^2x^4+81\,a^4bx^2+47\,a^5+60\,\left(a^2b^3x^6+3\,a^3b^2x^4+3\,a^4bx^2+a^5\right)\log\left(bx^2+a\right)}{12\,(b^9x^6+3\,ab^8x^4+3\,a^2b^7x^2+a^3b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 - 15*a*b^4*x^8 - 63*a^2*b^3*x^6 - 9*a^3*b^2*x^4
+ 81*a^4*b*x^2 + 47*a^5 + 60*(a^2*b^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4
*b*x^2 + a^5)*log(b*x^2 + a))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*
x^2 + a^3*b^6)
```

Sympy [A] time = 2.77952, size = 100, normalized size = 1.1

$$\frac{5a^2\log\left(a+bx^2\right)}{b^6}-\frac{2ax^2}{b^5}+\frac{47a^5+105a^4bx^2+60a^3b^2x^4}{12a^3b^6+36a^2b^7x^2+36ab^8x^4+12b^9x^6}+\frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 5*a**2*log(a + b*x**2)/b**6 - 2*a*x**2/b**5 + (47*a**5 + 105*a**4
*b*x**2 + 60*a**3*b**2*x**4)/(12*a**3*b**6 + 36*a**2*b**7*x**2 +
36*a*b**8*x**4 + 12*b**9*x**6) + x**4/(4*b**4)
```

GIAC/XCAS [A] time = 0.271753, size = 123, normalized size = 1.35

$$\frac{5\,a^2\ln\left(\left|bx^2+a\right|\right)}{b^6}+\frac{b^4x^4-8\,ab^3x^2}{4\,b^8}-\frac{110\,a^2b^3x^6+270\,a^3b^2x^4+225\,a^4bx^2+63\,a^5}{12\,(bx^2+a)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*ln(abs(b*x^2 + a))/b^6 + 1/4*(b^4*x^4 - 8*a*b^3*x^2)/b^8 -
1/12*(110*a^2*b^3*x^6 + 270*a^3*b^2*x^4 + 225*a^4*b*x^2 + 63*a^5)
/((b*x^2 + a)^3*b^6)
```

3.494 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=77

$$-\frac{a^4}{6b^5(a+bx^2)^3}+\frac{a^3}{b^5(a+bx^2)^2}-\frac{3a^2}{b^5(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^5}+\frac{x^2}{2b^4}$$

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2)
 - (3*a^2)/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rubi [A] time = 0.167811, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^4}{6b^5(a+bx^2)^3}+\frac{a^3}{b^5(a+bx^2)^2}-\frac{3a^2}{b^5(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] x^2/(2*b^4) - a^4/(6*b^5*(a + b*x^2)^3) + a^3/(b^5*(a + b*x^2)^2)
 - (3*a^2)/(b^5*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^4}{6b^5(a+bx^2)^3}+\frac{a^3}{b^5(a+bx^2)^2}-\frac{3a^2}{b^5(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^5}+\frac{b^4\int^{x^2}\frac{1}{b^8}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -a**4/(6*b**5*(a + b*x**2)**3) + a**3/(b**5*(a + b*x**2)**2) - 3*
a**2/(b**5*(a + b*x**2)) - 2*a*log(a + b*x**2)/b**5 + b**4*Integr
al(b**(-8), (x, x**2))/2
```

Mathematica [A] time = 0.0884273, size = 59, normalized size = 0.77

$$-\frac{\frac{a^2(13a^2+30abx^2+18b^2x^4)}{(a+bx^2)^3}+12a\log(a+bx^2)-3bx^2}{6b^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(-3*b*x^2 + (a^2*(13*a^2 + 30*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)
^3 + 12*a*Log[a + b*x^2])/(6*b^5)
```

Maple [A] time = 0.016, size = 74, normalized size = 1.

$$\frac{x^2}{2\,b^4}-\frac{a^4}{6\,b^5\,(bx^2+a)^3}+\frac{a^3}{b^5\,(bx^2+a)^2}-3\,\frac{a^2}{b^5\,(bx^2+a)}-2\,\frac{a\ln(bx^2+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/2*x^2/b^4-1/6*a^4/b^5/(b*x^2+a)^3+a^3/b^5/(b*x^2+a)^2-3*a^2/b^5
/(b*x^2+a)-2*a*ln(b*x^2+a)/b^5
```

Maxima [A] time = 0.697124, size = 119, normalized size = 1.55

$$-\frac{18\,a^2b^2x^4+30\,a^3bx^2+13\,a^4}{6\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)}+\frac{x^2}{2\,b^4}-\frac{2\,a\log(bx^2+a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(18*a^2*b^2*x^4 + 30*a^3*b*x^2 + 13*a^4)/(b^8*x^6 + 3*a*b^7*
x^4 + 3*a^2*b^6*x^2 + a^3*b^5) + 1/2*x^2/b^4 - 2*a*log(b*x^2 + a)
/b^5
```

Fricas [A] time = 0.256851, size = 167, normalized size = 2.17

$$\frac{3\,b^4x^8+9\,ab^3x^6-9\,a^2b^2x^4-27\,a^3bx^2-13\,a^4-12\,(ab^3x^6+3\,a^2b^2x^4+3\,a^3bx^2+a^4)\log(bx^2+a)}{6\,(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^4*x^8 + 9*a*b^3*x^6 - 9*a^2*b^2*x^4 - 27*a^3*b*x^2 - 13*
a^4 - 12*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*log(b*x^
2 + a))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [A] time = 2.64225, size = 88, normalized size = 1.14

$$-\frac{2a\log\left(a+bx^2\right)}{b^5}-\frac{13a^4+30a^3bx^2+18a^2b^2x^4}{6a^3b^5+18a^2b^6x^2+18ab^7x^4+6b^8x^6}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -2*a*log(a + b*x**2)/b**5 - (13*a**4 + 30*a**3*b*x**2 + 18*a**2*b
**2*x**4)/(6*a**3*b**5 + 18*a**2*b**6*x**2 + 18*a*b**7*x**4 + 6*b
**8*x**6) + x**2/(2*b**4)
```

GIAC/XCAS [A] time = 0.271643, size = 99, normalized size = 1.29

$$\frac{x^2}{2\,b^4}-\frac{2\,a\ln\left(\left|bx^2+a\right|\right)}{b^5}+\frac{22\,ab^3x^6+48\,a^2b^2x^4+36\,a^3bx^2+9\,a^4}{6\,(bx^2+a)^3b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^4 - 2*a*ln(abs(b*x^2 + a))/b^5 + 1/6*(22*a*b^3*x^6 + 48
*a^2*b^2*x^4 + 36*a^3*b*x^2 + 9*a^4)/((b*x^2 + a)^3*b^5)
```

3.495 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=71

$$\frac{a^3}{6b^4\,(a+bx^2)^3}-\frac{3a^2}{4b^4\,(a+bx^2)^2}+\frac{3a}{2b^4\,(a+bx^2)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)
/(2*b^4*(a + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rubi [A] time = 0.144904, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^3}{6b^4\,(a+bx^2)^3}-\frac{3a^2}{4b^4\,(a+bx^2)^2}+\frac{3a}{2b^4\,(a+bx^2)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] a^3/(6*b^4*(a + b*x^2)^3) - (3*a^2)/(4*b^4*(a + b*x^2)^2) + (3*a)
/(2*b^4*(a + b*x^2)) + Log[a + b*x^2]/(2*b^4)
```

Rubi in Sympy [A] time = 26.1401, size = 63, normalized size = 0.89

$$\frac{a^3}{6b^4\,(a+bx^2)^3}-\frac{3a^2}{4b^4\,(a+bx^2)^2}+\frac{3a}{2b^4\,(a+bx^2)}+\frac{\log\left(a+bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**3/(6*b**4*(a + b*x**2)**3) - 3*a**2/(4*b**4*(a + b*x**2)**2) +
 3*a/(2*b**4*(a + b*x**2)) + log(a + b*x**2)/(2*b**4)
```

Mathematica [A] time = 0.0315634, size = 50, normalized size = 0.7

$$\frac{\frac{a\left(11a^2+27abx^2+18b^2x^4\right)}{(a+bx^2)^3}+6\log\left(a+bx^2\right)}{12b^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x^2 + 18*b^2*x^4))/(a + b*x^2)^3 + 6*Log[a +
 b*x^2])/(12*b^4)
```

Maple [A] time = 0.013, size = 64, normalized size = 0.9

$$\frac{a^3}{6\,b^4\,(bx^2+a)^3}-\frac{3\,a^2}{4\,b^4\,(bx^2+a)^2}+\frac{3\,a}{2\,b^4\,(bx^2+a)}+\frac{\ln\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a^3/b^4/(b*x^2+a)^3-3/4*a^2/b^4/(b*x^2+a)^2+3/2*a/b^4/(b*x^2+
a)+1/2*ln(b*x^2+a)/b^4
```

MaximA [A] time = 0.68667, size = 104, normalized size = 1.46

$$\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3}{12\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}+\frac{\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3)/(b^7*x^6 + 3*a*b^6*x^
4 + 3*a^2*b^5*x^2 + a^3*b^4) + 1/2*log(b*x^2 + a)/b^4
```

Fricas [A] time = 0.252448, size = 138, normalized size = 1.94

$$\frac{18\,ab^2x^4+27\,a^2bx^2+11\,a^3+6\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{12\,(b^7x^6+3\,ab^6x^4+3\,a^2b^5x^2+a^3b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(18*a*b^2*x^4 + 27*a^2*b*x^2 + 11*a^3 + 6*(b^3*x^6 + 3*a*b^2
*x^4 + 3*a^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^7*x^6 + 3*a*b^6*x^4
+ 3*a^2*b^5*x^2 + a^3*b^4)
```

Sympy [A] time = 2.28705, size = 76, normalized size = 1.07

$$\frac{11a^3 + 27a^2bx^2 + 18ab^2x^4}{12a^3b^4 + 36a^2b^5x^2 + 36ab^6x^4 + 12b^7x^6} + \frac{\log\left(a + bx^2\right)}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x**2 + 18*a*b**2*x**4)/(12*a**3*b**4 + 36*a*
*2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + log(a + b*x**2)/(
2*b**4)
```

GIAC/XCAS [A] time = 0.271078, size = 72, normalized size = 1.01

$$\frac{\ln\left(\left|bx^2 + a\right|\right)}{2\,b^4} - \frac{11\,b^2x^6 + 15\,abx^4 + 6\,a^2x^2}{12\,(bx^2 + a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(b*x^2 + a))/b^4 - 1/12*(11*b^2*x^6 + 15*a*b*x^4 + 6*a^
2*x^2)/((b*x^2 + a)^3*b^3)
```

3.496 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a(a+bx^2)^3}$$

```
[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rubi [A] time = 0.0254265, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{x^6}{6a(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]

[Out] x^6/(6*a*(a + b*x^2)^3)
```

Rubi in Sympy [A] time = 9.01914, size = 14, normalized size = 0.74

$$\frac{x^6}{6a(a+bx^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)

[Out] x**6/(6*a*(a + b*x**2)**3)
```

Mathematica [A] time = 0.0220468, size = 35, normalized size = 1.84

$$-\frac{a^2+3abx^2+3b^2x^4}{6b^3(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a^2 + 3*a*b*x^2 + 3*b^2*x^4)/(6*b^3*(a + b*x^2)^3)
```

Maple [B] time = 0.012, size = 48, normalized size = 2.5

$$-\frac{a^2}{6\,b^3\,(bx^2+a)^3}+\frac{a}{2\,b^3\,(bx^2+a)^2}-\frac{1}{(2\,bx^2+2\,a)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/6*a^2/b^3/(b*x^2+a)^3+1/2*a/b^3/(b*x^2+a)^2-1/2/(b*x^2+a)/b^3
```

Maxima [A] time = 0.683812, size = 78, normalized size = 4.11

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2
*b^4*x^2 + a^3*b^3)
```

Fricas [A] time = 0.248892, size = 78, normalized size = 4.11

$$-\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2
*b^4*x^2 + a^3*b^3)
```

Sympy [A] time = 2.12276, size = 60, normalized size = 3.16

$$-\frac{a^2 + 3abx^2 + 3b^2x^4}{6a^3b^3 + 18a^2b^4x^2 + 18ab^5x^4 + 6b^6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -(a**2 + 3*a*b*x**2 + 3*b**2*x**4)/(6*a**3*b**3 + 18*a**2*b**4*x*
*2 + 18*a*b**5*x**4 + 6*b**6*x**6)
```

GIAC/XCAS [A] time = 0.269799, size = 45, normalized size = 2.37

$$-\frac{3\,b^2x^4 + 3\,abx^2 + a^2}{6\,(bx^2 + a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/((b*x^2 + a)^3*b^3)
```

3.497 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=34

$$\frac{a}{6b^2(a+bx^2)^3}-\frac{1}{4b^2(a+bx^2)^2}$$

```
[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rubi [A] time = 0.0733785, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a}{6b^2(a+bx^2)^3}-\frac{1}{4b^2(a+bx^2)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]

[Out] a/(6*b^2*(a + b*x^2)^3) - 1/(4*b^2*(a + b*x^2)^2)
```

Rubi in Sympy [A] time = 16.5691, size = 29, normalized size = 0.85

$$\frac{a}{6b^2(a+bx^2)^3}-\frac{1}{4b^2(a+bx^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)

[Out] a/(6*b**2*(a + b*x**2)**3) - 1/(4*b**2*(a + b*x**2)**2)
```

Mathematica [A] time = 0.0122969, size = 24, normalized size = 0.71

$$-\frac{a+3bx^2}{12b^2(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(a + 3*b*x^2)/(12*b^2*(a + b*x^2)^3)
```

Maple [A] time = 0.011, size = 31, normalized size = 0.9

$$\frac{a}{6\,b^2\,(bx^2+a)^3}-\frac{1}{4\,b^2\,(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*a/b^2/(b*x^2+a)^3-1/4/b^2/(b*x^2+a)^2
```

Maxima [A] time = 0.678458, size = 63, normalized size = 1.85

$$-\frac{3\,bx^2+a}{12\,(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*
b^2)
```

Fricas [A] time = 0.248675, size = 63, normalized size = 1.85

$$-\frac{3\,bx^2+a}{12\,(b^5x^6+3\,ab^4x^4+3\,a^2b^3x^2+a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/12*(3*b*x^2 + a)/(b^5*x^6 + 3*a*b^4*x^4 + 3*a^2*b^3*x^2 + a^3*
b^2)
```

Sympy [A] time = 1.9843, size = 48, normalized size = 1.41

$$-\frac{a+3bx^2}{12a^3b^2+36a^2b^3x^2+36ab^4x^4+12b^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -(a + 3*b*x**2)/(12*a**3*b**2 + 36*a**2*b**3*x**2 + 36*a*b**4*x**
4 + 12*b**5*x**6)
```

GIAC/XCAS [A] time = 0.269813, size = 30, normalized size = 0.88

$$-\frac{3\,bx^2+a}{12\,(bx^2+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] -1/12*(3*b*x^2 + a)/((b*x^2 + a)^3*b^2)
```

3.498 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=16

$$-\frac{1}{6b(a+bx^2)^3}$$

```
[Out] -1/(6*b*(a + b*x^2)^3)
```

Rubi [A] time = 0.0149842, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]

[Out] -1/(6*b*(a + b*x^2)^3)
```

Rubi in Sympy [A] time = 7.01099, size = 14, normalized size = 0.88

$$-\frac{1}{6b(a+bx^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)

[Out] -1/(6*b*(a + b*x**2)**3)
```

Mathematica [A] time = 0.00496614, size = 16, normalized size = 1.

$$-\frac{1}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]

[Out] -1/(6*b*(a + b*x^2)^3)
```

Maple [A] time = 0.006, size = 15, normalized size = 0.9

$$-\frac{1}{6\,b\,(bx^2+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)

[Out] -1/6/b/(b*x^2+a)^3
```

Maxima [A] time = 0.692449, size = 50, normalized size = 3.12

$$-\frac{1}{6\,(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")

[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Fricas [A] time = 0.258716, size = 50, normalized size = 3.12

$$-\frac{1}{6\,(b^4x^6+3\,ab^3x^4+3\,a^2b^2x^2+a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")

[Out] -1/6/(b^4*x^6 + 3*a*b^3*x^4 + 3*a^2*b^2*x^2 + a^3*b)
```

Sympy [A] time = 1.89188, size = 39, normalized size = 2.44

$$-\frac{1}{6a^3b + 18a^2b^2x^2 + 18ab^3x^4 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -1/(6*a**3*b + 18*a**2*b**2*x**2 + 18*a*b**3*x**4 + 6*b**4*x**6)
```

GIAC/XCAS [A] time = 0.269485, size = 19, normalized size = 1.19

$$-\frac{1}{6\,(bx^2 + a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] -1/6/((b*x^2 + a)^3*b)
```

3.499 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{6a\left(a+bx^2\right)^3}$$

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b
*x^2)) + Log[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rubi [A] time = 0.155505, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\log\left(a+bx^2\right)}{2a^4}+\frac{\log(x)}{a^4}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] 1/(6*a*(a + b*x^2)^3) + 1/(4*a^2*(a + b*x^2)^2) + 1/(2*a^3*(a + b
*x^2)) + Log[x]/a^4 - Log[a + b*x^2]/(2*a^4)
```

Rubi in Sympy [A] time = 28.9016, size = 65, normalized size = 0.93

$$\frac{1}{6a\left(a+bx^2\right)^3}+\frac{1}{4a^2\left(a+bx^2\right)^2}+\frac{1}{2a^3\left(a+bx^2\right)}+\frac{\log\left(x^2\right)}{2a^4}-\frac{\log\left(a+bx^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 1/(6*a*(a + b*x**2)**3) + 1/(4*a**2*(a + b*x**2)**2) + 1/(2*a**3*
(a + b*x**2)) + log(x**2)/(2*a**4) - log(a + b*x**2)/(2*a**4)
```

Mathematica [A] time = 0.0747477, size = 54, normalized size = 0.77

$$\frac{\frac{a\left(11a^2+15abx^2+6b^2x^4\right)}{\left(a+bx^2\right)^3}-6\log\left(a+bx^2\right)+12\log(x)}{12a^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x^2 + 6*b^2*x^4))/(a + b*x^2)^3 + 12*Log[x]
- 6*Log[a + b*x^2])/(12*a^4)
```

Maple [A]   time = 0.018, size = 63, normalized size = 0.9

$$\frac{1}{6\,a\left(bx^{2}+a\right)^{3}}+\frac{1}{4\,a^{2}\left(bx^{2}+a\right)^{2}}+\frac{1}{2\,a^{3}\left(bx^{2}+a\right)}+\frac{\ln\left(x\right)}{a^{4}}-\frac{\ln\left(bx^{2}+a\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6/a/(b*x^2+a)^3+1/4/a^2/(b*x^2+a)^2+1/2/a^3/(b*x^2+a)+ln(x)/a^4
-1/2*ln(b*x^2+a)/a^4
```

Maxima [A]   time = 0.693224, size = 111, normalized size = 1.59

$$\frac{6\,b^{2}x^{4}+15\,abx^{2}+11\,a^{2}}{12\left(a^{3}b^{3}x^{6}+3\,a^{4}b^{2}x^{4}+3\,a^{5}bx^{2}+a^{6}\right)}-\frac{\log\left(bx^{2}+a\right)}{2\,a^{4}}+\frac{\log\left(x^{2}\right)}{2\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x),x, algorithm="maxima")
```

```
[Out] 1/12*(6*b^2*x^4 + 15*a*b*x^2 + 11*a^2)/(a^3*b^3*x^6 + 3*a^4*b^2*x
^4 + 3*a^5*b*x^2 + a^6) - 1/2*log(b*x^2 + a)/a^4 + 1/2*log(x^2)/a
^4
```

Fricas [A]   time = 0.269195, size = 181, normalized size = 2.59

$$\frac{6\,ab^{2}x^{4}+15\,a^{2}bx^{2}+11\,a^{3}-6\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\log\left(bx^{2}+a\right)+12\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\log\left(x\right)}{12\left(a^{4}b^{3}x^{6}+3\,a^{5}b^{2}x^{4}+3\,a^{6}bx^{2}+a^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x),x, algorithm="fricas")
```

```
[Out] 1/12*(6*a*b^2*x^4 + 15*a^2*b*x^2 + 11*a^3 - 6*(b^3*x^6 + 3*a*b^2*
x^4 + 3*a^2*b*x^2 + a^3)*log(b*x^2 + a) + 12*(b^3*x^6 + 3*a*b^2*x
^4 + 3*a^2*b*x^2 + a^3)*log(x))/(a^4*b^3*x^6 + 3*a^5*b^2*x^4 + 3*
a^6*b*x^2 + a^7)
```

Sympy [A] time = 2.95668, size = 80, normalized size = 1.14

$$\frac{11a^2 + 15abx^2 + 6b^2x^4}{12a^6 + 36a^5bx^2 + 36a^4b^2x^4 + 12a^3b^3x^6} + \frac{\log(x)}{a^4} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (11*a**2 + 15*a*b*x**2 + 6*b**2*x**4)/(12*a**6 + 36*a**5*b*x**2 +
 36*a**4*b**2*x**4 + 12*a**3*b**3*x**6) + log(x)/a**4 - log(a/b +
 x**2)/(2*a**4)
```

GIAC/XCAS [A] time = 0.270831, size = 95, normalized size = 1.36

$$\frac{\ln\left(x^2\right)}{2\,a^4} - \frac{\ln\left(\left|bx^2 + a\right|\right)}{2\,a^4} + \frac{11\,b^3x^6 + 39\,ab^2x^4 + 48\,a^2bx^2 + 22\,a^3}{12\,(bx^2 + a)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2)/a^4 - 1/2*ln(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6 +
 39*a*b^2*x^4 + 48*a^2*b*x^2 + 22*a^3)/((b*x^2 + a)^3*a^4)
```

3.500 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=84

$$\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{1}{2a^4x^2}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}$$

```
[Out] -1/(2*a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2
) - (3*b)/(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b
*x^2])/a^5
```

Rubi [A] time = 0.185911, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{2b\log\left(a+bx^2\right)}{a^5}-\frac{4b\log(x)}{a^5}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{1}{2a^4x^2}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{b}{6a^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(2*a^4*x^2) - b/(6*a^2*(a + b*x^2)^3) - b/(2*a^3*(a + b*x^2)^2
) - (3*b)/(2*a^4*(a + b*x^2)) - (4*b*Log[x])/a^5 + (2*b*Log[a + b
*x^2])/a^5
```

Rubi in Sympy [A] time = 31.8662, size = 82, normalized size = 0.98

$$-\frac{b}{6a^2\left(a+bx^2\right)^3}-\frac{b}{2a^3\left(a+bx^2\right)^2}-\frac{3b}{2a^4\left(a+bx^2\right)}-\frac{1}{2a^4x^2}-\frac{2b\log\left(x^2\right)}{a^5}+\frac{2b\log\left(a+bx^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -b/(6*a**2*(a + b*x**2)**3) - b/(2*a**3*(a + b*x**2)**2) - 3*b/(2
*a**4*(a + b*x**2)) - 1/(2*a**4*x**2) - 2*b*log(x**2)/a**5 + 2*b*
log(a + b*x**2)/a**5
```

Mathematica [A] time = 0.118648, size = 70, normalized size = 0.83

$$-\frac{\frac{a(3a^3+22a^2bx^2+30ab^2x^4+12b^3x^6)}{x^2(a+bx^2)^3}-12b\log\left(a+bx^2\right)+24b\log(x)}{6a^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -((a*(3*a^3 + 22*a^2*b*x^2 + 30*a*b^2*x^4 + 12*b^3*x^6))/(x^2*(a
+ b*x^2)^3) + 24*b*Log[x] - 12*b*Log[a + b*x^2])/(6*a^5)
```

Maple [A] time = 0.023, size = 77, normalized size = 0.9

$$-\frac{1}{2\,a^4x^2}-\frac{b}{6\,a^2\,(bx^2+a)^3}-\frac{b}{2\,a^3\,(bx^2+a)^2}-\frac{3\,b}{2\,a^4\,(bx^2+a)}-4\,\frac{b\ln(x)}{a^5}+2\,\frac{b\ln\left(bx^2+a\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/2/a^4/x^2-1/6*b/a^2/(b*x^2+a)^3-1/2*b/a^3/(b*x^2+a)^2-3/2*b/a^
4/(b*x^2+a)-4*b*ln(x)/a^5+2*b*ln(b*x^2+a)/a^5
```

Maxima [A] time = 0.687942, size = 134, normalized size = 1.6

$$-\frac{12\,b^3x^6+30\,ab^2x^4+22\,a^2bx^2+3\,a^3}{6\,(a^4b^3x^8+3\,a^5b^2x^6+3\,a^6bx^4+a^7x^2)}+\frac{2\,b\log\left(bx^2+a\right)}{a^5}-\frac{2\,b\log\left(x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^3),x, algorithm="maxima")
```

```
[Out] -1/6*(12*b^3*x^6 + 30*a*b^2*x^4 + 22*a^2*b*x^2 + 3*a^3)/(a^4*b^3*
x^8 + 3*a^5*b^2*x^6 + 3*a^6*b*x^4 + a^7*x^2) + 2*b*log(b*x^2 + a)
/a^5 - 2*b*log(x^2)/a^5
```

Fricas [A] time = 0.269571, size = 220, normalized size = 2.62

$$-\frac{12\,ab^3x^6+30\,a^2b^2x^4+22\,a^3bx^2+3\,a^4-12\left(b^4x^8+3\,ab^3x^6+3\,a^2b^2x^4+a^3bx^2\right)\log\left(bx^2+a\right)+24\left(b^4x^8+3\,ab^3x^6+3\,a^2\right.}{6\,(a^5b^3x^8+3\,a^6b^2x^6+3\,a^7bx^4+a^8x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(12*a*b^3*x^6 + 30*a^2*b^2*x^4 + 22*a^3*b*x^2 + 3*a^4 - 12*(
b^4*x^8 + 3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(b*x^2 + a)
 + 24*(b^4*x^8 + 3*a*b^3*x^6 + 3*a^2*b^2*x^4 + a^3*b*x^2)*log(x))
/(a^5*b^3*x^8 + 3*a^6*b^2*x^6 + 3*a^7*b*x^4 + a^8*x^2)
```

Sympy [A] time = 4.83549, size = 100, normalized size = 1.19

$$-\frac{3a^3 + 22a^2bx^2 + 30ab^2x^4 + 12b^3x^6}{6a^7x^2 + 18a^6bx^4 + 18a^5b^2x^6 + 6a^4b^3x^8} - \frac{4b\log(x)}{a^5} + \frac{2b\log\left(\frac{a}{b} + x^2\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -(3*a**3 + 22*a**2*b*x**2 + 30*a*b**2*x**4 + 12*b**3*x**6)/(6*a**
7*x**2 + 18*a**6*b*x**4 + 18*a**5*b**2*x**6 + 6*a**4*b**3*x**8) -
 4*b*log(x)/a**5 + 2*b*log(a/b + x**2)/a**5
```

GIAC/XCAS [A] time = 0.271369, size = 126, normalized size = 1.5

$$-\frac{2\,b\ln\left(x^2\right)}{a^5} + \frac{2\,b\ln\left(\left|bx^2 + a\right|\right)}{a^5} + \frac{4\,bx^2 - a}{2\,a^5x^2} - \frac{22\,b^4x^6 + 75\,ab^3x^4 + 87\,a^2b^2x^2 + 35\,a^3b}{6\,(bx^2 + a)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^3),x, algorithm="giac")
```

```
[Out] -2*b*ln(x^2)/a^5 + 2*b*ln(abs(b*x^2 + a))/a^5 + 1/2*(4*b*x^2 - a)
/(a^5*x^2) - 1/6*(22*b^4*x^6 + 75*a*b^3*x^4 + 87*a^2*b^2*x^2 + 35
*a^3*b)/((b*x^2 + a)^3*a^5)
```

3.501 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=101

$$-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{2b}{a^5x^2}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4a^4x^4}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}$$

```
[Out] -1/(4*a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3
*b^2)/(4*a^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2
*Log[x])/a^6 - (5*b^2*Log[a + b*x^2])/a^6
```

Rubi [A] time = 0.215025, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{5b^2\log\left(a+bx^2\right)}{a^6}+\frac{10b^2\log(x)}{a^6}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{2b}{a^5x^2}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4a^4x^4}+\frac{b^2}{6a^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -1/(4*a^4*x^4) + (2*b)/(a^5*x^2) + b^2/(6*a^3*(a + b*x^2)^3) + (3
*b^2)/(4*a^4*(a + b*x^2)^2) + (3*b^2)/(a^5*(a + b*x^2)) + (10*b^2
*Log[x])/a^6 - (5*b^2*Log[a + b*x^2])/a^6
```

Rubi in Sympy [A] time = 37.915, size = 100, normalized size = 0.99

$$\frac{b^2}{6a^3\left(a+bx^2\right)^3}+\frac{3b^2}{4a^4\left(a+bx^2\right)^2}-\frac{1}{4a^4x^4}+\frac{3b^2}{a^5\left(a+bx^2\right)}+\frac{2b}{a^5x^2}+\frac{5b^2\log\left(x^2\right)}{a^6}-\frac{5b^2\log\left(a+bx^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] b**2/(6*a**3*(a + b*x**2)**3) + 3*b**2/(4*a**4*(a + b*x**2)**2) -
 1/(4*a**4*x**4) + 3*b**2/(a**5*(a + b*x**2)) + 2*b/(a**5*x**2) +
 5*b**2*log(x**2)/a**6 - 5*b**2*log(a + b*x**2)/a**6
```

Mathematica [A] time = 0.108827, size = 85, normalized size = 0.84

$$\frac{\frac{a(-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8)}{x^4(a+bx^2)^3} - 60b^2\log\left(a+bx^2\right) + 120b^2\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x^2 + 110*a^2*b^2*x^4 + 150*a*b^3*x^6 + 60
*b^4*x^8))/(x^4*(a + b*x^2)^3) + 120*b^2*Log[x] - 60*b^2*Log[a +
b*x^2])/(12*a^6)
```

Maple [A] time = 0.022, size = 96, normalized size = 1.

$$-\frac{1}{4\,a^4x^4}+2\,\frac{b}{a^5x^2}+\frac{b^2}{6\,a^3\,(bx^2+a)^3}+\frac{3\,b^2}{4\,a^4\,(bx^2+a)^2}+3\,\frac{b^2}{a^5\,(bx^2+a)}+10\,\frac{b^2\ln(x)}{a^6}-5\,\frac{b^2\ln\left(bx^2+a\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -1/4/a^4/x^4+2*b/a^5/x^2+1/6*b^2/a^3/(b*x^2+a)^3+3/4*b^2/a^4/(b*x
^2+a)^2+3*b^2/a^5/(b*x^2+a)+10*b^2*ln(x)/a^6-5*b^2*ln(b*x^2+a)/a^
6
```

Maxima [A] time = 0.696789, size = 154, normalized size = 1.52

$$\frac{60\,b^4x^8+150\,ab^3x^6+110\,a^2b^2x^4+15\,a^3bx^2-3\,a^4}{12\,(a^5b^3x^{10}+3\,a^6b^2x^8+3\,a^7bx^6+a^8x^4)}-\frac{5\,b^2\log\left(bx^2+a\right)}{a^6}+\frac{5\,b^2\log\left(x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^5),x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^8 + 150*a*b^3*x^6 + 110*a^2*b^2*x^4 + 15*a^3*b*x^2
 - 3*a^4)/(a^5*b^3*x^10 + 3*a^6*b^2*x^8 + 3*a^7*b*x^6 + a^8*x^4)
- 5*b^2*log(b*x^2 + a)/a^6 + 5*b^2*log(x^2)/a^6
```

Fricas [A] time = 0.271841, size = 240, normalized size = 2.38

$$\frac{60\,ab^4x^8+150\,a^2b^3x^6+110\,a^3b^2x^4+15\,a^4bx^2-3\,a^5-60\,\left(b^5x^{10}+3\,ab^4x^8+3\,a^2b^3x^6+a^3b^2x^4\right)\log\left(bx^2+a\right)+120\,\left(b^5x^{10}\right.}{12\,(a^6b^3x^{10}+3\,a^7b^2x^8+3\,a^8bx^6+a^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^5),x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^8 + 150*a^2*b^3*x^6 + 110*a^3*b^2*x^4 + 15*a^4*b
*x^2 - 3*a^5 - 60*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*x^6 + a^3*b
^2*x^4)*log(b*x^2 + a) + 120*(b^5*x^10 + 3*a*b^4*x^8 + 3*a^2*b^3*
x^6 + a^3*b^2*x^4)*log(x))/(a^6*b^3*x^10 + 3*a^7*b^2*x^8 + 3*a^8*
b*x^6 + a^9*x^4)
```

Sympy [A] time = 9.00198, size = 116, normalized size = 1.15

$$\frac{-3a^4+15a^3bx^2+110a^2b^2x^4+150ab^3x^6+60b^4x^8}{12a^8x^4+36a^7bx^6+36a^6b^2x^8+12a^5b^3x^{10}}+\frac{10b^2\log\left(x\right)}{a^6}-\frac{5b^2\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x**2 + 110*a**2*b**2*x**4 + 150*a*b**3*x**6
+ 60*b**4*x**8)/(12*a**8*x**4 + 36*a**7*b*x**6 + 36*a**6*b**2*x**
8 + 12*a**5*b**3*x**10) + 10*b**2*log(x)/a**6 - 5*b**2*log(a/b +
x**2)/a**6
```

GIAC/XCAS [A] time = 0.27134, size = 146, normalized size = 1.45

$$\frac{5\,b^2\ln\left(x^2\right)}{a^6}-\frac{5\,b^2\ln\left(\left|bx^2+a\right|\right)}{a^6}+\frac{110\,b^5x^6+366\,ab^4x^4+411\,a^2b^3x^2+157\,a^3b^2}{12\,\left(bx^2+a\right)^3a^6}-\frac{30\,b^2x^4-8\,abx^2+a^2}{4\,a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^5),x, algorithm="giac")
```

```
[Out] 5*b^2*ln(x^2)/a^6 - 5*b^2*ln(abs(b*x^2 + a))/a^6 + 1/12*(110*b^5*
x^6 + 366*a*b^4*x^4 + 411*a^2*b^3*x^2 + 157*a^3*b^2)/((b*x^2 + a)
^3*a^6) - 1/4*(30*b^2*x^4 - 8*a*b*x^2 + a^2)/(a^6*x^4)
```

3.502 $\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=117

$$-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}+\frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}-\frac{33x^7}{16b^3(a+bx^2)}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{x^{11}}{6b(a+bx^2)^3}+\frac{231x^5}{80b^4}$$

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) -
 x^11/(6*b*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33
*x^7)/(16*b^3*(a + b*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(16*b^(13/2))
```

Rubi [A] time = 0.184633, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}+\frac{231a^2x}{16b^6}-\frac{77ax^3}{16b^5}-\frac{33x^7}{16b^3(a+bx^2)}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{x^{11}}{6b(a+bx^2)^3}+\frac{231x^5}{80b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (231*a^2*x)/(16*b^6) - (77*a*x^3)/(16*b^5) + (231*x^5)/(80*b^4) -
 x^11/(6*b*(a + b*x^2)^3) - (11*x^9)/(24*b^2*(a + b*x^2)^2) - (33
*x^7)/(16*b^3*(a + b*x^2)) - (231*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(16*b^(13/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{231a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{13}{2}}}-\frac{77ax^3}{16b^5}-\frac{x^{11}}{6b(a+bx^2)^3}-\frac{11x^9}{24b^2(a+bx^2)^2}-\frac{33x^7}{16b^3(a+bx^2)}+\frac{231x^5}{80b^4}+\frac{231\int a^2\,dx}{16b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -231*a**(5/2)*atan(sqrt(b)*x/sqrt(a))/(16*b**(13/2)) - 77*a*x**3/
(16*b**5) - x**11/(6*b*(a + b*x**2)**3) - 11*x**9/(24*b**2*(a + b
*x**2)**2) - 33*x**7/(16*b**3*(a + b*x**2)) + 231*x**5/(80*b**4)
```

```
+ 231*Integral(a**2, x)/(16*b**6)
```

Mathematica [A] time = 0.105492, size = 99, normalized size = 0.85

$$\frac{3465a^5x+9240a^4bx^3+7623a^3b^2x^5+1584a^2b^3x^7-176ab^4x^9+48b^5x^{11}}{240b^6\left(a+bx^2\right)^3}-\frac{231a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (3465*a^5*x + 9240*a^4*b*x^3 + 7623*a^3*b^2*x^5 + 1584*a^2*b^3*x^
7 - 176*a*b^4*x^9 + 48*b^5*x^11)/(240*b^6*(a + b*x^2)^3) - (231*a
^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(13/2))
```

Maple [A] time = 0.015, size = 108, normalized size = 0.9

$$\frac{x^5}{5\,b^4}-\frac{4\,ax^3}{3\,b^5}+10\,\frac{a^2x}{b^6}+\frac{89\,a^3x^5}{16\,b^4\left(bx^2+a\right)^3}+\frac{59\,a^4x^3}{6\,b^5\left(bx^2+a\right)^3}+\frac{71\,a^5x}{16\,b^6\left(bx^2+a\right)^3}-\frac{231\,a^3}{16\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/5*x^5/b^4-4/3*a*x^3/b^5+10*a^2*x/b^6+89/16/b^4*a^3/(b*x^2+a)^3*
x^5+59/6/b^5*a^4/(b*x^2+a)^3*x^3+71/16/b^6*a^5/(b*x^2+a)^3*x-231/
16/b^6*a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273103, size = 1, normalized size = 0.01

$$\left[\frac{96\, b^{5} x^{11} - 352\, a b^{4} x^{9} + 3168\, a^{2} b^{3} x^{7} + 15246\, a^{3} b^{2} x^{5} + 18480\, a^{4} b x^{3} + 6930\, a^{5} x + 3465\, \left(a^{2} b^{3} x^{6} + 3\, a^{3} b^{2} x^{4} + 3\, a^{4} b x^{2} + a^{5}\right) \sqrt{}}{480\, \left(b^{9} x^{6} + 3\, a b^{8} x^{4} + 3\, a^{2} b^{7} x^{2} + a^{3} b^{6}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/480*(96*b^5*x^11 - 352*a*b^4*x^9 + 3168*a^2*b^3*x^7 + 15246*a^
3*b^2*x^5 + 18480*a^4*b*x^3 + 6930*a^5*x + 3465*(a^2*b^3*x^6 + 3*
a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sq
rt(-a/b) - a)/(b*x^2 + a)))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^
2 + a^3*b^6), 1/240*(48*b^5*x^11 - 176*a*b^4*x^9 + 1584*a^2*b^3*x
^7 + 7623*a^3*b^2*x^5 + 9240*a^4*b*x^3 + 3465*a^5*x - 3465*(a^2*b
^3*x^6 + 3*a^3*b^2*x^4 + 3*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(x/sq
rt(a/b)))/(b^9*x^6 + 3*a*b^8*x^4 + 3*a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [A] time = 2.90929, size = 172, normalized size = 1.47

$$\frac{10a^{2}x}{b^{6}} - \frac{4ax^{3}}{3b^{5}} + \frac{231\sqrt{-\frac{a^{5}}{b^{13}}}\log{\left(x - \frac{b^{6}\sqrt{-\frac{a^{5}}{b^{13}}}}{a^{2}} \right)}}{32} - \frac{231\sqrt{-\frac{a^{5}}{b^{13}}}\log{\left(x + \frac{b^{6}\sqrt{-\frac{a^{5}}{b^{13}}}}{a^{2}} \right)}}{32} + \frac{213a^{5}x + 472a^{4}bx^{3} + 267a^{3}b^{2}x^{5}}{48a^{3}b^{6} + 144a^{2}b^{7}x^{2} + 144ab^{8}x^{4} + 48b^{9}x^{6}} + \frac{x^{5}}{5b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 10*a**2*x/b**6 - 4*a*x**3/(3*b**5) + 231*sqrt(-a**5/b**13)*log(x
- b**6*sqrt(-a**5/b**13)/a**2)/32 - 231*sqrt(-a**5/b**13)*log(x +
 b**6*sqrt(-a**5/b**13)/a**2)/32 + (213*a**5*x + 472*a**4*b*x**3
+ 267*a**3*b**2*x**5)/(48*a**3*b**6 + 144*a**2*b**7*x**2 + 144*a*
b**8*x**4 + 48*b**9*x**6) + x**5/(5*b**4)
```

GIAC/XCAS [A] time = 0.270121, size = 130, normalized size = 1.11

$$-\frac{231\,a^{3}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^{6}}+\frac{267\,a^{3}b^{2}x^{5}+472\,a^{4}bx^{3}+213\,a^{5}x}{48\,(bx^{2}+a)^{3}b^{6}}+\frac{3\,b^{16}x^{5}-20\,ab^{15}x^{3}+150\,a^{2}b^{14}x}{15\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] -231/16*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/48*(267*a^3
*b^2*x^5 + 472*a^4*b*x^3 + 213*a^5*x)/((b*x^2 + a)^3*b^6) + 1/15*
(3*b^16*x^5 - 20*a*b^15*x^3 + 150*a^2*b^14*x)/b^20
```

3.503 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal. Leaf size=104

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{105ax}{16b^5}-\frac{21x^5}{16b^3(a+bx^2)}-\frac{3x^7}{8b^2(a+bx^2)^2}-\frac{x^9}{6b(a+bx^2)^3}+\frac{35x^3}{16b^4}$$

```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3)
 - (3*x^7)/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2))
+ (105*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```

Rubi [A] time = 0.158399, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{105a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{11/2}}-\frac{105ax}{16b^5}-\frac{21x^5}{16b^3(a+bx^2)}-\frac{3x^7}{8b^2(a+bx^2)^2}-\frac{x^9}{6b(a+bx^2)^3}+\frac{35x^3}{16b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-105*a*x)/(16*b^5) + (35*x^3)/(16*b^4) - x^9/(6*b*(a + b*x^2)^3)
 - (3*x^7)/(8*b^2*(a + b*x^2)^2) - (21*x^5)/(16*b^3*(a + b*x^2))
+ (105*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(11/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{105a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{11}{2}}}-\frac{x^9}{6b(a+bx^2)^3}-\frac{3x^7}{8b^2(a+bx^2)^2}-\frac{21x^5}{16b^3(a+bx^2)}+\frac{35x^3}{16b^4}-\frac{105\int a\,dx}{16b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 105*a**(3/2)*atan(sqrt(b)*x/sqrt(a))/(16*b**(11/2)) - x**9/(6*b*(
a + b*x**2)**3) - 3*x**7/(8*b**2*(a + b*x**2)**2) - 21*x**5/(16*b
**3*(a + b*x**2)) + 35*x**3/(16*b**4) - 105*Integral(a, x)/(16*b*
*5)
```

Mathematica [A] time = 0.0832548, size = 89, normalized size = 0.86

$$\frac{315a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\frac{\sqrt{b}x(-315a^4-840a^3bx^2-693a^2b^2x^4-144ab^3x^6+16b^4x^8)}{(a+bx^2)^3}}{48b^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((Sqrt[b]*x*(-315*a^4 - 840*a^3*b*x^2 - 693*a^2*b^2*x^4 - 144*a*b
^3*x^6 + 16*b^4*x^8))/(a + b*x^2)^3 + 315*a^(3/2)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(48*b^(11/2))
```

Maple [A] time = 0.015, size = 97, normalized size = 0.9

$$\frac{x^3}{3\,b^4}-4\,\frac{ax}{b^5}-\frac{55\,a^2x^5}{16\,b^3\,(bx^2+a)^3}-\frac{35\,a^3x^3}{6\,b^4\,(bx^2+a)^3}-\frac{41\,a^4x}{16\,b^5\,(bx^2+a)^3}+\frac{105\,a^2}{16\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/3*x^3/b^4-4*a*x/b^5-55/16/b^3*a^2/(b*x^2+a)^3*x^5-35/6/b^4*a^3/
(b*x^2+a)^3*x^3-41/16/b^5*a^4/(b*x^2+a)^3*x+105/16/b^5*a^2/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270931, size = 1, normalized size = 0.01

$$\left[\frac{32\,b^4x^9 - 288\,ab^3x^7 - 1386\,a^2b^2x^5 - 1680\,a^3bx^3 - 630\,a^4x + 315\,\left(ab^3x^6 + 3\,a^2b^2x^4 + 3\,a^3bx^2 + a^4\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}}{bx^2+a}\right.}{96\,(b^8x^6 + 3\,ab^7x^4 + 3\,a^2b^6x^2 + a^3b^5)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(32*b^4*x^9 - 288*a*b^3*x^7 - 1386*a^2*b^2*x^5 - 1680*a^3*b
*x^3 - 630*a^4*x + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 +
 a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))
/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5), 1/48*(16*b^4*
x^9 - 144*a*b^3*x^7 - 693*a^2*b^2*x^5 - 840*a^3*b*x^3 - 315*a^4*x
 + 315*(a*b^3*x^6 + 3*a^2*b^2*x^4 + 3*a^3*b*x^2 + a^4)*sqrt(a/b)*
arctan(x/sqrt(a/b)))/(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3
*b^5)]
```

Sympy [A] time = 2.78368, size = 155, normalized size = 1.49

$$-\frac{4ax}{b^5} - \frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32} + \frac{105\sqrt{-\frac{a^3}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^3}{b^{11}}}}{a}\right)}{32} - \frac{123a^4x + 280a^3bx^3 + 165a^2b^2x^5}{48a^3b^5 + 144a^2b^6x^2 + 144ab^7x^4 + 48b^8x^6} + \frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -4*a*x/b**5 - 105*sqrt(-a**3/b**11)*log(x - b**5*sqrt(-a**3/b**11
)/a)/32 + 105*sqrt(-a**3/b**11)*log(x + b**5*sqrt(-a**3/b**11)/a)
/32 - (123*a**4*x + 280*a**3*b*x**3 + 165*a**2*b**2*x**5)/(48*a**
3*b**5 + 144*a**2*b**6*x**2 + 144*a*b**7*x**4 + 48*b**8*x**6) + x
**3/(3*b**4)
```

GIAC/XCAS [A] time = 0.269506, size = 113, normalized size = 1.09

$$\frac{105\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^5} - \frac{165\,a^2b^2x^5 + 280\,a^3bx^3 + 123\,a^4x}{48\,(bx^2 + a)^3b^5} + \frac{b^8x^3 - 12\,ab^7x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 105/16*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/48*(165*a^2*
b^2*x^5 + 280*a^3*b*x^3 + 123*a^4*x)/((b*x^2 + a)^3*b^5) + 1/3*(b
^8*x^3 - 12*a*b^7*x)/b^12
```

3.504 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{35x^3}{48b^3(a+bx^2)}-\frac{7x^5}{24b^2(a+bx^2)^2}-\frac{x^7}{6b(a+bx^2)^3}+\frac{35x}{16b^4}$$

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a +
b*x^2)^2) - (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

Rubi [A] time = 0.135049, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}-\frac{35x^3}{48b^3(a+bx^2)}-\frac{7x^5}{24b^2(a+bx^2)^2}-\frac{x^7}{6b(a+bx^2)^3}+\frac{35x}{16b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (35*x)/(16*b^4) - x^7/(6*b*(a + b*x^2)^3) - (7*x^5)/(24*b^2*(a +
b*x^2)^2) - (35*x^3)/(48*b^3*(a + b*x^2)) - (35*Sqrt[a]*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(16*b^(9/2))
```

Rubi in Sympy [A] time = 30.9374, size = 85, normalized size = 0.91

$$-\frac{35\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{9}{2}}}-\frac{x^7}{6b(a+bx^2)^3}-\frac{7x^5}{24b^2(a+bx^2)^2}-\frac{35x^3}{48b^3(a+bx^2)}+\frac{35x}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -35*sqrt(a)*atan(sqrt(b)*x/sqrt(a))/(16*b**(9/2)) - x**7/(6*b*(a
+ b*x**2)**3) - 7*x**5/(24*b**2*(a + b*x**2)**2) - 35*x**3/(48*b*
*3*(a + b*x**2)) + 35*x/(16*b**4)
```

Mathematica [A] time = 0.0830746, size = 77, normalized size = 0.83

$$\frac{105a^3x+280a^2bx^3+231ab^2x^5+48b^3x^7}{48b^4\left(a+bx^2\right)^3}-\frac{35\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (105*a^3*x + 280*a^2*b*x^3 + 231*a*b^2*x^5 + 48*b^3*x^7)/(48*b^4*
(a + b*x^2)^3) - (35*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*b^(
9/2))
```

Maple [A] time = 0.014, size = 83, normalized size = 0.9

$$\frac{x}{b^4}+\frac{29\,ax^5}{16\,b^2\left(bx^2+a\right)^3}+\frac{17\,a^2x^3}{6\,b^3\left(bx^2+a\right)^3}+\frac{19\,a^3x}{16\,b^4\left(bx^2+a\right)^3}-\frac{35\,a}{16\,b^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] x/b^4+29/16/b^2*a/(b*x^2+a)^3*x^5+17/6/b^3*a^2/(b*x^2+a)^3*x^3+19
/16/b^4*a^3/(b*x^2+a)^3*x-35/16/b^4*a/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2787, size = 1, normalized size = 0.01

$$\left[\frac{96\,b^3x^7 + 462\,ab^2x^5 + 560\,a^2bx^3 + 210\,a^3x + 105\,\left(b^3x^6 + 3\,ab^2x^4 + 3\,a^2bx^2 + a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{96\,(b^7x^6 + 3\,ab^6x^4 + 3\,a^2b^5x^2 + a^3b^4)}, \frac{48\,b^3x^7 + 231\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b^3*x^7 + 462*a*b^2*x^5 + 560*a^2*b*x^3 + 210*a^3*x + 1
05*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*
x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^6 + 3*a*b^6*x^4
+ 3*a^2*b^5*x^2 + a^3*b^4), 1/48*(48*b^3*x^7 + 231*a*b^2*x^5 + 28
0*a^2*b*x^3 + 105*a^3*x - 105*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^
2 + a^3)*sqrt(a/b)*arctan(x/sqrt(a/b)))/(b^7*x^6 + 3*a*b^6*x^4 +
3*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [A] time = 2.62311, size = 131, normalized size = 1.41

$$\frac{35\sqrt{-\frac{a}{b^9}}\log\left(-b^4\sqrt{-\frac{a}{b^9}} + x\right)}{32} - \frac{35\sqrt{-\frac{a}{b^9}}\log\left(b^4\sqrt{-\frac{a}{b^9}} + x\right)}{32} + \frac{57a^3x + 136a^2bx^3 + 87ab^2x^5}{48a^3b^4 + 144a^2b^5x^2 + 144ab^6x^4 + 48b^7x^6} + \frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-a/b**9)*log(-b**4*sqrt(-a/b**9) + x)/32 - 35*sqrt(-a/b**
9)*log(b**4*sqrt(-a/b**9) + x)/32 + (57*a**3*x + 136*a**2*b*x**3
+ 87*a*b**2*x**5)/(48*a**3*b**4 + 144*a**2*b**5*x**2 + 144*a*b**6
*x**4 + 48*b**7*x**6) + x/b**4
```

GIAC/XCAS [A] time = 0.269393, size = 88, normalized size = 0.95

$$-\frac{35\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}b^4} + \frac{x}{b^4} + \frac{87\,ab^2x^5 + 136\,a^2bx^3 + 57\,a^3x}{48\,(bx^2 + a)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] -35/16*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + x/b^4 + 1/48*(87
*a*b^2*x^5 + 136*a^2*b*x^3 + 57*a^3*x)/((b*x^2 + a)^3*b^4)
```

3.505 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=83

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}-\frac{5x}{16b^3(a+bx^2)}-\frac{5x^3}{24b^2(a+bx^2)^2}-\frac{x^5}{6b(a+bx^2)^3}$$

```
[Out] -x^5/(6*b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)
/(16*b^3*(a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[
a]*b^(7/2))
```

Rubi [A] time = 0.11235, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}-\frac{5x}{16b^3(a+bx^2)}-\frac{5x^3}{24b^2(a+bx^2)^2}-\frac{x^5}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^5/(6*b*(a + b*x^2)^3) - (5*x^3)/(24*b^2*(a + b*x^2)^2) - (5*x)
/(16*b^3*(a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[
a]*b^(7/2))
```

Rubi in Sympy [A] time = 23.9564, size = 75, normalized size = 0.9

$$-\frac{x^5}{6b(a+bx^2)^3}-\frac{5x^3}{24b^2(a+bx^2)^2}-\frac{5x}{16b^3(a+bx^2)}+\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -x**5/(6*b*(a + b*x**2)**3) - 5*x**3/(24*b**2*(a + b*x**2)**2) -
5*x/(16*b**3*(a + b*x**2)) + 5*atan(sqrt(b)*x/sqrt(a))/(16*sqrt(a
)*b**(7/2))
```

Mathematica [A] time = 0.0688661, size = 66, normalized size = 0.8

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{a}b^{7/2}}-\frac{x\left(15a^2+40abx^2+33b^2x^4\right)}{48b^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(x*(15*a^2 + 40*a*b*x^2 + 33*b^2*x^4))/(48*b^3*(a + b*x^2)^3) +
(5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*Sqrt[a]*b^(7/2))
```

Maple [A] time = 0.013, size = 58, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^3}\left(-\frac{11\,x^5}{16\,b}-\frac{5\,ax^3}{6\,b^2}-\frac{5\,a^2x}{16\,b^3}\right)+\frac{5}{16\,b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (-11/16/b*x^5-5/6*a/b^2*x^3-5/16*a^2/b^3*x)/(b*x^2+a)^3+5/16/b^3/
(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274086, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\,a^{2}bx^{2}+a^{3}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)-2\left(33\,b^{2}x^{5}+40\,abx^{3}+15\,a^{2}x\right)\sqrt{-ab}}{96\left(b^{6}x^{6}+3\,ab^{5}x^{4}+3\,a^{2}b^{4}x^{2}+a^{3}b^{3}\right)\sqrt{-ab}},\frac{15\left(b^{3}x^{6}+3\,ab^{2}x^{4}+3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*log((2*a*b*
x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) - 2*(33*b^2*x^5 + 40*a*b
*x^3 + 15*a^2*x)*sqrt(-a*b))/((b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*
x^2 + a^3*b^3)*sqrt(-a*b)), 1/48*(15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*arctan(sqrt(a*b)*x/a) - (33*b^2*x^5 + 40*a*b*x^3
+ 15*a^2*x)*sqrt(a*b))/((b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
a^3*b^3)*sqrt(a*b))]
```

Sympy [A] time = 2.31076, size = 133, normalized size = 1.6

$$-\frac{5\sqrt{-\frac{1}{ab^{7}}}\log\left(-ab^{3}\sqrt{-\frac{1}{ab^{7}}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{ab^{7}}}\log\left(ab^{3}\sqrt{-\frac{1}{ab^{7}}}+x\right)}{32}-\frac{15a^{2}x+40abx^{3}+33b^{2}x^{5}}{48a^{3}b^{3}+144a^{2}b^{4}x^{2}+144ab^{5}x^{4}+48b^{6}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a*b**7))*log(-a*b**3*sqrt(-1/(a*b**7)) + x)/32 + 5*sq
rt(-1/(a*b**7))*log(a*b**3*sqrt(-1/(a*b**7)) + x)/32 - (15*a**2*x
 + 40*a*b*x**3 + 33*b**2*x**5)/(48*a**3*b**3 + 144*a**2*b**4*x**2
 + 144*a*b**5*x**4 + 48*b**6*x**6)
```

GIAC/XCAS [A] time = 0.270897, size = 76, normalized size = 0.92

$$\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}b^{3}}-\frac{33\,b^{2}x^{5}+40\,abx^{3}+15\,a^{2}x}{48\left(bx^{2}+a\right)^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/48*(33*b^2*x^5 + 4
0*a*b*x^3 + 15*a^2*x)/((b*x^2 + a)^3*b^3)
```

3.506 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=84

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2(a+bx^2)}-\frac{x}{8b^2(a+bx^2)^2}-\frac{x^3}{6b(a+bx^2)^3}$$

```
[Out] -x^3/(6*b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*
(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rubi [A] time = 0.106136, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{x}{16ab^2(a+bx^2)}-\frac{x}{8b^2(a+bx^2)^2}-\frac{x^3}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x^3/(6*b*(a + b*x^2)^3) - x/(8*b^2*(a + b*x^2)^2) + x/(16*a*b^2*
(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Rubi in Sympy [A] time = 21.8432, size = 70, normalized size = 0.83

$$-\frac{x^3}{6b(a+bx^2)^3}-\frac{x}{8b^2(a+bx^2)^2}+\frac{x}{16ab^2(a+bx^2)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -x**3/(6*b*(a + b*x**2)**3) - x/(8*b**2*(a + b*x**2)**2) + x/(16*
a*b**2*(a + b*x**2)) + atan(sqrt(b)*x/sqrt(a))/(16*a**(3/2)*b**(5
/2))
```

Mathematica [A] time = 0.0812664, size = 69, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{3/2}b^{5/2}}+\frac{-3a^2x-8abx^3+3b^2x^5}{48ab^2\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x - 8*a*b*x^3 + 3*b^2*x^5)/(48*a*b^2*(a + b*x^2)^3) + Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(3/2)*b^(5/2))
```

Maple [A] time = 0.012, size = 58, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^3}\left(\frac{x^5}{16\,a}-\frac{x^3}{6\,b}-\frac{ax}{16\,b^2}\right)+\frac{1}{16\,ab^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16/a*x^5-1/6/b*x^3-1/16*a/b^2*x)/(b*x^2+a)^3+1/16/a/b^2/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273506, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(3\,b^2x^5-8\,abx^3-3\,a^2x\right)\sqrt{-ab}}{96\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2\right)\sqrt{-ab}},\frac{3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2\right.}{48\left(ab^5x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*log((2*a*b*x
 + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(3*b^2*x^5 - 8*a*b*x^
3 - 3*a^2*x)*sqrt(-a*b))/((a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*
x^2 + a^4*b^2)*sqrt(-a*b)), 1/48*(3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3)*arctan(sqrt(a*b)*x/a) + (3*b^2*x^5 - 8*a*b*x^3 - 3
*a^2*x)*sqrt(a*b))/((a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 +
a^4*b^2)*sqrt(a*b))]
```

Sympy [A] time = 2.20087, size = 143, normalized size = 1.7

$$-\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^3b^5}}\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{32}+\frac{-3a^2x-8abx^3+3b^2x^5}{48a^4b^2+144a^3b^3x^2+144a^2b^4x^4+48ab^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/32
 + sqrt(-1/(a**3*b**5))*log(a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/3
2 + (-3*a**2*x - 8*a*b*x**3 + 3*b**2*x**5)/(48*a**4*b**2 + 144*a*
*3*b**3*x**2 + 144*a**2*b**4*x**4 + 48*a*b**5*x**6)
```

GIAC/XCAS [A] time = 0.270481, size = 84, normalized size = 1.

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}ab^2}+\frac{3\,b^2x^5-8\,abx^3-3\,a^2x}{48\,(bx^2+a)^3ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 1/48*(3*b^2*x^5 -
8*a*b*x^3 - 3*a^2*x)/((b*x^2 + a)^3*a*b^2)
```

3.507 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{x}{24ab\,(a+bx^2)^2}-\frac{x}{6b\,(a+bx^2)^3}$$

```
[Out] -x/(6*b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(
a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rubi [A] time = 0.100726, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{x}{24ab\,(a+bx^2)^2}-\frac{x}{6b\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -x/(6*b*(a + b*x^2)^3) + x/(24*a*b*(a + b*x^2)^2) + x/(16*a^2*b*(
a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Rubi in Sympy [A] time = 20.3255, size = 68, normalized size = 0.8

$$-\frac{x}{6b\,(a+bx^2)^3}+\frac{x}{24ab\,(a+bx^2)^2}+\frac{x}{16a^2b\,(a+bx^2)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -x/(6*b*(a + b*x**2)**3) + x/(24*a*b*(a + b*x**2)**2) + x/(16*a**
2*b*(a + b*x**2)) + atan(sqrt(b)*x/sqrt(a))/(16*a**(5/2)*b**(3/2)
)
```

Mathematica [A] time = 0.0755909, size = 69, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{5/2}b^{3/2}}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^2b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-3*a^2*x + 8*a*b*x^3 + 3*b^2*x^5)/(48*a^2*b*(a + b*x^2)^3) + Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/(16*a^(5/2)*b^(3/2))
```

Maple [A] time = 0.012, size = 58, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^3}\left(\frac{bx^5}{16\,a^2}+\frac{x^3}{6\,a}-\frac{x}{16\,b}\right)+\frac{1}{16\,a^2b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (1/16*b/a^2*x^5+1/6/a*x^3-1/16/b*x)/(b*x^2+a)^3+1/16/a^2/b/(a*b)^
(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.275281, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(3\,b^2x^5+8\,abx^3-3\,a^2x\right)\sqrt{-ab}}{96\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b\right)\sqrt{-ab}},\frac{3\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2\right.}{48\left(a^2b^4\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] [1/96*(3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*log((2*a*b*x
 + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(3*b^2*x^5 + 8*a*b*x^
3 - 3*a^2*x)*sqrt(-a*b))/((a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^
2*x^2 + a^5*b)*sqrt(-a*b)), 1/48*(3*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^
2*b*x^2 + a^3)*arctan(sqrt(a*b)*x/a) + (3*b^2*x^5 + 8*a*b*x^3 - 3
*a^2*x)*sqrt(a*b))/((a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2
+ a^5*b)*sqrt(a*b))]
```

Sympy [A] time = 2.15994, size = 139, normalized size = 1.64

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{\sqrt{-\frac{1}{a^5b^3}}\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{32}+\frac{-3a^2x+8abx^3+3b^2x^5}{48a^5b+144a^4b^2x^2+144a^3b^3x^4+48a^2b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 +
sqrt(-1/(a**5*b**3))*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/32 + (-
3*a**2*x + 8*a*b*x**3 + 3*b**2*x**5)/(48*a**5*b + 144*a**4*b**2*x
**2 + 144*a**3*b**3*x**4 + 48*a**2*b**4*x**6)
```

GIAC/XCAS [A] time = 0.271589, size = 84, normalized size = 0.99

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}a^2b}+\frac{3\,b^2x^5+8\,abx^3-3\,a^2x}{48\,(bx^2+a)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/48*(3*b^2*x^5 +
8*a*b*x^3 - 3*a^2*x)/((b*x^2 + a)^3*a^2*b)
```

3.508 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=79

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{x}{6a\left(a+bx^2\right)^3}$$

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*
a^3*(a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sq
rt[b])
```

Rubi [A] time = 0.0861244, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{x}{6a\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] x/(6*a*(a + b*x^2)^3) + (5*x)/(24*a^2*(a + b*x^2)^2) + (5*x)/(16*
a^3*(a + b*x^2)) + (5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sq
rt[b])
```

Rubi in Sympy [A] time = 22.887, size = 71, normalized size = 0.9

$$\frac{x}{6a\left(a+bx^2\right)^3}+\frac{5x}{24a^2\left(a+bx^2\right)^2}+\frac{5x}{16a^3\left(a+bx^2\right)}+\frac{5\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{\frac{7}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] x/(6*a*(a + b*x**2)**3) + 5*x/(24*a**2*(a + b*x**2)**2) + 5*x/(16
*a**3*(a + b*x**2)) + 5*atan(sqrt(b)*x/sqrt(a))/(16*a**(7/2)*sqrt
(b))
```

Mathematica [A] time = 0.0638648, size = 66, normalized size = 0.84

$$\frac{5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{7/2}\sqrt{b}}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^3\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2),x]
```

```
[Out] (33*a^2*x + 40*a*b*x^3 + 15*b^2*x^5)/(48*a^3*(a + b*x^2)^3) + (5*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(7/2)*Sqrt[b])
```

Maple [A] time = 0.006, size = 66, normalized size = 0.8

$$\frac{x}{6\,a\left(bx^2+a\right)^3}+\frac{5\,x}{24\,a^2\left(bx^2+a\right)^2}+\frac{5\,x}{16\,a^3\left(bx^2+a\right)}+\frac{5}{16\,a^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 1/6*x/a/(b*x^2+a)^3+5/24*x/a^2/(b*x^2+a)^2+5/16*x/a^3/(b*x^2+a)+5
/16/a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273967, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(15\,b^2x^5+40\,abx^3+33\,a^2x\right)\sqrt{-ab}}{96\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)\sqrt{-ab}},\frac{15\left(b^3x^6+3\,ab^2x^4+3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*log((2*a*b*
x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(15*b^2*x^5 + 40*a*b
*x^3 + 33*a^2*x)*sqrt(-a*b))/((a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^
5*b*x^2 + a^6)*sqrt(-a*b)), 1/48*(15*(b^3*x^6 + 3*a*b^2*x^4 + 3*a
^2*b*x^2 + a^3)*arctan(sqrt(a*b)*x/a) + (15*b^2*x^5 + 40*a*b*x^3
+ 33*a^2*x)*sqrt(a*b))/((a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^
2 + a^6)*sqrt(a*b))]
```

Sympy [A] time = 2.20807, size = 129, normalized size = 1.63

$$-\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{5\sqrt{-\frac{1}{a^7b}}\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{32}+\frac{33a^2x+40abx^3+15b^2x^5}{48a^6+144a^5bx^2+144a^4b^2x^4+48a^3b^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -5*sqrt(-1/(a**7*b))*log(-a**4*sqrt(-1/(a**7*b)) + x)/32 + 5*sqrt
(-1/(a**7*b))*log(a**4*sqrt(-1/(a**7*b)) + x)/32 + (33*a**2*x + 4
0*a*b*x**3 + 15*b**2*x**5)/(48*a**6 + 144*a**5*b*x**2 + 144*a**4*
b**2*x**4 + 48*a**3*b**3*x**6)
```

GIAC/XCAS [A] time = 0.269367, size = 76, normalized size = 0.96

$$\frac{5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{ab}a^3}+\frac{15\,b^2x^5+40\,abx^3+33\,a^2x}{48\,(bx^2+a)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2),x, algorithm="giac")
```

```
[Out] 5/16*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/48*(15*b^2*x^5 + 4
0*a*b*x^3 + 33*a^2*x)/((b*x^2 + a)^3*a^3)
```

3.509 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=95

$$-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{35}{48a^3x(a+bx^2)}+\frac{7}{24a^2x(a+bx^2)^2}+\frac{1}{6ax(a+bx^2)^3}$$

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2
)^2) + 35/(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(16*a^(9/2))
```

Rubi [A] time = 0.138077, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{35}{16a^4x}+\frac{35}{48a^3x(a+bx^2)}+\frac{7}{24a^2x(a+bx^2)^2}+\frac{1}{6ax(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x) + 1/(6*a*x*(a + b*x^2)^3) + 7/(24*a^2*x*(a + b*x^2
)^2) + 35/(48*a^3*x*(a + b*x^2)) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(16*a^(9/2))
```

Rubi in Sympy [A] time = 32.7319, size = 82, normalized size = 0.86

$$\frac{1}{6ax(a+bx^2)^3}+\frac{7}{24a^2x(a+bx^2)^2}+\frac{35}{48a^3x(a+bx^2)}-\frac{35}{16a^4x}-\frac{35\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 1/(6*a*x*(a + b*x**2)**3) + 7/(24*a**2*x*(a + b*x**2)**2) + 35/(4
8*a**3*x*(a + b*x**2)) - 35/(16*a**4*x) - 35*sqrt(b)*atan(sqrt(b)
*x/sqrt(a))/(16*a**(9/2))
```

Mathematica [A] time = 0.0779194, size = 79, normalized size = 0.83

$$-\frac{35\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{9/2}}-\frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^4x\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -(48*a^3 + 231*a^2*b*x^2 + 280*a*b^2*x^4 + 105*b^3*x^6)/(48*a^4*x
*(a + b*x^2)^3) - (35*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^
(9/2))
```

Maple [A] time = 0.017, size = 86, normalized size = 0.9

$$-\frac{19\,b^3x^5}{16\,a^4\left(bx^2+a\right)^3}-\frac{17\,b^2x^3}{6\,a^3\left(bx^2+a\right)^3}-\frac{29\,bx}{16\,a^2\left(bx^2+a\right)^3}-\frac{35\,b}{16\,a^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -19/16/a^4*b^3/(b*x^2+a)^3*x^5-17/6/a^3*b^2/(b*x^2+a)^3*x^3-29/16
/a^2*b/(b*x^2+a)^3*x-35/16/a^4*b/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/
2))-1/a^4/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.277659, size = 1, normalized size = 0.01

$$\left[-\frac{210\,b^3x^6+560\,ab^2x^4+462\,a^2bx^2+96\,a^3-105\,\left(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{96\,(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x)},\right.$$
$$\left.-\frac{105\,b^3x^6+280\,ab^2x^4+231\,a^2bx^2+48\,a^3+105\,\left(b^3x^7+3\,ab^2x^5+3\,a^2bx^3+a^3x\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)}{48\,(a^4b^3x^7+3\,a^5b^2x^5+3\,a^6bx^3+a^7x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^2),x, algorithm="fricas")
```

```
[Out] [-1/96*(210*b^3*x^6 + 560*a*b^2*x^4 + 462*a^2*b*x^2 + 96*a^3 - 10
5*(b^3*x^7 + 3*a*b^2*x^5 + 3*a^2*b*x^3 + a^3*x)*sqrt(-b/a)*log((b
*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b^3*x^7 + 3*a^5*b
^2*x^5 + 3*a^6*b*x^3 + a^7*x), -1/48*(105*b^3*x^6 + 280*a*b^2*x^4
 + 231*a^2*b*x^2 + 48*a^3 + 105*(b^3*x^7 + 3*a*b^2*x^5 + 3*a^2*b*
x^3 + a^3*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^4*b^3*x^7 +
3*a^5*b^2*x^5 + 3*a^6*b*x^3 + a^7*x)]
```

Sympy [A] time = 3.62115, size = 138, normalized size = 1.45

$$\frac{35\sqrt{-\frac{b}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}-\frac{35\sqrt{-\frac{b}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}}{b}+x\right)}{32}-\frac{48a^3+231a^2bx^2+280ab^2x^4+105b^3x^6}{48a^7x+144a^6bx^3+144a^5b^2x^5+48a^4b^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 35*sqrt(-b/a**9)*log(-a**5*sqrt(-b/a**9)/b + x)/32 - 35*sqrt(-b/a
**9)*log(a**5*sqrt(-b/a**9)/b + x)/32 - (48*a**3 + 231*a**2*b*x**
2 + 280*a*b**2*x**4 + 105*b**3*x**6)/(48*a**7*x + 144*a**6*b*x**3
 + 144*a**5*b**2*x**5 + 48*a**4*b**3*x**7)
```

GIAC/XCAS [A] time = 0.271286, size = 92, normalized size = 0.97

$$-\frac{35\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^4}-\frac{1}{a^4x}-\frac{57\,b^3x^5+136\,ab^2x^3+87\,a^2bx}{48\,(bx^2+a)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^2),x, algorithm="giac")
```

```
[Out] -35/16*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/(a^4*x) - 1/48
*(57*b^3*x^5 + 136*a*b^2*x^3 + 87*a^2*b*x)/((b*x^2 + a)^3*a^4)
```

3.510 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=106

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{21}{16a^3x^3(a+bx^2)}+\frac{3}{8a^2x^3(a+bx^2)^2}+\frac{1}{6ax^3(a+bx^2)^3}$$

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3)
 + 3/(8*a^2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (1
05*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rubi [A] time = 0.16781, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{105b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{11/2}}+\frac{105b}{16a^5x}-\frac{35}{16a^4x^3}+\frac{21}{16a^3x^3(a+bx^2)}+\frac{3}{8a^2x^3(a+bx^2)^2}+\frac{1}{6ax^3(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -35/(16*a^4*x^3) + (105*b)/(16*a^5*x) + 1/(6*a*x^3*(a + b*x^2)^3)
 + 3/(8*a^2*x^3*(a + b*x^2)^2) + 21/(16*a^3*x^3*(a + b*x^2)) + (1
05*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(11/2))
```

Rubi in Sympy [A] time = 39.4297, size = 99, normalized size = 0.93

$$\frac{1}{6ax^3(a+bx^2)^3}+\frac{3}{8a^2x^3(a+bx^2)^2}+\frac{21}{16a^3x^3(a+bx^2)}-\frac{35}{16a^4x^3}+\frac{105b}{16a^5x}+\frac{105b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 1/(6*a*x**3*(a + b*x**2)**3) + 3/(8*a**2*x**3*(a + b*x**2)**2) +
21/(16*a**3*x**3*(a + b*x**2)) - 35/(16*a**4*x**3) + 105*b/(16*a*
*5*x) + 105*b**(3/2)*atan(sqrt(b)*x/sqrt(a))/(16*a**(11/2))
```

Mathematica [A] time = 0.0882574, size = 91, normalized size = 0.86

$$\frac{\frac{\sqrt{a}(-16a^4+144a^3bx^2+693a^2b^2x^4+840ab^3x^6+315b^4x^8)}{x^3(a+bx^2)^3}+315b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{48a^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] ((Sqrt[a]*(-16*a^4 + 144*a^3*b*x^2 + 693*a^2*b^2*x^4 + 840*a*b^3*
x^6 + 315*b^4*x^8))/(x^3*(a + b*x^2)^3) + 315*b^(3/2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(48*a^(11/2))
```

Maple [A] time = 0.02, size = 99, normalized size = 0.9

$$\frac{41\,b^4x^5}{16\,a^5\,(bx^2+a)^3}+\frac{35\,b^3x^3}{6\,a^4\,(bx^2+a)^3}+\frac{55\,b^2x}{16\,a^3\,(bx^2+a)^3}+\frac{105\,b^2}{16\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{3\,a^4x^3}+4\,\frac{b}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 41/16/a^5*b^4/(b*x^2+a)^3*x^5+35/6/a^4*b^3/(b*x^2+a)^3*x^3+55/16/
a^3*b^2/(b*x^2+a)^3*x+105/16/a^5*b^2/(a*b)^(1/2)*arctan(x*b/(a*b)
^(1/2))-1/3/a^4/x^3+4*b/a^5/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279044, size = 1, normalized size = 0.01

$$\left[\frac{630\,b^4x^8 + 1680\,ab^3x^6 + 1386\,a^2b^2x^4 + 288\,a^3bx^2 - 32\,a^4 + 315\,\left(b^4x^9 + 3\,ab^3x^7 + 3\,a^2b^2x^5 + a^3bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}}{bx^2+a}\right.}{96\,\left(a^5b^3x^9 + 3\,a^6b^2x^7 + 3\,a^7bx^5 + a^8x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^4),x, algorithm="fricas")
```

```
[Out] [1/96*(630*b^4*x^8 + 1680*a*b^3*x^6 + 1386*a^2*b^2*x^4 + 288*a^3*
b*x^2 - 32*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3
*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
))/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 + 3*a^7*b*x^5 + a^8*x^3), 1/48*(3
15*b^4*x^8 + 840*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16
*a^4 + 315*(b^4*x^9 + 3*a*b^3*x^7 + 3*a^2*b^2*x^5 + a^3*b*x^3)*sq
rt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^5*b^3*x^9 + 3*a^6*b^2*x^7 +
 3*a^7*b*x^5 + a^8*x^3)]
```

Sympy [A] time = 6.30935, size = 162, normalized size = 1.53

$$-\frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2} + x\right)}{32} + \frac{105\sqrt{-\frac{b^3}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}}{b^2} + x\right)}{32} + \frac{-16a^4 + 144a^3bx^2 + 693a^2b^2x^4 + 840ab^3x^6 + 315b^4x^8}{48a^8x^3 + 144a^7bx^5 + 144a^6b^2x^7 + 48a^5b^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -105*sqrt(-b**3/a**11)*log(-a**6*sqrt(-b**3/a**11)/b**2 + x)/32 +
 105*sqrt(-b**3/a**11)*log(a**6*sqrt(-b**3/a**11)/b**2 + x)/32 +
(-16*a**4 + 144*a**3*b*x**2 + 693*a**2*b**2*x**4 + 840*a*b**3*x**
6 + 315*b**4*x**8)/(48*a**8*x**3 + 144*a**7*b*x**5 + 144*a**6*b**
2*x**7 + 48*a**5*b**3*x**9)
```

GIAC/XCAS [A] time = 0.270933, size = 111, normalized size = 1.05

$$\frac{105\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{ab}a^5} + \frac{315\,b^4x^8 + 840\,ab^3x^6 + 693\,a^2b^2x^4 + 144\,a^3bx^2 - 16\,a^4}{48\,\left(bx^3 + ax\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^4),x, algorithm="giac")
```

```
[Out] 105/16*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/48*(315*b^4*
x^8 + 840*a*b^3*x^6 + 693*a^2*b^2*x^4 + 144*a^3*b*x^2 - 16*a^4)/(
(b*x^3 + a*x)^3*a^5)
```

3.511 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=119

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{231b^2}{16a^6x}+\frac{77b}{16a^5x^3}-\frac{231}{80a^4x^5}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) +
1/(6*a*x^5*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(1
6*a^3*x^5*(a + b*x^2)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(16*a^(13/2))
```

Rubi [A] time = 0.198518, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{231b^2}{16a^6x}+\frac{77b}{16a^5x^3}-\frac{231}{80a^4x^5}+\frac{33}{16a^3x^5\left(a+bx^2\right)}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -231/(80*a^4*x^5) + (77*b)/(16*a^5*x^3) - (231*b^2)/(16*a^6*x) +
1/(6*a*x^5*(a + b*x^2)^3) + 11/(24*a^2*x^5*(a + b*x^2)^2) + 33/(1
6*a^3*x^5*(a + b*x^2)) - (231*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(16*a^(13/2))
```

Rubi in Sympy [A] time = 46.3349, size = 112, normalized size = 0.94

$$\frac{1}{6ax^5\left(a+bx^2\right)^3}+\frac{11}{24a^2x^5\left(a+bx^2\right)^2}+\frac{33}{16a^3x^5\left(a+bx^2\right)}-\frac{231}{80a^4x^5}+\frac{77b}{16a^5x^3}-\frac{231b^2}{16a^6x}-\frac{231b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 1/(6*a*x**5*(a + b*x**2)**3) + 11/(24*a**2*x**5*(a + b*x**2)**2)
+ 33/(16*a**3*x**5*(a + b*x**2)) - 231/(80*a**4*x**5) + 77*b/(16*
a**5*x**3) - 231*b**2/(16*a**6*x) - 231*b**(5/2)*atan(sqrt(b)*x/s
```

```
qrt(a))/(16*a**(13/2))
```

Mathematica [A] time = 0.09485, size = 101, normalized size = 0.85

$$-\frac{231b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16a^{13/2}}-\frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4x^8+3465b^5x^{10}}{240a^6x^5\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -(48*a^5 - 176*a^4*b*x^2 + 1584*a^3*b^2*x^4 + 7623*a^2*b^3*x^6 +
9240*a*b^4*x^8 + 3465*b^5*x^10)/(240*a^6*x^5*(a + b*x^2)^3) - (23
1*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(16*a^(13/2))
```

Maple [A] time = 0.02, size = 110, normalized size = 0.9

$$-\frac{71\,b^5x^5}{16\,a^6\left(bx^2+a\right)^3}-\frac{59\,b^4x^3}{6\,a^5\left(bx^2+a\right)^3}-\frac{89\,b^3x}{16\,a^4\left(bx^2+a\right)^3}-\frac{231\,b^3}{16\,a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{5\,a^4x^5}-10\,\frac{b^2}{a^6x}+\frac{4\,b}{3\,a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -71/16/a^6*b^5/(b*x^2+a)^3*x^5-59/6/a^5*b^4/(b*x^2+a)^3*x^3-89/16
/a^4*b^3/(b*x^2+a)^3*x-231/16/a^6*b^3/(a*b)^(1/2)*arctan(x*b/(a*b
)^(1/2))-1/5/a^4/x^5-10*b^2/a^6/x+4/3*b/a^5/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27934, size = 1, normalized size = 0.01

$$\left[-\frac{6930\, b^5 x^{10} + 18480\, a b^4 x^8 + 15246\, a^2 b^3 x^6 + 3168\, a^3 b^2 x^4 - 352\, a^4 b x^2 + 96\, a^5 - 3465 \left(b^5 x^{11} + 3\, a b^4 x^9 + 3\, a^2 b^3 x^7 + a^3 b^2 x^5\right.}{480 \left(a^6 b^3 x^{11} + 3\, a^7 b^2 x^9 + 3\, a^8 b x^7 + a^9 x^5\right)}\right.$$

$$-\frac{3465\, b^5 x^{10} + 9240\, a b^4 x^8 + 7623\, a^2 b^3 x^6 + 1584\, a^3 b^2 x^4 - 176\, a^4 b x^2 + 48\, a^5 + 3465 \left(b^5 x^{11} + 3\, a b^4 x^9 + 3\, a^2 b^3 x^7 + a^3 b^2 x^5\right) \sqrt{}}{240 \left(a^6 b^3 x^{11} + 3\, a^7 b^2 x^9 + 3\, a^8 b x^7 + a^9 x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^6),x, algorithm="fricas")
```

```
[Out] [-1/480*(6930*b^5*x^10 + 18480*a*b^4*x^8 + 15246*a^2*b^3*x^6 + 31
68*a^3*b^2*x^4 - 352*a^4*b*x^2 + 96*a^5 - 3465*(b^5*x^11 + 3*a*b^
4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3
*a^8*b*x^7 + a^9*x^5), -1/240*(3465*b^5*x^10 + 9240*a*b^4*x^8 + 7
623*a^2*b^3*x^6 + 1584*a^3*b^2*x^4 - 176*a^4*b*x^2 + 48*a^5 + 346
5*(b^5*x^11 + 3*a*b^4*x^9 + 3*a^2*b^3*x^7 + a^3*b^2*x^5)*sqrt(b/a
)*arctan(b*x/(a*sqrt(b/a))))/(a^6*b^3*x^11 + 3*a^7*b^2*x^9 + 3*a^
8*b*x^7 + a^9*x^5)]
```

Sympy [A] time = 12.6902, size = 173, normalized size = 1.45

$$\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{231\sqrt{-\frac{b^5}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}}{b^3}+x\right)}{32}-\frac{48a^5-176a^4bx^2+1584a^3b^2x^4+7623a^2b^3x^6+9240ab^4x^8+3465b^5x^{10}}{240a^9x^5+720a^8bx^7+720a^7b^2x^9+240a^6b^3x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 231*sqrt(-b**5/a**13)*log(-a**7*sqrt(-b**5/a**13)/b**3 + x)/32 -
231*sqrt(-b**5/a**13)*log(a**7*sqrt(-b**5/a**13)/b**3 + x)/32 - (
```

```
48*a**5 - 176*a**4*b*x**2 + 1584*a**3*b**2*x**4 + 7623*a**2*b**3*
x**6 + 9240*a*b**4*x**8 + 3465*b**5*x**10)/(240*a**9*x**5 + 720*a
**8*b*x**7 + 720*a**7*b**2*x**9 + 240*a**6*b**3*x**11)
```

GIAC/XCAS [A] time = 0.270256, size = 126, normalized size = 1.06

$$-\frac{231\, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{16\, \sqrt{ab} a^6} - \frac{213\, b^5 x^5 + 472\, ab^4 x^3 + 267\, a^2 b^3 x}{48\, (bx^2 + a)^3 a^6} - \frac{150\, b^2 x^4 - 20\, abx^2 + 3\, a^2}{15\, a^6 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*x^6),x, algorithm="giac")
```

```
[Out] -231/16*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/48*(213*b^5
*x^5 + 472*a*b^4*x^3 + 267*a^2*b^3*x)/((b*x^2 + a)^3*a^6) - 1/15*
(150*b^2*x^4 - 20*a*b*x^2 + 3*a^2)/(a^6*x^5)
```

3.512 $\int \frac{x^{15}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=133

$$\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

```
[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^
6)/(8*b^8*(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^
4)/(4*b^8*(a + b*x^2)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2
*Log[a + b*x^2])/(2*b^8)
```

Rubi [A] time = 0.308286, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{x^4}{4b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3*a*x^2)/b^7 + x^4/(4*b^6) + a^7/(10*b^8*(a + b*x^2)^5) - (7*a^
6)/(8*b^8*(a + b*x^2)^4) + (7*a^5)/(2*b^8*(a + b*x^2)^3) - (35*a^
4)/(4*b^8*(a + b*x^2)^2) + (35*a^3)/(2*b^8*(a + b*x^2)) + (21*a^2
*Log[a + b*x^2])/(2*b^8)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^7}{10b^8(a+bx^2)^5}-\frac{7a^6}{8b^8(a+bx^2)^4}+\frac{7a^5}{2b^8(a+bx^2)^3}-\frac{35a^4}{4b^8(a+bx^2)^2}+\frac{35a^3}{2b^8(a+bx^2)}+\frac{21a^2\log(a+bx^2)}{2b^8}-\frac{3ax^2}{b^7}+\frac{\int^{x^2} x\,dx}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**7/(10*b**8*(a + b*x**2)**5) - 7*a**6/(8*b**8*(a + b*x**2)**4)
+ 7*a**5/(2*b**8*(a + b*x**2)**3) - 35*a**4/(4*b**8*(a + b*x**2)*
*2) + 35*a**3/(2*b**8*(a + b*x**2)) + 21*a**2*log(a + b*x**2)/(2*
b**8) - 3*a*x**2/b**7 + Integral(x, (x, x**2))/(2*b**6)
```

Mathematica [A] time = 0.0462715, size = 114, normalized size = 0.86

$$\frac{459a^7+1875a^6bx^2+2700a^5b^2x^4+1300a^4b^3x^6-400a^3b^4x^8-500a^2b^5x^{10}+420a^2\left(a+bx^2\right)^5\log\left(a+bx^2\right)-70ab^6x^{12}+1}{40b^8\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^15/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (459*a^7 + 1875*a^6*b*x^2 + 2700*a^5*b^2*x^4 + 1300*a^4*b^3*x^6 -
 400*a^3*b^4*x^8 - 500*a^2*b^5*x^10 - 70*a*b^6*x^12 + 10*b^7*x^14
 + 420*a^2*(a + b*x^2)^5*Log[a + b*x^2])/(40*b^8*(a + b*x^2)^5)
```

Maple [A] time = 0.024, size = 120, normalized size = 0.9

$$-3\,{\frac{a{x}^{2}}{{b}^{7}}}+{\frac{{x}^{4}}{4\,{b}^{6}}}+{\frac{{a}^{7}}{10\,{b}^{8}\left(b{x}^{2}+a\right)^{5}}}-{\frac{7\,{a}^{6}}{8\,{b}^{8}\left(b{x}^{2}+a\right)^{4}}}+{\frac{7\,{a}^{5}}{2\,{b}^{8}\left(b{x}^{2}+a\right)^{3}}}-{\frac{35\,{a}^{4}}{4\,{b}^{8}\left(b{x}^{2}+a\right)^{2}}}+{\frac{35\,{a}^{3}}{2\,{b}^{8}\left(b{x}^{2}+a\right)}}+{\frac{21\,{a}^{2}\ln\left(b{x}^{2}+a\right)}{2\,{b}^{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^15/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -3*a*x^2/b^7+1/4*x^4/b^6+1/10*a^7/b^8/(b*x^2+a)^5-7/8*a^6/b^8/(b*
x^2+a)^4+7/2*a^5/b^8/(b*x^2+a)^3-35/4*a^4/b^8/(b*x^2+a)^2+35/2*a^
3/b^8/(b*x^2+a)+21/2*a^2*ln(b*x^2+a)/b^8
```

Maxima [A] time = 0.714016, size = 193, normalized size = 1.45

$$\frac{700\,a^{3}b^{4}x^{8}+2450\,a^{4}b^{3}x^{6}+3290\,a^{5}b^{2}x^{4}+1995\,a^{6}bx^{2}+459\,a^{7}}{40\,\left(b^{13}x^{10}+5\,ab^{12}x^{8}+10\,a^{2}b^{11}x^{6}+10\,a^{3}b^{10}x^{4}+5\,a^{4}b^{9}x^{2}+a^{5}b^{8}\right)}+\frac{21\,a^{2}\log\left(bx^{2}+a\right)}{2\,b^{8}}+\frac{bx^{4}-12\,ax^{2}}{4\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^15/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/40*(700*a^3*b^4*x^8 + 2450*a^4*b^3*x^6 + 3290*a^5*b^2*x^4 + 199
5*a^6*b*x^2 + 459*a^7)/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*b^11*x^
6 + 10*a^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8) + 21/2*a^2*log(b*x
^2 + a)/b^8 + 1/4*(b*x^4 - 12*a*x^2)/b^7
```

Fricas [A] time = 0.265403, size = 274, normalized size = 2.06

$$\frac{10\,b^7x^{14} - 70\,ab^6x^{12} - 500\,a^2b^5x^{10} - 400\,a^3b^4x^8 + 1300\,a^4b^3x^6 + 2700\,a^5b^2x^4 + 1875\,a^6bx^2 + 459\,a^7 + 420\,\left(a^2b^5x^{10} + 5\,a^3b\right.}{40\,\left(b^{13}x^{10} + 5\,ab^{12}x^8 + 10\,a^2b^{11}x^6 + 10\,a^3b^{10}x^4 + 5\,a^4b^9x^2 + a^5b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^15/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/40*(10*b^7*x^14 - 70*a*b^6*x^12 - 500*a^2*b^5*x^10 - 400*a^3*b^
4*x^8 + 1300*a^4*b^3*x^6 + 2700*a^5*b^2*x^4 + 1875*a^6*b*x^2 + 45
9*a^7 + 420*(a^2*b^5*x^10 + 5*a^3*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a
^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*log(b*x^2 + a))/(b^13*x^10 + 5*a*
b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^
5*b^8)
```

Sympy [A] time = 5.28281, size = 150, normalized size = 1.13

$$\frac{21a^2\log\left(a + bx^2\right)}{2b^8} - \frac{3ax^2}{b^7} + \frac{459a^7 + 1995a^6bx^2 + 3290a^5b^2x^4 + 2450a^4b^3x^6 + 700a^3b^4x^8}{40a^5b^8 + 200a^4b^9x^2 + 400a^3b^{10}x^4 + 400a^2b^{11}x^6 + 200ab^{12}x^8 + 40b^{13}x^{10}} + \frac{x^4}{4b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**15/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*log(a + b*x**2)/(2*b**8) - 3*a*x**2/b**7 + (459*a**7 + 19
95*a**6*b*x**2 + 3290*a**5*b**2*x**4 + 2450*a**4*b**3*x**6 + 700*
a**3*b**4*x**8)/(40*a**5*b**8 + 200*a**4*b**9*x**2 + 400*a**3*b**
10*x**4 + 400*a**2*b**11*x**6 + 200*a*b**12*x**8 + 40*b**13*x**10
) + x**4/(4*b**6)
```

GIAC/XCAS [A] time = 0.272786, size = 153, normalized size = 1.15

$$\frac{21\,a^2\ln\left(\left|bx^2+a\right|\right)}{2\,b^8}+\frac{b^6x^4-12\,ab^5x^2}{4\,b^{12}}-\frac{959\,a^2b^5x^{10}+4095\,a^3b^4x^8+7140\,a^4b^3x^6+6300\,a^5b^2x^4+2800\,a^6bx^2+500\,a^7}{40\,(bx^2+a)^5b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^15/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 21/2*a^2*ln(abs(b*x^2 + a))/b^8 + 1/4*(b^6*x^4 - 12*a*b^5*x^2)/b^
12 - 1/40*(959*a^2*b^5*x^10 + 4095*a^3*b^4*x^8 + 7140*a^4*b^3*x^6
 + 6300*a^5*b^2*x^4 + 2800*a^6*b*x^2 + 500*a^7)/((b*x^2 + a)^5*b^
8)
```

3.513 $\int \frac{x^{13}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=118

$$-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{2b^7(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^7}+\frac{x^2}{2b^6}$$

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*
x^2)^4) - (5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2
)^2) - (15*a^2)/(2*b^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rubi [A] time = 0.255735, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{2b^7(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^7}+\frac{x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] x^2/(2*b^6) - a^6/(10*b^7*(a + b*x^2)^5) + (3*a^5)/(4*b^7*(a + b*
x^2)^4) - (5*a^4)/(2*b^7*(a + b*x^2)^3) + (5*a^3)/(b^7*(a + b*x^2
)^2) - (15*a^2)/(2*b^7*(a + b*x^2)) - (3*a*Log[a + b*x^2])/b^7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^6}{10b^7(a+bx^2)^5}+\frac{3a^5}{4b^7(a+bx^2)^4}-\frac{5a^4}{2b^7(a+bx^2)^3}+\frac{5a^3}{b^7(a+bx^2)^2}-\frac{15a^2}{2b^7(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^7}+\frac{b^6\int^{x^2}\frac{1}{b^{12}}\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -a**6/(10*b**7*(a + b*x**2)**5) + 3*a**5/(4*b**7*(a + b*x**2)**4)
 - 5*a**4/(2*b**7*(a + b*x**2)**3) + 5*a**3/(b**7*(a + b*x**2)**2
) - 15*a**2/(2*b**7*(a + b*x**2)) - 3*a*log(a + b*x**2)/b**7 + b*
```

```
*6*Integral(b**(-12), (x, x**2))/2
```

Mathematica [A] time = 0.0507, size = 101, normalized size = 0.86

$$-\frac{87a^6+375a^5bx^2+600a^4b^2x^4+400a^3b^3x^6+50a^2b^4x^8-50ab^5x^{10}+60a\left(a+bx^2\right)^5\log\left(a+bx^2\right)-10b^6x^{12}}{20b^7\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^13/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(87*a^6 + 375*a^5*b*x^2 + 600*a^4*b^2*x^4 + 400*a^3*b^3*x^6 + 50
*a^2*b^4*x^8 - 50*a*b^5*x^10 - 10*b^6*x^12 + 60*a*(a + b*x^2)^5*L
og[a + b*x^2])/(20*b^7*(a + b*x^2)^5)
```

Maple [A] time = 0.02, size = 109, normalized size = 0.9

$$\frac{x^2}{2\,b^6}-\frac{a^6}{10\,b^7\left(bx^2+a\right)^5}+\frac{3\,a^5}{4\,b^7\left(bx^2+a\right)^4}-\frac{5\,a^4}{2\,b^7\left(bx^2+a\right)^3}+5\,\frac{a^3}{b^7\left(bx^2+a\right)^2}-\frac{15\,a^2}{2\,b^7\left(bx^2+a\right)}-3\,\frac{a\ln\left(bx^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^13/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*x^2/b^6-1/10*a^6/b^7/(b*x^2+a)^5+3/4*a^5/b^7/(b*x^2+a)^4-5/2*
a^4/b^7/(b*x^2+a)^3+5*a^3/b^7/(b*x^2+a)^2-15/2*a^2/b^7/(b*x^2+a)-
3*a*ln(b*x^2+a)/b^7
```

Maxima [A] time = 0.699634, size = 178, normalized size = 1.51

$$-\frac{150\,a^2b^4x^8+500\,a^3b^3x^6+650\,a^4b^2x^4+385\,a^5bx^2+87\,a^6}{20\left(b^{12}x^{10}+5\,ab^{11}x^8+10\,a^2b^{10}x^6+10\,a^3b^9x^4+5\,a^4b^8x^2+a^5b^7\right)}+\frac{x^2}{2\,b^6}-\frac{3\,a\log\left(bx^2+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/20*(150*a^2*b^4*x^8 + 500*a^3*b^3*x^6 + 650*a^4*b^2*x^4 + 385*
a^5*b*x^2 + 87*a^6)/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 +
```

```
 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7) + 1/2*x^2/b^6 - 3*a*lo
g(b*x^2 + a)/b^7
```

Fricas [A] time = 0.26371, size = 257, normalized size = 2.18

$$\frac{10\,b^6x^{12} + 50\,ab^5x^{10} - 50\,a^2b^4x^8 - 400\,a^3b^3x^6 - 600\,a^4b^2x^4 - 375\,a^5bx^2 - 87\,a^6 - 60\,\left(ab^5x^{10} + 5\,a^2b^4x^8 + 10\,a^3b^3x^6 + 10\,a}{20\,(b^{12}x^{10} + 5\,ab^{11}x^8 + 10\,a^2b^{10}x^6 + 10\,a^3b^9x^4 + 5\,a^4b^8x^2 + a^5b^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^6*x^12 + 50*a*b^5*x^10 - 50*a^2*b^4*x^8 - 400*a^3*b^3*
x^6 - 600*a^4*b^2*x^4 - 375*a^5*b*x^2 - 87*a^6 - 60*(a*b^5*x^10 +
 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4 + 5*a^5*b*x^2 +
a^6)*log(b*x^2 + a))/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6
+ 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)
```

Sympy [A] time = 5.14548, size = 136, normalized size = 1.15

$$-\frac{3a\log\left(a+bx^2\right)}{b^7} - \frac{87a^6 + 385a^5bx^2 + 650a^4b^2x^4 + 500a^3b^3x^6 + 150a^2b^4x^8}{20a^5b^7 + 100a^4b^8x^2 + 200a^3b^9x^4 + 200a^2b^{10}x^6 + 100ab^{11}x^8 + 20b^{12}x^{10}} + \frac{x^2}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**13/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*a*log(a + b*x**2)/b**7 - (87*a**6 + 385*a**5*b*x**2 + 650*a**4
*b**2*x**4 + 500*a**3*b**3*x**6 + 150*a**2*b**4*x**8)/(20*a**5*b*
*7 + 100*a**4*b**8*x**2 + 200*a**3*b**9*x**4 + 200*a**2*b**10*x**
6 + 100*a*b**11*x**8 + 20*b**12*x**10) + x**2/(2*b**6)
```

GIAC/XCAS [A] time = 0.271691, size = 128, normalized size = 1.08

$$\frac{x^2}{2\,b^6} - \frac{3\,a\ln\left(\left|bx^2+a\right|\right)}{b^7} + \frac{137\,ab^5x^{10} + 535\,a^2b^4x^8 + 870\,a^3b^3x^6 + 720\,a^4b^2x^4 + 300\,a^5bx^2 + 50\,a^6}{20\,(bx^2+a)^5b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^13/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2/b^6 - 3*a*ln(abs(b*x^2 + a))/b^7 + 1/20*(137*a*b^5*x^10 +
 535*a^2*b^4*x^8 + 870*a^3*b^3*x^6 + 720*a^4*b^2*x^4 + 300*a^5*b*
x^2 + 50*a^6)/((b*x^2 + a)^5*b^7)
```

3.514 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=109

$$\frac{a^5}{10b^6(a+bx^2)^5}-\frac{5a^4}{8b^6(a+bx^2)^4}+\frac{5a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{2b^6(a+bx^2)^2}+\frac{5a}{2b^6(a+bx^2)}+\frac{\log(a+bx^2)}{2b^6}$$

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a
^3)/(3*b^6*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)
/(2*b^6*(a + b*x^2)) + Log[a + b*x^2]/(2*b^6)
```

Rubi [A] time = 0.22197, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^5}{10b^6(a+bx^2)^5}-\frac{5a^4}{8b^6(a+bx^2)^4}+\frac{5a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{2b^6(a+bx^2)^2}+\frac{5a}{2b^6(a+bx^2)}+\frac{\log(a+bx^2)}{2b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] a^5/(10*b^6*(a + b*x^2)^5) - (5*a^4)/(8*b^6*(a + b*x^2)^4) + (5*a
^3)/(3*b^6*(a + b*x^2)^3) - (5*a^2)/(2*b^6*(a + b*x^2)^2) + (5*a)
/(2*b^6*(a + b*x^2)) + Log[a + b*x^2]/(2*b^6)
```

Rubi in Sympy [A] time = 36.1144, size = 100, normalized size = 0.92

$$\frac{a^5}{10b^6(a+bx^2)^5}-\frac{5a^4}{8b^6(a+bx^2)^4}+\frac{5a^3}{3b^6(a+bx^2)^3}-\frac{5a^2}{2b^6(a+bx^2)^2}+\frac{5a}{2b^6(a+bx^2)}+\frac{\log(a+bx^2)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**5/(10*b**6*(a + b*x**2)**5) - 5*a**4/(8*b**6*(a + b*x**2)**4)
+ 5*a**3/(3*b**6*(a + b*x**2)**3) - 5*a**2/(2*b**6*(a + b*x**2)**
2) + 5*a/(2*b**6*(a + b*x**2)) + log(a + b*x**2)/(2*b**6)
```

Mathematica [A] time = 0.0428848, size = 72, normalized size = 0.66

$$\frac{\frac{a\left(137a^4+625a^3bx^2+1100a^2b^2x^4+900ab^3x^6+300b^4x^8\right)}{(a+bx^2)^5}+60\log\left(a+bx^2\right)}{120b^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((a*(137*a^4 + 625*a^3*b*x^2 + 1100*a^2*b^2*x^4 + 900*a*b^3*x^6 +
 300*b^4*x^8))/(a + b*x^2)^5 + 60*Log[a + b*x^2])/(120*b^6)
```

Maple [A] time = 0.016, size = 98, normalized size = 0.9

$$\frac{a^5}{10\,b^6\,(bx^2+a)^5}-\frac{5\,a^4}{8\,b^6\,(bx^2+a)^4}+\frac{5\,a^3}{3\,b^6\,(bx^2+a)^3}-\frac{5\,a^2}{2\,b^6\,(bx^2+a)^2}+\frac{5\,a}{2\,b^6\,(bx^2+a)}+\frac{\ln\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a^5/b^6/(b*x^2+a)^5-5/8*a^4/b^6/(b*x^2+a)^4+5/3*a^3/b^6/(b*x
^2+a)^3-5/2*a^2/b^6/(b*x^2+a)^2+5/2*a/b^6/(b*x^2+a)+1/2*ln(b*x^2+
a)/b^6
```

Maxima [A] time = 0.732851, size = 163, normalized size = 1.5

$$\frac{300\,ab^4x^8+900\,a^2b^3x^6+1100\,a^3b^2x^4+625\,a^4bx^2+137\,a^5}{120\,(b^{11}x^{10}+5\,ab^{10}x^8+10\,a^2b^9x^6+10\,a^3b^8x^4+5\,a^4b^7x^2+a^5b^6)}+\frac{\log\left(bx^2+a\right)}{2\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a
^4*b*x^2 + 137*a^5)/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 +
10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6) + 1/2*log(b*x^2 + a)/b^
6
```

Fricas [A] time = 0.267725, size = 227, normalized size = 2.08

$$\frac{300\,ab^4x^8 + 900\,a^2b^3x^6 + 1100\,a^3b^2x^4 + 625\,a^4bx^2 + 137\,a^5 + 60\,\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5\right)\,\mathrm{lo}}{120\,(b^{11}x^{10} + 5\,ab^{10}x^8 + 10\,a^2b^9x^6 + 10\,a^3b^8x^4 + 5\,a^4b^7x^2 + a^5b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(300*a*b^4*x^8 + 900*a^2*b^3*x^6 + 1100*a^3*b^2*x^4 + 625*a
^4*b*x^2 + 137*a^5 + 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6
+ 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^11*x^10
+ 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2
+ a^5*b^6)
```

Sympy [A] time = 4.62984, size = 124, normalized size = 1.14

$$\frac{137a^5 + 625a^4bx^2 + 1100a^3b^2x^4 + 900a^2b^3x^6 + 300ab^4x^8}{120a^5b^6 + 600a^4b^7x^2 + 1200a^3b^8x^4 + 1200a^2b^9x^6 + 600ab^{10}x^8 + 120b^{11}x^{10}} + \frac{\log\left(a + bx^2\right)}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**5 + 625*a**4*b*x**2 + 1100*a**3*b**2*x**4 + 900*a**2*b**3
*x**6 + 300*a*b**4*x**8)/(120*a**5*b**6 + 600*a**4*b**7*x**2 + 12
00*a**3*b**8*x**4 + 1200*a**2*b**9*x**6 + 600*a*b**10*x**8 + 120*
b**11*x**10) + log(a + b*x**2)/(2*b**6)
```

GIAC/XCAS [A] time = 0.272478, size = 101, normalized size = 0.93

$$\frac{\ln\left(\left|bx^2 + a\right|\right)}{2\,b^6} - \frac{137\,b^4x^{10} + 385\,ab^3x^8 + 470\,a^2b^2x^6 + 270\,a^3bx^4 + 60\,a^4x^2}{120\,(bx^2 + a)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(b*x^2 + a))/b^6 - 1/120*(137*b^4*x^10 + 385*a*b^3*x^8
+ 470*a^2*b^2*x^6 + 270*a^3*b*x^4 + 60*a^4*x^2)/((b*x^2 + a)^5*b^
5)
```

3.515 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10a\,(a+bx^2)^5}$$

```
[Out] x^10/(10*a*(a + b*x^2)^5)
```

Rubi [A] time = 0.0254028, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{x^{10}}{10a\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] x^10/(10*a*(a + b*x^2)^5)
```

Rubi in Sympy [A] time = 8.09373, size = 14, normalized size = 0.74

$$\frac{x^{10}}{10a\,(a+bx^2)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] x**10/(10*a*(a + b*x**2)**5)
```

Mathematica [B] time = 0.0277281, size = 57, normalized size = 3.

$$-\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10b^5\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^4 + 5*a^3*b*x^2 + 10*a^2*b^2*x^4 + 10*a*b^3*x^6 + 5*b^4*x^8)/
(10*b^5*(a + b*x^2)^5)
```

Maple [B] time = 0.012, size = 81, normalized size = 4.3

$$-\frac{a^2}{b^5 (bx^2+a)^3}+\frac{a^3}{2\,b^5 (bx^2+a)^4}-\frac{a^4}{10\,b^5 (bx^2+a)^5}+\frac{a}{b^5 (bx^2+a)^2}-\frac{1}{(2\,bx^2+2\,a)\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -a^2/b^5/(b*x^2+a)^3+1/2*a^3/b^5/(b*x^2+a)^4-1/10*a^4/b^5/(b*x^2+
a)^5+a/b^5/(b*x^2+a)^2-1/2/(b*x^2+a)/b^5
```

Maxima [A] time = 0.703241, size = 138, normalized size = 7.26

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 +
a^4)/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 +
 5*a^4*b^6*x^2 + a^5*b^5)
```

Fricas [A] time = 0.260259, size = 138, normalized size = 7.26

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 +
a^4)/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 +
 5*a^4*b^6*x^2 + a^5*b^5)
```

Sympy [A] time = 4.33424, size = 107, normalized size = 5.63

$$-\frac{a^4+5a^3bx^2+10a^2b^2x^4+10ab^3x^6+5b^4x^8}{10a^5b^5+50a^4b^6x^2+100a^3b^7x^4+100a^2b^8x^6+50ab^9x^8+10b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a**4 + 5*a**3*b*x**2 + 10*a**2*b**2*x**4 + 10*a*b**3*x**6 + 5*b
**4*x**8)/(10*a**5*b**5 + 50*a**4*b**6*x**2 + 100*a**3*b**7*x**4
+ 100*a**2*b**8*x**6 + 50*a*b**9*x**8 + 10*b**10*x**10)
```

GIAC/XCAS [A] time = 0.272949, size = 74, normalized size = 3.89

$$-\frac{5\,b^4x^8+10\,ab^3x^6+10\,a^2b^2x^4+5\,a^3bx^2+a^4}{10\,(bx^2+a)^5b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10*(5*b^4*x^8 + 10*a*b^3*x^6 + 10*a^2*b^2*x^4 + 5*a^3*b*x^2 +
a^4)/((b*x^2 + a)^5*b^5)
```

3.516 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=39

$$\frac{x^8}{40a^2 (a + bx^2)^4} + \frac{x^8}{10a (a + bx^2)^5}$$

```
[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rubi [A] time = 0.0759035, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{x^8}{40a^2 (a + bx^2)^4} + \frac{x^8}{10a (a + bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] x^8/(10*a*(a + b*x^2)^5) + x^8/(40*a^2*(a + b*x^2)^4)
```

Rubi in Sympy [A] time = 14.6502, size = 31, normalized size = 0.79

$$\frac{x^8}{10a (a + bx^2)^5} + \frac{x^8}{40a^2 (a + bx^2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] x**8/(10*a*(a + b*x**2)**5) + x**8/(40*a**2*(a + b*x**2)**4)
```

Mathematica [A] time = 0.024256, size = 46, normalized size = 1.18

$$-\frac{a^3 + 5a^2bx^2 + 10ab^2x^4 + 10b^3x^6}{40b^4 (a + bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^3 + 5*a^2*b*x^2 + 10*a*b^2*x^4 + 10*b^3*x^6)/(40*b^4*(a + b*x
^2)^5)
```

Maple [A] time = 0.012, size = 65, normalized size = 1.7

$$\frac{a}{2\,b^4\,(bx^2+a)^3}-\frac{1}{4\,(bx^2+a)^2\,b^4}+\frac{a^3}{10\,b^4\,(bx^2+a)^5}-\frac{3\,a^2}{8\,b^4\,(bx^2+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/2*a/b^4/(b*x^2+a)^3-1/4/(b*x^2+a)^2/b^4+1/10*a^3/b^4/(b*x^2+a)^
5-3/8*a^2/b^4/(b*x^2+a)^4
```

Maxima [A] time = 0.689163, size = 123, normalized size = 3.15

$$-\frac{10\,b^3x^6+10\,ab^2x^4+5\,a^2bx^2+a^3}{40\,(b^9x^{10}+5\,ab^8x^8+10\,a^2b^7x^6+10\,a^3b^6x^4+5\,a^4b^5x^2+a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 +
 5*a*b^8*x^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 +
a^5*b^4)
```

Fricas [A] time = 0.259617, size = 123, normalized size = 3.15

$$-\frac{10\,b^3x^6+10\,ab^2x^4+5\,a^2bx^2+a^3}{40\,(b^9x^{10}+5\,ab^8x^8+10\,a^2b^7x^6+10\,a^3b^6x^4+5\,a^4b^5x^2+a^5b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/(b^9*x^10 +
 5*a*b^8*x^8 + 10*a^2*b^7*x^6 + 10*a^3*b^6*x^4 + 5*a^4*b^5*x^2 +
a^5*b^4)
```

Sympy [A] time = 4.10639, size = 95, normalized size = 2.44

$$-\frac{a^3 + 5a^2bx^2 + 10ab^2x^4 + 10b^3x^6}{40a^5b^4 + 200a^4b^5x^2 + 400a^3b^6x^4 + 400a^2b^7x^6 + 200ab^8x^8 + 40b^9x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a**3 + 5*a**2*b*x**2 + 10*a*b**2*x**4 + 10*b**3*x**6)/(40*a**5*
b**4 + 200*a**4*b**5*x**2 + 400*a**3*b**6*x**4 + 400*a**2*b**7*x*
*6 + 200*a*b**8*x**8 + 40*b**9*x**10)
```

GIAC/XCAS [A] time = 0.273355, size = 59, normalized size = 1.51

$$-\frac{10\, b^3x^6 + 10\, ab^2x^4 + 5\, a^2bx^2 + a^3}{40\,(bx^2 + a)^5 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6 + 10*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/((b*x^2 + a
)^5*b^4)
```

3.517 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=53

$$-\frac{a^2}{10b^3(a+bx^2)^5}+\frac{a}{4b^3(a+bx^2)^4}-\frac{1}{6b^3(a+bx^2)^3}$$

```
[Out] -a^2/(10*b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*
(a + b*x^2)^3)
```

Rubi [A] time = 0.112587, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{a^2}{10b^3(a+bx^2)^5}+\frac{a}{4b^3(a+bx^2)^4}-\frac{1}{6b^3(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -a^2/(10*b^3*(a + b*x^2)^5) + a/(4*b^3*(a + b*x^2)^4) - 1/(6*b^3*
(a + b*x^2)^3)
```

Rubi in Sympy [A] time = 19.4327, size = 46, normalized size = 0.87

$$-\frac{a^2}{10b^3(a+bx^2)^5}+\frac{a}{4b^3(a+bx^2)^4}-\frac{1}{6b^3(a+bx^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -a**2/(10*b**3*(a + b*x**2)**5) + a/(4*b**3*(a + b*x**2)**4) - 1/
(6*b**3*(a + b*x**2)**3)
```

Mathematica [A] time = 0.0243526, size = 35, normalized size = 0.66

$$-\frac{a^2+5abx^2+10b^2x^4}{60b^3(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a^2 + 5*a*b*x^2 + 10*b^2*x^4)/(60*b^3*(a + b*x^2)^5)
```

Maple [A] time = 0.011, size = 48, normalized size = 0.9

$$-\frac{a^2}{10\,b^3\,(bx^2+a)^5}+\frac{a}{4\,b^3\,(bx^2+a)^4}-\frac{1}{6\,b^3\,(bx^2+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10*a^2/b^3/(b*x^2+a)^5+1/4*a/b^3/(b*x^2+a)^4-1/6/b^3/(b*x^2+a)
^3
```

Maxima [A] time = 0.700852, size = 108, normalized size = 2.04

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10
*a^2*b^6*x^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Fricas [A] time = 0.26238, size = 108, normalized size = 2.04

$$-\frac{10\,b^2x^4+5\,abx^2+a^2}{60\,(b^8x^{10}+5\,ab^7x^8+10\,a^2b^6x^6+10\,a^3b^5x^4+5\,a^4b^4x^2+a^5b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/(b^8*x^10 + 5*a*b^7*x^8 + 10
*a^2*b^6*x^6 + 10*a^3*b^5*x^4 + 5*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [A] time = 3.96229, size = 83, normalized size = 1.57

$$-\frac{a^2 + 5abx^2 + 10b^2x^4}{60a^5b^3 + 300a^4b^4x^2 + 600a^3b^5x^4 + 600a^2b^6x^6 + 300ab^7x^8 + 60b^8x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a**2 + 5*a*b*x**2 + 10*b**2*x**4)/(60*a**5*b**3 + 300*a**4*b**4
*x**2 + 600*a**3*b**5*x**4 + 600*a**2*b**6*x**6 + 300*a*b**7*x**8
 + 60*b**8*x**10)
```

GIAC/XCAS [A] time = 0.270173, size = 45, normalized size = 0.85

$$-\frac{10\,b^2x^4 + 5\,abx^2 + a^2}{60\,(bx^2 + a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -1/60*(10*b^2*x^4 + 5*a*b*x^2 + a^2)/((b*x^2 + a)^5*b^3)
```

3.518 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^3} dx$

Optimal. Leaf size=34

$$\frac{a}{10b^2(a+bx^2)^5} - \frac{1}{8b^2(a+bx^2)^4}$$

```
[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rubi [A] time = 0.0754862, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{a}{10b^2(a+bx^2)^5} - \frac{1}{8b^2(a+bx^2)^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] a/(10*b^2*(a + b*x^2)^5) - 1/(8*b^2*(a + b*x^2)^4)
```

Rubi in Sympy [A] time = 14.9305, size = 29, normalized size = 0.85

$$\frac{a}{10b^2(a+bx^2)^5} - \frac{1}{8b^2(a+bx^2)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] a/(10*b**2*(a + b*x**2)**5) - 1/(8*b**2*(a + b*x**2)**4)
```

Mathematica [A] time = 0.0133692, size = 24, normalized size = 0.71

$$-\frac{a+5bx^2}{40b^2(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(a + 5*b*x^2)/(40*b^2*(a + b*x^2)^5)
```

Maple [A] time = 0.011, size = 31, normalized size = 0.9

$$\frac{a}{10\,b^2\,(bx^2+a)^5}-\frac{1}{8\,b^2\,(bx^2+a)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*a/b^2/(b*x^2+a)^5-1/8/b^2/(b*x^2+a)^4
```

Maxima [A] time = 0.697199, size = 93, normalized size = 2.74

$$-\frac{5\,bx^2+a}{40\,(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10
*a^3*b^4*x^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Fricas [A] time = 0.25778, size = 93, normalized size = 2.74

$$-\frac{5\,bx^2+a}{40\,(b^7x^{10}+5\,ab^6x^8+10\,a^2b^5x^6+10\,a^3b^4x^4+5\,a^4b^3x^2+a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + a)/(b^7*x^10 + 5*a*b^6*x^8 + 10*a^2*b^5*x^6 + 10
*a^3*b^4*x^4 + 5*a^4*b^3*x^2 + a^5*b^2)
```

Sympy [A] time = 3.77835, size = 71, normalized size = 2.09

$$-\frac{a+5bx^2}{40a^5b^2+200a^4b^3x^2+400a^3b^4x^4+400a^2b^5x^6+200ab^6x^8+40b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(a + 5*b*x**2)/(40*a**5*b**2 + 200*a**4*b**3*x**2 + 400*a**3*b**
4*x**4 + 400*a**2*b**5*x**6 + 200*a*b**6*x**8 + 40*b**7*x**10)
```

GIAC/XCAS [A] time = 0.271799, size = 30, normalized size = 0.88

$$-\frac{5\,bx^2+a}{40\,(bx^2+a)^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2 + a)/((b*x^2 + a)^5*b^2)
```

3.519 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{10b\,(a+bx^2)^5}$$

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Rubi [A] time = 0.0142325, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{1}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] -1/(10*b*(a + b*x^2)^5)
```

Rubi in Sympy [A] time = 6.31496, size = 14, normalized size = 0.88

$$-\frac{1}{10b\,(a+bx^2)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] -1/(10*b*(a + b*x**2)**5)
```

Mathematica [A] time = 0.00526628, size = 16, normalized size = 1.

$$-\frac{1}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -1/(10*b*(a + b*x^2)^5)
```

Maple [A] time = 0.006, size = 15, normalized size = 0.9

$$-\frac{1}{10\,b\,(bx^2+a)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/10/b/(b*x^2+a)^5
```

Maxima [A] time = 0.689691, size = 80, normalized size = 5.

$$-\frac{1}{10\,(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 +
 5*a^4*b^2*x^2 + a^5*b)
```

Fricas [A] time = 0.248255, size = 80, normalized size = 5.

$$-\frac{1}{10\,(b^6x^{10}+5\,ab^5x^8+10\,a^2b^4x^6+10\,a^3b^3x^4+5\,a^4b^2x^2+a^5b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/10/(b^6*x^10 + 5*a*b^5*x^8 + 10*a^2*b^4*x^6 + 10*a^3*b^3*x^4 +
 5*a^4*b^2*x^2 + a^5*b)
```

Sympy [A] time = 3.64774, size = 63, normalized size = 3.94

$$-\frac{1}{10a^5b + 50a^4b^2x^2 + 100a^3b^3x^4 + 100a^2b^4x^6 + 50ab^5x^8 + 10b^6x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -1/(10*a**5*b + 50*a**4*b**2*x**2 + 100*a**3*b**3*x**4 + 100*a**2
*b**4*x**6 + 50*a*b**5*x**8 + 10*b**6*x**10)
```

GIAC/XCAS [A] time = 0.269741, size = 19, normalized size = 1.19

$$-\frac{1}{10\left(bx^2 + a\right)^5 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -1/10/((b*x^2 + a)^5*b)
```

3.520 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=102

$$-\frac{\log(a+bx^2)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{2a^5(a+bx^2)}+\frac{1}{4a^4(a+bx^2)^2}+\frac{1}{6a^3(a+bx^2)^3}+\frac{1}{8a^2(a+bx^2)^4}+\frac{1}{10a(a+bx^2)^5}$$

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a +
b*x^2)^3) + 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log
[x]/a^6 - Log[a + b*x^2]/(2*a^6)
```

Rubi [A] time = 0.239805, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\log(a+bx^2)}{2a^6}+\frac{\log(x)}{a^6}+\frac{1}{2a^5(a+bx^2)}+\frac{1}{4a^4(a+bx^2)^2}+\frac{1}{6a^3(a+bx^2)^3}+\frac{1}{8a^2(a+bx^2)^4}+\frac{1}{10a(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] 1/(10*a*(a + b*x^2)^5) + 1/(8*a^2*(a + b*x^2)^4) + 1/(6*a^3*(a +
b*x^2)^3) + 1/(4*a^4*(a + b*x^2)^2) + 1/(2*a^5*(a + b*x^2)) + Log
[x]/a^6 - Log[a + b*x^2]/(2*a^6)
```

Rubi in Sympy [A] time = 35.9625, size = 95, normalized size = 0.93

$$\frac{1}{10a(a+bx^2)^5}+\frac{1}{8a^2(a+bx^2)^4}+\frac{1}{6a^3(a+bx^2)^3}+\frac{1}{4a^4(a+bx^2)^2}+\frac{1}{2a^5(a+bx^2)}+\frac{\log(x^2)}{2a^6}-\frac{\log(a+bx^2)}{2a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 1/(10*a*(a + b*x**2)**5) + 1/(8*a**2*(a + b*x**2)**4) + 1/(6*a**3
*(a + b*x**2)**3) + 1/(4*a**4*(a + b*x**2)**2) + 1/(2*a**5*(a + b
*x**2)) + log(x**2)/(2*a**6) - log(a + b*x**2)/(2*a**6)
```

Mathematica [A] time = 0.0946772, size = 76, normalized size = 0.75

$$\frac{\frac{a(137a^4+385a^3bx^2+470a^2b^2x^4+270ab^3x^6+60b^4x^8)}{(a+bx^2)^5}-60\log\left(a+bx^2\right)+120\log(x)}{120a^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(137*a^4 + 385*a^3*b*x^2 + 470*a^2*b^2*x^4 + 270*a*b^3*x^6 +
60*b^4*x^8))/(a + b*x^2)^5 + 120*Log[x] - 60*Log[a + b*x^2])/(120
*a^6)
```

Maple [A] time = 0.021, size = 91, normalized size = 0.9

$$\frac{1}{10\,a\,(bx^2+a)^5}+\frac{1}{8\,a^2\,(bx^2+a)^4}+\frac{1}{6\,a^3\,(bx^2+a)^3}+\frac{1}{4\,a^4\,(bx^2+a)^2}+\frac{1}{2\,a^5\,(bx^2+a)}+\frac{\ln(x)}{a^6}-\frac{\ln\left(bx^2+a\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10/a/(b*x^2+a)^5+1/8/a^2/(b*x^2+a)^4+1/6/a^3/(b*x^2+a)^3+1/4/a^
4/(b*x^2+a)^2+1/2/a^5/(b*x^2+a)+ln(x)/a^6-1/2*ln(b*x^2+a)/a^6
```

Maxima [A] time = 0.693334, size = 170, normalized size = 1.67

$$\frac{60\,b^4x^8+270\,ab^3x^6+470\,a^2b^2x^4+385\,a^3bx^2+137\,a^4}{120\,(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10})}-\frac{\log\left(bx^2+a\right)}{2\,a^6}+\frac{\log\left(x^2\right)}{2\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x),x, algorithm="maxima")
```

```
[Out] 1/120*(60*b^4*x^8 + 270*a*b^3*x^6 + 470*a^2*b^2*x^4 + 385*a^3*b*x
^2 + 137*a^4)/(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10
*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10) - 1/2*log(b*x^2 + a)/a^6 + 1/2
*log(x^2)/a^6
```

Fricas [A] time = 0.26622, size = 300, normalized size = 2.94

$$\frac{60\,ab^{4}x^{8} + 270\,a^{2}b^{3}x^{6} + 470\,a^{3}b^{2}x^{4} + 385\,a^{4}bx^{2} + 137\,a^{5} - 60\,\left(b^{5}x^{10} + 5\,ab^{4}x^{8} + 10\,a^{2}b^{3}x^{6} + 10\,a^{3}b^{2}x^{4} + 5\,a^{4}bx^{2} + a^{5}\right)\log}{120\,(a^{6}b^{5}x^{10} + 5\,a^{7}b^{4}x^{8} + 10\,a^{8}b^{3}x^{6} + 10\,a^{9}b^{2}x^{4} + 5\,a^{1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x),x, algorithm="fricas")
```

```
[Out] 1/120*(60*a*b^4*x^8 + 270*a^2*b^3*x^6 + 470*a^3*b^2*x^4 + 385*a^4
*b*x^2 + 137*a^5 - 60*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 +
10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*log(b*x^2 + a) + 120*(b^5*x^1
0 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 +
 a^5)*log(x))/(a^6*b^5*x^10 + 5*a^7*b^4*x^8 + 10*a^8*b^3*x^6 + 10
*a^9*b^2*x^4 + 5*a^10*b*x^2 + a^11)
```

Sympy [A] time = 11.2931, size = 128, normalized size = 1.25

$$\frac{137a^{4} + 385a^{3}bx^{2} + 470a^{2}b^{2}x^{4} + 270ab^{3}x^{6} + 60b^{4}x^{8}}{120a^{10} + 600a^{9}bx^{2} + 1200a^{8}b^{2}x^{4} + 1200a^{7}b^{3}x^{6} + 600a^{6}b^{4}x^{8} + 120a^{5}b^{5}x^{10}} + \frac{\log(x)}{a^{6}} - \frac{\log\left(\frac{a}{b} + x^{2}\right)}{2a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (137*a**4 + 385*a**3*b*x**2 + 470*a**2*b**2*x**4 + 270*a*b**3*x**
6 + 60*b**4*x**8)/(120*a**10 + 600*a**9*b*x**2 + 1200*a**8*b**2*x
**4 + 1200*a**7*b**3*x**6 + 600*a**6*b**4*x**8 + 120*a**5*b**5*x*
*10) + log(x)/a**6 - log(a/b + x**2)/(2*a**6)
```

GIAC/XCAS [A] time = 0.273273, size = 124, normalized size = 1.22

$$\frac{\ln\left(x^{2}\right)}{2\,a^{6}} - \frac{\ln\left(\left|bx^{2} + a\right|\right)}{2\,a^{6}} + \frac{137\,b^{5}x^{10} + 745\,ab^{4}x^{8} + 1640\,a^{2}b^{3}x^{6} + 1840\,a^{3}b^{2}x^{4} + 1070\,a^{4}bx^{2} + 274\,a^{5}}{120\,(bx^{2} + a)^{5}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(x^2)/a^6 - 1/2*ln(abs(b*x^2 + a))/a^6 + 1/120*(137*b^5*x^1
0 + 745*a*b^4*x^8 + 1640*a^2*b^3*x^6 + 1840*a^3*b^2*x^4 + 1070*a^
4*b*x^2 + 274*a^5)/((b*x^2 + a)^5*a^6)
```

3.521 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=116

$$\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{1}{2a^6x^2}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}$$

```
[Out] -1/(2*a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^
4) - b/(2*a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a
^6*(a + b*x^2)) - (6*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rubi [A] time = 0.28339, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{3b\log\left(a+bx^2\right)}{a^7}-\frac{6b\log(x)}{a^7}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{1}{2a^6x^2}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{10a^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(2*a^6*x^2) - b/(10*a^2*(a + b*x^2)^5) - b/(4*a^3*(a + b*x^2)^
4) - b/(2*a^4*(a + b*x^2)^3) - b/(a^5*(a + b*x^2)^2) - (5*b)/(2*a
^6*(a + b*x^2)) - (6*b*Log[x])/a^7 + (3*b*Log[a + b*x^2])/a^7
```

Rubi in Sympy [A] time = 43.2684, size = 110, normalized size = 0.95

$$-\frac{b}{10a^2\left(a+bx^2\right)^5}-\frac{b}{4a^3\left(a+bx^2\right)^4}-\frac{b}{2a^4\left(a+bx^2\right)^3}-\frac{b}{a^5\left(a+bx^2\right)^2}-\frac{5b}{2a^6\left(a+bx^2\right)}-\frac{1}{2a^6x^2}-\frac{3b\log\left(x^2\right)}{a^7}+\frac{3b\log\left(a+bx^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -b/(10*a**2*(a + b*x**2)**5) - b/(4*a**3*(a + b*x**2)**4) - b/(2*
a**4*(a + b*x**2)**3) - b/(a**5*(a + b*x**2)**2) - 5*b/(2*a**6*(a
 + b*x**2)) - 1/(2*a**6*x**2) - 3*b*log(x**2)/a**7 + 3*b*log(a +
b*x**2)/a**7
```

Mathematica [A] time = 0.154105, size = 92, normalized size = 0.79

$$-\frac{\frac{a\left(10a^5+137a^4bx^2+385a^3b^2x^4+470a^2b^3x^6+270ab^4x^8+60b^5x^{10}\right)}{x^2(a+bx^2)^5}-60b\log\left(a+bx^2\right)+120b\log(x)}{20a^7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -((a*(10*a^5 + 137*a^4*b*x^2 + 385*a^3*b^2*x^4 + 470*a^2*b^3*x^6
+ 270*a*b^4*x^8 + 60*b^5*x^10))/(x^2*(a + b*x^2)^5) + 120*b*Log[x
] - 60*b*Log[a + b*x^2])/(20*a^7)
```

Maple [A] time = 0.024, size = 107, normalized size = 0.9

$$-\frac{1}{2\,a^6x^2}-\frac{b}{10\,a^2\left(bx^2+a\right)^5}-\frac{b}{4\,a^3\left(bx^2+a\right)^4}-\frac{b}{2\,a^4\left(bx^2+a\right)^3}-\frac{b}{a^5\left(bx^2+a\right)^2}-\frac{5\,b}{2\,a^6\left(bx^2+a\right)}-6\,\frac{b\ln(x)}{a^7}+3\,\frac{b\ln\left(bx^2+a\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/2/a^6/x^2-1/10*b/a^2/(b*x^2+a)^5-1/4*b/a^3/(b*x^2+a)^4-1/2*b/a
^4/(b*x^2+a)^3-b/a^5/(b*x^2+a)^2-5/2*b/a^6/(b*x^2+a)-6*b*ln(x)/a^
7+3*b*ln(b*x^2+a)/a^7
```

Maxima [A] time = 0.699523, size = 193, normalized size = 1.66

$$-\frac{60\,b^5x^{10}+270\,ab^4x^8+470\,a^2b^3x^6+385\,a^3b^2x^4+137\,a^4bx^2+10\,a^5}{20\left(a^6b^5x^{12}+5\,a^7b^4x^{10}+10\,a^8b^3x^8+10\,a^9b^2x^6+5\,a^{10}bx^4+a^{11}x^2\right)}+\frac{3\,b\log\left(bx^2+a\right)}{a^7}-\frac{3\,b\log\left(x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^3),x, algorithm="maxima")
```

```
[Out] -1/20*(60*b^5*x^10 + 270*a*b^4*x^8 + 470*a^2*b^3*x^6 + 385*a^3*b^
2*x^4 + 137*a^4*b*x^2 + 10*a^5)/(a^6*b^5*x^12 + 5*a^7*b^4*x^10 +
10*a^8*b^3*x^8 + 10*a^9*b^2*x^6 + 5*a^10*b*x^4 + a^11*x^2) + 3*b*
log(b*x^2 + a)/a^7 - 3*b*log(x^2)/a^7
```

Fricas [A] time = 0.264739, size = 339, normalized size = 2.92

$$-\frac{60\,ab^5x^{10} + 270\,a^2b^4x^8 + 470\,a^3b^3x^6 + 385\,a^4b^2x^4 + 137\,a^5bx^2 + 10\,a^6 - 60\left(b^6x^{12} + 5\,ab^5x^{10} + 10\,a^2b^4x^8 + 10\,a^3b^3x^6 + 5\,a\right.}{20\left(a^7b^5x^{12} + 5\,a^8b^4x^{10} + 10\,a^9b^3x^8 + 10\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^3),x, algorithm="fricas")
```

```
[Out] -1/20*(60*a*b^5*x^10 + 270*a^2*b^4*x^8 + 470*a^3*b^3*x^6 + 385*a^
4*b^2*x^4 + 137*a^5*b*x^2 + 10*a^6 - 60*(b^6*x^12 + 5*a*b^5*x^10
+ 10*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*lo
g(b*x^2 + a) + 120*(b^6*x^12 + 5*a*b^5*x^10 + 10*a^2*b^4*x^8 + 10
*a^3*b^3*x^6 + 5*a^4*b^2*x^4 + a^5*b*x^2)*log(x))/(a^7*b^5*x^12 +
 5*a^8*b^4*x^10 + 10*a^9*b^3*x^8 + 10*a^10*b^2*x^6 + 5*a^11*b*x^4
 + a^12*x^2)
```

Sympy [A] time = 26.8549, size = 148, normalized size = 1.28

$$-\frac{10a^5 + 137a^4bx^2 + 385a^3b^2x^4 + 470a^2b^3x^6 + 270ab^4x^8 + 60b^5x^{10}}{20a^{11}x^2 + 100a^{10}bx^4 + 200a^9b^2x^6 + 200a^8b^3x^8 + 100a^7b^4x^{10} + 20a^6b^5x^{12}} - \frac{6b\log(x)}{a^7} + \frac{3b\log\left(\frac{a}{b} + x^2\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -(10*a**5 + 137*a**4*b*x**2 + 385*a**3*b**2*x**4 + 470*a**2*b**3*
x**6 + 270*a*b**4*x**8 + 60*b**5*x**10)/(20*a**11*x**2 + 100*a**1
0*b*x**4 + 200*a**9*b**2*x**6 + 200*a**8*b**3*x**8 + 100*a**7*b**
4*x**10 + 20*a**6*b**5*x**12) - 6*b*log(x)/a**7 + 3*b*log(a/b + x
**2)/a**7
```

GIAC/XCAS [A] time = 0.272588, size = 155, normalized size = 1.34

$$-\frac{3\,b\ln\left(x^2\right)}{a^7}+\frac{3\,b\ln\left(\left|bx^2+a\right|\right)}{a^7}+\frac{6\,bx^2-a}{2\,a^7x^2}-\frac{137\,b^6x^{10}+735\,ab^5x^8+1590\,a^2b^4x^6+1740\,a^3b^3x^4+970\,a^4b^2x^2+224\,a^5b}{20\,\left(bx^2+a\right)^5a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^3),x, algorithm="giac")
```

```
[Out] -3*b*ln(x^2)/a^7 + 3*b*ln(abs(b*x^2 + a))/a^7 + 1/2*(6*b*x^2 - a)
/(a^7*x^2) - 1/20*(137*b^6*x^10 + 735*a*b^5*x^8 + 1590*a^2*b^4*x^
6 + 1740*a^3*b^3*x^4 + 970*a^4*b^2*x^2 + 224*a^5*b)/((b*x^2 + a)^
5*a^7)
```

3.522 $\int \frac{1}{x^5(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}+\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{3b}{a^7x^2}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}-\frac{1}{4a^6x^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}$$

```
[Out] -1/(4*a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (
3*b^2)/(8*a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/
(2*a^6*(a + b*x^2)^2) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Lo
g[x])/a^8 - (21*b^2*Log[a + b*x^2])/(2*a^8)
```

Rubi [A] time = 0.31955, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}+\frac{21b^2\log(x)}{a^8}+\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{3b}{a^7x^2}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}-\frac{1}{4a^6x^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{10a^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1/(4*a^6*x^4) + (3*b)/(a^7*x^2) + b^2/(10*a^3*(a + b*x^2)^5) + (
3*b^2)/(8*a^4*(a + b*x^2)^4) + b^2/(a^5*(a + b*x^2)^3) + (5*b^2)/
(2*a^6*(a + b*x^2)^2) + (15*b^2)/(2*a^7*(a + b*x^2)) + (21*b^2*Lo
g[x])/a^8 - (21*b^2*Log[a + b*x^2])/(2*a^8)
```

Rubi in Sympy [A] time = 56.7045, size = 139, normalized size = 0.99

$$\frac{b^2}{10a^3\left(a+bx^2\right)^5}+\frac{3b^2}{8a^4\left(a+bx^2\right)^4}+\frac{b^2}{a^5\left(a+bx^2\right)^3}+\frac{5b^2}{2a^6\left(a+bx^2\right)^2}-\frac{1}{4a^6x^4}+\frac{15b^2}{2a^7\left(a+bx^2\right)}+\frac{3b}{a^7x^2}+\frac{21b^2\log\left(x^2\right)}{2a^8}-\frac{21b^2\log\left(a+bx^2\right)}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] b**2/(10*a**3*(a + b*x**2)**5) + 3*b**2/(8*a**4*(a + b*x**2)**4)
+ b**2/(a**5*(a + b*x**2)**3) + 5*b**2/(2*a**6*(a + b*x**2)**2) -
 1/(4*a**6*x**4) + 15*b**2/(2*a**7*(a + b*x**2)) + 3*b/(a**7*x**2
) + 21*b**2*log(x**2)/(2*a**8) - 21*b**2*log(a + b*x**2)/(2*a**8)
```

Mathematica [A] time = 0.109802, size = 107, normalized size = 0.76

$$\frac{\frac{a\left(-10a^6+70a^5bx^2+959a^4b^2x^4+2695a^3b^3x^6+3290a^2b^4x^8+1890ab^5x^{10}+420b^6x^{12}\right)}{x^4(a+bx^2)^5}-420b^2\log\left(a+bx^2\right)+840b^2\log(x)}{40a^8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((a*(-10*a^6 + 70*a^5*b*x^2 + 959*a^4*b^2*x^4 + 2695*a^3*b^3*x^6
+ 3290*a^2*b^4*x^8 + 1890*a*b^5*x^10 + 420*b^6*x^12))/(x^4*(a + b
*x^2)^5) + 840*b^2*Log[x] - 420*b^2*Log[a + b*x^2])/(40*a^8)
```

Maple [A] time = 0.025, size = 129, normalized size = 0.9

$$-\frac{1}{4\,a^6x^4}+3\,\frac{b}{a^7x^2}+\frac{b^2}{10\,a^3\,(bx^2+a)^5}+\frac{3\,b^2}{8\,a^4\,(bx^2+a)^4}+\frac{b^2}{a^5\,(bx^2+a)^3}+\frac{5\,b^2}{2\,a^6\,(bx^2+a)^2}+\frac{15\,b^2}{2\,a^7\,(bx^2+a)}+21\,\frac{b^2\ln(x)}{a^8}-\frac{21\,b^2\ln\left(bx^2+a\right)}{2\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1/4/a^6/x^4+3*b/a^7/x^2+1/10*b^2/a^3/(b*x^2+a)^5+3/8*b^2/a^4/(b*
x^2+a)^4+b^2/a^5/(b*x^2+a)^3+5/2*b^2/a^6/(b*x^2+a)^2+15/2*b^2/a^7
/(b*x^2+a)+21*b^2*ln(x)/a^8-21/2*b^2*ln(b*x^2+a)/a^8
```

Maxima [A] time = 0.705082, size = 213, normalized size = 1.52

$$\frac{420\,b^6x^{12}+1890\,ab^5x^{10}+3290\,a^2b^4x^8+2695\,a^3b^3x^6+959\,a^4b^2x^4+70\,a^5bx^2-10\,a^6}{40\,(a^7b^5x^{14}+5\,a^8b^4x^{12}+10\,a^9b^3x^{10}+10\,a^{10}b^2x^8+5\,a^{11}bx^6+a^{12}x^4)}-\frac{21\,b^2\log\left(bx^2+a\right)}{2\,a^8}+\frac{21\,b^2\log\left(x^2\right)}{2\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^5),x, algorithm="maxima")
```

```
[Out] 1/40*(420*b^6*x^12 + 1890*a*b^5*x^10 + 3290*a^2*b^4*x^8 + 2695*a^
3*b^3*x^6 + 959*a^4*b^2*x^4 + 70*a^5*b*x^2 - 10*a^6)/(a^7*b^5*x^1
4 + 5*a^8*b^4*x^12 + 10*a^9*b^3*x^10 + 10*a^10*b^2*x^8 + 5*a^11*b
*x^6 + a^12*x^4) - 21/2*b^2*log(b*x^2 + a)/a^8 + 21/2*b^2*log(x^2
)/a^8
```

Fricas [A] time = 0.263669, size = 359, normalized size = 2.56

$$\frac{420\,ab^6x^{12} + 1890\,a^2b^5x^{10} + 3290\,a^3b^4x^8 + 2695\,a^4b^3x^6 + 959\,a^5b^2x^4 + 70\,a^6bx^2 - 10\,a^7 - 420\,\left(b^7x^{14} + 5\,ab^6x^{12} + 10\,a^2b^5x\right.}{40\,(a^8b^5x^{14} + 5\,a^9b^4x^{12} + 10\,a^{10}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^5),x, algorithm="fricas")
```

```
[Out] 1/40*(420*a*b^6*x^12 + 1890*a^2*b^5*x^10 + 3290*a^3*b^4*x^8 + 269
5*a^4*b^3*x^6 + 959*a^5*b^2*x^4 + 70*a^6*b*x^2 - 10*a^7 - 420*(b^
7*x^14 + 5*a*b^6*x^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*
b^3*x^6 + a^5*b^2*x^4)*log(b*x^2 + a) + 840*(b^7*x^14 + 5*a*b^6*x
^12 + 10*a^2*b^5*x^10 + 10*a^3*b^4*x^8 + 5*a^4*b^3*x^6 + a^5*b^2*
x^4)*log(x))/(a^8*b^5*x^14 + 5*a^9*b^4*x^12 + 10*a^10*b^3*x^10 +
10*a^11*b^2*x^8 + 5*a^12*b*x^6 + a^13*x^4)
```

Sympy [A] time = 59.9091, size = 165, normalized size = 1.18

$$\frac{-10a^6 + 70a^5bx^2 + 959a^4b^2x^4 + 2695a^3b^3x^6 + 3290a^2b^4x^8 + 1890ab^5x^{10} + 420b^6x^{12}}{40a^{12}x^4 + 200a^{11}bx^6 + 400a^{10}b^2x^8 + 400a^9b^3x^{10} + 200a^8b^4x^{12} + 40a^7b^5x^{14}} + \frac{21b^2\log{(x)}}{a^8} - \frac{21b^2\log{\left(\frac{a}{b} + x^2\right)}}{2a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (-10*a**6 + 70*a**5*b*x**2 + 959*a**4*b**2*x**4 + 2695*a**3*b**3*
x**6 + 3290*a**2*b**4*x**8 + 1890*a*b**5*x**10 + 420*b**6*x**12)/
(40*a**12*x**4 + 200*a**11*b*x**6 + 400*a**10*b**2*x**8 + 400*a**
9*b**3*x**10 + 200*a**8*b**4*x**12 + 40*a**7*b**5*x**14) + 21*b**
2*log(x)/a**8 - 21*b**2*log(a/b + x**2)/(2*a**8)
```

GIAC/XCAS [A] time = 0.272207, size = 176, normalized size = 1.26

$$\frac{21\,b^2\ln\left(x^2\right)}{2\,a^8} - \frac{21\,b^2\ln\left(\left|bx^2+a\right|\right)}{2\,a^8} - \frac{63\,b^2x^4 - 12\,abx^2 + a^2}{4\,a^8x^4} + \frac{959\,b^7x^{10} + 5095\,ab^6x^8 + 10890\,a^2b^5x^6 + 11730\,a^3b^4x^4 + 6390\,a^4b^3x^2 + 1418\,a^5b^2}{40\,(bx^2+a)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^5),x, algorithm="giac")
```

```
[Out] 21/2*b^2*ln(x^2)/a^8 - 21/2*b^2*ln(abs(b*x^2 + a))/a^8 - 1/4*(63*
b^2*x^4 - 12*a*b*x^2 + a^2)/(a^8*x^4) + 1/40*(959*b^7*x^10 + 5095
*a*b^6*x^8 + 10890*a^2*b^5*x^6 + 11730*a^3*b^4*x^4 + 6390*a^4*b^3
*x^2 + 1418*a^5*b^2)/((b*x^2 + a)^5*a^8)
```

3.523 $\int \frac{x^{16}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{17/2}}+\frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}-\frac{1287x^7}{256b^5(a+bx^2)}-\frac{143x^9}{128b^4(a+bx^2)^2}-\frac{13x^{11}}{32b^3(a+bx^2)^3}-\frac{3x^{13}}{16b^2(a+bx^2)^4}-\frac{x^{15}}{10b(a+bx^2)^5}+\frac{9009x^5}{1280b^6}$$

```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(128
0*b^6) - x^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)
^4) - (13*x^11)/(32*b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a +
b*x^2)^2) - (1287*x^7)/(256*b^5*(a + b*x^2)) - (9009*a^(5/2)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```

Rubi [A] time = 0.270116, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{9009a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{17/2}}+\frac{9009a^2x}{256b^8}-\frac{3003ax^3}{256b^7}-\frac{1287x^7}{256b^5(a+bx^2)}-\frac{143x^9}{128b^4(a+bx^2)^2}-\frac{13x^{11}}{32b^3(a+bx^2)^3}-\frac{3x^{13}}{16b^2(a+bx^2)^4}-\frac{x^{15}}{10b(a+bx^2)^5}+\frac{9009x^5}{1280b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] (9009*a^2*x)/(256*b^8) - (3003*a*x^3)/(256*b^7) + (9009*x^5)/(128
0*b^6) - x^15/(10*b*(a + b*x^2)^5) - (3*x^13)/(16*b^2*(a + b*x^2)
^4) - (13*x^11)/(32*b^3*(a + b*x^2)^3) - (143*x^9)/(128*b^4*(a +
b*x^2)^2) - (1287*x^7)/(256*b^5*(a + b*x^2)) - (9009*a^(5/2)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(17/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{9009a^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{17}{2}}}-\frac{3003ax^3}{256b^7}-\frac{x^{15}}{10b(a+bx^2)^5}-\frac{3x^{13}}{16b^2(a+bx^2)^4}-\frac{13x^{11}}{32b^3(a+bx^2)^3}-\frac{143x^9}{128b^4(a+bx^2)^2}-\frac{1287x^7}{256b^5(a+bx^2)}+\frac{9009x^5}{1280b^6}+\frac{9009\int a^2\,dx}{256b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -9009*a**(5/2)*atan(sqrt(b)*x/sqrt(a))/(256*b**(17/2)) - 3003*a*x
**3/(256*b**7) - x**15/(10*b*(a + b*x**2)**5) - 3*x**13/(16*b**2*
(a + b*x**2)**4) - 13*x**11/(32*b**3*(a + b*x**2)**3) - 143*x**9/
(128*b**4*(a + b*x**2)**2) - 1287*x**7/(256*b**5*(a + b*x**2)) +
9009*x**5/(1280*b**6) + 9009*Integral(a**2, x)/(256*b**8)
```

Mathematica [A] time = 0.117025, size = 122, normalized size = 0.79

$$\frac{\frac{\sqrt{b}x\left(45045a^7+210210a^6bx^2+384384a^5b^2x^4+338910a^4b^3x^6+137995a^3b^4x^8+16640a^2b^5x^{10}-1280ab^6x^{12}+256b^7x^{14}\right)}{(a+bx^2)^5}-45045a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1280b^{17/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^16/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(45045*a^7 + 210210*a^6*b*x^2 + 384384*a^5*b^2*x^4 +
338910*a^4*b^3*x^6 + 137995*a^3*b^4*x^8 + 16640*a^2*b^5*x^10 - 12
80*a*b^6*x^12 + 256*b^7*x^14))/(a + b*x^2)^5 - 45045*a^(5/2)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(17/2))
```

Maple [A] time = 0.021, size = 148, normalized size = 1.

$$\frac{x^5}{5\,b^6}-2\,\frac{ax^3}{b^7}+21\,\frac{a^2x}{b^8}+\frac{5327\,a^3x^9}{256\,b^4\,(bx^2+a)^5}+\frac{9443\,a^4x^7}{128\,b^5\,(bx^2+a)^5}+\frac{1001\,a^5x^5}{10\,b^6\,(bx^2+a)^5}+\frac{7837\,a^6x^3}{128\,b^7\,(bx^2+a)^5}+\frac{3633\,a^7x}{256\,b^8\,(bx^2+a)^5}-\frac{9009\,a^3}{256\,b^8}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^16/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/5*x^5/b^6-2*a*x^3/b^7+21*a^2*x/b^8+5327/256/b^4*a^3/(b*x^2+a)^5
*x^9+9443/128/b^5*a^4/(b*x^2+a)^5*x^7+1001/10/b^6*a^5/(b*x^2+a)^5
*x^5+7837/128/b^7*a^6/(b*x^2+a)^5*x^3+3633/256/b^8*a^7/(b*x^2+a)^
5*x-9009/256/b^8*a^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^16/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267753, size = 1, normalized size = 0.01

$$\left[\frac{512\,b^7x^{15} - 2560\,ab^6x^{13} + 33280\,a^2b^5x^{11} + 275990\,a^3b^4x^9 + 677820\,a^4b^3x^7 + 768768\,a^5b^2x^5 + 420420\,a^6bx^3 + 90090\,a^7x + \cdots}{2560\,(b^{13}x^{10} + 5\,ab^{12}x^8 + 10\,a^2b^{11}x^6 + 10\,a^3b^{10}x \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^16/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(512*b^7*x^15 - 2560*a*b^6*x^13 + 33280*a^2*b^5*x^11 + 27
5990*a^3*b^4*x^9 + 677820*a^4*b^3*x^7 + 768768*a^5*b^2*x^5 + 4204
20*a^6*b*x^3 + 90090*a^7*x + 45045*(a^2*b^5*x^10 + 5*a^3*b^4*x^8
+ 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*sqrt(-a/b)
*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^13*x^10 + 5*
a*b^12*x^8 + 10*a^2*b^11*x^6 + 10*a^3*b^10*x^4 + 5*a^4*b^9*x^2 +
a^5*b^8), 1/1280*(256*b^7*x^15 - 1280*a*b^6*x^13 + 16640*a^2*b^5*
x^11 + 137995*a^3*b^4*x^9 + 338910*a^4*b^3*x^7 + 384384*a^5*b^2*x
^5 + 210210*a^6*b*x^3 + 45045*a^7*x - 45045*(a^2*b^5*x^10 + 5*a^3
*b^4*x^8 + 10*a^4*b^3*x^6 + 10*a^5*b^2*x^4 + 5*a^6*b*x^2 + a^7)*s
qrt(a/b)*arctan(x/sqrt(a/b)))/(b^13*x^10 + 5*a*b^12*x^8 + 10*a^2*
b^11*x^6 + 10*a^3*b^10*x^4 + 5*a^4*b^9*x^2 + a^5*b^8)]
```

Sympy [A] time = 5.44054, size = 218, normalized size = 1.41

$$\frac{21a^2x}{b^8} - \frac{2ax^3}{b^7} + \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x - \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} - \frac{9009\sqrt{-\frac{a^5}{b^{17}}}\log\left(x + \frac{b^8\sqrt{-\frac{a^5}{b^{17}}}}{a^2}\right)}{512} + \frac{18165a^7x + 78370a^6bx^3 + 128128a^5b^2x^5 + 94430a^4b^3x^7 + 26635a^3b^4x^9}{1280a^5b^8 + 6400a^4b^9x^2 + 12800a^3b^{10}x^4 + 12800a^2b^{11}x^6 + 6400ab^{12}x^8 + 1280b^{13}x^{10}} + \frac{x^5}{5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**16/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 21*a**2*x/b**8 - 2*a*x**3/b**7 + 9009*sqrt(-a**5/b**17)*log(x - b
**8*sqrt(-a**5/b**17)/a**2)/512 - 9009*sqrt(-a**5/b**17)*log(x +
b**8*sqrt(-a**5/b**17)/a**2)/512 + (18165*a**7*x + 78370*a**6*b*x
**3 + 128128*a**5*b**2*x**5 + 94430*a**4*b**3*x**7 + 26635*a**3*b
**4*x**9)/(1280*a**5*b**8 + 6400*a**4*b**9*x**2 + 12800*a**3*b**1
0*x**4 + 12800*a**2*b**11*x**6 + 6400*a*b**12*x**8 + 1280*b**13*x
**10) + x**5/(5*b**6)
```

GIAC/XCAS [A] time = 0.27536, size = 158, normalized size = 1.02

$$-\frac{9009\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^8}+\frac{26635\,a^3b^4x^9+94430\,a^4b^3x^7+128128\,a^5b^2x^5+78370\,a^6bx^3+18165\,a^7x}{1280\,(bx^2+a)^5b^8}+\frac{b^{24}x^5-10\,ab^{23}x^3+105\,a^2b^{22}x}{5\,b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^16/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -9009/256*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^8) + 1/1280*(266
35*a^3*b^4*x^9 + 94430*a^4*b^3*x^7 + 128128*a^5*b^2*x^5 + 78370*a
^6*b*x^3 + 18165*a^7*x)/((b*x^2 + a)^5*b^8) + 1/5*(b^24*x^5 - 10*
a*b^23*x^3 + 105*a^2*b^22*x)/b^30
```

3.524 $\int \frac{x^{14}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=142

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{3003ax}{256b^7}-\frac{3003x^5}{1280b^5(a+bx^2)}-\frac{429x^7}{640b^4(a+bx^2)^2}-\frac{143x^9}{480b^3(a+bx^2)^3}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{x^{13}}{10b(a+bx^2)^5}+\frac{1001x^3}{256b^6}$$

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*
x^2)^5) - (13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(
a + b*x^2)^3) - (429*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1
280*b^5*(a + b*x^2)) + (3003*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(256*b^(15/2))
```

Rubi [A] time = 0.242471, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{3003a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{15/2}}-\frac{3003ax}{256b^7}-\frac{3003x^5}{1280b^5(a+bx^2)}-\frac{429x^7}{640b^4(a+bx^2)^2}-\frac{143x^9}{480b^3(a+bx^2)^3}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{x^{13}}{10b(a+bx^2)^5}+\frac{1001x^3}{256b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-3003*a*x)/(256*b^7) + (1001*x^3)/(256*b^6) - x^13/(10*b*(a + b*
x^2)^5) - (13*x^11)/(80*b^2*(a + b*x^2)^4) - (143*x^9)/(480*b^3*(
a + b*x^2)^3) - (429*x^7)/(640*b^4*(a + b*x^2)^2) - (3003*x^5)/(1
280*b^5*(a + b*x^2)) + (3003*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(256*b^(15/2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3003a^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{15}{2}}}-\frac{x^{13}}{10b(a+bx^2)^5}-\frac{13x^{11}}{80b^2(a+bx^2)^4}-\frac{143x^9}{480b^3(a+bx^2)^3}-\frac{429x^7}{640b^4(a+bx^2)^2}-\frac{3003x^5}{1280b^5(a+bx^2)}+\frac{1001x^3}{256b^6}-\frac{3003\int a\,dx}{256b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 3003*a**(3/2)*atan(sqrt(b)*x/sqrt(a))/(256*b**(15/2)) - x**13/(10
*b*(a + b*x**2)**5) - 13*x**11/(80*b**2*(a + b*x**2)**4) - 143*x*
*9/(480*b**3*(a + b*x**2)**3) - 429*x**7/(640*b**4*(a + b*x**2)**
2) - 3003*x**5/(1280*b**5*(a + b*x**2)) + 1001*x**3/(256*b**6) -
3003*Integral(a, x)/(256*b**7)
```

Mathematica [A] time = 0.104396, size = 111, normalized size = 0.78

$$\frac{45045a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\frac{\sqrt{b}x\left(-45045a^6-210210a^5bx^2-384384a^4b^2x^4-338910a^3b^3x^6-137995a^2b^4x^8-16640ab^5x^{10}+1280b^6x^{12}\right)}{(a+bx^2)^5}}{3840b^{15/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^14/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(-45045*a^6 - 210210*a^5*b*x^2 - 384384*a^4*b^2*x^4 -
 338910*a^3*b^3*x^6 - 137995*a^2*b^4*x^8 - 16640*a*b^5*x^10 + 128
0*b^6*x^12))/(a + b*x^2)^5 + 45045*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(3840*b^(15/2))
```

Maple [A] time = 0.019, size = 137, normalized size = 1.

$$\frac{x^3}{3\,b^6}-6\,\frac{ax}{b^7}-\frac{2373\,a^2x^9}{256\,b^3\,(bx^2+a)^5}-\frac{12131\,a^3x^7}{384\,b^4\,(bx^2+a)^5}-\frac{1253\,a^4x^5}{30\,b^5\,(bx^2+a)^5}-\frac{9629\,a^5x^3}{384\,b^6\,(bx^2+a)^5}-\frac{1467\,a^6x}{256\,b^7\,(bx^2+a)^5}+\frac{3003\,a^2}{256\,b^7}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^14/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/3*x^3/b^6-6*a*x/b^7-2373/256/b^3*a^2/(b*x^2+a)^5*x^9-12131/384/
b^4*a^3/(b*x^2+a)^5*x^7-1253/30/b^5*a^4/(b*x^2+a)^5*x^5-9629/384/
b^6*a^5/(b*x^2+a)^5*x^3-1467/256/b^7*a^6/(b*x^2+a)^5*x+3003/256/b
^7*a^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269229, size = 1, normalized size = 0.01

$$\left[\frac{2560\,b^6x^{13} - 33280\,ab^5x^{11} - 275990\,a^2b^4x^9 - 677820\,a^3b^3x^7 - 768768\,a^4b^2x^5 - 420420\,a^5bx^3 - 90090\,a^6x + 45045\,(ab^5x}{7680\,(b^{12}x^{10} + 5\,ab^{11}x^8 + 10\,a^2b^{10}x^6 + 10\,a^3b^9x^4 + 5\,a^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(2560*b^6*x^13 - 33280*a*b^5*x^11 - 275990*a^2*b^4*x^9 -
677820*a^3*b^3*x^7 - 768768*a^4*b^2*x^5 - 420420*a^5*b*x^3 - 9009
0*a^6*x + 45045*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10
*a^4*b^2*x^4 + 5*a^5*b*x^2 + a^6)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)))/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b
^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7), 1/3840*(1280
*b^6*x^13 - 16640*a*b^5*x^11 - 137995*a^2*b^4*x^9 - 338910*a^3*b^
3*x^7 - 384384*a^4*b^2*x^5 - 210210*a^5*b*x^3 - 45045*a^6*x + 450
45*(a*b^5*x^10 + 5*a^2*b^4*x^8 + 10*a^3*b^3*x^6 + 10*a^4*b^2*x^4
+ 5*a^5*b*x^2 + a^6)*sqrt(a/b)*arctan(x/sqrt(a/b)))/(b^12*x^10 +
5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 +
 a^5*b^7)]
```

Sympy [A] time = 5.28372, size = 202, normalized size = 1.42

$$-\frac{6ax}{b^7} - \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x - \frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} + \frac{3003\sqrt{-\frac{a^3}{b^{15}}}\log\left(x + \frac{b^7\sqrt{-\frac{a^3}{b^{15}}}}{a}\right)}{512} - \frac{22005a^6x + 96290a^5bx^3 + 160384a^4b^2x^5 + 121310a^3b^3x^7 + 35595a^2b^4x^9}{3840a^5b^7 + 19200a^4b^8x^2 + 38400a^3b^9x^4 + 38400a^2b^{10}x^6 + 19200ab^{11}x^8 + 3840b^{12}x^{10}} + \frac{x^3}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**14/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -6*a*x/b**7 - 3003*sqrt(-a**3/b**15)*log(x - b**7*sqrt(-a**3/b**1
5)/a)/512 + 3003*sqrt(-a**3/b**15)*log(x + b**7*sqrt(-a**3/b**15)
/a)/512 - (22005*a**6*x + 96290*a**5*b*x**3 + 160384*a**4*b**2*x*
*5 + 121310*a**3*b**3*x**7 + 35595*a**2*b**4*x**9)/(3840*a**5*b**
7 + 19200*a**4*b**8*x**2 + 38400*a**3*b**9*x**4 + 38400*a**2*b**1
0*x**6 + 19200*a*b**11*x**8 + 3840*b**12*x**10) + x**3/(3*b**6)
```

GIAC/XCAS [A] time = 0.271406, size = 143, normalized size = 1.01

$$\frac{3003\,a^{2}\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^{7}} - \frac{35595\,a^{2}b^{4}x^{9} + 121310\,a^{3}b^{3}x^{7} + 160384\,a^{4}b^{2}x^{5} + 96290\,a^{5}bx^{3} + 22005\,a^{6}x}{3840\,(bx^{2} + a)^{5}b^{7}} + \frac{b^{12}x^{3} - 18\,ab^{11}x}{3\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^14/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 3003/256*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) - 1/3840*(3559
5*a^2*b^4*x^9 + 121310*a^3*b^3*x^7 + 160384*a^4*b^2*x^5 + 96290*a
^5*b*x^3 + 22005*a^6*x)/((b*x^2 + a)^5*b^7) + 1/3*(b^12*x^3 - 18*
a*b^11*x)/b^18
```

3.525 $\int \frac{x^{12}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=131

$$-\frac{693\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{231x^3}{256b^5(a+bx^2)}-\frac{231x^5}{640b^4(a+bx^2)^2}-\frac{33x^7}{160b^3(a+bx^2)^3}-\frac{11x^9}{80b^2(a+bx^2)^4}-\frac{x^{11}}{10b(a+bx^2)^5}+\frac{693x}{256b^6}$$

```
[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*
(a + b*x^2)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(64
0*b^4*(a + b*x^2)^2) - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqr
t[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(13/2))
```

Rubi [A] time = 0.203562, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{693\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{13/2}}-\frac{231x^3}{256b^5(a+bx^2)}-\frac{231x^5}{640b^4(a+bx^2)^2}-\frac{33x^7}{160b^3(a+bx^2)^3}-\frac{11x^9}{80b^2(a+bx^2)^4}-\frac{x^{11}}{10b(a+bx^2)^5}+\frac{693x}{256b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] (693*x)/(256*b^6) - x^11/(10*b*(a + b*x^2)^5) - (11*x^9)/(80*b^2*
(a + b*x^2)^4) - (33*x^7)/(160*b^3*(a + b*x^2)^3) - (231*x^5)/(64
0*b^4*(a + b*x^2)^2) - (231*x^3)/(256*b^5*(a + b*x^2)) - (693*Sqr
t[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*b^(13/2))
```

Rubi in Sympy [A] time = 42.8991, size = 122, normalized size = 0.93

$$-\frac{693\sqrt{a}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{13}{2}}}-\frac{x^{11}}{10b(a+bx^2)^5}-\frac{11x^9}{80b^2(a+bx^2)^4}-\frac{33x^7}{160b^3(a+bx^2)^3}-\frac{231x^5}{640b^4(a+bx^2)^2}-\frac{231x^3}{256b^5(a+bx^2)}+\frac{693x}{256b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -693*sqrt(a)*atan(sqrt(b)*x/sqrt(a))/(256*b**(13/2)) - x**11/(10*
b*(a + b*x**2)**5) - 11*x**9/(80*b**2*(a + b*x**2)**4) - 33*x**7/
(160*b**3*(a + b*x**2)**3) - 231*x**5/(640*b**4*(a + b*x**2)**2)
- 231*x**3/(256*b**5*(a + b*x**2)) + 693*x/(256*b**6)
```

Mathematica [A] time = 0.0970371, size = 100, normalized size = 0.76

$$\frac{\frac{\sqrt{b}x(3465a^5+16170a^4bx^2+29568a^3b^2x^4+26070a^2b^3x^6+10615ab^4x^8+1280b^5x^{10})}{(a+bx^2)^5}-3465\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1280b^{13/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^12/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((Sqrt[b]*x*(3465*a^5 + 16170*a^4*b*x^2 + 29568*a^3*b^2*x^4 + 260
70*a^2*b^3*x^6 + 10615*a*b^4*x^8 + 1280*b^5*x^10))/(a + b*x^2)^5
- 3465*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(1280*b^(13/2))
```

Maple [A] time = 0.02, size = 123, normalized size = 0.9

$$\frac{x}{b^6}+\frac{843\,ax^9}{256\,b^2\,(bx^2+a)^5}+\frac{1327\,a^2x^7}{128\,b^3\,(bx^2+a)^5}+\frac{131\,a^3x^5}{10\,b^4\,(bx^2+a)^5}+\frac{977\,a^4x^3}{128\,b^5\,(bx^2+a)^5}+\frac{437\,a^5x}{256\,b^6\,(bx^2+a)^5}-\frac{693\,a}{256\,b^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] x/b^6+843/256/b^2*a/(b*x^2+a)^5*x^9+1327/128/b^3*a^2/(b*x^2+a)^5*
x^7+131/10/b^4*a^3/(b*x^2+a)^5*x^5+977/128/b^5*a^4/(b*x^2+a)^5*x^
3+437/256/b^6*a^5/(b*x^2+a)^5*x-693/256/b^6*a/(a*b)^(1/2)*arctan(
x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267743, size = 1, normalized size = 0.01

$$\left[\frac{2560\,b^5x^{11} + 21230\,ab^4x^9 + 52140\,a^2b^3x^7 + 59136\,a^3b^2x^5 + 32340\,a^4bx^3 + 6930\,a^5x + 3465\,\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + \right.}{2560\,(b^{11}x^{10} + 5\,ab^{10}x^8 + 10\,a^2b^9x^6 + 10\,a^3b^8x^4 + 5\,a^4b^7x^2 + a^5b^6)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(2560*b^5*x^11 + 21230*a*b^4*x^9 + 52140*a^2*b^3*x^7 + 59
136*a^3*b^2*x^5 + 32340*a^4*b*x^3 + 6930*a^5*x + 3465*(b^5*x^10 +
 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^
5)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b
^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4
*b^7*x^2 + a^5*b^6), 1/1280*(1280*b^5*x^11 + 10615*a*b^4*x^9 + 26
070*a^2*b^3*x^7 + 29568*a^3*b^2*x^5 + 16170*a^4*b*x^3 + 3465*a^5*
x - 3465*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^
4 + 5*a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(x/sqrt(a/b)))/(b^11*x^10
+ 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2
+ a^5*b^6)]
```

Sympy [A] time = 5.02746, size = 178, normalized size = 1.36

$$\frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(-b^6\sqrt{-\frac{a}{b^{13}}} + x\right)}{512} - \frac{693\sqrt{-\frac{a}{b^{13}}}\log\left(b^6\sqrt{-\frac{a}{b^{13}}} + x\right)}{512} + \frac{2185a^5x + 9770a^4bx^3 + 16768a^3b^2x^5 + 13270a^2b^3x^7 + 4215ab^4x^9}{1280a^5b^6 + 6400a^4b^7x^2 + 12800a^3b^8x^4 + 12800a^2b^9x^6 + 6400ab^{10}x^8 + 1280b^{11}x^{10}} + \frac{x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-a/b**13)*log(-b**6*sqrt(-a/b**13) + x)/512 - 693*sqrt(-
a/b**13)*log(b**6*sqrt(-a/b**13) + x)/512 + (2185*a**5*x + 9770*a
```

```
**4*b*x**3 + 16768*a**3*b**2*x**5 + 13270*a**2*b**3*x**7 + 4215*a
*b**4*x**9)/(1280*a**5*b**6 + 6400*a**4*b**7*x**2 + 12800*a**3*b*
*8*x**4 + 12800*a**2*b**9*x**6 + 6400*a*b**10*x**8 + 1280*b**11*x
**10) + x/b**6
```

GIAC/XCAS [A] time = 0.271582, size = 117, normalized size = 0.89

$$-\frac{693\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}b^6}+\frac{x}{b^6}+\frac{4215\,ab^4x^9+13270\,a^2b^3x^7+16768\,a^3b^2x^5+9770\,a^4bx^3+2185\,a^5x}{1280\,(bx^2+a)^5b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^12/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -693/256*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + x/b^6 + 1/1280
*(4215*a*b^4*x^9 + 13270*a^2*b^3*x^7 + 16768*a^3*b^2*x^5 + 9770*a
^4*b*x^3 + 2185*a^5*x)/((b*x^2 + a)^5*b^6)
```

3.526 $\int \frac{x^{10}}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=121

$$\frac{63 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}-\frac{63x}{256b^5\,(a+bx^2)}-\frac{21x^3}{128b^4\,(a+bx^2)^2}-\frac{21x^5}{160b^3\,(a+bx^2)^3}-\frac{9x^7}{80b^2\,(a+bx^2)^4}-\frac{x^9}{10b\,(a+bx^2)^5}$$

```
[Out] -x^9/(10*b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*
x^5)/(160*b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) -
 (63*x)/(256*b^5*(a + b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(256*Sqrt[a]*b^(11/2))
```

Rubi [A] time = 0.184699, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{63 \tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}-\frac{63x}{256b^5\,(a+bx^2)}-\frac{21x^3}{128b^4\,(a+bx^2)^2}-\frac{21x^5}{160b^3\,(a+bx^2)^3}-\frac{9x^7}{80b^2\,(a+bx^2)^4}-\frac{x^9}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^9/(10*b*(a + b*x^2)^5) - (9*x^7)/(80*b^2*(a + b*x^2)^4) - (21*
x^5)/(160*b^3*(a + b*x^2)^3) - (21*x^3)/(128*b^4*(a + b*x^2)^2) -
 (63*x)/(256*b^5*(a + b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(256*Sqrt[a]*b^(11/2))
```

Rubi in Sympy [A] time = 36.3423, size = 112, normalized size = 0.93

$$-\frac{x^9}{10b\,(a+bx^2)^5}-\frac{9x^7}{80b^2\,(a+bx^2)^4}-\frac{21x^5}{160b^3\,(a+bx^2)^3}-\frac{21x^3}{128b^4\,(a+bx^2)^2}-\frac{63x}{256b^5\,(a+bx^2)}+\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -x**9/(10*b*(a + b*x**2)**5) - 9*x**7/(80*b**2*(a + b*x**2)**4) -
 21*x**5/(160*b**3*(a + b*x**2)**3) - 21*x**3/(128*b**4*(a + b*x*
*2)**2) - 63*x/(256*b**5*(a + b*x**2)) + 63*atan(sqrt(b)*x/sqrt(a
```

```
))/(256*sqrt(a)*b**(11/2))
```

Mathematica [A] time = 0.0877502, size = 88, normalized size = 0.73

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{a}b^{11/2}}-\frac{x\left(315a^4+1470a^3bx^2+2688a^2b^2x^4+2370ab^3x^6+965b^4x^8\right)}{1280b^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(x*(315*a^4 + 1470*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 2370*a*b^3*x^6
 + 965*b^4*x^8))/(1280*b^5*(a + b*x^2)^5) + (63*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(256*Sqrt[a]*b^(11/2))
```

Maple [A] time = 0.017, size = 80, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^5}\left(-\frac{193\,x^9}{256\,b}-\frac{237\,ax^7}{128\,b^2}-\frac{21\,a^2x^5}{10\,b^3}-\frac{147\,a^3x^3}{128\,b^4}-\frac{63\,a^4x}{256\,b^5}\right)+\frac{63}{256\,b^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (-193/256/b*x^9-237/128*a/b^2*x^7-21/10*a^2/b^3*x^5-147/128*a^3/b
^4*x^3-63/256*a^4/b^5*x)/(b*x^2+a)^5+63/256/b^5/(a*b)^(1/2)*arcta
n(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272994, size = 1, normalized size = 0.01

$$\left[\frac{315\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)-2\left(965\,b^4x^9+2370\,ab^3x^7+2688\,a\right.}{2560\left(b^{10}x^{10}+5\,ab^9x^8+10\,a^2b^8x^6+10\,a^3b^7x^4+5\,a^4b^6x^2+a^5b^5\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^
2*x^4 + 5*a^4*b*x^2 + a^5)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))
/(b*x^2 + a)) - 2*(965*b^4*x^9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^
5 + 1470*a^3*b*x^3 + 315*a^4*x)*sqrt(-a*b))/((b^10*x^10 + 5*a*b^9
*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
*sqrt(-a*b)), 1/1280*(315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^
6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*arctan(sqrt(a*b)*x/a) - (
965*b^4*x^9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3
+ 315*a^4*x)*sqrt(a*b))/((b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^
6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*sqrt(a*b))]
```

Sympy [A] time = 4.61379, size = 180, normalized size = 1.49

$$-\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{ab^{11}}}\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{512}-\frac{315a^4x+1470a^3bx^3+2688a^2b^2x^5+2370ab^3x^7+965b^4x^9}{1280a^5b^5+6400a^4b^6x^2+12800a^3b^7x^4+12800a^2b^8x^6+6400ab^9x^8+1280b^{10}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a*b**11))*log(-a*b**5*sqrt(-1/(a*b**11)) + x)/512 +
63*sqrt(-1/(a*b**11))*log(a*b**5*sqrt(-1/(a*b**11)) + x)/512 - (3
15*a**4*x + 1470*a**3*b*x**3 + 2688*a**2*b**2*x**5 + 2370*a*b**3*
x**7 + 965*b**4*x**9)/(1280*a**5*b**5 + 6400*a**4*b**6*x**2 + 128
00*a**3*b**7*x**4 + 12800*a**2*b**8*x**6 + 6400*a*b**9*x**8 + 128
0*b**10*x**10)
```

GIAC/XCAS [A] time = 0.271078, size = 105, normalized size = 0.87

$$\frac{63 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \sqrt{ab}b^5} - \frac{965 \, b^4 x^9 + 2370 \, ab^3 x^7 + 2688 \, a^2 b^2 x^5 + 1470 \, a^3 b x^3 + 315 \, a^4 x}{1280 \, (bx^2 + a)^5 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/1280*(965*b^4*x^
9 + 2370*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 1470*a^3*b*x^3 + 315*a^4*
x)/((b*x^2 + a)^5*b^5)
```

3.527 $\int \frac{x^8}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=122

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

```
[Out] -x^7/(10*b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x
^3)/(96*b^3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x
)/(256*a*b^4*(a + b*x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*
a^(3/2)*b^(9/2))
```

Rubi [A] time = 0.182228, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}+\frac{7x}{256ab^4\left(a+bx^2\right)}-\frac{7x}{128b^4\left(a+bx^2\right)^2}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{x^7}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^7/(10*b*(a + b*x^2)^5) - (7*x^5)/(80*b^2*(a + b*x^2)^4) - (7*x
^3)/(96*b^3*(a + b*x^2)^3) - (7*x)/(128*b^4*(a + b*x^2)^2) + (7*x
)/(256*a*b^4*(a + b*x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*
a^(3/2)*b^(9/2))
```

Rubi in Sympy [A] time = 35.0716, size = 112, normalized size = 0.92

$$-\frac{x^7}{10b\left(a+bx^2\right)^5}-\frac{7x^5}{80b^2\left(a+bx^2\right)^4}-\frac{7x^3}{96b^3\left(a+bx^2\right)^3}-\frac{7x}{128b^4\left(a+bx^2\right)^2}+\frac{7x}{256ab^4\left(a+bx^2\right)}+\frac{7\,\text{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{3}{2}}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -x**7/(10*b*(a + b*x**2)**5) - 7*x**5/(80*b**2*(a + b*x**2)**4) -
 7*x**3/(96*b**3*(a + b*x**2)**3) - 7*x/(128*b**4*(a + b*x**2)**2
) + 7*x/(256*a*b**4*(a + b*x**2)) + 7*atan(sqrt(b)*x/sqrt(a))/(25
```

```
6*a**(3/2)*b**(9/2))
```

Mathematica [A] time = 0.105656, size = 91, normalized size = 0.75

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{3/2}b^{9/2}}-\frac{x\left(105a^4+490a^3bx^2+896a^2b^2x^4+790ab^3x^6-105b^4x^8\right)}{3840ab^4\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(x*(105*a^4 + 490*a^3*b*x^2 + 896*a^2*b^2*x^4 + 790*a*b^3*x^6 -
105*b^4*x^8))/(3840*a*b^4*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(256*a^(3/2)*b^(9/2))
```

Maple [A] time = 0.015, size = 80, normalized size = 0.7

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{7\,x^9}{256\,a}-\frac{79\,x^7}{384\,b}-\frac{7\,ax^5}{30\,b^2}-\frac{49\,a^2x^3}{384\,b^3}-\frac{7\,a^3x}{256\,b^4}\right)+\frac{7}{256\,ab^4}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256/a*x^9-79/384/b*x^7-7/30*a/b^2*x^5-49/384*a^2/b^3*x^3-7/256
*a^3/b^4*x)/(b*x^2+a)^5+7/256/a/b^4/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270399, size = 1, normalized size = 0.01

$$\left[\frac{105\left(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\left(105\,b^{4}x^{9}-790\,ab^{3}x^{7}-896\,a^{2}b\right.}{7680\left(ab^{9}x^{10}+5\,a^{2}b^{8}x^{8}+10\,a^{3}b^{7}x^{6}+10\,a^{4}b^{6}x^{4}+5\,a^{5}b^{5}x^{2}+a^{6}b^{4}\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^
2*x^4 + 5*a^4*b*x^2 + a^5)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))
/(b*x^2 + a)) + 2*(105*b^4*x^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5
- 490*a^3*b*x^3 - 105*a^4*x)*sqrt(-a*b))/((a*b^9*x^10 + 5*a^2*b^8
*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)
*sqrt(-a*b)), 1/3840*(105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^
6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*arctan(sqrt(a*b)*x/a) + (
105*b^4*x^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 1
05*a^4*x)*sqrt(a*b))/((a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^
6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*sqrt(a*b))]
```

Sympy [A] time = 4.49052, size = 194, normalized size = 1.59

$$-\frac{7\sqrt{-\frac{1}{a^{3}b^{9}}}\log\left(-a^{2}b^{4}\sqrt{-\frac{1}{a^{3}b^{9}}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^{3}b^{9}}}\log\left(a^{2}b^{4}\sqrt{-\frac{1}{a^{3}b^{9}}}+x\right)}{512}+\frac{-105a^{4}x-490a^{3}bx^{3}-896a^{2}b^{2}x^{5}-790ab^{3}x^{7}+105b^{4}x^{9}}{3840a^{6}b^{4}+19200a^{5}b^{5}x^{2}+38400a^{4}b^{6}x^{4}+38400a^{3}b^{7}x^{6}+19200a^{2}b^{8}x^{8}+3840ab^{9}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**3*b**9))*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/
512 + 7*sqrt(-1/(a**3*b**9))*log(a**2*b**4*sqrt(-1/(a**3*b**9)) +
 x)/512 + (-105*a**4*x - 490*a**3*b*x**3 - 896*a**2*b**2*x**5 - 7
90*a*b**3*x**7 + 105*b**4*x**9)/(3840*a**6*b**4 + 19200*a**5*b**5
*x**2 + 38400*a**4*b**6*x**4 + 38400*a**3*b**7*x**6 + 19200*a**2*
b**8*x**8 + 3840*a*b**9*x**10)
```

GIAC/XCAS [A] time = 0.271863, size = 113, normalized size = 0.93

$$\frac{7 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \sqrt{ab}ab^4} + \frac{105 \, b^4 x^9 - 790 \, ab^3 x^7 - 896 \, a^2 b^2 x^5 - 490 \, a^3 b x^3 - 105 \, a^4 x}{3840 \, (bx^2 + a)^5 ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4) + 1/3840*(105*b^4*x
^9 - 790*a*b^3*x^7 - 896*a^2*b^2*x^5 - 490*a^3*b*x^3 - 105*a^4*x)
/((b*x^2 + a)^5*a*b^4)
```

3.528 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^3} \, dx$

Optimal. Leaf size=123

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{3x}{256a^2b^3(a+bx^2)}+\frac{x}{128ab^3(a+bx^2)^2}-\frac{x}{32b^3(a+bx^2)^3}-\frac{x^3}{16b^2(a+bx^2)^4}-\frac{x^5}{10b(a+bx^2)^5}$$

```
[Out] -x^5/(10*b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^
3*(a + b*x^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b
^3*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^
(7/2))
```

Rubi [A] time = 0.180164, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{3x}{256a^2b^3(a+bx^2)}+\frac{x}{128ab^3(a+bx^2)^2}-\frac{x}{32b^3(a+bx^2)^3}-\frac{x^3}{16b^2(a+bx^2)^4}-\frac{x^5}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^5/(10*b*(a + b*x^2)^5) - x^3/(16*b^2*(a + b*x^2)^4) - x/(32*b^
3*(a + b*x^2)^3) + x/(128*a*b^3*(a + b*x^2)^2) + (3*x)/(256*a^2*b
^3*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(5/2)*b^
(7/2))
```

Rubi in Sympy [A] time = 33.9606, size = 109, normalized size = 0.89

$$-\frac{x^5}{10b(a+bx^2)^5}-\frac{x^3}{16b^2(a+bx^2)^4}-\frac{x}{32b^3(a+bx^2)^3}+\frac{x}{128ab^3(a+bx^2)^2}+\frac{3x}{256a^2b^3(a+bx^2)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{5}{2}}b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -x**5/(10*b*(a + b*x**2)**5) - x**3/(16*b**2*(a + b*x**2)**4) - x
/(32*b**3*(a + b*x**2)**3) + x/(128*a*b**3*(a + b*x**2)**2) + 3*x
/(256*a**2*b**3*(a + b*x**2)) + 3*atan(sqrt(b)*x/sqrt(a))/(256*a*
```

```
*(5/2)*b**(7/2))
```

Mathematica [A] time = 0.105152, size = 91, normalized size = 0.74

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{5/2}b^{7/2}}+\frac{-15a^4x-70a^3bx^3-128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^2b^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 - 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b
^4*x^9)/(1280*a^2*b^3*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(256*a^(5/2)*b^(7/2))
```

Maple [A] time = 0.015, size = 78, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{3\,bx^9}{256\,a^2}+\frac{7\,x^7}{128\,a}-\frac{x^5}{10\,b}-\frac{7\,ax^3}{128\,b^2}-\frac{3\,a^2x}{256\,b^3}\right)+\frac{3}{256\,a^2b^3}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256*b/a^2*x^9+7/128/a*x^7-1/10/b*x^5-7/128*a/b^2*x^3-3/256*a^2
/b^3*x)/(b*x^2+a)^5+3/256/a^2/b^3/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272864, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\left(15\,b^{4}x^{9}+70\,ab^{3}x^{7}-128\,a^{2}b^{2}x^{5}\right.}{2560\left(a^{2}b^{8}x^{10}+5\,a^{3}b^{7}x^{8}+10\,a^{4}b^{6}x^{6}+10\,a^{5}b^{5}x^{4}+5\,a^{6}b^{4}x^{2}+a^{7}b^{3}\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2
*x^4 + 5*a^4*b*x^2 + a^5)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/
(b*x^2 + a)) + 2*(15*b^4*x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 7
0*a^3*b*x^3 - 15*a^4*x)*sqrt(-a*b))/((a^2*b^8*x^10 + 5*a^3*b^7*x^
8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*sq
rt(-a*b)), 1/1280*(15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 +
10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*arctan(sqrt(a*b)*x/a) + (15*b
^4*x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x
)*sqrt(a*b))/((a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10
*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*sqrt(a*b))]
```

Sympy [A] time = 4.26422, size = 196, normalized size = 1.59

$$-\frac{3\sqrt{-\frac{1}{a^{5}b^{7}}}\log\left(-a^{3}b^{3}\sqrt{-\frac{1}{a^{5}b^{7}}}+x\right)}{512}+\frac{3\sqrt{-\frac{1}{a^{5}b^{7}}}\log\left(a^{3}b^{3}\sqrt{-\frac{1}{a^{5}b^{7}}}+x\right)}{512}+\frac{-15a^{4}x-70a^{3}bx^{3}-128a^{2}b^{2}x^{5}+70ab^{3}x^{7}+15b^{4}x^{9}}{1280a^{7}b^{3}+6400a^{6}b^{4}x^{2}+12800a^{5}b^{5}x^{4}+12800a^{4}b^{6}x^{6}+6400a^{3}b^{7}x^{8}+1280a^{2}b^{8}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*b**7))*log(-a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/
512 + 3*sqrt(-1/(a**5*b**7))*log(a**3*b**3*sqrt(-1/(a**5*b**7)) +
 x)/512 + (-15*a**4*x - 70*a**3*b*x**3 - 128*a**2*b**2*x**5 + 70*
a*b**3*x**7 + 15*b**4*x**9)/(1280*a**7*b**3 + 6400*a**6*b**4*x**2
 + 12800*a**5*b**5*x**4 + 12800*a**4*b**6*x**6 + 6400*a**3*b**7*x
**8 + 1280*a**2*b**8*x**10)
```

GIAC/XCAS [A] time = 0.270989, size = 113, normalized size = 0.92

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \sqrt{ab}a^2b^3} + \frac{15 b^4x^9 + 70 ab^3x^7 - 128 a^2b^2x^5 - 70 a^3bx^3 - 15 a^4x}{1280 (bx^2 + a)^5 a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^3) + 1/1280*(15*b^4*
x^9 + 70*a*b^3*x^7 - 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/(
(b*x^2 + a)^5*a^2*b^3)
```

3.529 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=124

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{3x}{256a^3b^2(a+bx^2)}+\frac{x}{128a^2b^2(a+bx^2)^2}+\frac{x}{160ab^2(a+bx^2)^3}-\frac{3x}{80b^2(a+bx^2)^4}-\frac{x^3}{10b(a+bx^2)^5}$$

```
[Out] -x^3/(10*b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160
*a*b^2*(a + b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(25
6*a^3*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(
7/2)*b^(5/2))
```

Rubi [A] time = 0.176896, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{3x}{256a^3b^2(a+bx^2)}+\frac{x}{128a^2b^2(a+bx^2)^2}+\frac{x}{160ab^2(a+bx^2)^3}-\frac{3x}{80b^2(a+bx^2)^4}-\frac{x^3}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x^3/(10*b*(a + b*x^2)^5) - (3*x)/(80*b^2*(a + b*x^2)^4) + x/(160
*a*b^2*(a + b*x^2)^3) + x/(128*a^2*b^2*(a + b*x^2)^2) + (3*x)/(25
6*a^3*b^2*(a + b*x^2)) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(
7/2)*b^(5/2))
```

Rubi in Sympy [A] time = 32.5596, size = 112, normalized size = 0.9

$$-\frac{x^3}{10b(a+bx^2)^5}-\frac{3x}{80b^2(a+bx^2)^4}+\frac{x}{160ab^2(a+bx^2)^3}+\frac{x}{128a^2b^2(a+bx^2)^2}+\frac{3x}{256a^3b^2(a+bx^2)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{7}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -x**3/(10*b*(a + b*x**2)**5) - 3*x/(80*b**2*(a + b*x**2)**4) + x/
(160*a*b**2*(a + b*x**2)**3) + x/(128*a**2*b**2*(a + b*x**2)**2)
+ 3*x/(256*a**3*b**2*(a + b*x**2)) + 3*atan(sqrt(b)*x/sqrt(a))/(2
56*a**(7/2)*b**(5/2))
```

Mathematica [A] time = 0.0969715, size = 91, normalized size = 0.73

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{7/2}b^{5/2}}+\frac{-15a^4x-70a^3bx^3+128a^2b^2x^5+70ab^3x^7+15b^4x^9}{1280a^3b^2\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-15*a^4*x - 70*a^3*b*x^3 + 128*a^2*b^2*x^5 + 70*a*b^3*x^7 + 15*b
^4*x^9)/(1280*a^3*b^2*(a + b*x^2)^5) + (3*ArcTan[(Sqrt[b]*x)/Sqrt
[a]])/(256*a^(7/2)*b^(5/2))
```

Maple [A] time = 0.015, size = 78, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{3\,b^2x^9}{256\,a^3}+\frac{7\,bx^7}{128\,a^2}+\frac{x^5}{10\,a}-\frac{7\,x^3}{128\,b}-\frac{3\,ax}{256\,b^2}\right)+\frac{3}{256\,a^3b^2}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (3/256*b^2/a^3*x^9+7/128*b/a^2*x^7+1/10/a*x^5-7/128/b*x^3-3/256*a
/b^2*x)/(b*x^2+a)^5+3/256/a^3/b^2/(a*b)^(1/2)*arctan(x*b/(a*b)^(1
/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26658, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^5x^{10} + 5\,ab^4x^8 + 10\,a^2b^3x^6 + 10\,a^3b^2x^4 + 5\,a^4bx^2 + a^5\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right) + 2\left(15\,b^4x^9 + 70\,ab^3x^7 + 128\,a^2b^2x^5\right.}{2560\,(a^3b^7x^{10} + 5\,a^4b^6x^8 + 10\,a^5b^5x^6 + 10\,a^6b^4x^4 + 5\,a^7b^3x^2 + a^8b^2)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/2560*(15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2
*x^4 + 5*a^4*b*x^2 + a^5)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/
(b*x^2 + a)) + 2*(15*b^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 7
0*a^3*b*x^3 - 15*a^4*x)*sqrt(-a*b))/((a^3*b^7*x^10 + 5*a^4*b^6*x^
8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*sq
rt(-a*b)), 1/1280*(15*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 +
10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*arctan(sqrt(a*b)*x/a) + (15*b
^4*x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x
)*sqrt(a*b))/((a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10
*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*sqrt(a*b))]
```

Sympy [A] time = 4.14675, size = 196, normalized size = 1.58

$$-\frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}} + x\right)}{512} + \frac{3\sqrt{-\frac{1}{a^7b^5}}\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}} + x\right)}{512} + \frac{-15a^4x - 70a^3bx^3 + 128a^2b^2x^5 + 70ab^3x^7 + 15b^4x^9}{1280a^8b^2 + 6400a^7b^3x^2 + 12800a^6b^4x^4 + 12800a^5b^5x^6 + 6400a^4b^6x^8 + 1280a^3b^7x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3*sqrt(-1/(a**7*b**5))*log(-a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/
512 + 3*sqrt(-1/(a**7*b**5))*log(a**4*b**2*sqrt(-1/(a**7*b**5)) +
 x)/512 + (-15*a**4*x - 70*a**3*b*x**3 + 128*a**2*b**2*x**5 + 70*
a*b**3*x**7 + 15*b**4*x**9)/(1280*a**8*b**2 + 6400*a**7*b**3*x**2
 + 12800*a**6*b**4*x**4 + 12800*a**5*b**5*x**6 + 6400*a**4*b**6*x
```

```
**8 + 1280*a**3*b**7*x**10)
```

GIAC/XCAS [A] time = 0.269804, size = 113, normalized size = 0.91

$$\frac{3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \sqrt{ab}a^3b^2} + \frac{15 b^4x^9 + 70 ab^3x^7 + 128 a^2b^2x^5 - 70 a^3bx^3 - 15 a^4x}{1280 (bx^2 + a)^5 a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 3/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2) + 1/1280*(15*b^4*
x^9 + 70*a*b^3*x^7 + 128*a^2*b^2*x^5 - 70*a^3*b*x^3 - 15*a^4*x)/(
(b*x^2 + a)^5*a^3*b^2)
```

3.530 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=125

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

```
[Out] -x/(10*b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a
^2*b*(a + b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(25
6*a^4*b*(a + b*x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/
2)*b^(3/2))
```

Rubi [A] time = 0.167364, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{x}{80ab\left(a+bx^2\right)^4}-\frac{x}{10b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -x/(10*b*(a + b*x^2)^5) + x/(80*a*b*(a + b*x^2)^4) + (7*x)/(480*a
^2*b*(a + b*x^2)^3) + (7*x)/(384*a^3*b*(a + b*x^2)^2) + (7*x)/(25
6*a^4*b*(a + b*x^2)) + (7*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(9/
2)*b^(3/2))
```

Rubi in Sympy [A] time = 31.4254, size = 109, normalized size = 0.87

$$-\frac{x}{10b\left(a+bx^2\right)^5}+\frac{x}{80ab\left(a+bx^2\right)^4}+\frac{7x}{480a^2b\left(a+bx^2\right)^3}+\frac{7x}{384a^3b\left(a+bx^2\right)^2}+\frac{7x}{256a^4b\left(a+bx^2\right)}+\frac{7\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{9}{2}}b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -x/(10*b*(a + b*x**2)**5) + x/(80*a*b*(a + b*x**2)**4) + 7*x/(480
*a**2*b*(a + b*x**2)**3) + 7*x/(384*a**3*b*(a + b*x**2)**2) + 7*x
/(256*a**4*b*(a + b*x**2)) + 7*atan(sqrt(b)*x/sqrt(a))/(256*a**(9
```

```
/2)*b**(3/2))
```

Mathematica [A] time = 0.0983973, size = 91, normalized size = 0.73

$$\frac{7\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{9/2}b^{3/2}}+\frac{-105a^4x+790a^3bx^3+896a^2b^2x^5+490ab^3x^7+105b^4x^9}{3840a^4b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-105*a^4*x + 790*a^3*b*x^3 + 896*a^2*b^2*x^5 + 490*a*b^3*x^7 + 1
05*b^4*x^9)/(3840*a^4*b*(a + b*x^2)^5) + (7*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(256*a^(9/2)*b^(3/2))
```

Maple [A] time = 0.015, size = 80, normalized size = 0.6

$$\frac{1}{\left(bx^2+a\right)^5}\left(\frac{7\,b^3x^9}{256\,a^4}+\frac{49\,b^2x^7}{384\,a^3}+\frac{7\,bx^5}{30\,a^2}+\frac{79\,x^3}{384\,a}-\frac{7\,x}{256\,b}\right)+\frac{7}{256\,a^4b}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (7/256*b^3/a^4*x^9+49/384*b^2/a^3*x^7+7/30*b/a^2*x^5+79/384/a*x^3
-7/256/b*x)/(b*x^2+a)^5+7/256/a^4/b/(a*b)^(1/2)*arctan(x*b/(a*b)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266787, size = 1, normalized size = 0.01

$$\left[\frac{105\left(b^{5}x^{10}+5\,ab^{4}x^{8}+10\,a^{2}b^{3}x^{6}+10\,a^{3}b^{2}x^{4}+5\,a^{4}bx^{2}+a^{5}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\left(105\,b^{4}x^{9}+490\,ab^{3}x^{7}+896\,a^{2}b^{2}\right.}{7680\left(a^{4}b^{6}x^{10}+5\,a^{5}b^{5}x^{8}+10\,a^{6}b^{4}x^{6}+10\,a^{7}b^{3}x^{4}+5\,a^{8}b^{2}x^{2}+a^{9}b\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] [1/7680*(105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^
2*x^4 + 5*a^4*b*x^2 + a^5)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))
/(b*x^2 + a)) + 2*(105*b^4*x^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5
+ 790*a^3*b*x^3 - 105*a^4*x)*sqrt(-a*b))/((a^4*b^6*x^10 + 5*a^5*b
^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
*sqrt(-a*b)), 1/3840*(105*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^
6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*arctan(sqrt(a*b)*x/a) + (
105*b^4*x^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 1
05*a^4*x)*sqrt(a*b))/((a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*
x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*sqrt(a*b))]
```

Sympy [A] time = 4.06985, size = 190, normalized size = 1.52

$$-\frac{7\sqrt{-\frac{1}{a^{9}b^{3}}}\log\left(-a^{5}b\sqrt{-\frac{1}{a^{9}b^{3}}}+x\right)}{512}+\frac{7\sqrt{-\frac{1}{a^{9}b^{3}}}\log\left(a^{5}b\sqrt{-\frac{1}{a^{9}b^{3}}}+x\right)}{512}+\frac{-105a^{4}x+790a^{3}bx^{3}+896a^{2}b^{2}x^{5}+490ab^{3}x^{7}+105b^{4}x^{9}}{3840a^{9}b+19200a^{8}b^{2}x^{2}+38400a^{7}b^{3}x^{4}+38400a^{6}b^{4}x^{6}+19200a^{5}b^{5}x^{8}+3840a^{4}b^{6}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -7*sqrt(-1/(a**9*b**3))*log(-a**5*b*sqrt(-1/(a**9*b**3)) + x)/512
 + 7*sqrt(-1/(a**9*b**3))*log(a**5*b*sqrt(-1/(a**9*b**3)) + x)/51
2 + (-105*a**4*x + 790*a**3*b*x**3 + 896*a**2*b**2*x**5 + 490*a*b
**3*x**7 + 105*b**4*x**9)/(3840*a**9*b + 19200*a**8*b**2*x**2 + 3
8400*a**7*b**3*x**4 + 38400*a**6*b**4*x**6 + 19200*a**5*b**5*x**8
 + 3840*a**4*b**6*x**10)
```

GIAC/XCAS [A] time = 0.270129, size = 113, normalized size = 0.9

$$\frac{7 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256 \sqrt{ab}a^4b} + \frac{105 b^4x^9 + 490 ab^3x^7 + 896 a^2b^2x^5 + 790 a^3bx^3 - 105 a^4x}{3840 (bx^2 + a)^5 a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 7/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b) + 1/3840*(105*b^4*x
^9 + 490*a*b^3*x^7 + 896*a^2*b^2*x^5 + 790*a^3*b*x^3 - 105*a^4*x)
/((b*x^2 + a)^5*a^4*b)
```

3.531 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=113

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{63x}{256a^5(a+bx^2)}+\frac{21x}{128a^4(a+bx^2)^2}+\frac{21x}{160a^3(a+bx^2)^3}+\frac{9x}{80a^2(a+bx^2)^4}+\frac{x}{10a(a+bx^2)^5}$$

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(1
60*a^3*(a + b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(
256*a^5*(a + b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(1
1/2)*Sqrt[b])
```

Rubi [A] time = 0.145872, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{63\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{11/2}\sqrt{b}}+\frac{63x}{256a^5(a+bx^2)}+\frac{21x}{128a^4(a+bx^2)^2}+\frac{21x}{160a^3(a+bx^2)^3}+\frac{9x}{80a^2(a+bx^2)^4}+\frac{x}{10a(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] x/(10*a*(a + b*x^2)^5) + (9*x)/(80*a^2*(a + b*x^2)^4) + (21*x)/(1
60*a^3*(a + b*x^2)^3) + (21*x)/(128*a^4*(a + b*x^2)^2) + (63*x)/(
256*a^5*(a + b*x^2)) + (63*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(1
1/2)*Sqrt[b])
```

Rubi in Sympy [A] time = 27.4248, size = 105, normalized size = 0.93

$$\frac{x}{10a(a+bx^2)^5}+\frac{9x}{80a^2(a+bx^2)^4}+\frac{21x}{160a^3(a+bx^2)^3}+\frac{21x}{128a^4(a+bx^2)^2}+\frac{63x}{256a^5(a+bx^2)}+\frac{63\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{11}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] x/(10*a*(a + b*x**2)**5) + 9*x/(80*a**2*(a + b*x**2)**4) + 21*x/(
160*a**3*(a + b*x**2)**3) + 21*x/(128*a**4*(a + b*x**2)**2) + 63*
x/(256*a**5*(a + b*x**2)) + 63*atan(sqrt(b)*x/sqrt(a))/(256*a**(1
```

```
1/2)*sqrt(b))
```

Mathematica [A] time = 0.0804735, size = 89, normalized size = 0.79

$$\frac{\frac{\sqrt{a}x(965a^4+2370a^3bx^2+2688a^2b^2x^4+1470ab^3x^6+315b^4x^8)}{(a+bx^2)^5}+\frac{315\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{1280a^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3),x]
```

```
[Out] ((Sqrt[a]*x*(965*a^4 + 2370*a^3*b*x^2 + 2688*a^2*b^2*x^4 + 1470*a
*b^3*x^6 + 315*b^4*x^8))/(a + b*x^2)^5 + (315*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/Sqrt[b])/(1280*a^(11/2))
```

Maple [A] time = 0.006, size = 96, normalized size = 0.9

$$\frac{x}{10\,a\,(bx^2+a)^5}+\frac{9\,x}{80\,a^2\,(bx^2+a)^4}+\frac{21\,x}{160\,a^3\,(bx^2+a)^3}+\frac{21\,x}{128\,a^4\,(bx^2+a)^2}+\frac{63\,x}{256\,a^5\,(bx^2+a)}+\frac{63}{256\,a^5}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1/10*x/a/(b*x^2+a)^5+9/80*x/a^2/(b*x^2+a)^4+21/160*x/a^3/(b*x^2+a
)^3+21/128*x/a^4/(b*x^2+a)^2+63/256*x/a^5/(b*x^2+a)+63/256/a^5/(a
*b)^(1/2)*arctan(x*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265708, size = 1, normalized size = 0.01

$$\left[\frac{315\left(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(315\,b^4x^9+1470\,ab^3x^7+2688\,a\right.}{2560\left(a^5b^5x^{10}+5\,a^6b^4x^8+10\,a^7b^3x^6+10\,a^8b^2x^4+5\,a^9bx^2+a^{10}\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3),x, algorithm="fricas")
```

```
[Out] [1/2560*(315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^
2*x^4 + 5*a^4*b*x^2 + a^5)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))
/(b*x^2 + a)) + 2*(315*b^4*x^9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^
5 + 2370*a^3*b*x^3 + 965*a^4*x)*sqrt(-a*b))/((a^5*b^5*x^10 + 5*a^
6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)
*sqrt(-a*b)), 1/1280*(315*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^
6 + 10*a^3*b^2*x^4 + 5*a^4*b*x^2 + a^5)*arctan(sqrt(a*b)*x/a) + (
315*b^4*x^9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3
+ 965*a^4*x)*sqrt(a*b))/((a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b
^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*sqrt(a*b))]
```

Sympy [A] time = 4.12812, size = 177, normalized size = 1.57

$$-\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(-a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{63\sqrt{-\frac{1}{a^{11}b}}\log\left(a^6\sqrt{-\frac{1}{a^{11}b}}+x\right)}{512}+\frac{965a^4x+2370a^3bx^3+2688a^2b^2x^5+1470ab^3x^7+315b^4x^9}{1280a^{10}+6400a^9bx^2+12800a^8b^2x^4+12800a^7b^3x^6+6400a^6b^4x^8+1280a^5b^5x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -63*sqrt(-1/(a**11*b))*log(-a**6*sqrt(-1/(a**11*b)) + x)/512 + 63
*sqrt(-1/(a**11*b))*log(a**6*sqrt(-1/(a**11*b)) + x)/512 + (965*a
**4*x + 2370*a**3*b*x**3 + 2688*a**2*b**2*x**5 + 1470*a*b**3*x**7
 + 315*b**4*x**9)/(1280*a**10 + 6400*a**9*b*x**2 + 12800*a**8*b**
2*x**4 + 12800*a**7*b**3*x**6 + 6400*a**6*b**4*x**8 + 1280*a**5*b
**5*x**10)
```

GIAC/XCAS [A] time = 0.26998, size = 105, normalized size = 0.93

$$\frac{63\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^{5}}+\frac{315\,b^{4}x^{9}+1470\,ab^{3}x^{7}+2688\,a^{2}b^{2}x^{5}+2370\,a^{3}bx^{3}+965\,a^{4}x}{1280\,(bx^{2}+a)^{5}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3),x, algorithm="giac")
```

```
[Out] 63/256*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/1280*(315*b^4*x^
9 + 1470*a*b^3*x^7 + 2688*a^2*b^2*x^5 + 2370*a^3*b*x^3 + 965*a^4*
x)/((b*x^2 + a)^5*a^5)
```

3.532 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=133

$$-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{231}{256a^5x(a+bx^2)}+\frac{231}{640a^4x(a+bx^2)^2}+\frac{33}{160a^3x(a+bx^2)^3}+\frac{11}{80a^2x(a+bx^2)^4}+\frac{1}{10ax(a+bx^2)^5}$$

```
[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b
*x^2)^4) + 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x
^2)^2) + 231/(256*a^5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rubi [A] time = 0.229162, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{693}{256a^6x}+\frac{231}{256a^5x(a+bx^2)}+\frac{231}{640a^4x(a+bx^2)^2}+\frac{33}{160a^3x(a+bx^2)^3}+\frac{11}{80a^2x(a+bx^2)^4}+\frac{1}{10ax(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -693/(256*a^6*x) + 1/(10*a*x*(a + b*x^2)^5) + 11/(80*a^2*x*(a + b
*x^2)^4) + 33/(160*a^3*x*(a + b*x^2)^3) + 231/(640*a^4*x*(a + b*x
^2)^2) + 231/(256*a^5*x*(a + b*x^2)) - (693*Sqrt[b]*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Rubi in Sympy [A] time = 50.2141, size = 116, normalized size = 0.87

$$\frac{1}{10ax(a+bx^2)^5}+\frac{11}{80a^2x(a+bx^2)^4}+\frac{33}{160a^3x(a+bx^2)^3}+\frac{231}{640a^4x(a+bx^2)^2}+\frac{231}{256a^5x(a+bx^2)}-\frac{693}{256a^6x}-\frac{693\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 1/(10*a*x*(a + b*x**2)**5) + 11/(80*a**2*x*(a + b*x**2)**4) + 33/
(160*a**3*x*(a + b*x**2)**3) + 231/(640*a**4*x*(a + b*x**2)**2) +
 231/(256*a**5*x*(a + b*x**2)) - 693/(256*a**6*x) - 693*sqrt(b)*a
tan(sqrt(b)*x/sqrt(a))/(256*a**(13/2))
```

Mathematica [A] time = 0.101219, size = 101, normalized size = 0.76

$$-\frac{693\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{13/2}}-\frac{1280a^5+10615a^4bx^2+26070a^3b^2x^4+29568a^2b^3x^6+16170ab^4x^8+3465b^5x^{10}}{1280a^6x\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -(1280*a^5 + 10615*a^4*b*x^2 + 26070*a^3*b^2*x^4 + 29568*a^2*b^3*
x^6 + 16170*a*b^4*x^8 + 3465*b^5*x^10)/(1280*a^6*x*(a + b*x^2)^5)
 - (693*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(13/2))
```

Maple [A] time = 0.021, size = 126, normalized size = 1.

$$-\frac{437\,b^5x^9}{256\,a^6\left(bx^2+a\right)^5}-\frac{977\,b^4x^7}{128\,a^5\left(bx^2+a\right)^5}-\frac{131\,b^3x^5}{10\,a^4\left(bx^2+a\right)^5}-\frac{1327\,b^2x^3}{128\,a^3\left(bx^2+a\right)^5}-\frac{843\,bx}{256\,a^2\left(bx^2+a\right)^5}-\frac{693\,b}{256\,a^6}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{a^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -437/256/a^6*b^5/(b*x^2+a)^5*x^9-977/128/a^5*b^4/(b*x^2+a)^5*x^7-
131/10/a^4*b^3/(b*x^2+a)^5*x^5-1327/128/a^3*b^2/(b*x^2+a)^5*x^3-8
43/256/a^2*b/(b*x^2+a)^5*x-693/256/a^6*b/(a*b)^(1/2)*arctan(x*b/(
a*b)^(1/2))-1/a^6/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269713, size = 1, normalized size = 0.01

$$\left[-\frac{6930\,b^5x^{10} + 32340\,ab^4x^8 + 59136\,a^2b^3x^6 + 52140\,a^3b^2x^4 + 21230\,a^4bx^2 + 2560\,a^5 - 3465\,\left(b^5x^{11} + 5\,ab^4x^9 + 10\,a^2b^3x^7 + \cdots}{2560\,(a^6b^5x^{11} + 5\,a^7b^4x^9 + 10\,a^8b^3x^7 + 10\,a^9b^2x^5 + 5\,a^{10}bx^3 + a^{11}x\cdots}\right.$$

$$-\frac{3465\,b^5x^{10} + 16170\,ab^4x^8 + 29568\,a^2b^3x^6 + 26070\,a^3b^2x^4 + 10615\,a^4bx^2 + 1280\,a^5 + 3465\,\left(b^5x^{11} + 5\,ab^4x^9 + 10\,a^2b^3x^7 + 1\cdots}{1280\,(a^6b^5x^{11} + 5\,a^7b^4x^9 + 10\,a^8b^3x^7 + 10\,a^9b^2x^5 + 5\,a^{10}bx^3 + a^{11}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(6930*b^5*x^10 + 32340*a*b^4*x^8 + 59136*a^2*b^3*x^6 + 5
2140*a^3*b^2*x^4 + 21230*a^4*b*x^2 + 2560*a^5 - 3465*(b^5*x^11 +
5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^2*x^5 + 5*a^4*b*x^3 + a^5
*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(
a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 +
5*a^10*b*x^3 + a^11*x), -1/1280*(3465*b^5*x^10 + 16170*a*b^4*x^8
+ 29568*a^2*b^3*x^6 + 26070*a^3*b^2*x^4 + 10615*a^4*b*x^2 + 1280*
a^5 + 3465*(b^5*x^11 + 5*a*b^4*x^9 + 10*a^2*b^3*x^7 + 10*a^3*b^2*
x^5 + 5*a^4*b*x^3 + a^5*x)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(
a^6*b^5*x^11 + 5*a^7*b^4*x^9 + 10*a^8*b^3*x^7 + 10*a^9*b^2*x^5 +
5*a^10*b*x^3 + a^11*x)]
```

Sympy [A] time = 17.7159, size = 185, normalized size = 1.39

$$\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(-\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{693\sqrt{-\frac{b}{a^{13}}}\log\left(\frac{a^7\sqrt{-\frac{b}{a^{13}}}}{b}+x\right)}{512}-\frac{1280a^5+10615a^4bx^2+26070a^3b^2x^4+29568a^2b^3x^6+16170ab^4x^8+3465b^5x^{10}}{1280a^{11}x+6400a^{10}bx^3+12800a^9b^2x^5+12800a^8b^3x^7+6400a^7b^4x^9+1280a^6b^5x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 693*sqrt(-b/a**13)*log(-a**7*sqrt(-b/a**13)/b + x)/512 - 693*sqrt
(-b/a**13)*log(a**7*sqrt(-b/a**13)/b + x)/512 - (1280*a**5 + 1061
5*a**4*b*x**2 + 26070*a**3*b**2*x**4 + 29568*a**2*b**3*x**6 + 161
70*a*b**4*x**8 + 3465*b**5*x**10)/(1280*a**11*x + 6400*a**10*b*x*
*3 + 12800*a**9*b**2*x**5 + 12800*a**8*b**3*x**7 + 6400*a**7*b**4
*x**9 + 1280*a**6*b**5*x**11)
```

GIAC/XCAS [A] time = 0.270653, size = 122, normalized size = 0.92

$$-\frac{693\, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab} a^{6}} - \frac{1}{a^{6} x} - \frac{2185\, b^{5} x^{9} + 9770\, a b^{4} x^{7} + 16768\, a^{2} b^{3} x^{5} + 13270\, a^{3} b^{2} x^{3} + 4215\, a^{4} b x}{1280\, (b x^{2} + a)^{5} a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^2),x, algorithm="giac")
```

```
[Out] -693/256*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) - 1/(a^6*x) - 1/
1280*(2185*b^5*x^9 + 9770*a*b^4*x^7 + 16768*a^2*b^3*x^5 + 13270*a
^3*b^2*x^3 + 4215*a^4*b*x)/((b*x^2 + a)^5*a^6)
```

3.533 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=144

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003b}{256a^7x}-\frac{1001}{256a^6x^3}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x
^2)^5) + 13/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*
x^2)^3) + 429/(640*a^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a
 + b*x^2)) + (3003*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(1
5/2))
```

Rubi [A] time = 0.256513, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{3003b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{15/2}}+\frac{3003b}{256a^7x}-\frac{1001}{256a^6x^3}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{1}{10ax^3\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -1001/(256*a^6*x^3) + (3003*b)/(256*a^7*x) + 1/(10*a*x^3*(a + b*x
^2)^5) + 13/(80*a^2*x^3*(a + b*x^2)^4) + 143/(480*a^3*x^3*(a + b*
x^2)^3) + 429/(640*a^4*x^3*(a + b*x^2)^2) + 3003/(1280*a^5*x^3*(a
 + b*x^2)) + (3003*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(1
5/2))
```

Rubi in Sympy [A] time = 58.8596, size = 136, normalized size = 0.94

$$\frac{1}{10ax^3\left(a+bx^2\right)^5}+\frac{13}{80a^2x^3\left(a+bx^2\right)^4}+\frac{143}{480a^3x^3\left(a+bx^2\right)^3}+\frac{429}{640a^4x^3\left(a+bx^2\right)^2}+\frac{3003}{1280a^5x^3\left(a+bx^2\right)}-\frac{1001}{256a^6x^3}+\frac{3003b}{256a^7x}+\frac{3003b^{\frac{3}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 1/(10*a*x**3*(a + b*x**2)**5) + 13/(80*a**2*x**3*(a + b*x**2)**4)
 + 143/(480*a**3*x**3*(a + b*x**2)**3) + 429/(640*a**4*x**3*(a +
b*x**2)**2) + 3003/(1280*a**5*x**3*(a + b*x**2)) - 1001/(256*a**6
*x**3) + 3003*b/(256*a**7*x) + 3003*b**(3/2)*atan(sqrt(b)*x/sqrt(
a))/(256*a**(15/2))
```

Mathematica [A] time = 0.115468, size = 113, normalized size = 0.78

$$\frac{\frac{\sqrt{a}\left(-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}\right)}{x^3\left(a+bx^2\right)^5}+45045b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3840a^{15/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] ((Sqrt[a]*(-1280*a^6 + 16640*a^5*b*x^2 + 137995*a^4*b^2*x^4 + 338
910*a^3*b^3*x^6 + 384384*a^2*b^4*x^8 + 210210*a*b^5*x^10 + 45045*
b^6*x^12))/(x^3*(a + b*x^2)^5) + 45045*b^(3/2)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(3840*a^(15/2))
```

Maple [A] time = 0.024, size = 139, normalized size = 1.

$$\frac{1467\,b^6x^9}{256\,a^7\left(bx^2+a\right)^5}+\frac{9629\,b^5x^7}{384\,a^6\left(bx^2+a\right)^5}+\frac{1253\,b^4x^5}{30\,a^5\left(bx^2+a\right)^5}+\frac{12131\,b^3x^3}{384\,a^4\left(bx^2+a\right)^5}+\frac{2373\,b^2x}{256\,a^3\left(bx^2+a\right)^5}+\frac{3003\,b^2}{256\,a^7}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{3\,a^6x^3}+6\,\frac{b}{a^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 1467/256/a^7*b^6/(b*x^2+a)^5*x^9+9629/384/a^6*b^5/(b*x^2+a)^5*x^7
+1253/30/a^5*b^4/(b*x^2+a)^5*x^5+12131/384/a^4*b^3/(b*x^2+a)^5*x^
3+2373/256/a^3*b^2/(b*x^2+a)^5*x+3003/256/a^7*b^2/(a*b)^(1/2)*arc
tan(x*b/(a*b)^(1/2))-1/3/a^6/x^3+6*b/a^7/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27208, size = 1, normalized size = 0.01

$$\left[\frac{90090\,b^6x^{12} + 420420\,ab^5x^{10} + 768768\,a^2b^4x^8 + 677820\,a^3b^3x^6 + 275990\,a^4b^2x^4 + 33280\,a^5bx^2 - 2560\,a^6 + 45045\,(b^6x^{13} +}{7680\,(a^7b^5x^{13} + 5\,a^8b^4x^{11} + 10\,a^9b^3x^9 + 10\,a^{10}b^2x^7 + 5\,a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^4),x, algorithm="fricas")
```

```
[Out] [1/7680*(90090*b^6*x^12 + 420420*a*b^5*x^10 + 768768*a^2*b^4*x^8
+ 677820*a^3*b^3*x^6 + 275990*a^4*b^2*x^4 + 33280*a^5*b*x^2 - 256
0*a^6 + 45045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*
b^3*x^7 + 5*a^4*b^2*x^5 + a^5*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^5*x^13 + 5*a^8*b^4*x^11 +
10*a^9*b^3*x^9 + 10*a^10*b^2*x^7 + 5*a^11*b*x^5 + a^12*x^3), 1/38
40*(45045*b^6*x^12 + 210210*a*b^5*x^10 + 384384*a^2*b^4*x^8 + 338
910*a^3*b^3*x^6 + 137995*a^4*b^2*x^4 + 16640*a^5*b*x^2 - 1280*a^6
 + 45045*(b^6*x^13 + 5*a*b^5*x^11 + 10*a^2*b^4*x^9 + 10*a^3*b^3*x
^7 + 5*a^4*b^2*x^5 + a^5*b*x^3)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a)
)))/(a^7*b^5*x^13 + 5*a^8*b^4*x^11 + 10*a^9*b^3*x^9 + 10*a^10*b^2
*x^7 + 5*a^11*b*x^5 + a^12*x^3)]
```

Sympy [A] time = 40.1873, size = 209, normalized size = 1.45

$$-\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(-\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{3003\sqrt{-\frac{b^3}{a^{15}}}\log\left(\frac{a^8\sqrt{-\frac{b^3}{a^{15}}}}{b^2}+x\right)}{512}+\frac{-1280a^6+16640a^5bx^2+137995a^4b^2x^4+338910a^3b^3x^6+384384a^2b^4x^8+210210ab^5x^{10}+45045b^6x^{12}}{3840a^{12}x^3+19200a^{11}bx^5+38400a^{10}b^2x^7+38400a^9b^3x^9+19200a^8b^4x^{11}+3840a^7b^5x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] -3003*sqrt(-b**3/a**15)*log(-a**8*sqrt(-b**3/a**15)/b**2 + x)/512
 + 3003*sqrt(-b**3/a**15)*log(a**8*sqrt(-b**3/a**15)/b**2 + x)/51
2 + (-1280*a**6 + 16640*a**5*b*x**2 + 137995*a**4*b**2*x**4 + 338
910*a**3*b**3*x**6 + 384384*a**2*b**4*x**8 + 210210*a*b**5*x**10
+ 45045*b**6*x**12)/(3840*a**12*x**3 + 19200*a**11*b*x**5 + 38400
*a**10*b**2*x**7 + 38400*a**9*b**3*x**9 + 19200*a**8*b**4*x**11 +
 3840*a**7*b**5*x**13)
```

GIAC/XCAS [A] time = 0.271375, size = 140, normalized size = 0.97

$$\frac{3003\, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\, \sqrt{ab} a^7} + \frac{18\, bx^2 - a}{3\, a^7 x^3} + \frac{22005\, b^6 x^9 + 96290\, ab^5 x^7 + 160384\, a^2 b^4 x^5 + 121310\, a^3 b^3 x^3 + 35595\, a^4 b^2 x}{3840\, (bx^2 + a)^5 a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^4),x, algorithm="giac")
```

```
[Out] 3003/256*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7) + 1/3*(18*b*x^
2 - a)/(a^7*x^3) + 1/3840*(22005*b^6*x^9 + 96290*a*b^5*x^7 + 1603
84*a^2*b^4*x^5 + 121310*a^3*b^3*x^3 + 35595*a^4*b^2*x)/((b*x^2 +
a)^5*a^7)
```

3.534 $\int \frac{1}{x^6(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=157

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}-\frac{9009b^2}{256a^8x}+\frac{3003b}{256a^7x^3}-\frac{9009}{1280a^6x^5}+\frac{1287}{256a^5x^5(a+bx^2)}+\frac{143}{128a^4x^5(a+bx^2)^2}+\frac{13}{32a^3x^5(a+bx^2)^3}+\frac{3}{16a^2x^5(a+bx^2)^4}+\frac{1}{10ax^5(a+bx^2)^5}$$

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a
^8*x) + 1/(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4)
 + 13/(32*a^3*x^5*(a + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2
) + 1287/(256*a^5*x^5*(a + b*x^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rubi [A] time = 0.292419, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}-\frac{9009b^2}{256a^8x}+\frac{3003b}{256a^7x^3}-\frac{9009}{1280a^6x^5}+\frac{1287}{256a^5x^5(a+bx^2)}+\frac{143}{128a^4x^5(a+bx^2)^2}+\frac{13}{32a^3x^5(a+bx^2)^3}+\frac{3}{16a^2x^5(a+bx^2)^4}+\frac{1}{10ax^5(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -9009/(1280*a^6*x^5) + (3003*b)/(256*a^7*x^3) - (9009*b^2)/(256*a
^8*x) + 1/(10*a*x^5*(a + b*x^2)^5) + 3/(16*a^2*x^5*(a + b*x^2)^4)
 + 13/(32*a^3*x^5*(a + b*x^2)^3) + 143/(128*a^4*x^5*(a + b*x^2)^2
) + 1287/(256*a^5*x^5*(a + b*x^2)) - (9009*b^(5/2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(256*a^(17/2))
```

Rubi in Sympy [A] time = 67.1471, size = 150, normalized size = 0.96

$$\frac{1}{10ax^5(a+bx^2)^5}+\frac{3}{16a^2x^5(a+bx^2)^4}+\frac{13}{32a^3x^5(a+bx^2)^3}+\frac{143}{128a^4x^5(a+bx^2)^2}+\frac{1287}{256a^5x^5(a+bx^2)}-\frac{9009}{1280a^6x^5}+\frac{3003b}{256a^7x^3}-\frac{9009b^2}{256a^8x}-\frac{9009b^{\frac{5}{2}}\operatorname{atan}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 1/(10*a*x**5*(a + b*x**2)**5) + 3/(16*a**2*x**5*(a + b*x**2)**4)
+ 13/(32*a**3*x**5*(a + b*x**2)**3) + 143/(128*a**4*x**5*(a + b*x
**2)**2) + 1287/(256*a**5*x**5*(a + b*x**2)) - 9009/(1280*a**6*x*
*5) + 3003*b/(256*a**7*x**3) - 9009*b**2/(256*a**8*x) - 9009*b**(
5/2)*atan(sqrt(b)*x/sqrt(a))/(256*a**(17/2))
```

Mathematica [A] time = 0.117757, size = 123, normalized size = 0.78

$$-\frac{9009b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256a^{17/2}}-\frac{256a^7-1280a^6bx^2+16640a^5b^2x^4+137995a^4b^3x^6+338910a^3b^4x^8+384384a^2b^5x^{10}+210210ab^6x^{12}+45045b^7x^{14}}{1280a^8x^5\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -(256*a^7 - 1280*a^6*b*x^2 + 16640*a^5*b^2*x^4 + 137995*a^4*b^3*x
^6 + 338910*a^3*b^4*x^8 + 384384*a^2*b^5*x^10 + 210210*a*b^6*x^12
 + 45045*b^7*x^14)/(1280*a^8*x^5*(a + b*x^2)^5) - (9009*b^(5/2)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(256*a^(17/2))
```

Maple [A] time = 0.025, size = 150, normalized size = 1.

$$-\frac{3633\,b^7x^9}{256\,a^8\left(bx^2+a\right)^5}-\frac{7837\,b^6x^7}{128\,a^7\left(bx^2+a\right)^5}-\frac{1001\,b^5x^5}{10\,a^6\left(bx^2+a\right)^5}-\frac{9443\,b^4x^3}{128\,a^5\left(bx^2+a\right)^5}-\frac{5327\,b^3x}{256\,a^4\left(bx^2+a\right)^5}-\frac{9009\,b^3}{256\,a^8}\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{5\,a^6x^5}-21\,\frac{b^2}{a^8x}+2\,\frac{b}{a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^6/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -3633/256/a^8*b^7/(b*x^2+a)^5*x^9-7837/128/a^7*b^6/(b*x^2+a)^5*x^
7-1001/10/a^6*b^5/(b*x^2+a)^5*x^5-9443/128/a^5*b^4/(b*x^2+a)^5*x^
3-5327/256/a^4*b^3/(b*x^2+a)^5*x-9009/256/a^8*b^3/(a*b)^(1/2)*arc
tan(x*b/(a*b)^(1/2))-1/5/a^6/x^5-21*b^2/a^8/x+2*b/a^7/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^6),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272516, size = 1, normalized size = 0.01

$$\Bigg[-\frac{90090\,b^7x^{14} + 420420\,ab^6x^{12} + 768768\,a^2b^5x^{10} + 677820\,a^3b^4x^8 + 275990\,a^4b^3x^6 + 33280\,a^5b^2x^4 - 2560\,a^6bx^2 + 512\,a^7 -}{2560\,(a^8b^5x^{15} + 5\,a^9b^4x^{13} + 10\,a^{10}b^3x^{11} + 10\,a^1}$$

$$-\frac{45045\,b^7x^{14} + 210210\,ab^6x^{12} + 384384\,a^2b^5x^{10} + 338910\,a^3b^4x^8 + 137995\,a^4b^3x^6 + 16640\,a^5b^2x^4 - 1280\,a^6bx^2 + 256\,a^7 +}{1280\,(a^8b^5x^{15} + 5\,a^9b^4x^{13} + 10\,a^{10}b^3x^{11} + 10\,a^{11}b^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^6),x, algorithm="fricas")
```

```
[Out] [-1/2560*(90090*b^7*x^14 + 420420*a*b^6*x^12 + 768768*a^2*b^5*x^1
0 + 677820*a^3*b^4*x^8 + 275990*a^4*b^3*x^6 + 33280*a^5*b^2*x^4 -
 2560*a^6*b*x^2 + 512*a^7 - 45045*(b^7*x^15 + 5*a*b^6*x^13 + 10*a
^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7 + a^5*b^2*x^5)*sqrt(
-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^8*b^5*x
^15 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*b^2*x^9 + 5*a^1
2*b*x^7 + a^13*x^5), -1/1280*(45045*b^7*x^14 + 210210*a*b^6*x^12
+ 384384*a^2*b^5*x^10 + 338910*a^3*b^4*x^8 + 137995*a^4*b^3*x^6 +
 16640*a^5*b^2*x^4 - 1280*a^6*b*x^2 + 256*a^7 + 45045*(b^7*x^15 +
 5*a*b^6*x^13 + 10*a^2*b^5*x^11 + 10*a^3*b^4*x^9 + 5*a^4*b^3*x^7
+ a^5*b^2*x^5)*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))))/(a^8*b^5*x^15
 + 5*a^9*b^4*x^13 + 10*a^10*b^3*x^11 + 10*a^11*b^2*x^9 + 5*a^12*b
*x^7 + a^13*x^5)]
```

Sympy [A] time = 88.0252, size = 221, normalized size = 1.41

$$\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(-\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{9009\sqrt{-\frac{b^5}{a^{17}}}\log\left(\frac{a^9\sqrt{-\frac{b^5}{a^{17}}}}{b^3}+x\right)}{512}-\frac{256a^7-1280a^6bx^2+16640a^5b^2x^4+137995a^4b^3x^6+338910a^3b^4x^8+384384a^2b^5x^{10}+210210ab^6x^{12}+45045b^7x^{14}}{1280a^{13}x^5+6400a^{12}bx^7+12800a^{11}b^2x^9+12800a^{10}b^3x^{11}+6400a^9b^4x^{13}+1280a^8b^5x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**6/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 9009*sqrt(-b**5/a**17)*log(-a**9*sqrt(-b**5/a**17)/b**3 + x)/512
- 9009*sqrt(-b**5/a**17)*log(a**9*sqrt(-b**5/a**17)/b**3 + x)/512
 - (256*a**7 - 1280*a**6*b*x**2 + 16640*a**5*b**2*x**4 + 137995*a
**4*b**3*x**6 + 338910*a**3*b**4*x**8 + 384384*a**2*b**5*x**10 +
210210*a*b**6*x**12 + 45045*b**7*x**14)/(1280*a**13*x**5 + 6400*a
**12*b*x**7 + 12800*a**11*b**2*x**9 + 12800*a**10*b**3*x**11 + 64
00*a**9*b**4*x**13 + 1280*a**8*b**5*x**15)
```

GIAC/XCAS [A] time = 0.272427, size = 155, normalized size = 0.99

$$-\frac{9009\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{256\,\sqrt{ab}a^8}-\frac{45045\,b^7x^{14}+210210\,ab^6x^{12}+384384\,a^2b^5x^{10}+338910\,a^3b^4x^8+137995\,a^4b^3x^6+16640\,a^5b^2x^4-1280\,a^6bx^2+256\,a^7}{1280\,(bx^3+ax)^5a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*x^6),x, algorithm="giac")
```

```
[Out] -9009/256*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8) - 1/1280*(450
45*b^7*x^14 + 210210*a*b^6*x^12 + 384384*a^2*b^5*x^10 + 338910*a^
3*b^4*x^8 + 137995*a^4*b^3*x^6 + 16640*a^5*b^2*x^4 - 1280*a^6*b*x
^2 + 256*a^7)/((b*x^3 + a*x)^5*a^8)
```

3.535 $\int \frac{1}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=19

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0109537, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + 2*x^2 + x^4)^(-1),x]

[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi in Sympy [A] time = 2.54353, size = 12, normalized size = 0.63

$$\frac{x}{2(x^2+1)} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**4+2*x**2+1),x)

[Out] x/(2*(x**2 + 1)) + atan(x)/2
```

Mathematica [A] time = 0.0100036, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1} + \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + 2*x^2 + x^4)^(-1),x]
```

```
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$\frac{x}{2\,x^{2}+2}+\frac{\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^4+2*x^2+1),x)
```

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 0.76385, size = 20, normalized size = 1.05

$$\frac{x}{2\,(x^{2}+1)}+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.256915, size = 26, normalized size = 1.37

$$\frac{\left(x^{2}+1\right)\arctan\left(x\right)+x}{2\,(x^{2}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.191109, size = 12, normalized size = 0.63

$$\frac{x}{2x^{2}+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**4+2*x**2+1),x)
```

```
[Out] x/(2*x**2 + 2) + atan(x)/2
```

GIAC/XCAS [A] time = 0.268799, size = 20, normalized size = 1.05

$$\frac{x}{2\,(x^2+1)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.536 $\int \frac{x}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=11

$$-\frac{1}{2\left(x^2+1\right)}$$

```
[Out] -1/(2*(1 + x^2))
```

Rubi [A] time = 0.00759608, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2))
```

Rubi in Sympy [A] time = 2.74399, size = 8, normalized size = 0.73

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+2*x**2+1),x)
```

```
[Out] -1/(2*(x**2 + 1))
```

Mathematica [A] time = 0.00225236, size = 11, normalized size = 1.

$$-\frac{1}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + 2*x^2 + x^4),x]
```

```
[Out] -1/(2*(1 + x^2))
```

Maple [A] time = 0.004, size = 10, normalized size = 0.9

$$-\frac{1}{2\,x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+1),x)

[Out] -1/2/(x^2+1)
```

Maxima [A] time = 0.697025, size = 12, normalized size = 1.09

$$-\frac{1}{2\,(x^{2}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4 + 2*x^2 + 1),x, algorithm="maxima")

[Out] -1/2/(x^2 + 1)
```

Fricas [A] time = 0.248907, size = 12, normalized size = 1.09

$$-\frac{1}{2\,(x^{2}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4 + 2*x^2 + 1),x, algorithm="fricas")

[Out] -1/2/(x^2 + 1)
```

Sympy [A] time = 0.140513, size = 8, normalized size = 0.73

$$-\frac{1}{2x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+2*x**2+1),x)
```

```
[Out] -1/(2*x**2 + 2)
```

GIAC/XCAS [A] time = 0.26804, size = 12, normalized size = 1.09

$$-\frac{1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 + 1)
```

3.537 $\int \frac{x^2}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0171044, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 + 2*x^2 + x^4),x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi in Sympy [A] time = 4.04892, size = 12, normalized size = 0.63

$$-\frac{x}{2(x^2+1)} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4+2*x**2+1),x)

[Out] -x/(2*(x**2 + 1)) + atan(x)/2
```

Mathematica [A] time = 0.0127296, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(1 + 2*x^2 + x^4),x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Maple [A] time = 0.009, size = 16, normalized size = 0.8

$$-\frac{x}{2\,x^2+2}+\frac{\arctan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4+2*x^2+1),x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 0.759362, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.254105, size = 28, normalized size = 1.47

$$\frac{\left(x^2+1\right)\arctan\left(x\right)-x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] time = 0.187322, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4+2*x**2+1),x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

GIAC/XCAS [A] time = 0.270292, size = 20, normalized size = 1.05

$$-\frac{x}{2\,(x^2+1)}+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.538 $\int \frac{x^3}{1+2x^2+x^4} \, dx$

Optimal. Leaf size=22

$$\frac{1}{2(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2
```

Rubi [A] time = 0.0294682, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{1}{2(x^2+1)} + \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(1 + 2*x^2 + x^4),x]

[Out] 1/(2*(1 + x^2)) + Log[1 + x^2]/2
```

Rubi in Sympy [A] time = 4.95612, size = 15, normalized size = 0.68

$$\frac{\log\left(x^2+1\right)}{2} + \frac{1}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**4+2*x**2+1),x)

[Out] log(x**2 + 1)/2 + 1/(2*(x**2 + 1))
```

Mathematica [A] time = 0.00729017, size = 18, normalized size = 0.82

$$\frac{1}{2}\left(\frac{1}{x^2+1} + \log\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(1 + 2*x^2 + x^4),x]
```

```
[Out] ((1 + x^2)^(-1) + Log[1 + x^2])/2
```

Maple [A] time = 0.008, size = 19, normalized size = 0.9

$$\frac{1}{2\,x^2+2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^4+2*x^2+1),x)
```

```
[Out] 1/2/(x^2+1)+1/2*ln(x^2+1)
```

Maxima [A] time = 0.690458, size = 24, normalized size = 1.09

$$\frac{1}{2\,(x^2+1)}+\frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^4 + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 + 1) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.251669, size = 31, normalized size = 1.41

$$\frac{\left(x^2+1\right)\log\left(x^2+1\right)+1}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^4 + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*log(x^2 + 1) + 1)/(x^2 + 1)
```

Sympy [A] time = 0.154311, size = 15, normalized size = 0.68

$$\frac{\log\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**4+2*x**2+1),x)
```

```
[Out] log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

GIAC/XCAS [A] time = 0.270716, size = 24, normalized size = 1.09

$$\frac{1}{2\left(x^2+1\right)}+\frac{1}{2}\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^4 + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] 1/2/(x^2 + 1) + 1/2*ln(x^2 + 1)
```

3.539 $\int \frac{x}{81-18x^2+x^4} \, dx$

Optimal. Leaf size=13

$$\frac{1}{2(9-x^2)}$$

```
[Out] 1/(2*(9 - x^2))
```

Rubi [A] time = 0.00847123, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{2(9-x^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(81 - 18*x^2 + x^4),x]

[Out] 1/(2*(9 - x^2))
```

Rubi in Sympy [A] time = 2.86744, size = 7, normalized size = 0.54

$$\frac{1}{2(-x^2+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4-18*x**2+81),x)

[Out] 1/(2*(-x**2 + 9))
```

Mathematica [A] time = 0.00347502, size = 11, normalized size = 0.85

$$-\frac{1}{2(x^2-9)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(81 - 18*x^2 + x^4),x]
```

```
[Out] -1/(2*(-9 + x^2))
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$-\frac{1}{2\,x^2-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4-18*x^2+81),x)

[Out] -1/2/(x^2-9)
```

Maxima [A] time = 0.695305, size = 12, normalized size = 0.92

$$-\frac{1}{2\,(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 18*x^2 + 81),x, algorithm="maxima")

[Out] -1/2/(x^2 - 9)
```

Fricas [A] time = 0.247928, size = 12, normalized size = 0.92

$$-\frac{1}{2\,(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 18*x^2 + 81),x, algorithm="fricas")

[Out] -1/2/(x^2 - 9)
```

Sympy [A] time = 0.144972, size = 8, normalized size = 0.62

$$-\frac{1}{2x^2-18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4-18*x**2+81),x)
```

```
[Out] -1/(2*x**2 - 18)
```

GIAC/XCAS [A] time = 0.268517, size = 12, normalized size = 0.92

$$-\frac{1}{2(x^2-9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 - 18*x^2 + 81),x, algorithm="giac")
```

```
[Out] -1/2/(x^2 - 9)
```

3.540 $\int \frac{x^3}{16-8x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

```
[Out] 2/(4 - x^2) + Log[4 - x^2]/2
```

Rubi [A] time = 0.0377494, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$\frac{2}{4-x^2}+\frac{1}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] 2/(4 - x^2) + Log[4 - x^2]/2
```

Rubi in Sympy [A] time = 5.31184, size = 14, normalized size = 0.58

$$\frac{\log\left(-x^2+4\right)}{2}+\frac{2}{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(x**4-8*x**2+16),x)
```

```
[Out] log(-x**2 + 4)/2 + 2/(-x**2 + 4)
```

Mathematica [A] time = 0.00922287, size = 20, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-4\right)-\frac{2}{x^2-4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(16 - 8*x^2 + x^4),x]
```

```
[Out] -2/(-4 + x^2) + Log[-4 + x^2]/2
```

Maple [A] time = 0.009, size = 19, normalized size = 0.8

$$\frac{\ln\left(x^2-4\right)}{2}-2\left(x^2-4\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(x^4-8*x^2+16),x)
```

```
[Out] 1/2*ln(x^2-4)-2/(x^2-4)
```

Maxima [A] time = 0.68607, size = 24, normalized size = 1.

$$-\frac{2}{x^2-4}+\frac{1}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^4 - 8*x^2 + 16),x, algorithm="maxima")
```

```
[Out] -2/(x^2 - 4) + 1/2*log(x^2 - 4)
```

Fricas [A] time = 0.254725, size = 31, normalized size = 1.29

$$\frac{\left(x^2-4\right)\log\left(x^2-4\right)-4}{2\left(x^2-4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^4 - 8*x^2 + 16),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4)*log(x^2 - 4) - 4)/(x^2 - 4)
```

Sympy [A] time = 0.16383, size = 14, normalized size = 0.58

$$\frac{\log\left(x^2-4\right)}{2}-\frac{2}{x^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(x**4-8*x**2+16),x)
```

```
[Out] log(x**2 - 4)/2 - 2/(x**2 - 4)
```

GIAC/XCAS [A] time = 0.270128, size = 26, normalized size = 1.08

$$-\frac{2}{x^2-4}+\frac{1}{2}\ln\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(x^4 - 8*x^2 + 16),x, algorithm="giac")
```

```
[Out] -2/(x^2 - 4) + 1/2*ln(abs(x^2 - 4))
```

3.541 $\int x^5\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}$$

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rubi [A] time = 0.165053, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{bx^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{ax^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b*x^8*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0213285, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(4ax^6+3bx^8)}{24(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(4*a*x^6 + 3*b*x^8))/(24*(a + b*x^2))
```

Maple [A]   time = 0.007, size = 36, normalized size = 0.5

$$\frac{x^6\left(3\,bx^2+4\,a\right)}{24\,bx^2+24\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/24*x^6*(3*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.255105, size = 18, normalized size = 0.23

$$\frac{1}{8}\,bx^8+\frac{1}{6}\,ax^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/8*b*x^8 + 1/6*a*x^6
```

Sympy [A] time = 0.168967, size = 12, normalized size = 0.15

$$\frac{ax^6}{6} + \frac{bx^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**6/6 + b*x**8/8
```

GIAC/XCAS [A] time = 0.26868, size = 39, normalized size = 0.49

$$\frac{1}{8}\,bx^8\text{sign}\left(bx^2 + a\right) + \frac{1}{6}\,ax^6\text{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^5,x, algorithm="giac")
```

```
[Out] 1/8*b*x^8*sign(b*x^2 + a) + 1/6*a*x^6*sign(b*x^2 + a)
```

3.542 $\int x^3\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=67

$$\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2}-\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

```
[Out] -(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + (a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2)/(6*b^2)
```

Rubi [A] time = 0.131118, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{(a^2+2abx^2+b^2x^4)^{3/2}}{6b^2}-\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b^2) + (a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2)/(6*b^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0151035, size = 39, normalized size = 0.58

$$\frac{\sqrt{(a+bx^2)^2}\,(3ax^4+2bx^6)}{12(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x^4 + 2*b*x^6))/(12*(a + b*x^2))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$\frac{x^4 \left(2 b x^2 + 3 a\right)}{12 b x^2 + 12 a} \sqrt{\left(b x^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/12*x^4*(2*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25494, size = 18, normalized size = 0.27

$$\frac{1}{6} b x^6 + \frac{1}{4} a x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A] time = 0.166098, size = 12, normalized size = 0.18

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

GIAC/XCAS [A] time = 0.269719, size = 31, normalized size = 0.46

$$\frac{1}{12}\left(2\,bx^6 + 3\,ax^4\right)\operatorname{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/12*(2*b*x^6 + 3*a*x^4)*sign(b*x^2 + a)
```

3.543 $\int x\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rubi [A] time = 0.0656612, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*b)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{(a+bx^2)^2}\int^{a+bx^2} x\,dx}{2b\,(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt((a + b*x**2)**2)*Integral(x, (x, a + b*x**2))/(2*b*(a + b*x*
*2))
```

Mathematica [A] time = 0.012167, size = 38, normalized size = 1.06

$$\frac{\sqrt{(a+bx^2)^2}\,(2ax^2+bx^4)}{4\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]

[Out] (Sqrt[(a + b*x^2)^2]*(2*a*x^2 + b*x^4))/(4*(a + b*x^2))
```

Maple [A] time = 0.062, size = 35, normalized size = 1.

$$\frac{x^2\left(bx^2+2\,a\right)}{4\,bx^2+4\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*((b*x^2+a)^2)^(1/2),x)

[Out] 1/4*x^2*(b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258766, size = 18, normalized size = 0.5

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x,x, algorithm="fricas")

[Out] 1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] time = 0.167063, size = 12, normalized size = 0.33

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

GIAC/XCAS [A] time = 0.269295, size = 30, normalized size = 0.83

$$\frac{1}{4} \left(bx^4 + 2\,ax^2\right) \operatorname{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x,x, algorithm="giac")
```

```
[Out] 1/4*(b*x^4 + 2*a*x^2)*sign(b*x^2 + a)
```

3.544 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=75

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0681036, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (a*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^2)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x, x)
```

Mathematica [A] time = 0.0182458, size = 37, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,(2a\log(x)+bx^2)}{2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2 + 2*a*Log[x]))/(2*(a + b*x^2))
```

Maple [A] time = 0.012, size = 34, normalized size = 0.5

$$\frac{bx^2 + 2a\ln(x)}{2bx^2 + 2a}\sqrt{(bx^2 + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(b*x^2+2*a*ln(x))/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259295, size = 15, normalized size = 0.2

$$\frac{1}{2}bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.214455, size = 10, normalized size = 0.13

$$a\log(x) + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] a*log(x) + b*x**2/2
```

GIAC/XCAS [A] time = 0.269507, size = 41, normalized size = 0.55

$$\frac{1}{2}\,bx^2\mathrm{sign}\left(bx^2 + a\right) + \frac{1}{2}\,a\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*sign(b*x^2 + a) + 1/2*a*ln(x^2)*sign(b*x^2 + a)
```

3.545 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=75

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.0682473, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**3, x)
```

Mathematica [A] time = 0.0161505, size = 39, normalized size = 0.52

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a-2bx^2\log(x)\right)}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^3,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a - 2*b*x^2*Log[x]))/(2*x^2*(a + b*x^2))
```

Maple [A] time = 0.012, size = 38, normalized size = 0.5

$$\frac{2\,b\ln(x)\,x^2-a}{(2\,bx^2+2\,a)\,x^2}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^3,x)
```

```
[Out] 1/2*((b*x^2+a)^2)^(1/2)*(2*b*ln(x)*x^2-a)/(b*x^2+a)/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259938, size = 23, normalized size = 0.31

$$\frac{2\,bx^2\log(x)-a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A] time = 1.0711, size = 10, normalized size = 0.13

$$-\frac{a}{2x^2} + b\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x)
```

GIAC/XCAS [A] time = 0.269956, size = 61, normalized size = 0.81

$$\frac{1}{2}\,b\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2+a\right) - \frac{bx^2\mathrm{sign}\left(bx^2+a\right) + a\mathrm{sign}\left(bx^2+a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*b*ln(x^2)*sign(b*x^2 + a) - 1/2*(b*x^2*sign(b*x^2 + a) + a*si
gn(b*x^2 + a))/x^2
```

3.546 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^5}\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^4)
```

Rubi [A] time = 0.101289, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]

[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^4)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^2)^2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**5,x)

[Out] Integral(sqrt((a + b*x**2)**2)/x**5, x)
```

Mathematica [A] time = 0.0138953, size = 37, normalized size = 0.95

$$-\frac{\sqrt{(a+bx^2)^2}\,(a+2bx^2)}{4x^4\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a + 2*b*x^2))/(4*x^4*(a + b*x^2))
```

Maple [A] time = 0.013, size = 34, normalized size = 0.9

$$-\frac{2\,b{x}^{2}+a}{4\,{x}^{4} \left(b{x}^{2}+a \right) }\sqrt{ \left(b{x}^{2}+a \right) ^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^5,x)
```

```
[Out] -1/4*(2*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^4/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256595, size = 18, normalized size = 0.46

$$-\frac{2\,b x^{2} + a}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/x^4
```

Sympy [A] time = 1.11847, size = 14, normalized size = 0.36

$$-\frac{a+2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**5,x)
```

```
[Out] -(a + 2*b*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.269185, size = 41, normalized size = 1.05

$$-\frac{2\,bx^2\operatorname{sign}\left(bx^2+a\right)+a\operatorname{sign}\left(bx^2+a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2*sign(b*x^2 + a) + a*sign(b*x^2 + a))/x^4
```

3.547 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^7}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^6) + (a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rubi [A] time = 0.0565666, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{12a^2x^6}-\frac{\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]
```

```
[Out] -((a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*a*x^6) + (a^2 +
 2*a*b*x^2 + b^2*x^4)^(3/2)/(12*a^2*x^6)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**7, x)
```

Mathematica [A] time = 0.0172471, size = 39, normalized size = 0.54

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(2a+3bx^2\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^7,x]

[Out] -(Sqrt[(a + b*x^2)^2]*(2*a + 3*b*x^2))/(12*x^6*(a + b*x^2))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$-\frac{3\,bx^2+2\,a}{12\,x^6\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^7,x)

[Out] -1/12*(3*b*x^2+2*a)*((b*x^2+a)^2)^(1/2)/x^6/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256392, size = 20, normalized size = 0.28

$$-\frac{3\,bx^2+2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^7,x, algorithm="fricas")

[Out] -1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [A] time = 1.1752, size = 15, normalized size = 0.21

$$-\frac{2a+3bx^2}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**7,x)

[Out] -(2*a + 3*b*x**2)/(12*x**6)
```

GIAC/XCAS [A] time = 0.269263, size = 42, normalized size = 0.58

$$-\frac{3\,bx^2\mathrm{sign}\left(bx^2+a\right)+2\,a\mathrm{sign}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^7,x, algorithm="giac")

[Out] -1/12*(3*b*x^2*sign(b*x^2 + a) + 2*a*sign(b*x^2 + a))/x^6
```

3.548 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^9}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rubi [A] time = 0.144664, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**9, x)
```

Mathematica [A] time = 0.0137151, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+4bx^2\right)}{24x^8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a + 4*b*x^2))/(24*x^8*(a + b*x^2))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$-\frac{4\,bx^{2}+3\,a}{24\,x^{8}\,(bx^{2}+a)}\sqrt{(bx^{2}+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^9,x)
```

```
[Out] -1/24*(4*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^8/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.255692, size = 20, normalized size = 0.25

$$-\frac{4\,bx^{2}+3\,a}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + 3*a)/x^8
```

Sympy [A] time = 1.24565, size = 15, normalized size = 0.19

$$-\frac{3a+4bx^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**9,x)
```

```
[Out] -(3*a + 4*b*x**2)/(24*x**8)
```

GIAC/XCAS [A] time = 0.269581, size = 42, normalized size = 0.53

$$-\frac{4\,bx^2\mathrm{sign}\left(bx^2+a\right)+3\,a\mathrm{sign}\left(bx^2+a\right)}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^9,x, algorithm="giac")
```

```
[Out] -1/24*(4*b*x^2*sign(b*x^2 + a) + 3*a*sign(b*x^2 + a))/x^8
```

3.549 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{11}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi [A] time = 0.14649, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^2)^2}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**11, x)
```

Mathematica [A] time = 0.0140511, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^2)^2}\,(4a+5bx^2)}{40x^{10}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(4*a + 5*b*x^2))/(40*x^10*(a + b*x^2))
```

Maple [A] time = 0.008, size = 36, normalized size = 0.5

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^11,x)
```

```
[Out] -1/40*(5*b*x^2+4*a)*((b*x^2+a)^2)^(1/2)/x^10/(b*x^2+a)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256938, size = 20, normalized size = 0.25

$$-\frac{5\,bx^2+4\,a}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(5*b*x^2 + 4*a)/x^10
```

Sympy [A] time = 1.28125, size = 15, normalized size = 0.19

$$-\frac{4a+5bx^2}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**11,x)
```

```
[Out] -(4*a + 5*b*x**2)/(40*x**10)
```

GIAC/XCAS [A] time = 0.269367, size = 42, normalized size = 0.53

$$-\frac{5\,bx^2\mathrm{sign}\left(bx^2+a\right)+4\,a\mathrm{sign}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(5*b*x^2*sign(b*x^2 + a) + 4*a*sign(b*x^2 + a))/x^10
```

3.550 $\int x^4\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rubi [A] time = 0.0727897, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ax^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (b*x^7*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0118893, size = 39, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\left(7ax^5+5bx^7\right)}{35(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(7*a*x^5 + 5*b*x^7))/(35*(a + b*x^2))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$\frac{x^5\left(5\,bx^2+7\,a\right)}{35\,bx^2+35\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/35*x^5*(5*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.695197, size = 18, normalized size = 0.23

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Fricas [A] time = 0.255577, size = 18, normalized size = 0.23

$$\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/7*b*x^7 + 1/5*a*x^5
```

Sympy [A] time = 0.165378, size = 12, normalized size = 0.15

$$\frac{ax^5}{5} + \frac{bx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x**5/5 + b*x**7/7
```

GIAC/XCAS [A] time = 0.269305, size = 39, normalized size = 0.49

$$\frac{1}{7}\,bx^7\mathrm{sign}\left(bx^2 + a\right) + \frac{1}{5}\,ax^5\mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^4,x, algorithm="giac")
```

```
[Out] 1/7*b*x^7*sign(b*x^2 + a) + 1/5*a*x^5*sign(b*x^2 + a)
```

3.551 $\int x^2\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=79

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi [A] time = 0.0741372, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5(a+bx^2)}+\frac{ax^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b*x^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0122918, size = 39, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(5ax^3+3bx^5\right)}{15\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(5*a*x^3 + 3*b*x^5))/(15*(a + b*x^2))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$\frac{x^3 \left(3 bx^2 + 5 a\right)}{15 bx^2 + 15 a} \sqrt{\left(bx^2 + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/15*x^3*(3*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.692213, size = 18, normalized size = 0.23

$$\frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 0.25614, size = 18, normalized size = 0.23

$$\frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] time = 0.165286, size = 12, normalized size = 0.15

$$\frac{ax^3}{3} + \frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*((b*x**2+a)**2)**(1/2),x)

[Out] a*x**3/3 + b*x**5/5
```

GIAC/XCAS [A] time = 0.268559, size = 39, normalized size = 0.49

$$\frac{1}{5}\,bx^5\text{sign}\left(bx^2 + a\right) + \frac{1}{3}\,ax^3\text{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*x^2,x, algorithm="giac")

[Out] 1/5*b*x^5*sign(b*x^2 + a) + 1/3*a*x^3*sign(b*x^2 + a)
```

3.552 $\int \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=74

$$\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi [A] time = 0.0403233, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{ax\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*x^3*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0161419, size = 36, normalized size = 0.49

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(3ax+bx^3\right)}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]

[Out] (Sqrt[(a + b*x^2)^2]*(3*a*x + b*x^3))/(3*(a + b*x^2))
```

Maple [A] time = 0.003, size = 33, normalized size = 0.5

$$\frac{x\left(bx^2+3\,a\right)}{3\,bx^2+3\,a}\sqrt{\left(bx^2+a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2),x)

[Out] 1/3*x*(b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.703227, size = 14, normalized size = 0.19

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2),x, algorithm="maxima")

[Out] 1/3*b*x^3 + a*x
```

Fricas [A] time = 0.254874, size = 14, normalized size = 0.19

$$\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2),x, algorithm="fricas")

[Out] 1/3*b*x^3 + a*x
```

Sympy [A] time = 0.153532, size = 8, normalized size = 0.11

$$ax + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a*x + b*x**3/3
```

GIAC/XCAS [A] time = 0.268615, size = 27, normalized size = 0.36

$$\frac{1}{3}\left(bx^3 + 3\,ax\right)\text{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/3*(b*x^3 + 3*a*x)*sign(b*x^2 + a)
```

3.553 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=72

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi [A] time = 0.0672847, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] -((a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (b*x*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**2, x)
```

Mathematica [A] time = 0.0137778, size = 35, normalized size = 0.49

$$\frac{\left(bx^2-a\right)\sqrt{\left(a+bx^2\right)^2}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^2,x]
```

```
[Out] ((-a + b*x^2)*Sqrt[(a + b*x^2)^2])/(x*(a + b*x^2))
```

Maple [A]   time = 0.003, size = 34, normalized size = 0.5

$$-\frac{-bx^2+a}{(bx^2+a)\,x}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^2,x)
```

```
[Out] -(-b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/x
```

Maxima [A]   time = 0.706572, size = 18, normalized size = 0.25

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^2,x, algorithm="maxima")
```

```
[Out] (b*x^2 - a)/x
```

Fricas [A]   time = 0.265509, size = 18, normalized size = 0.25

$$\frac{bx^2-a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^2,x, algorithm="fricas")
```

```
[Out] (b*x^2 - a)/x
```

Sympy [A] time = 1.01229, size = 5, normalized size = 0.07

$$-\frac{a}{x} + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**2,x)

[Out] -a/x + b*x
```

GIAC/XCAS [A] time = 0.270535, size = 35, normalized size = 0.49

$$bx\text{sign}\left(bx^2 + a\right) - \frac{a\text{sign}\left(bx^2 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^2,x, algorithm="giac")

[Out] b*x*sign(b*x^2 + a) - a*sign(b*x^2 + a)/x
```

3.554 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^4}\,dx$

Optimal. Leaf size=77

$$-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi [A] time = 0.0704801, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**4, x)
```

Mathematica [A] time = 0.0129212, size = 37, normalized size = 0.48

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(a+3bx^2\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^4,x]

[Out] -(Sqrt[(a + b*x^2)^2]*(a + 3*b*x^2))/(3*x^3*(a + b*x^2))
```

Maple [A] time = 0.004, size = 34, normalized size = 0.4

$$-\frac{3\,bx^2+a}{3\,x^3\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^4,x)

[Out] -1/3*(3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/x^3/(b*x^2+a)
```

Maxima [A] time = 0.697982, size = 18, normalized size = 0.23

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^4,x, algorithm="maxima")

[Out] -1/3*(3*b*x^2 + a)/x^3
```

Fricas [A] time = 0.264528, size = 18, normalized size = 0.23

$$-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^4,x, algorithm="fricas")

[Out] -1/3*(3*b*x^2 + a)/x^3
```

Sympy [A] time = 1.1055, size = 14, normalized size = 0.18

$$-\frac{a+3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**4,x)

[Out] -(a + 3*b*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.270003, size = 41, normalized size = 0.53

$$-\frac{3\,bx^2\mathrm{sign}\left(bx^2+a\right)+a\mathrm{sign}\left(bx^2+a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^4,x, algorithm="giac")

[Out] -1/3*(3*b*x^2*sign(b*x^2 + a) + a*sign(b*x^2 + a))/x^3
```

3.555 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^6}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi [A] time = 0.0701777, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\left(a+bx^2\right)^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**6, x)
```

Mathematica [A] time = 0.0130489, size = 39, normalized size = 0.49

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a+5bx^2\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^6,x]

[Out] -(Sqrt[(a + b*x^2)^2]*(3*a + 5*b*x^2))/(15*x^5*(a + b*x^2))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.5

$$-\frac{5\,bx^2+3\,a}{15\,x^5\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^6,x)

[Out] -1/15*(5*b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/x^5/(b*x^2+a)
```

Maxima [A] time = 0.697821, size = 20, normalized size = 0.25

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^6,x, algorithm="maxima")

[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Fricas [A] time = 0.263699, size = 20, normalized size = 0.25

$$-\frac{5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^6,x, algorithm="fricas")

[Out] -1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [A] time = 1.16266, size = 15, normalized size = 0.19

$$-\frac{3a+5bx^2}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**6,x)

[Out] -(3*a + 5*b*x**2)/(15*x**5)
```

GIAC/XCAS [A] time = 0.27023, size = 42, normalized size = 0.53

$$-\frac{5\,bx^2\mathrm{sign}\left(bx^2+a\right)+3\,a\mathrm{sign}\left(bx^2+a\right)}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^6,x, algorithm="giac")

[Out] -1/15*(5*b*x^2*sign(b*x^2 + a) + 3*a*sign(b*x^2 + a))/x^5
```

3.556 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^8} dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi [A] time = 0.0698571, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^2)^2}}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**8,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**8, x)
```

Mathematica [A] time = 0.0141125, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^2)^2}\,(5a+7bx^2)}{35x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^8,x]

[Out] -(Sqrt[(a + b*x^2)^2]*(5*a + 7*b*x^2))/(35*x^7*(a + b*x^2))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$-\frac{7\,b x^2+5\,a}{35\,x^7\,(b x^2+a)}\sqrt{(b x^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^8,x)

[Out] -1/35*(7*b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/x^7/(b*x^2+a)
```

Maxima [A] time = 0.699372, size = 20, normalized size = 0.25

$$-\frac{7\,b x^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^8,x, algorithm="maxima")

[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

Fricas [A] time = 0.264843, size = 20, normalized size = 0.25

$$-\frac{7\,b x^2+5\,a}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^8,x, algorithm="fricas")

[Out] -1/35*(7*b*x^2 + 5*a)/x^7
```

Sympy [A] time = 1.21385, size = 15, normalized size = 0.19

$$-\frac{5a + 7bx^2}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**8,x)

[Out] -(5*a + 7*b*x**2)/(35*x**7)
```

GIAC/XCAS [A] time = 0.270571, size = 42, normalized size = 0.53

$$-\frac{7\,bx^2\text{sign}\left(bx^2 + a\right) + 5\,a\text{sign}\left(bx^2 + a\right)}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^8,x, algorithm="giac")

[Out] -1/35*(7*b*x^2*sign(b*x^2 + a) + 5*a*sign(b*x^2 + a))/x^7
```

3.557 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{x^{10}}\,dx$

Optimal. Leaf size=79

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi [A] time = 0.0714989, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]
```

```
[Out] -(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(a+bx^2)^2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

```
[Out] Integral(sqrt((a + b*x**2)**2)/x**10, x)
```

Mathematica [A] time = 0.0139439, size = 39, normalized size = 0.49

$$-\frac{\sqrt{(a+bx^2)^2}\,(7a+9bx^2)}{63x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/x^10,x]

[Out] -(Sqrt[(a + b*x^2)^2]*(7*a + 9*b*x^2))/(63*x^9*(a + b*x^2))
```

Maple [A] time = 0.005, size = 36, normalized size = 0.5

$$-\frac{9\,bx^2+7\,a}{63\,x^9\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/x^10,x)

[Out] -1/63*(9*b*x^2+7*a)*((b*x^2+a)^2)^(1/2)/x^9/(b*x^2+a)
```

Maxima [A] time = 0.689296, size = 20, normalized size = 0.25

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^10,x, algorithm="maxima")

[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

Fricas [A] time = 0.266367, size = 20, normalized size = 0.25

$$-\frac{9\,bx^2+7\,a}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^10,x, algorithm="fricas")

[Out] -1/63*(9*b*x^2 + 7*a)/x^9
```

Sympy [A] time = 1.26879, size = 15, normalized size = 0.19

$$-\frac{7a + 9bx^2}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/x**10,x)
```

```
[Out] -(7*a + 9*b*x**2)/(63*x**9)
```

GIAC/XCAS [A] time = 0.270789, size = 42, normalized size = 0.53

$$-\frac{9\,bx^2\mathrm{sign}\left(bx^2 + a\right) + 7\,a\mathrm{sign}\left(bx^2 + a\right)}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/x^10,x, algorithm="giac")
```

```
[Out] -1/63*(9*b*x^2*sign(b*x^2 + a) + 7*a*sign(b*x^2 + a))/x^9
```

3.558 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^
2*b*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*
b^2*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3
*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2))
```

Rubi [A] time = 0.281651, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3ab^2x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^2bx^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{b^3x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16\left(a+bx^2\right)}+\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^
2*b*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a*
b^2*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (b^3
*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.1633, size = 133, normalized size = 0.8

$$\frac{a^3x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{560\left(a+bx^2\right)}+\frac{a^2x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{112}+\frac{3ax^{10}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{112}+\frac{x^{10}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] a**3*x**10*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(560*(a + b*x**2))
 + a**2*x**10*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/112 + 3*a*x**10
*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/112 + x**10*(a*
*2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/16
```

Mathematica [A] time = 0.0323573, size = 61, normalized size = 0.37

$$\frac{x^{10}\sqrt{(a+bx^2)^2}\left(56a^3+140a^2bx^2+120ab^2x^4+35b^3x^6\right)}{560\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(56*a^3 + 140*a^2*b*x^2 + 120*a*b^2*x^4
 + 35*b^3*x^6))/(560*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$\frac{x^{10}\left(35\,b^3x^6+120\,ab^2x^4+140\,a^2bx^2+56\,a^3\right)}{560\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/560*x^10*(35*b^3*x^6+120*a*b^2*x^4+140*a^2*b*x^2+56*a^3)*((b*x^
2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266724, size = 47, normalized size = 0.28

$$\frac{1}{16} b^3 x^{16} + \frac{3}{14} a b^2 x^{14} + \frac{1}{4} a^2 b x^{12} + \frac{1}{10} a^3 x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^9,x, algorithm="fricas")
```

```
[Out] 1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.269299, size = 90, normalized size = 0.54

$$\frac{1}{16} b^3 x^{16} \mathrm{sign}\left(bx^2 + a\right) + \frac{3}{14} a b^2 x^{14} \mathrm{sign}\left(bx^2 + a\right) + \frac{1}{4} a^2 b x^{12} \mathrm{sign}\left(bx^2 + a\right) + \frac{1}{10} a^3 x^{10} \mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^9,x, algorithm="giac")
```

```
[Out] 1/16*b^3*x^16*sign(b*x^2 + a) + 3/14*a*b^2*x^14*sign(b*x^2 + a) +
 1/4*a^2*b*x^12*sign(b*x^2 + a) + 1/10*a^3*x^10*sign(b*x^2 + a)
```

3.559 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^
2*b*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b
^2*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x
^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2))
```

Rubi [A] time = 0.270207, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{ab^2x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{3a^2bx^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10\left(a+bx^2\right)}+\frac{b^3x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14\left(a+bx^2\right)}+\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (3*a^
2*b*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a*b
^2*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^3*x
^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.1272, size = 131, normalized size = 0.78

$$\frac{a^3x^8\sqrt{a^2+2abx^2+b^2x^4}}{280\left(a+bx^2\right)}+\frac{a^2x^8\sqrt{a^2+2abx^2+b^2x^4}}{70}+\frac{ax^8\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{28}+\frac{x^8\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] a**3*x**8*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(280*(a + b*x**2))
+ a**2*x**8*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/70 + a*x**8*(a +
b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/28 + x**8*(a**2 + 2*a
*b*x**2 + b**2*x**4)**(3/2)/14
```

Mathematica [A] time = 0.0269624, size = 61, normalized size = 0.37

$$\frac{x^8\sqrt{(a+bx^2)^2}\left(35a^3+84a^2bx^2+70ab^2x^4+20b^3x^6\right)}{280\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(35*a^3 + 84*a^2*b*x^2 + 70*a*b^2*x^4 +
20*b^3*x^6))/(280*(a + b*x^2))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.4

$$\frac{x^8\left(20\,b^3x^6+70\,ab^2x^4+84\,a^2bx^2+35\,a^3\right)}{280\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/280*x^8*(20*b^3*x^6+70*a*b^2*x^4+84*a^2*b*x^2+35*a^3)*((b*x^2+a
)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267843, size = 47, normalized size = 0.28

$$\frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^7,x, algorithm="fricas")
```

```
[Out] 1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.270687, size = 90, normalized size = 0.54

$$\frac{1}{14}\,b^3x^{14}\mathrm{sign}\left(bx^2+a\right) + \frac{1}{4}\,ab^2x^{12}\mathrm{sign}\left(bx^2+a\right) + \frac{3}{10}\,a^2bx^{10}\mathrm{sign}\left(bx^2+a\right) + \frac{1}{8}\,a^3x^8\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^7,x, algorithm="giac")
```

```
[Out] 1/14*b^3*x^14*sign(b*x^2 + a) + 1/4*a*b^2*x^12*sign(b*x^2 + a) +
3/10*a^2*b*x^10*sign(b*x^2 + a) + 1/8*a^3*x^8*sign(b*x^2 + a)
```

3.560 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=106

$$\frac{(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{5b^3}+\frac{a^2(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^3}$$

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^3) - (a*
(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(5*b^3) + ((a + b*x^2)*(a^2 +
2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^3)
```

Rubi [A] time = 0.208403, antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^4}{5b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}{8b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^2*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^3) - (a*
(a + b*x^2)^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*b^3) + ((a + b*
x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3)
```

Rubi in Sympy [A] time = 20.742, size = 107, normalized size = 1.01

$$\frac{a^2(2a+2bx^2)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{48b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{30b^3}+\frac{x^4(2a+2bx^2)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] a**2*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(48*
b**3) - a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(30*b**3) + x**4
*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(24*b)
```

Mathematica [A] time = 0.0263781, size = 61, normalized size = 0.58

$$\frac{x^6\sqrt{(a+bx^2)^2}\,(20a^3+45a^2bx^2+36ab^2x^4+10b^3x^6)}{120\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 +
10*b^3*x^6))/(120*(a + b*x^2))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.6

$$\frac{x^6\left(10\,b^3x^6+36\,ab^2x^4+45\,a^2bx^2+20\,a^3\right)}{120\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a
)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265603, size = 47, normalized size = 0.44

$$\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.271237, size = 90, normalized size = 0.85

$$\frac{1}{12}\, b^3 x^{12} \mathrm{sign}\left(bx^2 + a\right) + \frac{3}{10}\, ab^2 x^{10} \mathrm{sign}\left(bx^2 + a\right) + \frac{3}{8}\, a^2 b x^8 \mathrm{sign}\left(bx^2 + a\right) + \frac{1}{6}\, a^3 x^6 \mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/12*b^3*x^12*sign(b*x^2 + a) + 3/10*a*b^2*x^10*sign(b*x^2 + a) +
 3/8*a^2*b*x^8*sign(b*x^2 + a) + 1/6*a^3*x^6*sign(b*x^2 + a)
```

3.561 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^2) + (a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(10*b^2)
```

Rubi [A] time = 0.128327, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{10b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b^2) + (a^2
 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(10*b^2)
```

Rubi in Sympy [A] time = 13.6864, size = 65, normalized size = 0.97

$$-\frac{a\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{16b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] -a*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(16*b*
*2) + (a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(10*b**2)
```

Mathematica [A] time = 0.0278878, size = 61, normalized size = 0.91

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(10a^3+20a^2bx^2+15ab^2x^4+4b^3x^6\right)}{40\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 +
4*b^3*x^6))/(40*(a + b*x^2))
```

Maple [A]   time = 0.009, size = 58, normalized size = 0.9

$$\frac{x^4\left(4b^3x^6+15ab^2x^4+20a^2bx^2+10a^3\right)}{40\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/40*x^4*(4*b^3*x^6+15*a*b^2*x^4+20*a^2*b*x^2+10*a^3)*((b*x^2+a)^
2)^(3/2)/(b*x^2+a)^3
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A]   time = 0.264811, size = 47, normalized size = 0.7

$$\frac{1}{10}b^3x^{10}+\frac{3}{8}ab^2x^8+\frac{1}{2}a^2bx^6+\frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\left(a + b x^2 \right)^2 \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.26998, size = 61, normalized size = 0.91

$$\frac{1}{40} \left(4\, b^3 x^{10} + 15\, a b^2 x^8 + 20\, a^2 b x^6 + 10\, a^3 x^4 \right) \mathrm{sign}\left(b x^2 + a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/40*(4*b^3*x^10 + 15*a*b^2*x^8 + 20*a^2*b*x^6 + 10*a^3*x^4)*sign
(b*x^2 + a)
```

3.562 $\int x\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rubi [A] time = 0.0648148, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]

[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*b)
```

Rubi in Sympy [A] time = 8.17256, size = 34, normalized size = 0.94

$$\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] (2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(16*b)
```

Mathematica [A] time = 0.0222523, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(3/2))/(8*b)
```

Maple [A] time = 0.007, size = 57, normalized size = 1.6

$$\frac{x^{2}\left(b^{3}x^{6}+4\,ab^{2}x^{4}+6\,a^{2}bx^{2}+4\,a^{3}\right)}{8\,\left(bx^{2}+a\right)^{3}}\left(\left(bx^{2}+a\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^(3/
2)/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265308, size = 47, normalized size = 1.31

$$\frac{1}{8}\,b^{3}x^{8}+\frac{1}{2}\,ab^{2}x^{6}+\frac{3}{4}\,a^{2}bx^{4}+\frac{1}{2}\,a^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] 1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.269433, size = 59, normalized size = 1.64

$$\frac{1}{8}\left(b^3x^8+4\,ab^2x^6+6\,a^2bx^4+4\,a^3x^2\right)\operatorname{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*x^8 + 4*a*b^2*x^6 + 6*a^2*b*x^4 + 4*a^3*x^2)*sign(b*x^2
+ a)
```

3.563 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=163

$$\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (
3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b
^3*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.134498, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3ab^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{3a^2bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{a^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (3*a^2*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (
3*a*b^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b
^3*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (a^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 16.7836, size = 117, normalized size = 0.72

$$\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}}{2}+\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
```

```
[Out] a**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2) + a*
*2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 + a*(a + b*x**2)*sqrt(a*
*2 + 2*a*b*x**2 + b**2*x**4)/4 + (a**2 + 2*a*b*x**2 + b**2*x**4)*
*(3/2)/6
```

Mathematica [A] time = 0.034978, size = 60, normalized size = 0.37

$$\frac{\sqrt{(a+bx^2)^2}\left(12a^3\log(x)+bx^2\left(18a^2+9abx^2+2b^2x^4\right)\right)}{12\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12
*a^3*Log[x]))/(12*(a + b*x^2))
```

Maple [A] time = 0.012, size = 57, normalized size = 0.4

$$\frac{2\,b^3x^6+9\,ax^4b^2+18\,a^2bx^2+12\,a^3\ln(x)}{12\,(bx^2+a)^3}\left((bx^2+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+9*a*x^4*b^2+18*a^2*b*x^2+12*a
^3*ln(x))/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272004, size = 45, normalized size = 0.28

$$\frac{1}{6}b^3x^6 + \frac{3}{4}ab^2x^4 + \frac{3}{2}a^2bx^2 + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x, x)
```

GIAC/XCAS [A] time = 0.270201, size = 92, normalized size = 0.56

$$\frac{1}{6}b^3x^6\text{sign}\left(bx^2 + a\right) + \frac{3}{4}ab^2x^4\text{sign}\left(bx^2 + a\right) + \frac{3}{2}a^2bx^2\text{sign}\left(bx^2 + a\right) + \frac{1}{2}a^3\ln\left(x^2\right)\,\text{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6*sign(b*x^2 + a) + 3/4*a*b^2*x^4*sign(b*x^2 + a) + 3/2
*a^2*b*x^2*sign(b*x^2 + a) + 1/2*a^3*ln(x^2)*sign(b*x^2 + a)
```

3.564 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=164

$$\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (3*a
*b^2*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.143993, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{3ab^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{3a^2b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (3*a
*b^2*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^3*
x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (3*a^2*b*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 16.6011, size = 131, normalized size = 0.8

$$\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}\log\left(x\right)}{a+bx^2}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}}{2}-\frac{3a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4x^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
```

```
[Out] 3*a**2*b*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2)
+ 3*a*b*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 - 3*a*(a + b*x**2)*
sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*x**2) + (a**2 + 2*a*b*x**2
 + b**2*x**4)**(3/2)/(4*x**2)
```

Mathematica [A] time = 0.0379282, size = 62, normalized size = 0.38

$$\frac{\sqrt{(a+bx^2)^2}\left(-2a^3+12a^2bx^2\log(x)+6ab^2x^4+b^3x^6\right)}{4x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^3 + 6*a*b^2*x^4 + b^3*x^6 + 12*a^2*b*x
^2*Log[x]))/(4*x^2*(a + b*x^2))
```

Maple [A] time = 0.017, size = 59, normalized size = 0.4

$$\frac{b^3x^6+6\,ax^4b^2+12\,a^2b\ln(x)\,x^2-2\,a^3}{4\,(bx^2+a)^3\,x^2}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^3,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(b^3*x^6+6*a*x^4*b^2+12*a^2*b*ln(x)*x^2-2
*a^3)/(b*x^2+a)^3/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261715, size = 51, normalized size = 0.31

$$\frac{b^3x^6 + 6\,ab^2x^4 + 12\,a^2bx^2\log\left(x\right) - 2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.271289, size = 117, normalized size = 0.71

$$\frac{1}{4}\,b^3x^4\mathrm{sign}\left(bx^2 + a\right) + \frac{3}{2}\,ab^2x^2\mathrm{sign}\left(bx^2 + a\right) + \frac{3}{2}\,a^2b\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right) - \frac{3\,a^2bx^2\mathrm{sign}\left(bx^2 + a\right) + a^3\mathrm{sign}\left(bx^2 + a\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*sign(b*x^2 + a) + 3/2*a*b^2*x^2*sign(b*x^2 + a) + 3/2
*a^2*b*ln(x^2)*sign(b*x^2 + a) - 1/2*(3*a^2*b*x^2*sign(b*x^2 + a)
 + a^3*sign(b*x^2 + a))/x^2
```

3.565 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^5} dx$

Optimal. Leaf size=164

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*
x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.140148, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{3ab^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*
x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (3*a*b^2*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 16.8287, size = 129, normalized size = 0.79

$$\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}+\frac{3a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4x^4}+\frac{3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2}-\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
```

```
[Out] 3*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2)
+ 3*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*x**4) +
 3*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 - (a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/2)/x**4
```

Mathematica [A] time = 0.0301459, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(a^3+6a^2bx^2-12ab^2x^4\log(x)-2b^3x^6\right)}{4x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^5,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 6*a^2*b*x^2 - 2*b^3*x^6 - 12*a*b^2*x
^4*Log[x]))/(4*x^4*(a + b*x^2))
```

Maple [A] time = 0.018, size = 60, normalized size = 0.4

$$\frac{2\,b^3x^6+12\,ab^2\ln(x)\,x^4-6\,a^2bx^2-a^3}{4\,(bx^2+a)^3\,x^4}\left((bx^2+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^5,x)
```

```
[Out] 1/4*((b*x^2+a)^2)^(3/2)*(2*b^3*x^6+12*a*b^2*ln(x)*x^4-6*a^2*b*x^2
-a^3)/(b*x^2+a)^3/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263001, size = 53, normalized size = 0.32

$$\frac{2\,b^3x^6 + 12\,ab^2x^4\log\left(x\right) - 6\,a^2bx^2 - a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**5, x)
```

GIAC/XCAS [A] time = 0.27347, size = 117, normalized size = 0.71

$$\frac{1}{2}\,b^3x^2\mathrm{sign}\left(bx^2 + a\right) + \frac{3}{2}\,ab^2\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right) - \frac{9\,ab^2x^4\mathrm{sign}\left(bx^2 + a\right) + 6\,a^2bx^2\mathrm{sign}\left(bx^2 + a\right) + a^3\mathrm{sign}\left(bx^2 + a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*sign(b*x^2 + a) + 3/2*a*b^2*ln(x^2)*sign(b*x^2 + a) -
 1/4*(9*a*b^2*x^4*sign(b*x^2 + a) + 6*a^2*b*x^2*sign(b*x^2 + a) +
 a^3*sign(b*x^2 + a))/x^4
```

3.566 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.138473, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 22.3598, size = 138, normalized size = 0.85

$$-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4x^6}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}\log\left(x\right)}{a+bx^2}-\frac{5\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
```

```
[Out] -a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*x**2*(a + b*x**2))
 + a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*x**6) +
```

```
b**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2) - 5*
(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(12*x**6)
```

Mathematica [A] time = 0.0418087, size = 63, normalized size = 0.39

$$-\frac{\sqrt{(a+bx^2)^2}\left(a\left(2a^2+9abx^2+18b^2x^4\right)-12b^3x^6\log(x)\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^7,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a*(2*a^2 + 9*a*b*x^2 + 18*b^2*x^4) - 12*b^
3*x^6*Log[x]))/(12*x^6*(a + b*x^2))
```

Maple [A] time = 0.017, size = 60, normalized size = 0.4

$$\frac{12\,b^3\ln(x)\,x^6-18\,ax^4b^2-9\,a^2bx^2-2\,a^3}{12\,(bx^2+a)^3\,x^6}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(3/2)*(12*b^3*ln(x)*x^6-18*a*x^4*b^2-9*a^2*b*x
^2-2*a^3)/(b*x^2+a)^3/x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265472, size = 53, normalized size = 0.33

$$\frac{12\,b^{3}x^{6}\log\left(x\right)-18\,ab^{2}x^{4}-9\,a^{2}bx^{2}-2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**7, x)
```

GIAC/XCAS [A] time = 0.272081, size = 117, normalized size = 0.72

$$\frac{1}{2}\,b^{3}\ln\left(x^{2}\right)\,\mathrm{sign}\left(bx^{2}+a\right)-\frac{11\,b^{3}x^{6}\mathrm{sign}\left(bx^{2}+a\right)+18\,ab^{2}x^{4}\mathrm{sign}\left(bx^{2}+a\right)+9\,a^{2}bx^{2}\mathrm{sign}\left(bx^{2}+a\right)+2\,a^{3}\mathrm{sign}\left(bx^{2}+a\right)}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/2*b^3*ln(x^2)*sign(b*x^2 + a) - 1/12*(11*b^3*x^6*sign(b*x^2 + a
) + 18*a*b^2*x^4*sign(b*x^2 + a) + 9*a^2*b*x^2*sign(b*x^2 + a) +
2*a^3*sign(b*x^2 + a))/x^6
```

3.567 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

```
[Out] -((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*a*x^8)
```

Rubi [A] time = 0.105306, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]

[Out] -((a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*a*x^8)
```

Rubi in Sympy [A] time = 8.19731, size = 39, normalized size = 0.95

$$-\frac{(2a+2bx^2)\,(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{16ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)

[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(16*a*x*
*8)
```

Mathematica [A] time = 0.0268799, size = 59, normalized size = 1.44

$$-\frac{\sqrt{(a+bx^2)^2}\,(a^3+4a^2bx^2+6ab^2x^4+4b^3x^6)}{8x^8\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 4*a^2*b*x^2 + 6*a*b^2*x^4 + 4*b^3*x^
6))/(8*x^8*(a + b*x^2))
```

Maple [A] time = 0.01, size = 56, normalized size = 1.4

$$-\frac{4 b^3 x^6 + 6 a x^4 b^2 + 4 a^2 b x^2 + a^3}{8 x^8 (b x^2 + a)^3} \left((b x^2 + a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^9,x)
```

```
[Out] -1/8*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/
x^8/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26178, size = 47, normalized size = 1.15

$$-\frac{4 b^3 x^6 + 6 a b^2 x^4 + 4 a^2 b x^2 + a^3}{8 x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^9}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**9, x)
```

GIAC/XCAS [A] time = 0.269549, size = 92, normalized size = 2.24

$$-\frac{4\,b^3x^6\mathrm{sign}\left(bx^2+a\right)+6\,ab^2x^4\mathrm{sign}\left(bx^2+a\right)+4\,a^2bx^2\mathrm{sign}\left(bx^2+a\right)+a^3\mathrm{sign}\left(bx^2+a\right)}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(4*b^3*x^6*sign(b*x^2 + a) + 6*a*b^2*x^4*sign(b*x^2 + a) + 4
*a^2*b*x^2*sign(b*x^2 + a) + a^3*sign(b*x^2 + a))/x^8
```

3.568 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*a*x^10) + (a^
2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rubi [A] time = 0.0568258, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{40a^2x^{10}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{8ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(8*a*x^10) + (a^
2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(40*a^2*x^10)
```

Rubi in Sympy [A] time = 8.49242, size = 68, normalized size = 0.94

$$-\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{16ax^{10}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{40a^2x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

```
[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(16*a*x*
*10) + (a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(40*a**2*x**10)
```

Mathematica [A] time = 0.0237827, size = 61, normalized size = 0.85

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(4a^3+15a^2bx^2+20ab^2x^4+10b^3x^6\right)}{40x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(4*a^3 + 15*a^2*b*x^2 + 20*a*b^2*x^4 + 10*b
^3*x^6))/(40*x^10*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.8

$$-\frac{10\,b^3x^6+20\,ax^4b^2+15\,a^2bx^2+4\,a^3}{40\,x^{10}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^11,x)
```

```
[Out] -1/40*(10*b^3*x^6+20*a*b^2*x^4+15*a^2*b*x^2+4*a^3)*((b*x^2+a)^2)^
(3/2)/x^10/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263509, size = 50, normalized size = 0.69

$$-\frac{10\,b^3x^6+20\,ab^2x^4+15\,a^2bx^2+4\,a^3}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] -1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**11, x)
```

GIAC/XCAS [A] time = 0.271014, size = 93, normalized size = 1.29

$$-\frac{10\,b^3x^6\mathrm{sign}\left(bx^2+a\right)+20\,ab^2x^4\mathrm{sign}\left(bx^2+a\right)+15\,a^2bx^2\mathrm{sign}\left(bx^2+a\right)+4\,a^3\mathrm{sign}\left(bx^2+a\right)}{40\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] -1/40*(10*b^3*x^6*sign(b*x^2 + a) + 20*a*b^2*x^4*sign(b*x^2 + a)
+ 15*a^2*b*x^2*sign(b*x^2 + a) + 4*a^3*sign(b*x^2 + a))/x^10
```

3.569 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b
^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rubi [A] time = 0.253587, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (b
^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2))
```

Rubi in Sympy [A] time = 15.2065, size = 112, normalized size = 0.67

$$-\frac{(2a+2bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{24ax^{12}}+\frac{b(2a+2bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{48a^2x^{10}}-\frac{b(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{120a^3x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

```
[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(24*a*x*
*12) + b*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/
(48*a**2*x**10) - b*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(120*a
**3*x**10)
```

Mathematica [A] time = 0.0236637, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(10a^3+36a^2bx^2+45ab^2x^4+20b^3x^6)}{120x^{12}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(10*a^3 + 36*a^2*b*x^2 + 45*a*b^2*x^4 + 20*
b^3*x^6))/(120*x^12*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{20\,b^3x^6+45\,ax^4b^2+36\,a^2bx^2+10\,a^3}{120\,x^{12}\,(bx^2+a)^3}\left((bx^2+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^13,x)
```

```
[Out] -1/120*(20*b^3*x^6+45*a*b^2*x^4+36*a^2*b*x^2+10*a^3)*((b*x^2+a)^2
)^(3/2)/x^12/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264136, size = 50, normalized size = 0.3

$$-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**13, x)
```

GIAC/XCAS [A] time = 0.271521, size = 93, normalized size = 0.56

$$-\frac{20\,b^3x^6\text{sign}\left(bx^2+a\right)+45\,ab^2x^4\text{sign}\left(bx^2+a\right)+36\,a^2bx^2\text{sign}\left(bx^2+a\right)+10\,a^3\text{sign}\left(bx^2+a\right)}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(20*b^3*x^6*sign(b*x^2 + a) + 45*a*b^2*x^4*sign(b*x^2 + a)
 + 36*a^2*b*x^2*sign(b*x^2 + a) + 10*a^3*sign(b*x^2 + a))/x^12
```

3.570 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{15}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi [A] time = 0.256836, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (3*a
*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (b^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.8263, size = 133, normalized size = 0.8

$$\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{280x^{10}(a+bx^2)}+\frac{a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{28x^{14}}-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}}{56x^{10}}-\frac{3(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{28x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

```
[Out] a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(280*x**10*(a + b*x**2
)) + a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(28*x**14
) - b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(56*x**10) - 3*(a**2
 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(28*x**14)
```

Mathematica [A] time = 0.028362, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(20a^3+70a^2bx^2+84ab^2x^4+35b^3x^6)}{280x^{14}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(20*a^3 + 70*a^2*b*x^2 + 84*a*b^2*x^4 + 35*
b^3*x^6))/(280*x^14*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{35\,b^3x^6+84\,ax^4b^2+70\,a^2bx^2+20\,a^3}{280\,x^{14}\,(bx^2+a)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^15,x)
```

```
[Out] -1/280*(35*b^3*x^6+84*a*b^2*x^4+70*a^2*b*x^2+20*a^3)*((b*x^2+a)^2
)^(3/2)/x^14/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261451, size = 50, normalized size = 0.3

$$-\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**15, x)
```

GIAC/XCAS [A] time = 0.270106, size = 93, normalized size = 0.56

$$-\frac{35\,b^3x^6\mathrm{sign}\left(bx^2+a\right)+84\,ab^2x^4\mathrm{sign}\left(bx^2+a\right)+70\,a^2bx^2\mathrm{sign}\left(bx^2+a\right)+20\,a^3\mathrm{sign}\left(bx^2+a\right)}{280\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/280*(35*b^3*x^6*sign(b*x^2 + a) + 84*a*b^2*x^4*sign(b*x^2 + a)
 + 70*a^2*b*x^2*sign(b*x^2 + a) + 20*a^3*sign(b*x^2 + a))/x^14
```

3.571 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{17}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (
a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2))
```

Rubi [A] time = 0.254031, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{4x^{12}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) - (
a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^12*(a + b*x^2)) - (b^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.9319, size = 136, normalized size = 0.81

$$\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{560x^{12}\left(a+bx^2\right)}+\frac{3a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{112x^{16}}-\frac{3b^2\sqrt{a^2+2abx^2+b^2x^4}}{280x^{12}}-\frac{5\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{56x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

```
[Out] a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(560*x**12*(a + b*x**2
)) + 3*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(112*x*
```

```
*16) - 3*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(280*x**12) - 5
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(56*x**16)
```

Mathematica [A] time = 0.0237277, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(35a^3+120a^2bx^2+140ab^2x^4+56b^3x^6)}{560x^{16}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(35*a^3 + 120*a^2*b*x^2 + 140*a*b^2*x^4 + 5
6*b^3*x^6))/(560*x^16*(a + b*x^2))
```

Maple [A] time = 0.011, size = 58, normalized size = 0.4

$$-\frac{56\,b^3x^6+140\,ax^4b^2+120\,a^2bx^2+35\,a^3}{560\,x^{16}\,(bx^2+a)^3}\left((bx^2+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^17,x)
```

```
[Out] -1/560*(56*b^3*x^6+140*a*b^2*x^4+120*a^2*b*x^2+35*a^3)*((b*x^2+a)
^2)^(3/2)/x^16/(b*x^2+a)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258006, size = 50, normalized size = 0.3

$$-\frac{56\,b^3x^6 + 140\,ab^2x^4 + 120\,a^2bx^2 + 35\,a^3}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/560*(56*b^3*x^6 + 140*a*b^2*x^4 + 120*a^2*b*x^2 + 35*a^3)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**17, x)
```

GIAC/XCAS [A] time = 0.270698, size = 93, normalized size = 0.56

$$-\frac{56\,b^3x^6\mathrm{sign}\left(bx^2 + a\right) + 140\,ab^2x^4\mathrm{sign}\left(bx^2 + a\right) + 120\,a^2bx^2\mathrm{sign}\left(bx^2 + a\right) + 35\,a^3\mathrm{sign}\left(bx^2 + a\right)}{560\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/560*(56*b^3*x^6*sign(b*x^2 + a) + 140*a*b^2*x^4*sign(b*x^2 + a
) + 120*a^2*b*x^2*sign(b*x^2 + a) + 35*a^3*sign(b*x^2 + a))/x^16
```

3.572 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^
2*b*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a
*b^2*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^
3*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a + b*x^2))
```

Rubi [A] time = 0.138043, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3ab^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{3a^2bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\left(a+bx^2\right)}+\frac{a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (3*a^
2*b*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (3*a
*b^2*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (b^
3*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.0474, size = 136, normalized size = 0.81

$$\frac{16a^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{6435\left(a+bx^2\right)}+\frac{8a^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{715}+\frac{2ax^9\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{65}+\frac{x^9\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 16*a**3*x**9*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(6435*(a + b*x**
2)) + 8*a**2*x**9*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/715 + 2*a*x
**9*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/65 + x**9*(a
**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/15
```

Mathematica [A] time = 0.0291085, size = 61, normalized size = 0.37

$$\frac{x^9\sqrt{(a+bx^2)^2}\left(715a^3+1755a^2bx^2+1485ab^2x^4+429b^3x^6\right)}{6435\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(715*a^3 + 1755*a^2*b*x^2 + 1485*a*b^2*x
^4 + 429*b^3*x^6))/(6435*(a + b*x^2))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$\frac{x^9\left(429\,b^3x^6+1485\,ax^4b^2+1755\,a^2bx^2+715\,a^3\right)}{6435\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/6435*x^9*(429*b^3*x^6+1485*a*b^2*x^4+1755*a^2*b*x^2+715*a^3)*((
b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.700886, size = 47, normalized size = 0.28

$$\frac{1}{15}\,b^3x^{15}+\frac{3}{13}\,ab^2x^{13}+\frac{3}{11}\,a^2bx^{11}+\frac{1}{9}\,a^3x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Fricas [A] time = 0.26105, size = 47, normalized size = 0.28

$$\frac{1}{15}\,b^{3}x^{15}+\frac{3}{13}\,ab^{2}x^{13}+\frac{3}{11}\,a^{2}bx^{11}+\frac{1}{9}\,a^{3}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/15*b^3*x^15 + 3/13*a*b^2*x^13 + 3/11*a^2*b*x^11 + 1/9*a^3*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{8}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.271582, size = 90, normalized size = 0.54

$$\frac{1}{15}\,b^{3}x^{15}\mathrm{sign}\left(bx^{2}+a\right)+\frac{3}{13}\,ab^{2}x^{13}\mathrm{sign}\left(bx^{2}+a\right)+\frac{3}{11}\,a^{2}bx^{11}\mathrm{sign}\left(bx^{2}+a\right)+\frac{1}{9}\,a^{3}x^{9}\mathrm{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^8,x, algorithm="giac")
```

```
[Out] 1/15*b^3*x^15*sign(b*x^2 + a) + 3/13*a*b^2*x^13*sign(b*x^2 + a) +
 3/11*a^2*b*x^11*sign(b*x^2 + a) + 1/9*a^3*x^9*sign(b*x^2 + a)
```

3.573 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*
b*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2
*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^
13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2))
```

Rubi [A] time = 0.132912, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3ab^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^2bx^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\left(a+bx^2\right)}+\frac{a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a^2*
b*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a*b^2
*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^3*x^
13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.2463, size = 136, normalized size = 0.81

$$\frac{16a^3x^7\sqrt{a^2+2abx^2+b^2x^4}}{3003\left(a+bx^2\right)}+\frac{8a^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{429}+\frac{6ax^7\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{143}+\frac{x^7\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 16*a**3*x**7*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3003*(a + b*x**
2)) + 8*a**2*x**7*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/429 + 6*a*x
**7*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/143 + x**7*(
a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/13
```

Mathematica [A] time = 0.0298419, size = 61, normalized size = 0.37

$$\frac{x^7\sqrt{(a+bx^2)^2}\left(429a^3+1001a^2bx^2+819ab^2x^4+231b^3x^6\right)}{3003\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(429*a^3 + 1001*a^2*b*x^2 + 819*a*b^2*x^
4 + 231*b^3*x^6))/(3003*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$\frac{x^7\left(231\,b^3x^6+819\,ax^4b^2+1001\,a^2bx^2+429\,a^3\right)}{3003\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/3003*x^7*(231*b^3*x^6+819*a*b^2*x^4+1001*a^2*b*x^2+429*a^3)*((b
*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.702122, size = 47, normalized size = 0.28

$$\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Fricas [A] time = 0.256437, size = 47, normalized size = 0.28

$$\frac{1}{13}b^3x^{13} + \frac{3}{11}ab^2x^{11} + \frac{1}{3}a^2bx^9 + \frac{1}{7}a^3x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.270074, size = 90, normalized size = 0.54

$$\frac{1}{13}b^3x^{13}\operatorname{sign}\left(bx^2 + a\right) + \frac{3}{11}ab^2x^{11}\operatorname{sign}\left(bx^2 + a\right) + \frac{1}{3}a^2bx^9\operatorname{sign}\left(bx^2 + a\right) + \frac{1}{7}a^3x^7\operatorname{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^6,x, algorithm="giac")
```

```
[Out] 1/13*b^3*x^13*sign(b*x^2 + a) + 3/11*a*b^2*x^11*sign(b*x^2 + a) +
 1/3*a^2*b*x^9*sign(b*x^2 + a) + 1/7*a^3*x^7*sign(b*x^2 + a)
```

3.574 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=167

$$\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^
2*b*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2
*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2))
```

Rubi [A] time = 0.132073, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{ab^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{3a^2bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\left(a+bx^2\right)}+\frac{a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a^
2*b*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (a*b^2
*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^3*x^11
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.4673, size = 136, normalized size = 0.81

$$\frac{16a^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{1155\left(a+bx^2\right)}+\frac{8a^2x^5\sqrt{a^2+2abx^2+b^2x^4}}{231}+\frac{2ax^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{33}+\frac{x^5\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 16*a**3*x**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1155*(a + b*x**
2)) + 8*a**2*x**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/231 + 2*a*x
**5*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/33 + x**5*(a
**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/11
```

Mathematica [A] time = 0.0275083, size = 61, normalized size = 0.37

$$\frac{x^5\sqrt{(a+bx^2)^2}\left(231a^3+495a^2bx^2+385ab^2x^4+105b^3x^6\right)}{1155\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4
 + 105*b^3*x^6))/(1155*(a + b*x^2))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$\frac{x^5\left(105\,b^3x^6+385\,ax^4b^2+495\,a^2bx^2+231\,a^3\right)}{1155\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*
x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.69538, size = 47, normalized size = 0.28

$$\frac{1}{11}\,b^3x^{11}+\frac{1}{3}\,ab^2x^9+\frac{3}{7}\,a^2bx^7+\frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Fricas [A] time = 0.260619, size = 47, normalized size = 0.28

$$\frac{1}{11}\,b^{3}x^{11}+\frac{1}{3}\,ab^{2}x^{9}+\frac{3}{7}\,a^{2}bx^{7}+\frac{1}{5}\,a^{3}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{4}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.271606, size = 90, normalized size = 0.54

$$\frac{1}{11}\,b^{3}x^{11}\mathrm{sign}\left(bx^{2}+a\right)+\frac{1}{3}\,ab^{2}x^{9}\mathrm{sign}\left(bx^{2}+a\right)+\frac{3}{7}\,a^{2}bx^{7}\mathrm{sign}\left(bx^{2}+a\right)+\frac{1}{5}\,a^{3}x^{5}\mathrm{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/11*b^3*x^11*sign(b*x^2 + a) + 1/3*a*b^2*x^9*sign(b*x^2 + a) + 3
/7*a^2*b*x^7*sign(b*x^2 + a) + 1/5*a^3*x^5*sign(b*x^2 + a)
```

3.575 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=167

$$\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^
2*b*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b
^2*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^
9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rubi [A] time = 0.131793, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{3ab^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\left(a+bx^2\right)}+\frac{3a^2bx^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\left(a+bx^2\right)}+\frac{a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (3*a^
2*b*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (3*a*b
^2*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^3*x^
9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.6938, size = 136, normalized size = 0.81

$$\frac{16a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{315\left(a+bx^2\right)}+\frac{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{105}+\frac{2ax^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{21}+\frac{x^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 16*a**3*x**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(315*(a + b*x**2
)) + 8*a**2*x**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/105 + 2*a*x*
*3*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/21 + x**3*(a*
*2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/9
```

Mathematica [A] time = 0.026133, size = 61, normalized size = 0.37

$$\frac{\sqrt{(a+bx^2)^2}\left(105a^3x^3+189a^2bx^5+135ab^2x^7+35b^3x^9\right)}{315\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7
 + 35*b^3*x^9))/(315*(a + b*x^2))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$\frac{x^3\left(35\,b^3x^6+135\,ax^4b^2+189\,a^2bx^2+105\,a^3\right)}{315\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^
2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.695685, size = 47, normalized size = 0.28

$$\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Fricas [A] time = 0.26612, size = 47, normalized size = 0.28

$$\frac{1}{9}\,b^{3}x^{9}+\frac{3}{7}\,ab^{2}x^{7}+\frac{3}{5}\,a^{2}bx^{5}+\frac{1}{3}\,a^{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{2}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.27157, size = 90, normalized size = 0.54

$$\frac{1}{9}\,b^{3}x^{9}\mathrm{sign}\left(bx^{2}+a\right)+\frac{3}{7}\,ab^{2}x^{7}\mathrm{sign}\left(bx^{2}+a\right)+\frac{3}{5}\,a^{2}bx^{5}\mathrm{sign}\left(bx^{2}+a\right)+\frac{1}{3}\,a^{3}x^{3}\mathrm{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^3*x^9*sign(b*x^2 + a) + 3/7*a*b^2*x^7*sign(b*x^2 + a) + 3/5
*a^2*b*x^5*sign(b*x^2 + a) + 1/3*a^3*x^3*sign(b*x^2 + a)
```

3.576 $\int \left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=159

$$\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}$$

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*
x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x
^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^
7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(7*(a + b*x^2)^3)
```

Rubi [A] time = 0.0920367, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{3ab^2x^5\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{5\left(a+bx^2\right)^3}+\frac{a^2bx^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}+\frac{b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{7\left(a+bx^2\right)^3}+\frac{a^3x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*
x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x
^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^
7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))/(7*(a + b*x^2)^3)
```

Rubi in Sympy [A] time = 30.4051, size = 129, normalized size = 0.81

$$\frac{16a^3x\sqrt{a^2+2abx^2+b^2x^4}}{35\left(a+bx^2\right)}+\frac{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}{35}+\frac{6ax\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{35}+\frac{x\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 16*a**3*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(35*(a + b*x**2)) +
 8*a**2*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/35 + 6*a*x*(a + b*x
**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/35 + x*(a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/2)/7
```

Mathematica [A] time = 0.024495, size = 59, normalized size = 0.37

$$\frac{\sqrt{(a+bx^2)^2}\left(35a^3x+35a^2bx^3+21ab^2x^5+5b^3x^7\right)}{35\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*
b^3*x^7))/(35*(a + b*x^2))
```

Maple [A] time = 0.005, size = 56, normalized size = 0.4

$$\frac{x\left(5\,b^3x^6+21\,ax^4b^2+35\,a^2bx^2+35\,a^3\right)}{35\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2)
^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.697572, size = 42, normalized size = 0.26

$$\frac{1}{7}\,b^3x^7+\frac{3}{5}\,ab^2x^5+a^2bx^3+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Fricas [A] time = 0.26107, size = 42, normalized size = 0.26

$$\frac{1}{7}b^3x^7 + \frac{3}{5}ab^2x^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.269959, size = 85, normalized size = 0.53

$$\frac{1}{7}b^3x^7\operatorname{sign}\left(bx^2 + a\right) + \frac{3}{5}ab^2x^5\operatorname{sign}\left(bx^2 + a\right) + a^2bx^3\operatorname{sign}\left(bx^2 + a\right) + a^3x\operatorname{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7*sign(b*x^2 + a) + 3/5*a*b^2*x^5*sign(b*x^2 + a) + a^2
*b*x^3*sign(b*x^2 + a) + a^3*x*sign(b*x^2 + a)
```

3.577 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=158

$$\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2
*b*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi [A] time = 0.124167, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{3a^2bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{ab^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] -((a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (3*a^2
*b*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (a*b^2*x^3*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^5*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.6794, size = 128, normalized size = 0.81

$$-\frac{16a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x\left(a+bx^2\right)}+\frac{8a^2\sqrt{a^2+2abx^2+b^2x^4}}{5x}+\frac{2a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{5x}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
```

```
[Out] -16*a**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*x*(a + b*x**2)) +
 8*a**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*x) + 2*a*(a + b*x*
```

```
*2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*x) + (a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/2)/(5*x)
```

Mathematica [A] time = 0.0271576, size = 60, normalized size = 0.38

$$\frac{\sqrt{(a+bx^2)^2}\,(-5a^3+15a^2bx^2+5ab^2x^4+b^3x^6)}{5x\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x
^6))/(5*x*(a + b*x^2))
```

Maple [A] time = 0.009, size = 58, normalized size = 0.4

$$-\frac{-b^3x^6-5\,ax^4b^2-15\,a^2bx^2+5\,a^3}{5\,x\,(bx^2+a)^3}\left((bx^2+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^2,x)
```

```
[Out] -1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^(3/2
)/x/(b*x^2+a)^3
```

Maxima [A] time = 0.68978, size = 49, normalized size = 0.31

$$\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Fricas [A] time = 0.258021, size = 49, normalized size = 0.31

$$\frac{b^3x^6 + 5\,ab^2x^4 + 15\,a^2bx^2 - 5\,a^3}{5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.271259, size = 86, normalized size = 0.54

$$\frac{1}{5}\,b^3x^5\mathrm{sign}\left(bx^2 + a\right) + ab^2x^3\mathrm{sign}\left(bx^2 + a\right) + 3\,a^2bx\mathrm{sign}\left(bx^2 + a\right) - \frac{a^3\mathrm{sign}\left(bx^2 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5*sign(b*x^2 + a) + a*b^2*x^3*sign(b*x^2 + a) + 3*a^2*b
*x*sign(b*x^2 + a) - a^3*sign(b*x^2 + a)/x
```

3.578 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=161

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*
x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi [A] time = 0.125021, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}+\frac{3ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (3*a*b^2*
x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^3*x^3*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi in Sympy [A] time = 24.2465, size = 134, normalized size = 0.83

$$\frac{16ab^2x\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{2a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{x^3}+\frac{8b^2x\sqrt{a^2+2abx^2+b^2x^4}}{3}-\frac{7\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

```
[Out] 16*a*b**2*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3*(a + b*x**2))
+ 2*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/x**3 + 8*b
```

```
**2*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/3 - 7*(a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/2)/(3*x**3)
```

Mathematica [A] time = 0.0250204, size = 59, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(a^3+9a^2bx^2-9ab^2x^4-b^3x^6\right)}{3x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^4,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 9*a^2*b*x^2 - 9*a*b^2*x^4 - b^3*x^6)
)/(3*x^3*(a + b*x^2))
```

Maple [A] time = 0.009, size = 56, normalized size = 0.4

$$-\frac{-b^3x^6-9\,ax^4b^2+9\,a^2bx^2+a^3}{3\,x^3\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^4,x)
```

```
[Out] -1/3*(-b^3*x^6-9*a*b^2*x^4+9*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2)/x
^3/(b*x^2+a)^3
```

Maxima [A] time = 0.687994, size = 49, normalized size = 0.3

$$\frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Fricas [A] time = 0.259887, size = 49, normalized size = 0.3

$$\frac{b^3x^6 + 9\,ab^2x^4 - 9\,a^2bx^2 - a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**4, x)
```

GIAC/XCAS [A] time = 0.269993, size = 90, normalized size = 0.56

$$\frac{1}{3}\,b^3x^3\mathrm{sign}\left(bx^2 + a\right) + 3\,ab^2x\mathrm{sign}\left(bx^2 + a\right) - \frac{9\,a^2bx^2\mathrm{sign}\left(bx^2 + a\right) + a^3\mathrm{sign}\left(bx^2 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*sign(b*x^2 + a) + 3*a*b^2*x*sign(b*x^2 + a) - 1/3*(9*
a^2*b*x^2*sign(b*x^2 + a) + a^3*sign(b*x^2 + a))/x^3
```

3.579 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=158

$$-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a^2
*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi [A] time = 0.126478, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{x^3\left(a+bx^2\right)}+\frac{b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a^2
*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (3*a*b^2*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (b^3*x*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi in Sympy [A] time = 16.6686, size = 134, normalized size = 0.85

$$-\frac{16ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x\left(a+bx^2\right)}+\frac{2a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{5x^5}+\frac{8b^2\sqrt{a^2+2abx^2+b^2x^4}}{5x}-\frac{3\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

```
[Out] -16*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*x*(a + b*x**2))
 + 2*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*x**5)
```

```
+ 8*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*x) - 3*(a**2 + 2*
a*b*x**2 + b**2*x**4)**(3/2)/(5*x**5)
```

Mathematica [A] time = 0.0260665, size = 59, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(a^3+5a^2bx^2+15ab^2x^4-5b^3x^6\right)}{5x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^6,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^3 + 5*a^2*b*x^2 + 15*a*b^2*x^4 - 5*b^3*x
^6))/(5*x^5*(a + b*x^2))
```

Maple [A] time = 0.008, size = 56, normalized size = 0.4

$$-\frac{-5\,b^3x^6+15\,ax^4b^2+5\,a^2bx^2+a^3}{5\,x^5\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^6,x)
```

```
[Out] -1/5*(-5*b^3*x^6+15*a*b^2*x^4+5*a^2*b*x^2+a^3)*((b*x^2+a)^2)^(3/2
)/x^5/(b*x^2+a)^3
```

Maxima [A] time = 0.694968, size = 50, normalized size = 0.32

$$\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Fricas [A] time = 0.256894, size = 50, normalized size = 0.32

$$\frac{5\,b^3x^6 - 15\,ab^2x^4 - 5\,a^2bx^2 - a^3}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**6, x)
```

GIAC/XCAS [A] time = 0.269123, size = 89, normalized size = 0.56

$$b^3x\mathrm{sign}\left(bx^2 + a\right) - \frac{15\,ab^2x^4\mathrm{sign}\left(bx^2 + a\right) + 5\,a^2bx^2\mathrm{sign}\left(bx^2 + a\right) + a^3\mathrm{sign}\left(bx^2 + a\right)}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] b^3*x*sign(b*x^2 + a) - 1/5*(15*a*b^2*x^4*sign(b*x^2 + a) + 5*a^2
*b*x^2*sign(b*x^2 + a) + a^3*sign(b*x^2 + a))/x^5
```

3.580 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^8} dx$

Optimal. Leaf size=163

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi [A] time = 0.125733, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{x^3(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (a*b^
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^3*(a + b*x^2)) - (b^3*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.9677, size = 138, normalized size = 0.85

$$\frac{16ab^2\sqrt{a^2+2abx^2+b^2x^4}}{35x^3(a+bx^2)}+\frac{6a(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{35x^7}-\frac{24b^2\sqrt{a^2+2abx^2+b^2x^4}}{35x^3}-\frac{11(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{35x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
```

```
[Out] 16*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(35*x**3*(a + b*x**
2)) + 6*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(35*x*
```

```
*7) - 24*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(35*x**3) - 11*
(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(35*x**7)
```

Mathematica [A] time = 0.0280996, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\,(5a^3+21a^2bx^2+35ab^2x^4+35b^3x^6)}{35x^7\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(5*a^3 + 21*a^2*b*x^2 + 35*a*b^2*x^4 + 35*b
^3*x^6))/(35*x^7*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{35\,b^3x^6+35\,ax^4b^2+21\,a^2bx^2+5\,a^3}{35\,x^7\,(bx^2+a)^3}\left((bx^2+a)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^8,x)
```

```
[Out] -1/35*(35*b^3*x^6+35*a*b^2*x^4+21*a^2*b*x^2+5*a^3)*((b*x^2+a)^2)^
(3/2)/x^7/(b*x^2+a)^3
```

Maxima [A] time = 0.700061, size = 50, normalized size = 0.31

$$-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Fricas [A] time = 0.258444, size = 50, normalized size = 0.31

$$-\frac{35\,b^{3}x^{6}+35\,ab^{2}x^{4}+21\,a^{2}bx^{2}+5\,a^{3}}{35\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**8, x)
```

GIAC/XCAS [A] time = 0.269976, size = 93, normalized size = 0.57

$$-\frac{35\,b^{3}x^{6}\mathrm{sign}\left(bx^{2}+a\right)+35\,ab^{2}x^{4}\mathrm{sign}\left(bx^{2}+a\right)+21\,a^{2}bx^{2}\mathrm{sign}\left(bx^{2}+a\right)+5\,a^{3}\mathrm{sign}\left(bx^{2}+a\right)}{35\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] -1/35*(35*b^3*x^6*sign(b*x^2 + a) + 35*a*b^2*x^4*sign(b*x^2 + a)
+ 21*a^2*b*x^2*sign(b*x^2 + a) + 5*a^3*sign(b*x^2 + a))/x^7
```

3.581 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{10}} dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi [A] time = 0.12864, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (3*a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (b^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi in Sympy [A] time = 17.0033, size = 138, normalized size = 0.83

$$\frac{16ab^2\sqrt{a^2+2abx^2+b^2x^4}}{315x^5(a+bx^2)}+\frac{2a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{21x^9}-\frac{8b^2\sqrt{a^2+2abx^2+b^2x^4}}{63x^5}-\frac{13\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
```

```
[Out] 16*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(315*x**5*(a + b*x*
*2)) + 2*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(21*x
```

```
**9) - 8*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(63*x**5) - 13*
(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(63*x**9)
```

Mathematica [A] time = 0.0234023, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(35a^3+135a^2bx^2+189ab^2x^4+105b^3x^6\right)}{315x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(35*a^3 + 135*a^2*b*x^2 + 189*a*b^2*x^4 + 1
05*b^3*x^6))/(315*x^9*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{105\,b^3x^6+189\,ax^4b^2+135\,a^2bx^2+35\,a^3}{315\,x^9\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^10,x)
```

```
[Out] -1/315*(105*b^3*x^6+189*a*b^2*x^4+135*a^2*b*x^2+35*a^3)*((b*x^2+a
)^2)^(3/2)/x^9/(b*x^2+a)^3
```

Maxima [A] time = 0.698724, size = 50, normalized size = 0.3

$$-\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Fricas [A] time = 0.257522, size = 50, normalized size = 0.3

$$-\frac{105\,b^3x^6 + 189\,ab^2x^4 + 135\,a^2bx^2 + 35\,a^3}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**10, x)
```

GIAC/XCAS [A] time = 0.270743, size = 93, normalized size = 0.56

$$-\frac{105\,b^3x^6\mathrm{sign}\left(bx^2+a\right) + 189\,ab^2x^4\mathrm{sign}\left(bx^2+a\right) + 135\,a^2bx^2\mathrm{sign}\left(bx^2+a\right) + 35\,a^3\mathrm{sign}\left(bx^2+a\right)}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(105*b^3*x^6*sign(b*x^2 + a) + 189*a*b^2*x^4*sign(b*x^2 +
a) + 135*a^2*b*x^2*sign(b*x^2 + a) + 35*a^3*sign(b*x^2 + a))/x^9
```

3.582 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{x^{12}}\,dx$

Optimal. Leaf size=167

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi [A] time = 0.125694, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{a^2b\sqrt{a^2+2abx^2+b^2x^4}}{3x^9(a+bx^2)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{5x^5(a+bx^2)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (a
^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (3*a*
b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (b^3*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.7292, size = 138, normalized size = 0.83

$$\frac{16ab^2\sqrt{a^2+2abx^2+b^2x^4}}{1155x^7(a+bx^2)}+\frac{2a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{33x^{11}}-\frac{8b^2\sqrt{a^2+2abx^2+b^2x^4}}{165x^7}-\frac{5\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{33x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
```

```
[Out] 16*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1155*x**7*(a + b*x
**2)) + 2*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(33*
```

```
x**11) - 8*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(165*x**7) -
5*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(33*x**11)
```

Mathematica [A] time = 0.0287521, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(105a^3+385a^2bx^2+495ab^2x^4+231b^3x^6\right)}{1155x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(105*a^3 + 385*a^2*b*x^2 + 495*a*b^2*x^4 +
231*b^3*x^6))/(1155*x^11*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{231\,b^3x^6+495\,ax^4b^2+385\,a^2bx^2+105\,a^3}{1155\,x^{11}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^12,x)
```

```
[Out] -1/1155*(231*b^3*x^6+495*a*b^2*x^4+385*a^2*b*x^2+105*a^3)*((b*x^2
+a)^2)^(3/2)/x^11/(b*x^2+a)^3
```

Maxima [A] time = 0.692885, size = 50, normalized size = 0.3

$$-\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x
^11
```

Fricas [A] time = 0.260481, size = 50, normalized size = 0.3

$$-\frac{231 \, b^{3} x^{6} + 495 \, a b^{2} x^{4} + 385 \, a^{2} b x^{2} + 105 \, a^{3}}{1155 \, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x
^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**12, x)
```

GIAC/XCAS [A] time = 0.270948, size = 93, normalized size = 0.56

$$-\frac{231 \, b^{3} x^{6} \mathrm{sign}\left(b x^{2} + a\right) + 495 \, a b^{2} x^{4} \mathrm{sign}\left(b x^{2} + a\right) + 385 \, a^{2} b x^{2} \mathrm{sign}\left(b x^{2} + a\right) + 105 \, a^{3} \mathrm{sign}\left(b x^{2} + a\right)}{1155 \, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/1155*(231*b^3*x^6*sign(b*x^2 + a) + 495*a*b^2*x^4*sign(b*x^2 +
 a) + 385*a^2*b*x^2*sign(b*x^2 + a) + 105*a^3*sign(b*x^2 + a))/x^
11
```

3.583 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{14}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (
a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi [A] time = 0.128521, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^9\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (
a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^9*(a + b*x^2)) - (b^3
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.7848, size = 138, normalized size = 0.83

$$\frac{16ab^2\sqrt{a^2+2abx^2+b^2x^4}}{3003x^9\left(a+bx^2\right)}+\frac{6a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{143x^{13}}-\frac{24b^2\sqrt{a^2+2abx^2+b^2x^4}}{1001x^9}-\frac{17\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{143x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
```

```
[Out] 16*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3003*x**9*(a + b*x
**2)) + 6*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(143
```

```
*x**13) - 24*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1001*x**9)
 - 17*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(143*x**13)
```

Mathematica [A] time = 0.0238637, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(231a^3+819a^2bx^2+1001ab^2x^4+429b^3x^6\right)}{3003x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(231*a^3 + 819*a^2*b*x^2 + 1001*a*b^2*x^4 +
 429*b^3*x^6))/(3003*x^13*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{429\,b^3x^6+1001\,ax^4b^2+819\,a^2bx^2+231\,a^3}{3003\,x^{13}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^14,x)
```

```
[Out] -1/3003*(429*b^3*x^6+1001*a*b^2*x^4+819*a^2*b*x^2+231*a^3)*((b*x^
2+a)^2)^(3/2)/x^13/(b*x^2+a)^3
```

Maxima [A] time = 0.691072, size = 50, normalized size = 0.3

$$-\frac{429\,b^3x^6+1001\,ab^2x^4+819\,a^2bx^2+231\,a^3}{3003\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/
x^13
```

Fricas [A] time = 0.257893, size = 50, normalized size = 0.3

$$-\frac{429\, b^{3} x^{6} + 1001\, a b^{2} x^{4} + 819\, a^{2} b x^{2} + 231\, a^{3}}{3003\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/3003*(429*b^3*x^6 + 1001*a*b^2*x^4 + 819*a^2*b*x^2 + 231*a^3)/
x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}}{x^{14}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**14, x)
```

GIAC/XCAS [A] time = 0.26926, size = 93, normalized size = 0.56

$$-\frac{429\, b^{3} x^{6} \operatorname{sign}\left(b x^{2} + a\right) + 1001\, a b^{2} x^{4} \operatorname{sign}\left(b x^{2} + a\right) + 819\, a^{2} b x^{2} \operatorname{sign}\left(b x^{2} + a\right) + 231\, a^{3} \operatorname{sign}\left(b x^{2} + a\right)}{3003\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/3003*(429*b^3*x^6*sign(b*x^2 + a) + 1001*a*b^2*x^4*sign(b*x^2
+ a) + 819*a^2*b*x^2*sign(b*x^2 + a) + 231*a^3*sign(b*x^2 + a))/x
^13
```

3.584 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{x^{16}}\,dx$

Optimal. Leaf size=167

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}$$

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) -
(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2))
```

Rubi [A] time = 0.129317, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (3
*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (
3*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) -
(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2))
```

Rubi in Sympy [A] time = 16.784, size = 138, normalized size = 0.83

$$\frac{16ab^2\sqrt{a^2+2abx^2+b^2x^4}}{6435x^{11}\left(a+bx^2\right)}+\frac{2a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{65x^{15}}-\frac{8b^2\sqrt{a^2+2abx^2+b^2x^4}}{585x^{11}}-\frac{19\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{195x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
```

```
[Out] 16*a*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(6435*x**11*(a + b*
x**2)) + 2*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(65
```

```
*x**15) - 8*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(585*x**11)
- 19*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(195*x**15)
```

Mathematica [A] time = 0.02388, size = 61, normalized size = 0.37

$$-\frac{\sqrt{(a+bx^2)^2}\left(429a^3+1485a^2bx^2+1755ab^2x^4+715b^3x^6\right)}{6435x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(429*a^3 + 1485*a^2*b*x^2 + 1755*a*b^2*x^4
+ 715*b^3*x^6))/(6435*x^15*(a + b*x^2))
```

Maple [A] time = 0.01, size = 58, normalized size = 0.4

$$-\frac{715\,b^3x^6+1755\,ax^4b^2+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/x^16,x)
```

```
[Out] -1/6435*(715*b^3*x^6+1755*a*b^2*x^4+1485*a^2*b*x^2+429*a^3)*((b*x
^2+a)^2)^(3/2)/x^15/(b*x^2+a)^3
```

Maxima [A] time = 0.696674, size = 50, normalized size = 0.3

$$-\frac{715\,b^3x^6+1755\,ab^2x^4+1485\,a^2bx^2+429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)
/x^15
```

Fricas [A] time = 0.257435, size = 50, normalized size = 0.3

$$-\frac{715\,b^3x^6 + 1755\,ab^2x^4 + 1485\,a^2bx^2 + 429\,a^3}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/6435*(715*b^3*x^6 + 1755*a*b^2*x^4 + 1485*a^2*b*x^2 + 429*a^3)
/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/x**16, x)
```

GIAC/XCAS [A] time = 0.271383, size = 93, normalized size = 0.56

$$-\frac{715\,b^3x^6\mathrm{sign}\left(bx^2 + a\right) + 1755\,ab^2x^4\mathrm{sign}\left(bx^2 + a\right) + 1485\,a^2bx^2\mathrm{sign}\left(bx^2 + a\right) + 429\,a^3\mathrm{sign}\left(bx^2 + a\right)}{6435\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/6435*(715*b^3*x^6*sign(b*x^2 + a) + 1755*a*b^2*x^4*sign(b*x^2
+ a) + 1485*a^2*b*x^2*sign(b*x^2 + a) + 429*a^3*sign(b*x^2 + a))/
x^15
```

3.585 $\int x^{13}\left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24(a+bx^2)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22(a+bx^2)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14(a+bx^2)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16(a+bx^2)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}$$

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*
a^4*b*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5
*a^3*b^2*x^18*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) +
(a^2*b^3*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) +
(5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a + b*x^2)) +
 (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a + b*x^2))
```

Rubi [A] time = 0.397697, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^{24}\sqrt{a^2+2abx^2+b^2x^4}}{24(a+bx^2)}+\frac{5ab^4x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22(a+bx^2)}+\frac{a^2b^3x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14(a+bx^2)}+\frac{5a^4bx^{16}\sqrt{a^2+2abx^2+b^2x^4}}{16(a+bx^2)}+\frac{5a^3b^2x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5*
a^4*b*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*(a + b*x^2)) + (5
*a^3*b^2*x^18*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) +
(a^2*b^3*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) +
(5*a*b^4*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a + b*x^2)) +
 (b^5*x^24*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*(a + b*x^2))
```

Rubi in Sympy [A] time = 27.0025, size = 201, normalized size = 0.79

$$\frac{a^5x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{11088(a+bx^2)}+\frac{a^4x^{14}\sqrt{a^2+2abx^2+b^2x^4}}{1584}+\frac{a^3x^{14}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{396}+\frac{a^2x^{14}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{132}+\frac{5ax^{14}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{264}+\frac{x^{14}\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a**5*x**14*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(11088*(a + b*x**2
)) + a**4*x**14*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/1584 + a**3*x
**14*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/396 + a**2*
x**14*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/132 + 5*a*x**14*(a +
 b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/264 + x**14*(a**2
 + 2*a*b*x**2 + b**2*x**4)**(5/2)/24
```

Mathematica [A] time = 0.0427107, size = 83, normalized size = 0.33

$$\frac{x^{14}\sqrt{(a+bx^2)^2}\left(792a^5+3465a^4bx^2+6160a^3b^2x^4+5544a^2b^3x^6+2520ab^4x^8+462b^5x^{10}\right)}{11088\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^13*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^14*Sqrt[(a + b*x^2)^2]*(792*a^5 + 3465*a^4*b*x^2 + 6160*a^3*b^
2*x^4 + 5544*a^2*b^3*x^6 + 2520*a*b^4*x^8 + 462*b^5*x^10))/(11088
*(a + b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$\frac{x^{14}\left(462\,b^5x^{10}+2520\,ab^4x^8+5544\,a^2b^3x^6+6160\,a^3b^2x^4+3465\,a^4bx^2+792\,a^5\right)}{11088\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^13*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/11088*x^14*(462*b^5*x^10+2520*a*b^4*x^8+5544*a^2*b^3*x^6+6160*a
^3*b^2*x^4+3465*a^4*b*x^2+792*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^
5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.259608, size = 77, normalized size = 0.3

$$\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^13,x, algorithm="fricas")
```

```
[Out] 1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*
x^18 + 5/16*a^4*b*x^16 + 1/14*a^5*x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{13}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**13*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.270971, size = 142, normalized size = 0.56

$$\frac{1}{24}\,b^5x^{24}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{22}\,ab^4x^{22}\mathrm{sign}\left(bx^2+a\right)+\frac{1}{2}\,a^2b^3x^{20}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{9}\,a^3b^2x^{18}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{16}\,a^4bx^{16}\mathrm{sign}\left(bx^2+a\right)+\frac{1}{14}\,a^5x^{14}\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^13,x, algorithm="giac")
```

```
[Out] 1/24*b^5*x^24*sign(b*x^2 + a) + 5/22*a*b^4*x^22*sign(b*x^2 + a) +
 1/2*a^2*b^3*x^20*sign(b*x^2 + a) + 5/9*a^3*b^2*x^18*sign(b*x^2 +
 a) + 5/16*a^4*b*x^16*sign(b*x^2 + a) + 1/14*a^5*x^14*sign(b*x^2
+ a)
```

3.586 $\int x^{11} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22(a+bx^2)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12(a+bx^2)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14(a+bx^2)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}$$

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*
a^4*b*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5
*a^3*b^2*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) +
(5*a^2*b^3*x^18*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
+ (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) +
(b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a + b*x^2))
```

Rubi [A] time = 0.380891, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^{22}\sqrt{a^2+2abx^2+b^2x^4}}{22(a+bx^2)}+\frac{ab^4x^{20}\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^{18}\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{12(a+bx^2)}+\frac{5a^4bx^{14}\sqrt{a^2+2abx^2+b^2x^4}}{14(a+bx^2)}+\frac{5a^3b^2x^{16}\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^12*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*(a + b*x^2)) + (5*
a^4*b*x^14*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*(a + b*x^2)) + (5
*a^3*b^2*x^16*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) +
(5*a^2*b^3*x^18*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
+ (a*b^4*x^20*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) +
(b^5*x^22*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.9294, size = 199, normalized size = 0.78

$$\frac{a^5x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{5544(a+bx^2)}+\frac{a^4x^{12}\sqrt{a^2+2abx^2+b^2x^4}}{924}+\frac{a^3x^{12}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{264}+\frac{a^2x^{12}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{99}+\frac{ax^{12}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{44}+\frac{x^{12}\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a**5*x**12*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5544*(a + b*x**2)
) + a**4*x**12*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/924 + a**3*x**
12*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/264 + a**2*x*
*12*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/99 + a*x**12*(a + b*x*
*2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/44 + x**12*(a**2 + 2*a
*b*x**2 + b**2*x**4)**(5/2)/22
```

Mathematica [A] time = 0.0354557, size = 83, normalized size = 0.33

$$\frac{x^{12}\sqrt{(a+bx^2)^2}\left(462a^5+1980a^4bx^2+3465a^3b^2x^4+3080a^2b^3x^6+1386ab^4x^8+252b^5x^{10}\right)}{5544\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^12*Sqrt[(a + b*x^2)^2]*(462*a^5 + 1980*a^4*b*x^2 + 3465*a^3*b^
2*x^4 + 3080*a^2*b^3*x^6 + 1386*a*b^4*x^8 + 252*b^5*x^10))/(5544*
(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$\frac{x^{12}\left(252\,b^5x^{10}+1386\,ab^4x^8+3080\,a^2b^3x^6+3465\,a^3b^2x^4+1980\,a^4bx^2+462\,a^5\right)}{5544\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/5544*x^12*(252*b^5*x^10+1386*a*b^4*x^8+3080*a^2*b^3*x^6+3465*a^
3*b^2*x^4+1980*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.25975, size = 77, normalized size = 0.3

$$\frac{1}{22}\,b^5x^{22} + \frac{1}{4}\,ab^4x^{20} + \frac{5}{9}\,a^2b^3x^{18} + \frac{5}{8}\,a^3b^2x^{16} + \frac{5}{14}\,a^4bx^{14} + \frac{1}{12}\,a^5x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^11,x, algorithm="fricas")
```

```
[Out] 1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x
^16 + 5/14*a^4*b*x^14 + 1/12*a^5*x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{11}\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.270208, size = 142, normalized size = 0.56

$$\frac{1}{22}\,b^5x^{22}\mathrm{sign}\left(bx^2 + a\right) + \frac{1}{4}\,ab^4x^{20}\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{9}\,a^2b^3x^{18}\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{8}\,a^3b^2x^{16}\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{14}\,a^4bx^{14}\mathrm{sign}\left(bx^2 + a\right) + \frac{1}{12}\,a^5x^{12}\mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^11,x, algorithm="giac")
```

```
[Out] 1/22*b^5*x^22*sign(b*x^2 + a) + 1/4*a*b^4*x^20*sign(b*x^2 + a) +
5/9*a^2*b^3*x^18*sign(b*x^2 + a) + 5/8*a^3*b^2*x^16*sign(b*x^2 +
a) + 5/14*a^4*b*x^14*sign(b*x^2 + a) + 1/12*a^5*x^12*sign(b*x^2 +
 a)
```

3.587 $\int x^9 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=201

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{8b^5}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^5}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{7b^5}$$

```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2
*a^3*(a + b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*
a^2*(a + b*x^2)^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a
*(a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*b^5) + ((a + b
*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*b^5)
```

Rubi [A] time = 0.317753, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^9}{20b^5}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^8}{9b^5}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{8b^5}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^5}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{7b^5}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^4*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^5) - (2
*a^3*(a + b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^5) + (3*
a^2*(a + b*x^2)^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*b^5) - (2*a
*(a + b*x^2)^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*b^5) + ((a + b
*x^2)^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*b^5)
```

Rubi in Sympy [A] time = 37.3168, size = 196, normalized size = 0.98

$$\frac{a^4\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{720b^5}-\frac{a^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{420b^5}+\frac{a^2x^4\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{240b^3}-\frac{ax^6\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{90b^2}+\frac{x^8\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{40b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a**4*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(720
*b**5) - a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(7/2)/(420*b**5) +
 a**2*x**4*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
)/(240*b**3) - a*x**6*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*
x**4)**(5/2)/(90*b**2) + x**8*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2
 + b**2*x**4)**(5/2)/(40*b)
```

Mathematica [A] time = 0.03989, size = 83, normalized size = 0.41

$$\frac{x^{10}\sqrt{\left(a+bx^2\right)^2}\left(252a^5+1050a^4bx^2+1800a^3b^2x^4+1575a^2b^3x^6+700ab^4x^8+126b^5x^{10}\right)}{2520\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^10*Sqrt[(a + b*x^2)^2]*(252*a^5 + 1050*a^4*b*x^2 + 1800*a^3*b^
2*x^4 + 1575*a^2*b^3*x^6 + 700*a*b^4*x^8 + 126*b^5*x^10))/(2520*(
a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.4

$$\frac{x^{10}\left(126\,b^5x^{10}+700\,ab^4x^8+1575\,a^2b^3x^6+1800\,a^3b^2x^4+1050\,a^4bx^2+252\,a^5\right)}{2520\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2520*x^10*(126*b^5*x^10+700*a*b^4*x^8+1575*a^2*b^3*x^6+1800*a^3
*b^2*x^4+1050*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.257458, size = 77, normalized size = 0.38

$$\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^9,x, algorithm="fricas")
```

```
[Out] 1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*
x^14 + 5/12*a^4*b*x^12 + 1/10*a^5*x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^9\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.270341, size = 142, normalized size = 0.71

$$\frac{1}{20}\,b^5x^{20}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{18}\,ab^4x^{18}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{8}\,a^2b^3x^{16}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{7}\,a^3b^2x^{14}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{12}\,a^4bx^{12}\mathrm{sign}\left(bx^2+a\right)+\frac{1}{10}\,a^5x^{10}\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^9,x, algorithm="giac")
```

```
[Out] 1/20*b^5*x^20*sign(b*x^2 + a) + 5/18*a*b^4*x^18*sign(b*x^2 + a) +
 5/8*a^2*b^3*x^16*sign(b*x^2 + a) + 5/7*a^3*b^2*x^14*sign(b*x^2 +
 a) + 5/12*a^4*b*x^12*sign(b*x^2 + a) + 1/10*a^5*x^10*sign(b*x^2
+ a)
```

3.588 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{14b^4}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^4}$$

```
[Out] -(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^4) + (
3*a^2*(a + b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (
3*a*(a + b*x^2)^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a
 + b*x^2)^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*b^4)
```

Rubi [A] time = 0.292674, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^8}{18b^4}-\frac{3a\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^7}{16b^4}+\frac{3a^2\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^6}{14b^4}-\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^5}{12b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a^3*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^4) + (
3*a^2*(a + b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*b^4) - (
3*a*(a + b*x^2)^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^4) + ((a
 + b*x^2)^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*b^4)
```

Rubi in Sympy [A] time = 28.4076, size = 151, normalized size = 0.94

$$-\frac{a^3\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{288b^4}+\frac{a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{168b^4}-\frac{ax^4\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{96b^2}+\frac{x^6\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{36b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] -a**3*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(28
8*b**4) + a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(7/2)/(168*b**4)
- a*x**4*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/
(96*b**2) + x**6*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)
**(5/2)/(36*b)
```

Mathematica [A] time = 0.0348257, size = 83, normalized size = 0.52

$$\frac{x^8\sqrt{(a+bx^2)^2}\left(126a^5+504a^4bx^2+840a^3b^2x^4+720a^2b^3x^6+315ab^4x^8+56b^5x^{10}\right)}{1008\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^8*Sqrt[(a + b*x^2)^2]*(126*a^5 + 504*a^4*b*x^2 + 840*a^3*b^2*x
^4 + 720*a^2*b^3*x^6 + 315*a*b^4*x^8 + 56*b^5*x^10))/(1008*(a + b
*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.5

$$\frac{x^8\left(56\,b^5x^{10}+315\,ab^4x^8+720\,a^2b^3x^6+840\,a^3b^2x^4+504\,a^4bx^2+126\,a^5\right)}{1008\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/1008*x^8*(56*b^5*x^10+315*a*b^4*x^8+720*a^2*b^3*x^6+840*a^3*b^2
*x^4+504*a^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258676, size = 77, normalized size = 0.48

$$\frac{1}{18}\,b^5x^{18} + \frac{5}{16}\,ab^4x^{16} + \frac{5}{7}\,a^2b^3x^{14} + \frac{5}{6}\,a^3b^2x^{12} + \frac{1}{2}\,a^4bx^{10} + \frac{1}{8}\,a^5x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^7,x, algorithm="fricas")
```

```
[Out] 1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*
x^12 + 1/2*a^4*b*x^10 + 1/8*a^5*x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.270583, size = 142, normalized size = 0.89

$$\frac{1}{18}\,b^5x^{18}\mathrm{sign}\left(bx^2+a\right) + \frac{5}{16}\,ab^4x^{16}\mathrm{sign}\left(bx^2+a\right) + \frac{5}{7}\,a^2b^3x^{14}\mathrm{sign}\left(bx^2+a\right) + \frac{5}{6}\,a^3b^2x^{12}\mathrm{sign}\left(bx^2+a\right) + \frac{1}{2}\,a^4bx^{10}\mathrm{sign}\left(bx^2+a\right) + \frac{1}{8}\,a^5x^8\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^7,x, algorithm="giac")
```

```
[Out] 1/18*b^5*x^18*sign(b*x^2 + a) + 5/16*a*b^4*x^16*sign(b*x^2 + a) +
 5/7*a^2*b^3*x^14*sign(b*x^2 + a) + 5/6*a^3*b^2*x^12*sign(b*x^2 +
 a) + 1/2*a^4*b*x^10*sign(b*x^2 + a) + 1/8*a^5*x^8*sign(b*x^2 + a
)
```

3.589 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=119

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a
*(a + b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b
*x^2)^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^3)
```

Rubi [A] time = 0.238568, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^7}{16b^3}-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^6}{7b^3}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{12b^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*b^3) - (a
*(a + b*x^2)^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*b^3) + ((a + b
*x^2)^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*b^3)
```

Rubi in Sympy [A] time = 20.591, size = 107, normalized size = 0.9

$$\frac{a^2\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{96b^3}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{56b^3}+\frac{x^4\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{32b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a**2*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(96*
b**3) - a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(7/2)/(56*b**3) + x**4
*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(32*b)
```

Mathematica [A] time = 0.0370608, size = 83, normalized size = 0.7

$$\frac{x^6\sqrt{(a+bx^2)^2}\left(56a^5+210a^4bx^2+336a^3b^2x^4+280a^2b^3x^6+120ab^4x^8+21b^5x^{10}\right)}{336\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^6*Sqrt[(a + b*x^2)^2]*(56*a^5 + 210*a^4*b*x^2 + 336*a^3*b^2*x^
4 + 280*a^2*b^3*x^6 + 120*a*b^4*x^8 + 21*b^5*x^10))/(336*(a + b*x
^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.7

$$\frac{x^6\left(21\,b^5x^{10}+120\,ab^4x^8+280\,a^2b^3x^6+336\,a^3b^2x^4+210\,a^4bx^2+56\,a^5\right)}{336\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/336*x^6*(21*b^5*x^10+120*a*b^4*x^8+280*a^2*b^3*x^6+336*a^3*b^2*
x^4+210*a^4*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256256, size = 76, normalized size = 0.64

$$\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10
 + 5/8*a^4*b*x^8 + 1/6*a^5*x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.271189, size = 140, normalized size = 1.18

$$\frac{1}{16}\,b^5x^{16}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{14}\,ab^4x^{14}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{6}\,a^2b^3x^{12}\mathrm{sign}\left(bx^2+a\right)+a^3b^2x^{10}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{8}\,a^4bx^8\mathrm{sign}\left(bx^2+a\right)+\frac{1}{6}\,a^5x^6\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/16*b^5*x^16*sign(b*x^2 + a) + 5/14*a*b^4*x^14*sign(b*x^2 + a) +
 5/6*a^2*b^3*x^12*sign(b*x^2 + a) + a^3*b^2*x^10*sign(b*x^2 + a)
+ 5/8*a^4*b*x^8*sign(b*x^2 + a) + 1/6*a^5*x^6*sign(b*x^2 + a)
```

3.590 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=67

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^2) + (a^
2 + 2*a*b*x^2 + b^2*x^4)^(7/2)/(14*b^2)
```

Rubi [A] time = 0.129822, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{14b^2}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b^2) + (a^
2 + 2*a*b*x^2 + b^2*x^4)^(7/2)/(14*b^2)
```

Rubi in Sympy [A] time = 13.5696, size = 65, normalized size = 0.97

$$-\frac{a\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{24b^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{14b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] -a*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(24*b*
*2) + (a**2 + 2*a*b*x**2 + b**2*x**4)**(7/2)/(14*b**2)
```

Mathematica [A] time = 0.039988, size = 83, normalized size = 1.24

$$\frac{x^4\sqrt{\left(a+bx^2\right)^2}\left(21a^5+70a^4bx^2+105a^3b^2x^4+84a^2b^3x^6+35ab^4x^8+6b^5x^{10}\right)}{84\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^4*Sqrt[(a + b*x^2)^2]*(21*a^5 + 70*a^4*b*x^2 + 105*a^3*b^2*x^4
 + 84*a^2*b^3*x^6 + 35*a*b^4*x^8 + 6*b^5*x^10))/(84*(a + b*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 1.2

$$\frac{x^4\left(6\,b^5x^{10}+35\,ab^4x^8+84\,a^2b^3x^6+105\,a^3b^2x^4+70\,a^4bx^2+21\,a^5\right)}{84\,(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/84*x^4*(6*b^5*x^10+35*a*b^4*x^8+84*a^2*b^3*x^6+105*a^3*b^2*x^4+
70*a^4*b*x^2+21*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.256163, size = 76, normalized size = 1.13

$$\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8
+ 5/6*a^4*b*x^6 + 1/4*a^5*x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \left(\left(a + b x^2 \right)^2 \right)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.269848, size = 90, normalized size = 1.34

$$\frac{1}{84} \left(6\, b^5 x^{14} + 35\, a b^4 x^{12} + 84\, a^2 b^3 x^{10} + 105\, a^3 b^2 x^8 + 70\, a^4 b x^6 + 21\, a^5 x^4 \right) \mathrm{sign}\left(b x^2 + a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/84*(6*b^5*x^14 + 35*a*b^4*x^12 + 84*a^2*b^3*x^10 + 105*a^3*b^2*
x^8 + 70*a^4*b*x^6 + 21*a^5*x^4)*sign(b*x^2 + a)
```

3.591 $\int x \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rubi [A] time = 0.0652141, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]

[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*b)
```

Rubi in Sympy [A] time = 8.1513, size = 34, normalized size = 0.94

$$\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)

[Out] (2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(24*b)
```

Mathematica [A] time = 0.0312767, size = 27, normalized size = 0.75

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^{5/2}}{12b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^(5/2))/(12*b)
```

Maple [B] time = 0.007, size = 79, normalized size = 2.2

$$\frac{x^2 \left(b^5 x^{10} + 6\,ab^4 x^8 + 15\,a^2 b^3 x^6 + 20\,a^3 b^2 x^4 + 15\,a^4 b x^2 + 6\,a^5\right)}{12\,\left(bx^2 + a\right)^5} \left(\left(bx^2 + a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/12*x^2*(b^5*x^10+6*a*b^4*x^8+15*a^2*b^3*x^6+20*a^3*b^2*x^4+15*a
^4*b*x^2+6*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.258133, size = 77, normalized size = 2.14

$$\frac{1}{12}\,b^5 x^{12} + \frac{1}{2}\,ab^4 x^{10} + \frac{5}{4}\,a^2 b^3 x^8 + \frac{5}{3}\,a^3 b^2 x^6 + \frac{5}{4}\,a^4 b x^4 + \frac{1}{2}\,a^5 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x,x, algorithm="fricas")
```

```
[Out] 1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^
6 + 5/4*a^4*b*x^4 + 1/2*a^5*x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.269438, size = 89, normalized size = 2.47

$$\frac{1}{12}\left(b^{5}x^{12}+6\,ab^{4}x^{10}+15\,a^{2}b^{3}x^{8}+20\,a^{3}b^{2}x^{6}+15\,a^{4}bx^{4}+6\,a^{5}x^{2}\right)\operatorname{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x,x, algorithm="giac")
```

```
[Out] 1/12*(b^5*x^12 + 6*a*b^4*x^10 + 15*a^2*b^3*x^8 + 20*a^3*b^2*x^6 +
 15*a^4*b*x^4 + 6*a^5*x^2)*sign(b*x^2 + a)
```

3.592 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x} dx$

Optimal. Leaf size=251

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{a^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (
5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) +
(5*a^2*b^3*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) +
 (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) +
(b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^
5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.192227, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{b^5x^{10}\sqrt{a^2+2abx^2+b^2x^4}}{10(a+bx^2)}+\frac{5ab^4x^8\sqrt{a^2+2abx^2+b^2x^4}}{8(a+bx^2)}+\frac{5a^2b^3x^6\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{a^5\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^4bx^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5a^3b^2x^4\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (5*a^4*b*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (
5*a^3*b^2*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) +
(5*a^2*b^3*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) +
 (5*a*b^4*x^8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) +
(b^5*x^10*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*(a + b*x^2)) + (a^
5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 26.0527, size = 178, normalized size = 0.71

$$\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}}{2}+\frac{a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4}+\frac{a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{6}+\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{8}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

```
[Out] a**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2) + a*
*4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 + a**3*(a + b*x**2)*sqrt
(a**2 + 2*a*b*x**2 + b**2*x**4)/4 + a**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**(3/2)/6 + a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)
**(3/2)/8 + (a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/10
```

Mathematica [A] time = 0.0446216, size = 82, normalized size = 0.33

$$\frac{\sqrt{(a+bx^2)^2}\left(120a^5\log(x)+bx^2\left(300a^4+300a^3bx^2+200a^2b^2x^4+75ab^3x^6+12b^4x^8\right)\right)}{120\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(b*x^2*(300*a^4 + 300*a^3*b*x^2 + 200*a^2*b^
2*x^4 + 75*a*b^3*x^6 + 12*b^4*x^8) + 120*a^5*Log[x]))/(120*(a + b
*x^2))
```

Maple [A] time = 0.013, size = 79, normalized size = 0.3

$$\frac{12\,b^5x^{10}+75\,ab^4x^8+200\,a^2b^3x^6+300\,a^3b^2x^4+300\,a^4bx^2+120\,a^5\ln(x)}{120\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+75*a*b^4*x^8+200*a^2*b^3*x
^6+300*a^3*b^2*x^4+300*a^4*b*x^2+120*a^5*ln(x))/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265407, size = 74, normalized size = 0.29

$$\frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + a^5\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4
 + 5/2*a^4*b*x^2 + a^5*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x, x)
```

GIAC/XCAS [A] time = 0.271755, size = 143, normalized size = 0.57

$$\frac{1}{10}\,b^5x^{10}\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{8}\,ab^4x^8\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{3}\,a^2b^3x^6\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{2}\,a^3b^2x^4\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{2}\,a^4bx^2\mathrm{sign}\left(bx^2 + a\right) + \frac{1}{2}\,a^5\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10*sign(b*x^2 + a) + 5/8*a*b^4*x^8*sign(b*x^2 + a) + 5
/3*a^2*b^3*x^6*sign(b*x^2 + a) + 5/2*a^3*b^2*x^4*sign(b*x^2 + a)
+ 5/2*a^4*b*x^2*sign(b*x^2 + a) + 1/2*a^5*ln(x^2)*sign(b*x^2 + a)
```

3.593 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^3}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a
^3*b^2*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*
b^3*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b
^4*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^
8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.207094, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{b^5x^8\sqrt{a^2+2abx^2+b^2x^4}}{8\left(a+bx^2\right)}+\frac{5ab^4x^6\sqrt{a^2+2abx^2+b^2x^4}}{6\left(a+bx^2\right)}+\frac{5a^2b^3x^4\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{2x^2\left(a+bx^2\right)}+\frac{5a^4b\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{5a^3b^2x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a
^3*b^2*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a^2*
b^3*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b
^4*x^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (b^5*x^
8*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*(a + b*x^2)) + (5*a^4*b*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 26.0027, size = 199, normalized size = 0.8

$$\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}\log\left(x\right)}{a+bx^2}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}}{2}+\frac{5a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4}+\frac{5ab\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{6}-\frac{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{8x^2}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

```
[Out] 5*a**4*b*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2)
+ 5*a**3*b*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 + 5*a**2*b*(a +
b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/4 + 5*a*b*(a**2 + 2*a
*b*x**2 + b**2*x**4)**(3/2)/6 - 5*a*(a + b*x**2)*(a**2 + 2*a*b*x*
*2 + b**2*x**4)**(3/2)/(8*x**2) + (a**2 + 2*a*b*x**2 + b**2*x**4)
**(5/2)/(8*x**2)
```

Mathematica [A] time = 0.0491727, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\left(-12a^5+120a^4bx^2\log(x)+120a^3b^2x^4+60a^2b^3x^6+20ab^4x^8+3b^5x^{10}\right)}{24x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-12*a^5 + 120*a^3*b^2*x^4 + 60*a^2*b^3*x^6
+ 20*a*b^4*x^8 + 3*b^5*x^10 + 120*a^4*b*x^2*Log[x]))/(24*x^2*(a +
 b*x^2))
```

Maple [A] time = 0.017, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4b\ln(x)\,x^2-12\,a^5}{24\,(bx^2+a)^5\,x^2}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^3,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+20*a*b^4*x^8+60*a^2*b^3*x^6+
120*a^3*b^2*x^4+120*a^4*b*ln(x)*x^2-12*a^5)/(b*x^2+a)^5/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269034, size = 82, normalized size = 0.33

$$\frac{3\,b^{5}x^{10}+20\,ab^{4}x^{8}+60\,a^{2}b^{3}x^{6}+120\,a^{3}b^{2}x^{4}+120\,a^{4}bx^{2}\log\left(x\right)-12\,a^{5}}{24\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^
4 + 120*a^4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**3, x)
```

GIAC/XCAS [A] time = 0.272321, size = 169, normalized size = 0.68

$$\frac{1}{8}\,b^{5}x^{8}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{6}\,ab^{4}x^{6}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{2}\,a^{2}b^{3}x^{4}\mathrm{sign}\left(bx^{2}+a\right)+5\,a^{3}b^{2}x^{2}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{2}\,a^{4}b\ln\left(x^{2}\right)\mathrm{sign}\left(bx^{2}+a\right)-\frac{5\,a^{4}bx^{2}\mathrm{sign}\left(bx^{2}+a\right)+a^{5}\mathrm{sign}\left(bx^{2}+a\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8*sign(b*x^2 + a) + 5/6*a*b^4*x^6*sign(b*x^2 + a) + 5/2
*a^2*b^3*x^4*sign(b*x^2 + a) + 5*a^3*b^2*x^2*sign(b*x^2 + a) + 5/
2*a^4*b*ln(x^2)*sign(b*x^2 + a) - 1/2*(5*a^4*b*x^2*sign(b*x^2 + a
) + a^5*sign(b*x^2 + a))/x^2
```

3.594 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^5} \, dx$

Optimal. Leaf size=250

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^
2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4
*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (10*a^3*b^2*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.204374, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{b^5x^6\sqrt{a^2+2abx^2+b^2x^4}}{6(a+bx^2)}+\frac{5ab^4x^4\sqrt{a^2+2abx^2+b^2x^4}}{4(a+bx^2)}+\frac{5a^2b^3x^2\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{4x^4(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)}+\frac{10a^3b^2\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (5*a^
2*b^3*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4
*x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (b^5*x^6*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*(a + b*x^2)) + (10*a^3*b^2*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 26.7312, size = 202, normalized size = 0.81

$$\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}+5a^2b^2\sqrt{a^2+2abx^2+b^2x^4}+\frac{5ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{2}+\frac{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{4x^4}+\frac{5b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{3}-\frac{3\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

```
[Out] 10*a**3*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x*
*2) + 5*a**2*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4) + 5*a*b**2*
(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 + 5*a*(a + b*x
**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(4*x**4) + 5*b**2*(a*
*2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/3 - 3*(a**2 + 2*a*b*x**2 + b*
*2*x**4)**(5/2)/(2*x**4)
```

Mathematica [A] time = 0.0484329, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\left(-3a^5-30a^4bx^2+120a^3b^2x^4\log(x)+60a^2b^3x^6+15ab^4x^8+2b^5x^{10}\right)}{12x^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^5,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 30*a^4*b*x^2 + 60*a^2*b^3*x^6 + 15
*a*b^4*x^8 + 2*b^5*x^10 + 120*a^3*b^2*x^4*Log[x]))/(12*x^4*(a + b
*x^2))
```

Maple [A] time = 0.018, size = 82, normalized size = 0.3

$$\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2\ln(x)\,x^4-30\,a^4bx^2-3\,a^5}{12\,(bx^2+a)^5\,x^4}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^5,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(2*b^5*x^10+15*a*b^4*x^8+60*a^2*b^3*x^6+
120*a^3*b^2*ln(x)*x^4-30*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273281, size = 82, normalized size = 0.33

$$\frac{2\,b^5x^{10} + 15\,ab^4x^8 + 60\,a^2b^3x^6 + 120\,a^3b^2x^4\log\left(x\right) - 30\,a^4bx^2 - 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^
4*log(x) - 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**5,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**5, x)
```

GIAC/XCAS [A] time = 0.272269, size = 171, normalized size = 0.68

$$\frac{1}{6}\,b^5x^6\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{4}\,ab^4x^4\mathrm{sign}\left(bx^2 + a\right) + 5\,a^2b^3x^2\mathrm{sign}\left(bx^2 + a\right) + 5\,a^3b^2\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right) - \frac{30\,a^3b^2x^4\mathrm{sign}\left(bx^2 + a\right) + 10\,a^4bx^2\mathrm{sign}\left(bx^2 + a\right) + a^5\mathrm{sign}\left(bx^2 + a\right)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^5*x^6*sign(b*x^2 + a) + 5/4*a*b^4*x^4*sign(b*x^2 + a) + 5*a
^2*b^3*x^2*sign(b*x^2 + a) + 5*a^3*b^2*ln(x^2)*sign(b*x^2 + a) -
1/4*(30*a^3*b^2*x^4*sign(b*x^2 + a) + 10*a^4*b*x^2*sign(b*x^2 + a
) + a^5*sign(b*x^2 + a))/x^4
```

3.595 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^7}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^
3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b
^4*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (10*a^2*b^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.204852, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^4\sqrt{a^2+2abx^2+b^2x^4}}{4\left(a+bx^2\right)}+\frac{5ab^4x^2\sqrt{a^2+2abx^2+b^2x^4}}{2\left(a+bx^2\right)}+\frac{10a^2b^3\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{6x^6\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^4\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```


```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^4*(a + b*x^2)) - (5*a^
3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (5*a*b
^4*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (b^5*x^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*(a + b*x^2)) + (10*a^2*b^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 26.6674, size = 207, normalized size = 0.83

$$\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}\log\left(x\right)}{a+bx^2}+5ab^3\sqrt{a^2+2abx^2+b^2x^4}-\frac{5ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{2x^2}+\frac{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{12x^6}+\frac{5b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{6x^2}-\frac{7\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

```
[Out] 10*a**2*b**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x*
*2) + 5*a*b**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4) - 5*a*b**2*(a
+ b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*x**2) + 5*a*(a +
 b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(12*x**6) + 5*b**
2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(6*x**2) - 7*(a**2 + 2*a
*b*x**2 + b**2*x**4)**(5/2)/(12*x**6)
```

Mathematica [A] time = 0.0421725, size = 85, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\left(-2a^5-15a^4bx^2-60a^3b^2x^4+120a^2b^3x^6\log(x)+30ab^4x^8+3b^5x^{10}\right)}{12x^6\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^7,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-2*a^5 - 15*a^4*b*x^2 - 60*a^3*b^2*x^4 + 30
*a*b^4*x^8 + 3*b^5*x^10 + 120*a^2*b^3*x^6*Log[x]))/(12*x^6*(a + b
*x^2))
```

Maple [A] time = 0.018, size = 82, normalized size = 0.3

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3\ln(x)\,x^6-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,(bx^2+a)^5\,x^6}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^7,x)
```

```
[Out] 1/12*((b*x^2+a)^2)^(5/2)*(3*b^5*x^10+30*a*b^4*x^8+120*a^2*b^3*ln(
x)*x^6-60*a^3*b^2*x^4-15*a^4*b*x^2-2*a^5)/(b*x^2+a)^5/x^6
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269117, size = 82, normalized size = 0.33

$$\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log\left(x\right)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3
*b^2*x^4 - 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**7,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**7, x)
```

GIAC/XCAS [A] time = 0.271616, size = 173, normalized size = 0.69

$$\frac{1}{4}\,b^5x^4\mathrm{sign}\left(bx^2+a\right)+\frac{5}{2}\,ab^4x^2\mathrm{sign}\left(bx^2+a\right)+5\,a^2b^3\ln\left(x^2\right)\mathrm{sign}\left(bx^2+a\right)-\frac{110\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+60\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+15\,a^4bx^2\mathrm{sign}\left(bx^2+a\right)+2\,a^5\mathrm{sign}\left(bx^2+a\right)}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4*sign(b*x^2 + a) + 5/2*a*b^4*x^2*sign(b*x^2 + a) + 5*a
^2*b^3*ln(x^2)*sign(b*x^2 + a) - 1/12*(110*a^2*b^3*x^6*sign(b*x^2
 + a) + 60*a^3*b^2*x^4*sign(b*x^2 + a) + 15*a^4*b*x^2*sign(b*x^2
+ a) + 2*a^5*sign(b*x^2 + a))/x^6
```

3.596 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^9}\,dx$

Optimal. Leaf size=250

$$\frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^
3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a
^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*
x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.202516, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b^5x^2\sqrt{a^2+2abx^2+b^2x^4}}{2(a+bx^2)}+\frac{5ab^4\log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^2(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{6x^6(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^6*(a + b*x^2)) - (5*a^
3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (5*a
^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^2*(a + b*x^2)) + (b^5*
x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*(a + b*x^2)) + (5*a*b^4*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 26.6625, size = 204, normalized size = 0.82

$$\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2}+\frac{5ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4x^4}+\frac{5a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{24x^8}+\frac{5b^4\sqrt{a^2+2abx^2+b^2x^4}}{2}-\frac{5b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{3x^4}-\frac{(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

```
[Out] 5*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2)
+ 5*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*x*
*4) + 5*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(24
*x**8) + 5*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2 - 5*b**2*(a
**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(3*x**4) - (a**2 + 2*a*b*x**
2 + b**2*x**4)**(5/2)/(3*x**8)
```

Mathematica [A] time = 0.0426819, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\left(3a^5+20a^4bx^2+60a^3b^2x^4+120a^2b^3x^6-120ab^4x^8\log(x)-12b^5x^{10}\right)}{24x^8(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^9,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a^5 + 20*a^4*b*x^2 + 60*a^3*b^2*x^4 + 12
0*a^2*b^3*x^6 - 12*b^5*x^10 - 120*a*b^4*x^8*Log[x]))/(24*x^8*(a +
 b*x^2))
```

Maple [A] time = 0.018, size = 82, normalized size = 0.3

$$\frac{12\,b^5x^{10}+120\,ab^4\ln(x)\,x^8-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,(bx^2+a)^5\,x^8}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^9,x)
```

```
[Out] 1/24*((b*x^2+a)^2)^(5/2)*(12*b^5*x^10+120*a*b^4*ln(x)*x^8-120*a^2
*b^3*x^6-60*a^3*b^2*x^4-20*a^4*b*x^2-3*a^5)/(b*x^2+a)^5/x^8
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271051, size = 82, normalized size = 0.33

$$\frac{12\,b^5x^{10} + 120\,ab^4x^8\log\left(x\right) - 120\,a^2b^3x^6 - 60\,a^3b^2x^4 - 20\,a^4bx^2 - 3\,a^5}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a
^3*b^2*x^4 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**9,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**9, x)
```

GIAC/XCAS [A] time = 0.272312, size = 170, normalized size = 0.68

$$\frac{1}{2}\,b^{5}x^{2}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{2}\,ab^{4}\ln\left(x^{2}\right)\,\mathrm{sign}\left(bx^{2}+a\right)-\frac{125\,ab^{4}x^{8}\mathrm{sign}\left(bx^{2}+a\right)+120\,a^{2}b^{3}x^{6}\mathrm{sign}\left(bx^{2}+a\right)+60\,a^{3}b^{2}x^{4}\mathrm{sign}\left(bx^{2}+a\right)+20\,a^{4}bx^{2}\mathrm{sign}\left(bx^{2}+a\right)+3\,a^{5}\mathrm{sign}\left(bx^{2}+a\right)}{24\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2*sign(b*x^2 + a) + 5/2*a*b^4*ln(x^2)*sign(b*x^2 + a) -
 1/24*(125*a*b^4*x^8*sign(b*x^2 + a) + 120*a^2*b^3*x^6*sign(b*x^2
 + a) + 60*a^3*b^2*x^4*sign(b*x^2 + a) + 20*a^4*b*x^2*sign(b*x^2
+ a) + 3*a^5*sign(b*x^2 + a))/x^8
```

3.597 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{11}} \, dx$

Optimal. Leaf size=251

$$\frac{b^5 \log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)} - \frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b
^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi [A] time = 0.197469, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{b^5 \log(x)\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} - \frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} - \frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{2x^4(a+bx^2)} - \frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)} - \frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{8x^8(a+bx^2)} - \frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^6(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2)) - (5*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^6*(a + b*x^2)) - (5
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^4*(a + b*x^2)) - (
5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^2*(a + b*x^2)) + (b
^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*Log[x])/(a + b*x^2)
```

Rubi in Sympy [A] time = 32.1492, size = 209, normalized size = 0.83

$$-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^2(a+bx^2)} + \frac{ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4x^6} + \frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{8x^{10}} + \frac{b^5\sqrt{a^2+2abx^2+b^2x^4}\log(x)}{a+bx^2} - \frac{5b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{12x^6} - \frac{9\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{40x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

```
[Out] -a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*x**2*(a + b*x**2))
 + a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4*x**
6) + a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(8*x**
10) + b**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x)/(a + b*x**2
) - 5*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(12*x**6) - 9*(
a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(40*x**10)
```

Mathematica [A] time = 0.0525758, size = 85, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\left(a\left(12a^4+75a^3bx^2+200a^2b^2x^4+300ab^3x^6+300b^4x^8\right)-120b^5x^{10}\log(x)\right)}{120x^{10}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^11,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a*(12*a^4 + 75*a^3*b*x^2 + 200*a^2*b^2*x^4
 + 300*a*b^3*x^6 + 300*b^4*x^8) - 120*b^5*x^10*Log[x]))/(120*x^10
*(a + b*x^2))
```

Maple [A] time = 0.019, size = 82, normalized size = 0.3

$$\frac{120\,b^5\ln(x)\,x^{10}-300\,ab^4x^8-300\,a^2b^3x^6-200\,a^3b^2x^4-75\,a^4bx^2-12\,a^5}{120\,\left(bx^2+a\right)^5x^{10}}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^11,x)
```

```
[Out] 1/120*((b*x^2+a)^2)^(5/2)*(120*b^5*ln(x)*x^10-300*a*b^4*x^8-300*a
^2*b^3*x^6-200*a^3*b^2*x^4-75*a^4*b*x^2-12*a^5)/(b*x^2+a)^5/x^10
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272675, size = 82, normalized size = 0.33

$$\frac{120\,b^5x^{10}\log\left(x\right) - 300\,ab^4x^8 - 300\,a^2b^3x^6 - 200\,a^3b^2x^4 - 75\,a^4bx^2 - 12\,a^5}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 20
0*a^3*b^2*x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**11,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**11, x)
```

GIAC/XCAS [A] time = 0.27188, size = 169, normalized size = 0.67

$$\frac{1}{2}\,b^5\ln\left(x^2\right)\,\mathrm{sign}\left(bx^2 + a\right) - \frac{137\,b^5x^{10}\mathrm{sign}\left(bx^2 + a\right) + 300\,ab^4x^8\mathrm{sign}\left(bx^2 + a\right) + 300\,a^2b^3x^6\mathrm{sign}\left(bx^2 + a\right) + 200\,a^3b^2x^4\mathrm{sign}\left(bx^2 + a\right) + 75\,a^4bx^2\mathrm{sign}}{120\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^11,x, algorithm="giac")
```

```
[Out] 1/2*b^5*ln(x^2)*sign(b*x^2 + a) - 1/120*(137*b^5*x^10*sign(b*x^2
+ a) + 300*a*b^4*x^8*sign(b*x^2 + a) + 300*a^2*b^3*x^6*sign(b*x^2
 + a) + 200*a^3*b^2*x^4*sign(b*x^2 + a) + 75*a^4*b*x^2*sign(b*x^2
 + a) + 12*a^5*sign(b*x^2 + a))/x^10
```

3.598 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{13}}\,dx$

Optimal. Leaf size=41

$$-\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*a*x^12)
```

Rubi [A] time = 0.104701, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{(a+bx^2)^5\sqrt{a^2+2abx^2+b^2x^4}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*a*x^12)
```

Rubi in Sympy [A] time = 8.23687, size = 39, normalized size = 0.95

$$-\frac{(2a+2bx^2)(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{24ax^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)
```

```
[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(24*a*x*
*12)
```

Mathematica [A] time = 0.0302659, size = 81, normalized size = 1.98

$$-\frac{\sqrt{(a+bx^2)^2}\left(a^5+6a^4bx^2+15a^3b^2x^4+20a^2b^3x^6+15ab^4x^8+6b^5x^{10}\right)}{12x^{12}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^13,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(a^5 + 6*a^4*b*x^2 + 15*a^3*b^2*x^4 + 20*a^
2*b^3*x^6 + 15*a*b^4*x^8 + 6*b^5*x^10))/(12*x^12*(a + b*x^2))
```

Maple [B] time = 0.01, size = 78, normalized size = 1.9

$$-\frac{6 b^5x^{10}+15 ab^4x^8+20 a^2b^3x^6+15 a^3b^2x^4+6 a^4bx^2+a^5}{12 x^{12} \left(bx^2+a\right)^5} \left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^13,x)
```

```
[Out] -1/12*(6*b^5*x^10+15*a*b^4*x^8+20*a^2*b^3*x^6+15*a^3*b^2*x^4+6*a^
4*b*x^2+a^5)*((b*x^2+a)^2)^(5/2)/x^12/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267498, size = 77, normalized size = 1.88

$$-\frac{6 b^5x^{10}+15 ab^4x^8+20 a^2b^3x^6+15 a^3b^2x^4+6 a^4bx^2+a^5}{12 x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^13,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^
4 + 6*a^4*b*x^2 + a^5)/x^12
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**13,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**13, x)
```

GIAC/XCAS [A] time = 0.27369, size = 143, normalized size = 3.49

$$-\frac{6\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right)+15\,ab^4x^8\mathrm{sign}\left(bx^2+a\right)+20\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+15\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+6\,a^4bx^2\mathrm{sign}\left(bx^2+\right.}{12\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^13,x, algorithm="giac")
```

```
[Out] -1/12*(6*b^5*x^10*sign(b*x^2 + a) + 15*a*b^4*x^8*sign(b*x^2 + a)
+ 20*a^2*b^3*x^6*sign(b*x^2 + a) + 15*a^3*b^2*x^4*sign(b*x^2 + a)
 + 6*a^4*b*x^2*sign(b*x^2 + a) + a^5*sign(b*x^2 + a))/x^12
```

3.599 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{15}}\,dx$

Optimal. Leaf size=72

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}$$

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*a*x^14) + (a
^2 + 2*a*b*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rubi [A] time = 0.0565695, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{\left(a^2+2abx^2+b^2x^4\right)^{7/2}}{84a^2x^{14}}-\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{12ax^{14}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(12*a*x^14) + (a
^2 + 2*a*b*x^2 + b^2*x^4)^(7/2)/(84*a^2*x^14)
```

Rubi in Sympy [A] time = 8.48485, size = 68, normalized size = 0.94

$$-\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{24ax^{14}}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{84a^2x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

```
[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(24*a*x*
*14) + (a**2 + 2*a*b*x**2 + b**2*x**4)**(7/2)/(84*a**2*x**14)
```

Mathematica [A] time = 0.0447752, size = 83, normalized size = 1.15

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(6a^5+35a^4bx^2+84a^3b^2x^4+105a^2b^3x^6+70ab^4x^8+21b^5x^{10}\right)}{84x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^15,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(6*a^5 + 35*a^4*b*x^2 + 84*a^3*b^2*x^4 + 10
5*a^2*b^3*x^6 + 70*a*b^4*x^8 + 21*b^5*x^10))/(84*x^14*(a + b*x^2)
)
```

Maple [A] time = 0.011, size = 80, normalized size = 1.1

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}\,(bx^2+a)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^15,x)
```

```
[Out] -1/84*(21*b^5*x^10+70*a*b^4*x^8+105*a^2*b^3*x^6+84*a^3*b^2*x^4+35
*a^4*b*x^2+6*a^5)*((b*x^2+a)^2)^(5/2)/x^14/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^15,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266162, size = 80, normalized size = 1.11

$$-\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^15,x, algorithm="fricas")
```

```
[Out] -1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*
x^4 + 35*a^4*b*x^2 + 6*a^5)/x^14
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{15}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**15,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**15, x)
```

GIAC/XCAS [A] time = 0.274544, size = 144, normalized size = 2.

$$-\frac{21\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right)+70\,ab^4x^8\mathrm{sign}\left(bx^2+a\right)+105\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+84\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+35\,a^4bx^2\mathrm{sign}\left(b\right.}{84\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^15,x, algorithm="giac")
```

```
[Out] -1/84*(21*b^5*x^10*sign(b*x^2 + a) + 70*a*b^4*x^8*sign(b*x^2 + a)
 + 105*a^2*b^3*x^6*sign(b*x^2 + a) + 84*a^3*b^2*x^4*sign(b*x^2 +
a) + 35*a^4*b*x^2*sign(b*x^2 + a) + 6*a^5*sign(b*x^2 + a))/x^14
```

3.600 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{17}}\,dx$

Optimal. Leaf size=128

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{16ax^{16}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{336a^3x^{12}}$$

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*a*x^16) + (b
*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (
b^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rubi [A] time = 0.228735, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{16ax^{16}}+\frac{b\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{56a^2x^{14}}-\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\left(a+bx^2\right)^5}{336a^3x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -((a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*a*x^16) + (b
*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(56*a^2*x^14) - (
b^2*(a + b*x^2)^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(336*a^3*x^12)
```

Rubi in Sympy [A] time = 15.2611, size = 112, normalized size = 0.88

$$-\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{32ax^{16}}+\frac{b\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{96a^2x^{14}}-\frac{b\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{336a^3x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

```
[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(32*a*x*
*16) + b*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/
(96*a**2*x**14) - b*(a**2 + 2*a*b*x**2 + b**2*x**4)**(7/2)/(336*a
**3*x**14)
```

Mathematica [A] time = 0.0365187, size = 83, normalized size = 0.65

$$-\frac{\sqrt{(a+bx^2)^2}\,(21a^5+120a^4bx^2+280a^3b^2x^4+336a^2b^3x^6+210ab^4x^8+56b^5x^{10})}{336x^{16}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^17,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(21*a^5 + 120*a^4*b*x^2 + 280*a^3*b^2*x^4 +
 336*a^2*b^3*x^6 + 210*a*b^4*x^8 + 56*b^5*x^10))/(336*x^16*(a + b
*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.6

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}\,(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^17,x)
```

```
[Out] -1/336*(56*b^5*x^10+210*a*b^4*x^8+336*a^2*b^3*x^6+280*a^3*b^2*x^4
+120*a^4*b*x^2+21*a^5)*((b*x^2+a)^2)^(5/2)/x^16/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^17,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267972, size = 80, normalized size = 0.62

$$-\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^17,x, algorithm="fricas")
```

```
[Out] -1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b
^2*x^4 + 120*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{17}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**17,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**17, x)
```

GIAC/XCAS [A] time = 0.272898, size = 144, normalized size = 1.12

$$-\frac{56\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right)+210\,ab^4x^8\mathrm{sign}\left(bx^2+a\right)+336\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+280\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+120\,a^4bx^2\mathrm{sign}}{336\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^17,x, algorithm="giac")
```

```
[Out] -1/336*(56*b^5*x^10*sign(b*x^2 + a) + 210*a*b^4*x^8*sign(b*x^2 +
a) + 336*a^2*b^3*x^6*sign(b*x^2 + a) + 280*a^3*b^2*x^4*sign(b*x^2
 + a) + 120*a^4*b*x^2*sign(b*x^2 + a) + 21*a^5*sign(b*x^2 + a))/x
^16
```

3.601 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{19}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) -
 (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^12*(a + b*x^2))
 - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^10*(a + b*x^2)) -
 (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi [A] time = 0.360323, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{8x^8\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2x^{10}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{6x^{12}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2)) -
 (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(6*x^12*(a + b*x^2))
 - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^10*(a + b*x^2)) -
 (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^8*(a + b*x^2))
```

Rubi in Sympy [A] time = 23.2613, size = 158, normalized size = 0.62

$$-\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{36ax^{18}}+\frac{b\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{96a^2x^{16}}-\frac{b^2\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{288a^3x^{14}}+\frac{b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{7}{2}}}{1008a^4x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

```
[Out] -(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(36*a*x*
*18) + b*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/
(96*a**2*x**16) - b**2*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2
*x**4)**(5/2)/(288*a**3*x**14) + b**2*(a**2 + 2*a*b*x**2 + b**2*x
**4)**(7/2)/(1008*a**4*x**14)
```

Mathematica [A] time = 0.0308624, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(56a^5+315a^4bx^2+720a^3b^2x^4+840a^2b^3x^6+504ab^4x^8+126b^5x^{10}\right)}{1008x^{18}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^19,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(56*a^5 + 315*a^4*b*x^2 + 720*a^3*b^2*x^4 +
 840*a^2*b^3*x^6 + 504*a*b^4*x^8 + 126*b^5*x^10))/(1008*x^18*(a +
 b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^19,x)
```

```
[Out] -1/1008*(126*b^5*x^10+504*a*b^4*x^8+840*a^2*b^3*x^6+720*a^3*b^2*x
^4+315*a^4*b*x^2+56*a^5)*((b*x^2+a)^2)^(5/2)/x^18/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^19,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26662, size = 80, normalized size = 0.31

$$-\frac{126\,b^5x^{10} + 504\,ab^4x^8 + 840\,a^2b^3x^6 + 720\,a^3b^2x^4 + 315\,a^4bx^2 + 56\,a^5}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^19,x, algorithm="fricas")
```

```
[Out] -1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3
*b^2*x^4 + 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{19}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**19,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**19, x)
```

GIAC/XCAS [A] time = 0.271549, size = 144, normalized size = 0.56

$$-\frac{126\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right) + 504\,ab^4x^8\mathrm{sign}\left(bx^2+a\right) + 840\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right) + 720\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right) + 315\,a^4bx^2\mathrm{sig}}{1008\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^19,x, algorithm="giac")
```

```
[Out] -1/1008*(126*b^5*x^10*sign(b*x^2 + a) + 504*a*b^4*x^8*sign(b*x^2
+ a) + 840*a^2*b^3*x^6*sign(b*x^2 + a) + 720*a^3*b^2*x^4*sign(b*x
```

```
^2 + a) + 315*a^4*b*x^2*sign(b*x^2 + a) + 56*a^5*sign(b*x^2 + a))
/x^18
```

3.602 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{21}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*x^20*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) -
 (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2))
 - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)
) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2))
```

Rubi [A] time = 0.362513, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{10x^{10}(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^{14}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{20x^{20}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{18x^{18}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(20*x^20*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(18*x^18*(a + b*x^2)) - (
5*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) -
 (5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^14*(a + b*x^2))
 - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2)
) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(10*x^10*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.3236, size = 204, normalized size = 0.8

$$\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2520x^{12}(a+bx^2)}+\frac{ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{168x^{16}}+\frac{a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{36x^{20}}-\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}}{420x^{12}}-\frac{5b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{252x^{16}}-\frac{7(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{90x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

```
[Out] a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2520*x**12*(a + b*x**
2)) + a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(16
8*x**16) + a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/
(36*x**20) - b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(420*x**12)
 - 5*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(252*x**16) - 7*
(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(90*x**20)
```

Mathematica [A] time = 0.034506, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(126a^5+700a^4bx^2+1575a^3b^2x^4+1800a^2b^3x^6+1050ab^4x^8+252b^5x^{10}\right)}{2520x^{20}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^21,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(126*a^5 + 700*a^4*b*x^2 + 1575*a^3*b^2*x^4
 + 1800*a^2*b^3*x^6 + 1050*a*b^4*x^8 + 252*b^5*x^10))/(2520*x^20*
(a + b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^21,x)
```

```
[Out] -1/2520*(252*b^5*x^10+1050*a*b^4*x^8+1800*a^2*b^3*x^6+1575*a^3*b^
2*x^4+700*a^4*b*x^2+126*a^5)*((b*x^2+a)^2)^(5/2)/x^20/(b*x^2+a)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^21,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266272, size = 80, normalized size = 0.31

$$-\frac{252\,b^{5}x^{10}+1050\,ab^{4}x^{8}+1800\,a^{2}b^{3}x^{6}+1575\,a^{3}b^{2}x^{4}+700\,a^{4}bx^{2}+126\,a^{5}}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^21,x, algorithm="fricas")
```

```
[Out] -1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*
a^3*b^2*x^4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{21}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**21,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**21, x)
```

GIAC/XCAS [A] time = 0.272575, size = 144, normalized size = 0.56

$$-\frac{252\,b^{5}x^{10}\mathrm{sign}\left(bx^{2}+a\right)+1050\,ab^{4}x^{8}\mathrm{sign}\left(bx^{2}+a\right)+1800\,a^{2}b^{3}x^{6}\mathrm{sign}\left(bx^{2}+a\right)+1575\,a^{3}b^{2}x^{4}\mathrm{sign}\left(bx^{2}+a\right)+700\,a^{4}bx}{2520\,x^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^21,x, algorithm="giac")
```

```
[Out] -1/2520*(252*b^5*x^10*sign(b*x^2 + a) + 1050*a*b^4*x^8*sign(b*x^2
 + a) + 1800*a^2*b^3*x^6*sign(b*x^2 + a) + 1575*a^3*b^2*x^4*sign(
b*x^2 + a) + 700*a^4*b*x^2*sign(b*x^2 + a) + 126*a^5*sign(b*x^2 +
 a))/x^20
```

3.603 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{23}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) -
(5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) -
 (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2))
```

Rubi [A] time = 0.365483, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{12x^{12}(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}(a+bx^2)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{8x^{16}(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{4x^{20}(a+bx^2)}-\frac{5a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*x^20*(a + b*x^2)) - (5*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) - (5
*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(8*x^16*(a + b*x^2)) -
(5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2)) -
 (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(12*x^12*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.5308, size = 202, normalized size = 0.79

$$\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{5544x^{14}(a+bx^2)}+\frac{ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{264x^{18}}+\frac{a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{44x^{22}}-\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}}{792x^{14}}-\frac{b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{72x^{18}}-\frac{3(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{44x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

```
[Out] a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5544*x**14*(a + b*x**
2)) + a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(26
4*x**18) + a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/
(44*x**22) - b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(792*x**14)
 - b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(72*x**18) - 3*(a*
*2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(44*x**22)
```

Mathematica [A] time = 0.0351357, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(252a^5+1386a^4bx^2+3080a^3b^2x^4+3465a^2b^3x^6+1980ab^4x^8+462b^5x^{10}\right)}{5544x^{22}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^23,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(252*a^5 + 1386*a^4*b*x^2 + 3080*a^3*b^2*x^
4 + 3465*a^2*b^3*x^6 + 1980*a*b^4*x^8 + 462*b^5*x^10))/(5544*x^22
*(a + b*x^2))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{462\,b^5x^{10}+1980\,ab^4x^8+3465\,a^2b^3x^6+3080\,a^3b^2x^4+1386\,a^4bx^2+252\,a^5}{5544\,x^{22}\,(bx^2+a)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^23,x)
```

```
[Out] -1/5544*(462*b^5*x^10+1980*a*b^4*x^8+3465*a^2*b^3*x^6+3080*a^3*b^
2*x^4+1386*a^4*b*x^2+252*a^5)*((b*x^2+a)^2)^(5/2)/x^22/(b*x^2+a)^
5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^23,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267657, size = 80, normalized size = 0.31

$$-\frac{462\,b^5x^{10} + 1980\,ab^4x^8 + 3465\,a^2b^3x^6 + 3080\,a^3b^2x^4 + 1386\,a^4bx^2 + 252\,a^5}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^23,x, algorithm="fricas")
```

```
[Out] -1/5544*(462*b^5*x^10 + 1980*a*b^4*x^8 + 3465*a^2*b^3*x^6 + 3080*
a^3*b^2*x^4 + 1386*a^4*b*x^2 + 252*a^5)/x^22
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**23,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273884, size = 144, normalized size = 0.56

$$-\frac{462\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right) + 1980\,ab^4x^8\mathrm{sign}\left(bx^2+a\right) + 3465\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right) + 3080\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right) + 1386\,a^4b}{5544\,x^{22}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^23,x, algorithm="giac")
```

```
[Out] -1/5544*(462*b^5*x^10*sign(b*x^2 + a) + 1980*a*b^4*x^8*sign(b*x^2
 + a) + 3465*a^2*b^3*x^6*sign(b*x^2 + a) + 3080*a^3*b^2*x^4*sign(
```

```
b*x^2 + a) + 1386*a^4*b*x^2*sign(b*x^2 + a) + 252*a^5*sign(b*x^2
+ a))/x^22
```

3.604 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{25}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*x^24*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (
a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (
5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) -
 (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2))
- (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2))
```

Rubi [A] time = 0.366966, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{14x^{14}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{16x^{16}\left(a+bx^2\right)}-\frac{5a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^{18}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{24x^{24}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{22x^{22}\left(a+bx^2\right)}-\frac{a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{2x^{20}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(24*x^24*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(22*x^22*(a + b*x^2)) - (
a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*x^20*(a + b*x^2)) - (
5*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^18*(a + b*x^2)) -
 (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(16*x^16*(a + b*x^2))
- (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(14*x^14*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.4063, size = 204, normalized size = 0.8

$$\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11088x^{16}\left(a+bx^2\right)}+\frac{ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{396x^{20}}+\frac{5a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{264x^{24}}-\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}}{1386x^{16}}-\frac{b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{99x^{20}}-\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{33x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
```

```
[Out] a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(11088*x**16*(a + b*x*
*2)) + a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3
96*x**20) + 5*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/
2)/(264*x**24) - b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1386*x
**16) - b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(99*x**20) -
2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(33*x**24)
```

Mathematica [A] time = 0.0399038, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(462a^5+2520a^4bx^2+5544a^3b^2x^4+6160a^2b^3x^6+3465ab^4x^8+792b^5x^{10})}{11088x^{24}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^25,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(462*a^5 + 2520*a^4*b*x^2 + 5544*a^3*b^2*x^
4 + 6160*a^2*b^3*x^6 + 3465*a*b^4*x^8 + 792*b^5*x^10))/(11088*x^2
4*(a + b*x^2))
```

Maple [A] time = 0.013, size = 80, normalized size = 0.3

$$-\frac{792\,b^5x^{10}+3465\,ab^4x^8+6160\,a^2b^3x^6+5544\,a^3b^2x^4+2520\,a^4bx^2+462\,a^5}{11088\,x^{24}\,(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^25,x)
```

```
[Out] -1/11088*(792*b^5*x^10+3465*a*b^4*x^8+6160*a^2*b^3*x^6+5544*a^3*b
^2*x^4+2520*a^4*b*x^2+462*a^5)*((b*x^2+a)^2)^(5/2)/x^24/(b*x^2+a)
^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^25,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.263522, size = 80, normalized size = 0.31

$$-\frac{792\, b^{5} x^{10} + 3465\, a b^{4} x^{8} + 6160\, a^{2} b^{3} x^{6} + 5544\, a^{3} b^{2} x^{4} + 2520\, a^{4} b x^{2} + 462\, a^{5}}{11088\, x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^25,x, algorithm="fricas")
```

```
[Out] -1/11088*(792*b^5*x^10 + 3465*a*b^4*x^8 + 6160*a^2*b^3*x^6 + 5544
*a^3*b^2*x^4 + 2520*a^4*b*x^2 + 462*a^5)/x^24
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**25,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.272949, size = 144, normalized size = 0.56

$$-\frac{792\, b^{5} x^{10} \mathrm{sign}\left(b x^{2} + a\right) + 3465\, a b^{4} x^{8} \mathrm{sign}\left(b x^{2} + a\right) + 6160\, a^{2} b^{3} x^{6} \mathrm{sign}\left(b x^{2} + a\right) + 5544\, a^{3} b^{2} x^{4} \mathrm{sign}\left(b x^{2} + a\right) + 2520\, a^{4} b}{11088\, x^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^25,x, algorithm="giac")
```

```
[Out] -1/11088*(792*b^5*x^10*sign(b*x^2 + a) + 3465*a*b^4*x^8*sign(b*x^
2 + a) + 6160*a^2*b^3*x^6*sign(b*x^2 + a) + 5544*a^3*b^2*x^4*sign
```

```
(b*x^2 + a) + 2520*a^4*b*x^2*sign(b*x^2 + a) + 462*a^5*sign(b*x^2
 + a))/x^24
```

3.605 $\int x^{12} \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23(a+bx^2)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21(a+bx^2)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)}+\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}$$

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^
4*b*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a
^3*b^2*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (
10*a^2*b^3*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*(a + b*x^2))
 + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*(a + b*x^2)
) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*(a + b*x^2))
```

Rubi [A] time = 0.189802, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{23}\sqrt{a^2+2abx^2+b^2x^4}}{23(a+bx^2)}+\frac{5ab^4x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21(a+bx^2)}+\frac{10a^2b^3x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)}+\frac{a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{a^4bx^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{10a^3b^2x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (a^
4*b*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (10*a
^3*b^2*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)) + (
10*a^2*b^3*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*(a + b*x^2))
 + (5*a*b^4*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*(a + b*x^2)
) + (b^5*x^23*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.6059, size = 207, normalized size = 0.81

$$\frac{256a^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{2028117(a+bx^2)}+\frac{128a^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{156009}+\frac{160a^3x^{13}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{52003}+\frac{80a^2x^{13}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{9177}+\frac{10ax^{13}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{483}+\frac{x^{13}\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x**13*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2028117*(a +
b*x**2)) + 128*a**4*x**13*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/156
009 + 160*a**3*x**13*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x
**4)/52003 + 80*a**2*x**13*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)
/9177 + 10*a*x**13*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**
(3/2)/483 + x**13*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/23
```

Mathematica [A] time = 0.0387963, size = 83, normalized size = 0.33

$$\frac{x^{13}\sqrt{(a+bx^2)^2}\left(156009a^5+676039a^4bx^2+1193010a^3b^2x^4+1067430a^2b^3x^6+482885ab^4x^8+88179b^5x^{10}\right)}{2028117\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^12*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^13*Sqrt[(a + b*x^2)^2]*(156009*a^5 + 676039*a^4*b*x^2 + 119301
0*a^3*b^2*x^4 + 1067430*a^2*b^3*x^6 + 482885*a*b^4*x^8 + 88179*b^
5*x^10))/(2028117*(a + b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$\frac{x^{13}\left(88179\,b^5x^{10}+482885\,ab^4x^8+1067430\,a^2b^3x^6+1193010\,a^3b^2x^4+676039\,a^4bx^2+156009\,a^5\right)}{2028117\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^12*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/2028117*x^13*(88179*b^5*x^10+482885*a*b^4*x^8+1067430*a^2*b^3*x
^6+1193010*a^3*b^2*x^4+676039*a^4*b*x^2+156009*a^5)*((b*x^2+a)^2)
^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.699748, size = 77, normalized size = 0.3

$$\frac{1}{23}\,b^5x^{23}+\frac{5}{21}\,ab^4x^{21}+\frac{10}{19}\,a^2b^3x^{19}+\frac{10}{17}\,a^3b^2x^{17}+\frac{1}{3}\,a^4bx^{15}+\frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^12,x, algorithm="maxima")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*
b^2*x^17 + 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Fricas [A] time = 0.25787, size = 77, normalized size = 0.3

$$\frac{1}{23}\,b^5x^{23} + \frac{5}{21}\,ab^4x^{21} + \frac{10}{19}\,a^2b^3x^{19} + \frac{10}{17}\,a^3b^2x^{17} + \frac{1}{3}\,a^4bx^{15} + \frac{1}{13}\,a^5x^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^12,x, algorithm="fricas")
```

```
[Out] 1/23*b^5*x^23 + 5/21*a*b^4*x^21 + 10/19*a^2*b^3*x^19 + 10/17*a^3*
b^2*x^17 + 1/3*a^4*b*x^15 + 1/13*a^5*x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{12}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**12*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**12*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.272574, size = 142, normalized size = 0.56

$$\frac{1}{23}\,b^5x^{23}\mathrm{sign}\left(bx^2+a\right) + \frac{5}{21}\,ab^4x^{21}\mathrm{sign}\left(bx^2+a\right) + \frac{10}{19}\,a^2b^3x^{19}\mathrm{sign}\left(bx^2+a\right) + \frac{10}{17}\,a^3b^2x^{17}\mathrm{sign}\left(bx^2+a\right) + \frac{1}{3}\,a^4bx^{15}\mathrm{sign}\left(bx^2+a\right) + \frac{1}{13}\,a^5x^{13}\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^12,x, algorithm="giac")
```

```
[Out] 1/23*b^5*x^23*sign(b*x^2 + a) + 5/21*a*b^4*x^21*sign(b*x^2 + a) +
 10/19*a^2*b^3*x^19*sign(b*x^2 + a) + 10/17*a^3*b^2*x^17*sign(b*x
^2 + a) + 1/3*a^4*b*x^15*sign(b*x^2 + a) + 1/13*a^5*x^13*sign(b*x
^2 + a)
```

3.606 $\int x^{10} \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21(a+bx^2)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}+\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*
a^4*b*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2
*a^3*b^2*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) +
(10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)
) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*(a + b*x^2
)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*(a + b*x^2))
```

Rubi [A] time = 0.184808, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^{21}\sqrt{a^2+2abx^2+b^2x^4}}{21(a+bx^2)}+\frac{5ab^4x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)}+\frac{10a^2b^3x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}+\frac{a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{5a^4bx^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}+\frac{2a^3b^2x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (5*
a^4*b*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) + (2
*a^3*b^2*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) +
(10*a^2*b^3*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2)
) + (5*a*b^4*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*(a + b*x^2
)) + (b^5*x^21*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.508, size = 207, normalized size = 0.81

$$\frac{256a^5x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{969969(a+bx^2)}+\frac{128a^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{88179}+\frac{32a^3x^{11}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{6783}+\frac{80a^2x^{11}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{6783}+\frac{10ax^{11}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{399}+\frac{x^{11}\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x**11*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(969969*(a + b
*x**2)) + 128*a**4*x**11*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/8817
9 + 32*a**3*x**11*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4
)/6783 + 80*a**2*x**11*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/678
3 + 10*a*x**11*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2
)/399 + x**11*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/21
```

Mathematica [A] time = 0.035721, size = 83, normalized size = 0.33

$$\frac{x^{11}\sqrt{(a+bx^2)^2}\,\left(88179a^5+373065a^4bx^2+646646a^3b^2x^4+570570a^2b^3x^6+255255ab^4x^8+46189b^5x^{10}\right)}{969969\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^11*Sqrt[(a + b*x^2)^2]*(88179*a^5 + 373065*a^4*b*x^2 + 646646*
a^3*b^2*x^4 + 570570*a^2*b^3*x^6 + 255255*a*b^4*x^8 + 46189*b^5*x
^10))/(969969*(a + b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$\frac{x^{11}\left(46189\,b^5x^{10}+255255\,ab^4x^8+570570\,a^2b^3x^6+646646\,a^3b^2x^4+373065\,a^4bx^2+88179\,a^5\right)}{969969\,(bx^2+a)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/969969*x^11*(46189*b^5*x^10+255255*a*b^4*x^8+570570*a^2*b^3*x^6
+646646*a^3*b^2*x^4+373065*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/
2)/(b*x^2+a)^5
```

Maxima [A] time = 0.68635, size = 77, normalized size = 0.3

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^
2*x^15 + 5/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Fricas [A] time = 0.258596, size = 77, normalized size = 0.3

$$\frac{1}{21}\,b^5x^{21}+\frac{5}{19}\,ab^4x^{19}+\frac{10}{17}\,a^2b^3x^{17}+\frac{2}{3}\,a^3b^2x^{15}+\frac{5}{13}\,a^4bx^{13}+\frac{1}{11}\,a^5x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^10,x, algorithm="fricas")
```

```
[Out] 1/21*b^5*x^21 + 5/19*a*b^4*x^19 + 10/17*a^2*b^3*x^17 + 2/3*a^3*b^
2*x^15 + 5/13*a^4*b*x^13 + 1/11*a^5*x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{10}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**10*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.273178, size = 142, normalized size = 0.56

$$\frac{1}{21}\,b^5x^{21}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{19}\,ab^4x^{19}\mathrm{sign}\left(bx^2+a\right)+\frac{10}{17}\,a^2b^3x^{17}\mathrm{sign}\left(bx^2+a\right)+\frac{2}{3}\,a^3b^2x^{15}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{13}\,a^4bx^{13}\mathrm{sign}\left(bx^2+a\right)+\frac{1}{11}\,a^5x^{11}\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^10,x, algorithm="giac")
```

```
[Out] 1/21*b^5*x^21*sign(b*x^2 + a) + 5/19*a*b^4*x^19*sign(b*x^2 + a) +
 10/17*a^2*b^3*x^17*sign(b*x^2 + a) + 2/3*a^3*b^2*x^15*sign(b*x^2
 + a) + 5/13*a^4*b*x^13*sign(b*x^2 + a) + 1/11*a^5*x^11*sign(b*x^
2 + a)
```

3.607 $\int x^8 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}$$

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^
4*b*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*
a^3*b^2*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) +
(2*a^2*b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
+ (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2))
 + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*(a + b*x^2))
```

Rubi [A] time = 0.186029, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{19}\sqrt{a^2+2abx^2+b^2x^4}}{19(a+bx^2)}+\frac{5ab^4x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17(a+bx^2)}+\frac{2a^2b^3x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{5a^4bx^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11(a+bx^2)}+\frac{10a^3b^2x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a^
4*b*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (10*
a^3*b^2*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) +
(2*a^2*b^3*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
+ (5*a*b^4*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2))
 + (b^5*x^19*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.6469, size = 207, normalized size = 0.81

$$\frac{256a^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{415701(a+bx^2)}+\frac{128a^4x^9\sqrt{a^2+2abx^2+b^2x^4}}{46189}+\frac{32a^3x^9\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{4199}+\frac{16a^2x^9\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{969}+\frac{10ax^9\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{323}+\frac{x^9\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x**9*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(415701*(a + b*
x**2)) + 128*a**4*x**9*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/46189
+ 32*a**3*x**9*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/4
199 + 16*a**2*x**9*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/969 + 1
0*a*x**9*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/323
+ x**9*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/19
```

Mathematica [A] time = 0.0348026, size = 83, normalized size = 0.33

$$\frac{x^9\sqrt{(a+bx^2)^2}\left(46189a^5+188955a^4bx^2+319770a^3b^2x^4+277134a^2b^3x^6+122265ab^4x^8+21879b^5x^{10}\right)}{415701\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^9*Sqrt[(a + b*x^2)^2]*(46189*a^5 + 188955*a^4*b*x^2 + 319770*a
^3*b^2*x^4 + 277134*a^2*b^3*x^6 + 122265*a*b^4*x^8 + 21879*b^5*x^
10))/(415701*(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$\frac{x^9\left(21879\,b^5x^{10}+122265\,ab^4x^8+277134\,a^2b^3x^6+319770\,a^3b^2x^4+188955\,a^4bx^2+46189\,a^5\right)}{415701\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/415701*x^9*(21879*b^5*x^10+122265*a*b^4*x^8+277134*a^2*b^3*x^6+
319770*a^3*b^2*x^4+188955*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2
)/(b*x^2+a)^5
```

Maxima [A] time = 0.70015, size = 77, normalized size = 0.3

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^8,x, algorithm="maxima")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^
2*x^13 + 5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Fricas [A] time = 0.264247, size = 77, normalized size = 0.3

$$\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^8,x, algorithm="fricas")
```

```
[Out] 1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^
2*x^13 + 5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^8\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**8*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.27287, size = 142, normalized size = 0.56

$$\frac{1}{19}\,b^5x^{19}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{17}\,ab^4x^{17}\mathrm{sign}\left(bx^2+a\right)+\frac{2}{3}\,a^2b^3x^{15}\mathrm{sign}\left(bx^2+a\right)+\frac{10}{13}\,a^3b^2x^{13}\mathrm{sign}\left(bx^2+a\right)+\frac{5}{11}\,a^4bx^{11}\mathrm{sign}\left(bx^2+a\right)+\frac{1}{9}\,a^5x^9\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^8,x, algorithm="giac")
```

```
[Out] 1/19*b^5*x^19*sign(b*x^2 + a) + 5/17*a*b^4*x^17*sign(b*x^2 + a) +
 2/3*a^2*b^3*x^15*sign(b*x^2 + a) + 10/13*a^3*b^2*x^13*sign(b*x^2
 + a) + 5/11*a^4*b*x^11*sign(b*x^2 + a) + 1/9*a^5*x^9*sign(b*x^2
+ a)
```

3.608 $\int x^6 \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=255

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\,(a+bx^2)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}$$

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^
4*b*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^
3*b^2*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (1
0*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2))
+ (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) +
(b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2))
```

Rubi [A] time = 0.184916, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{17}\sqrt{a^2+2abx^2+b^2x^4}}{17\,(a+bx^2)}+\frac{ab^4x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{10a^2b^3x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{5a^4bx^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{10a^3b^2x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (5*a^
4*b*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*a^
3*b^2*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (1
0*a^2*b^3*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2))
+ (a*b^4*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) +
(b^5*x^17*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.4365, size = 207, normalized size = 0.81

$$\frac{256a^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{153153\,(a+bx^2)}+\frac{128a^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{21879}+\frac{32a^3x^7\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{2431}+\frac{16a^2x^7\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{663}+\frac{2ax^7\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{51}+\frac{x^7\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x**7*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(153153*(a + b*
x**2)) + 128*a**4*x**7*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/21879
+ 32*a**3*x**7*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/2
431 + 16*a**2*x**7*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/663 + 2
*a*x**7*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/51 +
x**7*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/17
```

Mathematica [A] time = 0.0359517, size = 83, normalized size = 0.33

$$\frac{x^7\sqrt{(a+bx^2)^2}\left(21879a^5+85085a^4bx^2+139230a^3b^2x^4+117810a^2b^3x^6+51051ab^4x^8+9009b^5x^{10}\right)}{153153\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^7*Sqrt[(a + b*x^2)^2]*(21879*a^5 + 85085*a^4*b*x^2 + 139230*a^
3*b^2*x^4 + 117810*a^2*b^3*x^6 + 51051*a*b^4*x^8 + 9009*b^5*x^10)
)/(153153*(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$\frac{x^7\left(9009\,b^5x^{10}+51051\,ab^4x^8+117810\,a^2b^3x^6+139230\,a^3b^2x^4+85085\,a^4bx^2+21879\,a^5\right)}{153153\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/153153*x^7*(9009*b^5*x^10+51051*a*b^4*x^8+117810*a^2*b^3*x^6+13
9230*a^3*b^2*x^4+85085*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/(
b*x^2+a)^5
```

Maxima [A] time = 0.697638, size = 77, normalized size = 0.3

$$\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^6,x, algorithm="maxima")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b
^2*x^11 + 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Fricas [A] time = 0.260199, size = 77, normalized size = 0.3

$$\frac{1}{17}\,b^{5}x^{17}+\frac{1}{3}\,ab^{4}x^{15}+\frac{10}{13}\,a^{2}b^{3}x^{13}+\frac{10}{11}\,a^{3}b^{2}x^{11}+\frac{5}{9}\,a^{4}bx^{9}+\frac{1}{7}\,a^{5}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^6,x, algorithm="fricas")
```

```
[Out] 1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b
^2*x^11 + 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{6}\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.271728, size = 142, normalized size = 0.56

$$\frac{1}{17}\,b^{5}x^{17}\mathrm{sign}\left(bx^{2}+a\right)+\frac{1}{3}\,ab^{4}x^{15}\mathrm{sign}\left(bx^{2}+a\right)+\frac{10}{13}\,a^{2}b^{3}x^{13}\mathrm{sign}\left(bx^{2}+a\right)+\frac{10}{11}\,a^{3}b^{2}x^{11}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{9}\,a^{4}bx^{9}\mathrm{sign}\left(bx^{2}+a\right)+\frac{1}{7}\,a^{5}x^{7}\mathrm{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^6,x, algorithm="giac")
```

```
[Out] 1/17*b^5*x^17*sign(b*x^2 + a) + 1/3*a*b^4*x^15*sign(b*x^2 + a) +
10/13*a^2*b^3*x^13*sign(b*x^2 + a) + 10/11*a^3*b^2*x^11*sign(b*x^
2 + a) + 5/9*a^4*b*x^9*sign(b*x^2 + a) + 1/7*a^5*x^7*sign(b*x^2 +
 a)
```

3.609 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=255

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\,(a+bx^2)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}$$

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^
4*b*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^
3*b^2*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*
a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) +
(5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) +
 (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a + b*x^2))
```

Rubi [A] time = 0.185073, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{15}\sqrt{a^2+2abx^2+b^2x^4}}{15\,(a+bx^2)}+\frac{5ab^4x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{10a^2b^3x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\,(a+bx^2)}+\frac{5a^4bx^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}+\frac{10a^3b^2x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2)) + (5*a^
4*b*x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^
3*b^2*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (10*
a^2*b^3*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) +
(5*a*b^4*x^13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2)) +
 (b^5*x^15*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*(a + b*x^2))
```

Rubi in Sympy [A] time = 27.4097, size = 207, normalized size = 0.81

$$\frac{256a^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{45045\,(a+bx^2)}+\frac{128a^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{9009}+\frac{32a^3x^5\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{1287}+\frac{16a^2x^5\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{429}+\frac{2ax^5\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{39}+\frac{x^5\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(45045*(a + b*x
**2)) + 128*a**4*x**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/9009 +
32*a**3*x**5*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/128
7 + 16*a**2*x**5*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/429 + 2*a
*x**5*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/39 + x*
*5*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/15
```

Mathematica [A] time = 0.0349085, size = 83, normalized size = 0.33

$$\frac{x^5\sqrt{(a+bx^2)^2}\left(9009a^5+32175a^4bx^2+50050a^3b^2x^4+40950a^2b^3x^6+17325ab^4x^8+3003b^5x^{10}\right)}{45045\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^5*Sqrt[(a + b*x^2)^2]*(9009*a^5 + 32175*a^4*b*x^2 + 50050*a^3*
b^2*x^4 + 40950*a^2*b^3*x^6 + 17325*a*b^4*x^8 + 3003*b^5*x^10))/(
45045*(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$\frac{x^5\left(3003\,b^5x^{10}+17325\,ab^4x^8+40950\,a^2b^3x^6+50050\,a^3b^2x^4+32175\,a^4bx^2+9009\,a^5\right)}{45045\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/45045*x^5*(3003*b^5*x^10+17325*a*b^4*x^8+40950*a^2*b^3*x^6+5005
0*a^3*b^2*x^4+32175*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^
2+a)^5
```

Maxima [A] time = 0.701013, size = 77, normalized size = 0.3

$$\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^4,x, algorithm="maxima")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b
^2*x^9 + 5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Fricas [A] time = 0.259928, size = 77, normalized size = 0.3

$$\frac{1}{15}\,b^5x^{15} + \frac{5}{13}\,ab^4x^{13} + \frac{10}{11}\,a^2b^3x^{11} + \frac{10}{9}\,a^3b^2x^9 + \frac{5}{7}\,a^4bx^7 + \frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^4,x, algorithm="fricas")
```

```
[Out] 1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b
^2*x^9 + 5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.270671, size = 142, normalized size = 0.56

$$\frac{1}{15}\,b^5x^{15}\mathrm{sign}\left(bx^2+a\right) + \frac{5}{13}\,ab^4x^{13}\mathrm{sign}\left(bx^2+a\right) + \frac{10}{11}\,a^2b^3x^{11}\mathrm{sign}\left(bx^2+a\right) + \frac{10}{9}\,a^3b^2x^9\mathrm{sign}\left(bx^2+a\right) + \frac{5}{7}\,a^4bx^7\mathrm{sign}\left(bx^2+a\right) + \frac{1}{5}\,a^5x^5\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^4,x, algorithm="giac")
```

```
[Out] 1/15*b^5*x^15*sign(b*x^2 + a) + 5/13*a*b^4*x^13*sign(b*x^2 + a) +
 10/11*a^2*b^3*x^11*sign(b*x^2 + a) + 10/9*a^3*b^2*x^9*sign(b*x^2
 + a) + 5/7*a^4*b*x^7*sign(b*x^2 + a) + 1/5*a^5*x^5*sign(b*x^2 +
a)
```

3.610 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=252

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}$$

```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*
b*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*
x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^
3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4
*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^5*x^
13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2))
```

Rubi [A] time = 0.183144, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^{13}\sqrt{a^2+2abx^2+b^2x^4}}{13\,(a+bx^2)}+\frac{5ab^4x^{11}\sqrt{a^2+2abx^2+b^2x^4}}{11\,(a+bx^2)}+\frac{10a^2b^3x^9\sqrt{a^2+2abx^2+b^2x^4}}{9\,(a+bx^2)}+\frac{a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\,(a+bx^2)}+\frac{a^4bx^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^7\sqrt{a^2+2abx^2+b^2x^4}}{7\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a^4*
b*x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*
x^7*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (10*a^2*b^
3*x^9*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2)) + (5*a*b^4
*x^11*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*(a + b*x^2)) + (b^5*x^
13*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*(a + b*x^2))
```

Rubi in Sympy [A] time = 27.6286, size = 207, normalized size = 0.82

$$\frac{256a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{9009\,(a+bx^2)}+\frac{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3003}+\frac{160a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{3003}+\frac{80a^2x^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{1287}+\frac{10ax^3\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{143}+\frac{x^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(9009*(a + b*x*
*2)) + 128*a**4*x**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/3003 + 1
60*a**3*x**3*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/300
3 + 80*a**2*x**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/1287 + 10
*a*x**3*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/143 +
 x**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/13
```

Mathematica [A] time = 0.0349802, size = 83, normalized size = 0.33

$$\frac{x^3\sqrt{(a+bx^2)^2}\left(3003a^5+9009a^4bx^2+12870a^3b^2x^4+10010a^2b^3x^6+4095ab^4x^8+693b^5x^{10}\right)}{9009\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x^3*Sqrt[(a + b*x^2)^2]*(3003*a^5 + 9009*a^4*b*x^2 + 12870*a^3*b
^2*x^4 + 10010*a^2*b^3*x^6 + 4095*a*b^4*x^8 + 693*b^5*x^10))/(900
9*(a + b*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$\frac{x^3\left(693\,b^5x^{10}+4095\,ab^4x^8+10010\,a^2b^3x^6+12870\,a^3b^2x^4+9009\,a^4bx^2+3003\,a^5\right)}{9009\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/9009*x^3*(693*b^5*x^10+4095*a*b^4*x^8+10010*a^2*b^3*x^6+12870*a
^3*b^2*x^4+9009*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)
^5
```

Maxima [A] time = 0.700195, size = 76, normalized size = 0.3

$$\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2
*x^7 + a^4*b*x^5 + 1/3*a^5*x^3
```

Fricas [A] time = 0.261192, size = 76, normalized size = 0.3

$$\frac{1}{13}\,b^5x^{13} + \frac{5}{11}\,ab^4x^{11} + \frac{10}{9}\,a^2b^3x^9 + \frac{10}{7}\,a^3b^2x^7 + a^4bx^5 + \frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^2,x, algorithm="fricas")
```

```
[Out] 1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2
*x^7 + a^4*b*x^5 + 1/3*a^5*x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.271591, size = 140, normalized size = 0.56

$$\frac{1}{13}\,b^5x^{13}\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{11}\,ab^4x^{11}\mathrm{sign}\left(bx^2 + a\right) + \frac{10}{9}\,a^2b^3x^9\mathrm{sign}\left(bx^2 + a\right) + \frac{10}{7}\,a^3b^2x^7\mathrm{sign}\left(bx^2 + a\right) + a^4bx^5\mathrm{sign}\left(bx^2 + a\right) + \frac{1}{3}\,a^5x^3\mathrm{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^2,x, algorithm="giac")
```

```
[Out] 1/13*b^5*x^13*sign(b*x^2 + a) + 5/11*a*b^4*x^11*sign(b*x^2 + a) +
 10/9*a^2*b^3*x^9*sign(b*x^2 + a) + 10/7*a^3*b^2*x^7*sign(b*x^2 +
 a) + a^4*b*x^5*sign(b*x^2 + a) + 1/3*a^5*x^3*sign(b*x^2 + a)
```

3.611 $\int \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=248

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}$$

```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*
b*x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a
^3*b^2*x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10
*a^2*b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(7*(a + b*x^2)^5)
 + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(9*(a + b*x^2)
^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(11*(a + b*x^2
)^5)
```

Rubi [A] time = 0.141133, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{b^5x^{11}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{11\left(a+bx^2\right)^5}+\frac{5ab^4x^9\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{9\left(a+bx^2\right)^5}+\frac{10a^2b^3x^7\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{7\left(a+bx^2\right)^5}+\frac{a^5x\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}+\frac{5a^4bx^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{3\left(a+bx^2\right)^5}+\frac{2a^3b^2x^5\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^5*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (5*a^4*
b*x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(3*(a + b*x^2)^5) + (2*a
^3*b^2*x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(a + b*x^2)^5 + (10
*a^2*b^3*x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(7*(a + b*x^2)^5)
 + (5*a*b^4*x^9*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(9*(a + b*x^2)
^5) + (b^5*x^11*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))/(11*(a + b*x^2
)^5)
```

Rubi in Sympy [A] time = 46.4279, size = 197, normalized size = 0.79

$$\frac{256a^5x\sqrt{a^2+2abx^2+b^2x^4}}{693\,(a+bx^2)}+\frac{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}{693}+\frac{32a^3x\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{231}+\frac{80a^2x\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{693}+\frac{10ax\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{99}+\frac{x\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 256*a**5*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(693*(a + b*x**2))
 + 128*a**4*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/693 + 32*a**3*x
*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/231 + 80*a**2*x
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/693 + 10*a*x*(a + b*x**2)
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/99 + x*(a**2 + 2*a*b*x**2
 + b**2*x**4)**(5/2)/11
```

Mathematica [A] time = 0.0320447, size = 81, normalized size = 0.33

$$\frac{\sqrt{(a+bx^2)^2}\left(693a^5x+1155a^4bx^3+1386a^3b^2x^5+990a^2b^3x^7+385ab^4x^9+63b^5x^{11}\right)}{693\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(693*a^5*x + 1155*a^4*b*x^3 + 1386*a^3*b^2*x
^5 + 990*a^2*b^3*x^7 + 385*a*b^4*x^9 + 63*b^5*x^11))/(693*(a + b*
x^2))
```

Maple [A] time = 0.006, size = 78, normalized size = 0.3

$$\frac{x\left(63\,b^5x^{10}+385\,ab^4x^8+990\,a^2b^3x^6+1386\,a^3b^2x^4+1155\,a^4bx^2+693\,a^5\right)}{693\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/693*x*(63*b^5*x^10+385*a*b^4*x^8+990*a^2*b^3*x^6+1386*a^3*b^2*x
^4+1155*a^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.693496, size = 73, normalized size = 0.29

$$\frac{1}{11}\,b^5x^{11} + \frac{5}{9}\,ab^4x^9 + \frac{10}{7}\,a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}\,a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5
+ 5/3*a^4*b*x^3 + a^5*x
```

Fricas [A] time = 0.255879, size = 73, normalized size = 0.29

$$\frac{1}{11}\,b^5x^{11} + \frac{5}{9}\,ab^4x^9 + \frac{10}{7}\,a^2b^3x^7 + 2\,a^3b^2x^5 + \frac{5}{3}\,a^4bx^3 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5
+ 5/3*a^4*b*x^3 + a^5*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2), x)
```

GIAC/XCAS [A] time = 0.271644, size = 138, normalized size = 0.56

$$\frac{1}{11}\,b^{5}x^{11}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{9}\,ab^{4}x^{9}\mathrm{sign}\left(bx^{2}+a\right)+\frac{10}{7}\,a^{2}b^{3}x^{7}\mathrm{sign}\left(bx^{2}+a\right)+2\,a^{3}b^{2}x^{5}\mathrm{sign}\left(bx^{2}+a\right)+\frac{5}{3}\,a^{4}bx^{3}\mathrm{sign}\left(bx^{2}+a\right)+a^{5}x\mathrm{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11*sign(b*x^2 + a) + 5/9*a*b^4*x^9*sign(b*x^2 + a) + 1
0/7*a^2*b^3*x^7*sign(b*x^2 + a) + 2*a^3*b^2*x^5*sign(b*x^2 + a) +
 5/3*a^4*b*x^3*sign(b*x^2 + a) + a^5*x*sign(b*x^2 + a)
```

3.612 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4
*b*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x
^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*
x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^7*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^5*x^9*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rubi [A] time = 0.17594, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^9\sqrt{a^2+2abx^2+b^2x^4}}{9(a+bx^2)}+\frac{5ab^4x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{2a^2b^3x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{5a^4bx\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^3b^2x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] -((a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))) + (5*a^4
*b*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^3*b^2*x
^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (2*a^2*b^3*
x^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^7*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2)) + (b^5*x^9*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(9*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.1457, size = 196, normalized size = 0.79

$$-\frac{256a^5\sqrt{a^2+2abx^2+b^2x^4}}{63x(a+bx^2)}+\frac{128a^4\sqrt{a^2+2abx^2+b^2x^4}}{63x}+\frac{32a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{63x}+\frac{16a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{63x}+\frac{10a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{63x}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
```

```
[Out] -256*a**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(63*x*(a + b*x**2))
 + 128*a**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(63*x) + 32*a**3*
(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(63*x) + 16*a**2
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(63*x) + 10*a*(a + b*x**2
)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(63*x) + (a**2 + 2*a*b*x
**2 + b**2*x**4)**(5/2)/(9*x)
```

Mathematica [A] time = 0.0390482, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\left(-63a^5+315a^4bx^2+210a^3b^2x^4+126a^2b^3x^6+45ab^4x^8+7b^5x^{10}\right)}{63x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-63*a^5 + 315*a^4*b*x^2 + 210*a^3*b^2*x^4 +
 126*a^2*b^3*x^6 + 45*a*b^4*x^8 + 7*b^5*x^10))/(63*x*(a + b*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{10}-45\,ab^4x^8-126\,a^2b^3x^6-210\,a^3b^2x^4-315\,a^4bx^2+63\,a^5}{63\,x\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^2,x)
```

```
[Out] -1/63*(-7*b^5*x^10-45*a*b^4*x^8-126*a^2*b^3*x^6-210*a^3*b^2*x^4-3
15*a^4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x/(b*x^2+a)^5
```

Maxima [A] time = 0.705841, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x
^4 + 315*a^4*b*x^2 - 63*a^5)/x
```

Fricas [A] time = 0.259916, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{10} + 45\,ab^4x^8 + 126\,a^2b^3x^6 + 210\,a^3b^2x^4 + 315\,a^4bx^2 - 63\,a^5}{63\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x
^4 + 315*a^4*b*x^2 - 63*a^5)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**2, x)
```

GIAC/XCAS [A] time = 0.271463, size = 139, normalized size = 0.56

$$\frac{1}{9}\,b^5x^9\mathrm{sign}\left(bx^2 + a\right) + \frac{5}{7}\,ab^4x^7\mathrm{sign}\left(bx^2 + a\right) + 2\,a^2b^3x^5\mathrm{sign}\left(bx^2 + a\right) + \frac{10}{3}\,a^3b^2x^3\mathrm{sign}\left(bx^2 + a\right) + 5\,a^4bx\mathrm{sign}\left(bx^2 + a\right) - \frac{a^5\mathrm{sign}\left(bx^2 + a\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9*sign(b*x^2 + a) + 5/7*a*b^4*x^7*sign(b*x^2 + a) + 2*a
^2*b^3*x^5*sign(b*x^2 + a) + 10/3*a^3*b^2*x^3*sign(b*x^2 + a) + 5
*a^4*b*x*sign(b*x^2 + a) - a^5*sign(b*x^2 + a)/x
```

3.613 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^4}\,dx$

Optimal. Leaf size=246

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b
^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x
^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^5*x^7*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rubi [A] time = 0.17662, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^7\sqrt{a^2+2abx^2+b^2x^4}}{7(a+bx^2)}+\frac{ab^4x^5\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{10a^2b^3x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}+\frac{10a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^3*b
^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (10*a^2*b^3*x
^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (a*b^4*x^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^5*x^7*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])/(7*(a + b*x^2))
```

Rubi in Sympy [A] time = 40.4399, size = 211, normalized size = 0.86

$$\frac{256a^3b^2x\sqrt{a^2+2abx^2+b^2x^4}}{21(a+bx^2)}+\frac{128a^2b^2x\sqrt{a^2+2abx^2+b^2x^4}}{21}+\frac{32ab^2x\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{7}+\frac{10a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{3x^3}+\frac{80b^2x\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{21}-\frac{11\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
```

```
[Out] 256*a**3*b**2*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(21*(a + b*x*
*2)) + 128*a**2*b**2*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/21 + 3
2*a*b**2*x*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/7 + 1
0*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(3*x**3)
+ 80*b**2*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/21 - 11*(a**2
+ 2*a*b*x**2 + b**2*x**4)**(5/2)/(3*x**3)
```

Mathematica [A] time = 0.0383423, size = 83, normalized size = 0.34

$$\frac{\sqrt{(a+bx^2)^2}\left(-7a^5-105a^4bx^2+210a^3b^2x^4+70a^2b^3x^6+21ab^4x^8+3b^5x^{10}\right)}{21x^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^4,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-7*a^5 - 105*a^4*b*x^2 + 210*a^3*b^2*x^4 +
70*a^2*b^3*x^6 + 21*a*b^4*x^8 + 3*b^5*x^10))/(21*x^3*(a + b*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-3\,b^5x^{10}-21\,ab^4x^8-70\,a^2b^3x^6-210\,a^3b^2x^4+105\,a^4bx^2+7\,a^5}{21\,x^3\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^4,x)
```

```
[Out] -1/21*(-3*b^5*x^10-21*a*b^4*x^8-70*a^2*b^3*x^6-210*a^3*b^2*x^4+10
5*a^4*b*x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^3/(b*x^2+a)^5
```

Maxima [A] time = 0.693884, size = 80, normalized size = 0.33

$$\frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^
4 - 105*a^4*b*x^2 - 7*a^5)/x^3
```

Fricas [A] time = 0.263063, size = 80, normalized size = 0.33

$$\frac{3\,b^5x^{10} + 21\,ab^4x^8 + 70\,a^2b^3x^6 + 210\,a^3b^2x^4 - 105\,a^4bx^2 - 7\,a^5}{21\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^
4 - 105*a^4*b*x^2 - 7*a^5)/x^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**4, x)
```

GIAC/XCAS [A] time = 0.271885, size = 140, normalized size = 0.57

$$\frac{1}{7}\,b^5x^7\text{sign}\left(bx^2 + a\right) + ab^4x^5\text{sign}\left(bx^2 + a\right) + \frac{10}{3}\,a^2b^3x^3\text{sign}\left(bx^2 + a\right) + 10\,a^3b^2x\text{sign}\left(bx^2 + a\right) - \frac{15\,a^4bx^2\text{sign}\left(bx^2 + a\right) + a^5\text{sign}\left(bx^2 + a\right)}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7*sign(b*x^2 + a) + a*b^4*x^5*sign(b*x^2 + a) + 10/3*a^
2*b^3*x^3*sign(b*x^2 + a) + 10*a^3*b^2*x*sign(b*x^2 + a) - 1/3*(1
5*a^4*b*x^2*sign(b*x^2 + a) + a^5*sign(b*x^2 + a))/x^3
```

3.614 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^6}\,dx$

Optimal. Leaf size=249

$$\frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2
*b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^5*x^5*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi [A] time = 0.177201, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{b^5x^5\sqrt{a^2+2abx^2+b^2x^4}}{5\left(a+bx^2\right)}+\frac{5ab^4x^3\sqrt{a^2+2abx^2+b^2x^4}}{3\left(a+bx^2\right)}+\frac{10a^2b^3x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a
^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (10*a^2
*b^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (5*a*b^4*x^
3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2)) + (b^5*x^5*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.7095, size = 209, normalized size = 0.84

$$-\frac{256a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{15x\left(a+bx^2\right)}+\frac{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}{15x}+\frac{32ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{15x}+\frac{2a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{3x^5}+\frac{16b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{15x}-\frac{13\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
```

```
[Out] -256*a**3*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(15*x*(a + b*x
**2)) + 128*a**2*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(15*x)
+ 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(15*
x) + 2*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(3*x
**5) + 16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(15*x) - 13
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(15*x**5)
```

Mathematica [A] time = 0.0449608, size = 83, normalized size = 0.33

$$\frac{\sqrt{(a+bx^2)^2}\left(-3a^5-25a^4bx^2-150a^3b^2x^4+150a^2b^3x^6+25ab^4x^8+3b^5x^{10}\right)}{15x^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^6,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(-3*a^5 - 25*a^4*b*x^2 - 150*a^3*b^2*x^4 + 1
50*a^2*b^3*x^6 + 25*a*b^4*x^8 + 3*b^5*x^10))/(15*x^5*(a + b*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-3\,b^5x^{10}-25\,ab^4x^8-150\,a^2b^3x^6+150\,a^3b^2x^4+25\,a^4bx^2+3\,a^5}{15\,x^5\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^6,x)
```

```
[Out] -1/15*(-3*b^5*x^10-25*a*b^4*x^8-150*a^2*b^3*x^6+150*a^3*b^2*x^4+2
5*a^4*b*x^2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^5/(b*x^2+a)^5
```

Maxima [A] time = 0.69996, size = 80, normalized size = 0.32

$$\frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x
^4 - 25*a^4*b*x^2 - 3*a^5)/x^5
```

Fricas [A] time = 0.262312, size = 80, normalized size = 0.32

$$\frac{3 b^5 x^{10} + 25 a b^4 x^8 + 150 a^2 b^3 x^6 - 150 a^3 b^2 x^4 - 25 a^4 b x^2 - 3 a^5}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x
^4 - 25*a^4*b*x^2 - 3*a^5)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^2\right)^2\right)^{\frac{5}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**6,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**6, x)
```

GIAC/XCAS [A] time = 0.272048, size = 143, normalized size = 0.57

$$\frac{1}{5} b^5 x^5 \mathrm{sign}\left(b x^2 + a\right) + \frac{5}{3} a b^4 x^3 \mathrm{sign}\left(b x^2 + a\right) + 10 a^2 b^3 x \mathrm{sign}\left(b x^2 + a\right) - \frac{150 a^3 b^2 x^4 \mathrm{sign}\left(b x^2 + a\right) + 25 a^4 b x^2 \mathrm{sign}\left(b x^2 + a\right) + 3 a^5 \mathrm{sign}\left(b x^2 + a\right)}{15 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5*sign(b*x^2 + a) + 5/3*a*b^4*x^3*sign(b*x^2 + a) + 10*
a^2*b^3*x*sign(b*x^2 + a) - 1/15*(150*a^3*b^2*x^4*sign(b*x^2 + a)
 + 25*a^4*b*x^2*sign(b*x^2 + a) + 3*a^5*sign(b*x^2 + a))/x^5
```

3.615 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^8}\,dx$

Optimal. Leaf size=247

$$\frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a^4
*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b
^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2
*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*
x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^5*x^3*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi [A] time = 0.177523, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x^3\sqrt{a^2+2abx^2+b^2x^4}}{3(a+bx^2)}+\frac{5ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (a^4
*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^3*b
^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (10*a^2
*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (5*a*b^4*
x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b^5*x^3*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(3*(a + b*x^2))
```

Rubi in Sympy [A] time = 33.3664, size = 209, normalized size = 0.85

$$\frac{256ab^4x\sqrt{a^2+2abx^2+b^2x^4}}{21\left(a+bx^2\right)}+\frac{32ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{7x^3}+\frac{2a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{7x^7}+\frac{128b^4x\sqrt{a^2+2abx^2+b^2x^4}}{21}-\frac{16b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{3x^3}-\frac{3\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
```

```
[Out] 256*a*b**4*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(21*(a + b*x**2)
) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(7
*x**3) + 2*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/
(7*x**7) + 128*b**4*x*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/21 - 16
*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(3*x**3) - 3*(a**2 +
 2*a*b*x**2 + b**2*x**4)**(5/2)/(7*x**7)
```

Mathematica [A] time = 0.031702, size = 83, normalized size = 0.34

$$-\frac{\sqrt{\left(a+bx^2\right)^2}\left(3a^5+21a^4bx^2+70a^3b^2x^4+210a^2b^3x^6-105ab^4x^8-7b^5x^{10}\right)}{21x^7\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^8,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3*a^5 + 21*a^4*b*x^2 + 70*a^3*b^2*x^4 + 21
0*a^2*b^3*x^6 - 105*a*b^4*x^8 - 7*b^5*x^10))/(21*x^7*(a + b*x^2))
```

Maple [A] time = 0.009, size = 80, normalized size = 0.3

$$-\frac{-7\,b^5x^{10}-105\,ab^4x^8+210\,a^2b^3x^6+70\,a^3b^2x^4+21\,a^4bx^2+3\,a^5}{21\,x^7\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^8,x)
```

```
[Out] -1/21*(-7*b^5*x^10-105*a*b^4*x^8+210*a^2*b^3*x^6+70*a^3*b^2*x^4+2
1*a^4*b*x^2+3*a^5)*((b*x^2+a)^2)^(5/2)/x^7/(b*x^2+a)^5
```

Maxima [A] time = 0.69963, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{10} + 105\,ab^4x^8 - 210\,a^2b^3x^6 - 70\,a^3b^2x^4 - 21\,a^4bx^2 - 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^8,x, algorithm="maxima")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x
^4 - 21*a^4*b*x^2 - 3*a^5)/x^7
```

Fricas [A] time = 0.258653, size = 80, normalized size = 0.32

$$\frac{7\,b^5x^{10} + 105\,ab^4x^8 - 210\,a^2b^3x^6 - 70\,a^3b^2x^4 - 21\,a^4bx^2 - 3\,a^5}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x
^4 - 21*a^4*b*x^2 - 3*a^5)/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**8,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**8, x)
```

GIAC/XCAS [A] time = 0.273708, size = 143, normalized size = 0.58

$$\frac{1}{3}\,b^5x^3\mathrm{sign}\left(bx^2+a\right)+5\,ab^4x\mathrm{sign}\left(bx^2+a\right)-\frac{210\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+70\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+21\,a^4bx^2\mathrm{sign}\left(bx^2+a\right)+3\,a^5\mathrm{sign}\left(bx^2+a\right)}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3*sign(b*x^2 + a) + 5*a*b^4*x*sign(b*x^2 + a) - 1/21*(2
10*a^2*b^3*x^6*sign(b*x^2 + a) + 70*a^3*b^2*x^4*sign(b*x^2 + a) +
 21*a^4*b*x^2*sign(b*x^2 + a) + 3*a^5*sign(b*x^2 + a))/x^7
```

3.616 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{x^{10}} dx$

Optimal. Leaf size=246

$$\frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^
3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^
2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a
*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (b^5*x*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi [A] time = 0.173745, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{b^5x\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (5*a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (2*a^
3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (10*a^
2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (5*a
*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2)) + (b^5*x*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2)
```

Rubi in Sympy [A] time = 26.4847, size = 207, normalized size = 0.84

$$-\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{63x(a+bx^2)}+\frac{32ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{63x^5}+\frac{10a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{63x^9}+\frac{128b^4\sqrt{a^2+2abx^2+b^2x^4}}{63x}-\frac{16b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{21x^5}-\frac{17(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{63x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

```
[Out] -256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(63*x*(a + b*x**2
)) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(
63*x**5) + 10*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/
2)/(63*x**9) + 128*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(63*x
) - 16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(21*x**5) - 17
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(63*x**9)
```

Mathematica [A] time = 0.0312822, size = 83, normalized size = 0.34

$$-\frac{\sqrt{(a+bx^2)^2}\left(7a^5+45a^4bx^2+126a^3b^2x^4+210a^2b^3x^6+315ab^4x^8-63b^5x^{10}\right)}{63x^9(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^10,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(7*a^5 + 45*a^4*b*x^2 + 126*a^3*b^2*x^4 + 2
10*a^2*b^3*x^6 + 315*a*b^4*x^8 - 63*b^5*x^10))/(63*x^9*(a + b*x^2
))
```

Maple [A] time = 0.008, size = 80, normalized size = 0.3

$$-\frac{-63\,b^5x^{10}+315\,ab^4x^8+210\,a^2b^3x^6+126\,a^3b^2x^4+45\,a^4bx^2+7\,a^5}{63\,x^9(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^10,x)
```

```
[Out] -1/63*(-63*b^5*x^10+315*a*b^4*x^8+210*a^2*b^3*x^6+126*a^3*b^2*x^4
+45*a^4*b*x^2+7*a^5)*((b*x^2+a)^2)^(5/2)/x^9/(b*x^2+a)^5
```

Maxima [A] time = 0.71963, size = 80, normalized size = 0.33

$$\frac{63\,b^5x^{10} - 315\,ab^4x^8 - 210\,a^2b^3x^6 - 126\,a^3b^2x^4 - 45\,a^4bx^2 - 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^10,x, algorithm="maxima")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2
*x^4 - 45*a^4*b*x^2 - 7*a^5)/x^9
```

Fricas [A] time = 0.263454, size = 80, normalized size = 0.33

$$\frac{63\,b^5x^{10} - 315\,ab^4x^8 - 210\,a^2b^3x^6 - 126\,a^3b^2x^4 - 45\,a^4bx^2 - 7\,a^5}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^10,x, algorithm="fricas")
```

```
[Out] 1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2
*x^4 - 45*a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**10,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**10, x)
```

GIAC/XCAS [A] time = 0.271795, size = 142, normalized size = 0.58

$$b^5 x\operatorname{sign}\left(bx^2+a\right) - \frac{315\,ab^4x^8\operatorname{sign}\left(bx^2+a\right) + 210\,a^2b^3x^6\operatorname{sign}\left(bx^2+a\right) + 126\,a^3b^2x^4\operatorname{sign}\left(bx^2+a\right) + 45\,a^4bx^2\operatorname{sign}\left(bx^2+a\right) + 7\,a^5\operatorname{sign}\left(bx^2\right.}{63\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^10,x, algorithm="giac")
```

```
[Out] b^5*x*sign(b*x^2 + a) - 1/63*(315*a*b^4*x^8*sign(b*x^2 + a) + 210
*a^2*b^3*x^6*sign(b*x^2 + a) + 126*a^3*b^2*x^4*sign(b*x^2 + a) +
45*a^4*b*x^2*sign(b*x^2 + a) + 7*a^5*sign(b*x^2 + a))/x^9
```

3.617 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{12}} \, dx$

Optimal. Leaf size=251

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10
*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (
2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (5
*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b^
5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi [A] time = 0.17755, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{x(a+bx^2)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^3(a+bx^2)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{x^5(a+bx^2)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}(a+bx^2)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{9x^9(a+bx^2)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{7x^7(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) - (10
*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)) - (
2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (5
*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2)) - (b^
5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.7293, size = 211, normalized size = 0.84

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{693x^3(a+bx^2)}+\frac{32ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{231x^7}+\frac{10a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{99x^{11}}-\frac{128b^4\sqrt{a^2+2abx^2+b^2x^4}}{231x^3}-\frac{16b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{63x^7}-\frac{19(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{99x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(693*x**3*(a + b*x
**2)) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4
)/(231*x**7) + 10*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)*
*(3/2)/(99*x**11) - 128*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/
(231*x**3) - 16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(63*x
**7) - 19*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(99*x**11)
```

Mathematica [A] time = 0.0337643, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(63a^5+385a^4bx^2+990a^3b^2x^4+1386a^2b^3x^6+1155ab^4x^8+693b^5x^{10}\right)}{693x^{11}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^12,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(63*a^5 + 385*a^4*b*x^2 + 990*a^3*b^2*x^4 +
 1386*a^2*b^3*x^6 + 1155*a*b^4*x^8 + 693*b^5*x^10))/(693*x^11*(a
+ b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^12,x)
```

```
[Out] -1/693*(693*b^5*x^10+1155*a*b^4*x^8+1386*a^2*b^3*x^6+990*a^3*b^2*
x^4+385*a^4*b*x^2+63*a^5)*((b*x^2+a)^2)^(5/2)/x^11/(b*x^2+a)^5
```

Maxima [A] time = 0.700499, size = 80, normalized size = 0.32

$$-\frac{693\, b^5 x^{10} + 1155\, ab^4 x^8 + 1386\, a^2 b^3 x^6 + 990\, a^3 b^2 x^4 + 385\, a^4 b x^2 + 63\, a^5}{693\, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^12,x, algorithm="maxima")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^
3*b^2*x^4 + 385*a^4*b*x^2 + 63*a^5)/x^11
```

Fricas [A] time = 0.260375, size = 80, normalized size = 0.32

$$-\frac{693\, b^5 x^{10} + 1155\, ab^4 x^8 + 1386\, a^2 b^3 x^6 + 990\, a^3 b^2 x^4 + 385\, a^4 b x^2 + 63\, a^5}{693\, x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^12,x, algorithm="fricas")
```

```
[Out] -1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^
3*b^2*x^4 + 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}{x^{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**12,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**12, x)
```

GIAC/XCAS [A] time = 0.2716, size = 144, normalized size = 0.57

$$-\frac{693\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right)+1155\,ab^4x^8\mathrm{sign}\left(bx^2+a\right)+1386\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+990\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+385\,a^4bx^2\mathrm{s}}{693\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^12,x, algorithm="giac")
```

```
[Out] -1/693*(693*b^5*x^10*sign(b*x^2 + a) + 1155*a*b^4*x^8*sign(b*x^2
+ a) + 1386*a^2*b^3*x^6*sign(b*x^2 + a) + 990*a^3*b^2*x^4*sign(b*
x^2 + a) + 385*a^4*b*x^2*sign(b*x^2 + a) + 63*a^5*sign(b*x^2 + a)
)/x^11
```

3.618 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{14}}\,dx$

Optimal. Leaf size=253

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
 - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (b
^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi [A] time = 0.180109, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{3x^3\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{x^5\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
 - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(x^5*(a + b*x^2)) - (b
^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^3*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.4285, size = 211, normalized size = 0.83

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9009x^5(a+bx^2)}+\frac{160ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{3003x^9}+\frac{10a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{143x^{13}}-\frac{640b^4\sqrt{a^2+2abx^2+b^2x^4}}{9009x^5}-\frac{80b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{693x^9}-\frac{21\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{143x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(9009*x**5*(a + b*
x**2)) + 160*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x*
*4)/(3003*x**9) + 10*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**
4)**(3/2)/(143*x**13) - 640*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x*
*4)/(9009*x**5) - 80*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/
(693*x**9) - 21*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(143*x**13
)
```

Mathematica [A] time = 0.0299904, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(693a^5+4095a^4bx^2+10010a^3b^2x^4+12870a^2b^3x^6+9009ab^4x^8+3003b^5x^{10}\right)}{9009x^{13}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^14,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(693*a^5 + 4095*a^4*b*x^2 + 10010*a^3*b^2*x
^4 + 12870*a^2*b^3*x^6 + 9009*a*b^4*x^8 + 3003*b^5*x^10))/(9009*x
^13*(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}\,(bx^2+a)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^14,x)
```

```
[Out] -1/9009*(3003*b^5*x^10+9009*a*b^4*x^8+12870*a^2*b^3*x^6+10010*a^3
*b^2*x^4+4095*a^4*b*x^2+693*a^5)*((b*x^2+a)^2)^(5/2)/x^13/(b*x^2+
a)^5
```

Maxima [A] time = 0.694521, size = 80, normalized size = 0.32

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^14,x, algorithm="maxima")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 100
10*a^3*b^2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Fricas [A] time = 0.259252, size = 80, normalized size = 0.32

$$-\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^14,x, algorithm="fricas")
```

```
[Out] -1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 100
10*a^3*b^2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{14}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**14,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**14, x)
```

GIAC/XCAS [A] time = 0.272979, size = 144, normalized size = 0.57

$$-\frac{3003\, b^5 x^{10} \operatorname{sign}\left(bx^2+a\right) + 9009\, ab^4 x^8 \operatorname{sign}\left(bx^2+a\right) + 12870\, a^2 b^3 x^6 \operatorname{sign}\left(bx^2+a\right) + 10010\, a^3 b^2 x^4 \operatorname{sign}\left(bx^2+a\right) + 4095}{9009\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^14,x, algorithm="giac")
```

```
[Out] -1/9009*(3003*b^5*x^10*sign(b*x^2 + a) + 9009*a*b^4*x^8*sign(b*x^
2 + a) + 12870*a^2*b^3*x^6*sign(b*x^2 + a) + 10010*a^3*b^2*x^4*si
gn(b*x^2 + a) + 4095*a^4*b*x^2*sign(b*x^2 + a) + 693*a^5*sign(b*x
^2 + a))/x^13
```

3.619 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{16}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2))
 - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)
) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi [A] time = 0.177928, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{5x^5\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{15x^{15}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*x^15*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2))
 - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2)
) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*x^5*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.535, size = 211, normalized size = 0.83

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{45045x^7(a+bx^2)}+\frac{32ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{1287x^{11}}+\frac{2a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{39x^{15}}-\frac{128b^4\sqrt{a^2+2abx^2+b^2x^4}}{6435x^7}-\frac{80b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{1287x^{11}}-\frac{23(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{195x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(45045*x**7*(a + b
*x**2)) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x*
*4)/(1287*x**11) + 2*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**
4)**(3/2)/(39*x**15) - 128*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**
4)/(6435*x**7) - 80*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(
1287*x**11) - 23*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(195*x**1
5)
```

Mathematica [A] time = 0.0335128, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(3003a^5+17325a^4bx^2+40950a^3b^2x^4+50050a^2b^3x^6+32175ab^4x^8+9009b^5x^{10}\right)}{45045x^{15}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^16,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(3003*a^5 + 17325*a^4*b*x^2 + 40950*a^3*b^2
*x^4 + 50050*a^2*b^3*x^6 + 32175*a*b^4*x^8 + 9009*b^5*x^10))/(450
45*x^15*(a + b*x^2))
```

Maple [A] time = 0.01, size = 80, normalized size = 0.3

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^16,x)
```

```
[Out] -1/45045*(9009*b^5*x^10+32175*a*b^4*x^8+50050*a^2*b^3*x^6+40950*a
^3*b^2*x^4+17325*a^4*b*x^2+3003*a^5)*((b*x^2+a)^2)^(5/2)/x^15/(b*
x^2+a)^5
```

Maxima [A] time = 0.708736, size = 80, normalized size = 0.31

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^16,x, algorithm="maxima")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 4
0950*a^3*b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Fricas [A] time = 0.259888, size = 80, normalized size = 0.31

$$-\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^16,x, algorithm="fricas")
```

```
[Out] -1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 4
0950*a^3*b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{16}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**16,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**16, x)
```

GIAC/XCAS [A] time = 0.272736, size = 144, normalized size = 0.56

$$-\frac{9009\, b^5 x^{10} \operatorname{sign}\left(bx^2+a\right) + 32175\, ab^4 x^8 \operatorname{sign}\left(bx^2+a\right) + 50050\, a^2 b^3 x^6 \operatorname{sign}\left(bx^2+a\right) + 40950\, a^3 b^2 x^4 \operatorname{sign}\left(bx^2+a\right) + 1732}{45045\, x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^16,x, algorithm="giac")
```

```
[Out] -1/45045*(9009*b^5*x^10*sign(b*x^2 + a) + 32175*a*b^4*x^8*sign(b*
x^2 + a) + 50050*a^2*b^3*x^6*sign(b*x^2 + a) + 40950*a^3*b^2*x^4*
sign(b*x^2 + a) + 17325*a^4*b*x^2*sign(b*x^2 + a) + 3003*a^5*sign
(b*x^2 + a))/x^15
```

3.620 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{18}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) -
(10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)
) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2))
 - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi [A] time = 0.178327, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{7x^7\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^4b\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (a
^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) - (10*
a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2)) -
(10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2)
) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2))
 - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*x^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.4553, size = 211, normalized size = 0.83

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{153153x^9\,(a+bx^2)}+\frac{32ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{2431x^{13}}+\frac{2a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{51x^{17}}-\frac{128b^4\sqrt{a^2+2abx^2+b^2x^4}}{17017x^9}-\frac{16b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{429x^{13}}-\frac{5\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{51x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(153153*x**9*(a +
b*x**2)) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x
**4)/(2431*x**13) + 2*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x*
*4)**(3/2)/(51*x**17) - 128*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x*
*4)/(17017*x**9) - 16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)
/(429*x**13) - 5*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(51*x**17
)
```

Mathematica [A] time = 0.0307603, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(9009a^5+51051a^4bx^2+117810a^3b^2x^4+139230a^2b^3x^6+85085ab^4x^8+21879b^5x^{10}\right)}{153153x^{17}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^18,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(9009*a^5 + 51051*a^4*b*x^2 + 117810*a^3*b^
2*x^4 + 139230*a^2*b^3*x^6 + 85085*a*b^4*x^8 + 21879*b^5*x^10))/(
153153*x^17*(a + b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}\,(bx^2+a)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^18,x)
```

```
[Out] -1/153153*(21879*b^5*x^10+85085*a*b^4*x^8+139230*a^2*b^3*x^6+1178
10*a^3*b^2*x^4+51051*a^4*b*x^2+9009*a^5)*((b*x^2+a)^2)^(5/2)/x^17
/(b*x^2+a)^5
```

Maxima [A] time = 0.689808, size = 80, normalized size = 0.31

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^18,x, algorithm="maxima")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6
+ 117810*a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Fricas [A] time = 0.259087, size = 80, normalized size = 0.31

$$-\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^18,x, algorithm="fricas")
```

```
[Out] -1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6
+ 117810*a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}{x^{18}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**18,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**18, x)
```

GIAC/XCAS [A] time = 0.274105, size = 144, normalized size = 0.56

$$-\frac{21879\,b^5x^{10}\mathrm{sign}\left(bx^2+a\right)+85085\,ab^4x^8\mathrm{sign}\left(bx^2+a\right)+139230\,a^2b^3x^6\mathrm{sign}\left(bx^2+a\right)+117810\,a^3b^2x^4\mathrm{sign}\left(bx^2+a\right)+5}{153153\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^18,x, algorithm="giac")
```

```
[Out] -1/153153*(21879*b^5*x^10*sign(b*x^2 + a) + 85085*a*b^4*x^8*sign(
b*x^2 + a) + 139230*a^2*b^3*x^6*sign(b*x^2 + a) + 117810*a^3*b^2*
x^4*sign(b*x^2 + a) + 51051*a^4*b*x^2*sign(b*x^2 + a) + 9009*a^5*
sign(b*x^2 + a))/x^17
```

3.621 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{20}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (
2*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^
2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2))
```

Rubi [A] time = 0.18172, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{9x^9\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{2a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2)) - (
2*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2)) -
 (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^
2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*x^9*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.4678, size = 211, normalized size = 0.83

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{415701x^{11}(a+bx^2)}+\frac{32ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4199x^{15}}+\frac{10a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{323x^{19}}-\frac{128b^4\sqrt{a^2+2abx^2+b^2x^4}}{37791x^{11}}-\frac{16b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{663x^{15}}-\frac{27(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{323x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(415701*x**11*(a +
 b*x**2)) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*
x**4)/(4199*x**15) + 10*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*
x**4)**(3/2)/(323*x**19) - 128*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2
*x**4)/(37791*x**11) - 16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(
3/2)/(663*x**15) - 27*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(323
*x**19)
```

Mathematica [A] time = 0.0347217, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(21879a^5+122265a^4bx^2+277134a^3b^2x^4+319770a^2b^3x^6+188955ab^4x^8+46189b^5x^{10}\right)}{415701x^{19}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^20,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(21879*a^5 + 122265*a^4*b*x^2 + 277134*a^3*
b^2*x^4 + 319770*a^2*b^3*x^6 + 188955*a*b^4*x^8 + 46189*b^5*x^10)
)/(415701*x^19*(a + b*x^2))
```

Maple [A] time = 0.011, size = 80, normalized size = 0.3

$$-\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^20,x)
```

```
[Out] -1/415701*(46189*b^5*x^10+188955*a*b^4*x^8+319770*a^2*b^3*x^6+277
134*a^3*b^2*x^4+122265*a^4*b*x^2+21879*a^5)*((b*x^2+a)^2)^(5/2)/x
^19/(b*x^2+a)^5
```

Maxima [A] time = 0.706611, size = 80, normalized size = 0.31

$$-\frac{46189\, b^{5} x^{10} + 188955\, a b^{4} x^{8} + 319770\, a^{2} b^{3} x^{6} + 277134\, a^{3} b^{2} x^{4} + 122265\, a^{4} b x^{2} + 21879\, a^{5}}{415701\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^20,x, algorithm="maxima")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6
 + 277134*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Fricas [A] time = 0.259164, size = 80, normalized size = 0.31

$$-\frac{46189\, b^{5} x^{10} + 188955\, a b^{4} x^{8} + 319770\, a^{2} b^{3} x^{6} + 277134\, a^{3} b^{2} x^{4} + 122265\, a^{4} b x^{2} + 21879\, a^{5}}{415701\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^20,x, algorithm="fricas")
```

```
[Out] -1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6
 + 277134*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{5}{2}}}{x^{20}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**20,x)
```

```
[Out] Integral(((a + b*x**2)**2)**(5/2)/x**20, x)
```

GIAC/XCAS [A] time = 0.272039, size = 144, normalized size = 0.56

$$-\frac{46189\, b^5 x^{10} \operatorname{sign}\left(bx^2+a\right) + 188955\, ab^4 x^8 \operatorname{sign}\left(bx^2+a\right) + 319770\, a^2 b^3 x^6 \operatorname{sign}\left(bx^2+a\right) + 277134\, a^3 b^2 x^4 \operatorname{sign}\left(bx^2+a\right) + }{415701\, x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^20,x, algorithm="giac")
```

```
[Out] -1/415701*(46189*b^5*x^10*sign(b*x^2 + a) + 188955*a*b^4*x^8*sign
(b*x^2 + a) + 319770*a^2*b^3*x^6*sign(b*x^2 + a) + 277134*a^3*b^2
*x^4*sign(b*x^2 + a) + 122265*a^4*b*x^2*sign(b*x^2 + a) + 21879*a
^5*sign(b*x^2 + a))/x^19
```

3.622 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{22}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2))
 - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^
2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2))
```

Rubi [A] time = 0.179276, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{11x^{11}\left(a+bx^2\right)}-\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{2a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x^2))
 - (2*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2
)) - (5*a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^
2)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*x^11*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.3625, size = 211, normalized size = 0.83

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{969969x^{13}(a+bx^2)}+\frac{32ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{6783x^{17}}+\frac{10a(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{399x^{21}}-\frac{128b^4\sqrt{a^2+2abx^2+b^2x^4}}{74613x^{13}}-\frac{16b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{969x^{17}}-\frac{29(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{399x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(969969*x**13*(a +
 b*x**2)) + 32*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*
x**4)/(6783*x**17) + 10*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*
x**4)**(3/2)/(399*x**21) - 128*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2
*x**4)/(74613*x**13) - 16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(
3/2)/(969*x**17) - 29*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(399
*x**21)
```

Mathematica [A] time = 0.0309283, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\,(46189a^5+255255a^4bx^2+570570a^3b^2x^4+646646a^2b^3x^6+373065ab^4x^8+88179b^5x^{10})}{969969x^{21}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^22,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(46189*a^5 + 255255*a^4*b*x^2 + 570570*a^3*
b^2*x^4 + 646646*a^2*b^3*x^6 + 373065*a*b^4*x^8 + 88179*b^5*x^10)
)/(969969*x^21*(a + b*x^2))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{88179\,b^5x^{10}+373065\,ab^4x^8+646646\,a^2b^3x^6+570570\,a^3b^2x^4+255255\,a^4bx^2+46189\,a^5}{969969\,x^{21}\,(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^22,x)
```

```
[Out] -1/969969*(88179*b^5*x^10+373065*a*b^4*x^8+646646*a^2*b^3*x^6+570
570*a^3*b^2*x^4+255255*a^4*b*x^2+46189*a^5)*((b*x^2+a)^2)^(5/2)/x
^21/(b*x^2+a)^5
```

Maxima [A] time = 0.698523, size = 80, normalized size = 0.31

$$-\frac{88179\,b^5x^{10} + 373065\,ab^4x^8 + 646646\,a^2b^3x^6 + 570570\,a^3b^2x^4 + 255255\,a^4bx^2 + 46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^22,x, algorithm="maxima")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6
 + 570570*a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

Fricas [A] time = 0.259814, size = 80, normalized size = 0.31

$$-\frac{88179\,b^5x^{10} + 373065\,ab^4x^8 + 646646\,a^2b^3x^6 + 570570\,a^3b^2x^4 + 255255\,a^4bx^2 + 46189\,a^5}{969969\,x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^22,x, algorithm="fricas")
```

```
[Out] -1/969969*(88179*b^5*x^10 + 373065*a*b^4*x^8 + 646646*a^2*b^3*x^6
 + 570570*a^3*b^2*x^4 + 255255*a^4*b*x^2 + 46189*a^5)/x^21
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**22,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273466, size = 144, normalized size = 0.56

$$-\frac{88179\, b^5 x^{10} \mathrm{sign}\left(bx^2+a\right) + 373065\, ab^4 x^8 \mathrm{sign}\left(bx^2+a\right) + 646646\, a^2 b^3 x^6 \mathrm{sign}\left(bx^2+a\right) + 570570\, a^3 b^2 x^4 \mathrm{sign}\left(bx^2+a\right) +}{969969\, x^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^22,x, algorithm="giac")
```

```
[Out] -1/969969*(88179*b^5*x^10*sign(b*x^2 + a) + 373065*a*b^4*x^8*sign
(b*x^2 + a) + 646646*a^2*b^3*x^6*sign(b*x^2 + a) + 570570*a^3*b^2
*x^4*sign(b*x^2 + a) + 255255*a^4*b*x^2*sign(b*x^2 + a) + 46189*a
^5*sign(b*x^2 + a))/x^21
```

3.623 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{5/2}}{x^{24}}\,dx$

Optimal. Leaf size=255

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}$$

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*x^23*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2))
 - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x
^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2
)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2))
```

Rubi [A] time = 0.180125, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}}{13x^{13}\left(a+bx^2\right)}-\frac{ab^4\sqrt{a^2+2abx^2+b^2x^4}}{3x^{15}\left(a+bx^2\right)}-\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}}{17x^{17}\left(a+bx^2\right)}-\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}}{23x^{23}\left(a+bx^2\right)}-\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}}{21x^{21}\left(a+bx^2\right)}-\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}}{19x^{19}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*x^23*(a + b*x^2)) - (5
*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*x^21*(a + b*x^2)) - (
10*a^3*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*x^19*(a + b*x^2))
 - (10*a^2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*x^17*(a + b*x
^2)) - (a*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*x^15*(a + b*x^2
)) - (b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*x^13*(a + b*x^2))
```

Rubi in Sympy [A] time = 26.4433, size = 211, normalized size = 0.83

$$\frac{256ab^4\sqrt{a^2+2abx^2+b^2x^4}}{2028117x^{15}(a+bx^2)}+\frac{160ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{52003x^{19}}+\frac{10a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{483x^{23}}-\frac{640b^4\sqrt{a^2+2abx^2+b^2x^4}}{676039x^{15}}-\frac{80b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{6783x^{19}}-\frac{31\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{483x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

```
[Out] 256*a*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2028117*x**15*(a
+ b*x**2)) + 160*a*b**2*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**
2*x**4)/(52003*x**19) + 10*a*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b*
*2*x**4)**(3/2)/(483*x**23) - 640*b**4*sqrt(a**2 + 2*a*b*x**2 + b
**2*x**4)/(676039*x**15) - 80*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4
)**(3/2)/(6783*x**19) - 31*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)
/(483*x**23)
```

Mathematica [A] time = 0.0358403, size = 83, normalized size = 0.33

$$-\frac{\sqrt{(a+bx^2)^2}\left(88179a^5+482885a^4bx^2+1067430a^3b^2x^4+1193010a^2b^3x^6+676039ab^4x^8+156009b^5x^{10}\right)}{2028117x^{23}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/x^24,x]
```

```
[Out] -(Sqrt[(a + b*x^2)^2]*(88179*a^5 + 482885*a^4*b*x^2 + 1067430*a^3
*b^2*x^4 + 1193010*a^2*b^3*x^6 + 676039*a*b^4*x^8 + 156009*b^5*x^
10))/(2028117*x^23*(a + b*x^2))
```

Maple [A] time = 0.012, size = 80, normalized size = 0.3

$$-\frac{156009\,b^5x^{10}+676039\,ab^4x^8+1193010\,a^2b^3x^6+1067430\,a^3b^2x^4+482885\,a^4bx^2+88179\,a^5}{2028117\,x^{23}\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/x^24,x)
```

```
[Out] -1/2028117*(156009*b^5*x^10+676039*a*b^4*x^8+1193010*a^2*b^3*x^6+
1067430*a^3*b^2*x^4+482885*a^4*b*x^2+88179*a^5)*((b*x^2+a)^2)^(5/
2)/x^23/(b*x^2+a)^5
```

Maxima [A] time = 0.705122, size = 80, normalized size = 0.31

$$-\frac{156009\, b^5x^{10} + 676039\, ab^4x^8 + 1193010\, a^2b^3x^6 + 1067430\, a^3b^2x^4 + 482885\, a^4bx^2 + 88179\, a^5}{2028117\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^24,x, algorithm="maxima")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*
x^6 + 1067430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

Fricas [A] time = 0.258587, size = 80, normalized size = 0.31

$$-\frac{156009\, b^5x^{10} + 676039\, ab^4x^8 + 1193010\, a^2b^3x^6 + 1067430\, a^3b^2x^4 + 482885\, a^4bx^2 + 88179\, a^5}{2028117\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^24,x, algorithm="fricas")
```

```
[Out] -1/2028117*(156009*b^5*x^10 + 676039*a*b^4*x^8 + 1193010*a^2*b^3*
x^6 + 1067430*a^3*b^2*x^4 + 482885*a^4*b*x^2 + 88179*a^5)/x^23
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/x**24,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271771, size = 144, normalized size = 0.56

$$-\frac{156009\, b^5 x^{10} \mathrm{sign}\left(bx^2+a\right) + 676039\, ab^4 x^8 \mathrm{sign}\left(bx^2+a\right) + 1193010\, a^2 b^3 x^6 \mathrm{sign}\left(bx^2+a\right) + 1067430\, a^3 b^2 x^4 \mathrm{sign}\left(bx^2+a\right)}{2028117\, x^{23}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/x^24,x, algorithm="giac")
```

```
[Out] -1/2028117*(156009*b^5*x^10*sign(b*x^2 + a) + 676039*a*b^4*x^8*si
gn(b*x^2 + a) + 1193010*a^2*b^3*x^6*sign(b*x^2 + a) + 1067430*a^3
*b^2*x^4*sign(b*x^2 + a) + 482885*a^4*b*x^2*sign(b*x^2 + a) + 881
79*a^5*sign(b*x^2 + a))/x^23
```

3.624 $\int \frac{x^5}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=127

$$\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(a*x^2*(a + b*x^2))/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x
^4*(a + b*x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a +
 b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.247639, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x^4\left(a+bx^2\right)}{4b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{ax^2\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^2\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(a*x^2*(a + b*x^2))/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x
^4*(a + b*x^2))/(4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^2*(a +
 b*x^2)*Log[a + b*x^2])/(2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0414004, size = 55, normalized size = 0.43

$$\frac{(a+bx^2)\left(2a^2\log(a+bx^2)+bx^2(bx^2-2a)\right)}{4b^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2*(-2*a + b*x^2) + 2*a^2*Log[a + b*x^2]))/(4*b^
3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.011, size = 52, normalized size = 0.4

$$\frac{(bx^2+a)\left(b^2x^4-2\,abx^2+2\,a^2\ln(bx^2+a)\right)}{4\,b^3}\frac{1}{\sqrt{(bx^2+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))/((b*x^2+a)^2)
^(1/2)/b^3
```

Maxima [A] time = 0.693782, size = 62, normalized size = 0.49

$$\frac{x^4}{4\sqrt{b^2}}-\frac{abx^2}{2\,(b^2)^{\frac{3}{2}}}+\frac{a^2b^2\log\left(x^2+\frac{a}{b}\right)}{2\,(b^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4/sqrt(b^2) - 1/2*a*b*x^2/(b^2)^(3/2) + 1/2*a^2*b^2*log(x^2
 + a/b)/(b^2)^(5/2)
```

Fricas [A] time = 0.257979, size = 45, normalized size = 0.35

$$\frac{b^2x^4-2\,abx^2+2\,a^2\log(bx^2+a)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [A] time = 1.22657, size = 32, normalized size = 0.25

$$\frac{a^2\log\left(a+bx^2\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

GIAC/XCAS [A] time = 0.271777, size = 80, normalized size = 0.63

$$\frac{a^2\ln\left(\left|bx^2+a\right|\right)\,\mathrm{sign}\left(bx^2+a\right)}{2\,b^3}+\frac{bx^4\mathrm{sign}\left(bx^2+a\right)-2\,ax^2\mathrm{sign}\left(bx^2+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/2*a^2*ln(abs(b*x^2 + a))*sign(b*x^2 + a)/b^3 + 1/4*(b*x^4*sign(
b*x^2 + a) - 2*a*x^2*sign(b*x^2 + a))/b^2
```

3.625 $\int \frac{x^3}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a +
b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.146885, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}-\frac{a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(2*b^2) - (a*(a + b*x^2)*Log[a +
b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.020955, size = 44, normalized size = 0.59

$$\frac{\left(a+bx^2\right)\left(bx^2-a\log\left(a+bx^2\right)\right)}{2b^2\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(b*x^2 - a*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2)^
2])
```

Maple [A] time = 0.011, size = 41, normalized size = 0.6

$$-\frac{\left(bx^2+a\right)\left(-bx^2+a\ln\left(bx^2+a\right)\right)}{2\,b^2}\frac{1}{\sqrt{\left(bx^2+a\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(-b*x^2+a*ln(b*x^2+a))/((b*x^2+a)^2)^(1/2)/b^2
```

Maxima [A] time = 0.704752, size = 63, normalized size = 0.84

$$-\frac{a\sqrt{\frac{1}{b^2}}\log\left(x^2+\frac{a}{b}\right)}{2\,b}+\frac{\sqrt{b^2x^4+2\,abx^2+a^2}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] -1/2*a*sqrt(b^(-2))*log(x^2 + a/b)/b + 1/2*sqrt(b^2*x^4 + 2*a*b*x
^2 + a^2)/b^2
```

Fricas [A] time = 0.257117, size = 30, normalized size = 0.4

$$\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A] time = 1.16651, size = 20, normalized size = 0.27

$$-\frac{a\log\left(a+bx^2\right)}{2b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

GIAC/XCAS [A] time = 0.270255, size = 45, normalized size = 0.6

$$\frac{1}{2}\left(\frac{x^2}{b}-\frac{a\ln\left(\left|bx^2+a\right|\right)}{b^2}\right)\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/2*(x^2/b - a*ln(abs(b*x^2 + a))/b^2)*sign(b*x^2 + a)
```

3.626 $\int \frac{x}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)
```

Rubi [A] time = 0.0786467, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)
```

Rubi in Sympy [A] time = 3.98391, size = 29, normalized size = 0.66

$$\frac{\sqrt{(a+bx^2)^2}\log(a+bx^2)}{2b(a+bx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt((a + b*x**2)**2)*log(a + b*x**2)/(2*b*(a + b*x**2))
```

Mathematica [A] time = 0.0118819, size = 35, normalized size = 0.8

$$\frac{(a+bx^2)\log(a+bx^2)}{2b\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*Log[a + b*x^2])/(2*b*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.007, size = 32, normalized size = 0.7

$$\frac{(bx^2+a)\ln(bx^2+a)}{2b}\frac{1}{\sqrt{(bx^2+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*ln(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)
```

Maxima [A] time = 0.698751, size = 23, normalized size = 0.52

$$\frac{1}{2}\sqrt{\frac{1}{b^2}}\log\left(x^2+\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b^(-2))*log(x^2 + a/b)
```

Fricas [A] time = 0.258657, size = 18, normalized size = 0.41

$$\frac{\log(bx^2+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(b*x^2 + a)/b
```

Sympy [A] time = 0.292081, size = 10, normalized size = 0.23

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/((b*x**2+a)**2)**(1/2),x)

[Out] log(a + b*x**2)/(2*b)
```

GIAC/XCAS [A] time = 0.270073, size = 30, normalized size = 0.68

$$\frac{\ln\left(\left|bx^2+a\right|\right)\,\mathrm{sign}\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt((b*x^2 + a)^2),x, algorithm="giac")

[Out] 1/2*ln(abs(b*x^2 + a))*sign(b*x^2 + a)/b
```

3.627 $\int \frac{1}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=80

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a +
b*x^2)*Log[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.103734, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192

$$\frac{\log(x)\left(a+bx^2\right)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((a +
b*x^2)*Log[a + b*x^2])/(2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a + b*x**2)**2)), x)
```

Mathematica [A] time = 0.0195548, size = 42, normalized size = 0.52

$$\frac{\left(a+bx^2\right)\left(2\log(x)-\log\left(a+bx^2\right)\right)}{2a\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] ((a + b*x^2)*(2*Log[x] - Log[a + b*x^2]))/(2*a*Sqrt[(a + b*x^2)^2
])
```

Maple [A] time = 0.012, size = 37, normalized size = 0.5

$$-\frac{\left(bx^{2}+a\right)\left(\ln\left(bx^{2}+a\right)-2\ln\left(x\right)\right)}{2a}\frac{1}{\sqrt{\left(bx^{2}+a\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)*(ln(b*x^2+a)-2*ln(x))/((b*x^2+a)^2)^(1/2)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.261634, size = 24, normalized size = 0.3

$$-\frac{\log\left(bx^{2}+a\right)-2\log\left(x\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x),x, algorithm="fricas")
```

```
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A] time = 0.569871, size = 15, normalized size = 0.19

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/((b*x**2+a)**2)**(1/2),x)

[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

GIAC/XCAS [A] time = 0.271064, size = 45, normalized size = 0.56

$$\frac{1}{2}\left(\frac{\ln\left(x^2\right)}{a} - \frac{\ln\left(\left|bx^2 + a\right|\right)}{a}\right)\operatorname{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x),x, algorithm="giac")

[Out] 1/2*(ln(x^2)/a - ln(abs(b*x^2 + a))/a)*sign(b*x^2 + a)
```

3.628 $\int \frac{1}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=122

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(a + b*x^2)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a +
b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*
x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.135872, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{a+bx^2}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b\log(x)\left(a+bx^2\right)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(2*a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*(a +
b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a + b*
x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt((a + b*x**2)**2)), x)
```

Mathematica [A] time = 0.0300493, size = 54, normalized size = 0.44

$$-\frac{(a+bx^2)\left(-bx^2\log\left(a+bx^2\right)+a+2bx^2\log(x)\right)}{2a^2x^2\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)*(a + 2*b*x^2*Log[x] - b*x^2*Log[a + b*x^2]))/(2*a^2
*x^2*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.016, size = 52, normalized size = 0.4

$$\frac{(bx^2+a)\left(b\ln\left(bx^2+a\right)x^2-2b\ln(x)x^2-a\right)}{2a^2x^2}\frac{1}{\sqrt{(bx^2+a)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/2*(b*x^2+a)*(b*ln(b*x^2+a)*x^2-2*b*ln(x)*x^2-a)/((b*x^2+a)^2)^(
1/2)/x^2/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.262705, size = 45, normalized size = 0.37

$$\frac{bx^2\log\left(bx^2+a\right)-2bx^2\log(x)-a}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A] time = 1.59665, size = 31, normalized size = 0.25

$$-\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

GIAC/XCAS [A] time = 0.271726, size = 70, normalized size = 0.57

$$-\frac{1}{2}\left(\frac{b\ln\left(x^2\right)}{a^2} - \frac{b\ln\left(\left|bx^2 + a\right|\right)}{a^2} - \frac{bx^2 - a}{a^2x^2}\right)\operatorname{sign}\left(bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*(b*ln(x^2)/a^2 - b*ln(abs(b*x^2 + a))/a^2 - (b*x^2 - a)/(a^2
*x^2))*sign(b*x^2 + a)
```

3.629 $\int \frac{x^4}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=129

$$-\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3
*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a
 + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

Rubi [A] time = 0.139212, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{ax\left(a+bx^2\right)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^3\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((a*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) + (x^3
*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/2)*(a
 + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0486806, size = 66, normalized size = 0.51

$$\frac{(a+bx^2)\left(3a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{b}x\left(bx^2-3a\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(-3*a + b*x^2) + 3*a^(3/2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.012, size = 63, normalized size = 0.5

$$\frac{bx^2+a}{3\,b^2}\left(\sqrt{ab}x^3b-3\,\sqrt{ab}xa+3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*((a*b)^(1/2)*x^3*b-3*(a*b)^(1/2)*x*a+3*a^2*arctan(x
*b/(a*b)^(1/2)))/((b*x^2+a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264613, size = 1, normalized size = 0.01

$$\left[\frac{2\,bx^3+3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,ax}{6\,b^2},\frac{bx^3+3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-3\,ax}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(x/s
qrt(a/b)) - 3*a*x)/b^2]
```

Sympy [A] time = 1.27894, size = 80, normalized size = 0.62

$$-\frac{ax}{b^2}-\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x-\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x+\frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 +
 sqrt(-a**3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

GIAC/XCAS [A] time = 0.273083, size = 86, normalized size = 0.67

$$\frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\mathrm{sign}\left(bx^2+a\right)}{\sqrt{ab}b^2}+\frac{b^2x^3\mathrm{sign}\left(bx^2+a\right)-3\,abx\mathrm{sign}\left(bx^2+a\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] a^2*arctan(b*x/sqrt(a*b))*sign(b*x^2 + a)/(sqrt(a*b)*b^2) + 1/3*(
b^2*x^3*sign(b*x^2 + a) - 3*a*b*x*sign(b*x^2 + a))/b^3
```

3.630 $\int \frac{x^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=89

$$\frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a
 + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

Rubi [A] time = 0.104396, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{x\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(a
 + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(3/2)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0327067, size = 54, normalized size = 0.61

$$\frac{(a+bx^2)\left(\sqrt{b}x-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{b^{3/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x - Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(
b^(3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.01, size = 48, normalized size = 0.5

$$\frac{bx^2+a}{b}\left(x\sqrt{ab}-a\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] (b*x^2+a)*(x*(a*b)^(1/2)-a*arctan(x*b/(a*b)^(1/2)))/((b*x^2+a)^2)
^(1/2)/b/(a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268572, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{x}{\sqrt{\frac{a}{b}}}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a))
+ 2*x)/b, -(sqrt(a/b)*arctan(x/sqrt(a/b)) - x)/b]
```

Sympy [A] time = 1.19954, size = 56, normalized size = 0.63

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*s
qrt(-a/b**3) + x)/2 + x/b
```

GIAC/XCAS [A] time = 0.271855, size = 57, normalized size = 0.64

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sign}\left(bx^2+a\right)}{\sqrt{ab}b}+\frac{x\operatorname{sign}\left(bx^2+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))*sign(b*x^2 + a)/(sqrt(a*b)*b) + x*sign(b
*x^2 + a)/b
```

3.631 $\int \frac{1}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=53

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0412784, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**2)**2), x)
```

Mathematica [A] time = 0.0235722, size = 44, normalized size = 0.83

$$\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*Sqrt[(
a + b*x^2)^2])
```

Maple [A] time = 0.006, size = 34, normalized size = 0.6

$$(bx^2+a)\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/(a*b)^(1/2)*arctan(x*b/(a*b)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266776, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)}{2\sqrt{-ab}},\ \frac{\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a))/sqrt(-a*
b), arctan(sqrt(a*b)*x/a)/sqrt(a*b)]
```

Sympy [A] time = 0.363505, size = 53, normalized size = 1.

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log
(a*sqrt(-1/(a*b)) + x)/2
```

GIAC/XCAS [A] time = 0.272995, size = 31, normalized size = 0.58

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)\operatorname{sign}\left(bx^2+a\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))*sign(b*x^2 + a)/sqrt(a*b)
```

3.632 $\int \frac{1}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi [A] time = 0.102713, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a+bx^2}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - (Sqrt[b]*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt((a + b*x**2)**2)), x)
```

Mathematica [A] time = 0.0269867, size = 56, normalized size = 0.61

$$-\frac{(a+bx^2)\left(\sqrt{b}x\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+\sqrt{a}\right)}{a^{3/2}x\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*(Sqrt[a] + Sqrt[b]*x*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))
/(a^(3/2)*x*Sqrt[(a + b*x^2)^2]))
```

Maple [A] time = 0.014, size = 50, normalized size = 0.5

$$-\frac{bx^2+a}{ax}\left(b\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x+\sqrt{ab}\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -(b*x^2+a)*(b*arctan(x*b/(a*b)^(1/2))*x+(a*b)^(1/2))/((b*x^2+a)^2
)^(1/2)/a/(a*b)^(1/2)/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270888, size = 1, normalized size = 0.01

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)
) - 2)/(a*x), -(x*sqrt(b/a)*arctan(b*x/(a*sqrt(b/a))) + 1)/(a*x)]
```

Sympy [A] time = 1.33737, size = 65, normalized size = 0.71

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*lo
g(a**2*sqrt(-b/a**3)/b + x)/2 - 1/(a*x)
```

GIAC/XCAS [A] time = 0.275917, size = 50, normalized size = 0.54

$$-\left(\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{1}{ax}\right)\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^2),x, algorithm="giac")
```

```
[Out] -(b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/(a*x))*sign(b*x^2 + a
)
```

3.633 $\int \frac{1}{x^4\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=130

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a +
b*x^2))/(a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])
```

Rubi [A] time = 0.140357, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{b\left(a+bx^2\right)}{a^2x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{3ax^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(a + b*x^2)/(3*a*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*(a +
b*x^2))/(a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(3/2)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{\left(a+bx^2\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt((a + b*x**2)**2)), x)
```

Mathematica [A] time = 0.0437113, size = 70, normalized size = 0.54

$$-\frac{(a+bx^2)\left(\sqrt{a}\,(a-3bx^2)-3b^{3/2}x^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\right)}{3a^{5/2}x^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((a + b*x^2)*(Sqrt[a]*(a - 3*b*x^2) - 3*b^(3/2)*x^3*ArcTan[(Sqrt
[b]*x)/Sqrt[a]]))/(3*a^(5/2)*x^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.016, size = 69, normalized size = 0.5

$$\frac{bx^2+a}{3\,a^2x^3}\left(3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3+3\,b\sqrt{ab}x^2-a\sqrt{ab}\right)\frac{1}{\sqrt{(bx^2+a)^2}}\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/3*(b*x^2+a)*(3*b^2*arctan(x*b/(a*b)^(1/2))*x^3+3*b*(a*b)^(1/2)*
x^2-a*(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a^2/(a*b)^(1/2)/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270313, size = 1, normalized size = 0.01

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3}, \frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^4),x, algorithm="fricas")
```

```
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^
2 + a)) + 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan
(b*x/(a*sqrt(b/a))) + 3*b*x^2 - a)/(a^2*x^3)]
```

Sympy [A] time = 1.49538, size = 87, normalized size = 0.67

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-
b**3/a**5)*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2
)/(3*a**2*x**3)
```

GIAC/XCAS [A] time = 0.28209, size = 68, normalized size = 0.52

$$\frac{1}{3}\left(\frac{3\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{3\,bx^2-a}{a^2x^3}\right)\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*x^4),x, algorithm="giac")
```

```
[Out] 1/3*(3*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + (3*b*x^2 - a)/
(a^2*x^3))*sign(b*x^2 + a)
```

3.634 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a
+ b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*
b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b
*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.310168, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{3a^2}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^3}{4b^4\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*a^2)/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^3/(4*b^4*(a
+ b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*
b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*a*(a + b*x^2)*Log[a + b
*x^2])/(2*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 24.0764, size = 141, normalized size = 0.89

$$\frac{ax^4\left(a+bx^2\right)}{4b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}-\frac{3a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^4}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\sqrt{a^2+2abx^2+b^2x^4}}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] a*x**4*(a + b*x**2)/(4*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2
)) - 3*a*(a + b*x**2)*log(a + b*x**2)/(2*b**4*sqrt(a**2 + 2*a*b*x
**2 + b**2*x**4)) - x**4/(b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4
)) + 3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*b**4)
```

Mathematica [A] time = 0.0500946, size = 81, normalized size = 0.51

$$\frac{-5a^3-4a^2bx^2+4ab^2x^4-6a\left(a+bx^2\right)^2\log\left(a+bx^2\right)+2b^3x^6}{4b^4\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*a^3 - 4*a^2*b*x^2 + 4*a*b^2*x^4 + 2*b^3*x^6 - 6*a*(a + b*x^2)
^2*Log[a + b*x^2])/(4*b^4*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.024, size = 103, normalized size = 0.7

$$-\frac{\left(-2\,b^3x^6+6\,\ln\left(bx^2+a\right)x^4ab^2-4\,ax^4b^2+12\,\ln\left(bx^2+a\right)x^2a^2b+4\,a^2bx^2+6\,\ln\left(bx^2+a\right)a^3+5\,a^3\right)\left(bx^2+a\right)}{4\,b^4}\left(\left(bx^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(-2*b^3*x^6+6*ln(b*x^2+a)*x^4*a*b^2-4*a*x^4*b^2+12*ln(b*x^2+
a)*x^2*a^2*b+4*a^2*b*x^2+6*ln(b*x^2+a)*a^3+5*a^3)*(b*x^2+a)/b^4/(
(b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 0.691343, size = 198, normalized size = 1.25

$$\frac{x^4}{2\sqrt{b^2x^4+2\,abx^2+a^2}b^2}-\frac{3\,a^2x^2}{\left(b^2\right)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^2}-\frac{3\,a\log\left(x^2+\frac{a}{b}\right)}{2\left(b^2\right)^{\frac{3}{2}}b}-\frac{9\,a^3b}{4\left(b^2\right)^{\frac{7}{2}}\left(x^2+\frac{a}{b}\right)^2}+\frac{a^2}{\sqrt{b^2x^4+2\,abx^2+a^2}b^4}-\frac{a^3}{2\left(b^2\right)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^4/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^2) - 3*a^2*x^2/((b^2)^
(5/2)*(x^2 + a/b)^2) - 3/2*a*log(x^2 + a/b)/((b^2)^(3/2)*b) - 9/4
*a^3*b/((b^2)^(7/2)*(x^2 + a/b)^2) + a^2/(sqrt(b^2*x^4 + 2*a*b*x^
2 + a^2)*b^4) - 1/2*a^3/((b^2)^(3/2)*(x^2 + a/b)^2*b^3)
```

Fricas [A] time = 0.261056, size = 123, normalized size = 0.78

$$\frac{2\,b^3x^6 + 4\,ab^2x^4 - 4\,a^2bx^2 - 5\,a^3 - 6\left(ab^2x^4 + 2\,a^2bx^2 + a^3\right)\log\left(bx^2 + a\right)}{4\left(b^6x^4 + 2\,ab^5x^2 + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4
 + 2*a^2*b*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^
2*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.639064, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.635 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=113

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(
2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.248354, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$-\frac{a^2}{4b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a}{b^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] a/(b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^2/(4*b^3*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])/(
2*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 17.4784, size = 112, normalized size = 0.99

$$\frac{ax^2\left(a+bx^2\right)}{4b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}-\frac{3x^2}{4b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] a*x**2*(a + b*x**2)/(4*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2
)) - 3*x**2/(4*b**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)) + (a + b
*x**2)*log(a + b*x**2)/(2*b**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4
))
```

Mathematica [A] time = 0.0422013, size = 61, normalized size = 0.54

$$\frac{a\left(3a+4bx^2\right)+2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4b^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a*(3*a + 4*b*x^2) + 2*(a + b*x^2)^2*Log[a + b*x^2])/(4*b^3*(a +
b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.022, size = 81, normalized size = 0.7

$$\frac{\left(2\ln\left(bx^2+a\right)x^4b^2+4\ln\left(bx^2+a\right)x^2ab+4\,abx^2+2\,a^2\ln\left(bx^2+a\right)+3\,a^2\right)\left(bx^2+a\right)}{4\,b^3}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(2*ln(b*x^2+a)*x^4*b^2+4*ln(b*x^2+a)*x^2*a*b+4*a*b*x^2+2*a^2*
ln(b*x^2+a)+3*a^2)*(b*x^2+a)/b^3/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 0.698568, size = 86, normalized size = 0.76

$$\frac{abx^2}{\left(b^2\right)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^2}+\frac{\log\left(x^2+\frac{a}{b}\right)}{2\left(b^2\right)^{\frac{3}{2}}}+\frac{3\,a^2b^2}{4\left(b^2\right)^{\frac{7}{2}}\left(x^2+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] a*b*x^2/((x^2 + a/b)^2*(b^2)^(5/2)) + 1/2*log(x^2 + a/b)/(b^2)^(3
/2) + 3/4*a^2*b^2/((b^2)^(7/2)*(x^2 + a/b)^2)
```

Fricas [A] time = 0.262293, size = 93, normalized size = 0.82

$$\frac{4\,abx^2+3\,a^2+2\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)}{4\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2
+ a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.623328, size = 4, normalized size = 0.04

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.636 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^4}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x^4/(4*a*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.131287, antiderivative size = 69, normalized size of antiderivative = 1.68, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{a}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a/(4*b^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 8.38562, size = 37, normalized size = 0.9

$$\frac{x^4\left(2a+2bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] x**4*(2*a + 2*b*x**2)/(8*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)
)
```

Mathematica [A] time = 0.0227758, size = 39, normalized size = 0.95

$$\frac{-a-2bx^2}{4b^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-a - 2*b*x^2)/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.01, size = 32, normalized size = 0.8

$$-\frac{(bx^2+a)(2bx^2+a)}{4b^2}\left((bx^2+a)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(2*b*x^2+a)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 0.705099, size = 65, normalized size = 1.59

$$-\frac{1}{2\sqrt{b^2x^4+2abx^2+a^2}b^2}+\frac{a}{4(b^2)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2/(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*b^2) + 1/4*a/((b^2)^(3/2)*(
x^2 + a/b)^2*b)
```

Fricas [A] time = 0.257924, size = 49, normalized size = 1.2

$$-\frac{2bx^2+a}{4(b^4x^4+2ab^3x^2+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.659879, size = 4, normalized size = 0.1

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.637 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0671289, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{1}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]

[Out] -1/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 8.10381, size = 36, normalized size = 0.95

$$-\frac{2a+2bx^2}{8b\,(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] -(2*a + 2*b*x**2)/(8*b*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2))
```

Mathematica [A] time = 0.0137682, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{4b\left((a+bx^2)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a + b*x^2)/(4*b*((a + b*x^2)^2)^(3/2))
```

Maple [A] time = 0.006, size = 24, normalized size = 0.6

$$-\frac{bx^2+a}{4b}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(b*x^2+a)/b/((b*x^2+a)^2)^(3/2)
```

Maxima [A] time = 0.693813, size = 24, normalized size = 0.63

$$-\frac{1}{4\left(b^2\right)^{\frac{3}{2}}\left(x^2+\frac{a}{b}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4/((b^2)^(3/2)*(x^2 + a/b)^2)
```

Fricas [A] time = 0.256803, size = 35, normalized size = 0.92

$$-\frac{1}{4\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.636729, size = 4, normalized size = 0.11

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.638 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=147

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.205017, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{1}{4a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 1/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*(a + b*x^2)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^3*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 27.6618, size = 144, normalized size = 0.98

$$\frac{2a+2bx^2}{8a\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}+\frac{1}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\log\left(x^2\right)}{2a^3\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\log\left(a+bx^2\right)}{2a^3\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] (2*a + 2*b*x**2)/(8*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)) + 1
/(2*a**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)) + sqrt(a**2 + 2*a*b
```

```
*x**2 + b**2*x**4)*log(x**2)/(2*a**3*(a + b*x**2)) - sqrt(a**2 +
2*a*b*x**2 + b**2*x**4)*log(a + b*x**2)/(2*a**3*(a + b*x**2))
```

Mathematica [A] time = 0.0441868, size = 74, normalized size = 0.5

$$\frac{a\left(3a+2bx^2\right)+4\log(x)\left(a+bx^2\right)^2-2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a + 2*b*x^2) + 4*(a + b*x^2)^2*Log[x] - 2*(a + b*x^2)^2*Log
[a + b*x^2])/(4*a^3*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.024, size = 107, normalized size = 0.7

$$-\frac{\left(2\ln\left(bx^2+a\right)x^4b^2-4\ln(x)x^4b^2+4\ln\left(bx^2+a\right)x^2ab-8\ln(x)x^2ab-2abx^2+2a^2\ln\left(bx^2+a\right)-4a^2\ln(x)-3a^2\right)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*x^4*b^2-4*ln(x)*x^4*b^2+4*ln(b*x^2+a)*x^2*a*b
-8*ln(x)*x^2*a*b-2*a*b*x^2+2*a^2*ln(b*x^2+a)-4*a^2*ln(x)-3*a^2)*(
b*x^2+a)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268827, size = 122, normalized size = 0.83

$$\frac{2\,abx^2+3\,a^2-2\,\left(b^2x^4+2\,abx^2+a^2\right)\log\left(bx^2+a\right)+4\,\left(b^2x^4+2\,abx^2+a^2\right)\log\left(x\right)}{4\,(a^3b^2x^4+2\,a^4bx^2+a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2
+ a) + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4
*b*x^2 + a^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.639614, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.639 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) + (3*b*(a + b*x^2)*Log[a + b*x^2])/(2*
a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.242029, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{b}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\log(x)\left(a+bx^2\right)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -(b/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - b/(4*a^2*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^3*x^2*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - (3*b*(a + b*x^2)*Log[x])/(a^4*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) + (3*b*(a + b*x^2)*Log[a + b*x^2])/(2*
a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 34.7448, size = 190, normalized size = 1.01

$$\frac{2a+2bx^2}{8ax^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}+\frac{3}{4a^2x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b\sqrt{a^2+2abx^2+b^2x^4}\log\left(x^2\right)}{2a^4\left(a+bx^2\right)}+\frac{3b\sqrt{a^2+2abx^2+b^2x^4}\log\left(a+bx^2\right)}{2a^4\left(a+bx^2\right)}-\frac{3\sqrt{a^2+2abx^2+b^2x^4}}{2a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] (2*a + 2*b*x**2)/(8*a*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)
) + 3/(4*a**2*x**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)) - 3*b*sqr
t(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x**2)/(2*a**4*(a + b*x**2))
+ 3*b*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(a + b*x**2)/(2*a**4
*(a + b*x**2)) - 3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*a**4*x*
*2)
```

Mathematica [A] time = 0.0702177, size = 97, normalized size = 0.51

$$\frac{-a\left(2a^2+9abx^2+6b^2x^4\right)-12bx^2\log(x)\left(a+bx^2\right)^2+6bx^2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4)) - 12*b*x^2*(a + b*x^2)^2*Lo
g[x] + 6*b*x^2*(a + b*x^2)^2*Log[a + b*x^2])/(4*a^4*x^2*(a + b*x^
2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.027, size = 133, normalized size = 0.7

$$\frac{\left(6\,\ln\left(bx^2+a\right)x^6b^3-12\,b^3\ln(x)\,x^6+12\,\ln\left(bx^2+a\right)x^4ab^2-24\,ab^2\ln(x)\,x^4-6\,ax^4b^2+6\,\ln\left(bx^2+a\right)x^2a^2b-12\,a^2b\ln}{4\,x^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/4*(6*ln(b*x^2+a)*x^6*b^3-12*b^3*ln(x)*x^6+12*ln(b*x^2+a)*x^4*a*
b^2-24*a*b^2*ln(x)*x^4-6*a*x^4*b^2+6*ln(b*x^2+a)*x^2*a^2*b-12*a^2
*b*ln(x)*x^2-9*a^2*b*x^2-2*a^3)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(
3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.266921, size = 161, normalized size = 0.85

$$-\frac{6\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3 - 6\,\left(b^3x^6 + 2\,ab^2x^4 + a^2bx^2\right)\log\left(bx^2 + a\right) + 12\,\left(b^3x^6 + 2\,ab^2x^4 + a^2bx^2\right)\log\left(x\right)}{4\,\left(a^4b^2x^6 + 2\,a^5bx^4 + a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^
4 + a^2*b*x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b
*x^2)*log(x))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.678956, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.640 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{3/2}} \, dx$

Optimal. Leaf size=128

$$-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*
x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rubi [A] time = 0.151702, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{3x}{8b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-3*x)/(8*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(4*b*(a + b*
x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0607869, size = 84, normalized size = 0.66

$$\frac{3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\left(3a+5bx^2\right)}{8\sqrt{a}b^{5/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a + 5*b*x^2)) + 3*(a + b*x^2)^2*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(5/2)*(a + b*x^2)*Sqrt[(a + b*x^
2)^2])
```

Maple [A] time = 0.021, size = 97, normalized size = 0.8

$$-\frac{bx^2+a}{8\,b^2}\left(-3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4b^2+5\,\sqrt{ab}x^3b-6\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2ab+3\,\sqrt{ab}xa-3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{ab}}\left(\left(bx^2+a\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(-3*arctan(x*b/(a*b)^(1/2))*x^4*b^2+5*(a*b)^(1/2)*x^3*b-6*ar
ctan(x*b/(a*b)^(1/2))*x^2*a*b+3*(a*b)^(1/2)*x*a-3*a^2*arctan(x*b/
(a*b)^(1/2)))*(b*x^2+a)/(a*b)^(1/2)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268007, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)-2\left(5\,bx^{3}+3\,ax\right)\sqrt{-ab}}{16\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)\sqrt{-ab}},\frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)-\left(5\,bx^{3}+\right.}{8\left(b^{4}x^{4}+2\,ab^{3}x^{2}+a^{2}b^{2}\right)\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*log((2*a*b*x + (b*x^2 - a)*s
qrt(-a*b))/(b*x^2 + a)) - 2*(5*b*x^3 + 3*a*x)*sqrt(-a*b))/((b^4*x
^4 + 2*a*b^3*x^2 + a^2*b^2)*sqrt(-a*b)), 1/8*(3*(b^2*x^4 + 2*a*b*
x^2 + a^2)*arctan(sqrt(a*b)*x/a) - (5*b*x^3 + 3*a*x)*sqrt(a*b))/(
(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{4}}{\left(\left(a+bx^{2}\right)^{2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.694764, size = 4, normalized size = 0.03

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.641 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=129

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(3/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.148118, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{x}{8ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{4b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] x/(8*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(4*b*(a + b*x^2)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(3/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0546982, size = 81, normalized size = 0.63

$$\frac{\sqrt{a}\sqrt{b}x\left(bx^2-a\right)+\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-a + b*x^2) + (a + b*x^2)^2*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(8*a^(3/2)*b^(3/2)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.021, size = 97, normalized size = 0.8

$$\frac{bx^2+a}{8\,ab}\left(\arctan\left(bx\frac{1}{\sqrt{ab}}\right)x^4b^2+\sqrt{ab}x^3b+2\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2ab-\sqrt{ab}xa+a^2\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{ab}}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(arctan(x*b/(a*b)^(1/2))*x^4*b^2+(a*b)^(1/2)*x^3*b+2*arctan(x
*b/(a*b)^(1/2))*x^2*a*b-(a*b)^(1/2)*x*a+a^2*arctan(x*b/(a*b)^(1/2
)))*(b*x^2+a)/(a*b)^(1/2)/b/a/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273689, size = 1, normalized size = 0.01

$$\left[\frac{\left(b^{2} x^{4}+2 a b x^{2}+a^{2}\right) \log \left(\frac{2 a b x+\left(b x^{2}-a\right) \sqrt{-a b}}{b x^{2}+a}\right)+2\left(b x^{3}-a x\right) \sqrt{-a b}}{16\left(a b^{3} x^{4}+2 a^{2} b^{2} x^{2}+a^{3} b\right) \sqrt{-a b}}, \frac{\left(b^{2} x^{4}+2 a b x^{2}+a^{2}\right) \arctan \left(\frac{\sqrt{a b} x}{a}\right)+\left(b x^{3}-a x\right) \sqrt{a b}}{8\left(a b^{3} x^{4}+2 a^{2} b^{2} x^{2}+a^{3} b\right) \sqrt{a b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*x^4 + 2*a*b*x^2 + a^2)*log((2*a*b*x + (b*x^2 - a)*sqr
t(-a*b))/(b*x^2 + a)) + 2*(b*x^3 - a*x)*sqrt(-a*b))/((a*b^3*x^4 +
 2*a^2*b^2*x^2 + a^3*b)*sqrt(-a*b)), 1/8*((b^2*x^4 + 2*a*b*x^2 +
a^2)*arctan(sqrt(a*b)*x/a) + (b*x^3 - a*x)*sqrt(a*b))/((a*b^3*x^4
 + 2*a^2*b^2*x^2 + a^3*b)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{\left(\left(a+b x^{2}\right)^{2}\right)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.650853, size = 4, normalized size = 0.03

$$sage_{0} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.642 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{3/2}} \, dx$

Optimal. Leaf size=135

$$\frac{3x(a+bx^2)^2}{8a^2(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{x(a+bx^2)}{4a(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{3(a+bx^2)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}(a^2+2abx^2+b^2x^4)^{3/2}}$$

```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a
 + b*x^2)^2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a +
b*x^2)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2
*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.101372, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{3x(a+bx^2)^2}{8a^2(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{x(a+bx^2)}{4a(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{3(a+bx^2)^3\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}(a^2+2abx^2+b^2x^4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (x*(a + b*x^2))/(4*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*x*(a
 + b*x^2)^2)/(8*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + (3*(a +
b*x^2)^3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a^2 + 2
*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0485606, size = 83, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{b}x\left(5a+3bx^2\right)+3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(5*a + 3*b*x^2) + 3*(a + b*x^2)^2*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*(a + b*x^2)*Sqrt[(a + b*x^2)^
2])
```

Maple [A] time = 0.011, size = 97, normalized size = 0.7

$$\frac{bx^2+a}{8\,a^2}\left(3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4b^2+3\,\sqrt{ab}x^3b+6\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^2ab+5\,\sqrt{ab}xa+3\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)\right)\frac{1}{\sqrt{ab}}\left(\left(bx^2+a\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/8*(3*arctan(x*b/(a*b)^(1/2))*x^4*b^2+3*(a*b)^(1/2)*x^3*b+6*arct
an(x*b/(a*b)^(1/2))*x^2*a*b+5*(a*b)^(1/2)*x*a+3*a^2*arctan(x*b/(a
*b)^(1/2)))*(b*x^2+a)/(a*b)^(1/2)/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269753, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\log\left(\frac{2\,abx+\left(bx^{2}-a\right)\sqrt{-ab}}{bx^{2}+a}\right)+2\left(3\,bx^{3}+5\,ax\right)\sqrt{-ab}}{16\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)\sqrt{-ab}},\ \frac{3\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\arctan\left(\frac{\sqrt{ab}x}{a}\right)+\left(3\,bx^{3}+\cdots\right.}{8\left(a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}\right)\sqrt{ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*log((2*a*b*x + (b*x^2 - a)*s
qrt(-a*b))/(b*x^2 + a)) + 2*(3*b*x^3 + 5*a*x)*sqrt(-a*b))/((a^2*b
^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(-a*b)), 1/8*(3*(b^2*x^4 + 2*a*b*
x^2 + a^2)*arctan(sqrt(a*b)*x/a) + (3*b*x^3 + 5*a*x)*sqrt(a*b))/(
(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.622283, size = 4, normalized size = 0.03

$$sage_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.643 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=169

$$\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{15\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\,(a+bx^2)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.185066, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{5}{8a^2x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{15\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15\,(a+bx^2)}{8a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 5/(8*a^2*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*(a + b*x^2))/(8*a^3*x*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (15*Sqrt[b]*(a + b*x^2)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

[Out] Exception raised: RecursionError

Mathematica [A] time = 0.0631704, size = 93, normalized size = 0.55

$$\frac{-\sqrt{a}\left(8a^2+25abx^2+15b^2x^4\right)-15\sqrt{b}x\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (-(Sqrt[a]*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)) - 15*Sqrt[b]*x*(a +
 b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*x*(a + b*x^2)*S
qrt[(a + b*x^2)^2])
```

Maple [A] time = 0.024, size = 119, normalized size = 0.7

$$-\frac{bx^2+a}{8\,a^3x}\left(15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5b^3+15\,\sqrt{ab}x^4b^2+30\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3ab^2+25\,\sqrt{ab}x^2ab+15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)xa^2b+8\,\sqrt{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/8*(15*arctan(x*b/(a*b)^(1/2))*x^5*b^3+15*(a*b)^(1/2)*x^4*b^2+3
0*arctan(x*b/(a*b)^(1/2))*x^3*a*b^2+25*(a*b)^(1/2)*x^2*a*b+15*arc
tan(x*b/(a*b)^(1/2))*x*a^2*b+8*(a*b)^(1/2)*a^2)*(b*x^2+a)/x/(a*b)
^(1/2)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273984, size = 1, normalized size = 0.01

$$\left[-\frac{30\,b^2x^4+50\,abx^2-15\,\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\,\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)},\right.$$
$$\left.-\frac{15\,b^2x^4+25\,abx^2+15\,\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{bx}{a\sqrt{\frac{b}{a}}}\right)+8\,a^2}{8\,\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x
)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16
*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*
a*b*x^2 + 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(b/a)*arctan(b*x/(
a*sqrt(b/a))) + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.617207, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^2),x, algorithm="giac")

[Out] sage0*x
```

3.644 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=209

$$\frac{1}{4ax^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b(a+bx^2)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35(a+bx^2)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b
*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3
*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4
*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*b^(3/2)*(a + b*x^2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])
```

Rubi [A] time = 0.222925, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{1}{4ax^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7}{8a^2x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35b(a+bx^2)}{8a^4x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35(a+bx^2)}{24a^3x^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 7/(8*a^2*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*x^3*(a + b
*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*(a + b*x^2))/(24*a^3
*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*b*(a + b*x^2))/(8*a^4
*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*b^(3/2)*(a + b*x^2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0705342, size = 105, normalized size = 0.5

$$\frac{\sqrt{a}\left(-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6\right)+105b^{3/2}x^3\left(a+bx^2\right)^2\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{24a^{9/2}x^3\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6) +
105*b^(3/2)*x^3*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(24*a^
(9/2)*x^3*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.027, size = 139, normalized size = 0.7

$$\frac{bx^2+a}{24\,a^4x^3}\left(105\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^7b^4+105\,\sqrt{ab}x^6b^3+210\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5ab^3+175\,\sqrt{ab}x^4ab^2+105\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^3a^2b^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/24*(105*arctan(x*b/(a*b)^(1/2))*x^7*b^4+105*(a*b)^(1/2)*x^6*b^3
+210*arctan(x*b/(a*b)^(1/2))*x^5*a*b^3+175*(a*b)^(1/2)*x^4*a*b^2+
105*arctan(x*b/(a*b)^(1/2))*x^3*a^2*b^2+56*(a*b)^(1/2)*x^2*a^2*b-
8*(a*b)^(1/2)*a^3)*(b*x^2+a)/x^3/(a*b)^(1/2)/a^4/((b*x^2+a)^2)^(3
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^4),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.274802, size = 1, normalized size = 0.

$$\left[\frac{210\,b^3x^6 + 350\,ab^2x^4 + 112\,a^2bx^2 - 16\,a^3 + 105\,\left(b^3x^7 + 2\,ab^2x^5 + a^2bx^3\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{48\,(a^4b^2x^7 + 2\,a^5bx^5 + a^6x^3)}, \frac{105\,b^3x^6 + 175\,ab^2x^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^4),x, algorithm="fricas")

[Out] [1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105
*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*
x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*
x^3), 1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 +
105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(b/a)*arctan(b*x/(a*s
qrt(b/a))))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)

[Out] Integral(1/(x**4*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.619549, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.645 $\int \frac{x^{11}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=238

$$-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a +
b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b
*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*
x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*(a + b*x^2)*Log[a + b*x^2
])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.450005, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{5a^2}{b^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x^2\left(a+bx^2\right)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^5}{8b^6\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5a^4}{6b^6\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5a^3}{2b^6\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-5*a^2)/(b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + a^5/(8*b^6*(a +
b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a^4)/(6*b^6*(a + b
*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*a^3)/(2*b^6*(a + b*
x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (x^2*(a + b*x^2))/(2*b^5*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*a*(a + b*x^2)*Log[a + b*x^2
])/(2*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 35.0905, size = 216, normalized size = 0.91

$$\frac{ax^8\left(a+bx^2\right)}{8b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}+\frac{5ax^4\left(a+bx^2\right)}{12b^4\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}-\frac{5a\left(a+bx^2\right)\log\left(a+bx^2\right)}{2b^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^8}{3b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}-\frac{5x^4}{3b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{a^2+2abx^2+b^2x^4}}{2b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a*x**8*(a + b*x**2)/(8*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
)) + 5*a*x**4*(a + b*x**2)/(12*b**4*(a**2 + 2*a*b*x**2 + b**2*x**
4)**(3/2)) - 5*a*(a + b*x**2)*log(a + b*x**2)/(2*b**6*sqrt(a**2 +
 2*a*b*x**2 + b**2*x**4)) - x**8/(3*b**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**(3/2)) - 5*x**4/(3*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x*
*4)) + 5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*b**6)
```

Mathematica [A] time = 0.06648, size = 103, normalized size = 0.43

$$\frac{-77a^5-248a^4bx^2-252a^3b^2x^4-48a^2b^3x^6+48ab^4x^8-60a\left(a+bx^2\right)^4\log\left(a+bx^2\right)+12b^5x^{10}}{24b^6\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-77*a^5 - 248*a^4*b*x^2 - 252*a^3*b^2*x^4 - 48*a^2*b^3*x^6 + 48*
a*b^4*x^8 + 12*b^5*x^10 - 60*a*(a + b*x^2)^4*Log[a + b*x^2])/(24*
b^6*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.027, size = 163, normalized size = 0.7

$$-\frac{\left(-12\,b^5x^{10}+60\,\ln\left(bx^2+a\right)x^8ab^4-48\,ab^4x^8+240\,\ln\left(bx^2+a\right)x^6a^2b^3+48\,a^2b^3x^6+360\,\ln\left(bx^2+a\right)x^4a^3b^2+252\,a^3b}{24\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(-12*b^5*x^10+60*ln(b*x^2+a)*x^8*a*b^4-48*a*b^4*x^8+240*ln(
b*x^2+a)*x^6*a^2*b^3+48*a^2*b^3*x^6+360*ln(b*x^2+a)*x^4*a^3*b^2+2
52*a^3*b^2*x^4+240*ln(b*x^2+a)*x^2*a^4*b+248*a^4*b*x^2+60*ln(b*x^
2+a)*a^5+77*a^5)*(b*x^2+a)/b^6/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.705676, size = 161, normalized size = 0.68

$$\frac{12 \, b^{5} x^{10} + 48 \, a b^{4} x^{8} - 48 \, a^{2} b^{3} x^{6} - 252 \, a^{3} b^{2} x^{4} - 248 \, a^{4} b x^{2} - 77 \, a^{5}}{24 \, {\left(b^{10} x^{8} + 4 \, a b^{9} x^{6} + 6 \, a^{2} b^{8} x^{4} + 4 \, a^{3} b^{7} x^{2} + a^{4} b^{6}\right)}} - \frac{5 \, a \log \left(b x^{2} + a\right)}{2 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x
^4 - 248*a^4*b*x^2 - 77*a^5)/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*
x^4 + 4*a^3*b^7*x^2 + a^4*b^6) - 5/2*a*log(b*x^2 + a)/b^6
```

Fricas [A] time = 0.261759, size = 212, normalized size = 0.89

$$\frac{12 \, b^{5} x^{10} + 48 \, a b^{4} x^{8} - 48 \, a^{2} b^{3} x^{6} - 252 \, a^{3} b^{2} x^{4} - 248 \, a^{4} b x^{2} - 77 \, a^{5} - 60 \, {\left(a b^{4} x^{8} + 4 \, a^{2} b^{3} x^{6} + 6 \, a^{3} b^{2} x^{4} + 4 \, a^{4} b x^{2} + a^{5}\right)} \log \left(b \right.}{24 \, {\left(b^{10} x^{8} + 4 \, a b^{9} x^{6} + 6 \, a^{2} b^{8} x^{4} + 4 \, a^{3} b^{7} x^{2} + a^{4} b^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(12*b^5*x^10 + 48*a*b^4*x^8 - 48*a^2*b^3*x^6 - 252*a^3*b^2*x
^4 - 248*a^4*b*x^2 - 77*a^5 - 60*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a
^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^10*x^8 + 4*a*b
^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{11}}{\left(\left(a + b x^{2}\right)^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**11/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.620322, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.646 $\int \frac{x^9}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{3a^2}{2b^5(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log(a+bx^2)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x
^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^
2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])
/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.377918, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{3a^2}{2b^5(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a}{b^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log(a+bx^2)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^4}{8b^5(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2a^3}{3b^5(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*a)/(b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - a^4/(8*b^5*(a + b*x
^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2*a^3)/(3*b^5*(a + b*x^
2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*a^2)/(2*b^5*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[a + b*x^2])
/(2*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 28.0512, size = 184, normalized size = 0.94

$$\frac{ax^6(a+bx^2)}{8b^2(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}+\frac{ax^2(a+bx^2)}{4b^4(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}-\frac{7x^6}{24b^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}-\frac{3x^2}{4b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log(a+bx^2)}{2b^5\sqrt{a^2+2abx^2+b^2x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a*x**6*(a + b*x**2)/(8*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
)) + a*x**2*(a + b*x**2)/(4*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4)*
*(3/2)) - 7*x**6/(24*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2))
 - 3*x**2/(4*b**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)) + (a + b*x
**2)*log(a + b*x**2)/(2*b**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4))
```

Mathematica [A] time = 0.0554598, size = 83, normalized size = 0.42

$$\frac{a\left(25a^3+88a^2bx^2+108ab^2x^4+48b^3x^6\right)+12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24b^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a*(25*a^3 + 88*a^2*b*x^2 + 108*a*b^2*x^4 + 48*b^3*x^6) + 12*(a +
 b*x^2)^4*Log[a + b*x^2])/(24*b^5*(a + b*x^2)^3*Sqrt[(a + b*x^2)^
2])
```

Maple [A] time = 0.024, size = 141, normalized size = 0.7

$$\frac{\left(12\ln\left(bx^2+a\right)x^8b^4+48\ln\left(bx^2+a\right)x^6ab^3+48ab^3x^6+72\ln\left(bx^2+a\right)x^4a^2b^2+108a^2b^2x^4+48\ln\left(bx^2+a\right)x^2a^3b+8\right.}{24b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(12*ln(b*x^2+a)*x^8*b^4+48*ln(b*x^2+a)*x^6*a*b^3+48*a*b^3*x^
6+72*ln(b*x^2+a)*x^4*a^2*b^2+108*a^2*b^2*x^4+48*ln(b*x^2+a)*x^2*a
^3*b+88*a^3*b*x^2+12*ln(b*x^2+a)*a^4+25*a^4)*(b*x^2+a)/b^5/((b*x^
2+a)^2)^(5/2)
```

Maxima [A] time = 0.702838, size = 134, normalized size = 0.68

$$\frac{48\,ab^3x^6+108\,a^2b^2x^4+88\,a^3bx^2+25\,a^4}{24\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}+\frac{\log\left(bx^2+a\right)}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4)/(b^
9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5) +
1/2*log(b*x^2 + a)/b^5
```

Fricas [A] time = 0.260762, size = 182, normalized size = 0.93

$$\frac{48\,ab^3x^6 + 108\,a^2b^2x^4 + 88\,a^3bx^2 + 25\,a^4 + 12\,\left(b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4\right)\log\left(bx^2 + a\right)}{24\,(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(48*a*b^3*x^6 + 108*a^2*b^2*x^4 + 88*a^3*b*x^2 + 25*a^4 + 12
*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(
b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^
2 + a^4*b^5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**9/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.620414, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")

[Out] sage0*x
```

3.647 $\int \frac{x^7}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{x^8}{8a\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.119058, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{x^8}{8a\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^8/(8*a*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 8.45338, size = 37, normalized size = 0.9

$$\frac{x^8\left(2a+2bx^2\right)}{16a\,(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] x**8*(2*a + 2*b*x**2)/(16*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
))
```

Mathematica [A] time = 0.030128, size = 61, normalized size = 1.49

$$\frac{-a^3-4a^2bx^2-6ab^2x^4-4b^3x^6}{8b^4\,(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^3 - 4*a^2*b*x^2 - 6*a*b^2*x^4 - 4*b^3*x^6)/(8*b^4*(a + b*x^2)
^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.011, size = 54, normalized size = 1.3

$$-\frac{\left(bx^{2}+a\right)\left(4\,b^{3}x^{6}+6\,ax^{4}b^{2}+4\,a^{2}bx^{2}+a^{3}\right)}{8\,b^{4}}\left(\left(bx^{2}+a\right)^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8*(b*x^2+a)*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)/b^4/((b*x^
2+a)^2)^(5/2)
```

Maxima [A] time = 0.698362, size = 197, normalized size = 4.8

$$-\frac{x^{4}}{2\,(b^{2}x^{4}+2\,abx^{2}+a^{2})^{\frac{3}{2}}b^{2}}-\frac{a^{2}}{3\,(b^{2}x^{4}+2\,abx^{2}+a^{2})^{\frac{3}{2}}b^{4}}+\frac{a^{2}}{3\,(b^{2})^{\frac{7}{2}}\left(x^{2}+\frac{a}{b}\right)^{3}}-\frac{a}{4\,(b^{2})^{\frac{5}{2}}\left(x^{2}+\frac{a}{b}\right)^{2}b}-\frac{a^{3}b}{8\,(b^{2})^{\frac{9}{2}}\left(x^{2}+\frac{a}{b}\right)^{4}}+\frac{a^{3}}{4\,(b^{2})^{\frac{5}{2}}\left(x^{2}+\frac{a}{b}\right)^{4}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^4/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*b^2) - 1/3*a^2/((b^2*
x^4 + 2*a*b*x^2 + a^2)^(3/2)*b^4) + 1/3*a^2/((b^2)^(7/2)*(x^2 + a
/b)^3) - 1/4*a/((b^2)^(5/2)*(x^2 + a/b)^2*b) - 1/8*a^3*b/((b^2)^(
9/2)*(x^2 + a/b)^4) + 1/4*a^3/((b^2)^(5/2)*(x^2 + a/b)^4*b^3)
```

Fricas [A] time = 0.258123, size = 108, normalized size = 2.63

$$-\frac{4\,b^{3}x^{6}+6\,ab^{2}x^{4}+4\,a^{2}bx^{2}+a^{3}}{8\,(b^{8}x^{8}+4\,ab^{7}x^{6}+6\,a^{2}b^{6}x^{4}+4\,a^{3}b^{5}x^{2}+a^{4}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/(b^8*x^8 + 4*a
*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**7/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.63871, size = 4, normalized size = 0.1

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.648 $\int \frac{x^5}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{x^6}{8a(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{x^6}{24a^2(a^2+2abx^2+b^2x^4)^{3/2}}$$

```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*
x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.0606992, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{x^6}{8a(a+bx^2)(a^2+2abx^2+b^2x^4)^{3/2}}+\frac{x^6}{24a^2(a^2+2abx^2+b^2x^4)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] x^6/(24*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + x^6/(8*a*(a + b*
x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi in Sympy [A] time = 8.62612, size = 68, normalized size = 0.92

$$\frac{x^6(2a+2bx^2)}{16a(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}+\frac{x^6}{24a^2(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] x**6*(2*a + 2*b*x**2)/(16*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
)) + x**6/(24*a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2))
```

Mathematica [A] time = 0.0341559, size = 50, normalized size = 0.68

$$\frac{-a^2-4abx^2-6b^2x^4}{24b^3(a+bx^2)^3\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a^2 - 4*a*b*x^2 - 6*b^2*x^4)/(24*b^3*(a + b*x^2)^3*Sqrt[(a + b*
x^2)^2])
```

Maple [A] time = 0.009, size = 43, normalized size = 0.6

$$-\frac{\left(bx^{2}+a\right)\left(6\,b^{2}x^{4}+4\,abx^{2}+a^{2}\right)}{24\,b^{3}}\left(\left(bx^{2}+a\right)^{2}\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(6*b^2*x^4+4*a*b*x^2+a^2)/b^3/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.691088, size = 85, normalized size = 1.15

$$-\frac{1}{4\left(b^{2}\right)^{\frac{5}{2}}\left(x^{2}+\frac{a}{b}\right)^{2}}+\frac{ab}{3\left(b^{2}\right)^{\frac{7}{2}}\left(x^{2}+\frac{a}{b}\right)^{3}}-\frac{a^{2}b^{2}}{8\left(b^{2}\right)^{\frac{9}{2}}\left(x^{2}+\frac{a}{b}\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4/((b^2)^(5/2)*(x^2 + a/b)^2) + 1/3*a*b/((b^2)^(7/2)*(x^2 + a/
b)^3) - 1/8*a^2*b^2/((b^2)^(9/2)*(x^2 + a/b)^4)
```

Fricas [A] time = 0.260328, size = 93, normalized size = 1.26

$$-\frac{6\,b^{2}x^{4}+4\,abx^{2}+a^{2}}{24\left(b^{7}x^{8}+4\,ab^{6}x^{6}+6\,a^{2}b^{5}x^{4}+4\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*b^2*x^4 + 4*a*b*x^2 + a^2)/(b^7*x^8 + 4*a*b^6*x^6 + 6*a^
2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**5/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.620702, size = 4, normalized size = 0.05

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.649 $\int \frac{x^3}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/(6*b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^
2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.131474, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{a}{8b^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(6*b^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)) + a/(8*b^2*(a + b*x^
2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi in Sympy [A] time = 13.6299, size = 66, normalized size = 0.96

$$\frac{a\left(2a+2bx^2\right)}{16b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}-\frac{1}{6b^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] a*(2*a + 2*b*x**2)/(16*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
)) - 1/(6*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2))
```

Mathematica [A] time = 0.0227492, size = 39, normalized size = 0.57

$$\frac{-a-4bx^2}{24b^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-a - 4*b*x^2)/(24*b^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.01, size = 32, normalized size = 0.5

$$-\frac{(bx^2+a)\,(4\,bx^2+a)}{24\,b^2}\left((bx^2+a)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(b*x^2+a)*(4*b*x^2+a)/b^2/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.694729, size = 65, normalized size = 0.94

$$-\frac{1}{6\,(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}b^2}+\frac{a}{8\,(b^2)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^4 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*b^2) + 1/8*a/((b^2)^(5/2)
*(x^2 + a/b)^4*b)
```

Fricas [A] time = 0.258823, size = 78, normalized size = 1.13

$$-\frac{4\,bx^2+a}{24\,(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/24*(4*b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x^4 + 4*a^
3*b^3*x^2 + a^4*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**3/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.610832, size = 4, normalized size = 0.06

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.650 $\int \frac{x}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

```
[Out] -1/(8*b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi [A] time = 0.0662134, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{1}{8b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -1/(8*b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2))
```

Rubi in Sympy [A] time = 8.21846, size = 36, normalized size = 0.95

$$-\frac{2a+2bx^2}{16b\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] -(2*a + 2*b*x**2)/(16*b*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2))
```

Mathematica [A] time = 0.0187446, size = 27, normalized size = 0.71

$$-\frac{a+bx^2}{8b\left(\left(a+bx^2\right)^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] -(a + b*x^2)/(8*b*((a + b*x^2)^2)^(5/2))
```

Maple [A] time = 0.008, size = 24, normalized size = 0.6

$$-\frac{bx^2+a}{8\,b}\left(\left(bx^2+a\right)^2\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8*(b*x^2+a)/b/((b*x^2+a)^2)^(5/2)
```

Maxima [A] time = 0.692021, size = 24, normalized size = 0.63

$$-\frac{1}{8\,(b^2)^{\frac{5}{2}}\left(x^2+\frac{a}{b}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8/((b^2)^(5/2)*(x^2 + a/b)^4)
```

Fricas [A] time = 0.255785, size = 65, normalized size = 1.71

$$-\frac{1}{8\,(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^2 + a^4
*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.619312, size = 4, normalized size = 0.11

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.651 $\int \frac{1}{x(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^5*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.293692, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115

$$\frac{1}{6a^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8a\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\log(x)\left(a+bx^2\right)}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^5\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*(a + b*x^2)^3*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(6*a^2*(a + b*x^2)^2*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a^3*(a + b*x^2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + ((a + b*x^2)*Log[x])/(a^5*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) - ((a + b*x^2)*Log[a + b*x^2])/(2*a^5*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 36.7566, size = 211, normalized size = 0.95

$$\frac{2a+2bx^2}{16a\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}+\frac{1}{6a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}+\frac{2a+2bx^2}{8a^3\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}+\frac{1}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\log\left(x^2\right)}{2a^5\left(a+bx^2\right)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\log\left(a+bx^2\right)}{2a^5\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] (2*a + 2*b*x**2)/(16*a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)) +
1/(6*a**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)) + (2*a + 2*b*x*
*2)/(8*a**3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)) + 1/(2*a**4*s
qrt(a**2 + 2*a*b*x**2 + b**2*x**4)) + sqrt(a**2 + 2*a*b*x**2 + b*
*2*x**4)*log(x**2)/(2*a**5*(a + b*x**2)) - sqrt(a**2 + 2*a*b*x**2
 + b**2*x**4)*log(a + b*x**2)/(2*a**5*(a + b*x**2))
```

Mathematica [A] time = 0.0741545, size = 96, normalized size = 0.43

$$\frac{a\left(25a^3+52a^2bx^2+42ab^2x^4+12b^3x^6\right)+24\log(x)\left(a+bx^2\right)^4-12\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^5\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (a*(25*a^3 + 52*a^2*b*x^2 + 42*a*b^2*x^4 + 12*b^3*x^6) + 24*(a +
b*x^2)^4*Log[x] - 12*(a + b*x^2)^4*Log[a + b*x^2])/(24*a^5*(a + b
*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.026, size = 193, normalized size = 0.9

$$-\frac{\left(12\ln\left(bx^2+a\right)x^8b^4-24\ln(x)x^8b^4+48\ln\left(bx^2+a\right)x^6ab^3-96\ln(x)x^6ab^3-12ab^3x^6+72\ln\left(bx^2+a\right)x^4a^2b^2-14}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24*(12*ln(b*x^2+a)*x^8*b^4-24*ln(x)*x^8*b^4+48*ln(b*x^2+a)*x^6
*a*b^3-96*ln(x)*x^6*a*b^3-12*a*b^3*x^6+72*ln(b*x^2+a)*x^4*a^2*b^2
-144*ln(x)*x^4*a^2*b^2-42*a^2*b^2*x^4+48*ln(b*x^2+a)*x^2*a^3*b-96
*ln(x)*x^2*a^3*b-52*a^3*b*x^2+12*ln(b*x^2+a)*a^4-24*a^4*ln(x)-25*
a^4)*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270221, size = 240, normalized size = 1.08

$$\frac{12\,ab^3x^6 + 42\,a^2b^2x^4 + 52\,a^3bx^2 + 25\,a^4 - 12\left(b^4x^8 + 4\,ab^3x^6 + 6\,a^2b^2x^4 + 4\,a^3bx^2 + a^4\right)\log\left(bx^2 + a\right) + 24\left(b^4x^8 + 4\,ab^3x^6\right.}{24\left(a^5b^4x^8 + 4\,a^6b^3x^6 + 6\,a^7b^2x^4 + 4\,a^8bx^2 + a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x),x, algorithm="fricas")
```

```
[Out] 1/24*(12*a*b^3*x^6 + 42*a^2*b^2*x^4 + 52*a^3*b*x^2 + 25*a^4 - 12*
(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*log(b
*x^2 + a) + 24*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x
^2 + a^4)*log(x))/(a^5*b^4*x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 +
4*a^8*b*x^2 + a^9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x*((a + b*x**2)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.620811, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.652 $\int \frac{1}{x^3(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=267

$$-\frac{b}{8a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\log(x)\left(a+bx^2\right)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^5x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b}{4a^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^
2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Log[x])/(a^6*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^6*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.339114, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$-\frac{b}{8a^2(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\log(x)\left(a+bx^2\right)}{a^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5b\left(a+bx^2\right)\log\left(a+bx^2\right)}{2a^6\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2b}{a^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a+bx^2}{2a^5x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3b}{4a^4(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b}{3a^3(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```


```
[Out] (-2*b)/(a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(8*a^2*(a + b*x^
2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - b/(3*a^3*(a + b*x^2)^2*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*b)/(4*a^4*(a + b*x^2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (a + b*x^2)/(2*a^5*x^2*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) - (5*b*(a + b*x^2)*Log[x])/(a^6*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + (5*b*(a + b*x^2)*Log[a + b*x^2])/(2*a^6*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 44.9027, size = 264, normalized size = 0.99

$$\frac{2a+2bx^2}{16ax^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}+\frac{5}{24a^2x^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}+\frac{5\left(2a+2bx^2\right)}{24a^3x^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}+\frac{5}{4a^4x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5b\sqrt{a^2+2abx^2+b^2x^4}\log\left(x^2\right)}{2a^6\left(a+bx^2\right)}+\frac{5b\sqrt{a^2+2abx^2+b^2x^4}\log\left(a+bx^2\right)}{2a^6\left(a+bx^2\right)}-\frac{5\sqrt{a^2+2abx^2+b^2x^4}}{2a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] (2*a + 2*b*x**2)/(16*a*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2
)) + 5/(24*a**2*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)) + 5*
(2*a + 2*b*x**2)/(24*a**3*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(
3/2)) + 5/(4*a**4*x**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)) - 5*b
*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(x**2)/(2*a**6*(a + b*x**
2)) + 5*b*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*log(a + b*x**2)/(2*
a**6*(a + b*x**2)) - 5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(2*a**
6*x**2)
```

Mathematica [A] time = 0.0851996, size = 119, normalized size = 0.45

$$\frac{-a\left(12a^4+125a^3bx^2+260a^2b^2x^4+210ab^3x^6+60b^4x^8\right)-120bx^2\log(x)\left(a+bx^2\right)^4+60bx^2\left(a+bx^2\right)^4\log\left(a+bx^2\right)}{24a^6x^2\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(a*(12*a^4 + 125*a^3*b*x^2 + 260*a^2*b^2*x^4 + 210*a*b^3*x^6 +
60*b^4*x^8)) - 120*b*x^2*(a + b*x^2)^4*Log[x] + 60*b*x^2*(a + b*x
^2)^4*Log[a + b*x^2])/(24*a^6*x^2*(a + b*x^2)^3*Sqrt[(a + b*x^2)^
2])
```

Maple [A] time = 0.03, size = 219, normalized size = 0.8

$$\left(60\ln\left(bx^2+a\right)x^{10}b^5-120\,b^5\ln(x)\,x^{10}+240\ln\left(bx^2+a\right)x^8ab^4-480\,ab^4\ln(x)\,x^8-60\,ab^4x^8+360\ln\left(bx^2+a\right)x^6a^2b^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24*(60*ln(b*x^2+a)*x^10*b^5-120*b^5*ln(x)*x^10+240*ln(b*x^2+a)*
x^8*a*b^4-480*a*b^4*ln(x)*x^8-60*a*b^4*x^8+360*ln(b*x^2+a)*x^6*a^
2*b^3-720*a^2*b^3*ln(x)*x^6-210*a^2*b^3*x^6+240*ln(b*x^2+a)*x^4*a
^3*b^2-480*a^3*b^2*ln(x)*x^4-260*a^3*b^2*x^4+60*ln(b*x^2+a)*x^2*a
^4*b-120*a^4*b*ln(x)*x^2-125*a^4*b*x^2-12*a^5)*(b*x^2+a)/x^2/a^6/
((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.2688, size = 279, normalized size = 1.04

$$-\frac{60\,ab^4x^8 + 210\,a^2b^3x^6 + 260\,a^3b^2x^4 + 125\,a^4bx^2 + 12\,a^5 - 60\,\left(b^5x^{10} + 4\,ab^4x^8 + 6\,a^2b^3x^6 + 4\,a^3b^2x^4 + a^4bx^2\right)\log\left(bx^2 + a\right.}{24\,(a^6b^4x^{10} + 4\,a^7b^3x^8 + 6\,a^8b^2x^6 + 4\,a^9bx^4 + a^{10}x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^3),x, algorithm="fricas")
```

```
[Out] -1/24*(60*a*b^4*x^8 + 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 + 125*a^4
*b*x^2 + 12*a^5 - 60*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*
a^3*b^2*x^4 + a^4*b*x^2)*log(b*x^2 + a) + 120*(b^5*x^10 + 4*a*b^4
*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^
4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**3*((a + b*x**2)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.618559, size = 4, normalized size = 0.01

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^3),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.653 $\int \frac{x^6}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=211

$$-\frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{3/2}b^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a +
 b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a +
 b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b
*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*(a + b*x^2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rubi [A] time = 0.245129, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{x^5}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x^3}{48b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5x}{64b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128ab^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{3/2}b^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^5/(8*b*(a +
 b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x^3)/(48*b^2*(a +
 b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*x)/(64*b^3*(a + b
*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*(a + b*x^2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(128*a^(3/2)*b^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0814978, size = 105, normalized size = 0.5

$$\frac{\sqrt{a}\sqrt{b}x\left(-15a^3-55a^2bx^2-73ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{3/2}b^{7/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 - 55*a^2*b*x^2 - 73*a*b^2*x^4 + 15*b^
3*x^6) + 15*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(3/
2)*b^(7/2)*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.023, size = 172, normalized size = 0.8

$$\frac{bx^2+a}{384\,ab^3}\left(15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+15\,\sqrt{ab}x^7b^3+60\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3-73\,\sqrt{ab}x^5ab^2+90\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2-55\,\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*arctan(x*b/(a*b)^(1/2))*x^8*b^4+15*(a*b)^(1/2)*x^7*b^3+
60*arctan(x*b/(a*b)^(1/2))*x^6*a*b^3-73*(a*b)^(1/2)*x^5*a*b^2+90*
arctan(x*b/(a*b)^(1/2))*x^4*a^2*b^2-55*(a*b)^(1/2)*x^3*a^2*b+60*a
rctan(x*b/(a*b)^(1/2))*x^2*a^3*b-15*(a*b)^(1/2)*x*a^3+15*arctan(x
*b/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)/b^3/a/((b*x^2+a)^2)^(5
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269621, size = 1, normalized size = 0.

$$\left[\frac{15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(15\,b^3x^7-73\,ab^2x^5-55\,a^2bx^3-15\,a^3x\right)\sqrt{-ab}}{768\left(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 +
 a^4)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(15
*b^3*x^7 - 73*a*b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)*sqrt(-a*b))/((
a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b
^3)*sqrt(-a*b)), 1/384*(15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4
 + 4*a^3*b*x^2 + a^4)*arctan(sqrt(a*b)*x/a) + (15*b^3*x^7 - 73*a*
b^2*x^5 - 55*a^2*b*x^3 - 15*a^3*x)*sqrt(a*b))/((a*b^7*x^8 + 4*a^2
*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**6/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.623443, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.654 $\int \frac{x^4}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=212

$$\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a
 + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x
^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 0.236176, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{x}{64ab^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3x}{128a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{16b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x^3}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (3*x)/(128*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x^3/(8*b*(a
 + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(16*b^2*(a + b*x
^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(64*a*b^2*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*(a + b*x^2)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(128*a^(5/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0801836, size = 105, normalized size = 0.5

$$\frac{\sqrt{a}\sqrt{b}x\left(-3a^3-11a^2bx^2+11ab^2x^4+3b^3x^6\right)+3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{5/2}b^{5/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-3*a^3 - 11*a^2*b*x^2 + 11*a*b^2*x^4 + 3*b^3*
x^6) + 3*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(5/2)*
b^(5/2)*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.032, size = 172, normalized size = 0.8

$$\frac{bx^2+a}{128\,a^2b^2}\left(3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+3\,\sqrt{ab}x^7b^3+12\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3+11\,\sqrt{ab}x^5ab^2+18\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2-11\,\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/128*(3*arctan(x*b/(a*b)^(1/2))*x^8*b^4+3*(a*b)^(1/2)*x^7*b^3+12
*arctan(x*b/(a*b)^(1/2))*x^6*a*b^3+11*(a*b)^(1/2)*x^5*a*b^2+18*ar
ctan(x*b/(a*b)^(1/2))*x^4*a^2*b^2-11*(a*b)^(1/2)*x^3*a^2*b+12*arc
tan(x*b/(a*b)^(1/2))*x^2*a^3*b-3*(a*b)^(1/2)*x*a^3+3*arctan(x*b/(
a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)/b^2/a^2/((b*x^2+a)^2)^(5/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27264, size = 1, normalized size = 0.

$$\left[\frac{3\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(3\,b^3x^7+11\,ab^2x^5-11\,a^2bx^3-3\,a^3x\right)\sqrt{-ab}}{256\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\sqrt{-ab}},\ \frac{3\,(\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 +
a^4)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(3*b
^3*x^7 + 11*a*b^2*x^5 - 11*a^2*b*x^3 - 3*a^3*x)*sqrt(-a*b))/((a^2
*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^
2)*sqrt(-a*b)), 1/128*(3*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 +
 4*a^3*b*x^2 + a^4)*arctan(sqrt(a*b)*x/a) + (3*b^3*x^7 + 11*a*b^2
*x^5 - 11*a^2*b*x^3 - 3*a^3*x)*sqrt(a*b))/((a^2*b^6*x^8 + 4*a^3*b
^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**4/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.614071, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.655 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{5x}{192a^2b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b
*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])
```

Rubi [A] time = 0.22966, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{5x}{192a^2b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x}{48ab\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5x}{128a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (5*x)/(128*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - x/(8*b*(a + b
*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + x/(48*a*b*(a + b*x^2)^
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*x)/(192*a^2*b*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (5*(a + b*x^2)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0682047, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}x\left(-15a^3+73a^2bx^2+55ab^2x^4+15b^3x^6\right)+15\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{7/2}b^{3/2}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-15*a^3 + 73*a^2*b*x^2 + 55*a*b^2*x^4 + 15*b^
3*x^6) + 15*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(7/
2)*b^(3/2)*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.023, size = 172, normalized size = 0.8

$$\frac{bx^2+a}{384\,a^3b}\left(15\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+15\,\sqrt{ab}x^7b^3+60\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3+55\,\sqrt{ab}x^5ab^2+90\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2+73\,\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(15*arctan(x*b/(a*b)^(1/2))*x^8*b^4+15*(a*b)^(1/2)*x^7*b^3+
60*arctan(x*b/(a*b)^(1/2))*x^6*a*b^3+55*(a*b)^(1/2)*x^5*a*b^2+90*
arctan(x*b/(a*b)^(1/2))*x^4*a^2*b^2+73*(a*b)^(1/2)*x^3*a^2*b+60*a
rctan(x*b/(a*b)^(1/2))*x^2*a^3*b-15*(a*b)^(1/2)*x*a^3+15*arctan(x
*b/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)/b/a^3/((b*x^2+a)^2)^(5
/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270827, size = 1, normalized size = 0.

$$\left[\frac{15\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\log\left(\frac{2\,abx+(bx^2-a)\sqrt{-ab}}{bx^2+a}\right)+2\left(15\,b^3x^7+55\,ab^2x^5+73\,a^2bx^3-15\,a^3x\right)\sqrt{-ab}}{768\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)\sqrt{-ab}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 +
 a^4)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(15
*b^3*x^7 + 55*a*b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)*sqrt(-a*b))/((
a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7
*b)*sqrt(-a*b)), 1/384*(15*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4
 + 4*a^3*b*x^2 + a^4)*arctan(sqrt(a*b)*x/a) + (15*b^3*x^7 + 55*a*
b^2*x^5 + 73*a^2*b*x^3 - 15*a^3*x)*sqrt(a*b))/((a^3*b^5*x^8 + 4*a
^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.612447, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.656 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=213

$$\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a
 + b*x^2)^2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(
a + b*x^2)^3)/(192*a^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x
*(a + b*x^2)^4)/(128*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35
*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(9/2)*Sqrt[b]*
(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rubi [A] time = 0.1777, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{7x\left(a+bx^2\right)^2}{48a^2\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{x\left(a+bx^2\right)}{8a\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35\left(a+bx^2\right)^5\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^4}{128a^4\left(a^2+2abx^2+b^2x^4\right)^{5/2}}+\frac{35x\left(a+bx^2\right)^3}{192a^3\left(a^2+2abx^2+b^2x^4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (x*(a + b*x^2))/(8*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (7*x*(a
 + b*x^2)^2)/(48*a^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x*(
a + b*x^2)^3)/(192*a^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35*x
*(a + b*x^2)^4)/(128*a^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)) + (35
*(a + b*x^2)^5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(9/2)*Sqrt[b]*
(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2))
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0725017, size = 105, normalized size = 0.49

$$\frac{\sqrt{a}\sqrt{b}x\left(279a^3+511a^2bx^2+385ab^2x^4+105b^3x^6\right)+105\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{9/2}\sqrt{b}\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-5/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(279*a^3 + 511*a^2*b*x^2 + 385*a*b^2*x^4 + 105
*b^3*x^6) + 105*(a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a
^(9/2)*Sqrt[b]*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.012, size = 169, normalized size = 0.8

$$\frac{bx^2+a}{384\,a^4}\left(105\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^8b^4+105\,\sqrt{ab}x^7b^3+420\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^6ab^3+385\,\sqrt{ab}x^5ab^2+630\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^4a^2b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(105*arctan(x*b/(a*b)^(1/2))*x^8*b^4+105*(a*b)^(1/2)*x^7*b^
3+420*arctan(x*b/(a*b)^(1/2))*x^6*a*b^3+385*(a*b)^(1/2)*x^5*a*b^2
+630*arctan(x*b/(a*b)^(1/2))*x^4*a^2*b^2+511*(a*b)^(1/2)*x^3*a^2*
b+420*arctan(x*b/(a*b)^(1/2))*x^2*a^3*b+279*(a*b)^(1/2)*x*a^3+105
*arctan(x*b/(a*b)^(1/2))*a^4)*(b*x^2+a)/(a*b)^(1/2)/a^4/((b*x^2+a
)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269975, size = 1, normalized size = 0.

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\log\left(\frac{2\,abx+\left(bx^2-a\right)\sqrt{-ab}}{bx^2+a}\right)+2\left(105\,b^3x^7+385\,ab^2x^5+511\,a^2bx^3+279\,a^3x\right)\sqrt{-ab}}{768\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)\sqrt{-ab}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2
+ a^4)*log((2*a*b*x + (b*x^2 - a)*sqrt(-a*b))/(b*x^2 + a)) + 2*(1
05*b^3*x^7 + 385*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)*sqrt(-a*b
))/((a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 +
a^8)*sqrt(-a*b)), 1/384*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x
^4 + 4*a^3*b*x^2 + a^4)*arctan(sqrt(a*b)*x/a) + (105*b^3*x^7 + 38
5*a*b^2*x^5 + 511*a^2*b*x^3 + 279*a^3*x)*sqrt(a*b))/((a^4*b^4*x^8
 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*sqrt(a*b))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.60368, size = 4, normalized size = 0.02

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.657 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{3}{16a^2x\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}-\frac{315\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{315\,(a+bx^2)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b
*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2
)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*(a + b*x^2))/(128*a^5*x*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sqrt[b]*(a + b*x^2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])
```

Rubi [A] time = 0.285934, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{3}{16a^2x\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ax\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}-\frac{315\sqrt{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{315\,(a+bx^2)}{128a^5x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{105}{128a^4x\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21}{64a^3x\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 105/(128*a^4*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x*(a + b
*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 3/(16*a^2*x*(a + b*x^2
)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 21/(64*a^3*x*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*(a + b*x^2))/(128*a^5*x*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (315*Sqrt[b]*(a + b*x^2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/(128*a^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0862745, size = 115, normalized size = 0.46

$$\frac{-\sqrt{a}\left(128a^4+837a^3bx^2+1533a^2b^2x^4+1155ab^3x^6+315b^4x^8\right)-315\sqrt{b}x\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}x\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (-(Sqrt[a]*(128*a^4 + 837*a^3*b*x^2 + 1533*a^2*b^2*x^4 + 1155*a*b
^3*x^6 + 315*b^4*x^8)) - 315*Sqrt[b]*x*(a + b*x^2)^4*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(128*a^(11/2)*x*(a + b*x^2)^3*Sqrt[(a + b*x^2)^2
])
```

Maple [A] time = 0.028, size = 191, normalized size = 0.8

$$-\frac{bx^2+a}{128\,xa^5}\left(315\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^9b^5+315\,\sqrt{ab}x^8b^4+1260\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^7ab^4+1155\,\sqrt{ab}x^6ab^3+1890\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/128*(315*arctan(x*b/(a*b)^(1/2))*x^9*b^5+315*(a*b)^(1/2)*x^8*b
^4+1260*arctan(x*b/(a*b)^(1/2))*x^7*a*b^4+1155*(a*b)^(1/2)*x^6*a*
b^3+1890*arctan(x*b/(a*b)^(1/2))*x^5*a^2*b^3+1533*(a*b)^(1/2)*x^4
*a^2*b^2+1260*arctan(x*b/(a*b)^(1/2))*x^3*a^3*b^2+837*(a*b)^(1/2)
*x^2*a^3*b+315*arctan(x*b/(a*b)^(1/2))*x*a^4*b+128*(a*b)^(1/2)*a^
4)*(b*x^2+a)/x/(a*b)^(1/2)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278975, size = 1, normalized size = 0.

$$\left[-\frac{630\,b^4x^8 + 2310\,ab^3x^6 + 3066\,a^2b^2x^4 + 1674\,a^3bx^2 + 256\,a^4 - 315\left(b^4x^9 + 4\,ab^3x^7 + 6\,a^2b^2x^5 + 4\,a^3bx^3 + a^4x\right)\sqrt{-\frac{b}{a}}\log\left(\right.}{256\left(a^5b^4x^9 + 4\,a^6b^3x^7 + 6\,a^7b^2x^5 + 4\,a^8bx^3 + a^9x\right)}\right.$$

$$-\frac{315\,b^4x^8 + 1155\,ab^3x^6 + 1533\,a^2b^2x^4 + 837\,a^3bx^2 + 128\,a^4 + 315\left(b^4x^9 + 4\,ab^3x^7 + 6\,a^2b^2x^5 + 4\,a^3bx^3 + a^4x\right)\sqrt{\frac{b}{a}}\arctan\left(\right.}{128\left(a^5b^4x^9 + 4\,a^6b^3x^7 + 6\,a^7b^2x^5 + 4\,a^8bx^3 + a^9x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^2),x, algorithm="fricas")
```

```
[Out] [-1/256*(630*b^4*x^8 + 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 + 1674*a
^3*b*x^2 + 256*a^4 - 315*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 +
 4*a^3*b*x^3 + a^4*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4
*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x^8 + 1155*a*b^3*x^6 + 1533*
a^2*b^2*x^4 + 837*a^3*b*x^2 + 128*a^4 + 315*(b^4*x^9 + 4*a*b^3*x^
7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*sqrt(b/a)*arctan(b*x/(a*
sqrt(b/a))))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8
*b*x^3 + a^9*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.624943, size = 4, normalized size = 0.02

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.658 $\int \frac{1}{x^4(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=291

$$\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b\left(a+bx^2\right)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385\left(a+bx^2\right)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a
 + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a
+ b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a +
 b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (385*(a + b*x^2))/(128
*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*b*(a + b*x^2))/
(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*b^(3/2)*(a +
b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.339539, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{11}{48a^2x^3\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ax^3\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155b\left(a+bx^2\right)}{128a^6x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385\left(a+bx^2\right)}{128a^5x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{231}{128a^4x^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{33}{64a^3x^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 231/(128*a^4*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8*a*x^3*(a
 + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 11/(48*a^2*x^3*(a
+ b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 33/(64*a^3*x^3*(a +
 b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (385*(a + b*x^2))/(128
*a^5*x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*b*(a + b*x^2))/
(128*a^6*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*b^(3/2)*(a +
b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(13/2)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.116031, size = 127, normalized size = 0.44

$$\frac{\sqrt{a}\left(-128a^5+1408a^4bx^2+9207a^3b^2x^4+16863a^2b^3x^6+12705ab^4x^8+3465b^5x^{10}\right)+3465b^{3/2}x^3\left(a+bx^2\right)^4\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{384a^{13/2}x^3\left(a+bx^2\right)^3\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[a]*(-128*a^5 + 1408*a^4*b*x^2 + 9207*a^3*b^2*x^4 + 16863*a^
2*b^3*x^6 + 12705*a*b^4*x^8 + 3465*b^5*x^10) + 3465*b^(3/2)*x^3*(
a + b*x^2)^4*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(384*a^(13/2)*x^3*(a +
b*x^2)^3*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.031, size = 211, normalized size = 0.7

$$\frac{bx^2+a}{384\,a^6x^3}\left(3465\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^{11}b^6+3465\,\sqrt{ab}x^{10}b^5+13860\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)x^9ab^5+12705\,\sqrt{ab}x^8ab^4+20790\,\arctan\left(\frac{b}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/384*(3465*arctan(x*b/(a*b)^(1/2))*x^11*b^6+3465*(a*b)^(1/2)*x^1
0*b^5+13860*arctan(x*b/(a*b)^(1/2))*x^9*a*b^5+12705*(a*b)^(1/2)*x
^8*a*b^4+20790*arctan(x*b/(a*b)^(1/2))*x^7*a^2*b^4+16863*(a*b)^(1
/2)*x^6*a^2*b^3+13860*arctan(x*b/(a*b)^(1/2))*x^5*a^3*b^3+9207*(a
*b)^(1/2)*x^4*a^3*b^2+3465*arctan(x*b/(a*b)^(1/2))*x^3*a^4*b^2+14
08*(a*b)^(1/2)*x^2*a^4*b-128*(a*b)^(1/2)*a^5)*(b*x^2+a)/x^3/(a*b)
```

```
^(1/2)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276518, size = 1, normalized size = 0.

$$\left[\frac{6930\,b^5x^{10} + 25410\,ab^4x^8 + 33726\,a^2b^3x^6 + 18414\,a^3b^2x^4 + 2816\,a^4bx^2 - 256\,a^5 + 3465\left(b^5x^{11} + 4\,ab^4x^9 + 6\,a^2b^3x^7 + 4\,a^3b\right.}{768\left(a^6b^4x^{11} + 4\,a^7b^3x^9 + 6\,a^8b^2x^7 + 4\,a^9bx^5 + a^{10}x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^4),x, algorithm="fricas")
```

```
[Out] [1/768*(6930*b^5*x^10 + 25410*a*b^4*x^8 + 33726*a^2*b^3*x^6 + 184
14*a^3*b^2*x^4 + 2816*a^4*b*x^2 - 256*a^5 + 3465*(b^5*x^11 + 4*a*
b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-b/a)*l
og((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^4*x^11 + 4
*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), 1/384*(34
65*b^5*x^10 + 12705*a*b^4*x^8 + 16863*a^2*b^3*x^6 + 9207*a^3*b^2*
x^4 + 1408*a^4*b*x^2 - 128*a^5 + 3465*(b^5*x^11 + 4*a*b^4*x^9 + 6
*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(b/a)*arctan(b*x/(a
*sqrt(b/a))))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a
^9*b*x^5 + a^10*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/(x**4*((a + b*x**2)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.619341, size = 4, normalized size = 0.01

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.659 $\int \frac{x^2}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=298

$$\frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*x*(a + b*x^2))/(5*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)) + (3*3^
(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^
2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2
/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x
^2)/a)^(1/3))^2)])
```

Rubi [A] time = 0.517997, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{3x(a+bx^2)}{5b\sqrt[3]{a^2+2abx^2+b^2x^4}}+\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{5b^2x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(5*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)) + (3*3^
(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^
2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2
/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^
(1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x
^2)/a)^(1/3))^2)])
```

Rubi in Sympy [A] time = 33.4425, size = 345, normalized size = 1.16

$$\frac{3\cdot 3^{\frac{3}{4}} a \sqrt{\frac{a^{\frac{2}{3}} b^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{ab+b^2x^2}+(ab+b^2x^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right)+\sqrt[3]{ab+b^2x^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{ab+b^2x^2}\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{2}{3}} F\Bigl(\operatorname{asin}\Bigl(\frac{\sqrt[3]{a}\sqrt[3]{b}\left(1+\sqrt{3}\right)}{-\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right)}}{5bx\sqrt{-\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{ab+b^2x^2}\right)}{\left(\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right)+\sqrt[3]{ab+b^2x^2}\right)^2}}\left(ab+b^2x^2\right)^{\frac{4}{3}}}$$

$$+\frac{3x\left(a^2+2abx^2+b^2x^4\right)^{\frac{2}{3}}}{5b\left(a+bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] 3*3**(3/4)*a*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(a*b + b
**2*x**2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**(1/3)*(
-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*sqrt(-sqrt(3) + 2)*
(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))*(a**2 + 2*a*b*x**2
 + b**2*x**4)**(2/3)*elliptic_f(asin((a**(1/3)*b**(1/3)*(1 + sqrt
(3)) - (a*b + b**2*x**2)**(1/3))/(-a**(1/3)*b**(1/3)*(-1 + sqrt(3
)) - (a*b + b**2*x**2)**(1/3))), -7 + 4*sqrt(3))/(5*b*x*sqrt(-a**
(1/3)*b**(1/3)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))/(a*
*(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a
*b + b**2*x**2)**(4/3)) + 3*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(2
/3)/(5*b*(a + b*x**2))
```

Mathematica [C] time = 0.0698657, size = 64, normalized size = 0.21

$$\frac{3x\left(-a\left(\frac{bx^2}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3),x]
```

```
[Out] (3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2,
2/3, 3/2, -((b*x^2)/a)]))/(5*b*((a + b*x^2)^2)^(1/3))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt[3]{b^2x^4 + 2\,abx^2 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt[3]{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3), x)
```

3.660 $\int \frac{1}{\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=256

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b
*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)
^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2
)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(
1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))^2)]))
```

Rubi [A] time = 0.279655, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\,a\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{bx\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] -((3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b
*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)
```

```
^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2
)/a)^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(
1/3)*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^
2)/a)^(1/3))^2)]))
```

Rubi in Sympy [A] time = 29.5528, size = 304, normalized size = 1.19

$$-\frac{3^{\frac{3}{4}}\sqrt{\frac{a^{\frac{2}{3}}b^{\frac{2}{3}}+\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{ab+b^2x^2}+(ab+b^2x^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right)+\sqrt[3]{ab+b^2x^2}\right)^2}}\sqrt{-\sqrt{3}+2}\left(\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{ab+b^2x^2}\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{2}{3}}F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\sqrt[3]{b}\left(1+\sqrt{3}\right)-\sqrt[3]{\cdots}}{-\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right)-\cdots}\right.\right.}{x\sqrt{-\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b}-\sqrt[3]{ab+b^2x^2}\right)}{\left(\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right)+\sqrt[3]{ab+b^2x^2}\right)^2}}\left(ab+b^2x^2\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] -3**(3/4)*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(a*b + b**2
*x**2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**(1/3)*(-1
+ sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*sqrt(-sqrt(3) + 2)*(a*
*(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))*(a**2 + 2*a*b*x**2 +
b**2*x**4)**(2/3)*elliptic_f(asin((a**(1/3)*b**(1/3)*(1 + sqrt(3)
) - (a*b + b**2*x**2)**(1/3))/(-a**(1/3)*b**(1/3)*(-1 + sqrt(3))
- (a*b + b**2*x**2)**(1/3))), -7 + 4*sqrt(3))/(x*sqrt(-a**(1/3)*b
**(1/3)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))/(a**(1/3)*
b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a*b + b*
*2*x**2)**(4/3))
```

Mathematica [C] time = 0.0264652, size = 49, normalized size = 0.19

$$\frac{x\left(\frac{a+bx^2}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right)}{\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-1/3),x]
```

```
[Out] (x*((a + b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^
2)/a)])/((a + b*x^2)^2)^(1/3)
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{b^{2}x^{4}+2\,ab\,x^{2}+a^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2}x^{4}+2\,ab\,x^{2}+a^{2}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(b^{2}x^{4}+2\,ab\,x^{2}+a^{2}\right)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a^{2}+2abx^{2}+b^{2}x^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2 x^4 + 2\,abx^2 + a^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-1/3), x)
```

3.661 $\int \frac{1}{x^2\sqrt[3]{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=289

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((a + b*x^2)/(a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3))) + (Sqrt[2
- Sqrt[3]]*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt
[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3]
 - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4
*Sqrt[3]])/(3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1
 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2
)])
```

Rubi [A] time = 0.383851, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{\sqrt{2-\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{2/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt[4]{3}x\sqrt[3]{a^2+2abx^2+b^2x^4}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{a+bx^2}{ax\sqrt[3]{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] -((a + b*x^2)/(a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3))) + (Sqrt[2
- Sqrt[3]]*(1 + (b*x^2)/a)^(2/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt
[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3]
 - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4
*Sqrt[3]])/(3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)*Sqrt[-((1
 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2
)])
```

Rubi in Sympy [A] time = 33.1114, size = 340, normalized size = 1.18

$$\frac{3^{\frac{3}{4}} b \sqrt{\frac{a^{\frac{2}{3}} b^{\frac{2}{3}} + \sqrt[3]{a} \sqrt[3]{b} \sqrt[3]{ab + b^2x^2} + (ab+b^2x^2)^{\frac{2}{3}}}{\left(\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right) + \sqrt[3]{ab + b^2x^2}\right)^2}} \sqrt{-\sqrt{3}+2} \left(\sqrt[3]{a}\sqrt[3]{b} - \sqrt[3]{ab + b^2x^2}\right) \left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{2}{3}} F\left(\operatorname{asin}\left(\frac{\sqrt[3]{a}\sqrt[3]{b}\left(1+\sqrt{3}\right) - \sqrt[3]{\cdots}}{-\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right) - \cdots}\right.\right.}{3ax\sqrt{-\frac{\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}\sqrt[3]{b} - \sqrt[3]{ab + b^2x^2}\right)}{\left(\sqrt[3]{a}\sqrt[3]{b}\left(-1+\sqrt{3}\right) + \sqrt[3]{ab + b^2x^2}\right)^2}} \left(ab + b^2x^2\right)^{\frac{4}{3}}}$$

$$-\frac{\left(a^2 + 2abx^2 + b^2x^4\right)^{\frac{2}{3}}}{ax\left(a + bx^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] 3**(3/4)*b*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(a*b + b**
2*x**2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**(1/3)*(-1
 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*sqrt(-sqrt(3) + 2)*(a
**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))*(a**2 + 2*a*b*x**2 +
 b**2*x**4)**(2/3)*elliptic_f(asin((a**(1/3)*b**(1/3)*(1 + sqrt(3
)) - (a*b + b**2*x**2)**(1/3))/(-a**(1/3)*b**(1/3)*(-1 + sqrt(3))
 - (a*b + b**2*x**2)**(1/3))), -7 + 4*sqrt(3))/(3*a*x*sqrt(-a**(1
/3)*b**(1/3)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))/(a**(
1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a*b
 + b**2*x**2)**(4/3)) - (a**2 + 2*a*b*x**2 + b**2*x**4)**(2/3)/(a
*x*(a + b*x**2))
```

Mathematica [C] time = 0.0470247, size = 72, normalized size = 0.25

$$\frac{-bx^2\left(\frac{bx^2}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{3}{2};-\frac{bx^2}{a}\right) - 3\left(a+bx^2\right)}{3ax\sqrt[3]{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(1/3)),x]
```

```
[Out] (-3*(a + b*x^2) - b*x^2*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1
/2, 2/3, 3/2, -((b*x^2)/a)])/(3*a*x*((a + b*x^2)^2)^(1/3))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \frac{1}{\sqrt[3]{b^2x^4 + 2\,abx^2 + a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

```
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{3}}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt[3]{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(1/3),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(1/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^2x^4 + 2\,abx^2 + a^2)^{\frac{1}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(1/3)*x^2), x)
```

3.662 $\int \frac{x^2}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx$

Optimal. Leaf size=618

$$-\frac{9ax\left(\frac{bx^2}{a}+1\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{2}b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (-3*x*(a + b*x^2))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (9*a
*x*(1 + (b*x^2)/a)^(4/3))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*
(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(
1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] -
 (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*S
qrt[3]])/(4*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (
1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
- (3*3^(3/4)*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3)
)*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - S
qrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3]
- (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))],
-7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)
*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
)^(1/3))^2)])
```

Rubi [A] time = 0.967609, antiderivative size = 618, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{9ax\left(\frac{bx^2}{a}+1\right)^{4/3}}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{3x\left(a+bx^2\right)}{2b\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{3\,3^{3/4}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^2\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4b^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (-3*x*(a + b*x^2))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (9*a
*x*(1 + (b*x^2)/a)^(4/3))/(2*b*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*
(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) + (9*3^(1/4)*Sqrt[2 + Sqrt
[3]]*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(
1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] -
 (1 + (b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*S
qrt[3]])/(4*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (
1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
- (3*3^(3/4)*a^2*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3)
)*Sqrt[(1 + (1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - S
qrt[3] - (1 + (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3]
- (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))],
-7 + 4*Sqrt[3]])/(Sqrt[2]*b^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)
*Sqrt[-((1 - (1 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a
```

```
)^(1/3))^2)])
```

Rubi in Sympy [A] time = 71.7491, size = 736, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] 9*3**(1/4)*a**(1/3)*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(
a*b + b**2*x**2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**
(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*sqrt(sqrt(3)
 + 2)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))*(a**2 + 2*a*
b*x**2 + b**2*x**4)**(1/3)*elliptic_e(asin((a**(1/3)*b**(1/3)*(1
+ sqrt(3)) - (a*b + b**2*x**2)**(1/3))/(-a**(1/3)*b**(1/3)*(-1 +
sqrt(3)) - (a*b + b**2*x**2)**(1/3))), -7 + 4*sqrt(3))/(4*b**(5/3
)*x*sqrt(-a**(1/3)*b**(1/3)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2
)**(1/3))/(a**(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**
(1/3))**2)*(a*b + b**2*x**2)**(2/3)) - 3*sqrt(2)*3**(3/4)*a**(1/3
)*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(a*b + b**2*x**2)**
(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**(1/3)*(-1 + sqrt(3
)) + (a*b + b**2*x**2)**(1/3))**2)*(a**(1/3)*b**(1/3) - (a*b + b*
*2*x**2)**(1/3))*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)*elliptic_
f(asin((a**(1/3)*b**(1/3)*(1 + sqrt(3)) - (a*b + b**2*x**2)**(1/3
))/(-a**(1/3)*b**(1/3)*(-1 + sqrt(3)) - (a*b + b**2*x**2)**(1/3))
), -7 + 4*sqrt(3))/(2*b**(5/3)*x*sqrt(-a**(1/3)*b**(1/3)*(a**(1/3
)*b**(1/3) - (a*b + b**2*x**2)**(1/3))/(a**(1/3)*b**(1/3)*(-1 + s
qrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a*b + b**2*x**2)**(2/3))
 + 9*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*(a*b + b**2*x**2
)**(2/3)*(a**(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(
1/3))) - 3*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*b*(a + b*x
**2))
```

Mathematica [C] time = 0.0734777, size = 64, normalized size = 0.1

$$\frac{3x\left(a+bx^2\right)\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-1\right)}{2b\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3),x]
```

```
[Out] (3*x*(a + b*x^2)*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/
3, 1/2, 3/2, -((b*x^2)/a)]))/(2*b*((a + b*x^2)^2)^(2/3))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int x^2 \left(b^2x^4 + 2\,abx^2 + a^2\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left((a+bx^2)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral(x**2/((a + b*x**2)**2)**(2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(b^2x^4+2\,abx^2+a^2)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3), x)
```

3.663 $\int \frac{1}{(a^2+2abx^2+b^2x^4)^{2/3}}\,dx$

Optimal. Leaf size=609

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}+\frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}$$

$$+\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{2}bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*x*(a + b*x^2))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) + (3*x*
(1 + (b*x^2)/a)^(4/3))/(2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*
a*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1
+ (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/
a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]]
)/(4*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^
2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) + (3^(3/4
)*a*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (
1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2
)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3
]])/(Sqrt[2]*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1
 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rubi [A] time = 0.894752, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{3x\left(\frac{bx^2}{a}+1\right)^{4/3}}{2\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}+\frac{3x\left(a+bx^2\right)}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3^{3/4}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}a\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}\left(\frac{bx^2}{a}+1\right)^{4/3}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4bx\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] (3*x*(a + b*x^2))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) + (3*x*
(1 + (b*x^2)/a)^(4/3))/(2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1 -
Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*
a*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1
+ (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (
b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/
a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]]
)/(4*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^
2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) + (3^(3/4
)*a*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (
1 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 +
 (b*x^2)/a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2
)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3
]])/(Sqrt[2]*b*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1
 + (b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rubi in Sympy [A] time = 76.8475, size = 738, normalized size = 1.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] -3*b*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*a*(a*b + b**2*x*
*2)**(2/3)*(a**(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)*
*(1/3))) + 3*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*a*(a + b
*x**2)) - 3*3**(1/4)*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*
(a*b + b**2*x**2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b*
*(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*sqrt(sqrt(3
) + 2)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))*(a**2 + 2*a
*b*x**2 + b**2*x**4)**(1/3)*elliptic_e(asin((a**(1/3)*b**(1/3)*(1
 + sqrt(3)) - (a*b + b**2*x**2)**(1/3))/(-a**(1/3)*b**(1/3)*(-1 +
 sqrt(3)) - (a*b + b**2*x**2)**(1/3))), -7 + 4*sqrt(3))/(4*a**(2/
3)*b**(2/3)*x*sqrt(-a**(1/3)*b**(1/3)*(a**(1/3)*b**(1/3) - (a*b +
 b**2*x**2)**(1/3))/(a**(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b*
*2*x**2)**(1/3))**2)*(a*b + b**2*x**2)**(2/3)) + sqrt(2)*3**(3/4)
*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(a*b + b**2*x**2)**(
1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**(1/3)*(-1 + sqrt(3)
) + (a*b + b**2*x**2)**(1/3))**2)*(a**(1/3)*b**(1/3) - (a*b + b**
2*x**2)**(1/3))*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)*elliptic_f
(asin((a**(1/3)*b**(1/3)*(1 + sqrt(3)) - (a*b + b**2*x**2)**(1/3)
)/(-a**(1/3)*b**(1/3)*(-1 + sqrt(3)) - (a*b + b**2*x**2)**(1/3)))
, -7 + 4*sqrt(3))/(2*a**(2/3)*b**(2/3)*x*sqrt(-a**(1/3)*b**(1/3)*
(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))/(a**(1/3)*b**(1/3)
*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a*b + b**2*x**2)
**(2/3))
```

Mathematica [C] time = 0.0531306, size = 64, normalized size = 0.11

$$-\frac{x\left(a+bx^2\right)\left(\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)-3\right)}{2a\left(\left(a+bx^2\right)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(-2/3),x]
```

```
[Out] -(x*(a + b*x^2)*(-3 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3
, 1/2, 3/2, -((b*x^2)/a)]))/(2*a*((a + b*x^2)^2)^(2/3))
```

Maple [F] time = 0.018, size = 0, normalized size = 0.

$$\int \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{-\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)

[Out] int(1/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3),x, algorithm="maxima")

[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3),x, algorithm="fricas")

[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{2}+2abx^{2}+b^{2}x^{4}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**(-2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b^2x^4 + 2\,abx^2 + a^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(-2/3), x)
```

3.664 $\int \frac{1}{x^2(a^2+2abx^2+b^2x^4)^{2/3}}\,dx$

Optimal. Leaf size=649

$$-\frac{5(a+bx^2)^2}{2a^2x(a^2+2abx^2+b^2x^4)^{2/3}}+\frac{3(a+bx^2)}{2ax(a^2+2abx^2+b^2x^4)^{2/3}}-\frac{5bx\left(\frac{bx^2}{a}+1\right)^{4/3}}{2a(a^2+2abx^2+b^2x^4)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{5\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{\sqrt{2}\sqrt[4]{3}x(a^2+2abx^2+b^2x^4)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{4x(a^2+2abx^2+b^2x^4)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(a + b*x^2))/(2*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*(a
 + b*x^2)^2)/(2*a^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*b*x
*(1 + (b*x^2)/a)^(4/3))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1
 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1
 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)
/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]
])/(4*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2
)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (5*(1 +
(b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^
2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/
a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3
))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt
[2]*3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 +
```

```
(b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rubi [A] time = 1.05768, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{5\left(a+bx^2\right)^2}{2a^2x\left(a^2+2abx^2+b^2x^4\right)^{2/3}}+\frac{3\left(a+bx^2\right)}{2ax\left(a^2+2abx^2+b^2x^4\right)^{2/3}}-\frac{5bx\left(\frac{bx^2}{a}+1\right)^{4/3}}{2a\left(a^2+2abx^2+b^2x^4\right)^{2/3}\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)}-\frac{5\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{\sqrt{2}\sqrt[4]{3}x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}+\frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\frac{bx^2}{a}+1\right)^{4/3}\left(1-\sqrt[3]{\frac{bx^2}{a}+1}\right)\sqrt{\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}+\sqrt[3]{\frac{bx^2}{a}+1}+1}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-\sqrt[3]{\frac{bx^2}{a}+1}+\sqrt{3}+1}{-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1}\right)|-7+4\sqrt{3}\right)}{4x\left(a^2+2abx^2+b^2x^4\right)^{2/3}\sqrt{-\frac{1-\sqrt[3]{\frac{bx^2}{a}+1}}{\left(-\sqrt[3]{\frac{bx^2}{a}+1}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] (3*(a + b*x^2))/(2*a*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*(a
 + b*x^2)^2)/(2*a^2*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)) - (5*b*x
*(1 + (b*x^2)/a)^(4/3))/(2*a*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*(1
 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))) + (5*3^(1/4)*Sqrt[2 + Sqrt[3
]]*(1 + (b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1
 + (b*x^2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 +
(b*x^2)/a)^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)
/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]
```

```
])/(4*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 + (b*x^2
)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)]) - (5*(1 +
(b*x^2)/a)^(4/3)*(1 - (1 + (b*x^2)/a)^(1/3))*Sqrt[(1 + (1 + (b*x^
2)/a)^(1/3) + (1 + (b*x^2)/a)^(2/3))/(1 - Sqrt[3] - (1 + (b*x^2)/
a)^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 + (b*x^2)/a)^(1/3
))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt
[2]*3^(1/4)*x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)*Sqrt[-((1 - (1 +
(b*x^2)/a)^(1/3))/(1 - Sqrt[3] - (1 + (b*x^2)/a)^(1/3))^2)])
```

Rubi in Sympy [A] time = 88.5335, size = 774, normalized size = 1.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] 3*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*a*x*(a + b*x**2)) + 5
*b**2*x*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*a**2*(a*b + b**
2*x**2)**(2/3)*(a**(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x*
*2)**(1/3))) - 5*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)/(2*a**2*x
) + 5*3**(1/4)*b**(1/3)*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/
3)*(a*b + b**2*x**2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)
*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*sqrt(sqr
t(3) + 2)*(a**(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))*(a**2 +
2*a*b*x**2 + b**2*x**4)**(1/3)*elliptic_e(asin((a**(1/3)*b**(1/3)
*(1 + sqrt(3)) - (a*b + b**2*x**2)**(1/3))/(-a**(1/3)*b**(1/3)*(-
1 + sqrt(3)) - (a*b + b**2*x**2)**(1/3))), -7 + 4*sqrt(3))/(4*a**
(5/3)*x*sqrt(-a**(1/3)*b**(1/3)*(a**(1/3)*b**(1/3) - (a*b + b**2*
x**2)**(1/3))/(a**(1/3)*b**(1/3)*(-1 + sqrt(3)) + (a*b + b**2*x**
2)**(1/3))**2)*(a*b + b**2*x**2)**(2/3)) - 5*sqrt(2)*3**(3/4)*b**
(1/3)*sqrt((a**(2/3)*b**(2/3) + a**(1/3)*b**(1/3)*(a*b + b**2*x**
2)**(1/3) + (a*b + b**2*x**2)**(2/3))/(a**(1/3)*b**(1/3)*(-1 + sq
rt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a**(1/3)*b**(1/3) - (a*b
+ b**2*x**2)**(1/3))*(a**2 + 2*a*b*x**2 + b**2*x**4)**(1/3)*ellip
tic_f(asin((a**(1/3)*b**(1/3)*(1 + sqrt(3)) - (a*b + b**2*x**2)**
(1/3))/(-a**(1/3)*b**(1/3)*(-1 + sqrt(3)) - (a*b + b**2*x**2)**(1
/3))), -7 + 4*sqrt(3))/(6*a**(5/3)*x*sqrt(-a**(1/3)*b**(1/3)*(a**
(1/3)*b**(1/3) - (a*b + b**2*x**2)**(1/3))/(a**(1/3)*b**(1/3)*(-1
 + sqrt(3)) + (a*b + b**2*x**2)**(1/3))**2)*(a*b + b**2*x**2)**(2
/3))
```

Mathematica [C] time = 0.0664442, size = 79, normalized size = 0.12

$$-\frac{(a+bx^2)\left(-5bx^2\sqrt[3]{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{3}{2};-\frac{bx^2}{a}\right)+6a+15bx^2\right)}{6a^2x\left((a+bx^2)^2\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(2/3)),x]
```

```
[Out] -((a + b*x^2)*(6*a + 15*b*x^2 - 5*b*x^2*(1 + (b*x^2)/a)^(1/3)*Hyp
ergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)]))/(6*a^2*x*((a + b*x^
2)^2)^(2/3))
```

Maple [F] time = 0.089, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

```
[Out] int(1/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b^2x^4+2\,abx^2+a^2)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(2/3),x)
```

```
[Out] Integral(1/(x**2*((a + b*x**2)**2)**(2/3)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{2}{3}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(2/3)*x^2), x)
```

3.665 $\int (dx)^{5/2} \left(a^2 + 2abx^2 + b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{7/2}}{7d} + \frac{4ab(dx)^{11/2}}{11d^3} + \frac{2b^2(dx)^{15/2}}{15d^5}$$

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^
2*(d*x)^(15/2))/(15*d^5)
```

Rubi [A] time = 0.0424518, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{2a^2(dx)^{7/2}}{7d} + \frac{4ab(dx)^{11/2}}{11d^3} + \frac{2b^2(dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(7/2))/(7*d) + (4*a*b*(d*x)^(11/2))/(11*d^3) + (2*b^
2*(d*x)^(15/2))/(15*d^5)
```

Rubi in Sympy [A] time = 15.5656, size = 48, normalized size = 0.94

$$\frac{2a^2 (dx)^{\frac{7}{2}}}{7d} + \frac{4ab (dx)^{\frac{11}{2}}}{11d^3} + \frac{2b^2 (dx)^{\frac{15}{2}}}{15d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*(d*x)**(7/2)/(7*d) + 4*a*b*(d*x)**(11/2)/(11*d**3) + 2*b**
2*(d*x)**(15/2)/(15*d**5)
```

Mathematica [A] time = 0.0223038, size = 33, normalized size = 0.65

$$\frac{2x(dx)^{5/2} \left(165a^2 + 210abx^2 + 77b^2x^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(5/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Maple [A] time = 0.011, size = 30, normalized size = 0.6

$$\frac{2\,x\left(77\,b^{2}x^{4}+210\,ab{x}^{2}+165\,a^{2}\right)}{1155}\left(dx\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/1155*x*(77*b^2*x^4+210*a*b*x^2+165*a^2)*(d*x)^(5/2)
```

Maxima [A] time = 0.686051, size = 55, normalized size = 1.08

$$\frac{2\left(77\,\left(dx\right)^{\frac{15}{2}}b^{2}+210\,\left(dx\right)^{\frac{11}{2}}abd^{2}+165\,\left(dx\right)^{\frac{7}{2}}a^{2}d^{4}\right)}{1155\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(77*(d*x)^(15/2)*b^2 + 210*(d*x)^(11/2)*a*b*d^2 + 165*(d*x
)^(7/2)*a^2*d^4)/d^5
```

Fricas [A] time = 0.257135, size = 54, normalized size = 1.06

$$\frac{2}{1155}\left(77\,b^{2}d^{2}x^{7}+210\,abd^{2}x^{5}+165\,a^{2}d^{2}x^{3}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^2*d^2*x^7 + 210*a*b*d^2*x^5 + 165*a^2*d^2*x^3)*sqrt(
d*x)
```

Sympy [A] time = 8.29621, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{4abd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{2b^2d^{\frac{5}{2}}x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(5/2)*x**(7/2)/7 + 4*a*b*d**(5/2)*x**(11/2)/11 + 2*b**2
*d**(5/2)*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.26216, size = 65, normalized size = 1.27

$$\frac{2}{15}\sqrt{dx}b^2d^2x^7+\frac{4}{11}\sqrt{dx}abd^2x^5+\frac{2}{7}\sqrt{dx}a^2d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*sqrt(d*x)*b^2*d^2*x^7 + 4/11*sqrt(d*x)*a*b*d^2*x^5 + 2/7*sqr
t(d*x)*a^2*d^2*x^3
```

3.666 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*
(d*x)^(13/2))/(13*d^5)
```

Rubi [A] time = 0.0382421, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{2a^2(dx)^{5/2}}{5d}+\frac{4ab(dx)^{9/2}}{9d^3}+\frac{2b^2(dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(5/2))/(5*d) + (4*a*b*(d*x)^(9/2))/(9*d^3) + (2*b^2*
(d*x)^(13/2))/(13*d^5)
```

Rubi in Sympy [A] time = 15.5899, size = 48, normalized size = 0.94

$$\frac{2a^2\left(dx\right)^{\frac{5}{2}}}{5d}+\frac{4ab\left(dx\right)^{\frac{9}{2}}}{9d^3}+\frac{2b^2\left(dx\right)^{\frac{13}{2}}}{13d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*(d*x)**(5/2)/(5*d) + 4*a*b*(d*x)**(9/2)/(9*d**3) + 2*b**2*
(d*x)**(13/2)/(13*d**5)
```

Mathematica [A] time = 0.0182458, size = 33, normalized size = 0.65

$$\frac{2}{585}x(dx)^{3/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*(d*x)^(3/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Maple [A] time = 0.01, size = 30, normalized size = 0.6

$$\frac{2\,x\left(45\,b^{2}x^{4}+130\,ab x^{2}+117\,a^{2}\right)}{585}\left(dx\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/585*x*(45*b^2*x^4+130*a*b*x^2+117*a^2)*(d*x)^(3/2)
```

Maxima [A] time = 0.681239, size = 55, normalized size = 1.08

$$\frac{2\left(45\,\left(dx\right)^{\frac{13}{2}}b^{2}+130\,\left(dx\right)^{\frac{9}{2}}abd^{2}+117\,\left(dx\right)^{\frac{5}{2}}a^{2}d^{4}\right)}{585\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/585*(45*(d*x)^(13/2)*b^2 + 130*(d*x)^(9/2)*a*b*d^2 + 117*(d*x)^
(5/2)*a^2*d^4)/d^5
```

Fricas [A] time = 0.256475, size = 46, normalized size = 0.9

$$\frac{2}{585}\left(45\,b^{2}dx^{6}+130\,abdx^{4}+117\,a^{2}dx^{2}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^2*d*x^6 + 130*a*b*d*x^4 + 117*a^2*d*x^2)*sqrt(d*x)
```

Sympy [A] time = 3.81795, size = 49, normalized size = 0.96

$$\frac{2a^2d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{4abd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*a**2*d**(3/2)*x**(5/2)/5 + 4*a*b*d**(3/2)*x**(9/2)/9 + 2*b**2*d
**(3/2)*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.26309, size = 57, normalized size = 1.12

$$\frac{2}{13}\sqrt{dx}b^2dx^6+\frac{4}{9}\sqrt{dx}abdx^4+\frac{2}{5}\sqrt{dx}a^2dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/13*sqrt(d*x)*b^2*d*x^6 + 4/9*sqrt(d*x)*a*b*d*x^4 + 2/5*sqrt(d*x
)*a^2*d*x^2
```

3.667 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)\,dx$

Optimal. Leaf size=51

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*
(d*x)^(11/2))/(11*d^5)
```

Rubi [A] time = 0.0391867, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{2a^2(dx)^{3/2}}{3d}+\frac{4ab(dx)^{7/2}}{7d^3}+\frac{2b^2(dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*a^2*(d*x)^(3/2))/(3*d) + (4*a*b*(d*x)^(7/2))/(7*d^3) + (2*b^2*
(d*x)^(11/2))/(11*d^5)
```

Rubi in Sympy [A] time = 15.5266, size = 48, normalized size = 0.94

$$\frac{2a^2\,(dx)^{\frac{3}{2}}}{3d}+\frac{4ab\,(dx)^{\frac{7}{2}}}{7d^3}+\frac{2b^2\,(dx)^{\frac{11}{2}}}{11d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)*(d*x)**(1/2),x)
```

```
[Out] 2*a**2*(d*x)**(3/2)/(3*d) + 4*a*b*(d*x)**(7/2)/(7*d**3) + 2*b**2*
(d*x)**(11/2)/(11*d**5)
```

Mathematica [A] time = 0.0133564, size = 33, normalized size = 0.65

$$\frac{2}{231}x\sqrt{dx}\left(77a^2+66abx^2+21b^2x^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (2*x*Sqrt[d*x]*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Maple [A] time = 0.009, size = 30, normalized size = 0.6

$$\frac{2\,x\left(21\,b^{2}x^{4}+66\,ab x^{2}+77\,a^{2}\right)}{231}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)*(d*x)^(1/2),x)
```

```
[Out] 2/231*x*(21*b^2*x^4+66*a*b*x^2+77*a^2)*(d*x)^(1/2)
```

Maxima [A] time = 0.691685, size = 55, normalized size = 1.08

$$\frac{2\left(21\,(dx)^{\frac{11}{2}}\,b^{2}+66\,(dx)^{\frac{7}{2}}\,abd^{2}+77\,(dx)^{\frac{3}{2}}\,a^{2}d^{4}\right)}{231\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/231*(21*(d*x)^(11/2)*b^2 + 66*(d*x)^(7/2)*a*b*d^2 + 77*(d*x)^(3
/2)*a^2*d^4)/d^5
```

Fricas [A] time = 0.256995, size = 39, normalized size = 0.76

$$\frac{2}{231}\left(21\,b^{2}x^{5}+66\,abx^{3}+77\,a^{2}x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(d*x)
```

Sympy [A] time = 1.25467, size = 49, normalized size = 0.96

$$\frac{2a^2\sqrt{d}x^{\frac{3}{2}}}{3}+\frac{4ab\sqrt{d}x^{\frac{7}{2}}}{7}+\frac{2b^2\sqrt{d}x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)*(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(d)*x**(3/2)/3 + 4*a*b*sqrt(d)*x**(7/2)/7 + 2*b**2*sqr
t(d)*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.261993, size = 61, normalized size = 1.2

$$\frac{2\left(21\sqrt{dx}b^2dx^5+66\sqrt{dx}abdx^3+77\sqrt{dx}a^2dx\right)}{231\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/231*(21*sqrt(d*x)*b^2*d*x^5 + 66*sqrt(d*x)*a*b*d*x^3 + 77*sqrt(
d*x)*a^2*d*x)/d
```

3.668 $\int \frac{a^2+2abx^2+b^2x^4}{\sqrt{dx}}\,dx$

Optimal. Leaf size=49

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^
(9/2))/(9*d^5)
```

Rubi [A] time = 0.0390312, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab(dx)^{5/2}}{5d^3}+\frac{2b^2(dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*a^2*Sqrt[d*x])/d + (4*a*b*(d*x)^(5/2))/(5*d^3) + (2*b^2*(d*x)^
(9/2))/(9*d^5)
```

Rubi in Sympy [A] time = 15.5064, size = 46, normalized size = 0.94

$$\frac{2a^2\sqrt{dx}}{d}+\frac{4ab\,(dx)^{\frac{5}{2}}}{5d^3}+\frac{2b^2\,(dx)^{\frac{9}{2}}}{9d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(d*x)/d + 4*a*b*(d*x)**(5/2)/(5*d**3) + 2*b**2*(d*x)**
(9/2)/(9*d**5)
```

Mathematica [A] time = 0.0150856, size = 33, normalized size = 0.67

$$\frac{2\left(45a^2x+18abx^3+5b^2x^5\right)}{45\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/Sqrt[d*x],x]
```

```
[Out] (2*(45*a^2*x + 18*a*b*x^3 + 5*b^2*x^5))/(45*Sqrt[d*x])
```

Maple [A] time = 0.01, size = 30, normalized size = 0.6

$$\frac{\left(10\,b^{2}x^{4}+36\,abx^{2}+90\,a^{2}\right)x}{45}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 2/45*(5*b^2*x^4+18*a*b*x^2+45*a^2)*x/(d*x)^(1/2)
```

Maxima [A] time = 0.698087, size = 55, normalized size = 1.12

$$\frac{2\left(45\,\sqrt{dx}a^{2}+\frac{5\,(dx)^{\frac{9}{2}}\,b^{2}}{d^{4}}+\frac{18\,(dx)^{\frac{5}{2}}\,ab}{d^{2}}\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/45*(45*sqrt(d*x)*a^2 + 5*(d*x)^(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a
*b/d^2)/d
```

Fricas [A] time = 0.256735, size = 42, normalized size = 0.86

$$\frac{2\left(5\,b^{2}x^{4}+18\,abx^{2}+45\,a^{2}\right)\sqrt{dx}}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(d*x)/d
```

Sympy [A] time = 2.02231, size = 48, normalized size = 0.98

$$\frac{2a^2\sqrt{x}}{\sqrt{d}} + \frac{4abx^{\frac{5}{2}}}{5\sqrt{d}} + \frac{2b^2x^{\frac{9}{2}}}{9\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x)/sqrt(d) + 4*a*b*x**(5/2)/(5*sqrt(d)) + 2*b**2*x**(
9/2)/(9*sqrt(d))
```

GIAC/XCAS [A] time = 0.262484, size = 55, normalized size = 1.12

$$\frac{2\left(5\sqrt{dx}b^2x^4 + 18\sqrt{dx}abx^2 + 45\sqrt{dx}a^2\right)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/45*(5*sqrt(d*x)*b^2*x^4 + 18*sqrt(d*x)*a*b*x^2 + 45*sqrt(d*x)*a
^2)/d
```

3.669 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*
x)^(7/2))/(7*d^5)
```

Rubi [A] time = 0.0383538, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab(dx)^{3/2}}{3d^3}+\frac{2b^2(dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(d*Sqrt[d*x]) + (4*a*b*(d*x)^(3/2))/(3*d^3) + (2*b^2*(d*
x)^(7/2))/(7*d^5)
```

Rubi in Sympy [A] time = 15.5606, size = 46, normalized size = 0.94

$$-\frac{2a^2}{d\sqrt{dx}}+\frac{4ab\,(dx)^{\frac{3}{2}}}{3d^3}+\frac{2b^2\,(dx)^{\frac{7}{2}}}{7d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

```
[Out] -2*a**2/(d*sqrt(d*x)) + 4*a*b*(d*x)**(3/2)/(3*d**3) + 2*b**2*(d*x
)**(7/2)/(7*d**5)
```

Mathematica [A] time = 0.0184294, size = 33, normalized size = 0.67

$$\frac{2x\left(-21a^2+14abx^2+3b^2x^4\right)}{21(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-21*a^2 + 14*a*b*x^2 + 3*b^2*x^4))/(21*(d*x)^(3/2))
```

Maple [A] time = 0.009, size = 30, normalized size = 0.6

$$-\frac{\left(-6\,b^{2}x^{4}-28\,abx^{2}+42\,a^{2}\right)x}{21}\left(dx\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(3/2),x)
```

```
[Out] -2/21*(-3*b^2*x^4-14*a*b*x^2+21*a^2)*x/(d*x)^(3/2)
```

Maxima [A] time = 0.690431, size = 59, normalized size = 1.2

$$-\frac{2\left(\frac{21\,a^{2}}{\sqrt{dx}}-\frac{3\,(dx)^{\frac{7}{2}}b^{2}+14\,(dx)^{\frac{3}{2}}abd^{2}}{d^{4}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*(d*x)^(7/2)*b^2 + 14*(d*x)^(3/2)*a*b
*d^2)/d^4)/d
```

Fricas [A] time = 0.256204, size = 42, normalized size = 0.86

$$\frac{2\left(3\,b^{2}x^{4}+14\,abx^{2}-21\,a^{2}\right)}{21\,\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)/(sqrt(d*x)*d)
```

Sympy [A] time = 2.13637, size = 48, normalized size = 0.98

$$-\frac{2a^2}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(3/2),x)
```

```
[Out] -2*a**2/(d**(3/2)*sqrt(x)) + 4*a*b*x**(3/2)/(3*d**(3/2)) + 2*b**2
*x**(7/2)/(7*d**(3/2))
```

GIAC/XCAS [A] time = 0.261695, size = 69, normalized size = 1.41

$$-\frac{2\left(\frac{21\,a^2}{\sqrt{dx}}-\frac{3\,\sqrt{dx}b^2d^{27}x^3+14\,\sqrt{dx}abd^{27}x}{d^{28}}\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/21*(21*a^2/sqrt(d*x) - (3*sqrt(d*x)*b^2*d^27*x^3 + 14*sqrt(d*x
)*a*b*d^27*x)/d^28)/d
```

3.670 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)
^(5/2))/(5*d^5)
```

Rubi [A] time = 0.0376342, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{2a^2}{3d(dx)^{3/2}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2(dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*d*(d*x)^(3/2)) + (4*a*b*Sqrt[d*x])/d^3 + (2*b^2*(d*x)
^(5/2))/(5*d^5)
```

Rubi in Sympy [A] time = 15.5935, size = 46, normalized size = 0.94

$$-\frac{2a^2}{3d\,(dx)^{\frac{3}{2}}}+\frac{4ab\sqrt{dx}}{d^3}+\frac{2b^2\,(dx)^{\frac{5}{2}}}{5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
```

```
[Out] -2*a**2/(3*d*(d*x)**(3/2)) + 4*a*b*sqrt(d*x)/d**3 + 2*b**2*(d*x)*
*(5/2)/(5*d**5)
```

Mathematica [A] time = 0.0198114, size = 33, normalized size = 0.67

$$\frac{x\left(-10a^2+60abx^2+6b^2x^4\right)}{15(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(5/2),x]
```

```
[Out] (x*(-10*a^2 + 60*a*b*x^2 + 6*b^2*x^4))/(15*(d*x)^(5/2))
```

Maple [A] time = 0.01, size = 30, normalized size = 0.6

$$-\frac{\left(-6\,b^2x^4-60\,abx^2+10\,a^2\right)x}{15}\,(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(5/2),x)
```

```
[Out] -2/15*(-3*b^2*x^4-30*a*b*x^2+5*a^2)*x/(d*x)^(5/2)
```

Maxima [A] time = 0.695361, size = 58, normalized size = 1.18

$$-\frac{2\left(\frac{5\,a^2}{(dx)^{\frac{3}{2}}}-\frac{3\left((dx)^{\frac{5}{2}}b^2+10\,\sqrt{dx}abd^2\right)}{d^4}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/15*(5*a^2/(d*x)^(3/2) - 3*((d*x)^(5/2)*b^2 + 10*sqrt(d*x)*a*b*
d^2)/d^4)/d
```

Fricas [A] time = 0.256608, size = 46, normalized size = 0.94

$$\frac{2\left(3\,b^2x^4+30\,abx^2-5\,a^2\right)}{15\,\sqrt{dx}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)/(sqrt(d*x)*d^2*x)
```

Sympy [A] time = 2.99007, size = 48, normalized size = 0.98

$$-\frac{2a^2}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{4ab\sqrt{x}}{d^{\frac{5}{2}}}+\frac{2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(5/2),x)
```

```
[Out] -2*a**2/(3*d**(5/2)*x**(3/2)) + 4*a*b*sqrt(x)/d**(5/2) + 2*b**2*x
**(5/2)/(5*d**(5/2))
```

GIAC/XCAS [A] time = 0.261547, size = 72, normalized size = 1.47

$$-\frac{2\left(\frac{5\,a^2d}{\sqrt{dx}x}-\frac{3\left(\sqrt{dx}b^2d^{10}x^2+10\,\sqrt{dx}abd^{10}\right)}{d^{10}}\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/15*(5*a^2*d/(sqrt(d*x)*x) - 3*(sqrt(d*x)*b^2*d^10*x^2 + 10*sqr
t(d*x)*a*b*d^10)/d^10)/d^3
```

3.671 $\int \frac{a^2+2abx^2+b^2x^4}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*
x)^(3/2))/(3*d^5)
```

Rubi [A] time = 0.0374636, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038

$$-\frac{2a^2}{5d(dx)^{5/2}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2(dx)^{3/2}}{3d^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^2)/(5*d*(d*x)^(5/2)) - (4*a*b)/(d^3*Sqrt[d*x]) + (2*b^2*(d*
x)^(3/2))/(3*d^5)
```

Rubi in Sympy [A] time = 15.609, size = 46, normalized size = 0.94

$$-\frac{2a^2}{5d\,(dx)^{\frac{5}{2}}}-\frac{4ab}{d^3\sqrt{dx}}+\frac{2b^2\,(dx)^{\frac{3}{2}}}{3d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
```

```
[Out] -2*a**2/(5*d*(d*x)**(5/2)) - 4*a*b/(d**3*sqrt(d*x)) + 2*b**2*(d*x
)**(3/2)/(3*d**5)
```

Mathematica [A] time = 0.0196514, size = 38, normalized size = 0.78

$$\frac{2\sqrt{dx}\left(-3a^2-30abx^2+5b^2x^4\right)}{15d^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*d^4*x^3)
```

Maple [A] time = 0.01, size = 30, normalized size = 0.6

$$-\frac{\left(-10\,b^{2}x^{4}+60\,abx^{2}+6\,a^{2}\right)x}{15}\,(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(7/2),x)
```

```
[Out] -2/15*(-5*b^2*x^4+30*a*b*x^2+3*a^2)*x/(d*x)^(7/2)
```

Maxima [A] time = 0.685156, size = 63, normalized size = 1.29

$$\frac{2\left(\frac{5\,(dx)^{\frac{3}{2}}b^{2}}{d^{4}}-\frac{3\left(10\,abd^{2}x^{2}+a^{2}d^{2}\right)}{(dx)^{\frac{5}{2}}d^{2}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(d*x)^(3/2)*b^2/d^4 - 3*(10*a*b*d^2*x^2 + a^2*d^2)/((d*x)
^(5/2)*d^2))/d
```

Fricas [A] time = 0.256631, size = 46, normalized size = 0.94

$$\frac{2\left(5\,b^{2}x^{4}-30\,abx^{2}-3\,a^{2}\right)}{15\,\sqrt{dx}d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)/(sqrt(d*x)*d^3*x^2)
```

Sympy [A] time = 6.84073, size = 48, normalized size = 0.98

$$-\frac{2a^2}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{4ab}{d^{\frac{7}{2}}\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(7/2),x)
```

```
[Out] -2*a**2/(5*d**(7/2)*x**(5/2)) - 4*a*b/(d**(7/2)*sqrt(x)) + 2*b**2
*x**(3/2)/(3*d**(7/2))
```

GIAC/XCAS [A] time = 0.262285, size = 65, normalized size = 1.33

$$\frac{2\left(5\sqrt{dx}b^2x-\frac{3\left(10\,abd^3x^2+a^2d^3\right)}{\sqrt{dx}d^2x^2}\right)}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*sqrt(d*x)*b^2*x - 3*(10*a*b*d^3*x^2 + a^2*d^3)/(sqrt(d*x)
*d^2*x^2))/d^4
```

3.672 $\int (dx)^{5/2}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*
a^2*b^2*(d*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) +
 (2*b^4*(d*x)^(23/2))/(23*d^9)
```

Rubi [A] time = 0.115137, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2a^4(dx)^{7/2}}{7d}+\frac{8a^3b(dx)^{11/2}}{11d^3}+\frac{4a^2b^2(dx)^{15/2}}{5d^5}+\frac{8ab^3(dx)^{19/2}}{19d^7}+\frac{2b^4(dx)^{23/2}}{23d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(7/2))/(7*d) + (8*a^3*b*(d*x)^(11/2))/(11*d^3) + (4*
a^2*b^2*(d*x)^(15/2))/(5*d^5) + (8*a*b^3*(d*x)^(19/2))/(19*d^7) +
 (2*b^4*(d*x)^(23/2))/(23*d^9)
```

Rubi in Sympy [A] time = 25.3343, size = 88, normalized size = 0.97

$$\frac{2a^4\left(dx\right)^{\frac{7}{2}}}{7d}+\frac{8a^3b\left(dx\right)^{\frac{11}{2}}}{11d^3}+\frac{4a^2b^2\left(dx\right)^{\frac{15}{2}}}{5d^5}+\frac{8ab^3\left(dx\right)^{\frac{19}{2}}}{19d^7}+\frac{2b^4\left(dx\right)^{\frac{23}{2}}}{23d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*(d*x)**(7/2)/(7*d) + 8*a**3*b*(d*x)**(11/2)/(11*d**3) + 4*
a**2*b**2*(d*x)**(15/2)/(5*d**5) + 8*a*b**3*(d*x)**(19/2)/(19*d**
7) + 2*b**4*(d*x)**(23/2)/(23*d**9)
```

Mathematica [A] time = 0.0274376, size = 55, normalized size = 0.6

$$\frac{2x(dx)^{5/2}\left(24035a^4+61180a^3bx^2+67298a^2b^2x^4+35420ab^3x^6+7315b^4x^8\right)}{168245}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(24035*a^4 + 61180*a^3*b*x^2 + 67298*a^2*b^2*x^4
 + 35420*a*b^3*x^6 + 7315*b^4*x^8))/168245
```

Maple [A] time = 0.011, size = 52, normalized size = 0.6

$$\frac{2x\left(7315\,b^4x^8+35420\,ab^3x^6+67298\,a^2b^2x^4+61180\,a^3bx^2+24035\,a^4\right)}{168245}(dx)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/168245*x*(7315*b^4*x^8+35420*a*b^3*x^6+67298*a^2*b^2*x^4+61180*
a^3*b*x^2+24035*a^4)*(d*x)^(5/2)
```

Maxima [A] time = 0.682949, size = 99, normalized size = 1.09

$$\frac{2\left(7315\,(dx)^{\frac{23}{2}}b^4+35420\,(dx)^{\frac{19}{2}}ab^3d^2+67298\,(dx)^{\frac{15}{2}}a^2b^2d^4+61180\,(dx)^{\frac{11}{2}}a^3bd^6+24035\,(dx)^{\frac{7}{2}}a^4d^8\right)}{168245\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/168245*(7315*(d*x)^(23/2)*b^4 + 35420*(d*x)^(19/2)*a*b^3*d^2 +
67298*(d*x)^(15/2)*a^2*b^2*d^4 + 61180*(d*x)^(11/2)*a^3*b*d^6 + 2
4035*(d*x)^(7/2)*a^4*d^8)/d^9
```

Fricas [A] time = 0.257763, size = 92, normalized size = 1.01

$$\frac{2}{168245}\left(7315\,b^4d^2x^{11}+35420\,ab^3d^2x^9+67298\,a^2b^2d^2x^7+61180\,a^3bd^2x^5+24035\,a^4d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*b^4*d^2*x^11 + 35420*a*b^3*d^2*x^9 + 67298*a^2*b^2
*d^2*x^7 + 61180*a^3*b*d^2*x^5 + 24035*a^4*d^2*x^3)*sqrt(d*x)
```

Sympy [A] time = 18.714, size = 90, normalized size = 0.99

$$\frac{2a^4d^{\frac{5}{2}}x^{\frac{7}{2}}}{7}+\frac{8a^3bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11}+\frac{4a^2b^2d^{\frac{5}{2}}x^{\frac{15}{2}}}{5}+\frac{8ab^3d^{\frac{5}{2}}x^{\frac{19}{2}}}{19}+\frac{2b^4d^{\frac{5}{2}}x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(5/2)*x**(7/2)/7 + 8*a**3*b*d**(5/2)*x**(11/2)/11 + 4*a
**2*b**2*d**(5/2)*x**(15/2)/5 + 8*a*b**3*d**(5/2)*x**(19/2)/19 +
2*b**4*d**(5/2)*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.261846, size = 116, normalized size = 1.27

$$\frac{2}{23}\sqrt{dx}b^4d^2x^{11}+\frac{8}{19}\sqrt{dx}ab^3d^2x^9+\frac{4}{5}\sqrt{dx}a^2b^2d^2x^7+\frac{8}{11}\sqrt{dx}a^3bd^2x^5+\frac{2}{7}\sqrt{dx}a^4d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/23*sqrt(d*x)*b^4*d^2*x^11 + 8/19*sqrt(d*x)*a*b^3*d^2*x^9 + 4/5*
sqrt(d*x)*a^2*b^2*d^2*x^7 + 8/11*sqrt(d*x)*a^3*b*d^2*x^5 + 2/7*sq
rt(d*x)*a^4*d^2*x^3
```

3.673 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a
^2*b^2*(d*x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) +
 (2*b^4*(d*x)^(21/2))/(21*d^9)
```

Rubi [A] time = 0.103515, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2a^4(dx)^{5/2}}{5d}+\frac{8a^3b(dx)^{9/2}}{9d^3}+\frac{12a^2b^2(dx)^{13/2}}{13d^5}+\frac{8ab^3(dx)^{17/2}}{17d^7}+\frac{2b^4(dx)^{21/2}}{21d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(5/2))/(5*d) + (8*a^3*b*(d*x)^(9/2))/(9*d^3) + (12*a
^2*b^2*(d*x)^(13/2))/(13*d^5) + (8*a*b^3*(d*x)^(17/2))/(17*d^7) +
 (2*b^4*(d*x)^(21/2))/(21*d^9)
```

Rubi in Sympy [A] time = 25.6473, size = 88, normalized size = 0.97

$$\frac{2a^4\,(dx)^{\frac{5}{2}}}{5d}+\frac{8a^3b\,(dx)^{\frac{9}{2}}}{9d^3}+\frac{12a^2b^2\,(dx)^{\frac{13}{2}}}{13d^5}+\frac{8ab^3\,(dx)^{\frac{17}{2}}}{17d^7}+\frac{2b^4\,(dx)^{\frac{21}{2}}}{21d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*(d*x)**(5/2)/(5*d) + 8*a**3*b*(d*x)**(9/2)/(9*d**3) + 12*a
**2*b**2*(d*x)**(13/2)/(13*d**5) + 8*a*b**3*(d*x)**(17/2)/(17*d**
7) + 2*b**4*(d*x)**(21/2)/(21*d**9)
```

Mathematica [A] time = 0.0236864, size = 55, normalized size = 0.6

$$\frac{2x(dx)^{3/2}\left(13923a^4+30940a^3bx^2+32130a^2b^2x^4+16380ab^3x^6+3315b^4x^8\right)}{69615}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(13923*a^4 + 30940*a^3*b*x^2 + 32130*a^2*b^2*x^4
 + 16380*a*b^3*x^6 + 3315*b^4*x^8))/69615
```

Maple [A] time = 0.011, size = 52, normalized size = 0.6

$$\frac{2\,x\left(3315\,b^4x^8+16380\,ab^3x^6+32130\,a^2b^2x^4+30940\,a^3bx^2+13923\,a^4\right)}{69615}(dx)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/69615*x*(3315*b^4*x^8+16380*a*b^3*x^6+32130*a^2*b^2*x^4+30940*a
^3*b*x^2+13923*a^4)*(d*x)^(3/2)
```

Maxima [A] time = 0.681575, size = 99, normalized size = 1.09

$$\frac{2\left(3315\,(dx)^{\frac{21}{2}}\,b^4+16380\,(dx)^{\frac{17}{2}}\,ab^3d^2+32130\,(dx)^{\frac{13}{2}}\,a^2b^2d^4+30940\,(dx)^{\frac{9}{2}}\,a^3bd^6+13923\,(dx)^{\frac{5}{2}}\,a^4d^8\right)}{69615\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/69615*(3315*(d*x)^(21/2)*b^4 + 16380*(d*x)^(17/2)*a*b^3*d^2 + 3
2130*(d*x)^(13/2)*a^2*b^2*d^4 + 30940*(d*x)^(9/2)*a^3*b*d^6 + 139
23*(d*x)^(5/2)*a^4*d^8)/d^9
```

Fricas [A] time = 0.259796, size = 78, normalized size = 0.86

$$\frac{2}{69615}\left(3315\,b^4dx^{10}+16380\,ab^3dx^8+32130\,a^2b^2dx^6+30940\,a^3bdx^4+13923\,a^4dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*b^4*d*x^10 + 16380*a*b^3*d*x^8 + 32130*a^2*b^2*d*x^
6 + 30940*a^3*b*d*x^4 + 13923*a^4*d*x^2)*sqrt(d*x)
```

Sympy [A] time = 8.73631, size = 90, normalized size = 0.99

$$\frac{2a^4d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{8a^3bd^{\frac{3}{2}}x^{\frac{9}{2}}}{9}+\frac{12a^2b^2d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}+\frac{8ab^3d^{\frac{3}{2}}x^{\frac{17}{2}}}{17}+\frac{2b^4d^{\frac{3}{2}}x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 2*a**4*d**(3/2)*x**(5/2)/5 + 8*a**3*b*d**(3/2)*x**(9/2)/9 + 12*a*
*2*b**2*d**(3/2)*x**(13/2)/13 + 8*a*b**3*d**(3/2)*x**(17/2)/17 +
2*b**4*d**(3/2)*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.266414, size = 103, normalized size = 1.13

$$\frac{2}{21}\sqrt{dx}b^4dx^{10}+\frac{8}{17}\sqrt{dx}ab^3dx^8+\frac{12}{13}\sqrt{dx}a^2b^2dx^6+\frac{8}{9}\sqrt{dx}a^3bdx^4+\frac{2}{5}\sqrt{dx}a^4dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*sqrt(d*x)*b^4*d*x^10 + 8/17*sqrt(d*x)*a*b^3*d*x^8 + 12/13*sq
rt(d*x)*a^2*b^2*d*x^6 + 8/9*sqrt(d*x)*a^3*b*d*x^4 + 2/5*sqrt(d*x)
*a^4*d*x^2
```

3.674 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^2\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a
^2*b^2*(d*x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) +
 (2*b^4*(d*x)^(19/2))/(19*d^9)
```

Rubi [A] time = 0.103946, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2a^4(dx)^{3/2}}{3d}+\frac{8a^3b(dx)^{7/2}}{7d^3}+\frac{12a^2b^2(dx)^{11/2}}{11d^5}+\frac{8ab^3(dx)^{15/2}}{15d^7}+\frac{2b^4(dx)^{19/2}}{19d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*a^4*(d*x)^(3/2))/(3*d) + (8*a^3*b*(d*x)^(7/2))/(7*d^3) + (12*a
^2*b^2*(d*x)^(11/2))/(11*d^5) + (8*a*b^3*(d*x)^(15/2))/(15*d^7) +
 (2*b^4*(d*x)^(19/2))/(19*d^9)
```

Rubi in Sympy [A] time = 25.6687, size = 88, normalized size = 0.97

$$\frac{2a^4\left(dx\right)^{\frac{3}{2}}}{3d}+\frac{8a^3b\left(dx\right)^{\frac{7}{2}}}{7d^3}+\frac{12a^2b^2\left(dx\right)^{\frac{11}{2}}}{11d^5}+\frac{8ab^3\left(dx\right)^{\frac{15}{2}}}{15d^7}+\frac{2b^4\left(dx\right)^{\frac{19}{2}}}{19d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2*(d*x)**(1/2),x)
```

```
[Out] 2*a**4*(d*x)**(3/2)/(3*d) + 8*a**3*b*(d*x)**(7/2)/(7*d**3) + 12*a
**2*b**2*(d*x)**(11/2)/(11*d**5) + 8*a*b**3*(d*x)**(15/2)/(15*d**
7) + 2*b**4*(d*x)**(19/2)/(19*d**9)
```

Mathematica [A] time = 0.0171594, size = 55, normalized size = 0.6

$$\frac{2x\sqrt{dx}\left(7315a^4+12540a^3bx^2+11970a^2b^2x^4+5852ab^3x^6+1155b^4x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (2*x*Sqrt[d*x]*(7315*a^4 + 12540*a^3*b*x^2 + 11970*a^2*b^2*x^4 +
5852*a*b^3*x^6 + 1155*b^4*x^8))/21945
```

Maple [A] time = 0.01, size = 52, normalized size = 0.6

$$\frac{2x\left(1155\,b^{4}x^{8}+5852\,ab^{3}x^{6}+11970\,a^{2}b^{2}x^{4}+12540\,a^{3}bx^{2}+7315\,a^{4}\right)}{21945}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2*(d*x)^(1/2),x)
```

```
[Out] 2/21945*x*(1155*b^4*x^8+5852*a*b^3*x^6+11970*a^2*b^2*x^4+12540*a^
3*b*x^2+7315*a^4)*(d*x)^(1/2)
```

Maxima [A] time = 0.675555, size = 99, normalized size = 1.09

$$\frac{2\left(1155\,(dx)^{\frac{19}{2}}\,b^{4}+5852\,(dx)^{\frac{15}{2}}\,ab^{3}d^{2}+11970\,(dx)^{\frac{11}{2}}\,a^{2}b^{2}d^{4}+12540\,(dx)^{\frac{7}{2}}\,a^{3}bd^{6}+7315\,(dx)^{\frac{3}{2}}\,a^{4}d^{8}\right)}{21945\,d^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/21945*(1155*(d*x)^(19/2)*b^4 + 5852*(d*x)^(15/2)*a*b^3*d^2 + 11
970*(d*x)^(11/2)*a^2*b^2*d^4 + 12540*(d*x)^(7/2)*a^3*b*d^6 + 7315
*(d*x)^(3/2)*a^4*d^8)/d^9
```

Fricas [A] time = 0.256566, size = 69, normalized size = 0.76

$$\frac{2}{21945}\left(1155\,b^{4}x^{9}+5852\,ab^{3}x^{7}+11970\,a^{2}b^{2}x^{5}+12540\,a^{3}bx^{3}+7315\,a^{4}x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*b^4*x^9 + 5852*a*b^3*x^7 + 11970*a^2*b^2*x^5 + 1254
0*a^3*b*x^3 + 7315*a^4*x)*sqrt(d*x)
```

Sympy [A] time = 4.08404, size = 90, normalized size = 0.99

$$\frac{2a^4\sqrt{d}x^{\frac{3}{2}}}{3}+\frac{8a^3b\sqrt{d}x^{\frac{7}{2}}}{7}+\frac{12a^2b^2\sqrt{d}x^{\frac{11}{2}}}{11}+\frac{8ab^3\sqrt{d}x^{\frac{15}{2}}}{15}+\frac{2b^4\sqrt{d}x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2*(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(d)*x**(3/2)/3 + 8*a**3*b*sqrt(d)*x**(7/2)/7 + 12*a**2
*b**2*sqrt(d)*x**(11/2)/11 + 8*a*b**3*sqrt(d)*x**(15/2)/15 + 2*b*
*4*sqrt(d)*x**(19/2)/19
```

GIAC/XCAS [A] time = 0.264035, size = 107, normalized size = 1.18

$$\frac{2\left(1155\sqrt{dx}b^4dx^9+5852\sqrt{dx}ab^3dx^7+11970\sqrt{dx}a^2b^2dx^5+12540\sqrt{dx}a^3bdx^3+7315\sqrt{dx}a^4dx\right)}{21945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/21945*(1155*sqrt(d*x)*b^4*d*x^9 + 5852*sqrt(d*x)*a*b^3*d*x^7 +
11970*sqrt(d*x)*a^2*b^2*d*x^5 + 12540*sqrt(d*x)*a^3*b*d*x^3 + 731
5*sqrt(d*x)*a^4*d*x)/d
```

3.675 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{\sqrt{dx}}\,dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*
(d*x)^(9/2))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(
d*x)^(17/2))/(17*d^9)
```

Rubi [A] time = 0.107404, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b(dx)^{5/2}}{5d^3}+\frac{4a^2b^2(dx)^{9/2}}{3d^5}+\frac{8ab^3(dx)^{13/2}}{13d^7}+\frac{2b^4(dx)^{17/2}}{17d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*a^4*Sqrt[d*x])/d + (8*a^3*b*(d*x)^(5/2))/(5*d^3) + (4*a^2*b^2*
(d*x)^(9/2))/(3*d^5) + (8*a*b^3*(d*x)^(13/2))/(13*d^7) + (2*b^4*(
d*x)^(17/2))/(17*d^9)
```

Rubi in Sympy [A] time = 25.559, size = 87, normalized size = 0.98

$$\frac{2a^4\sqrt{dx}}{d}+\frac{8a^3b\,(dx)^{\frac{5}{2}}}{5d^3}+\frac{4a^2b^2\,(dx)^{\frac{9}{2}}}{3d^5}+\frac{8ab^3\,(dx)^{\frac{13}{2}}}{13d^7}+\frac{2b^4\,(dx)^{\frac{17}{2}}}{17d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(d*x)/d + 8*a**3*b*(d*x)**(5/2)/(5*d**3) + 4*a**2*b**2
*(d*x)**(9/2)/(3*d**5) + 8*a*b**3*(d*x)**(13/2)/(13*d**7) + 2*b**
4*(d*x)**(17/2)/(17*d**9)
```

Mathematica [A] time = 0.0193471, size = 55, normalized size = 0.62

$$\frac{2\left(3315a^4x+2652a^3bx^3+2210a^2b^2x^5+1020ab^3x^7+195b^4x^9\right)}{3315\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/Sqrt[d*x],x]
```

```
[Out] (2*(3315*a^4*x + 2652*a^3*b*x^3 + 2210*a^2*b^2*x^5 + 1020*a*b^3*x
^7 + 195*b^4*x^9))/(3315*Sqrt[d*x])
```

Maple [A] time = 0.01, size = 52, normalized size = 0.6

$$\frac{\left(390\,b^4x^8+2040\,ab^3x^6+4420\,a^2b^2x^4+5304\,a^3bx^2+6630\,a^4\right)x}{3315}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 2/3315*(195*b^4*x^8+1020*a*b^3*x^6+2210*a^2*b^2*x^4+2652*a^3*b*x^
2+3315*a^4)*x/(d*x)^(1/2)
```

Maxima [A] time = 0.690707, size = 122, normalized size = 1.37

$$\frac{2\left(9945\sqrt{dx}a^4+\frac{585\,(dx)^{\frac{17}{2}}b^4}{d^8}+\frac{3060\,(dx)^{\frac{13}{2}}ab^3}{d^6}+\frac{4420\,(dx)^{\frac{9}{2}}a^2b^2}{d^4}+442\left(\frac{5\,(dx)^{\frac{9}{2}}b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}ab}{d^2}\right)a^2\right)}{9945\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/9945*(9945*sqrt(d*x)*a^4 + 585*(d*x)^(17/2)*b^4/d^8 + 3060*(d*x
)^(13/2)*a*b^3/d^6 + 4420*(d*x)^(9/2)*a^2*b^2/d^4 + 442*(5*(d*x)^
(9/2)*b^2/d^4 + 18*(d*x)^(5/2)*a*b/d^2)*a^2)/d
```

Fricas [A] time = 0.2549, size = 72, normalized size = 0.81

$$\frac{2\,\left(195\,b^{4}x^{8} + 1020\,ab^{3}x^{6} + 2210\,a^{2}b^{2}x^{4} + 2652\,a^{3}bx^{2} + 3315\,a^{4}\right)\sqrt{dx}}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^4*x^8 + 1020*a*b^3*x^6 + 2210*a^2*b^2*x^4 + 2652*a^
3*b*x^2 + 3315*a^4)*sqrt(d*x)/d
```

Sympy [A] time = 4.52077, size = 88, normalized size = 0.99

$$\frac{2a^{4}\sqrt{x}}{\sqrt{d}} + \frac{8a^{3}bx^{\frac{5}{2}}}{5\sqrt{d}} + \frac{4a^{2}b^{2}x^{\frac{9}{2}}}{3\sqrt{d}} + \frac{8ab^{3}x^{\frac{13}{2}}}{13\sqrt{d}} + \frac{2b^{4}x^{\frac{17}{2}}}{17\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] 2*a**4*sqrt(x)/sqrt(d) + 8*a**3*b*x**(5/2)/(5*sqrt(d)) + 4*a**2*b
**2*x**(9/2)/(3*sqrt(d)) + 8*a*b**3*x**(13/2)/(13*sqrt(d)) + 2*b*
*4*x**(17/2)/(17*sqrt(d))
```

GIAC/XCAS [A] time = 0.263066, size = 99, normalized size = 1.11

$$\frac{2\left(195\,\sqrt{dx}b^{4}x^{8} + 1020\,\sqrt{dx}ab^{3}x^{6} + 2210\,\sqrt{dx}a^{2}b^{2}x^{4} + 2652\,\sqrt{dx}a^{3}bx^{2} + 3315\,\sqrt{dx}a^{4}\right)}{3315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/3315*(195*sqrt(d*x)*b^4*x^8 + 1020*sqrt(d*x)*a*b^3*x^6 + 2210*s
qrt(d*x)*a^2*b^2*x^4 + 2652*sqrt(d*x)*a^3*b*x^2 + 3315*sqrt(d*x)*
a^4)/d
```

3.676 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*
b^2*(d*x)^(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b
^4*(d*x)^(15/2))/(15*d^9)
```

Rubi [A] time = 0.104692, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b(dx)^{3/2}}{3d^3}+\frac{12a^2b^2(dx)^{7/2}}{7d^5}+\frac{8ab^3(dx)^{11/2}}{11d^7}+\frac{2b^4(dx)^{15/2}}{15d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(d*Sqrt[d*x]) + (8*a^3*b*(d*x)^(3/2))/(3*d^3) + (12*a^2*
b^2*(d*x)^(7/2))/(7*d^5) + (8*a*b^3*(d*x)^(11/2))/(11*d^7) + (2*b
^4*(d*x)^(15/2))/(15*d^9)
```

Rubi in Sympy [A] time = 25.8596, size = 87, normalized size = 0.98

$$-\frac{2a^4}{d\sqrt{dx}}+\frac{8a^3b\,(dx)^{\frac{3}{2}}}{3d^3}+\frac{12a^2b^2\,(dx)^{\frac{7}{2}}}{7d^5}+\frac{8ab^3\,(dx)^{\frac{11}{2}}}{11d^7}+\frac{2b^4\,(dx)^{\frac{15}{2}}}{15d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
```

```
[Out] -2*a**4/(d*sqrt(d*x)) + 8*a**3*b*(d*x)**(3/2)/(3*d**3) + 12*a**2*
b**2*(d*x)**(7/2)/(7*d**5) + 8*a*b**3*(d*x)**(11/2)/(11*d**7) + 2
*b**4*(d*x)**(15/2)/(15*d**9)
```

Mathematica [A] time = 0.0215086, size = 55, normalized size = 0.62

$$\frac{2x\left(-1155a^4+1540a^3bx^2+990a^2b^2x^4+420ab^3x^6+77b^4x^8\right)}{1155(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-1155*a^4 + 1540*a^3*b*x^2 + 990*a^2*b^2*x^4 + 420*a*b^3*x^
6 + 77*b^4*x^8))/(1155*(d*x)^(3/2))
```

Maple [A] time = 0.01, size = 52, normalized size = 0.6

$$-\frac{\left(-154\,b^4x^8-840\,ab^3x^6-1980\,a^2b^2x^4-3080\,a^3bx^2+2310\,a^4\right)x}{1155}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(3/2),x)
```

```
[Out] -2/1155*(-77*b^4*x^8-420*a*b^3*x^6-990*a^2*b^2*x^4-1540*a^3*b*x^2
+1155*a^4)*x/(d*x)^(3/2)
```

Maxima [A] time = 0.693294, size = 103, normalized size = 1.16

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,(dx)^{\frac{15}{2}}b^4+420\,(dx)^{\frac{11}{2}}ab^3d^2+990\,(dx)^{\frac{7}{2}}a^2b^2d^4+1540\,(dx)^{\frac{3}{2}}a^3bd^6}{d^8}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*(d*x)^(15/2)*b^4 + 420*(d*x)^(1
1/2)*a*b^3*d^2 + 990*(d*x)^(7/2)*a^2*b^2*d^4 + 1540*(d*x)^(3/2)*a
^3*b*d^6)/d^8)/d
```

Fricas [A] time = 0.260165, size = 72, normalized size = 0.81

$$\frac{2\left(77\,b^4x^8+420\,ab^3x^6+990\,a^2b^2x^4+1540\,a^3bx^2-1155\,a^4\right)}{1155\,\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^4*x^8 + 420*a*b^3*x^6 + 990*a^2*b^2*x^4 + 1540*a^3*b
*x^2 - 1155*a^4)/(sqrt(d*x)*d)
```

Sympy [A] time = 4.82865, size = 88, normalized size = 0.99

$$-\frac{2a^4}{d^{\frac{3}{2}}\sqrt{x}}+\frac{8a^3bx^{\frac{3}{2}}}{3d^{\frac{3}{2}}}+\frac{12a^2b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{8ab^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2b^4x^{\frac{15}{2}}}{15d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(3/2),x)
```

```
[Out] -2*a**4/(d**(3/2)*sqrt(x)) + 8*a**3*b*x**(3/2)/(3*d**(3/2)) + 12*
a**2*b**2*x**(7/2)/(7*d**(3/2)) + 8*a*b**3*x**(11/2)/(11*d**(3/2)
) + 2*b**4*x**(15/2)/(15*d**(3/2))
```

GIAC/XCAS [A] time = 0.264769, size = 120, normalized size = 1.35

$$-\frac{2\left(\frac{1155\,a^4}{\sqrt{dx}}-\frac{77\,\sqrt{dx}b^4d^{119}x^7+420\,\sqrt{dx}ab^3d^{119}x^5+990\,\sqrt{dx}a^2b^2d^{119}x^3+1540\,\sqrt{dx}a^3bd^{119}x}{d^{120}}\right)}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/1155*(1155*a^4/sqrt(d*x) - (77*sqrt(d*x)*b^4*d^119*x^7 + 420*s
qrt(d*x)*a*b^3*d^119*x^5 + 990*sqrt(d*x)*a^2*b^2*d^119*x^3 + 1540
*sqrt(d*x)*a^3*b*d^119*x)/d^120)/d
```

3.677 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^2}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^
2*(d*x)^(5/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(
d*x)^(13/2))/(13*d^9)
```

Rubi [A] time = 0.104559, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{2a^4}{3d(dx)^{3/2}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2(dx)^{5/2}}{5d^5}+\frac{8ab^3(dx)^{9/2}}{9d^7}+\frac{2b^4(dx)^{13/2}}{13d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*d*(d*x)^(3/2)) + (8*a^3*b*Sqrt[d*x])/d^3 + (12*a^2*b^
2*(d*x)^(5/2))/(5*d^5) + (8*a*b^3*(d*x)^(9/2))/(9*d^7) + (2*b^4*(
d*x)^(13/2))/(13*d^9)
```

Rubi in Sympy [A] time = 25.4228, size = 87, normalized size = 0.98

$$-\frac{2a^4}{3d\left(dx\right)^{\frac{3}{2}}}+\frac{8a^3b\sqrt{dx}}{d^3}+\frac{12a^2b^2\left(dx\right)^{\frac{5}{2}}}{5d^5}+\frac{8ab^3\left(dx\right)^{\frac{9}{2}}}{9d^7}+\frac{2b^4\left(dx\right)^{\frac{13}{2}}}{13d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**4/(3*d*(d*x)**(3/2)) + 8*a**3*b*sqrt(d*x)/d**3 + 12*a**2*b*
*2*(d*x)**(5/2)/(5*d**5) + 8*a*b**3*(d*x)**(9/2)/(9*d**7) + 2*b**
4*(d*x)**(13/2)/(13*d**9)
```

Mathematica [A] time = 0.0247744, size = 55, normalized size = 0.62

$$\frac{x\left(-390a^4+4680a^3bx^2+1404a^2b^2x^4+520ab^3x^6+90b^4x^8\right)}{585(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(5/2),x]
```

```
[Out] (x*(-390*a^4 + 4680*a^3*b*x^2 + 1404*a^2*b^2*x^4 + 520*a*b^3*x^6
+ 90*b^4*x^8))/(585*(d*x)^(5/2))
```

Maple [A] time = 0.01, size = 52, normalized size = 0.6

$$-\frac{\left(-90\,b^4x^8-520\,ab^3x^6-1404\,a^2b^2x^4-4680\,a^3bx^2+390\,a^4\right)x}{585}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(5/2),x)
```

```
[Out] -2/585*(-45*b^4*x^8-260*a*b^3*x^6-702*a^2*b^2*x^4-2340*a^3*b*x^2+
195*a^4)*x/(d*x)^(5/2)
```

Maxima [A] time = 0.690063, size = 103, normalized size = 1.16

$$-\frac{2\left(\frac{195\,a^4}{(dx)^{\frac{3}{2}}}-\frac{45\,(dx)^{\frac{13}{2}}\,b^4+260\,(dx)^{\frac{9}{2}}\,ab^3d^2+702\,(dx)^{\frac{5}{2}}\,a^2b^2d^4+2340\,\sqrt{dx}a^3bd^6}{d^8}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/585*(195*a^4/(d*x)^(3/2) - (45*(d*x)^(13/2)*b^4 + 260*(d*x)^(9
/2)*a*b^3*d^2 + 702*(d*x)^(5/2)*a^2*b^2*d^4 + 2340*sqrt(d*x)*a^3*
b*d^6)/d^8)/d
```

Fricas [A] time = 0.257524, size = 76, normalized size = 0.85

$$\frac{2\left(45\,b^4x^8+260\,ab^3x^6+702\,a^2b^2x^4+2340\,a^3bx^2-195\,a^4\right)}{585\,\sqrt{dx}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*b^4*x^8 + 260*a*b^3*x^6 + 702*a^2*b^2*x^4 + 2340*a^3*b*
x^2 - 195*a^4)/(sqrt(d*x)*d^2*x)
```

Sympy [A] time = 5.7893, size = 88, normalized size = 0.99

$$-\frac{2a^4}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{8a^3b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{12a^2b^2x^{\frac{5}{2}}}{5d^{\frac{5}{2}}}+\frac{8ab^3x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{2b^4x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(5/2),x)
```

```
[Out] -2*a**4/(3*d**(5/2)*x**(3/2)) + 8*a**3*b*sqrt(x)/d**(5/2) + 12*a*
*2*b**2*x**(5/2)/(5*d**(5/2)) + 8*a*b**3*x**(9/2)/(9*d**(5/2)) +
2*b**4*x**(13/2)/(13*d**(5/2))
```

GIAC/XCAS [A] time = 0.264385, size = 124, normalized size = 1.39

$$-\frac{2\left(\frac{195\,a^4d}{\sqrt{dx}x}-\frac{45\,\sqrt{dx}b^4d^{78}x^6+260\,\sqrt{dx}ab^3d^{78}x^4+702\,\sqrt{dx}a^2b^2d^{78}x^2+2340\,\sqrt{dx}a^3bd^{78}}{d^{78}}\right)}{585\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/585*(195*a^4*d/(sqrt(d*x)*x) - (45*sqrt(d*x)*b^4*d^78*x^6 + 26
0*sqrt(d*x)*a*b^3*d^78*x^4 + 702*sqrt(d*x)*a^2*b^2*d^78*x^2 + 234
0*sqrt(d*x)*a^3*b*d^78)/d^78)/d^3
```

3.678 $\int \frac{(a^2+2abx^2+b^2x^4)^2}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b
^2*(d*x)^(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x
)^(11/2))/(11*d^9)
```

Rubi [A] time = 0.102674, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{2a^4}{5d(dx)^{5/2}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2(dx)^{3/2}}{d^5}+\frac{8ab^3(dx)^{7/2}}{7d^7}+\frac{2b^4(dx)^{11/2}}{11d^9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (-2*a^4)/(5*d*(d*x)^(5/2)) - (8*a^3*b)/(d^3*Sqrt[d*x]) + (4*a^2*b
^2*(d*x)^(3/2))/d^5 + (8*a*b^3*(d*x)^(7/2))/(7*d^7) + (2*b^4*(d*x
)^(11/2))/(11*d^9)
```

Rubi in Sympy [A] time = 25.4637, size = 85, normalized size = 0.98

$$-\frac{2a^4}{5d\,(dx)^{\frac{5}{2}}}-\frac{8a^3b}{d^3\sqrt{dx}}+\frac{4a^2b^2\,(dx)^{\frac{3}{2}}}{d^5}+\frac{8ab^3\,(dx)^{\frac{7}{2}}}{7d^7}+\frac{2b^4\,(dx)^{\frac{11}{2}}}{11d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

```
[Out] -2*a**4/(5*d*(d*x)**(5/2)) - 8*a**3*b/(d**3*sqrt(d*x)) + 4*a**2*b
**2*(d*x)**(3/2)/d**5 + 8*a*b**3*(d*x)**(7/2)/(7*d**7) + 2*b**4*(
d*x)**(11/2)/(11*d**9)
```

Mathematica [A] time = 0.029913, size = 60, normalized size = 0.69

$$\frac{2\sqrt{dx}\left(-77a^4-1540a^3bx^2+770a^2b^2x^4+220ab^3x^6+35b^4x^8\right)}{385d^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^2/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-77*a^4 - 1540*a^3*b*x^2 + 770*a^2*b^2*x^4 + 220*a*
b^3*x^6 + 35*b^4*x^8))/(385*d^4*x^3)
```

Maple [A] time = 0.01, size = 52, normalized size = 0.6

$$-\frac{\left(-70\,b^4x^8-440\,ab^3x^6-1540\,a^2b^2x^4+3080\,a^3bx^2+154\,a^4\right)x}{385}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(7/2),x)
```

```
[Out] -2/385*(-35*b^4*x^8-220*a*b^3*x^6-770*a^2*b^2*x^4+1540*a^3*b*x^2+
77*a^4)*x/(d*x)^(7/2)
```

Maxima [A] time = 0.691664, size = 111, normalized size = 1.28

$$-\frac{2\left(\frac{77\,(20\,a^3bd^2x^2+a^4d^2)}{(dx)^{\frac{5}{2}}d^2}-\frac{5\left(7\,(dx)^{\frac{11}{2}}b^4+44\,(dx)^{\frac{7}{2}}ab^3d^2+154\,(dx)^{\frac{3}{2}}a^2b^2d^4\right)}{d^8}\right)}{385\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/385*(77*(20*a^3*b*d^2*x^2 + a^4*d^2)/((d*x)^(5/2)*d^2) - 5*(7*
(d*x)^(11/2)*b^4 + 44*(d*x)^(7/2)*a*b^3*d^2 + 154*(d*x)^(3/2)*a^2
*b^2*d^4)/d^8)/d
```

Fricas [A] time = 0.255457, size = 76, normalized size = 0.87

$$\frac{2\left(35\,b^4x^8+220\,ab^3x^6+770\,a^2b^2x^4-1540\,a^3bx^2-77\,a^4\right)}{385\,\sqrt{dx}d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/385*(35*b^4*x^8 + 220*a*b^3*x^6 + 770*a^2*b^2*x^4 - 1540*a^3*b*
x^2 - 77*a^4)/(sqrt(d*x)*d^3*x^2)
```

Sympy [A] time = 8.69829, size = 87, normalized size = 1.

$$-\frac{2a^4}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{8a^3b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{4a^2b^2x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{8ab^3x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{2b^4x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(7/2),x)
```

```
[Out] -2*a**4/(5*d**(7/2)*x**(5/2)) - 8*a**3*b/(d**(7/2)*sqrt(x)) + 4*a
**2*b**2*x**(3/2)/d**(7/2) + 8*a*b**3*x**(7/2)/(7*d**(7/2)) + 2*b
**4*x**(11/2)/(11*d**(7/2))
```

GIAC/XCAS [A] time = 0.262451, size = 128, normalized size = 1.47

$$-\frac{2\left(\frac{77\left(20\,a^3bd^3x^2+a^4d^3\right)}{\sqrt{dx}d^2x^2}-\frac{5\left(7\,\sqrt{dx}b^4d^{55}x^5+44\,\sqrt{dx}ab^3d^{55}x^3+154\,\sqrt{dx}a^2b^2d^{55}x\right)}{d^{55}}\right)}{385\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/385*(77*(20*a^3*b*d^3*x^2 + a^4*d^3)/(sqrt(d*x)*d^2*x^2) - 5*(
7*sqrt(d*x)*b^4*d^55*x^5 + 44*sqrt(d*x)*a*b^3*d^55*x^3 + 154*sqrt
(d*x)*a^2*b^2*d^55*x)/d^55)/d^4
```

3.679 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=129

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2
*a^4*b^2*(d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) +
 (30*a^2*b^4*(d*x)^(23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d
^11) + (2*b^6*(d*x)^(31/2))/(31*d^13)
```

Rubi [A] time = 0.17419, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2a^6(dx)^{7/2}}{7d}+\frac{12a^5b(dx)^{11/2}}{11d^3}+\frac{2a^4b^2(dx)^{15/2}}{d^5}+\frac{40a^3b^3(dx)^{19/2}}{19d^7}+\frac{30a^2b^4(dx)^{23/2}}{23d^9}+\frac{4ab^5(dx)^{27/2}}{9d^{11}}+\frac{2b^6(dx)^{31/2}}{31d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(7/2))/(7*d) + (12*a^5*b*(d*x)^(11/2))/(11*d^3) + (2
*a^4*b^2*(d*x)^(15/2))/d^5 + (40*a^3*b^3*(d*x)^(19/2))/(19*d^7) +
 (30*a^2*b^4*(d*x)^(23/2))/(23*d^9) + (4*a*b^5*(d*x)^(27/2))/(9*d
^11) + (2*b^6*(d*x)^(31/2))/(31*d^13)
```

Rubi in Sympy [A] time = 35.9814, size = 128, normalized size = 0.99

$$\frac{2a^6\,(dx)^{\frac{7}{2}}}{7d}+\frac{12a^5b\,(dx)^{\frac{11}{2}}}{11d^3}+\frac{2a^4b^2\,(dx)^{\frac{15}{2}}}{d^5}+\frac{40a^3b^3\,(dx)^{\frac{19}{2}}}{19d^7}+\frac{30a^2b^4\,(dx)^{\frac{23}{2}}}{23d^9}+\frac{4ab^5\,(dx)^{\frac{27}{2}}}{9d^{11}}+\frac{2b^6\,(dx)^{\frac{31}{2}}}{31d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*(d*x)**(7/2)/(7*d) + 12*a**5*b*(d*x)**(11/2)/(11*d**3) + 2
*a**4*b**2*(d*x)**(15/2)/d**5 + 40*a**3*b**3*(d*x)**(19/2)/(19*d*
*7) + 30*a**2*b**4*(d*x)**(23/2)/(23*d**9) + 4*a*b**5*(d*x)**(27/
2)/(9*d**11) + 2*b**6*(d*x)**(31/2)/(31*d**13)
```

Mathematica [A] time = 0.034641, size = 77, normalized size = 0.6

$$\frac{2x(dx)^{5/2}\left(1341153a^6+5120766a^5bx^2+9388071a^4b^2x^4+9882180a^3b^3x^6+6122655a^2b^4x^8+2086238ab^5x^{10}+302841b^6x^{1}\right.}{9388071}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(5/2)*(1341153*a^6 + 5120766*a^5*b*x^2 + 9388071*a^4*b
^2*x^4 + 9882180*a^3*b^3*x^6 + 6122655*a^2*b^4*x^8 + 2086238*a*b^
5*x^10 + 302841*b^6*x^12))/9388071
```

Maple [A] time = 0.011, size = 74, normalized size = 0.6

$$\frac{2\,x\left(302841\,b^6x^{12}+2086238\,ab^5x^{10}+6122655\,a^2b^4x^8+9882180\,a^3b^3x^6+9388071\,a^4b^2x^4+5120766\,a^5bx^2+1341153\,a^6\right)}{9388071}($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/9388071*x*(302841*b^6*x^12+2086238*a*b^5*x^10+6122655*a^2*b^4*x
^8+9882180*a^3*b^3*x^6+9388071*a^4*b^2*x^4+5120766*a^5*b*x^2+1341
153*a^6)*(d*x)^(5/2)
```

Maxima [A] time = 0.688189, size = 142, normalized size = 1.1

$$\frac{2\left(302841\,(dx)^{\frac{31}{2}}\,b^6+2086238\,(dx)^{\frac{27}{2}}\,ab^5d^2+6122655\,(dx)^{\frac{23}{2}}\,a^2b^4d^4+9882180\,(dx)^{\frac{19}{2}}\,a^3b^3d^6+9388071\,(dx)^{\frac{15}{2}}\,a^4b^2d^8+5\right.}{9388071\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9388071*(302841*(d*x)^(31/2)*b^6 + 2086238*(d*x)^(27/2)*a*b^5*d
^2 + 6122655*(d*x)^(23/2)*a^2*b^4*d^4 + 9882180*(d*x)^(19/2)*a^3*
b^3*d^6 + 9388071*(d*x)^(15/2)*a^4*b^2*d^8 + 5120766*(d*x)^(11/2)
*a^5*b*d^10 + 1341153*(d*x)^(7/2)*a^6*d^12)/d^13
```

Fricas [A] time = 0.256801, size = 130, normalized size = 1.01

$$\frac{2}{9388071}\left(302841\,b^6d^2x^{15} + 2086238\,ab^5d^2x^{13} + 6122655\,a^2b^4d^2x^{11} + 9882180\,a^3b^3d^2x^9 + 9388071\,a^4b^2d^2x^7 + 5120766\,a^5b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/9388071*(302841*b^6*d^2*x^15 + 2086238*a*b^5*d^2*x^13 + 6122655
*a^2*b^4*d^2*x^11 + 9882180*a^3*b^3*d^2*x^9 + 9388071*a^4*b^2*d^2
*x^7 + 5120766*a^5*b*d^2*x^5 + 1341153*a^6*d^2*x^3)*sqrt(d*x)
```

Sympy [A] time = 33.7286, size = 129, normalized size = 1.

$$\frac{2a^6d^{\frac{5}{2}}x^{\frac{7}{2}}}{7} + \frac{12a^5bd^{\frac{5}{2}}x^{\frac{11}{2}}}{11} + 2a^4b^2d^{\frac{5}{2}}x^{\frac{15}{2}} + \frac{40a^3b^3d^{\frac{5}{2}}x^{\frac{19}{2}}}{19} + \frac{30a^2b^4d^{\frac{5}{2}}x^{\frac{23}{2}}}{23} + \frac{4ab^5d^{\frac{5}{2}}x^{\frac{27}{2}}}{9} + \frac{2b^6d^{\frac{5}{2}}x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(5/2)*x**(7/2)/7 + 12*a**5*b*d**(5/2)*x**(11/2)/11 + 2*
a**4*b**2*d**(5/2)*x**(15/2) + 40*a**3*b**3*d**(5/2)*x**(19/2)/19
 + 30*a**2*b**4*d**(5/2)*x**(23/2)/23 + 4*a*b**5*d**(5/2)*x**(27/
2)/9 + 2*b**6*d**(5/2)*x**(31/2)/31
```

GIAC/XCAS [A] time = 0.265429, size = 167, normalized size = 1.29

$$\frac{2}{31}\sqrt{dx}b^6d^2x^{15} + \frac{4}{9}\sqrt{dx}ab^5d^2x^{13} + \frac{30}{23}\sqrt{dx}a^2b^4d^2x^{11} + \frac{40}{19}\sqrt{dx}a^3b^3d^2x^9 + 2\sqrt{dx}a^4b^2d^2x^7 + \frac{12}{11}\sqrt{dx}a^5bd^2x^5 + \frac{2}{7}\sqrt{dx}a^6d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/31*sqrt(d*x)*b^6*d^2*x^15 + 4/9*sqrt(d*x)*a*b^5*d^2*x^13 + 30/2
3*sqrt(d*x)*a^2*b^4*d^2*x^11 + 40/19*sqrt(d*x)*a^3*b^3*d^2*x^9 +
2*sqrt(d*x)*a^4*b^2*d^2*x^7 + 12/11*sqrt(d*x)*a^5*b*d^2*x^5 + 2/7
*sqrt(d*x)*a^6*d^2*x^3
```

3.680 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a
^4*b^2*(d*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7
) + (10*a^2*b^4*(d*x)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(
25*d^11) + (2*b^6*(d*x)^(29/2))/(29*d^13)
```

Rubi [A] time = 0.152198, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{2a^6(dx)^{5/2}}{5d}+\frac{4a^5b(dx)^{9/2}}{3d^3}+\frac{30a^4b^2(dx)^{13/2}}{13d^5}+\frac{40a^3b^3(dx)^{17/2}}{17d^7}+\frac{10a^2b^4(dx)^{21/2}}{7d^9}+\frac{12ab^5(dx)^{25/2}}{25d^{11}}+\frac{2b^6(dx)^{29/2}}{29d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(5/2))/(5*d) + (4*a^5*b*(d*x)^(9/2))/(3*d^3) + (30*a
^4*b^2*(d*x)^(13/2))/(13*d^5) + (40*a^3*b^3*(d*x)^(17/2))/(17*d^7
) + (10*a^2*b^4*(d*x)^(21/2))/(7*d^9) + (12*a*b^5*(d*x)^(25/2))/(
25*d^11) + (2*b^6*(d*x)^(29/2))/(29*d^13)
```

Rubi in Sympy [A] time = 36.0061, size = 129, normalized size = 0.98

$$\frac{2a^6\left(dx\right)^{\frac{5}{2}}}{5d}+\frac{4a^5b\left(dx\right)^{\frac{9}{2}}}{3d^3}+\frac{30a^4b^2\left(dx\right)^{\frac{13}{2}}}{13d^5}+\frac{40a^3b^3\left(dx\right)^{\frac{17}{2}}}{17d^7}+\frac{10a^2b^4\left(dx\right)^{\frac{21}{2}}}{7d^9}+\frac{12ab^5\left(dx\right)^{\frac{25}{2}}}{25d^{11}}+\frac{2b^6\left(dx\right)^{\frac{29}{2}}}{29d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*(d*x)**(5/2)/(5*d) + 4*a**5*b*(d*x)**(9/2)/(3*d**3) + 30*a
**4*b**2*(d*x)**(13/2)/(13*d**5) + 40*a**3*b**3*(d*x)**(17/2)/(17
*d**7) + 10*a**2*b**4*(d*x)**(21/2)/(7*d**9) + 12*a*b**5*(d*x)**(
25/2)/(25*d**11) + 2*b**6*(d*x)**(29/2)/(29*d**13)
```

Mathematica [A] time = 0.0297783, size = 77, normalized size = 0.59

$$\frac{2x(dx)^{3/2}\left(672945a^6+2243150a^5bx^2+3882375a^4b^2x^4+3958500a^3b^3x^6+2403375a^2b^4x^8+807534ab^5x^{10}+116025b^6x^{12}\right)}{3364725}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*(d*x)^(3/2)*(672945*a^6 + 2243150*a^5*b*x^2 + 3882375*a^4*b^
2*x^4 + 3958500*a^3*b^3*x^6 + 2403375*a^2*b^4*x^8 + 807534*a*b^5*
x^10 + 116025*b^6*x^12))/3364725
```

Maple [A] time = 0.011, size = 74, normalized size = 0.6

$$\frac{2x\left(116025b^6x^{12}+807534ab^5x^{10}+2403375a^2b^4x^8+3958500a^3b^3x^6+3882375a^4b^2x^4+2243150a^5bx^2+672945a^6\right)}{3364725}(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3364725*x*(116025*b^6*x^12+807534*a*b^5*x^10+2403375*a^2*b^4*x^
8+3958500*a^3*b^3*x^6+3882375*a^4*b^2*x^4+2243150*a^5*b*x^2+67294
5*a^6)*(d*x)^(3/2)
```

Maxima [A] time = 0.705166, size = 142, normalized size = 1.08

$$\frac{2\left(116025\,(dx)^{\frac{29}{2}}\,b^6+807534\,(dx)^{\frac{25}{2}}\,ab^5d^2+2403375\,(dx)^{\frac{21}{2}}\,a^2b^4d^4+3958500\,(dx)^{\frac{17}{2}}\,a^3b^3d^6+3882375\,(dx)^{\frac{13}{2}}\,a^4b^2d^8+22\right.}{3364725\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3364725*(116025*(d*x)^(29/2)*b^6 + 807534*(d*x)^(25/2)*a*b^5*d^
2 + 2403375*(d*x)^(21/2)*a^2*b^4*d^4 + 3958500*(d*x)^(17/2)*a^3*b
^3*d^6 + 3882375*(d*x)^(13/2)*a^4*b^2*d^8 + 2243150*(d*x)^(9/2)*a
^5*b*d^10 + 672945*(d*x)^(5/2)*a^6*d^12)/d^13
```

Fricas [A] time = 0.256682, size = 111, normalized size = 0.85

$$\frac{2}{3364725}\left(116025\,b^{6}dx^{14}+807534\,ab^{5}dx^{12}+2403375\,a^{2}b^{4}dx^{10}+3958500\,a^{3}b^{3}dx^{8}+3882375\,a^{4}b^{2}dx^{6}+2243150\,a^{5}bdx^{4}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*b^6*d*x^14 + 807534*a*b^5*d*x^12 + 2403375*a^2*
b^4*d*x^10 + 3958500*a^3*b^3*d*x^8 + 3882375*a^4*b^2*d*x^6 + 2243
150*a^5*b*d*x^4 + 672945*a^6*d*x^2)*sqrt(d*x)
```

Sympy [A] time = 19.3357, size = 131, normalized size = 1.

$$\frac{2a^{6}d^{\frac{3}{2}}x^{\frac{5}{2}}}{5}+\frac{4a^{5}bd^{\frac{3}{2}}x^{\frac{9}{2}}}{3}+\frac{30a^{4}b^{2}d^{\frac{3}{2}}x^{\frac{13}{2}}}{13}+\frac{40a^{3}b^{3}d^{\frac{3}{2}}x^{\frac{17}{2}}}{17}+\frac{10a^{2}b^{4}d^{\frac{3}{2}}x^{\frac{21}{2}}}{7}+\frac{12ab^{5}d^{\frac{3}{2}}x^{\frac{25}{2}}}{25}+\frac{2b^{6}d^{\frac{3}{2}}x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] 2*a**6*d**(3/2)*x**(5/2)/5 + 4*a**5*b*d**(3/2)*x**(9/2)/3 + 30*a*
*4*b**2*d**(3/2)*x**(13/2)/13 + 40*a**3*b**3*d**(3/2)*x**(17/2)/1
7 + 10*a**2*b**4*d**(3/2)*x**(21/2)/7 + 12*a*b**5*d**(3/2)*x**(25
/2)/25 + 2*b**6*d**(3/2)*x**(29/2)/29
```

GIAC/XCAS [A] time = 0.265365, size = 149, normalized size = 1.14

$$\frac{2}{29}\sqrt{dx}b^{6}dx^{14}+\frac{12}{25}\sqrt{dx}ab^{5}dx^{12}+\frac{10}{7}\sqrt{dx}a^{2}b^{4}dx^{10}+\frac{40}{17}\sqrt{dx}a^{3}b^{3}dx^{8}+\frac{30}{13}\sqrt{dx}a^{4}b^{2}dx^{6}+\frac{4}{3}\sqrt{dx}a^{5}bdx^{4}+\frac{2}{5}\sqrt{dx}a^{6}dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/29*sqrt(d*x)*b^6*d*x^14 + 12/25*sqrt(d*x)*a*b^5*d*x^12 + 10/7*s
qrt(d*x)*a^2*b^4*d*x^10 + 40/17*sqrt(d*x)*a^3*b^3*d*x^8 + 30/13*s
qrt(d*x)*a^4*b^2*d*x^6 + 4/3*sqrt(d*x)*a^5*b*d*x^4 + 2/5*sqrt(d*x
)*a^6*d*x^2
```

3.681 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=131

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*
a^4*b^2*(d*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7)
 + (30*a^2*b^4*(d*x)^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(
23*d^11) + (2*b^6*(d*x)^(27/2))/(27*d^13)
```

Rubi [A] time = 0.151779, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2a^6(dx)^{3/2}}{3d}+\frac{12a^5b(dx)^{7/2}}{7d^3}+\frac{30a^4b^2(dx)^{11/2}}{11d^5}+\frac{8a^3b^3(dx)^{15/2}}{3d^7}+\frac{30a^2b^4(dx)^{19/2}}{19d^9}+\frac{12ab^5(dx)^{23/2}}{23d^{11}}+\frac{2b^6(dx)^{27/2}}{27d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*a^6*(d*x)^(3/2))/(3*d) + (12*a^5*b*(d*x)^(7/2))/(7*d^3) + (30*
a^4*b^2*(d*x)^(11/2))/(11*d^5) + (8*a^3*b^3*(d*x)^(15/2))/(3*d^7)
 + (30*a^2*b^4*(d*x)^(19/2))/(19*d^9) + (12*a*b^5*(d*x)^(23/2))/(
23*d^11) + (2*b^6*(d*x)^(27/2))/(27*d^13)
```

Rubi in Sympy [A] time = 36.0538, size = 129, normalized size = 0.98

$$\frac{2a^6\,(dx)^{\frac{3}{2}}}{3d}+\frac{12a^5b\,(dx)^{\frac{7}{2}}}{7d^3}+\frac{30a^4b^2\,(dx)^{\frac{11}{2}}}{11d^5}+\frac{8a^3b^3\,(dx)^{\frac{15}{2}}}{3d^7}+\frac{30a^2b^4\,(dx)^{\frac{19}{2}}}{19d^9}+\frac{12ab^5\,(dx)^{\frac{23}{2}}}{23d^{11}}+\frac{2b^6\,(dx)^{\frac{27}{2}}}{27d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3*(d*x)**(1/2),x)
```

```
[Out] 2*a**6*(d*x)**(3/2)/(3*d) + 12*a**5*b*(d*x)**(7/2)/(7*d**3) + 30*
a**4*b**2*(d*x)**(11/2)/(11*d**5) + 8*a**3*b**3*(d*x)**(15/2)/(3*
d**7) + 30*a**2*b**4*(d*x)**(19/2)/(19*d**9) + 12*a*b**5*(d*x)**(
23/2)/(23*d**11) + 2*b**6*(d*x)**(27/2)/(27*d**13)
```

Mathematica [A] time = 0.0235069, size = 77, normalized size = 0.59

$$\frac{2x\sqrt{dx}\left(302841a^6+778734a^5bx^2+1238895a^4b^2x^4+1211364a^3b^3x^6+717255a^2b^4x^8+237006ab^5x^{10}+33649b^6x^{12}\right)}{908523}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (2*x*Sqrt[d*x]*(302841*a^6 + 778734*a^5*b*x^2 + 1238895*a^4*b^2*x
^4 + 1211364*a^3*b^3*x^6 + 717255*a^2*b^4*x^8 + 237006*a*b^5*x^10
 + 33649*b^6*x^12))/908523
```

Maple [A] time = 0.011, size = 74, normalized size = 0.6

$$\frac{2x\left(33649b^6x^{12}+237006ab^5x^{10}+717255a^2b^4x^8+1211364a^3b^3x^6+1238895a^4b^2x^4+778734a^5bx^2+302841a^6\right)}{908523}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3*(d*x)^(1/2),x)
```

```
[Out] 2/908523*x*(33649*b^6*x^12+237006*a*b^5*x^10+717255*a^2*b^4*x^8+1
211364*a^3*b^3*x^6+1238895*a^4*b^2*x^4+778734*a^5*b*x^2+302841*a^
6)*(d*x)^(1/2)
```

Maxima [A] time = 0.703632, size = 142, normalized size = 1.08

$$\frac{2\left(33649\,(dx)^{\frac{27}{2}}b^6+237006\,(dx)^{\frac{23}{2}}ab^5d^2+717255\,(dx)^{\frac{19}{2}}a^2b^4d^4+1211364\,(dx)^{\frac{15}{2}}a^3b^3d^6+1238895\,(dx)^{\frac{11}{2}}a^4b^2d^8+7787\right.}{908523\,d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/908523*(33649*(d*x)^(27/2)*b^6 + 237006*(d*x)^(23/2)*a*b^5*d^2
+ 717255*(d*x)^(19/2)*a^2*b^4*d^4 + 1211364*(d*x)^(15/2)*a^3*b^3*
d^6 + 1238895*(d*x)^(11/2)*a^4*b^2*d^8 + 778734*(d*x)^(7/2)*a^5*b
*d^10 + 302841*(d*x)^(3/2)*a^6*d^12)/d^13
```

Fricas [A] time = 0.257206, size = 99, normalized size = 0.76

$$\frac{2}{908523}\left(33649\,b^6x^{13}+237006\,ab^5x^{11}+717255\,a^2b^4x^9+1211364\,a^3b^3x^7+1238895\,a^4b^2x^5+778734\,a^5bx^3+302841\,a^6x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^6*x^13 + 237006*a*b^5*x^11 + 717255*a^2*b^4*x^9
 + 1211364*a^3*b^3*x^7 + 1238895*a^4*b^2*x^5 + 778734*a^5*b*x^3 +
 302841*a^6*x)*sqrt(d*x)
```

Sympy [A] time = 9.01914, size = 131, normalized size = 1.

$$\frac{2a^6\sqrt{d}x^{\frac{3}{2}}}{3}+\frac{12a^5b\sqrt{d}x^{\frac{7}{2}}}{7}+\frac{30a^4b^2\sqrt{d}x^{\frac{11}{2}}}{11}+\frac{8a^3b^3\sqrt{d}x^{\frac{15}{2}}}{3}+\frac{30a^2b^4\sqrt{d}x^{\frac{19}{2}}}{19}+\frac{12ab^5\sqrt{d}x^{\frac{23}{2}}}{23}+\frac{2b^6\sqrt{d}x^{\frac{27}{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3*(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(d)*x**(3/2)/3 + 12*a**5*b*sqrt(d)*x**(7/2)/7 + 30*a**
4*b**2*sqrt(d)*x**(11/2)/11 + 8*a**3*b**3*sqrt(d)*x**(15/2)/3 + 3
0*a**2*b**4*sqrt(d)*x**(19/2)/19 + 12*a*b**5*sqrt(d)*x**(23/2)/23
 + 2*b**6*sqrt(d)*x**(27/2)/27
```

GIAC/XCAS [A] time = 0.264919, size = 153, normalized size = 1.17

$$\frac{2\left(33649\,\sqrt{dx}b^6dx^{13}+237006\,\sqrt{dx}ab^5dx^{11}+717255\,\sqrt{dx}a^2b^4dx^9+1211364\,\sqrt{dx}a^3b^3dx^7+1238895\,\sqrt{dx}a^4b^2dx^5+778734\right.}{908523\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/908523*(33649*sqrt(d*x)*b^6*d*x^13 + 237006*sqrt(d*x)*a*b^5*d*x
^11 + 717255*sqrt(d*x)*a^2*b^4*d*x^9 + 1211364*sqrt(d*x)*a^3*b^3*
d*x^7 + 1238895*sqrt(d*x)*a^4*b^2*d*x^5 + 778734*sqrt(d*x)*a^5*b*
d*x^3 + 302841*sqrt(d*x)*a^6*d*x)/d
```

3.682 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{\sqrt{dx}}\,dx$

Optimal. Leaf size=129

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^
2*(d*x)^(9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30
*a^2*b^4*(d*x)^(17/2))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11)
 + (2*b^6*(d*x)^(25/2))/(25*d^13)
```

Rubi [A] time = 0.149391, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b(dx)^{5/2}}{5d^3}+\frac{10a^4b^2(dx)^{9/2}}{3d^5}+\frac{40a^3b^3(dx)^{13/2}}{13d^7}+\frac{30a^2b^4(dx)^{17/2}}{17d^9}+\frac{4ab^5(dx)^{21/2}}{7d^{11}}+\frac{2b^6(dx)^{25/2}}{25d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*a^6*Sqrt[d*x])/d + (12*a^5*b*(d*x)^(5/2))/(5*d^3) + (10*a^4*b^
2*(d*x)^(9/2))/(3*d^5) + (40*a^3*b^3*(d*x)^(13/2))/(13*d^7) + (30
*a^2*b^4*(d*x)^(17/2))/(17*d^9) + (4*a*b^5*(d*x)^(21/2))/(7*d^11)
 + (2*b^6*(d*x)^(25/2))/(25*d^13)
```

Rubi in Sympy [A] time = 36.0383, size = 128, normalized size = 0.99

$$\frac{2a^6\sqrt{dx}}{d}+\frac{12a^5b\,(dx)^{\frac{5}{2}}}{5d^3}+\frac{10a^4b^2\,(dx)^{\frac{9}{2}}}{3d^5}+\frac{40a^3b^3\,(dx)^{\frac{13}{2}}}{13d^7}+\frac{30a^2b^4\,(dx)^{\frac{17}{2}}}{17d^9}+\frac{4ab^5\,(dx)^{\frac{21}{2}}}{7d^{11}}+\frac{2b^6\,(dx)^{\frac{25}{2}}}{25d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(d*x)/d + 12*a**5*b*(d*x)**(5/2)/(5*d**3) + 10*a**4*b*
*2*(d*x)**(9/2)/(3*d**5) + 40*a**3*b**3*(d*x)**(13/2)/(13*d**7) +
 30*a**2*b**4*(d*x)**(17/2)/(17*d**9) + 4*a*b**5*(d*x)**(21/2)/(7
*d**11) + 2*b**6*(d*x)**(25/2)/(25*d**13)
```

Mathematica [A] time = 0.0246764, size = 77, normalized size = 0.6

$$\frac{2\left(116025a^6x+139230a^5bx^3+193375a^4b^2x^5+178500a^3b^3x^7+102375a^2b^4x^9+33150ab^5x^{11}+4641b^6x^{13}\right)}{116025\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/Sqrt[d*x],x]
```

```
[Out] (2*(116025*a^6*x + 139230*a^5*b*x^3 + 193375*a^4*b^2*x^5 + 178500
*a^3*b^3*x^7 + 102375*a^2*b^4*x^9 + 33150*a*b^5*x^11 + 4641*b^6*x
^13))/(116025*Sqrt[d*x])
```

Maple [A] time = 0.011, size = 74, normalized size = 0.6

$$\frac{\left(9282\,b^6x^{12}+66300\,ab^5x^{10}+204750\,a^2b^4x^8+357000\,a^3b^3x^6+386750\,a^4b^2x^4+278460\,a^5bx^2+232050\,a^6\right)x}{116025}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 2/116025*(4641*b^6*x^12+33150*a*b^5*x^10+102375*a^2*b^4*x^8+17850
0*a^3*b^3*x^6+193375*a^4*b^2*x^4+139230*a^5*b*x^2+116025*a^6)*x/(
d*x)^(1/2)
```

Maxima [A] time = 0.711111, size = 209, normalized size = 1.62

$$\frac{2\left(116025\sqrt{dx}a^6+\frac{4641\,(dx)^{\frac{25}{2}}\,b^6}{d^{12}}+\frac{33150\,(dx)^{\frac{21}{2}}\,ab^5}{d^{10}}+\frac{81900\,(dx)^{\frac{17}{2}}\,a^2b^4}{d^8}+\frac{71400\,(dx)^{\frac{13}{2}}\,a^3b^3}{d^6}+7735\left(\frac{5\,(dx)^{\frac{9}{2}}\,b^2}{d^4}+\frac{18\,(dx)^{\frac{5}{2}}\,ab}{d^2}\right)a^4+175\left(\cdots\right.}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/116025*(116025*sqrt(d*x)*a^6 + 4641*(d*x)^(25/2)*b^6/d^12 + 331
50*(d*x)^(21/2)*a*b^5/d^10 + 81900*(d*x)^(17/2)*a^2*b^4/d^8 + 714
00*(d*x)^(13/2)*a^3*b^3/d^6 + 7735*(5*(d*x)^(9/2)*b^2/d^4 + 18*(d
*x)^(5/2)*a*b/d^2)*a^4 + 175*(117*(d*x)^(17/2)*b^4/d^8 + 612*(d*x
```

```
)^(13/2)*a*b^3/d^6 + 884*(d*x)^(9/2)*a^2*b^2/d^4)*a^2)/d
```

Fricas [A] time = 0.255527, size = 101, normalized size = 0.78

$$\frac{2\left(4641\,b^{6}x^{12}+33150\,ab^{5}x^{10}+102375\,a^{2}b^{4}x^{8}+178500\,a^{3}b^{3}x^{6}+193375\,a^{4}b^{2}x^{4}+139230\,a^{5}bx^{2}+116025\,a^{6}\right)\sqrt{dx}}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*b^6*x^12 + 33150*a*b^5*x^10 + 102375*a^2*b^4*x^8 +
 178500*a^3*b^3*x^6 + 193375*a^4*b^2*x^4 + 139230*a^5*b*x^2 + 116
025*a^6)*sqrt(d*x)/d
```

Sympy [A] time = 9.93403, size = 129, normalized size = 1.

$$\frac{2a^{6}\sqrt{x}}{\sqrt{d}}+\frac{12a^{5}bx^{\frac{5}{2}}}{5\sqrt{d}}+\frac{10a^{4}b^{2}x^{\frac{9}{2}}}{3\sqrt{d}}+\frac{40a^{3}b^{3}x^{\frac{13}{2}}}{13\sqrt{d}}+\frac{30a^{2}b^{4}x^{\frac{17}{2}}}{17\sqrt{d}}+\frac{4ab^{5}x^{\frac{21}{2}}}{7\sqrt{d}}+\frac{2b^{6}x^{\frac{25}{2}}}{25\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] 2*a**6*sqrt(x)/sqrt(d) + 12*a**5*b*x**(5/2)/(5*sqrt(d)) + 10*a**4
*b**2*x**(9/2)/(3*sqrt(d)) + 40*a**3*b**3*x**(13/2)/(13*sqrt(d))
+ 30*a**2*b**4*x**(17/2)/(17*sqrt(d)) + 4*a*b**5*x**(21/2)/(7*sqr
t(d)) + 2*b**6*x**(25/2)/(25*sqrt(d))
```

GIAC/XCAS [A] time = 0.263063, size = 142, normalized size = 1.1

$$\frac{2\left(4641\,\sqrt{dx}b^{6}x^{12}+33150\,\sqrt{dx}ab^{5}x^{10}+102375\,\sqrt{dx}a^{2}b^{4}x^{8}+178500\,\sqrt{dx}a^{3}b^{3}x^{6}+193375\,\sqrt{dx}a^{4}b^{2}x^{4}+139230\,\sqrt{dx}a^{5}bx^{2}\right.}{116025\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/116025*(4641*sqrt(d*x)*b^6*x^12 + 33150*sqrt(d*x)*a*b^5*x^10 +
102375*sqrt(d*x)*a^2*b^4*x^8 + 178500*sqrt(d*x)*a^3*b^3*x^6 + 193
```

```
375*sqrt(d*x)*a^4*b^2*x^4 + 139230*sqrt(d*x)*a^5*b*x^2 + 116025*s
qrt(d*x)*a^6)/d
```

3.683 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^3}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*
(d*x)^(7/2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^
2*b^4*(d*x)^(15/2))/d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*
b^6*(d*x)^(23/2))/(23*d^13)
```

Rubi [A] time = 0.151483, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b(dx)^{3/2}}{d^3}+\frac{30a^4b^2(dx)^{7/2}}{7d^5}+\frac{40a^3b^3(dx)^{11/2}}{11d^7}+\frac{2a^2b^4(dx)^{15/2}}{d^9}+\frac{12ab^5(dx)^{19/2}}{19d^{11}}+\frac{2b^6(dx)^{23/2}}{23d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (-2*a^6)/(d*Sqrt[d*x]) + (4*a^5*b*(d*x)^(3/2))/d^3 + (30*a^4*b^2*
(d*x)^(7/2))/(7*d^5) + (40*a^3*b^3*(d*x)^(11/2))/(11*d^7) + (2*a^
2*b^4*(d*x)^(15/2))/d^9 + (12*a*b^5*(d*x)^(19/2))/(19*d^11) + (2*
b^6*(d*x)^(23/2))/(23*d^13)
```

Rubi in Sympy [A] time = 36.3074, size = 124, normalized size = 0.99

$$-\frac{2a^6}{d\sqrt{dx}}+\frac{4a^5b\,(dx)^{\frac{3}{2}}}{d^3}+\frac{30a^4b^2\,(dx)^{\frac{7}{2}}}{7d^5}+\frac{40a^3b^3\,(dx)^{\frac{11}{2}}}{11d^7}+\frac{2a^2b^4\,(dx)^{\frac{15}{2}}}{d^9}+\frac{12ab^5\,(dx)^{\frac{19}{2}}}{19d^{11}}+\frac{2b^6\,(dx)^{\frac{23}{2}}}{23d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

```
[Out] -2*a**6/(d*sqrt(d*x)) + 4*a**5*b*(d*x)**(3/2)/d**3 + 30*a**4*b**2
*(d*x)**(7/2)/(7*d**5) + 40*a**3*b**3*(d*x)**(11/2)/(11*d**7) + 2
*a**2*b**4*(d*x)**(15/2)/d**9 + 12*a*b**5*(d*x)**(19/2)/(19*d**11
) + 2*b**6*(d*x)**(23/2)/(23*d**13)
```

Mathematica [A] time = 0.026701, size = 77, normalized size = 0.62

$$\frac{2x\left(-33649a^6+67298a^5bx^2+72105a^4b^2x^4+61180a^3b^3x^6+33649a^2b^4x^8+10626ab^5x^{10}+1463b^6x^{12}\right)}{33649(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-33649*a^6 + 67298*a^5*b*x^2 + 72105*a^4*b^2*x^4 + 61180*a^
3*b^3*x^6 + 33649*a^2*b^4*x^8 + 10626*a*b^5*x^10 + 1463*b^6*x^12)
)/(33649*(d*x)^(3/2))
```

Maple [A] time = 0.011, size = 74, normalized size = 0.6

$$-\frac{\left(-2926\,b^6x^{12}-21252\,ab^5x^{10}-67298\,a^2b^4x^8-122360\,a^3b^3x^6-144210\,a^4b^2x^4-134596\,a^5bx^2+67298\,a^6\right)x}{33649}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(3/2),x)
```

```
[Out] -2/33649*(-1463*b^6*x^12-10626*a*b^5*x^10-33649*a^2*b^4*x^8-61180
*a^3*b^3*x^6-72105*a^4*b^2*x^4-67298*a^5*b*x^2+33649*a^6)*x/(d*x)
^(3/2)
```

Maxima [A] time = 0.704944, size = 146, normalized size = 1.17

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}}-\frac{1463\,(dx)^{\frac{23}{2}}\,b^6+10626\,(dx)^{\frac{19}{2}}\,ab^5d^2+33649\,(dx)^{\frac{15}{2}}\,a^2b^4d^4+61180\,(dx)^{\frac{11}{2}}\,a^3b^3d^6+72105\,(dx)^{\frac{7}{2}}\,a^4b^2d^8+67298\,(dx)^{\frac{3}{2}}\,a^5bd^{10}}{d^{12}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*(d*x)^(23/2)*b^6 + 10626*(d
*x)^(19/2)*a*b^5*d^2 + 33649*(d*x)^(15/2)*a^2*b^4*d^4 + 61180*(d*
x)^(11/2)*a^3*b^3*d^6 + 72105*(d*x)^(7/2)*a^4*b^2*d^8 + 67298*(d*
x)^(3/2)*a^5*b*d^10)/d^12)/d
```

Fricas [A] time = 0.257741, size = 101, normalized size = 0.81

$$\frac{2\left(1463\,b^6x^{12}+10626\,ab^5x^{10}+33649\,a^2b^4x^8+61180\,a^3b^3x^6+72105\,a^4b^2x^4+67298\,a^5bx^2-33649\,a^6\right)}{33649\,\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/33649*(1463*b^6*x^12 + 10626*a*b^5*x^10 + 33649*a^2*b^4*x^8 + 6
1180*a^3*b^3*x^6 + 72105*a^4*b^2*x^4 + 67298*a^5*b*x^2 - 33649*a^
6)/(sqrt(d*x)*d)
```

Sympy [A] time = 10.7921, size = 126, normalized size = 1.01

$$-\frac{2a^6}{d^{\frac{3}{2}}\sqrt{x}}+\frac{4a^5bx^{\frac{3}{2}}}{d^{\frac{3}{2}}}+\frac{30a^4b^2x^{\frac{7}{2}}}{7d^{\frac{3}{2}}}+\frac{40a^3b^3x^{\frac{11}{2}}}{11d^{\frac{3}{2}}}+\frac{2a^2b^4x^{\frac{15}{2}}}{d^{\frac{3}{2}}}+\frac{12ab^5x^{\frac{19}{2}}}{19d^{\frac{3}{2}}}+\frac{2b^6x^{\frac{23}{2}}}{23d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(3/2),x)
```

```
[Out] -2*a**6/(d**(3/2)*sqrt(x)) + 4*a**5*b*x**(3/2)/d**(3/2) + 30*a**4
*b**2*x**(7/2)/(7*d**(3/2)) + 40*a**3*b**3*x**(11/2)/(11*d**(3/2)
) + 2*a**2*b**4*x**(15/2)/d**(3/2) + 12*a*b**5*x**(19/2)/(19*d**(
3/2)) + 2*b**6*x**(23/2)/(23*d**(3/2))
```

GIAC/XCAS [A] time = 0.263694, size = 171, normalized size = 1.37

$$-\frac{2\left(\frac{33649\,a^6}{\sqrt{dx}}-\frac{1463\,\sqrt{dx}b^6d^{275}x^{11}+10626\,\sqrt{dx}ab^5d^{275}x^9+33649\,\sqrt{dx}a^2b^4d^{275}x^7+61180\,\sqrt{dx}a^3b^3d^{275}x^5+72105\,\sqrt{dx}a^4b^2d^{275}x^3+67298\,\sqrt{dx}a^5bd^{275}x}{d^{276}}\right)}{33649\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/33649*(33649*a^6/sqrt(d*x) - (1463*sqrt(d*x)*b^6*d^275*x^11 +
10626*sqrt(d*x)*a*b^5*d^275*x^9 + 33649*sqrt(d*x)*a^2*b^4*d^275*x
^7 + 61180*sqrt(d*x)*a^3*b^3*d^275*x^5 + 72105*sqrt(d*x)*a^4*b^2*
d^275*x^3 + 67298*sqrt(d*x)*a^5*b*d^275*x)/d^276)/d
```

3.684 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^
2*(d*x)^(5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b
^4*(d*x)^(13/2))/(13*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (
2*b^6*(d*x)^(21/2))/(21*d^13)
```

Rubi [A] time = 0.151208, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{2a^6}{3d(dx)^{3/2}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2(dx)^{5/2}}{d^5}+\frac{40a^3b^3(dx)^{9/2}}{9d^7}+\frac{30a^2b^4(dx)^{13/2}}{13d^9}+\frac{12ab^5(dx)^{17/2}}{17d^{11}}+\frac{2b^6(dx)^{21/2}}{21d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (-2*a^6)/(3*d*(d*x)^(3/2)) + (12*a^5*b*Sqrt[d*x])/d^3 + (6*a^4*b^
2*(d*x)^(5/2))/d^5 + (40*a^3*b^3*(d*x)^(9/2))/(9*d^7) + (30*a^2*b
^4*(d*x)^(13/2))/(13*d^9) + (12*a*b^5*(d*x)^(17/2))/(17*d^11) + (
2*b^6*(d*x)^(21/2))/(21*d^13)
```

Rubi in Sympy [A] time = 36.5308, size = 126, normalized size = 0.99

$$-\frac{2a^6}{3d\left(dx\right)^{\frac{3}{2}}}+\frac{12a^5b\sqrt{dx}}{d^3}+\frac{6a^4b^2\left(dx\right)^{\frac{5}{2}}}{d^5}+\frac{40a^3b^3\left(dx\right)^{\frac{9}{2}}}{9d^7}+\frac{30a^2b^4\left(dx\right)^{\frac{13}{2}}}{13d^9}+\frac{12ab^5\left(dx\right)^{\frac{17}{2}}}{17d^{11}}+\frac{2b^6\left(dx\right)^{\frac{21}{2}}}{21d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

```
[Out] -2*a**6/(3*d*(d*x)**(3/2)) + 12*a**5*b*sqrt(d*x)/d**3 + 6*a**4*b*
*2*(d*x)**(5/2)/d**5 + 40*a**3*b**3*(d*x)**(9/2)/(9*d**7) + 30*a*
*2*b**4*(d*x)**(13/2)/(13*d**9) + 12*a*b**5*(d*x)**(17/2)/(17*d**
11) + 2*b**6*(d*x)**(21/2)/(21*d**13)
```

Mathematica [A] time = 0.0303693, size = 77, normalized size = 0.61

$$\frac{2x\left(-4641a^6+83538a^5bx^2+41769a^4b^2x^4+30940a^3b^3x^6+16065a^2b^4x^8+4914ab^5x^{10}+663b^6x^{12}\right)}{13923(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(5/2),x]
```

```
[Out] (2*x*(-4641*a^6 + 83538*a^5*b*x^2 + 41769*a^4*b^2*x^4 + 30940*a^3
*b^3*x^6 + 16065*a^2*b^4*x^8 + 4914*a*b^5*x^10 + 663*b^6*x^12))/(
13923*(d*x)^(5/2))
```

Maple [A] time = 0.011, size = 74, normalized size = 0.6

$$-\frac{\left(-1326\,b^6x^{12}-9828\,ab^5x^{10}-32130\,a^2b^4x^8-61880\,a^3b^3x^6-83538\,a^4b^2x^4-167076\,a^5bx^2+9282\,a^6\right)x}{13923}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(5/2),x)
```

```
[Out] -2/13923*(-663*b^6*x^12-4914*a*b^5*x^10-16065*a^2*b^4*x^8-30940*a
^3*b^3*x^6-41769*a^4*b^2*x^4-83538*a^5*b*x^2+4641*a^6)*x/(d*x)^(5
/2)
```

Maxima [A] time = 0.710648, size = 146, normalized size = 1.15

$$-\frac{2\left(\frac{4641\,a^6}{(dx)^{\frac{3}{2}}}-\frac{663\,(dx)^{\frac{21}{2}}b^6+4914\,(dx)^{\frac{17}{2}}ab^5d^2+16065\,(dx)^{\frac{13}{2}}a^2b^4d^4+30940\,(dx)^{\frac{9}{2}}a^3b^3d^6+41769\,(dx)^{\frac{5}{2}}a^4b^2d^8+83538\,\sqrt{dx}a^5bd^{10}}{d^{12}}\right)}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/13923*(4641*a^6/(d*x)^(3/2) - (663*(d*x)^(21/2)*b^6 + 4914*(d*
x)^(17/2)*a*b^5*d^2 + 16065*(d*x)^(13/2)*a^2*b^4*d^4 + 30940*(d*x
)^(9/2)*a^3*b^3*d^6 + 41769*(d*x)^(5/2)*a^4*b^2*d^8 + 83538*sqrt(
d*x)*a^5*b*d^10)/d^12)/d
```

Fricas [A] time = 0.256352, size = 105, normalized size = 0.83

$$\frac{2\left(663\,b^{6}x^{12}+4914\,ab^{5}x^{10}+16065\,a^{2}b^{4}x^{8}+30940\,a^{3}b^{3}x^{6}+41769\,a^{4}b^{2}x^{4}+83538\,a^{5}bx^{2}-4641\,a^{6}\right)}{13923\,\sqrt{dx}d^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^6*x^12 + 4914*a*b^5*x^10 + 16065*a^2*b^4*x^8 + 309
40*a^3*b^3*x^6 + 41769*a^4*b^2*x^4 + 83538*a^5*b*x^2 - 4641*a^6)/
(sqrt(d*x)*d^2*x)
```

Sympy [A] time = 10.9888, size = 128, normalized size = 1.01

$$-\frac{2a^{6}}{3d^{\frac{5}{2}}x^{\frac{3}{2}}}+\frac{12a^{5}b\sqrt{x}}{d^{\frac{5}{2}}}+\frac{6a^{4}b^{2}x^{\frac{5}{2}}}{d^{\frac{5}{2}}}+\frac{40a^{3}b^{3}x^{\frac{9}{2}}}{9d^{\frac{5}{2}}}+\frac{30a^{2}b^{4}x^{\frac{13}{2}}}{13d^{\frac{5}{2}}}+\frac{12ab^{5}x^{\frac{17}{2}}}{17d^{\frac{5}{2}}}+\frac{2b^{6}x^{\frac{21}{2}}}{21d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(5/2),x)
```

```
[Out] -2*a**6/(3*d**(5/2)*x**(3/2)) + 12*a**5*b*sqrt(x)/d**(5/2) + 6*a*
*4*b**2*x**(5/2)/d**(5/2) + 40*a**3*b**3*x**(9/2)/(9*d**(5/2)) +
30*a**2*b**4*x**(13/2)/(13*d**(5/2)) + 12*a*b**5*x**(17/2)/(17*d*
*(5/2)) + 2*b**6*x**(21/2)/(21*d**(5/2))
```

GIAC/XCAS [A] time = 0.265795, size = 176, normalized size = 1.39

$$-\frac{2\left(\frac{4641\,a^{6}d}{\sqrt{dx}x}-\frac{663\,\sqrt{dx}b^{6}d^{210}x^{10}+4914\,\sqrt{dx}ab^{5}d^{210}x^{8}+16065\,\sqrt{dx}a^{2}b^{4}d^{210}x^{6}+30940\,\sqrt{dx}a^{3}b^{3}d^{210}x^{4}+41769\,\sqrt{dx}a^{4}b^{2}d^{210}x^{2}+83538\,\sqrt{dx}a^{5}bd^{210}}{d^{210}}\right)}{13923\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/13923*(4641*a^6*d/(sqrt(d*x)*x) - (663*sqrt(d*x)*b^6*d^210*x^1
0 + 4914*sqrt(d*x)*a*b^5*d^210*x^8 + 16065*sqrt(d*x)*a^2*b^4*d^21
0*x^6 + 30940*sqrt(d*x)*a^3*b^3*d^210*x^4 + 41769*sqrt(d*x)*a^4*b
^2*d^210*x^2 + 83538*sqrt(d*x)*a^5*b*d^210)/d^210)/d^3
```

3.685 $\int \frac{(a^2+2abx^2+b^2x^4)^3}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4
*b^2*(d*x)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^
2*b^4*(d*x)^(11/2))/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) +
(2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rubi [A] time = 0.151398, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{2a^6}{5d(dx)^{5/2}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2(dx)^{3/2}}{d^5}+\frac{40a^3b^3(dx)^{7/2}}{7d^7}+\frac{30a^2b^4(dx)^{11/2}}{11d^9}+\frac{4ab^5(dx)^{15/2}}{5d^{11}}+\frac{2b^6(dx)^{19/2}}{19d^{13}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (-2*a^6)/(5*d*(d*x)^(5/2)) - (12*a^5*b)/(d^3*Sqrt[d*x]) + (10*a^4
*b^2*(d*x)^(3/2))/d^5 + (40*a^3*b^3*(d*x)^(7/2))/(7*d^7) + (30*a^
2*b^4*(d*x)^(11/2))/(11*d^9) + (4*a*b^5*(d*x)^(15/2))/(5*d^11) +
(2*b^6*(d*x)^(19/2))/(19*d^13)
```

Rubi in Sympy [A] time = 36.4038, size = 126, normalized size = 0.99

$$-\frac{2a^6}{5d\,(dx)^{\frac{5}{2}}}-\frac{12a^5b}{d^3\sqrt{dx}}+\frac{10a^4b^2\,(dx)^{\frac{3}{2}}}{d^5}+\frac{40a^3b^3\,(dx)^{\frac{7}{2}}}{7d^7}+\frac{30a^2b^4\,(dx)^{\frac{11}{2}}}{11d^9}+\frac{4ab^5\,(dx)^{\frac{15}{2}}}{5d^{11}}+\frac{2b^6\,(dx)^{\frac{19}{2}}}{19d^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

```
[Out] -2*a**6/(5*d*(d*x)**(5/2)) - 12*a**5*b/(d**3*sqrt(d*x)) + 10*a**4
*b**2*(d*x)**(3/2)/d**5 + 40*a**3*b**3*(d*x)**(7/2)/(7*d**7) + 30
*a**2*b**4*(d*x)**(11/2)/(11*d**9) + 4*a*b**5*(d*x)**(15/2)/(5*d*
*11) + 2*b**6*(d*x)**(19/2)/(19*d**13)
```

Mathematica [A] time = 0.0286599, size = 82, normalized size = 0.65

$$\frac{2\sqrt{dx}\left(-1463a^6-43890a^5bx^2+36575a^4b^2x^4+20900a^3b^3x^6+9975a^2b^4x^8+2926ab^5x^{10}+385b^6x^{12}\right)}{7315d^4x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^3/(d*x)^(7/2),x]
```

```
[Out] (2*Sqrt[d*x]*(-1463*a^6 - 43890*a^5*b*x^2 + 36575*a^4*b^2*x^4 + 2
0900*a^3*b^3*x^6 + 9975*a^2*b^4*x^8 + 2926*a*b^5*x^10 + 385*b^6*x
^12))/(7315*d^4*x^3)
```

Maple [A] time = 0.012, size = 74, normalized size = 0.6

$$-\frac{\left(-770\,b^6x^{12}-5852\,ab^5x^{10}-19950\,a^2b^4x^8-41800\,a^3b^3x^6-73150\,a^4b^2x^4+87780\,a^5bx^2+2926\,a^6\right)x}{7315}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(7/2),x)
```

```
[Out] -2/7315*(-385*b^6*x^12-2926*a*b^5*x^10-9975*a^2*b^4*x^8-20900*a^3
*b^3*x^6-36575*a^4*b^2*x^4+43890*a^5*b*x^2+1463*a^6)*x/(d*x)^(7/2
)
```

Maxima [A] time = 0.716817, size = 154, normalized size = 1.21

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^2x^2+a^6d^2\right)}{(dx)^{\frac{5}{2}}d^2}-\frac{385\,(dx)^{\frac{19}{2}}b^6+2926\,(dx)^{\frac{15}{2}}ab^5d^2+9975\,(dx)^{\frac{11}{2}}a^2b^4d^4+20900\,(dx)^{\frac{7}{2}}a^3b^3d^6+36575\,(dx)^{\frac{3}{2}}a^4b^2d^8}{d^{12}}\right)}{7315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^2*x^2 + a^6*d^2)/((d*x)^(5/2)*d^2) - (3
85*(d*x)^(19/2)*b^6 + 2926*(d*x)^(15/2)*a*b^5*d^2 + 9975*(d*x)^(1
1/2)*a^2*b^4*d^4 + 20900*(d*x)^(7/2)*a^3*b^3*d^6 + 36575*(d*x)^(3
/2)*a^4*b^2*d^8)/d^12)/d
```

Fricas [A] time = 0.256063, size = 105, normalized size = 0.83

$$\frac{2\left(385\,b^6x^{12}+2926\,ab^5x^{10}+9975\,a^2b^4x^8+20900\,a^3b^3x^6+36575\,a^4b^2x^4-43890\,a^5bx^2-1463\,a^6\right)}{7315\,\sqrt{dx}d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^6*x^12 + 2926*a*b^5*x^10 + 9975*a^2*b^4*x^8 + 20900
*a^3*b^3*x^6 + 36575*a^4*b^2*x^4 - 43890*a^5*b*x^2 - 1463*a^6)/(s
qrt(d*x)*d^3*x^2)
```

Sympy [A] time = 15.9239, size = 128, normalized size = 1.01

$$-\frac{2a^6}{5d^{\frac{7}{2}}x^{\frac{5}{2}}}-\frac{12a^5b}{d^{\frac{7}{2}}\sqrt{x}}+\frac{10a^4b^2x^{\frac{3}{2}}}{d^{\frac{7}{2}}}+\frac{40a^3b^3x^{\frac{7}{2}}}{7d^{\frac{7}{2}}}+\frac{30a^2b^4x^{\frac{11}{2}}}{11d^{\frac{7}{2}}}+\frac{4ab^5x^{\frac{15}{2}}}{5d^{\frac{7}{2}}}+\frac{2b^6x^{\frac{19}{2}}}{19d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(7/2),x)
```

```
[Out] -2*a**6/(5*d**(7/2)*x**(5/2)) - 12*a**5*b/(d**(7/2)*sqrt(x)) + 10
*a**4*b**2*x**(3/2)/d**(7/2) + 40*a**3*b**3*x**(7/2)/(7*d**(7/2))
 + 30*a**2*b**4*x**(11/2)/(11*d**(7/2)) + 4*a*b**5*x**(15/2)/(5*d
**(7/2)) + 2*b**6*x**(19/2)/(19*d**(7/2))
```

GIAC/XCAS [A] time = 0.26871, size = 180, normalized size = 1.42

$$-\frac{2\left(\frac{1463\left(30\,a^5bd^3x^2+a^6d^3\right)}{\sqrt{dx}d^2x^2}-\frac{385\,\sqrt{dx}b^6d^{171}x^9+2926\,\sqrt{dx}ab^5d^{171}x^7+9975\,\sqrt{dx}a^2b^4d^{171}x^5+20900\,\sqrt{dx}a^3b^3d^{171}x^3+36575\,\sqrt{dx}a^4b^2d^{171}x}{d^{171}}\right)}{7315\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/7315*(1463*(30*a^5*b*d^3*x^2 + a^6*d^3)/(sqrt(d*x)*d^2*x^2) -
(385*sqrt(d*x)*b^6*d^171*x^9 + 2926*sqrt(d*x)*a*b^5*d^171*x^7 + 9
975*sqrt(d*x)*a^2*b^4*d^171*x^5 + 20900*sqrt(d*x)*a^3*b^3*d^171*x
^3 + 36575*sqrt(d*x)*a^4*b^2*d^171*x)/d^171)/d^4
```

3.686 $\int \frac{(dx)^{11/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=316

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}b^{13/4}}-\frac{9ad^5\sqrt{dx}}{2b^3}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{9d^3(dx)^{5/2}}{10b^2}$$

```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*
(d*x)^(9/2))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4
)) + (9*a^(5/4)*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) - (9*a^(5/4)*d^(11/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(8*Sqrt[2]*b^(13/4))
```

Rubi [A] time = 0.792247, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{9a^{5/4}d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{13/4}}-\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{13/4}}+\frac{9a^{5/4}d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}b^{13/4}}-\frac{9ad^5\sqrt{dx}}{2b^3}-\frac{d(dx)^{9/2}}{2b\left(a+bx^2\right)}+\frac{9d^3(dx)^{5/2}}{10b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (-9*a*d^5*Sqrt[d*x])/(2*b^3) + (9*d^3*(d*x)^(5/2))/(10*b^2) - (d*
(d*x)^(9/2))/(2*b*(a + b*x^2)) - (9*a^(5/4)*d^(11/2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4
```

```
)) + (9*a^(5/4)*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(13/4)) - (9*a^(5/4)*d^(11/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(8*Sqrt[2]*b^(13/4)) + (9*a^(5/4)*d^(11/2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(8*Sqrt[2]*b^(13/4))
```

Rubi in Sympy [A] time = 149.22, size = 298, normalized size = 0.94

$$-\frac{9\sqrt{2}a^{\frac{5}{4}}d^{\frac{11}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16b^{\frac{13}{4}}}+\frac{9\sqrt{2}a^{\frac{5}{4}}d^{\frac{11}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16b^{\frac{13}{4}}}-\frac{9\sqrt{2}a^{\frac{5}{4}}d^{\frac{11}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8b^{\frac{13}{4}}}+\frac{9\sqrt{2}a^{\frac{5}{4}}d^{\frac{11}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8b^{\frac{13}{4}}}-\frac{9ad^5\sqrt{dx}}{2b^3}-\frac{d\,(dx)^{\frac{9}{2}}}{2b\,(a+bx^2)}+\frac{9d^3\,(dx)^{\frac{5}{2}}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -9*sqrt(2)*a**(5/4)*d**(11/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt
(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*b**(13/4)) + 9*sqrt(
2)*a**(5/4)*d**(11/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(
d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*b**(13/4)) - 9*sqrt(2)*a**(5/
4)*d**(11/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d
)))/(8*b**(13/4)) + 9*sqrt(2)*a**(5/4)*d**(11/2)*atan(1 + sqrt(2)
*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*b**(13/4)) - 9*a*d**5*
sqrt(d*x)/(2*b**3) - d*(d*x)**(9/2)/(2*b*(a + b*x**2)) + 9*d**3*(
d*x)**(5/2)/(10*b**2)
```

Mathematica [A] time = 0.532438, size = 245, normalized size = 0.78

$$\frac{d^5\sqrt{dx}\left(-\frac{45\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}+\frac{45\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{90\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}}+\frac{90\sqrt{2}a^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{\,}}{\sqrt[4]{\,}}\right.}{\sqrt{x}}\right.}{80b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^5*Sqrt[d*x]*(-320*a*b^(1/4) + 32*b^(5/4)*x^2 - (40*a^2*b^(1/4)
)/(a + b*x^2) - (90*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)])/Sqrt[x] + (90*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x] - (45*Sqrt[2]*a^(5/4)*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] +
(45*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x])/Sqrt[x]))/(80*b^(13/4))
```

Maple [A] time = 0.035, size = 242, normalized size = 0.8

$$\frac{2\,d^3}{5\,b^2}(dx)^{\frac{5}{2}}-4\,\frac{ad^5\sqrt{dx}}{b^3}-\frac{d^7a^2}{2\,b^3\,(bd^2x^2+ad^2)}\sqrt{dx}+\frac{9\,ad^5\sqrt{2}}{16\,b^3}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{9\,ad^5\sqrt{2}}{8\,b^3}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{9\,ad^5\sqrt{2}}{8\,b^3}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/5*d^3*(d*x)^(5/2)/b^2-4*a*d^5*(d*x)^(1/2)/b^3-1/2*d^7/b^3*a^2*(
d*x)^(1/2)/(b*d^2*x^2+a*d^2)+9/16*d^5/b^3*a*(a*d^2/b)^(1/4)*2^(1/
2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(
d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+9/8*d^5
/b^3*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)+1)+9/8*d^5/b^3*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288267, size = 351, normalized size = 1.11

$$-\frac{180\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^4x^2+ab^3)\arctan\left(\frac{\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3}{\sqrt{dx}ad^5+\sqrt{a^2d^{11}x+\sqrt{-\frac{a^5d^{22}}{b^{13}}}b^6}}\right)-45\left(-\frac{a^5d^{22}}{b^{13}}\right)^{\frac{1}{4}}(b^4x^2+ab^3)\log\left(9\sqrt{dx}ad^5+9\left(-\frac{a^5d^{22}}{b^{13}}\right.\right.}{40\left(b^4x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] -1/40*(180*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*arctan((-a^5*
d^22/b^13)^(1/4)*b^3/(sqrt(d*x)*a*d^5 + sqrt(a^2*d^11*x + sqrt(-a
^5*d^22/b^13)*b^6))) - 45*(-a^5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3
)*log(9*sqrt(d*x)*a*d^5 + 9*(-a^5*d^22/b^13)^(1/4)*b^3) + 45*(-a^
5*d^22/b^13)^(1/4)*(b^4*x^2 + a*b^3)*log(9*sqrt(d*x)*a*d^5 - 9*(-
a^5*d^22/b^13)^(1/4)*b^3) - 4*(4*b^2*d^5*x^4 - 36*a*b*d^5*x^2 - 4
5*a^2*d^5)*sqrt(d*x))/(b^4*x^2 + a*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.275633, size = 406, normalized size = 1.28

$$-\frac{1}{80}\left(\frac{40\sqrt{dx}a^2d^3}{(bd^2x^2+ad^2)b^3}-\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4}-\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -1/80*(40*sqrt(d*x)*a^2*d^3/((b*d^2*x^2 + a*d^2)*b^3) - 90*sqrt(2
)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 - 90*sqrt(2)*(a*b^3*d^2)^(
1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*
x))/(a*d^2/b)^(1/4))/b^4 - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*ln(d*
x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^4 + 45*s
qrt(2)*(a*b^3*d^2)^(1/4)*a*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/b^4 - 32*(sqrt(d*x)*b^8*d^6*x^2 - 10*sqrt
(d*x)*a*b^7*d^6)/(b^10*d^5))*d^4
```

3.687 $\int \frac{(dx)^{9/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=298

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}b^{11/4}}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}+\frac{7d^3(dx)^{3/2}}{6b^2}$$

```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) +
 (7*a^(3/4)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(1
1/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4)) + (7
*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```

Rubi [A] time = 0.652996, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{7a^{3/4}d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{11/4}}+\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{11/4}}-\frac{7a^{3/4}d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}b^{11/4}}-\frac{d(dx)^{7/2}}{2b\left(a+bx^2\right)}+\frac{7d^3(dx)^{3/2}}{6b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (7*d^3*(d*x)^(3/2))/(6*b^2) - (d*(d*x)^(7/2))/(2*b*(a + b*x^2)) +
 (7*a^(3/4)*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(4*Sqrt[2]*b^(11/4)) - (7*a^(3/4)*d^(9/2)*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(1
```

```
1/4)) - (7*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4)) + (7
*a^(3/4)*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(11/4))
```

Rubi in Sympy [A] time = 131.164, size = 279, normalized size = 0.94

$$-\frac{7\sqrt{2}a^{\frac{3}{4}}d^{\frac{9}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16b^{\frac{11}{4}}}+\frac{7\sqrt{2}a^{\frac{3}{4}}d^{\frac{9}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16b^{\frac{11}{4}}}+\frac{7\sqrt{2}a^{\frac{3}{4}}d^{\frac{9}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8b^{\frac{11}{4}}}-\frac{7\sqrt{2}a^{\frac{3}{4}}d^{\frac{9}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8b^{\frac{11}{4}}}-\frac{d\,(dx)^{\frac{7}{2}}}{2b\,(a+bx^2)}+\frac{7d^3\,(dx)^{\frac{3}{2}}}{6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -7*sqrt(2)*a**(3/4)*d**(9/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(
d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*b**(11/4)) + 7*sqrt(2
)*a**(3/4)*d**(9/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*
x) + sqrt(a)*d + sqrt(b)*d*x)/(16*b**(11/4)) + 7*sqrt(2)*a**(3/4)
*d**(9/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))
/(8*b**(11/4)) - 7*sqrt(2)*a**(3/4)*d**(9/2)*atan(1 + sqrt(2)*b**
(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*b**(11/4)) - d*(d*x)**(7/2
)/(2*b*(a + b*x**2)) + 7*d**3*(d*x)**(3/2)/(6*b**2)
```

Mathematica [A] time = 0.351294, size = 227, normalized size = 0.76

$$\frac{d^4\sqrt{dx}\left(-21\sqrt{2}a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+21\sqrt{2}a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+42\sqrt{2}a^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{48b^{11/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^4*Sqrt[d*x]*(32*b^(3/4)*x^(3/2) + (24*a*b^(3/4)*x^(3/2))/(a +
b*x^2) + 42*Sqrt[2]*a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)] - 42*Sqrt[2]*a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
```

```
)/a^(1/4)] - 21*Sqrt[2]*a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[x] + Sqrt[b]*x] + 21*Sqrt[2]*a^(3/4)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(48*b^(11/4)*Sqrt[x])
```

Maple [A] time = 0.023, size = 226, normalized size = 0.8

$$\frac{2\,d^3}{3\,b^2}\,(dx)^{\frac{3}{2}}+\frac{d^5a}{2\,b^2\,(bd^2x^2+ad^2)}\,(dx)^{\frac{3}{2}} -\frac{7\,d^5a\sqrt{2}}{16\,b^3}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}} -\frac{7\,d^5a\sqrt{2}}{8\,b^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{7\,d^5a\sqrt{2}}{8\,b^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2/3*d^3*(d*x)^(3/2)/b^2+1/2*d^5*a/b^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^
2)-7/16*d^5*a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-7/8*d^5*a/b^3/(a*d^2/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-7/8*d^5*a/b^3/
(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281199, size = 351, normalized size = 1.18

$$-\frac{84\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\arctan\left(\frac{\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{3}{4}}b^8}{\sqrt{dx}a^2d^{13}+\sqrt{a^4d^{27}x-\sqrt{-\frac{a^3d^{18}}{b^{11}}}a^3b^5d^{18}}}\right)+21\left(-\frac{a^3d^{18}}{b^{11}}\right)^{\frac{1}{4}}\left(b^3x^2+ab^2\right)\log\left(343\sqrt{dx}a^2d^{13}+3\right.}{24\left(b^3x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] -1/24*(84*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*arctan((-a^3*d
^18/b^11)^(3/4)*b^8/(sqrt(d*x)*a^2*d^13 + sqrt(a^4*d^27*x - sqrt(
-a^3*d^18/b^11)*a^3*b^5*d^18))) + 21*(-a^3*d^18/b^11)^(1/4)*(b^3*
x^2 + a*b^2)*log(343*sqrt(d*x)*a^2*d^13 + 343*(-a^3*d^18/b^11)^(3
/4)*b^8) - 21*(-a^3*d^18/b^11)^(1/4)*(b^3*x^2 + a*b^2)*log(343*sq
rt(d*x)*a^2*d^13 - 343*(-a^3*d^18/b^11)^(3/4)*b^8) - 4*(4*b*d^4*x
^3 + 7*a*d^4*x)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{9}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(9/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.276171, size = 359, normalized size = 1.2

$$\frac{1}{48}\left(\frac{24\sqrt{dx}ad^3x}{(bd^2x^2+ad^2)b^2}+\frac{32\sqrt{dx}dx}{b^2}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5}-\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\right.}{2\left(\frac{ad^2}{b}\right)}\right.}{b^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/48*(24*sqrt(d*x)*a*d^3*x/((b*d^2*x^2 + a*d^2)*b^2) + 32*sqrt(d*
x)*d*x/b^2 - 42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 - 42*sqr
t(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 21*sqrt(2)*(a*b^3*d^2)^(
3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
b^5 - 21*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5)*d^3
```

3.688 $\int \frac{(dx)^{7/2}}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=298

$$\frac{5\sqrt[4]{a}d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}b^{9/4}}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d^3\sqrt{dx}}{2b^2}$$

```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (
5*a^(1/4)*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)
) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4)) - (5*a^(1
/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```

Rubi [A] time = 0.618619, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{5\sqrt[4]{a}d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}b^{9/4}}+\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}b^{9/4}}-\frac{d(dx)^{5/2}}{2b\left(a+bx^2\right)}+\frac{5d^3\sqrt{dx}}{2b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] (5*d^3*Sqrt[d*x])/(2*b^2) - (d*(d*x)^(5/2))/(2*b*(a + b*x^2)) + (
5*a^(1/4)*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)) - (5*a^(1/4)*d^(7/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*b^(9/4)
) + (5*a^(1/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4)) - (5*a^(1
/4)*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*b^(9/4))
```

Rubi in Sympy [A] time = 134.323, size = 279, normalized size = 0.94

$$\frac{5\sqrt{2}\sqrt[4]{a}d^{\frac{7}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16b^{\frac{9}{4}}} - \frac{5\sqrt{2}\sqrt[4]{a}d^{\frac{7}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16b^{\frac{9}{4}}} + \frac{5\sqrt{2}\sqrt[4]{a}d^{\frac{7}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8b^{\frac{9}{4}}} - \frac{5\sqrt{2}\sqrt[4]{a}d^{\frac{7}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8b^{\frac{9}{4}}} - \frac{d\,(dx)^{\frac{5}{2}}}{2b\,(a+bx^2)} + \frac{5d^3\sqrt{dx}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 5*sqrt(2)*a**(1/4)*d**(7/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d
)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*b**(9/4)) - 5*sqrt(2)*
a**(1/4)*d**(7/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x)
 + sqrt(a)*d + sqrt(b)*d*x)/(16*b**(9/4)) + 5*sqrt(2)*a**(1/4)*d*
*(7/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8
*b**(9/4)) - 5*sqrt(2)*a**(1/4)*d**(7/2)*atan(1 + sqrt(2)*b**(1/4
)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*b**(9/4)) - d*(d*x)**(5/2)/(2*
b*(a + b*x**2)) + 5*d**3*sqrt(d*x)/(2*b**2)
```

Mathematica [A] time = 0.368317, size = 232, normalized size = 0.78

$$\frac{d^3\sqrt{dx}\left(\frac{8a\sqrt[4]{b}}{a+bx^2} + \frac{5\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}} - \frac{5\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}} + \frac{10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}} - 10\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(\right.\right.}{16b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d^3*Sqrt[d*x]*(32*b^(1/4) + (8*a*b^(1/4))/(a + b*x^2) + (10*Sqrt
[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x
] - (10*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/Sqrt[x] + (5*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] - (5*Sqrt[2]*a^(1/4)*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(
16*b^(9/4))
```

Maple [A] time = 0.021, size = 223, normalized size = 0.8

$$2\,\frac{d^{3}\sqrt{dx}}{b^{2}}+\frac{d^{5}a}{2\,b^{2}\left(bd^{2}x^{2}+ad^{2}\right)}\sqrt{dx}-\frac{5\,d^{3}\sqrt{2}}{16\,b^{2}}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)-\frac{5\,d^{3}\sqrt{2}}{8\,b^{2}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)-\frac{5\,d^{3}\sqrt{2}}{8\,b^{2}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 2*d^3*(d*x)^(1/2)/b^2+1/2*d^5/b^2*a*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)
-5/16*d^3/b^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2)))-5/8*d^3/b^2*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-5/8*d^3/b^2*(a*d^2/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283242, size = 305, normalized size = 1.02

$$\frac{20\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\arctan\left(\frac{\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{1}{4}}b^{2}}{\sqrt{dx}d^{3}+\sqrt{d^{7}x+\sqrt{-\frac{ad^{14}}{b^{9}}}b^{4}}}\right)-5\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{1}{4}}\left(b^{3}x^{2}+ab^{2}\right)\log\left(5\sqrt{dx}d^{3}+5\left(-\frac{ad^{14}}{b^{9}}\right)^{\frac{1}{4}}b^{2}\right)+5\,(\cdots}{8\left(b^{3}x^{2}+ab^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(20*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*arctan((-a*d^14/b^9
)^(1/4)*b^2/(sqrt(d*x)*d^3 + sqrt(d^7*x + sqrt(-a*d^14/b^9)*b^4))
) - 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*b^2)*log(5*sqrt(d*x)*d^3 +
 5*(-a*d^14/b^9)^(1/4)*b^2) + 5*(-a*d^14/b^9)^(1/4)*(b^3*x^2 + a*
b^2)*log(5*sqrt(d*x)*d^3 - 5*(-a*d^14/b^9)^(1/4)*b^2) + 4*(4*b*d^
3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^3*x^2 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{7}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(7/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.27336, size = 362, normalized size = 1.21

$$\frac{1}{16}\left(\frac{8\sqrt{dx}ad^3}{(bd^2x^2+ad^2)b^2}-\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}-\frac{10\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/16*(8*sqrt(d*x)*a*d^3/((b*d^2*x^2 + a*d^2)*b^2) - 10*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 - 10*sqrt(2)*(a*b^3*d^2)^(1/4)*d
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 5*sqrt(2)*(a*b^
3*d^2)^(1/4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/b^3 + 32*sqrt(d*x)*d/b^2)*d^2
```

3.689 $\int \frac{(dx)^{5/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

```
[Out] -(d*(d*x)^(3/2))/(2*b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7
/4)) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*a^(1/4)*b^(7/4))
```

Rubi [A] time = 0.558677, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$\frac{3d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{3d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}+\frac{3d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}-\frac{d(dx)^{3/2}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```


```
[Out] -(d*(d*x)^(3/2))/(2*b*(a + b*x^2)) - (3*d^(5/2)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7
/4)) + (3*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(4*Sqrt[2]*a^(1/4)*b^(7/4)) + (3*d^(5/2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(8*Sqrt[2]*a^(1/4)*b^(7/4)) - (3*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]
*a^(1/4)*b^(7/4))
```

Rubi in Sympy [A] time = 117.827, size = 262, normalized size = 0.93

$$-\frac{d\,(dx)^{\frac{3}{2}}}{2b\,(a+bx^2)}+\frac{3\sqrt{2}d^{\frac{5}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16\sqrt[4]{a}b^{\frac{7}{4}}}-\frac{3\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16\sqrt[4]{a}b^{\frac{7}{4}}}-\frac{3\sqrt{2}d^{\frac{5}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8\sqrt[4]{a}b^{\frac{7}{4}}}+\frac{3\sqrt{2}d^{\frac{5}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8\sqrt[4]{a}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -d*(d*x)**(3/2)/(2*b*(a + b*x**2)) + 3*sqrt(2)*d**(5/2)*log(-sqrt
(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x
)/(16*a**(1/4)*b**(7/4)) - 3*sqrt(2)*d**(5/2)*log(sqrt(2)*a**(1/4
)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(1
/4)*b**(7/4)) - 3*sqrt(2)*d**(5/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt
(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(1/4)*b**(7/4)) + 3*sqrt(2)*d**(5
/2)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a*
*(1/4)*b**(7/4))
```

Mathematica [A] time = 0.428697, size = 211, normalized size = 0.75

$$\frac{(dx)^{5/2}\left(-\frac{8b^{3/4}x^{3/2}}{a+bx^2}+\frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}-\frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}-\frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}}+\frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{a}}\right)}{16b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(5/2)*((-8*b^(3/4)*x^(3/2))/(a + b*x^2) - (6*Sqrt[2]*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (6*Sqrt[2]*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(1/4) + (3*Sqrt[2]
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1
/4) - (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/a^(1/4)))/(16*b^(7/4)*x^(5/2))
```

Maple [A] time = 0.02, size = 209, normalized size = 0.7

$$-\frac{d^3}{2\,b\,(bd^2x^2+ad^2)}(dx)^{\frac{3}{2}} + \frac{3\,d^3\sqrt{2}}{16\,b^2}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}} + \frac{3\,d^3\sqrt{2}}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}} + \frac{3\,d^3\sqrt{2}}{8\,b^2}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)+3/16*d^3/b^2/(a*d^2/b)^(
1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)
))+3/8*d^3/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)+1)+3/8*d^3/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285622, size = 308, normalized size = 1.1

$$-\frac{4\sqrt{dx}d^2x-12\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{10}}{ab^7}\right)^{\frac{3}{4}}ab^5}{\sqrt{dx}d^7+\sqrt{d^{15}x-\sqrt{-\frac{d^{10}}{ab^7}}ab^3d^{10}}}\right)-3\left(b^2x^2+ab\right)\left(-\frac{d^{10}}{ab^7}\right)^{\frac{1}{4}}\log\left(27\sqrt{dx}d^7+27\left(-\right.\right.}{8\left(b^2x^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*sqrt(d*x)*d^2*x - 12*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4
)*arctan((-d^10/(a*b^7))^(3/4)*a*b^5/(sqrt(d*x)*d^7 + sqrt(d^15*x
 - sqrt(-d^10/(a*b^7))*a*b^3*d^10))) - 3*(b^2*x^2 + a*b)*(-d^10/(
a*b^7))^(1/4)*log(27*sqrt(d*x)*d^7 + 27*(-d^10/(a*b^7))^(3/4)*a*b
^5) + 3*(b^2*x^2 + a*b)*(-d^10/(a*b^7))^(1/4)*log(27*sqrt(d*x)*d^
7 - 27*(-d^10/(a*b^7))^(3/4)*a*b^5))/(b^2*x^2 + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.275834, size = 355, normalized size = 1.26

$$-\frac{1}{16}\left(\frac{8\sqrt{dx}d^3x}{(bd^2x^2+ad^2)b} - \frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4} - \frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4} + \frac{3\sqrt{\ }}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -1/16*(8*sqrt(d*x)*d^3*x/((b*d^2*x^2 + a*d^2)*b) - 6*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(a*b^4) - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/
b)^(1/4))/(a*b^4) + 3*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 3*sqrt(2)*(a
*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a*b^4))*d
```

3.690 $\int \frac{(dx)^{3/2}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=281

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

```
[Out] -(d*Sqrt[d*x])/(2*b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt
[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b
^(5/4))
```

Rubi [A] time = 0.526489, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}+\frac{d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{3/4}b^{5/4}}-\frac{d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{d\sqrt{dx}}{2b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] -(d*Sqrt[d*x])/(2*b*(a + b*x^2)) - (d^(3/2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
 + (d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[
d])])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) - (d^(3/2)*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt
[2]*a^(3/4)*b^(5/4)) + (d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(3/4)*b
^(5/4))
```

Rubi in Sympy [A] time = 121.918, size = 255, normalized size = 0.91

$$-\frac{d\sqrt{dx}}{2b(a+bx^2)}-\frac{\sqrt{2}d^{\frac{3}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}d^{\frac{3}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{3}{4}}b^{\frac{5}{4}}}-\frac{\sqrt{2}d^{\frac{3}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}+\frac{\sqrt{2}d^{\frac{3}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{3}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] -d*sqrt(d*x)/(2*b*(a + b*x**2)) - sqrt(2)*d**(3/2)*log(-sqrt(2)*a
**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16
*a**(3/4)*b**(5/4)) + sqrt(2)*d**(3/2)*log(sqrt(2)*a**(1/4)*b**(1
/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(3/4)*b**
(5/4)) - sqrt(2)*d**(3/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a*
*(1/4)*sqrt(d)))/(8*a**(3/4)*b**(5/4)) + sqrt(2)*d**(3/2)*atan(1
+ sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(3/4)*b**(
5/4))
```

Mathematica [A] time = 0.471166, size = 210, normalized size = 0.75

$$\frac{(dx)^{3/2}\left(-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}}-\frac{8\sqrt[4]{b}\sqrt{x}}{a+bx^2}\right)}{16b^{5/4}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(3/2)*((-8*b^(1/4)*Sqrt[x])/(a + b*x^2) - (2*Sqrt[2]*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) + (2*Sqrt[2]*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(3/4) - (Sqrt[2]*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(3/4
) + (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/a^(3/4)))/(16*b^(5/4)*x^(3/2))
```

Maple [A] time = 0.019, size = 212, normalized size = 0.8

$$-\frac{d^{3}}{2\,b\,(bd^{2}x^{2}+ad^{2})}\sqrt{dx}+\frac{d\sqrt{2}}{16\,ab}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)+\frac{d\sqrt{2}}{8\,ab}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)+\frac{d\sqrt{2}}{8\,ab}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -1/2*d^3/b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)+1/16*d/b*(a*d^2/b)^(1/4)
/a*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
+1/8*d/b*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)+1)+1/8*d/b*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282928, size = 279, normalized size = 0.99

$$-\frac{4\left(b^{2}x^{2}+ab\right)\left(-\frac{d^{6}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\arctan\left(\frac{ab\left(-\frac{d^{6}}{a^{3}b^{5}}\right)^{\frac{1}{4}}}{\sqrt{dx}d+\sqrt{a^{2}b^{2}\sqrt{-\frac{d^{6}}{a^{3}b^{5}}}+d^{3}x}}\right)-\left(b^{2}x^{2}+ab\right)\left(-\frac{d^{6}}{a^{3}b^{5}}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{d^{6}}{a^{3}b^{5}}\right)^{\frac{1}{4}}+\sqrt{dx}d\right)+\left(b^{2}x^{2}+a\ldots\right.}{8\left(b^{2}x^{2}+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] -1/8*(4*(b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*arctan(a*b*(-d^6/(
a^3*b^5))^(1/4)/(sqrt(d*x)*d + sqrt(a^2*b^2*sqrt(-d^6/(a^3*b^5))
+ d^3*x))) - (b^2*x^2 + a*b)*(-d^6/(a^3*b^5))^(1/4)*log(a*b*(-d^6
/(a^3*b^5))^(1/4) + sqrt(d*x)*d) + (b^2*x^2 + a*b)*(-d^6/(a^3*b^5
))^(1/4)*log(-a*b*(-d^6/(a^3*b^5))^(1/4) + sqrt(d*x)*d) + 4*sqrt(
d*x)*d)/(b^2*x^2 + a*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**2, x)
```

GIAC/XCAS [A] time = 0.273195, size = 355, normalized size = 1.26

$$-\frac{\sqrt{dx}d^3}{2\,(bd^2x^2+ad^2)b}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,ab^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*d^3/((b*d^2*x^2 + a*d^2)*b) + 1/8*sqrt(2)*(a*b^3*d
^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(
```

```
d*x))/(a*d^2/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3*d^2)^(1/4)*d*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^
2/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^2) - 1/16*
sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/(a*b^2)
```

3.691 $\int \frac{\sqrt{dx}}{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=283

$$\frac{\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4))
+ (Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[
2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^
(3/4))
```

Rubi [A] time = 0.551575, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$\frac{\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{5/4}b^{3/4}}-\frac{\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{5/4}b^{3/4}}+\frac{(dx)^{3/2}}{2ad\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d*x)^(3/2)/(2*a*d*(a + b*x^2)) - (Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(5/4)*b^(3/4))
+ (Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(4*Sqrt[2]*a^(5/4)*b^(3/4)) + (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[
2]*a^(5/4)*b^(3/4)) - (Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(5/4)*b^
(3/4))
```

Rubi in Sympy [A] time = 117.367, size = 255, normalized size = 0.9

$$\frac{(dx)^{\frac{3}{2}}}{2ad\,(a+bx^2)}+\frac{\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{5}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{5}{4}}b^{\frac{3}{4}}}-\frac{\sqrt{2}\sqrt{d}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{5}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\sqrt{d}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{5}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (d*x)**(3/2)/(2*a*d*(a + b*x**2)) + sqrt(2)*sqrt(d)*log(-sqrt(2)*
a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(1
6*a**(5/4)*b**(3/4)) - sqrt(2)*sqrt(d)*log(sqrt(2)*a**(1/4)*b**(1
/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(5/4)*b**
(3/4)) - sqrt(2)*sqrt(d)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**
(1/4)*sqrt(d)))/(8*a**(5/4)*b**(3/4)) + sqrt(2)*sqrt(d)*atan(1 +
sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(5/4)*b**(3/
4))
```

Mathematica [A] time = 0.476005, size = 210, normalized size = 0.74

$$\frac{\sqrt{dx}\left(\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}-\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}-\frac{2\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}}+\frac{2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{b^{3/4}}+\frac{8\sqrt[4]{a}x^{3/2}}{a+bx^2}\right)}{16a^{5/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (Sqrt[d*x]*((8*a^(1/4)*x^(3/2))/(a + b*x^2) - (2*Sqrt[2]*ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (2*Sqrt[2]*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (Sqrt[2]*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4) -
 (Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/b^(3/4)))/(16*a^(5/4)*Sqrt[x])
```

Maple [A] time = 0.016, size = 210, normalized size = 0.7

$$\frac{d}{2\,a\left(bd^{2}x^{2}+ad^{2}\right)}\left(dx\right)^{\frac{3}{2}}+\frac{d\sqrt{2}}{16\,ab}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{d\sqrt{2}}{8\,ab}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{d\sqrt{2}}{8\,ab}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] 1/2*d*(d*x)^(3/2)/a/(b*d^2*x^2+a*d^2)+1/16*d/a/b/(a*d^2/b)^(1/4)*
2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+1/
8*d/a/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)+1)+1/8*d/a/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283703, size = 293, normalized size = 1.04

$$\frac{4\left(abx^{2}+a^{2}\right)\left(-\frac{d^{2}}{a^{5}b^{3}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^{4}b^{2}\left(-\frac{d^{2}}{a^{5}b^{3}}\right)^{\frac{3}{4}}}{\sqrt{dx}d+\sqrt{-a^{3}bd^{2}\sqrt{-\frac{d^{2}}{a^{5}b^{3}}}+d^{3}x}}\right)+\left(abx^{2}+a^{2}\right)\left(-\frac{d^{2}}{a^{5}b^{3}}\right)^{\frac{1}{4}}\log\left(a^{4}b^{2}\left(-\frac{d^{2}}{a^{5}b^{3}}\right)^{\frac{3}{4}}+\sqrt{dx}d\right)-\left(abx^{2}\right.}{8\left(abx^{2}+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] 1/8*(4*(a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*arctan(a^4*b^2*(-d^
2/(a^5*b^3))^(3/4)/(sqrt(d*x)*d + sqrt(-a^3*b*d^2*sqrt(-d^2/(a^5*
b^3)) + d^3*x))) + (a*b*x^2 + a^2)*(-d^2/(a^5*b^3))^(1/4)*log(a^4
*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x)*d) - (a*b*x^2 + a^2)*(-d^
2/(a^5*b^3))^(1/4)*log(-a^4*b^2*(-d^2/(a^5*b^3))^(3/4) + sqrt(d*x
)*d) + 4*sqrt(d*x)*x)/(a*b*x^2 + a^2)
```

Sympy [A] time = 16.8004, size = 78, normalized size = 0.28

$$\frac{2d^3\,(dx)^{\frac{3}{2}}}{4a^2d^4+4abd^4x^2}+2d^3\operatorname{RootSum}\left(65536t^4a^5b^3d^{10}+1,\left(t\mapsto t\log\left(4096t^3a^4b^2d^8+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 2*d**3*(d*x)**(3/2)/(4*a**2*d**4 + 4*a*b*d**4*x**2) + 2*d**3*Root
Sum(65536*_t**4*a**5*b**3*d**10 + 1, Lambda(_t, _t*log(4096*_t**3
*a**4*b**2*d**8 + sqrt(d*x))))
```

GIAC/XCAS [A] time = 0.275803, size = 367, normalized size = 1.3

$$\frac{\sqrt{dx}d^2x}{2\,(bd^2x^2+ad^2)a}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3d}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3d}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2b^3d}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d^2*x/((b*d^2*x^2 + a*d^2)*a) + 1/8*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/(a^2*b^3*d) + 1/8*sqrt(2)*(a*b^3*d^2)^(3/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(a^2*b^3*d) - 1/16*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x
+ sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3*d)
+ 1/16*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3*d)
```

3.692 $\int \frac{1}{\sqrt{dx}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=283

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}}{2ad\,(a+bx^2)}$$

```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d])
+ (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a
^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1
/4)*Sqrt[d])
```

Rubi [A] time = 0.540323, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}-\frac{3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}\sqrt{d}}+\frac{\sqrt{dx}}{2ad\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] Sqrt[d*x]/(2*a*d*(a + b*x^2)) - (3*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d])
+ (3*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
4*Sqrt[2]*a^(7/4)*b^(1/4)*Sqrt[d]) - (3*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a
^(7/4)*b^(1/4)*Sqrt[d]) + (3*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(7/4)*b^(1
/4)*Sqrt[d])
```

Rubi in Sympy [A] time = 120.789, size = 262, normalized size = 0.93

$$\frac{\sqrt{dx}}{2ad(a+bx^2)} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{7}{4}}\sqrt[4]{b}\sqrt{d}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{7}{4}}\sqrt[4]{b}\sqrt{d}} - \frac{3\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{7}{4}}\sqrt[4]{b}\sqrt{d}} + \frac{3\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{7}{4}}\sqrt[4]{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] sqrt(d*x)/(2*a*d*(a + b*x**2)) - 3*sqrt(2)*log(-sqrt(2)*a**(1/4)*
b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(7/4
)*b**(1/4)*sqrt(d)) + 3*sqrt(2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqr
t(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(7/4)*b**(1/4)*s
qrt(d)) - 3*sqrt(2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)
*sqrt(d)))/(8*a**(7/4)*b**(1/4)*sqrt(d)) + 3*sqrt(2)*atan(1 + sqr
t(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(7/4)*b**(1/4)*
sqrt(d))
```

Mathematica [A] time = 0.428794, size = 211, normalized size = 0.75

$$\frac{\sqrt{x}\left(\frac{8a^{3/4}\sqrt{x}}{a+bx^2} - \frac{3\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}} + \frac{3\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}} - \frac{6\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{b}} + \frac{6\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt[4]{b}}\right)}{16a^{7/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (Sqrt[x]*((8*a^(3/4)*Sqrt[x])/(a + b*x^2) - (6*Sqrt[2]*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (6*Sqrt[2]*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) - (3*Sqrt[2]*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) +
 (3*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/b^(1/4)))/(16*a^(7/4)*Sqrt[d*x])
```

Maple [A] time = 0.015, size = 207, normalized size = 0.7

$$\frac{d}{2a(bd^2x^2+ad^2)}\sqrt{dx} + \frac{3\sqrt{2}}{16a^2d}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right) + \frac{3\sqrt{2}}{8a^2d}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right) + \frac{3\sqrt{2}}{8a^2d}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)/(d*x)^(1/2),x)
```

```
[Out] 1/2*d*(d*x)^(1/2)/a/(b*d^2*x^2+a*d^2)+3/16/d/a^2*(a*d^2/b)^(1/4)*
2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/
2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+3/
8/d/a^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)+1)+3/8/d/a^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28193, size = 288, normalized size = 1.02

$$-\frac{12\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}}{\sqrt{a^4d^2\sqrt{-\frac{1}{a^7bd^2}}+dx}+\sqrt{dx}}\right)-3\left(abdx^2+a^2d\right)\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}\log\left(a^2d\left(-\frac{1}{a^7bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)+}{8\left(abdx^2+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] -1/8*(12*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*arctan(a^2*d*
(-1/(a^7*b*d^2))^(1/4)/(sqrt(a^4*d^2*sqrt(-1/(a^7*b*d^2)) + d*x)
+ sqrt(d*x))) - 3*(a*b*d*x^2 + a^2*d)*(-1/(a^7*b*d^2))^(1/4)*log(
a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(d*x)) + 3*(a*b*d*x^2 + a^2*d)
*(-1/(a^7*b*d^2))^(1/4)*log(-a^2*d*(-1/(a^7*b*d^2))^(1/4) + sqrt(
d*x)) - 4*sqrt(d*x))/(a*b*d*x^2 + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.270159, size = 363, normalized size = 1.28

$$\frac{\sqrt{dx}d}{2\,(bd^2x^2+ad^2)a}+\frac{3\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2bd}+\frac{3\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2bd}+\frac{3\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2bd}-\frac{3\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*d/((b*d^2*x^2 + a*d^2)*a) + 3/8*sqrt(2)*(a*b^3*d^2)
^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))
```

```
/(a*d^2/b)^(1/4))/(a^2*b*d) + 3/8*sqrt(2)*(a*b^3*d^2)^(1/4)*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a^2*b*d) + 3/16*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x + sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b*d) - 3/16*sqr
t(2)*(a*b^3*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^2*b*d)
```

3.693 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=300

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{9/4}d^{3/2}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{9/4}d^{3/2}}-\frac{5}{2a^2d\sqrt{dx}}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (
5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2)) + (5*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```

Rubi [A] time = 0.639628, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{5\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{9/4}d^{3/2}}+\frac{5\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{9/4}d^{3/2}}-\frac{5\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{9/4}d^{3/2}}-\frac{5}{2a^2d\sqrt{dx}}+\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] -5/(2*a^2*d*Sqrt[d*x]) + 1/(2*a*d*Sqrt[d*x]*(a + b*x^2)) + (5*b^(
1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(
4*Sqrt[2]*a^(9/4)*d^(3/2)) - (5*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(9/4)*d^(3/2)) - (
5*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2)) + (5*b^(1/4)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(8*Sqrt[2]*a^(9/4)*d^(3/2))
```

Rubi in Sympy [A] time = 131.919, size = 279, normalized size = 0.93

$$\frac{1}{2ad\sqrt{dx}\,(a+bx^2)}-\frac{5}{2a^2d\sqrt{dx}}-\frac{5\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{9}{4}}d^{\frac{3}{2}}}+\frac{5\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{9}{4}}d^{\frac{3}{2}}}+\frac{5\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{9}{4}}d^{\frac{3}{2}}}-\frac{5\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{9}{4}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*d*sqrt(d*x)*(a + b*x**2)) - 5/(2*a**2*d*sqrt(d*x)) - 5*sqr
t(2)*b**(1/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) +
sqrt(a)*d + sqrt(b)*d*x)/(16*a**(9/4)*d**(3/2)) + 5*sqrt(2)*b**(1
/4)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d +
 sqrt(b)*d*x)/(16*a**(9/4)*d**(3/2)) + 5*sqrt(2)*b**(1/4)*atan(1
- sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(9/4)*d**(
3/2)) - 5*sqrt(2)*b**(1/4)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a
**(1/4)*sqrt(d)))/(8*a**(9/4)*d**(3/2))
```

Mathematica [A] time = 0.297847, size = 233, normalized size = 0.78

$$\frac{x\left(-\frac{8\sqrt[4]{a}bx^2}{a+bx^2}-5\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+10\sqrt{2}\sqrt[4]{b}\sqrt{x}\tan^{-1}\right.}{16a^{9/4}(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (x*(-32*a^(1/4) - (8*a^(1/4)*b*x^2)/(a + b*x^2) + 10*Sqrt[2]*b^(1
/4)*Sqrt[x]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 10*Sq
rt[2]*b^(1/4)*Sqrt[x]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)] - 5*Sqrt[2]*b^(1/4)*Sqrt[x]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*b^(1/4)*Sqrt[x]*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(16*a^(9/4)*(d*x
)^(3/2))
```

Maple [A] time = 0.027, size = 223, normalized size = 0.7

$$-2\frac{1}{a^2d\sqrt{dx}}-\frac{b}{2\,a^2d\,(bd^2x^2+ad^2)}(dx)^{\frac{3}{2}}-\frac{5\sqrt{2}}{16\,a^2d}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{5\sqrt{2}}{8\,a^2d}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{5\sqrt{2}}{8\,a^2d}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2/a^2/d/(d*x)^(1/2)-1/2/d*b/a^2*(d*x)^(3/2)/(b*d^2*x^2+a*d^2)-5/
16/d/a^2/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2)))-5/8/d/a^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2
^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-5/8/d/a^2/(a*d^2/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289882, size = 340, normalized size = 1.13

$$-\frac{20\,bx^2+20\left(a^2bdx^2+a^3d\right)\sqrt{dx}\left(-\frac{b}{a^9d^6}\right)^{\frac{1}{4}}\arctan\left(\frac{125\,a^7d^5\left(-\frac{b}{a^9d^6}\right)^{\frac{3}{4}}}{125\sqrt{dx}b+\sqrt{-15625\,a^5bd^4\sqrt{-\frac{b}{a^9d^6}}+15625\,b^2dx}}\right)+5\left(a^2bdx^2+a^3d\right)\sqrt{dx}\left(-\frac{b}{a^9d^6}\right.}{8\,(a^2bdx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/8*(20*b*x^2 + 20*(a^2*b*d*x^2 + a^3*d)*sqrt(d*x)*(-b/(a^9*d^6)
)^(1/4)*arctan(125*a^7*d^5*(-b/(a^9*d^6))^(3/4)/(125*sqrt(d*x)*b
+ sqrt(-15625*a^5*b*d^4*sqrt(-b/(a^9*d^6)) + 15625*b^2*d*x))) + 5
*(a^2*b*d*x^2 + a^3*d)*sqrt(d*x)*(-b/(a^9*d^6))^(1/4)*log(125*a^7
*d^5*(-b/(a^9*d^6))^(3/4) + 125*sqrt(d*x)*b) - 5*(a^2*b*d*x^2 + a
^3*d)*sqrt(d*x)*(-b/(a^9*d^6))^(1/4)*log(-125*a^7*d^5*(-b/(a^9*d^
6))^(3/4) + 125*sqrt(d*x)*b) + 16*a)/((a^2*b*d*x^2 + a^3*d)*sqrt(
d*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.273224, size = 397, normalized size = 1.32

$$-\frac{\frac{8\,(5\,bd^2x^2+4\,ad^2)}{\left(\sqrt{dx}bd^2x^2+\sqrt{dx}ad^2\right)a^2}+\frac{10\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}+\frac{10\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^2d^2}-\frac{5\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\cdots\right.\right.}{a^3b^2d\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/16*(8*(5*b*d^2*x^2 + 4*a*d^2)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x
)*a*d^2)*a^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*
d^2) + 10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2*d^2) - 5
*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
d*x) + sqrt(a*d^2/b))/(a^3*b^2*d^2) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)
*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3
*b^2*d^2))/d
```

3.694 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=300

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{11/4}d^{5/2}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{11/4}d^{5/2}}-\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}(a+bx^2)}$$

```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7
*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2
)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2)) - (7*b^
(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rubi [A] time = 0.644911, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{7b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}-\frac{7b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{8\sqrt{2}a^{11/4}d^{5/2}}+\frac{7b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{11/4}d^{5/2}}-\frac{7b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{11/4}d^{5/2}}-\frac{7}{6a^2d(dx)^{3/2}}+\frac{1}{2ad(dx)^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] -7/(6*a^2*d*(d*x)^(3/2)) + 1/(2*a*d*(d*x)^(3/2)*(a + b*x^2)) + (7
*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(4*Sqrt[2]*a^(11/4)*d^(5/2)) - (7*b^(3/4)*ArcTan[1 + (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(11/4)*d^(5/2
)) + (7*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2)) - (7*b^
(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(11/4)*d^(5/2))
```

Rubi in Sympy [A] time = 135.371, size = 279, normalized size = 0.93

$$\frac{1}{2ad\,(dx)^{\frac{3}{2}}\,(a+bx^2)}-\frac{7}{6a^2d\,(dx)^{\frac{3}{2}}}+\frac{7\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{11}{4}}d^{\frac{5}{2}}}-\frac{7\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{11}{4}}d^{\frac{5}{2}}}+\frac{7\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{11}{4}}d^{\frac{5}{2}}}-\frac{7\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{11}{4}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*d*(d*x)**(3/2)*(a + b*x**2)) - 7/(6*a**2*d*(d*x)**(3/2)) +
 7*sqrt(2)*b**(3/4)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d
*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(11/4)*d**(5/2)) - 7*sqrt(2
)*b**(3/4)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt
(a)*d + sqrt(b)*d*x)/(16*a**(11/4)*d**(5/2)) + 7*sqrt(2)*b**(3/4)
*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(1
1/4)*d**(5/2)) - 7*sqrt(2)*b**(3/4)*atan(1 + sqrt(2)*b**(1/4)*sqr
t(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(11/4)*d**(5/2))
```

Mathematica [A] time = 0.312241, size = 233, normalized size = 0.78

$$\frac{x\left(-\frac{24a^{3/4}bx^2}{a+bx^2}-32a^{3/4}+21\sqrt{2}b^{3/4}x^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-21\sqrt{2}b^{3/4}x^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+4\right.}{48a^{11/4}(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (x*(-32*a^(3/4) - (24*a^(3/4)*b*x^2)/(a + b*x^2) + 42*Sqrt[2]*b^(
3/4)*x^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 42*S
qrt[2]*b^(3/4)*x^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)] + 21*Sqrt[2]*b^(3/4)*x^(3/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x] - 21*Sqrt[2]*b^(3/4)*x^(3/2)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(48*a^(11/4)*
(d*x)^(5/2))
```

Maple [A] time = 0.023, size = 226, normalized size = 0.8

$$-\frac{2}{3\,a^{2}d}\left(dx\right)^{-\frac{3}{2}}-\frac{b}{2\,a^{2}d\left(bd^{2}x^{2}+ad^{2}\right)}\sqrt{dx}-\frac{7\,b\sqrt{2}}{16\,d^{3}a^{3}}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)-\frac{7\,b\sqrt{2}}{8\,d^{3}a^{3}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)-\frac{7\,b\sqrt{2}}{8\,d^{3}a^{3}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2/3/a^2/d/(d*x)^(3/2)-1/2/d/a^2*b*(d*x)^(1/2)/(b*d^2*x^2+a*d^2)-
7/16/d^3/a^3*b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))-7/8/d^3/a^3*b*(a*d^2/b)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-7/8/d^3/a^3*b*(a*
d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289106, size = 379, normalized size = 1.26

$$-\frac{28\,bx^{2}-84\left(a^{2}bd^{2}x^{3}+a^{3}d^{2}x\right)\sqrt{dx}\left(-\frac{b^{3}}{a^{11}d^{10}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^{3}d^{3}\left(-\frac{b^{3}}{a^{11}d^{10}}\right)^{\frac{1}{4}}}{\sqrt{dx}b+\sqrt{a^{6}d^{6}\sqrt{-\frac{b^{3}}{a^{11}d^{10}}}+b^{2}dx}}\right)+21\left(a^{2}bd^{2}x^{3}+a^{3}d^{2}x\right)\sqrt{dx}\left(-\frac{b^{3}}{a^{11}d^{10}}\right)}{24\left(a^{2}bd^{2}x^{3}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/24*(28*b*x^2 - 84*(a^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d*x)*(-b^3/
(a^11*d^10))^(1/4)*arctan(a^3*d^3*(-b^3/(a^11*d^10))^(1/4)/(sqrt(
d*x)*b + sqrt(a^6*d^6*sqrt(-b^3/(a^11*d^10)) + b^2*d*x))) + 21*(a
^2*b*d^2*x^3 + a^3*d^2*x)*sqrt(d*x)*(-b^3/(a^11*d^10))^(1/4)*log(
7*a^3*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) - 21*(a^2*b*d
^2*x^3 + a^3*d^2*x)*sqrt(d*x)*(-b^3/(a^11*d^10))^(1/4)*log(-7*a^3
*d^3*(-b^3/(a^11*d^10))^(1/4) + 7*sqrt(d*x)*b) + 16*a)/((a^2*b*d^
2*x^3 + a^3*d^2*x)*sqrt(d*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.271856, size = 373, normalized size = 1.24

$$-\frac{\sqrt{dx}b}{2\left(bd^2x^2+ad^2\right)a^2d}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8a^3d^3}-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^3d^3}+\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16a^3d^3}-\frac{2}{3\sqrt{dx}a^2d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(5/2)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(d*x)*b/((b*d^2*x^2 + a*d^2)*a^2*d) - 7/8*sqrt(2)*(a*b^3
*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/(a^3*d^3) - 7/8*sqrt(2)*(a*b^3*d^2)^(1/4)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^
2/b)^(1/4))/(a^3*d^3) - 7/16*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x + s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*d^3) + 7/1
6*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/(a^3*d^3) - 2/3/(sqrt(d*x)*a^2*d^2*x)
```

3.695 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)}\,dx$

Optimal. Leaf size=318

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{13/4}d^{7/2}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{13/4}d^{7/2}}+\frac{9b}{2a^3d^3\sqrt{dx}}-\frac{9}{10a^2d(dx)^{5/2}}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*
d*(d*x)^(5/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) +
(9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sq
rt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1
3/4)*d^(7/2))
```

Rubi [A] time = 0.720129, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{9b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{8\sqrt{2}a^{13/4}d^{7/2}}-\frac{9b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{4\sqrt{2}a^{13/4}d^{7/2}}+\frac{9b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{4\sqrt{2}a^{13/4}d^{7/2}}+\frac{9b}{2a^3d^3\sqrt{dx}}-\frac{9}{10a^2d(dx)^{5/2}}+\frac{1}{2ad(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```


```
[Out] -9/(10*a^2*d*(d*x)^(5/2)) + (9*b)/(2*a^3*d^3*Sqrt[d*x]) + 1/(2*a*
d*(d*x)^(5/2)*(a + b*x^2)) - (9*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) +
(9*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d
])])/(4*Sqrt[2]*a^(13/4)*d^(7/2)) + (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sq
```

```
rt[2]*a^(13/4)*d^(7/2)) - (9*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(8*Sqrt[2]*a^(1
3/4)*d^(7/2))
```

Rubi in Sympy [A] time = 147.993, size = 298, normalized size = 0.94

$$\frac{1}{2ad\left(dx\right)^{\frac{5}{2}}\left(a+bx^{2}\right)}-\frac{9}{10a^{2}d\left(dx\right)^{\frac{5}{2}}}+\frac{9b}{2a^{3}d^{3}\sqrt{dx}}+\frac{9\sqrt{2}b^{\frac{5}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{13}{4}}d^{\frac{7}{2}}}-\frac{9\sqrt{2}b^{\frac{5}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{16a^{\frac{13}{4}}d^{\frac{7}{2}}}-\frac{9\sqrt{2}b^{\frac{5}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{13}{4}}d^{\frac{7}{2}}}+\frac{9\sqrt{2}b^{\frac{5}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8a^{\frac{13}{4}}d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] 1/(2*a*d*(d*x)**(5/2)*(a + b*x**2)) - 9/(10*a**2*d*(d*x)**(5/2))
+ 9*b/(2*a**3*d**3*sqrt(d*x)) + 9*sqrt(2)*b**(5/4)*log(-sqrt(2)*a
**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16
*a**(13/4)*d**(7/2)) - 9*sqrt(2)*b**(5/4)*log(sqrt(2)*a**(1/4)*b*
*(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(16*a**(13/4)
*d**(7/2)) - 9*sqrt(2)*b**(5/4)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*
x)/(a**(1/4)*sqrt(d)))/(8*a**(13/4)*d**(7/2)) + 9*sqrt(2)*b**(5/4
)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(8*a**(
13/4)*d**(7/2))
```

Mathematica [A] time = 0.645192, size = 251, normalized size = 0.79

$$\frac{\sqrt{dx}\left(-32a^{5/4}+45\sqrt{2}b^{5/4}x^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-45\sqrt{2}b^{5/4}x^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-90\sqrt{2}b^{5/4}\right.}{80a^{13/4}d^{4}x^{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)),x]
```

```
[Out] (Sqrt[d*x]*(-32*a^(5/4) + 320*a^(1/4)*b*x^2 + (40*a^(1/4)*b^2*x^4
)/(a + b*x^2) - 90*Sqrt[2]*b^(5/4)*x^(5/2)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)] + 90*Sqrt[2]*b^(5/4)*x^(5/2)*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 45*Sqrt[2]*b^(5/4)*x^(5/2)*Lo
```

```
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 45*Sqr
t[2]*b^(5/4)*x^(5/2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x]))/(80*a^(13/4)*d^4*x^3)
```

Maple [A] time = 0.027, size = 242, normalized size = 0.8

$$-\frac{2}{5\,a^2d}\,(dx)^{-\frac{5}{2}}+4\,\frac{b}{a^3d^3\sqrt{dx}}+\frac{b^2}{2\,a^3d^3\,(bd^2x^2+ad^2)}\,(dx)^{\frac{3}{2}}+\frac{9\,b\sqrt{2}}{16\,a^3d^3}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{9\,b\sqrt{2}}{8\,a^3d^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{9\,b\sqrt{2}}{8\,a^3d^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] -2/5/a^2/d/(d*x)^(5/2)+4*b/a^3/d^3/(d*x)^(1/2)+1/2/d^3*b^2/a^3*(d
*x)^(3/2)/(b*d^2*x^2+a*d^2)+9/16/d^3*b/a^3/(a*d^2/b)^(1/4)*2^(1/2
)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+9/8/d^3*
b/a^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x
)^(1/2)+1)+9/8/d^3*b/a^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291155, size = 423, normalized size = 1.33

$$\frac{180\,b^2x^4 + 144\,abx^2 + 180\,\left(a^3bd^3x^4 + a^4d^3x^2\right)\sqrt{dx}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{1}{4}}\arctan\left(\frac{729\,a^{10}d^{11}\left(-\frac{b^5}{a^{13}d^{14}}\right)^{\frac{3}{4}}}{729\,\sqrt{dx}b^4+\sqrt{-531441\,a^7b^5d^8\sqrt{-\frac{b^5}{a^{13}d^{14}}}+531441\,b^8dx}}\right) + 45\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/40*(180*b^2*x^4 + 144*a*b*x^2 + 180*(a^3*b*d^3*x^4 + a^4*d^3*x^
2)*sqrt(d*x)*(-b^5/(a^13*d^14))^(1/4)*arctan(729*a^10*d^11*(-b^5/
(a^13*d^14))^(3/4)/(729*sqrt(d*x)*b^4 + sqrt(-531441*a^7*b^5*d^8*
sqrt(-b^5/(a^13*d^14)) + 531441*b^8*d*x))) + 45*(a^3*b*d^3*x^4 +
a^4*d^3*x^2)*sqrt(d*x)*(-b^5/(a^13*d^14))^(1/4)*log(729*a^10*d^11
*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 45*(a^3*b*d^3*x^
4 + a^4*d^3*x^2)*sqrt(d*x)*(-b^5/(a^13*d^14))^(1/4)*log(-729*a^10
*d^11*(-b^5/(a^13*d^14))^(3/4) + 729*sqrt(d*x)*b^4) - 16*a^2)/((a
^3*b*d^3*x^4 + a^4*d^3*x^2)*sqrt(d*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{7}{2}}\,(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral(1/((d*x)**(7/2)*(a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.274332, size = 414, normalized size = 1.3

$$\frac{\sqrt{dx}b^2x}{2\,(bd^2x^2+ad^2)a^3d^2}+\frac{9\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4bd^5}+\frac{9\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4bd^5}-\frac{9\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^4bd^5}+\frac{9\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\,\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{16\,a^4bd^5}+\frac{2\,\left(10\,bd^2x^2-ad^2\right)}{5\,\sqrt{dx}a^3d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^(7/2)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x)*b^2*x/((b*d^2*x^2 + a*d^2)*a^3*d^2) + 9/8*sqrt(2)*(
a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*
sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d^5) + 9/8*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x)
)/(a*d^2/b)^(1/4))/(a^4*b*d^5) - 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*l
n(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b
*d^5) + 9/16*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)
^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d^5) + 2/5*(10*b*d^2*x^2
 - a*d^2)/(sqrt(d*x)*a^3*d^5*x^2)
```

3.696 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=368

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{21/4}}-\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}b^{21/4}}-\frac{663ad^9\sqrt{dx}}{64b^5}-\frac{221d^5(dx)^{9/2}}{192b^3(a+bx^2)}-\frac{17d^3(dx)^{13/2}}{48b^2(a+bx^2)^2}-\frac{d(dx)^{17/2}}{6b(a+bx^2)^3}+\frac{663d^7(dx)^{5/2}}{320b^4}$$

```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4)
 - (d*(d*x)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(
48*b^2*(a + b*x^2)^2) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2
)) - (663*a^(5/4)*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) + (663*a^(5/4)*d^(19/
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqr
t[2]*b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4))
```

Rubi [A] time = 0.878508, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{663a^{5/4}d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{21/4}}-\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{21/4}}+\frac{663a^{5/4}d^{19/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}b^{21/4}}-\frac{663ad^9\sqrt{dx}}{64b^5}-\frac{221d^5(dx)^{9/2}}{192b^3(a+bx^2)}-\frac{17d^3(dx)^{13/2}}{48b^2(a+bx^2)^2}-\frac{d(dx)^{17/2}}{6b(a+bx^2)^3}+\frac{663d^7(dx)^{5/2}}{320b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (-663*a*d^9*Sqrt[d*x])/(64*b^5) + (663*d^7*(d*x)^(5/2))/(320*b^4)
 - (d*(d*x)^(17/2))/(6*b*(a + b*x^2)^3) - (17*d^3*(d*x)^(13/2))/(
48*b^2*(a + b*x^2)^2) - (221*d^5*(d*x)^(9/2))/(192*b^3*(a + b*x^2
)) - (663*a^(5/4)*d^(19/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(21/4)) + (663*a^(5/4)*d^(19/
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*b^(21/4)) - (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqr
t[2]*b^(21/4)) + (663*a^(5/4)*d^(19/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*
b^(21/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.355021, size = 283, normalized size = 0.77

$$\frac{d^9\sqrt{dx}\left(-\frac{9945\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}+\frac{9945\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{19890\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}}+\frac{19890\sqrt{2}a^{5/4}\,\mathrm{t}\cdots}{\cdots}\right.}{7680b^{21/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^9*Sqrt[d*x]*(-61440*a*b^(1/4) + 3072*b^(5/4)*x^2 - (1280*a^4*b
^(1/4))/(a + b*x^2)^3 + (7840*a^3*b^(1/4))/(a + b*x^2)^2 - (24680
*a^2*b^(1/4))/(a + b*x^2) - (19890*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x] + (19890*Sqrt[2]*a^(5/4)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x] - (9945*S
qrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x])/Sqrt[x] + (9945*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*a
```

```
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(7680*b^(21/4))
```

Maple [A] time = 0.031, size = 306, normalized size = 0.8

$$\frac{2d^7}{5b^4}(dx)^{\frac{5}{2}} - 8\frac{ad^9\sqrt{dx}}{b^5} - \frac{617\,d^{11}a^2}{192\,b^3\,(bd^2x^2+ad^2)^3}(dx)^{\frac{9}{2}} - \frac{173\,d^{13}a^3}{32\,b^4\,(bd^2x^2+ad^2)^3}(dx)^{\frac{5}{2}} - \frac{151\,d^{15}a^4}{64\,b^5\,(bd^2x^2+ad^2)^3}\sqrt{dx} + \frac{663\,ad^9\sqrt{2}}{512\,b^5}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right) + \frac{663\,ad^9\sqrt{2}}{256\,b^5}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right) + \frac{663\,ad^9\sqrt{2}}{256\,b^5}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/5*d^7*(d*x)^(5/2)/b^4-8*a*d^9*(d*x)^(1/2)/b^5-617/192*d^11/b^3*
a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(9/2)-173/32*d^13/b^4*a^3/(b*d^2*x^
2+a*d^2)^3*(d*x)^(5/2)-151/64*d^15/b^5*a^4/(b*d^2*x^2+a*d^2)^3*(d
*x)^(1/2)+663/512*d^9/b^5*a*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/256*d^9/b^5*a*(a*d^
2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+
663/256*d^9/b^5*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b
)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293018, size = 508, normalized size = 1.38

$$-\frac{39780\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3\,ab^7x^4+3\,a^2b^6x^2+a^3b^5\right)\arctan\left(\frac{\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}b^5}{\sqrt{dx}ad^9+\sqrt{a^2d^{19}x+\sqrt{-\frac{a^5d^{38}}{b^{21}}}b^{10}}}\right)-9945\left(-\frac{a^5d^{38}}{b^{21}}\right)^{\frac{1}{4}}\left(b^8x^6+3\,ab^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3840*(39780*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*
a^2*b^6*x^2 + a^3*b^5)*arctan((-a^5*d^38/b^21)^(1/4)*b^5/(sqrt(d*
x)*a*d^9 + sqrt(a^2*d^19*x + sqrt(-a^5*d^38/b^21)*b^10))) - 9945*
(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a
^3*b^5)*log(663*sqrt(d*x)*a*d^9 + 663*(-a^5*d^38/b^21)^(1/4)*b^5)
 + 9945*(-a^5*d^38/b^21)^(1/4)*(b^8*x^6 + 3*a*b^7*x^4 + 3*a^2*b^6
*x^2 + a^3*b^5)*log(663*sqrt(d*x)*a*d^9 - 663*(-a^5*d^38/b^21)^(1
/4)*b^5) - 4*(384*b^4*d^9*x^8 - 6528*a*b^3*d^9*x^6 - 24973*a^2*b^
2*d^9*x^4 - 27846*a^3*b*d^9*x^2 - 9945*a^4*d^9)*sqrt(d*x))/(b^8*x
^6 + 3*a*b^7*x^4 + 3*a^2*b^6*x^2 + a^3*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27964, size = 459, normalized size = 1.25

$$\frac{1}{7680}d^8\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6}+\frac{994}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/7680*d^8*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2
)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 +
19890*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^6 + 9945*sqrt(2
)*(a*b^3*d^2)^(1/4)*a*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/b^6 - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*ln(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^6 - 40*(
617*sqrt(d*x)*a^2*b^2*d^7*x^4 + 1038*sqrt(d*x)*a^3*b*d^7*x^2 + 45
3*sqrt(d*x)*a^4*d^7)/((b*d^2*x^2 + a*d^2)^3*b^5) + 3072*(sqrt(d*x
)*b^16*d^6*x^2 - 20*sqrt(d*x)*a*b^15*d^6)/(b^20*d^5))
```

3.697 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=350

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{19/4}}-\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}b^{19/4}}-\frac{55d^5(dx)^{7/2}}{64b^3(a+bx^2)}-\frac{5d^3(dx)^{11/2}}{16b^2(a+bx^2)^2}-\frac{d(dx)^{15/2}}{6b(a+bx^2)^3}+\frac{385d^7(dx)^{3/2}}{192b^4}$$

```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^
2)^3) - (5*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*
x)^(7/2))/(64*b^3*(a + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(
19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^
(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4))
```

Rubi [A] time = 0.823642, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{385a^{3/4}d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}b^{19/4}}+\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{19/4}}-\frac{385a^{3/4}d^{17/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}b^{19/4}}-\frac{55d^5(dx)^{7/2}}{64b^3(a+bx^2)}-\frac{5d^3(dx)^{11/2}}{16b^2(a+bx^2)^2}-\frac{d(dx)^{15/2}}{6b(a+bx^2)^3}+\frac{385d^7(dx)^{3/2}}{192b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (385*d^7*(d*x)^(3/2))/(192*b^4) - (d*(d*x)^(15/2))/(6*b*(a + b*x^
2)^3) - (5*d^3*(d*x)^(11/2))/(16*b^2*(a + b*x^2)^2) - (55*d^5*(d*
x)^(7/2))/(64*b^3*(a + b*x^2)) + (385*a^(3/4)*d^(17/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(
19/4)) - (385*a^(3/4)*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(19/4)) - (385*a^(3/4)*d^
(17/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4)) + (385*a^(3/4)*d^(17/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(19/4))
```

Rubi in Sympy [A] time = 163.661, size = 330, normalized size = 0.94

$$-\frac{385\sqrt{2}a^{\frac{3}{4}}d^{\frac{17}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512b^{\frac{19}{4}}}+\frac{385\sqrt{2}a^{\frac{3}{4}}d^{\frac{17}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512b^{\frac{19}{4}}}+\frac{385\sqrt{2}a^{\frac{3}{4}}d^{\frac{17}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256b^{\frac{19}{4}}}-\frac{385\sqrt{2}a^{\frac{3}{4}}d^{\frac{17}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256b^{\frac{19}{4}}}-\frac{d(dx)^{\frac{15}{2}}}{6b(a+bx^2)^3}-\frac{5d^3(dx)^{\frac{11}{2}}}{16b^2(a+bx^2)^2}-\frac{55d^5(dx)^{\frac{7}{2}}}{64b^3(a+bx^2)}+\frac{385d^7(dx)^{\frac{3}{2}}}{192b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -385*sqrt(2)*a**(3/4)*d**(17/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sq
rt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*b**(19/4)) + 385*
sqrt(2)*a**(3/4)*d**(17/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*
sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*b**(19/4)) + 385*sqrt(2
)*a**(3/4)*d**(17/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4
)*sqrt(d)))/(256*b**(19/4)) - 385*sqrt(2)*a**(3/4)*d**(17/2)*atan
(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*b**(19/4
)) - d*(d*x)**(15/2)/(6*b*(a + b*x**2)**3) - 5*d**3*(d*x)**(11/2)
/(16*b**2*(a + b*x**2)**2) - 55*d**5*(d*x)**(7/2)/(64*b**3*(a + b
*x**2)) + 385*d**7*(d*x)**(3/2)/(192*b**4)
```

Mathematica [A] time = 0.288963, size = 275, normalized size = 0.79

$$\frac{d^8\sqrt{dx}\left(-1155\sqrt{2}a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+1155\sqrt{2}a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2310\sqrt{2}a^{3/4}\tan^{-1}\left(\right.\right.}{1536b^{19/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^8*Sqrt[d*x]*(1024*b^(3/4)*x^(3/2) + (256*a^3*b^(3/4)*x^(3/2))/
(a + b*x^2)^3 - (1248*a^2*b^(3/4)*x^(3/2))/(a + b*x^2)^2 + (3048*
a*b^(3/4)*x^(3/2))/(a + b*x^2) + 2310*Sqrt[2]*a^(3/4)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2310*Sqrt[2]*a^(3/4)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 1155*Sqrt[2]*a^(3/4)*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 1155*Sq
rt[2]*a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x]))/(1536*b^(19/4)*Sqrt[x])
```

Maple [A] time = 0.031, size = 290, normalized size = 0.8

$$\frac{2\,d^7}{3\,b^4}(dx)^{\frac{3}{2}}+\frac{127\,d^9a}{64\,b^2\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{11}{2}}+\frac{101\,d^{11}a^2}{32\,b^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{7}{2}}+\frac{257\,d^{13}a^3}{192\,b^4\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{3}{2}}$$
$$-\frac{385\,d^9a\sqrt{2}}{512\,b^5}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$
$$-\frac{385\,d^9a\sqrt{2}}{256\,b^5}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{385\,d^9a\sqrt{2}}{256\,b^5}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2/3*d^7*(d*x)^(3/2)/b^4+127/64*d^9*a/b^2/(b*d^2*x^2+a*d^2)^3*(d*x
)^(11/2)+101/32*d^11*a^2/b^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)+257/
192*d^13*a^3/b^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)-385/512*d^9*a/b^
5/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2)))-385/256*d^9*a/b^5/(a*d^2/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-385/256*d^9*a/b^5/(a*d^2/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285312, size = 508, normalized size = 1.45

$$-\frac{4620 \left(-\frac{a^3 d^{34}}{b^{19}}\right)^{\frac{1}{4}} \left(b^7 x^6 + 3\, a b^6 x^4 + 3\, a^2 b^5 x^2 + a^3 b^4\right) \arctan\left(\frac{\left(-\frac{a^3 d^{34}}{b^{19}}\right)^{\frac{3}{4}} b^{14}}{\sqrt{dx}\, a^2 d^{25} + \sqrt{a^4 d^{51} x - \sqrt{-\frac{a^3 d^{34}}{b^{19}}}\, a^3 b^9 d^{34}}}\right) + 1155 \left(-\frac{a^3 d^{34}}{b^{19}}\right)^{\frac{1}{4}} \left(b^7 x^6 + 3 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(4620*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^
2*b^5*x^2 + a^3*b^4)*arctan((-a^3*d^34/b^19)^(3/4)*b^14/(sqrt(d*x
)*a^2*d^25 + sqrt(a^4*d^51*x - sqrt(-a^3*d^34/b^19)*a^3*b^9*d^34)
)) + 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b
^5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^2*d^25 + 57066625*(-a^
3*d^34/b^19)^(3/4)*b^14) - 1155*(-a^3*d^34/b^19)^(1/4)*(b^7*x^6 +
 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(57066625*sqrt(d*x)*a^
2*d^25 - 57066625*(-a^3*d^34/b^19)^(3/4)*b^14) - 4*(128*b^3*d^8*x
^7 + 765*a*b^2*d^8*x^5 + 990*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(
d*x))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.28108, size = 412, normalized size = 1.18

$$\frac{1}{1536}\,d^7\left(\frac{1024\,\sqrt{dx}dx}{b^4}-\frac{2310\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}-\frac{2310\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^7*(1024*sqrt(d*x)*d*x/b^4 - 2310*sqrt(2)*(a*b^3*d^2)^(3/
4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*
d^2/b)^(1/4))/b^7 - 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^
7 + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 1155*sqrt(2)*(a*b^3*d^2)^(3/4
)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7
 + 8*(381*sqrt(d*x)*a*b^2*d^7*x^5 + 606*sqrt(d*x)*a^2*b*d^7*x^3 +
 257*sqrt(d*x)*a^3*d^7*x)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

3.698 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=350

$$\frac{195\sqrt[4]{a}d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{17/4}} - \frac{195\sqrt[4]{a}d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{17/4}} + \frac{195\sqrt[4]{a}d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{17/4}} - \frac{195\sqrt[4]{a}d^{15/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}b^{17/4}} - \frac{39d^5(dx)^{5/2}}{64b^3(a+bx^2)} - \frac{13d^3(dx)^{9/2}}{48b^2(a+bx^2)^2} - \frac{d(dx)^{13/2}}{6b(a+bx^2)^3} + \frac{195d^7\sqrt{dx}}{64b^4}$$

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^
3) - (13*d^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^
(5/2))/(64*b^3*(a + b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/
4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (195*a^(1/4)*d^(15
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4))
```

Rubi [A] time = 0.803969, antiderivative size = 350, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{195\sqrt[4]{a}d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{17/4}} - \frac{195\sqrt[4]{a}d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}b^{17/4}} + \frac{195\sqrt[4]{a}d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}b^{17/4}} - \frac{195\sqrt[4]{a}d^{15/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}b^{17/4}} - \frac{39d^5(dx)^{5/2}}{64b^3(a+bx^2)} - \frac{13d^3(dx)^{9/2}}{48b^2(a+bx^2)^2} - \frac{d(dx)^{13/2}}{6b(a+bx^2)^3} + \frac{195d^7\sqrt{dx}}{64b^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (195*d^7*Sqrt[d*x])/(64*b^4) - (d*(d*x)^(13/2))/(6*b*(a + b*x^2)^
3) - (13*d^3*(d*x)^(9/2))/(48*b^2*(a + b*x^2)^2) - (39*d^5*(d*x)^
```

```
(5/2))/(64*b^3*(a + b*x^2)) + (195*a^(1/4)*d^(15/2)*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/
4)) - (195*a^(1/4)*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*b^(17/4)) + (195*a^(1/4)*d^(15
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4)) - (195*a^(1/4)*d^(15/2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(256*Sqrt[2]*b^(17/4))
```

Rubi in Sympy [A] time = 166.267, size = 330, normalized size = 0.94

$$\frac{195\sqrt{2}\sqrt[4]{a}d^{\frac{15}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512b^{\frac{17}{4}}} - \frac{195\sqrt{2}\sqrt[4]{a}d^{\frac{15}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512b^{\frac{17}{4}}} + \frac{195\sqrt{2}\sqrt[4]{a}d^{\frac{15}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256b^{\frac{17}{4}}} - \frac{195\sqrt{2}\sqrt[4]{a}d^{\frac{15}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256b^{\frac{17}{4}}} - \frac{d(dx)^{\frac{13}{2}}}{6b(a+bx^2)^3} - \frac{13d^3(dx)^{\frac{9}{2}}}{48b^2(a+bx^2)^2} - \frac{39d^5(dx)^{\frac{5}{2}}}{64b^3(a+bx^2)} + \frac{195d^7\sqrt{dx}}{64b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 195*sqrt(2)*a**(1/4)*d**(15/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqr
t(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*b**(17/4)) - 195*s
qrt(2)*a**(1/4)*d**(15/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*s
qrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*b**(17/4)) + 195*sqrt(2)
*a**(1/4)*d**(15/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)
*sqrt(d)))/(256*b**(17/4)) - 195*sqrt(2)*a**(1/4)*d**(15/2)*atan(
1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*b**(17/4)
) - d*(d*x)**(13/2)/(6*b*(a + b*x**2)**3) - 13*d**3*(d*x)**(9/2)/
(48*b**2*(a + b*x**2)**2) - 39*d**5*(d*x)**(5/2)/(64*b**3*(a + b*
x**2)) + 195*d**7*sqrt(d*x)/(64*b**4)
```

Mathematica [A] time = 0.275741, size = 270, normalized size = 0.77

$$\frac{d^7\sqrt{dx}\left(\frac{256a^3\sqrt[4]{b}}{(a+bx^2)^3} - \frac{1184a^2\sqrt[4]{b}}{(a+bx^2)^2} + \frac{2536a\sqrt[4]{b}}{a+bx^2} + \frac{585\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}} - \frac{585\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}} + \frac{1170\sqrt{2}\sqrt[4]{a}\,\mathrm{t}}{}\right.}{1536b^{17/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^7*Sqrt[d*x]*(3072*b^(1/4) + (256*a^3*b^(1/4))/(a + b*x^2)^3 -
(1184*a^2*b^(1/4))/(a + b*x^2)^2 + (2536*a*b^(1/4))/(a + b*x^2) +
 (1170*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)])/Sqrt[x] - (1170*Sqrt[2]*a^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)])/Sqrt[x] + (585*Sqrt[2]*a^(1/4)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] - (585*Sqrt[
2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/Sqrt[x]))/(1536*b^(17/4))
```

Maple [A] time = 0.03, size = 287, normalized size = 0.8

$$2\,\frac{d^7\sqrt{dx}}{b^4}+\frac{317\,d^9a}{192\,b^2\,(bd^2x^2+ad^2)^3}(dx)^{\frac{9}{2}}+\frac{81\,d^{11}a^2}{32\,b^3\,(bd^2x^2+ad^2)^3}(dx)^{\frac{5}{2}}+\frac{67\,d^{13}a^3}{64\,b^4\,(bd^2x^2+ad^2)^3}\sqrt{dx}$$
$$-\frac{195\,d^7\sqrt{2}}{512\,b^4}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$
$$-\frac{195\,d^7\sqrt{2}}{256\,b^4}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)-\frac{195\,d^7\sqrt{2}}{256\,b^4}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 2*d^7*(d*x)^(1/2)/b^4+317/192*d^9/b^2*a/(b*d^2*x^2+a*d^2)^3*(d*x)
^(9/2)+81/32*d^11/b^3*a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/2)+67/64*d
^13/b^4*a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)-195/512*d^7/b^4*(a*d^
2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2)))-195/256*d^7/b^4*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-195/256*d^7/b^4*(a*d^2/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285247, size = 462, normalized size = 1.32

$$2340 \left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}} \left(b^7x^6 + 3\,ab^6x^4 + 3\,a^2b^5x^2 + a^3b^4\right) \arctan\left(\frac{\left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}} b^4}{\sqrt{dx}d^7 + \sqrt{d^{15}x + \sqrt{-\frac{ad^{30}}{b^{17}}}\,b^8}}\right) - 585 \left(-\frac{ad^{30}}{b^{17}}\right)^{\frac{1}{4}} \left(b^7x^6 + 3\,ab^6x^4 + 3\,a^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(2340*(-a*d^30/b^17)^(1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b
^5*x^2 + a^3*b^4)*arctan((-a*d^30/b^17)^(1/4)*b^4/(sqrt(d*x)*d^7
+ sqrt(d^15*x + sqrt(-a*d^30/b^17)*b^8))) - 585*(-a*d^30/b^17)^(1
/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sqr
t(d*x)*d^7 + 195*(-a*d^30/b^17)^(1/4)*b^4) + 585*(-a*d^30/b^17)^(
1/4)*(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)*log(195*sq
rt(d*x)*d^7 - 195*(-a*d^30/b^17)^(1/4)*b^4) + 4*(384*b^3*d^7*x^6
+ 1469*a*b^2*d^7*x^4 + 1638*a^2*b*d^7*x^2 + 585*a^3*d^7)*sqrt(d*x
))/(b^7*x^6 + 3*a*b^6*x^4 + 3*a^2*b^5*x^2 + a^3*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277488, size = 414, normalized size = 1.18

$$-\frac{1}{1536}\,d^{6}\left(\frac{1170\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}}+\frac{1170\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{b^{5}}+\frac{585\,\sqrt{2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] -1/1536*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 11
70*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^5 + 585*sqrt(2)*(a*b
^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/b^5 - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x - sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^5 - 3072*sqrt(d*x)*
d/b^4 - 8*(317*sqrt(d*x)*a*b^2*d^7*x^4 + 486*sqrt(d*x)*a^2*b*d^7*
x^2 + 201*sqrt(d*x)*a^3*d^7)/((b*d^2*x^2 + a*d^2)^3*b^4))
```

3.699 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^2} \, dx$

Optimal. Leaf size=333

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}} + \frac{77d^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^5(dx)^{3/2}}{192b^3(a+bx^2)} - \frac{11d^3(dx)^{7/2}}{48b^2(a+bx^2)^2} - \frac{d(dx)^{11/2}}{6b(a+bx^2)^3}$$

```
[Out] -(d*(d*x)^(11/2))/(6*b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*
b^2*(a + b*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) -
 (77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1
/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^
(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4)
)
```

Rubi [A] time = 0.721298, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}} + \frac{77d^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}\sqrt[4]{a}b^{15/4}} - \frac{77d^5(dx)^{3/2}}{192b^3(a+bx^2)} - \frac{11d^3(dx)^{7/2}}{48b^2(a+bx^2)^2} - \frac{d(dx)^{11/2}}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(11/2))/(6*b*(a + b*x^2)^3) - (11*d^3*(d*x)^(7/2))/(48*
b^2*(a + b*x^2)^2) - (77*d^5*(d*x)^(3/2))/(192*b^3*(a + b*x^2)) -
```

```
(77*d^(13/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(128*Sqrt[2]*a^(1/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(1
/4)*b^(15/4)) + (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^
(15/4)) - (77*d^(13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(1/4)*b^(15/4)
)
```

Rubi in Sympy [A] time = 147.988, size = 313, normalized size = 0.94

$$-\frac{d\,(dx)^{\frac{11}{2}}}{6b\,(a+bx^2)^3}-\frac{11d^3\,(dx)^{\frac{7}{2}}}{48b^2\,(a+bx^2)^2}-\frac{77d^5\,(dx)^{\frac{3}{2}}}{192b^3\,(a+bx^2)}+\frac{77\sqrt{2}d^{\frac{13}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512\sqrt[4]{a}b^{\frac{15}{4}}}-\frac{77\sqrt{2}d^{\frac{13}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512\sqrt[4]{a}b^{\frac{15}{4}}}-\frac{77\sqrt{2}d^{\frac{13}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256\sqrt[4]{a}b^{\frac{15}{4}}}+\frac{77\sqrt{2}d^{\frac{13}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256\sqrt[4]{a}b^{\frac{15}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -d*(d*x)**(11/2)/(6*b*(a + b*x**2)**3) - 11*d**3*(d*x)**(7/2)/(48
*b**2*(a + b*x**2)**2) - 77*d**5*(d*x)**(3/2)/(192*b**3*(a + b*x*
*2)) + 77*sqrt(2)*d**(13/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d
)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(1/4)*b**(15/4)) -
 77*sqrt(2)*d**(13/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(
d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(1/4)*b**(15/4)) - 77*sqr
t(2)*d**(13/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt
(d)))/(256*a**(1/4)*b**(15/4)) + 77*sqrt(2)*d**(13/2)*atan(1 + sq
rt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(1/4)*b**(15
/4))
```

Mathematica [A] time = 0.270199, size = 260, normalized size = 0.78

$$\frac{d^6\sqrt{dx}\left(-\frac{256a^2b^{3/4}x^{3/2}}{(a+bx^2)^3}+\frac{864ab^{3/4}x^{3/2}}{(a+bx^2)^2}-\frac{1224b^{3/4}x^{3/2}}{a+bx^2}+\frac{231\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}-\frac{231\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}}-\frac{462\sqrt{2}\,\mathrm{ta}\cdots}{\cdots}\right.}{1536b^{15/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^6*Sqrt[d*x]*((-256*a^2*b^(3/4)*x^(3/2))/(a + b*x^2)^3 + (864*a
*b^(3/4)*x^(3/2))/(a + b*x^2)^2 - (1224*b^(3/4)*x^(3/2))/(a + b*x
^2) - (462*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])
/a^(1/4) + (462*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)])/a^(1/4) + (231*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/a^(1/4) - (231*Sqrt[2]*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(1/4)))/(1536*b^(15/4
)*Sqrt[x])
```

Maple [A] time = 0.027, size = 271, normalized size = 0.8

$$-\frac{51\,d^7}{64\,(bd^2x^2+ad^2)^3\,b}(dx)^{\frac{11}{2}}-\frac{33\,d^9a}{32\,(bd^2x^2+ad^2)^3\,b^2}(dx)^{\frac{7}{2}}-\frac{77\,d^{11}a^2}{192\,(bd^2x^2+ad^2)^3\,b^3}(dx)^{\frac{3}{2}}+\frac{77\,d^7\sqrt{2}}{512\,b^4}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{77\,d^7\sqrt{2}}{256\,b^4}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{77\,d^7\sqrt{2}}{256\,b^4}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -51/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(11/2)-33/32*d^9/(b*d^2*x^
2+a*d^2)^3/b^2*a*(d*x)^(7/2)-77/192*d^11/(b*d^2*x^2+a*d^2)^3/b^3*
a^2*(d*x)^(3/2)+77/512*d^7/b^4/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+77/256*d^7/b^4/(a*d^
2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+
77/256*d^7/b^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285922, size = 474, normalized size = 1.42

$$924\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{26}}{ab^{15}}\right)^{\frac{3}{4}}ab^{11}}{\sqrt{dx}\,d^{19}+\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{ab^{15}}}\,ab^7d^{26}}}\right)+231\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(924*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^
26/(a*b^15))^(1/4)*arctan((-d^26/(a*b^15))^(3/4)*a*b^11/(sqrt(d*x
)*d^19 + sqrt(d^39*x - sqrt(-d^26/(a*b^15))*a*b^7*d^26))) + 231*(
b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^26/(a*b^15))
^(1/4)*log(456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a*b^15))^(3/4)*
a*b^11) - 231*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(
-d^26/(a*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 - 456533*(-d^26/(
a*b^15))^(3/4)*a*b^11) - 4*(153*b^2*d^6*x^5 + 198*a*b*d^6*x^3 + 7
7*a^2*d^6*x)*sqrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 +
a^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278115, size = 408, normalized size = 1.23

$$\frac{1}{1536}d^5\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}-\frac{231\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^5*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 46
2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) - 231*sqrt(2)*(a*
b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a*b^6) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^6) - 8*(153*sqr
t(d*x)*b^2*d^7*x^5 + 198*sqrt(d*x)*a*b*d^7*x^3 + 77*sqrt(d*x)*a^2
*d^7*x)/((b*d^2*x^2 + a*d^2)^3*b^3))
```

3.700 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=333

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{3/4}b^{13/4}}-\frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)}-\frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2}-\frac{d(dx)^{9/2}}{6b(a+bx^2)^3}$$

```
[Out] -(d*(d*x)^(9/2))/(6*b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^
2*(a + b*x^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*
d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b
^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4
)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.715746, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{15d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{3/4}b^{13/4}}-\frac{15d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{3/4}b^{13/4}}+\frac{15d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{3/4}b^{13/4}}-\frac{15d^5\sqrt{dx}}{64b^3(a+bx^2)}-\frac{3d^3(dx)^{5/2}}{16b^2(a+bx^2)^2}-\frac{d(dx)^{9/2}}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(9/2))/(6*b*(a + b*x^2)^3) - (3*d^3*(d*x)^(5/2))/(16*b^
2*(a + b*x^2)^2) - (15*d^5*Sqrt[d*x])/(64*b^3*(a + b*x^2)) - (15*
```

```
d^(11/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(128*Sqrt[2]*a^(3/4)*b^(13/4)) + (15*d^(11/2)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(3/4)*b
^(13/4)) - (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4
)) + (15*d^(11/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(3/4)*b^(13/4))
```

Rubi in Sympy [A] time = 152.628, size = 313, normalized size = 0.94

$$-\frac{d\,(dx)^{\frac{9}{2}}}{6b\,(a+bx^2)^3}-\frac{3d^3\,(dx)^{\frac{5}{2}}}{16b^2\,(a+bx^2)^2}-\frac{15d^5\sqrt{dx}}{64b^3\,(a+bx^2)}-\frac{15\sqrt{2}d^{\frac{11}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{3}{4}}b^{\frac{13}{4}}}+\frac{15\sqrt{2}d^{\frac{11}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{3}{4}}b^{\frac{13}{4}}}-\frac{15\sqrt{2}d^{\frac{11}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{3}{4}}b^{\frac{13}{4}}}+\frac{15\sqrt{2}d^{\frac{11}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{3}{4}}b^{\frac{13}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -d*(d*x)**(9/2)/(6*b*(a + b*x**2)**3) - 3*d**3*(d*x)**(5/2)/(16*b
**2*(a + b*x**2)**2) - 15*d**5*sqrt(d*x)/(64*b**3*(a + b*x**2)) -
 15*sqrt(2)*d**(11/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt
(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(3/4)*b**(13/4)) + 15*sq
rt(2)*d**(11/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) +
 sqrt(a)*d + sqrt(b)*d*x)/(512*a**(3/4)*b**(13/4)) - 15*sqrt(2)*d
**(11/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/
(256*a**(3/4)*b**(13/4)) + 15*sqrt(2)*d**(11/2)*atan(1 + sqrt(2)*
b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(3/4)*b**(13/4))
```

Mathematica [A] time = 0.267893, size = 260, normalized size = 0.78

$$\frac{d^5\sqrt{dx}\left(-\frac{45\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\frac{45\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}-\frac{90\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\frac{90\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{3/4}\sqrt{x}}-\frac{256a\ldots}{(a+b\ldots}\right.}{1536b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-256*a^2*b^(1/4))/(a + b*x^2)^3 + (800*a*b^(1/4)
)/(a + b*x^2)^2 - (904*b^(1/4))/(a + b*x^2) - (90*Sqrt[2]*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sqrt[x]) + (90*S
qrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3/4)*Sq
rt[x]) - (45*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x
] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x]) + (45*Sqrt[2]*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x])))/(1
536*b^(13/4))
```

Maple [A] time = 0.027, size = 280, normalized size = 0.8

$$-\frac{113\,d^7}{192\,(bd^2x^2+ad^2)^3\,b}(dx)^{\frac{9}{2}}-\frac{21\,d^9a}{32\,(bd^2x^2+ad^2)^3\,b^2}(dx)^{\frac{5}{2}}-\frac{15\,d^{11}a^2}{64\,(bd^2x^2+ad^2)^3\,b^3}\sqrt{dx}+\frac{15\,d^5\sqrt{2}}{512\,ab^3}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{15\,d^5\sqrt{2}}{256\,ab^3}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{15\,d^5\sqrt{2}}{256\,ab^3}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -113/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(9/2)-21/32*d^9/(b*d^2*x
^2+a*d^2)^3/b^2*a*(d*x)^(5/2)-15/64*d^11/(b*d^2*x^2+a*d^2)^3/b^3*
a^2*(d*x)^(1/2)+15/512*d^5/b^3*(a*d^2/b)^(1/4)/a*2^(1/2)*ln((d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+15/256*d^5/b^3*(a*
d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)
+1)+15/256*d^5/b^3*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^
2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288441, size = 470, normalized size = 1.41

$$-\frac{180\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3b^3\right)\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{22}}{a^3b^{13}}\right)^{\frac{1}{4}}ab^3}{\sqrt{dx}d^5+\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^3b^{13}}}a^2b^6}}\right)-45\left(b^6x^6+3\,ab^5x^4+3\,a^2b^4x^2+a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(180*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d
^22/(a^3*b^13))^(1/4)*arctan((-d^22/(a^3*b^13))^(1/4)*a*b^3/(sqrt
(d*x)*d^5 + sqrt(d^11*x + sqrt(-d^22/(a^3*b^13))*a^2*b^6))) - 45*
(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/(a^3*b^1
3))^(1/4)*log(15*sqrt(d*x)*d^5 + 15*(-d^22/(a^3*b^13))^(1/4)*a*b^
3) + 45*(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)*(-d^22/
(a^3*b^13))^(1/4)*log(15*sqrt(d*x)*d^5 - 15*(-d^22/(a^3*b^13))^(1
/4)*a*b^3) + 4*(113*b^2*d^5*x^4 + 126*a*b*d^5*x^2 + 45*a^2*d^5)*s
qrt(d*x))/(b^6*x^6 + 3*a*b^5*x^4 + 3*a^2*b^4*x^2 + a^3*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27997, size = 412, normalized size = 1.24

$$\frac{1}{1536}d^4\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^4}+\frac{45\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^4*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 9
0*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2
/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^4) + 45*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sq
rt(a*d^2/b))/(a*b^4) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x - sq
rt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^4) - 8*(113
*sqrt(d*x)*b^2*d^7*x^4 + 126*sqrt(d*x)*a*b*d^7*x^2 + 45*sqrt(d*x)
*a^2*d^7)/((b*d^2*x^2 + a*d^2)^3*b^3))
```

3.701 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=336

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{5/4}b^{11/4}}+\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{5/4}b^{11/4}}+\frac{7d^3(dx)^{3/2}}{64ab^2(a+bx^2)}-\frac{7d^3(dx)^{3/2}}{48b^2(a+bx^2)^2}-\frac{d(dx)^{7/2}}{6b(a+bx^2)^3}$$

```
[Out] -(d*(d*x)^(7/2))/(6*b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^
2*(a + b*x^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (
7*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^
(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
- (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi [A] time = 0.735634, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{7d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{5/4}b^{11/4}}-\frac{7d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{5/4}b^{11/4}}+\frac{7d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{5/4}b^{11/4}}+\frac{7d^3(dx)^{3/2}}{64ab^2(a+bx^2)}-\frac{7d^3(dx)^{3/2}}{48b^2(a+bx^2)^2}-\frac{d(dx)^{7/2}}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] -(d*(d*x)^(7/2))/(6*b*(a + b*x^2)^3) - (7*d^3*(d*x)^(3/2))/(48*b^
2*(a + b*x^2)^2) + (7*d^3*(d*x)^(3/2))/(64*a*b^2*(a + b*x^2)) - (
7*d^(9/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d]
)])/(128*Sqrt[2]*a^(5/4)*b^(11/4)) + (7*d^(9/2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(5/4)*b^
```

```
(11/4)) + (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
- (7*d^(9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(5/4)*b^(11/4))
```

Rubi in Sympy [A] time = 149.427, size = 314, normalized size = 0.93

$$-\frac{d\,(dx)^{\frac{7}{2}}}{6b\,(a+bx^2)^3}-\frac{7d^3\,(dx)^{\frac{3}{2}}}{48b^2\,(a+bx^2)^2}+\frac{7d^3\,(dx)^{\frac{3}{2}}}{64ab^2\,(a+bx^2)}+\frac{7\sqrt{2}d^{\frac{9}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{5}{4}}b^{\frac{11}{4}}}-\frac{7\sqrt{2}d^{\frac{9}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{5}{4}}b^{\frac{11}{4}}}-\frac{7\sqrt{2}d^{\frac{9}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{5}{4}}b^{\frac{11}{4}}}+\frac{7\sqrt{2}d^{\frac{9}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{5}{4}}b^{\frac{11}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -d*(d*x)**(7/2)/(6*b*(a + b*x**2)**3) - 7*d**3*(d*x)**(3/2)/(48*b
**2*(a + b*x**2)**2) + 7*d**3*(d*x)**(3/2)/(64*a*b**2*(a + b*x**2
)) + 7*sqrt(2)*d**(9/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sq
rt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(5/4)*b**(11/4)) - 7*s
qrt(2)*d**(9/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) +
 sqrt(a)*d + sqrt(b)*d*x)/(512*a**(5/4)*b**(11/4)) - 7*sqrt(2)*d*
*(9/2)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(2
56*a**(5/4)*b**(11/4)) + 7*sqrt(2)*d**(9/2)*atan(1 + sqrt(2)*b**(
1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(5/4)*b**(11/4))
```

Mathematica [A] time = 0.287899, size = 260, normalized size = 0.77

$$\frac{d^4\sqrt{dx}\left(\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}}+\frac{168b^{3/\cdots}}{a^2+a\cdots}\right.}{1536b^{11/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^4*Sqrt[d*x]*((256*a*b^(3/4)*x^(3/2))/(a + b*x^2)^3 - (480*b^(3
/4)*x^(3/2))/(a + b*x^2)^2 + (168*b^(3/4)*x^(3/2))/(a^2 + a*b*x^2
```

```
) - (42*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^
(5/4) + (42*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
)/a^(5/4) + (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/a^(5/4) - (21*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4)))/(1536*b^(11/4)*Sqrt
[x])
```

Maple [A] time = 0.029, size = 277, normalized size = 0.8

$$\frac{7\,d^5}{64\,(bd^2x^2+ad^2)^3\,a}(dx)^{\frac{11}{2}}-\frac{3\,d^7}{32\,(bd^2x^2+ad^2)^3\,b}(dx)^{\frac{7}{2}}-\frac{7\,d^9a}{192\,(bd^2x^2+ad^2)^3\,b^2}(dx)^{\frac{3}{2}}+\frac{7\,d^5\sqrt{2}}{512\,ab^3}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{7\,d^5\sqrt{2}}{256\,ab^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{7\,d^5\sqrt{2}}{256\,ab^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(11/2)-3/32*d^7/(b*d^2*x^2+a
*d^2)^3/b*(d*x)^(7/2)-7/192*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(
3/2)+7/512*d^5/a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+7/256*d^5/a/b^3/(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+7/256*d^5
/a/b^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286097, size = 501, normalized size = 1.49

$$84\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{18}}{a^5b^{11}}\right)^{\frac{3}{4}}a^4b^8}{\sqrt{dx}d^{13}+\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{a^5b^{11}}}a^3b^5d^{18}}}\right)+21\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(84*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(
-d^18/(a^5*b^11))^(1/4)*arctan((-d^18/(a^5*b^11))^(3/4)*a^4*b^8/(
sqrt(d*x)*d^13 + sqrt(d^27*x - sqrt(-d^18/(a^5*b^11))*a^3*b^5*d^1
8))) + 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(
-d^18/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 + 343*(-d^18/(a^5*
b^11))^(3/4)*a^4*b^8) - 21*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3
*x^2 + a^4*b^2)*(-d^18/(a^5*b^11))^(1/4)*log(343*sqrt(d*x)*d^13 -
 343*(-d^18/(a^5*b^11))^(3/4)*a^4*b^8) + 4*(21*b^2*d^4*x^5 - 18*a
*b*d^4*x^3 - 7*a^2*d^4*x)*sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 +
 3*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281037, size = 412, normalized size = 1.23

$$\frac{1}{1536}d^3\left(\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{42\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^3*(42*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt
(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 4
2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) - 21*sqrt(2)*(a
*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a^2*b^5) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt
(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^5) + 8*(21*
sqrt(d*x)*b^2*d^7*x^5 - 18*sqrt(d*x)*a*b*d^7*x^3 - 7*sqrt(d*x)*a^
2*d^7*x)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

3.702 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^3\sqrt{dx}}{192ab^2(a+bx^2)}-\frac{5d^3\sqrt{dx}}{48b^2(a+bx^2)^2}-\frac{d(dx)^{5/2}}{6b(a+bx^2)^3}$$

```
[Out] -(d*(d*x)^(5/2))/(6*b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*
(a + b*x^2)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d
^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(128*Sqrt[2]*a^(7/4)*b^(9/4)) + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4
)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4)) + (5*d
^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.748829, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{5d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{7/4}b^{9/4}}-\frac{5d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{7/4}b^{9/4}}+\frac{5d^3\sqrt{dx}}{192ab^2(a+bx^2)}-\frac{5d^3\sqrt{dx}}{48b^2(a+bx^2)^2}-\frac{d(dx)^{5/2}}{6b(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(5/2))/(6*b*(a + b*x^2)^3) - (5*d^3*Sqrt[d*x])/(48*b^2*
(a + b*x^2)^2) + (5*d^3*Sqrt[d*x])/(192*a*b^2*(a + b*x^2)) - (5*d
^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(128*Sqrt[2]*a^(7/4)*b^(9/4)) + (5*d^(7/2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(7/4)*b^(9/4
```

```
)) - (5*d^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4)) + (5*d
^(7/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi in Sympy [A] time = 152.356, size = 314, normalized size = 0.93

$$-\frac{d\left(dx\right)^{\frac{5}{2}}}{6b\left(a+bx^{2}\right)^{3}}-\frac{5d^{3}\sqrt{dx}}{48b^{2}\left(a+bx^{2}\right)^{2}}+\frac{5d^{3}\sqrt{dx}}{192ab^{2}\left(a+bx^{2}\right)}-\frac{5\sqrt{2}d^{\frac{7}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{5\sqrt{2}d^{\frac{7}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{7}{4}}b^{\frac{9}{4}}}-\frac{5\sqrt{2}d^{\frac{7}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{7}{4}}b^{\frac{9}{4}}}+\frac{5\sqrt{2}d^{\frac{7}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{7}{4}}b^{\frac{9}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -d*(d*x)**(5/2)/(6*b*(a + b*x**2)**3) - 5*d**3*sqrt(d*x)/(48*b**2
*(a + b*x**2)**2) + 5*d**3*sqrt(d*x)/(192*a*b**2*(a + b*x**2)) -
5*sqrt(2)*d**(7/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*
x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(7/4)*b**(9/4)) + 5*sqrt(2)
*d**(7/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(
a)*d + sqrt(b)*d*x)/(512*a**(7/4)*b**(9/4)) - 5*sqrt(2)*d**(7/2)*
atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(
7/4)*b**(9/4)) + 5*sqrt(2)*d**(7/2)*atan(1 + sqrt(2)*b**(1/4)*sqr
t(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(7/4)*b**(9/4))
```

Mathematica [A] time = 0.325501, size = 260, normalized size = 0.77

$$\frac{d^{3}\sqrt{dx}\left(-\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}+\frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}-\frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}}+\frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}\sqrt{x}}+\frac{40\cdots}{a^{2}+a\cdots}\cdots\right)}{1536b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d^3*Sqrt[d*x]*((256*a*b^(1/4))/(a + b*x^2)^3 - (416*b^(1/4))/(a
+ b*x^2)^2 + (40*b^(1/4))/(a^2 + a*b*x^2) - (30*Sqrt[2]*ArcTan[1
```

```
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt[x]) + (30*Sqr
t[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqrt
[x]) - (15*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x])/(a^(7/4)*Sqrt[x]) + (15*Sqrt[2]*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x])))/(153
6*b^(9/4))
```

Maple [A] time = 0.027, size = 277, normalized size = 0.8

$$\frac{5\,d^5}{192\,(bd^2x^2+ad^2)^3\,a}(dx)^{\frac{9}{2}}-\frac{7\,d^7}{32\,(bd^2x^2+ad^2)^3\,b}(dx)^{\frac{5}{2}}-\frac{5\,d^9a}{64\,(bd^2x^2+ad^2)^3\,b^2}\sqrt{dx}+\frac{5\,d^3\sqrt{2}}{512\,a^2b^2}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{5\,d^3\sqrt{2}}{256\,a^2b^2}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{5\,d^3\sqrt{2}}{256\,a^2b^2}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/192*d^5/(b*d^2*x^2+a*d^2)^3/a*(d*x)^(9/2)-7/32*d^7/(b*d^2*x^2+a
*d^2)^3/b*(d*x)^(5/2)-5/64*d^9/(b*d^2*x^2+a*d^2)^3/b^2*a*(d*x)^(1
/2)+5/512*d^3/a^2/b^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+5/256*d^3/a^2/b^2*(a*d^2/b)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+5/256*
d^3/a^2/b^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290716, size = 494, normalized size = 1.47

$$-\frac{60\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a^4b^2\right)\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b^2\left(-\frac{d^{14}}{a^7b^9}\right)^{\frac{1}{4}}}{\sqrt{dx}d^3+\sqrt{a^4b^4\sqrt{-\frac{d^{14}}{a^7b^9}}+d^7x}}\right)-15\left(ab^5x^6+3\,a^2b^4x^4+3\,a^3b^3x^2+a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(60*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*
(-d^14/(a^7*b^9))^(1/4)*arctan(a^2*b^2*(-d^14/(a^7*b^9))^(1/4)/(s
qrt(d*x)*d^3 + sqrt(a^4*b^4*sqrt(-d^14/(a^7*b^9)) + d^7*x))) - 15
*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*(-d^14/(a^
7*b^9))^(1/4)*log(5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) + 5*sqrt(d*x)
*d^3) + 15*(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2)*
(-d^14/(a^7*b^9))^(1/4)*log(-5*a^2*b^2*(-d^14/(a^7*b^9))^(1/4) +
5*sqrt(d*x)*d^3) - 4*(5*b^2*d^3*x^4 - 42*a*b*d^3*x^2 - 15*a^2*d^3
)*sqrt(d*x))/(a*b^5*x^6 + 3*a^2*b^4*x^4 + 3*a^3*b^3*x^2 + a^4*b^2
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280278, size = 416, normalized size = 1.24

$$\frac{1}{1536}d^2\left(\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{30\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^3}+\frac{15\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d^2*(30*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) +
 30*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^3) + 15*sqrt(2
)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^2*b^3) - 15*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^3)
+ 8*(5*sqrt(d*x)*b^2*d^7*x^4 - 42*sqrt(d*x)*a*b*d^7*x^2 - 15*sqrt
(d*x)*a^2*d^7)/((b*d^2*x^2 + a*d^2)^3*a*b^2))
```

3.703 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=335

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{9/4}b^{7/4}}+\frac{5d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{9/4}b^{7/4}}+\frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)}+\frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

```
[Out] -(d*(d*x)^(3/2))/(6*b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a
 + b*x^2)^2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5
/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4))
+ (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) - (5*d^(5
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi [A] time = 0.741866, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{5d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{9/4}b^{7/4}}-\frac{5d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{9/4}b^{7/4}}+\frac{5d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{9/4}b^{7/4}}+\frac{5d(dx)^{3/2}}{64a^2b\,(a+bx^2)}+\frac{d(dx)^{3/2}}{16ab\,(a+bx^2)^2}-\frac{d(dx)^{3/2}}{6b\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] -(d*(d*x)^(3/2))/(6*b*(a + b*x^2)^3) + (d*(d*x)^(3/2))/(16*a*b*(a
 + b*x^2)^2) + (5*d*(d*x)^(3/2))/(64*a^2*b*(a + b*x^2)) - (5*d^(5
/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(9/4)*b^(7/4)) + (5*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(9/4)*b^(7/4))
```

```
+ (5*d^(5/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4)) - (5*d^(5
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(9/4)*b^(7/4))
```

Rubi in Sympy [A] time = 148.37, size = 309, normalized size = 0.92

$$-\frac{d(dx)^{\frac{3}{2}}}{6b(a+bx^2)^3}+\frac{d(dx)^{\frac{3}{2}}}{16ab(a+bx^2)^2}+\frac{5d(dx)^{\frac{3}{2}}}{64a^2b(a+bx^2)}+\frac{5\sqrt{2}d^{\frac{5}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{9}{4}}b^{\frac{7}{4}}}-\frac{5\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{9}{4}}b^{\frac{7}{4}}}-\frac{5\sqrt{2}d^{\frac{5}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{9}{4}}b^{\frac{7}{4}}}+\frac{5\sqrt{2}d^{\frac{5}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{9}{4}}b^{\frac{7}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -d*(d*x)**(3/2)/(6*b*(a + b*x**2)**3) + d*(d*x)**(3/2)/(16*a*b*(a
 + b*x**2)**2) + 5*d*(d*x)**(3/2)/(64*a**2*b*(a + b*x**2)) + 5*sq
rt(2)*d**(5/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) +
 sqrt(a)*d + sqrt(b)*d*x)/(512*a**(9/4)*b**(7/4)) - 5*sqrt(2)*d**
(5/2)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d
 + sqrt(b)*d*x)/(512*a**(9/4)*b**(7/4)) - 5*sqrt(2)*d**(5/2)*atan
(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(9/4)
*b**(7/4)) + 5*sqrt(2)*d**(5/2)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*
x)/(a**(1/4)*sqrt(d)))/(256*a**(9/4)*b**(7/4))
```

Mathematica [A] time = 0.327327, size = 259, normalized size = 0.77

$$\frac{(dx)^{5/2}\left(\frac{15\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}-\frac{15\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}-\frac{30\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}}+\frac{30\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}}+\frac{120b^{3/}}{a^2(a+}\right.}{1536b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(5/2)*((-256*b^(3/4)*x^(3/2))/(a + b*x^2)^3 + (96*b^(3/4)*
x^(3/2))/(a*(a + b*x^2)^2) + (120*b^(3/4)*x^(3/2))/(a^2*(a + b*x^
```

```
2)) - (30*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/
a^(9/4) + (30*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/a^(9/4) + (15*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x])/a^(9/4) - (15*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4)))/(1536*b^(7/4)*x^(
5/2))
```

Maple [A] time = 0.026, size = 277, normalized size = 0.8

$$\frac{5\,d^3b}{64\,(bd^2x^2+ad^2)^3\,a^2}(dx)^{\frac{11}{2}}+\frac{7\,d^5}{32\,(bd^2x^2+ad^2)^3\,a}(dx)^{\frac{7}{2}}-\frac{5\,d^7}{192\,(bd^2x^2+ad^2)^3\,b}(dx)^{\frac{3}{2}}+\frac{5\,d^3\sqrt{2}}{512\,a^2b^2}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{5\,d^3\sqrt{2}}{256\,a^2b^2}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{5\,d^3\sqrt{2}}{256\,a^2b^2}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 5/64*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(11/2)+7/32*d^5/(b*d^2*x
^2+a*d^2)^3/a*(d*x)^(7/2)-5/192*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(
3/2)+5/512*d^3/a^2/b^2/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+5/256*d^3/a^2/b^2/(a*d^2/b)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+5/256
*d^3/a^2/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288391, size = 505, normalized size = 1.51

$$\frac{60 \left(a^{2} b^{4} x^{6} + 3 \, a^{3} b^{3} x^{4} + 3 \, a^{4} b^{2} x^{2} + a^{5} b\right) \left(-\frac{d^{10}}{a^{9} b^{7}}\right)^{\frac{1}{4}} \arctan\left(\frac{125 \, a^{7} b^{5} \left(-\frac{d^{10}}{a^{9} b^{7}}\right)^{\frac{3}{4}}}{125 \, \sqrt{d x} d^{7} + \sqrt{-15625 \, a^{5} b^{3} d^{10} \sqrt{-\frac{d^{10}}{a^{9} b^{7}}} + 15625 \, d^{15} x}}\right) + 15 \left(a^{2} b^{4} x^{6} + 3 \, a^{3} b^{3} x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(60*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(
-d^10/(a^9*b^7))^(1/4)*arctan(125*a^7*b^5*(-d^10/(a^9*b^7))^(3/4)
/(125*sqrt(d*x)*d^7 + sqrt(-15625*a^5*b^3*d^10*sqrt(-d^10/(a^9*b^
7)) + 15625*d^15*x))) + 15*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b
^2*x^2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(125*a^7*b^5*(-d^10/(a
^9*b^7))^(3/4) + 125*sqrt(d*x)*d^7) - 15*(a^2*b^4*x^6 + 3*a^3*b^3
*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^10/(a^9*b^7))^(1/4)*log(-125*a^
7*b^5*(-d^10/(a^9*b^7))^(3/4) + 125*sqrt(d*x)*d^7) + 4*(15*b^2*d^
2*x^5 + 42*a*b*d^2*x^3 - 5*a^2*d^2*x)*sqrt(d*x))/(a^2*b^4*x^6 + 3
*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{(a + bx^2)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(5/2)/(a + b*x**2)**4, x)
```

GIAC/XCAS [A] time = 0.277815, size = 409, normalized size = 1.22

$$\frac{1}{1536} d \left(\frac{30 \sqrt{2} \left(ab^3 d^2\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} + 2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3 b^4} + \frac{30 \sqrt{2} \left(ab^3 d^2\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}} - 2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3 b^4} - \frac{15 \sqrt{2} \left(ab^3 d^2\right)^{\frac{3}{4}} \ln}{} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/1536*d*(30*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 30*
sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) - 15*sqrt(2)*(a*b
^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(a^3*b^4) + 15*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^4) + 8*(15*sq
rt(d*x)*b^2*d^7*x^5 + 42*sqrt(d*x)*a*b*d^7*x^3 - 5*sqrt(d*x)*a^2*
d^7*x)/((b*d^2*x^2 + a*d^2)^3*a^2*b))
```

3.704 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

```
[Out] -(d*Sqrt[d*x])/(6*b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b
*x^2)^2) + (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sq
rt[2]*a^(11/4)*b^(5/4)) + (7*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) - (
7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d^(3/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi [A] time = 0.736708, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{7d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{11/4}b^{5/4}}-\frac{7d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{11/4}b^{5/4}}+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2}-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```


```
[Out] -(d*Sqrt[d*x])/(6*b*(a + b*x^2)^3) + (d*Sqrt[d*x])/(48*a*b*(a + b
*x^2)^2) + (7*d*Sqrt[d*x])/(192*a^2*b*(a + b*x^2)) - (7*d^(3/2)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sq
rt[2]*a^(11/4)*b^(5/4)) + (7*d^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(11/4)*b^(5/4)) - (
```

```
7*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4)) + (7*d^(3/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(11/4)*b^(5/4))
```

Rubi in Sympy [A] time = 151.361, size = 309, normalized size = 0.92

$$-\frac{d\sqrt{dx}}{6b\left(a+bx^2\right)^3}+\frac{d\sqrt{dx}}{48ab\left(a+bx^2\right)^2}+\frac{7d\sqrt{dx}}{192a^2b\left(a+bx^2\right)}-\frac{7\sqrt{2}d^{\frac{3}{2}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{7\sqrt{2}d^{\frac{3}{2}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{11}{4}}b^{\frac{5}{4}}}-\frac{7\sqrt{2}d^{\frac{3}{2}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{11}{4}}b^{\frac{5}{4}}}+\frac{7\sqrt{2}d^{\frac{3}{2}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{11}{4}}b^{\frac{5}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] -d*sqrt(d*x)/(6*b*(a + b*x**2)**3) + d*sqrt(d*x)/(48*a*b*(a + b*x
**2)**2) + 7*d*sqrt(d*x)/(192*a**2*b*(a + b*x**2)) - 7*sqrt(2)*d*
*(3/2)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)
*d + sqrt(b)*d*x)/(512*a**(11/4)*b**(5/4)) + 7*sqrt(2)*d**(3/2)*l
og(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt
(b)*d*x)/(512*a**(11/4)*b**(5/4)) - 7*sqrt(2)*d**(3/2)*atan(1 - s
qrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(11/4)*b**(
5/4)) + 7*sqrt(2)*d**(3/2)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a
**(1/4)*sqrt(d)))/(256*a**(11/4)*b**(5/4))
```

Mathematica [A] time = 0.30085, size = 260, normalized size = 0.78

$$\frac{d\sqrt{dx}\left(-\frac{21\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}+\frac{21\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}-\frac{42\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\frac{42\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}\sqrt{x}}+\frac{56\sqrt[4]{\ldots}}{a^2(a+\ldots}\right.}{1536b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*Sqrt[d*x]*((-256*b^(1/4))/(a + b*x^2)^3 + (32*b^(1/4))/(a*(a +
 b*x^2)^2) + (56*b^(1/4))/(a^2*(a + b*x^2)) - (42*Sqrt[2]*ArcTan[
```

```
1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqrt[x]) + (42*
Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*
Sqrt[x]) - (21*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (21*Sqrt[2]*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]))
)/(1536*b^(5/4))
```

Maple [A] time = 0.026, size = 271, normalized size = 0.8

$$\frac{7\,d^3b}{192\,(bd^2x^2+ad^2)^3\,a^2}(dx)^{\frac{9}{2}}+\frac{3\,d^5}{32\,(bd^2x^2+ad^2)^3\,a}(dx)^{\frac{5}{2}}-\frac{7\,d^7}{64\,(bd^2x^2+ad^2)^3\,b}\sqrt{dx}$$
$$+\frac{7\,d\sqrt{2}}{512\,a^3b}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$
$$+\frac{7\,d\sqrt{2}}{256\,a^3b}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{7\,d\sqrt{2}}{256\,a^3b}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 7/192*d^3/(b*d^2*x^2+a*d^2)^3/a^2*b*(d*x)^(9/2)+3/32*d^5/(b*d^2*x
^2+a*d^2)^3/a*(d*x)^(5/2)-7/64*d^7/(b*d^2*x^2+a*d^2)^3/b*(d*x)^(1
/2)+7/512*d/a^3/b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+7/256*d/a^3/b*(a*d^2/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+7/256*d/a^3/b*
(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286952, size = 470, normalized size = 1.4

$$-\frac{84\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+a^5b\right)\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3b\left(-\frac{d^6}{a^{11}b^5}\right)^{\frac{1}{4}}}{\sqrt{dx}d+\sqrt{a^6b^2\sqrt{-\frac{d^6}{a^{11}b^5}}+d^3x}}\right)-21\left(a^2b^4x^6+3\,a^3b^3x^4+3\,a^4b^2x^2+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] -1/768*(84*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*
(-d^6/(a^11*b^5))^(1/4)*arctan(a^3*b*(-d^6/(a^11*b^5))^(1/4)/(sqr
t(d*x)*d + sqrt(a^6*b^2*sqrt(-d^6/(a^11*b^5)) + d^3*x))) - 21*(a^
2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a^11*b^
5))^(1/4)*log(7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x)*d) +
21*(a^2*b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)*(-d^6/(a
^11*b^5))^(1/4)*log(-7*a^3*b*(-d^6/(a^11*b^5))^(1/4) + 7*sqrt(d*x
)*d) - 4*(7*b^2*d*x^4 + 18*a*b*d*x^2 - 21*a^2*d)*sqrt(d*x))/(a^2*
b^4*x^6 + 3*a^3*b^3*x^4 + 3*a^4*b^2*x^2 + a^5*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{3}{2}}}{(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2)**4, x)
```

GIAC/XCAS [A] time = 0.279978, size = 409, normalized size = 1.22

$$\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^3b^2}+\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^3b^2}$$
$$+\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^3b^2}$$
$$-\frac{7\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^3b^2}$$
$$+\frac{7\sqrt{dx}b^2d^7x^4+18\sqrt{dx}abd^7x^2-21\sqrt{dx}a^2d^7}{192\left(bd^2x^2+ad^2\right)^3a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 7/256*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 7/256*sq
rt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^
(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^2) + 7/512*sqrt(2)*(
a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + s
qrt(a*d^2/b))/(a^3*b^2) - 7/512*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^2)
+ 1/192*(7*sqrt(d*x)*b^2*d^7*x^4 + 18*sqrt(d*x)*a*b*d^7*x^2 - 21*
sqrt(d*x)*a^2*d^7)/((b*d^2*x^2 + a*d^2)^3*a^2*b)
```

3.705 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=335

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{13/4}b^{3/4}}+\frac{15\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{13/4}b^{3/4}}+\frac{15(dx)^{3/2}}{64a^3d\,(a+bx^2)}+\frac{3(dx)^{3/2}}{16a^2d\,(a+bx^2)^2}+\frac{(dx)^{3/2}}{6ad\,(a+bx^2)^3}$$

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a
+ b*x^2)^2) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[
d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)
) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4)) - (15
*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rubi [A] time = 0.729592, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$\frac{15\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{13/4}b^{3/4}}-\frac{15\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{13/4}b^{3/4}}+\frac{15\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{13/4}b^{3/4}}+\frac{15(dx)^{3/2}}{64a^3d\,(a+bx^2)}+\frac{3(dx)^{3/2}}{16a^2d\,(a+bx^2)^2}+\frac{(dx)^{3/2}}{6ad\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*x)^(3/2)/(6*a*d*(a + b*x^2)^3) + (3*(d*x)^(3/2))/(16*a^2*d*(a
+ b*x^2)^2) + (15*(d*x)^(3/2))/(64*a^3*d*(a + b*x^2)) - (15*Sqrt[
d]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(12
8*Sqrt[2]*a^(13/4)*b^(3/4)) + (15*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(13/4)*b^(3/4)
```

```
) + (15*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4)) - (15
*Sqrt[d]*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(13/4)*b^(3/4))
```

Rubi in Sympy [A] time = 147.626, size = 309, normalized size = 0.92

$$\frac{(dx)^{\frac{3}{2}}}{6ad\left(a+bx^2\right)^3}+\frac{3\,(dx)^{\frac{3}{2}}}{16a^2d\left(a+bx^2\right)^2}+\frac{15\,(dx)^{\frac{3}{2}}}{64a^3d\left(a+bx^2\right)}+\frac{15\sqrt{2}\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{13}{4}}b^{\frac{3}{4}}}-\frac{15\sqrt{2}\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{13}{4}}b^{\frac{3}{4}}}-\frac{15\sqrt{2}\sqrt{d}\,\mathrm{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{13}{4}}b^{\frac{3}{4}}}+\frac{15\sqrt{2}\sqrt{d}\,\mathrm{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{13}{4}}b^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (d*x)**(3/2)/(6*a*d*(a + b*x**2)**3) + 3*(d*x)**(3/2)/(16*a**2*d*
(a + b*x**2)**2) + 15*(d*x)**(3/2)/(64*a**3*d*(a + b*x**2)) + 15*
sqrt(2)*sqrt(d)*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x)
+ sqrt(a)*d + sqrt(b)*d*x)/(512*a**(13/4)*b**(3/4)) - 15*sqrt(2)*
sqrt(d)*log(sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)
*d + sqrt(b)*d*x)/(512*a**(13/4)*b**(3/4)) - 15*sqrt(2)*sqrt(d)*a
tan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(1
3/4)*b**(3/4)) + 15*sqrt(2)*sqrt(d)*atan(1 + sqrt(2)*b**(1/4)*sqr
t(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(13/4)*b**(3/4))
```

Mathematica [A] time = 0.260261, size = 253, normalized size = 0.76

$$\frac{\sqrt{dx}\left(\frac{256a^{9/4}x^{3/2}}{\left(a+bx^2\right)^3}+\frac{288a^{5/4}x^{3/2}}{\left(a+bx^2\right)^2}+\frac{45\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}-\frac{45\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}-\frac{90\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{b^{3/4}}+90\sqrt{2}\,\mathrm{ta}\ldots\right.}{1536a^{13/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (Sqrt[d*x]*((256*a^(9/4)*x^(3/2))/(a + b*x^2)^3 + (288*a^(5/4)*x^
(3/2))/(a + b*x^2)^2 + (360*a^(1/4)*x^(3/2))/(a + b*x^2) - (90*Sq
rt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (9
0*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4)
+ (45*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqr
t[b]*x])/b^(3/4) - (45*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4)))/(1536*a^(13/4)*Sqrt[x])
```

Maple [A] time = 0.026, size = 272, normalized size = 0.8

$$\frac{15\,b^2d}{64\,(bd^2x^2+ad^2)^3\,a^3}(dx)^{\frac{11}{2}}+\frac{21\,d^3b}{32\,(bd^2x^2+ad^2)^3\,a^2}(dx)^{\frac{7}{2}}+\frac{113\,d^5}{192\,(bd^2x^2+ad^2)^3\,a}(dx)^{\frac{3}{2}}+\frac{15\,d\sqrt{2}}{512\,a^3b}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{15\,d\sqrt{2}}{256\,a^3b}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{15\,d\sqrt{2}}{256\,a^3b}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] 15/64*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(11/2)+21/32*d^3/(b*d^2
*x^2+a*d^2)^3/a^2*b*(d*x)^(7/2)+113/192*d^5/(b*d^2*x^2+a*d^2)^3/a
*(d*x)^(3/2)+15/512*d/a^3/b/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+15/256*d/a^3/b/(a*d^2/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+15/
256*d/a^3/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4
)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288892, size = 460, normalized size = 1.37

$$180\left(a^3b^3x^6+3\,a^4b^2x^4+3\,a^5bx^2+a^6\right)\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{3375\,a^{10}b^2\left(-\frac{d^2}{a^{13}b^3}\right)^{\frac{3}{4}}}{3375\,\sqrt{dx}\,d+\sqrt{-11390625\,a^7bd^2\sqrt{-\frac{d^2}{a^{13}b^3}}+11390625\,d^3x}}\right)+45\left(a^3b^3x^6+3\,a^4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] 1/768*(180*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b*x^2 + a^6)*(-d^
2/(a^13*b^3))^(1/4)*arctan(3375*a^10*b^2*(-d^2/(a^13*b^3))^(3/4)/
(3375*sqrt(d*x)*d + sqrt(-11390625*a^7*b*d^2*sqrt(-d^2/(a^13*b^3)
) + 11390625*d^3*x))) + 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^4 + 3*a^5*b
*x^2 + a^6)*(-d^2/(a^13*b^3))^(1/4)*log(3375*a^10*b^2*(-d^2/(a^13
*b^3))^(3/4) + 3375*sqrt(d*x)*d) - 45*(a^3*b^3*x^6 + 3*a^4*b^2*x^
4 + 3*a^5*b*x^2 + a^6)*(-d^2/(a^13*b^3))^(1/4)*log(-3375*a^10*b^2
*(-d^2/(a^13*b^3))^(3/4) + 3375*sqrt(d*x)*d) + 4*(45*b^2*x^5 + 12
6*a*b*x^3 + 113*a^2*x)*sqrt(d*x))/(a^3*b^3*x^6 + 3*a^4*b^2*x^4 +
3*a^5*b*x^2 + a^6)
```

Sympy [A] time = 136.218, size = 252, normalized size = 0.75

$$\frac{226a^2d^{11}(dx)^{\frac{3}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{252abd^9(dx)^{\frac{7}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+\frac{90b^2d^7(dx)^{\frac{11}{2}}}{384a^6d^{12}+1152a^5bd^{12}x^2+1152a^4b^2d^{12}x^4+384a^3b^3d^{12}x^6}+2d^7\operatorname{RootSum}\left(68719476736t^4a^{13}b^3d^{26}+50625,\left(t\mapsto t\log\left(\frac{134217728t^3a^{10}b^2d^{20}}{3375}+\sqrt{dx}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 226*a**2*d**11*(d*x)**(3/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x
**2 + 1152*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 252
*a*b*d**9*(d*x)**(7/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 +
```

```
 1152*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 90*b**2*
d**7*(d*x)**(11/2)/(384*a**6*d**12 + 1152*a**5*b*d**12*x**2 + 115
2*a**4*b**2*d**12*x**4 + 384*a**3*b**3*d**12*x**6) + 2*d**7*RootS
um(68719476736*_t**4*a**13*b**3*d**26 + 50625, Lambda(_t, _t*log(
134217728*_t**3*a**10*b**2*d**20/3375 + sqrt(d*x))))
```

GIAC/XCAS [A] time = 0.278439, size = 417, normalized size = 1.24

$$\frac{45\sqrt{dx}b^2d^6x^5 + 126\sqrt{dx}abd^6x^3 + 113\sqrt{dx}a^2d^6x}{192\left(bd^2x^2 + ad^2\right)^3a^3} + \frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4b^3d} + \frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4b^3d} - \frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx + \sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx} + \sqrt{\frac{ad^2}{b}}\right)}{512\,a^4b^3d} + \frac{15\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx - \sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx} + \sqrt{\frac{ad^2}{b}}\right)}{512\,a^4b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] 1/192*(45*sqrt(d*x)*b^2*d^6*x^5 + 126*sqrt(d*x)*a*b*d^6*x^3 + 113
*sqrt(d*x)*a^2*d^6*x)/((b*d^2*x^2 + a*d^2)^3*a^3) + 15/256*sqrt(2
)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) +
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3*d) + 15/256*sqrt(2)*(a*b^
3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3*d) - 15/512*sqrt(2)*(a*b^3*d^2)
^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^4*b^3*d) + 15/512*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)
*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^3*d)
```

3.706 $\int \frac{1}{\sqrt{dx}(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=335

$$-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}-\frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b
*x^2)^2) + (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^
(15/4)*b^(1/4)*Sqrt[d]) + (77*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) -
(77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rubi [A] time = 0.741447, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{77\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}-\frac{77\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{15/4}\sqrt[4]{b}\sqrt{d}}+\frac{77\sqrt{dx}}{192a^3d\left(a+bx^2\right)}+\frac{11\sqrt{dx}}{48a^2d\left(a+bx^2\right)^2}+\frac{\sqrt{dx}}{6ad\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] Sqrt[d*x]/(6*a*d*(a + b*x^2)^3) + (11*Sqrt[d*x])/(48*a^2*d*(a + b
*x^2)^2) + (77*Sqrt[d*x])/(192*a^3*d*(a + b*x^2)) - (77*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^
(15/4)*b^(1/4)*Sqrt[d]) + (77*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) -
```

```
(77*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d]) + (77*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(256*Sqrt[2]*a^(15/4)*b^(1/4)*Sqrt[d])
```

Rubi in Sympy [A] time = 151.186, size = 309, normalized size = 0.92

$$\frac{\sqrt{dx}}{6ad\left(a+bx^{2}\right)^{3}}+\frac{11\sqrt{dx}}{48a^{2}d\left(a+bx^{2}\right)^{2}}+\frac{77\sqrt{dx}}{192a^{3}d\left(a+bx^{2}\right)} - \frac{77\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{15}{4}}\sqrt[4]{b}\sqrt{d}}+\frac{77\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{15}{4}}\sqrt[4]{b}\sqrt{d}} - \frac{77\sqrt{2}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{15}{4}}\sqrt[4]{b}\sqrt{d}}+\frac{77\sqrt{2}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{15}{4}}\sqrt[4]{b}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] sqrt(d*x)/(6*a*d*(a + b*x**2)**3) + 11*sqrt(d*x)/(48*a**2*d*(a +
b*x**2)**2) + 77*sqrt(d*x)/(192*a**3*d*(a + b*x**2)) - 77*sqrt(2)
*log(-sqrt(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + s
qrt(b)*d*x)/(512*a**(15/4)*b**(1/4)*sqrt(d)) + 77*sqrt(2)*log(sqr
t(2)*a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*
x)/(512*a**(15/4)*b**(1/4)*sqrt(d)) - 77*sqrt(2)*atan(1 - sqrt(2)
*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(15/4)*b**(1/4)*s
qrt(d)) + 77*sqrt(2)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4
)*sqrt(d)))/(256*a**(15/4)*b**(1/4)*sqrt(d))
```

Mathematica [A] time = 0.270083, size = 253, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{256a^{11/4}\sqrt{x}}{\left(a+bx^{2}\right)^{3}}+\frac{352a^{7/4}\sqrt{x}}{\left(a+bx^{2}\right)^{2}}+\frac{616a^{3/4}\sqrt{x}}{a+bx^{2}}-\frac{231\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{231\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}-\frac{462\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a}}\right.}{\sqrt[4]{b}}\right.}{1536a^{15/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[x]*((256*a^(11/4)*Sqrt[x])/(a + b*x^2)^3 + (352*a^(7/4)*Sqr
t[x])/(a + b*x^2)^2 + (616*a^(3/4)*Sqrt[x])/(a + b*x^2) - (462*Sq
rt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (4
62*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4)
 - (231*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/b^(1/4) + (231*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4)))/(1536*a^(15/4)*Sqrt[d*x])
```

Maple [A] time = 0.026, size = 269, normalized size = 0.8

$$\frac{77\,b^2d}{192\,(bd^2x^2+ad^2)^3\,a^3}(dx)^{\frac{9}{2}}+\frac{33\,d^3b}{32\,(bd^2x^2+ad^2)^3\,a^2}(dx)^{\frac{5}{2}}+\frac{51\,d^5}{64\,(bd^2x^2+ad^2)^3\,a}\sqrt{dx}$$
$$+\frac{77\sqrt{2}}{512\,da^4}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$
$$+\frac{77\sqrt{2}}{256\,da^4}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{77\sqrt{2}}{256\,da^4}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^2/(d*x)^(1/2),x)
```

```
[Out] 77/192*d/(b*d^2*x^2+a*d^2)^3/a^3*b^2*(d*x)^(9/2)+33/32*d^3/(b*d^2
*x^2+a*d^2)^3/a^2*b*(d*x)^(5/2)+51/64*d^5/(b*d^2*x^2+a*d^2)^3/a*(
d*x)^(1/2)+77/512/d/a^4*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+77/256/d/a^4*(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+77/256/d/
a^4*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^
(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286165, size = 456, normalized size = 1.36

$$-\frac{924\left(a^{3}b^{3}dx^{6}+3\,a^{4}b^{2}dx^{4}+3\,a^{5}bdx^{2}+a^{6}d\right)\left(-\frac{1}{a^{15}bd^{2}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^{4}d\left(-\frac{1}{a^{15}bd^{2}}\right)^{\frac{1}{4}}}{\sqrt{a^{8}d^{2}\sqrt{-\frac{1}{a^{15}bd^{2}}}+dx}+\sqrt{dx}}\right)-231\left(a^{3}b^{3}dx^{6}+3\,a^{4}b^{2}dx^{4}+3\,\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] -1/768*(924*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^
6*d)*(-1/(a^15*b*d^2))^(1/4)*arctan(a^4*d*(-1/(a^15*b*d^2))^(1/4)
/(sqrt(a^8*d^2*sqrt(-1/(a^15*b*d^2)) + d*x) + sqrt(d*x))) - 231*(
a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1/(a^1
5*b*d^2))^(1/4)*log(a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*x)) +
231*(a^3*b^3*d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)*(-1
/(a^15*b*d^2))^(1/4)*log(-a^4*d*(-1/(a^15*b*d^2))^(1/4) + sqrt(d*
x)) - 4*(77*b^2*x^4 + 198*a*b*x^2 + 153*a^2)*sqrt(d*x))/(a^3*b^3*
d*x^6 + 3*a^4*b^2*d*x^4 + 3*a^5*b*d*x^2 + a^6*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\,(a+bx^2)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**2/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2)**4), x)
```

GIAC/XCAS [A] time = 0.274027, size = 416, normalized size = 1.24

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^4bd}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{512\,a^4bd}+\frac{77\sqrt{dx}b^2d^5x^4+198\sqrt{dx}abd^5x^2+153\sqrt{dx}a^2d^5}{192\left(bd^2x^2+ad^2\right)^3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] 77/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/256*sq
rt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b*d) + 77/512*sqrt(2)*(a
*b^3*d^2)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(a^4*b*d) - 77/512*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x -
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b*d) + 1/
192*(77*sqrt(d*x)*b^2*d^5*x^4 + 198*sqrt(d*x)*a*b*d^5*x^2 + 153*s
qrt(d*x)*a^2*d^5)/((b*d^2*x^2 + a*d^2)^3*a^3)
```

3.707 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=352

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}-\frac{195\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{17/4}d^{3/2}}-\frac{195}{64a^4d\sqrt{dx}}+\frac{39}{64a^3d\sqrt{dx}\,(a+bx^2)}+\frac{13}{48a^2d\sqrt{dx}\,(a+bx^2)^2}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 1
3/(48*a^2*d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a
+ b*x^2)) + (195*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*S
qrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]
*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17
/4)*d^(3/2))
```

Rubi [A] time = 0.852021, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{195\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{17/4}d^{3/2}}+\frac{195\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{17/4}d^{3/2}}-\frac{195\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{17/4}d^{3/2}}-\frac{195}{64a^4d\sqrt{dx}}+\frac{39}{64a^3d\sqrt{dx}\,(a+bx^2)}+\frac{13}{48a^2d\sqrt{dx}\,(a+bx^2)^2}+\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] -195/(64*a^4*d*Sqrt[d*x]) + 1/(6*a*d*Sqrt[d*x]*(a + b*x^2)^3) + 1
3/(48*a^2*d*Sqrt[d*x]*(a + b*x^2)^2) + 39/(64*a^3*d*Sqrt[d*x]*(a
+ b*x^2)) + (195*b^(1/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(
a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*S
```

```
qrt[2]*a^(17/4)*d^(3/2)) - (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]
*a^(17/4)*d^(3/2)) + (195*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(17
/4)*d^(3/2))
```

Rubi in Sympy [A] time = 165.108, size = 326, normalized size = 0.93

$$\frac{1}{6ad\sqrt{dx}\,(a+bx^2)^3}+\frac{13}{48a^2d\sqrt{dx}\,(a+bx^2)^2}+\frac{39}{64a^3d\sqrt{dx}\,(a+bx^2)}-\frac{195}{64a^4d\sqrt{dx}}-\frac{195\sqrt{2}\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{17}{4}}d^{\frac{3}{2}}}+\frac{195\sqrt{2}\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{17}{4}}d^{\frac{3}{2}}}+\frac{195\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{17}{4}}d^{\frac{3}{2}}}-\frac{195\sqrt{2}\sqrt[4]{b}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{17}{4}}d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 1/(6*a*d*sqrt(d*x)*(a + b*x**2)**3) + 13/(48*a**2*d*sqrt(d*x)*(a
+ b*x**2)**2) + 39/(64*a**3*d*sqrt(d*x)*(a + b*x**2)) - 195/(64*a
**4*d*sqrt(d*x)) - 195*sqrt(2)*b**(1/4)*log(-sqrt(2)*a**(1/4)*b**
(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(17/4)
*d**(3/2)) + 195*sqrt(2)*b**(1/4)*log(sqrt(2)*a**(1/4)*b**(1/4)*s
qrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(17/4)*d**(3/
2)) + 195*sqrt(2)*b**(1/4)*atan(1 - sqrt(2)*b**(1/4)*sqrt(d*x)/(a
**(1/4)*sqrt(d)))/(256*a**(17/4)*d**(3/2)) - 195*sqrt(2)*b**(1/4)
*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**
(17/4)*d**(3/2))
```

Mathematica [A] time = 0.254574, size = 273, normalized size = 0.78

$$\frac{x\left(-\frac{256a^{9/4}bx^2}{(a+bx^2)^3}-\frac{672a^{5/4}bx^2}{(a+bx^2)^2}-\frac{1608\sqrt[4]{a}bx^2}{a+bx^2}-585\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+585\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\right.\right.}{1536a^{17/4}(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (x*(-3072*a^(1/4) - (256*a^(9/4)*b*x^2)/(a + b*x^2)^3 - (672*a^(5
/4)*b*x^2)/(a + b*x^2)^2 - (1608*a^(1/4)*b*x^2)/(a + b*x^2) + 117
0*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)] - 1170*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*b^(1/4)*Sqrt[x]*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*b^(1/4)*
Sqrt[x]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
]))/(1536*a^(17/4)*(d*x)^(3/2))
```

Maple [A] time = 0.031, size = 285, normalized size = 0.8

$$
\begin{aligned}
&-2\frac{1}{a^4d\sqrt{dx}}-\frac{67\,b^3}{64\,a^4d\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{11}{2}}\\
&-\frac{81\,b^2d}{32\,a^3\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{7}{2}}-\frac{317\,d^3b}{192\,a^2\left(bd^2x^2+ad^2\right)^3}(dx)^{\frac{3}{2}}\\
&-\frac{195\sqrt{2}}{512\,a^4d}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}\\
&-\frac{195\sqrt{2}}{256\,a^4d}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{195\sqrt{2}}{256\,a^4d}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2/a^4/d/(d*x)^(1/2)-67/64/d*b^3/a^4/(b*d^2*x^2+a*d^2)^3*(d*x)^(1
1/2)-81/32*d*b^2/a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(7/2)-317/192*d^3*
b/a^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/2)-195/512/d/a^4/(a*d^2/b)^(1/
4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
-195/256/d/a^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)+1)-195/256/d/a^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296927, size = 500, normalized size = 1.42

$$-\frac{2340\,b^3x^6 + 6552\,ab^2x^4 + 5876\,a^2bx^2 + 1536\,a^3 + 2340\left(a^4b^3dx^6 + 3\,a^5b^2dx^4 + 3\,a^6bdx^2 + a^7d\right)\sqrt{dx}\left(-\frac{b}{a^{17}d^6}\right)^{\frac{1}{4}}\arctan\left(\frac{\quad}{741}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/768*(2340*b^3*x^6 + 6552*a*b^2*x^4 + 5876*a^2*b*x^2 + 1536*a^3
 + 2340*(a^4*b^3*d*x^6 + 3*a^5*b^2*d*x^4 + 3*a^6*b*d*x^2 + a^7*d)
*sqrt(d*x)*(-b/(a^17*d^6))^(1/4)*arctan(7414875*a^13*d^5*(-b/(a^1
7*d^6))^(3/4)/(7414875*sqrt(d*x)*b + sqrt(-54980371265625*a^9*b*d
^4*sqrt(-b/(a^17*d^6)) + 54980371265625*b^2*d*x))) + 585*(a^4*b^3
*d*x^6 + 3*a^5*b^2*d*x^4 + 3*a^6*b*d*x^2 + a^7*d)*sqrt(d*x)*(-b/(
a^17*d^6))^(1/4)*log(7414875*a^13*d^5*(-b/(a^17*d^6))^(3/4) + 741
4875*sqrt(d*x)*b) - 585*(a^4*b^3*d*x^6 + 3*a^5*b^2*d*x^4 + 3*a^6*
b*d*x^2 + a^7*d)*sqrt(d*x)*(-b/(a^17*d^6))^(1/4)*log(-7414875*a^1
3*d^5*(-b/(a^17*d^6))^(3/4) + 7414875*sqrt(d*x)*b))/((a^4*b^3*d*x
^6 + 3*a^5*b^2*d*x^4 + 3*a^6*b*d*x^2 + a^7*d)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277814, size = 441, normalized size = 1.25

$$-\frac{\frac{3072}{\sqrt{dx}a^4}+\frac{8\left(201\sqrt{dx}b^3d^5x^5+486\sqrt{dx}ab^2d^5x^3+317\sqrt{dx}a^2bd^5x\right)}{(bd^2x^2+ad^2)^3a^4}+\frac{1170\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^2d^2}+\frac{1170\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\cdots\right.\right.}{2\cdots}\right.}{a^5b^2d^2}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/1536*(3072/(sqrt(d*x)*a^4) + 8*(201*sqrt(d*x)*b^3*d^5*x^5 + 48
6*sqrt(d*x)*a*b^2*d^5*x^3 + 317*sqrt(d*x)*a^2*b*d^5*x)/((b*d^2*x^
2 + a*d^2)^3*a^4) + 1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqr
t(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
5*b^2*d^2) + 1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2*
d^2) - 585*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2*d^2) + 585*sqrt(2)*(a*b^
3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^5*b^2*d^2))/d
```

3.708 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^2} dx$

Optimal. Leaf size=352

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{19/4}d^{5/2}}+\frac{385b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{19/4}d^{5/2}}-\frac{385}{192a^4d(dx)^{3/2}}+\frac{55}{64a^3d(dx)^{3/2}(a+bx^2)}+\frac{5}{16a^2d(dx)^{3/2}(a+bx^2)^2}+\frac{1}{6ad(dx)^{3/2}(a+bx^2)^3}$$

```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3
) + 5/(16*a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(
3/2)*(a + b*x^2)) + (385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*
b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256
*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[
2]*a^(19/4)*d^(5/2))
```

Rubi [A] time = 0.825306, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{385b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{256\sqrt{2}a^{19/4}d^{5/2}}+\frac{385b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{19/4}d^{5/2}}-\frac{385b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{19/4}d^{5/2}}-\frac{385}{192a^4d(dx)^{3/2}}+\frac{55}{64a^3d(dx)^{3/2}(a+bx^2)}+\frac{5}{16a^2d(dx)^{3/2}(a+bx^2)^2}+\frac{1}{6ad(dx)^{3/2}(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```



```
[Out] -385/(192*a^4*d*(d*x)^(3/2)) + 1/(6*a*d*(d*x)^(3/2)*(a + b*x^2)^3
) + 5/(16*a^2*d*(d*x)^(3/2)*(a + b*x^2)^2) + 55/(64*a^3*d*(d*x)^(
3/2)*(a + b*x^2)) + (385*b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*
b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
```

```
)/(128*Sqrt[2]*a^(19/4)*d^(5/2)) + (385*b^(3/4)*Log[Sqrt[a]*Sqrt[
d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256
*Sqrt[2]*a^(19/4)*d^(5/2)) - (385*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[
2]*a^(19/4)*d^(5/2))
```

Rubi in Sympy [A] time = 169.256, size = 326, normalized size = 0.93

$$\frac{1}{6ad\left(dx\right)^{\frac{3}{2}}\left(a+bx^{2}\right)^{3}}+\frac{5}{16a^{2}d\left(dx\right)^{\frac{3}{2}}\left(a+bx^{2}\right)^{2}}+\frac{55}{64a^{3}d\left(dx\right)^{\frac{3}{2}}\left(a+bx^{2}\right)}-\frac{385}{192a^{4}d\left(dx\right)^{\frac{3}{2}}}+\frac{385\sqrt{2}b^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{19}{4}}d^{\frac{5}{2}}}-\frac{385\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{d}\sqrt{dx}+\sqrt{a}d+\sqrt{b}dx\right)}{512a^{\frac{19}{4}}d^{\frac{5}{2}}}+\frac{385\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{19}{4}}d^{\frac{5}{2}}}-\frac{385\sqrt{2}b^{\frac{3}{4}}\operatorname{atan}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{256a^{\frac{19}{4}}d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] 1/(6*a*d*(d*x)**(3/2)*(a + b*x**2)**3) + 5/(16*a**2*d*(d*x)**(3/2
)*(a + b*x**2)**2) + 55/(64*a**3*d*(d*x)**(3/2)*(a + b*x**2)) - 3
85/(192*a**4*d*(d*x)**(3/2)) + 385*sqrt(2)*b**(3/4)*log(-sqrt(2)*
a**(1/4)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(5
12*a**(19/4)*d**(5/2)) - 385*sqrt(2)*b**(3/4)*log(sqrt(2)*a**(1/4
)*b**(1/4)*sqrt(d)*sqrt(d*x) + sqrt(a)*d + sqrt(b)*d*x)/(512*a**(
19/4)*d**(5/2)) + 385*sqrt(2)*b**(3/4)*atan(1 - sqrt(2)*b**(1/4)*
sqrt(d*x)/(a**(1/4)*sqrt(d)))/(256*a**(19/4)*d**(5/2)) - 385*sqrt
(2)*b**(3/4)*atan(1 + sqrt(2)*b**(1/4)*sqrt(d*x)/(a**(1/4)*sqrt(d
)))/(256*a**(19/4)*d**(5/2))
```

Mathematica [A] time = 0.302912, size = 273, normalized size = 0.78

$$\frac{x\left(-\frac{256a^{11/4}bx^{2}}{\left(a+bx^{2}\right)^{3}}-\frac{736a^{7/4}bx^{2}}{\left(a+bx^{2}\right)^{2}}-\frac{2056a^{3/4}bx^{2}}{a+bx^{2}}-1024a^{3/4}+1155\sqrt{2}b^{3/4}x^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1155\sqrt{2}b^{3/4}x^{3/2}\ \cdots}{1536a^{19/4}(dx)\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (x*(-1024*a^(3/4) - (256*a^(11/4)*b*x^2)/(a + b*x^2)^3 - (736*a^(
7/4)*b*x^2)/(a + b*x^2)^2 - (2056*a^(3/4)*b*x^2)/(a + b*x^2) + 23
10*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)] - 2310*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)] + 1155*Sqrt[2]*b^(3/4)*x^(3/2)*Log[Sqrt[a] -
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1155*Sqrt[2]*b^(3/
4)*x^(3/2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x]))/(1536*a^(19/4)*(d*x)^(5/2))
```

Maple [A] time = 0.032, size = 288, normalized size = 0.8

$$-\frac{2}{3\,a^4 d}(dx)^{-\frac{3}{2}}-\frac{257\,b^3}{192\,a^4 d\,(bd^2x^2+ad^2)^3}(dx)^{\frac{9}{2}}-\frac{101\,b^2 d}{32\,a^3\,(bd^2x^2+ad^2)^3}(dx)^{\frac{5}{2}}-\frac{127\,d^3 b}{64\,a^2\,(bd^2x^2+ad^2)^3}\sqrt{dx}-\frac{385\,b\sqrt{2}}{512\,d^3a^5}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)-\frac{385\,b\sqrt{2}}{256\,d^3a^5}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)-\frac{385\,b\sqrt{2}}{256\,d^3a^5}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2/3/a^4/d/(d*x)^(3/2)-257/192/d/a^4*b^3/(b*d^2*x^2+a*d^2)^3*(d*x
)^(9/2)-101/32*d/a^3*b^2/(b*d^2*x^2+a*d^2)^3*(d*x)^(5/2)-127/64*d
^3/a^2*b/(b*d^2*x^2+a*d^2)^3*(d*x)^(1/2)-385/512/d^3/a^5*b*(a*d^2
/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))-385/256/d^3/a^5*b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-385/256/d^3/a^5*b*(a*d^2/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298918, size = 560, normalized size = 1.59

$$-\frac{1540\, b^3 x^6 + 3960\, a b^2 x^4 + 3060\, a^2 b x^2 + 512\, a^3 - 4620\, \left(a^4 b^3 d^2 x^7 + 3\, a^5 b^2 d^2 x^5 + 3\, a^6 b d^2 x^3 + a^7 d^2 x\right) \sqrt{dx} \left(-\frac{b^3}{a^{19} d^{10}}\right)^{\frac{1}{4}} \arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/768*(1540*b^3*x^6 + 3960*a*b^2*x^4 + 3060*a^2*b*x^2 + 512*a^3
- 4620*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a
^7*d^2*x)*sqrt(d*x)*(-b^3/(a^19*d^10))^(1/4)*arctan(a^5*d^3*(-b^3
/(a^19*d^10))^(1/4)/(sqrt(d*x)*b + sqrt(a^10*d^6*sqrt(-b^3/(a^19*
d^10)) + b^2*d*x))) + 1155*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 +
 3*a^6*b*d^2*x^3 + a^7*d^2*x)*sqrt(d*x)*(-b^3/(a^19*d^10))^(1/4)*
log(385*a^5*d^3*(-b^3/(a^19*d^10))^(1/4) + 385*sqrt(d*x)*b) - 115
5*(a^4*b^3*d^2*x^7 + 3*a^5*b^2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^
2*x)*sqrt(d*x)*(-b^3/(a^19*d^10))^(1/4)*log(-385*a^5*d^3*(-b^3/(a
^19*d^10))^(1/4) + 385*sqrt(d*x)*b))/((a^4*b^3*d^2*x^7 + 3*a^5*b^
2*d^2*x^5 + 3*a^6*b*d^2*x^3 + a^7*d^2*x)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273379, size = 416, normalized size = 1.18

$$-\frac{385 \sqrt{2} \left(ab^{3}d^{2}\right)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2 \sqrt{dx}\right)}{2 \left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{256 a^{5}d^{3}} - \frac{385 \sqrt{2} \left(ab^{3}d^{2}\right)^{\frac{1}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2 \sqrt{dx}\right)}{2 \left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{256 a^{5}d^{3}} - \frac{385 \sqrt{2} \left(ab^{3}d^{2}\right)^{\frac{1}{4}} \ln\left(dx + \sqrt{2} \left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx} + \sqrt{\frac{ad^{2}}{b}}\right)}{512 a^{5}d^{3}} + \frac{385 \sqrt{2} \left(ab^{3}d^{2}\right)^{\frac{1}{4}} \ln\left(dx - \sqrt{2} \left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}} \sqrt{dx} + \sqrt{\frac{ad^{2}}{b}}\right)}{512 a^{5}d^{3}} - \frac{385 b^{3}d^{6}x^{6} + 990 ab^{2}d^{6}x^{4} + 765 a^{2}bd^{6}x^{2} + 128 a^{3}d^{6}}{192 \left(\sqrt{dx}bd^{2}x^{2} + \sqrt{dx}ad^{2}\right)^{3} a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(5/2)),x, algorithm="giac")
```

```
[Out] -385/256*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/256
*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)
^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*d^3) - 385/512*sqrt(2
)*(a*b^3*d^2)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
sqrt(a*d^2/b))/(a^5*d^3) + 385/512*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*d^3)
 - 1/192*(385*b^3*d^6*x^6 + 990*a*b^2*d^6*x^4 + 765*a^2*b*d^6*x^2
 + 128*a^3*d^6)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^3*a^4*d)
```

3.709 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=370

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b}{64a^5d^3\sqrt{dx}}-\frac{663}{320a^4d(dx)^{5/2}}+\frac{221}{192a^3d(dx)^{5/2}(a+bx^2)}+\frac{17}{48a^2d(dx)^{5/2}(a+bx^2)^2}+\frac{1}{6ad(dx)^{5/2}(a+bx^2)^3}$$

```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1
/(6*a*d*(d*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 221/(192*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5
/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5/4)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/
2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) -
(663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rubi [A] time = 0.917915, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{663b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{256\sqrt{2}a^{21/4}d^{7/2}}-\frac{663b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{128\sqrt{2}a^{21/4}d^{7/2}}+\frac{663b}{64a^5d^3\sqrt{dx}}-\frac{663}{320a^4d(dx)^{5/2}}+\frac{221}{192a^3d(dx)^{5/2}(a+bx^2)}+\frac{17}{48a^2d(dx)^{5/2}(a+bx^2)^2}+\frac{1}{6ad(dx)^{5/2}(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```


```
[Out] -663/(320*a^4*d*(d*x)^(5/2)) + (663*b)/(64*a^5*d^3*Sqrt[d*x]) + 1
/(6*a*d*(d*x)^(5/2)*(a + b*x^2)^3) + 17/(48*a^2*d*(d*x)^(5/2)*(a
+ b*x^2)^2) + 221/(192*a^3*d*(d*x)^(5/2)*(a + b*x^2)) - (663*b^(5
/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(1
28*Sqrt[2]*a^(21/4)*d^(7/2)) + (663*b^(5/4)*ArcTan[1 + (Sqrt[2]*b
```

```
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(128*Sqrt[2]*a^(21/4)*d^(7/
2)) + (663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2)) -
(663*b^(5/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(256*Sqrt[2]*a^(21/4)*d^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.298428, size = 295, normalized size = 0.8

$$\frac{\sqrt{dx}\left(\frac{5280a^{5/4}b^2x^4}{(a+bx^2)^2}+\frac{1280a^{9/4}b^2x^4}{(a+bx^2)^3}-3072a^{5/4}+9945\sqrt{2}b^{5/4}x^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-9945\sqrt{2}b^{5/4}x^{5/2}\log\left(\sqrt{2}\sqrt[4]{}\right.\right.}{7\ldots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^2),x]
```

```
[Out] (Sqrt[d*x]*(-3072*a^(5/4) + 61440*a^(1/4)*b*x^2 + (1280*a^(9/4)*b
^2*x^4)/(a + b*x^2)^3 + (5280*a^(5/4)*b^2*x^4)/(a + b*x^2)^2 + (1
8120*a^(1/4)*b^2*x^4)/(a + b*x^2) - 19890*Sqrt[2]*b^(5/4)*x^(5/2)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 19890*Sqrt[2]*b^
(5/4)*x^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 994
5*Sqrt[2]*b^(5/4)*x^(5/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x] - 9945*Sqrt[2]*b^(5/4)*x^(5/2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(7680*a^(21/4)*d^4
*x^3)
```

Maple [A] time = 0.036, size = 304, normalized size = 0.8

$$-\frac{2}{5\,a^4d}\,(dx)^{-\frac{5}{2}}+8\,\frac{b}{a^5d^3\sqrt{dx}}+\frac{151\,b^4}{64\,a^5d^3\,(bd^2x^2+ad^2)^3}\,(dx)^{\frac{11}{2}}+\frac{173\,b^3}{32\,a^4d\,(bd^2x^2+ad^2)^3}\,(dx)^{\frac{7}{2}}+\frac{617\,b^2d}{192\,a^3\,(bd^2x^2+ad^2)^3}\,(dx)^{\frac{3}{2}}+\frac{663\,b\sqrt{2}}{512\,a^5d^3}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{663\,b\sqrt{2}}{256\,a^5d^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{663\,b\sqrt{2}}{256\,a^5d^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] -2/5/a^4/d/(d*x)^(5/2)+8*b/a^5/d^3/(d*x)^(1/2)+151/64/d^3*b^4/a^5
/(b*d^2*x^2+a*d^2)^3*(d*x)^(11/2)+173/32/d*b^3/a^4/(b*d^2*x^2+a*d
^2)^3*(d*x)^(7/2)+617/192*d*b^2/a^3/(b*d^2*x^2+a*d^2)^3*(d*x)^(3/
2)+663/512/d^3*b/a^5/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/256/d^3*b/a^5/(a*d^2/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+663/256
/d^3*b/a^5/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)
*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302338, size = 603, normalized size = 1.63

$$\frac{39780\, b^4 x^8 + 111384\, ab^3 x^6 + 99892\, a^2 b^2 x^4 + 26112\, a^3 bx^2 - 1536\, a^4 + 39780\, \left(a^5 b^3 d^3 x^8 + 3\, a^6 b^2 d^3 x^6 + 3\, a^7 bd^3 x^4 + a^8 d^3 x^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/3840*(39780*b^4*x^8 + 111384*a*b^3*x^6 + 99892*a^2*b^2*x^4 + 26
112*a^3*b*x^2 - 1536*a^4 + 39780*(a^5*b^3*d^3*x^8 + 3*a^6*b^2*d^3
*x^6 + 3*a^7*b*d^3*x^4 + a^8*d^3*x^2)*sqrt(d*x)*(-b^5/(a^21*d^14)
)^(1/4)*arctan(291434247*a^16*d^11*(-b^5/(a^21*d^14))^(3/4)/(2914
34247*sqrt(d*x)*b^4 + sqrt(-84933920324457009*a^11*b^5*d^8*sqrt(-
b^5/(a^21*d^14)) + 84933920324457009*b^8*d*x))) + 9945*(a^5*b^3*d
^3*x^8 + 3*a^6*b^2*d^3*x^6 + 3*a^7*b*d^3*x^4 + a^8*d^3*x^2)*sqrt(
d*x)*(-b^5/(a^21*d^14))^(1/4)*log(291434247*a^16*d^11*(-b^5/(a^21
*d^14))^(3/4) + 291434247*sqrt(d*x)*b^4) - 9945*(a^5*b^3*d^3*x^8
+ 3*a^6*b^2*d^3*x^6 + 3*a^7*b*d^3*x^4 + a^8*d^3*x^2)*sqrt(d*x)*(-
b^5/(a^21*d^14))^(1/4)*log(-291434247*a^16*d^11*(-b^5/(a^21*d^14)
)^(3/4) + 291434247*sqrt(d*x)*b^4))/((a^5*b^3*d^3*x^8 + 3*a^6*b^2
*d^3*x^6 + 3*a^7*b*d^3*x^4 + a^8*d^3*x^2)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274729, size = 471, normalized size = 1.27

$$\frac{663\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^{6}bd^{5}}+\frac{663\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{256\,a^{6}bd^{5}}-\frac{663\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{512\,a^{6}bd^{5}}+\frac{663\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{512\,a^{6}bd^{5}}+\frac{453\sqrt{dx}b^{4}d^{5}x^{5}+1038\sqrt{dx}ab^{3}d^{5}x^{3}+617\sqrt{dx}a^{2}b^{2}d^{5}x}{192\left(bd^{2}x^{2}+ad^{2}\right)^{3}a^{5}d^{3}}+\frac{2\left(20\,bd^{2}x^{2}-ad^{2}\right)}{5\sqrt{dx}a^{5}d^{5}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^(7/2)),x, algorithm="giac")
```

```
[Out] 663/256*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*
d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) + 663/25
6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d^5) - 663/512*sqr
t(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a^6*b*d^5) + 663/512*sqrt(2)*(a*b^3*d^2)^(3/4)
*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6
*b*d^5) + 1/192*(453*sqrt(d*x)*b^4*d^5*x^5 + 1038*sqrt(d*x)*a*b^3
*d^5*x^3 + 617*sqrt(d*x)*a^2*b^2*d^5*x)/((b*d^2*x^2 + a*d^2)^3*a^
5*d^3) + 2/5*(20*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^5*d^5*x^2)
```

3.710 $\int \frac{(dx)^{27/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=420

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{29/4}}-\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}b^{29/4}}-\frac{69615ad^{13}\sqrt{dx}}{4096b^7}-\frac{7735d^9(dx)^{9/2}}{4096b^5(a+bx^2)}-\frac{595d^7(dx)^{13/2}}{1024b^4(a+bx^2)^2}-\frac{35d^5(dx)^{17/2}}{128b^3(a+bx^2)^3}-\frac{5d^3(dx)^{21/2}}{32b^2(a+bx^2)^4}-\frac{d(dx)^{25/2}}{10b(a+bx^2)^5}+\frac{13923d^{11}(dx)^{5/2}}{4096b^6}$$

```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(
4096*b^6) - (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^
(21/2))/(32*b^2*(a + b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(
a + b*x^2)^3) - (595*d^7*(d*x)^(13/2))/(1024*b^4*(a + b*x^2)^2) -
 (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b*x^2)) - (69615*a^(5/4)*d
^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29
/4)) - (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/
4)) + (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4
))
```

Rubi [A] time = 1.09604, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{69615a^{5/4}d^{27/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{29/4}}-\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{29/4}}+\frac{69615a^{5/4}d^{27/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}b^{29/4}}-\frac{69615ad^{13}\sqrt{dx}}{4096b^7}-\frac{7735d^9(dx)^{9/2}}{4096b^5(a+bx^2)}-\frac{595d^7(dx)^{13/2}}{1024b^4(a+bx^2)^2}-\frac{35d^5(dx)^{17/2}}{128b^3(a+bx^2)^3}-\frac{5d^3(dx)^{21/2}}{32b^2(a+bx^2)^4}-\frac{d(dx)^{25/2}}{10b(a+bx^2)^5}+\frac{13923d^{11}(dx)^{5/2}}{4096b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (-69615*a*d^13*Sqrt[d*x])/(4096*b^7) + (13923*d^11*(d*x)^(5/2))/(
4096*b^6) - (d*(d*x)^(25/2))/(10*b*(a + b*x^2)^5) - (5*d^3*(d*x)^
(21/2))/(32*b^2*(a + b*x^2)^4) - (35*d^5*(d*x)^(17/2))/(128*b^3*(
a + b*x^2)^3) - (595*d^7*(d*x)^(13/2))/(1024*b^4*(a + b*x^2)^2) -
 (7735*d^9*(d*x)^(9/2))/(4096*b^5*(a + b*x^2)) - (69615*a^(5/4)*d
^(27/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(8192*Sqrt[2]*b^(29/4)) + (69615*a^(5/4)*d^(27/2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(29
/4)) - (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/
4)) + (69615*a^(5/4)*d^(27/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[
d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*b^(29/4
))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.431586, size = 321, normalized size = 0.76

$$\frac{d^{13}\sqrt{dx}\left(-\frac{348075\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}+\frac{348075\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{696150\sqrt{2}a^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{x}}+\frac{696150\cdots}{\cdots}\right)}{163840\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(27/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^13*Sqrt[d*x]*(-1966080*a*b^(1/4) + 65536*b^(5/4)*x^2 - (16384*
a^6*b^(1/4))/(a + b*x^2)^5 + (123904*a^5*b^(1/4))/(a + b*x^2)^4 -
 (418560*a^4*b^(1/4))/(a + b*x^2)^3 + (858080*a^3*b^(1/4))/(a + b
*x^2)^2 - (1365560*a^2*b^(1/4))/(a + b*x^2) - (696150*Sqrt[2]*a^(
```

```
5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x] + (69
6150*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/Sqrt[x] - (348075*Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x] + (348075*Sqrt[2]*a^(5/4)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt
[x]))/(163840*b^(29/4))
```

Maple [A] time = 0.042, size = 370, normalized size = 0.9

$$\frac{2\,d^{11}}{5\,b^6}(dx)^{\frac{5}{2}}-12\,\frac{ad^{13}\sqrt{dx}}{b^7}-\frac{20463\,d^{23}a^6}{4096\,b^7\,(bd^2x^2+ad^2)^5}\sqrt{dx}-\frac{56269\,d^{21}a^5}{2560\,b^6\,(bd^2x^2+ad^2)^5}(dx)^{\frac{5}{2}}$$
$$-\frac{75471\,d^{19}a^4}{2048\,b^5\,(bd^2x^2+ad^2)^5}(dx)^{\frac{9}{2}}-\frac{3597\,d^{17}a^3}{128\,b^4\,(bd^2x^2+ad^2)^5}(dx)^{\frac{13}{2}}-\frac{34139\,d^{15}a^2}{4096\,b^3\,(bd^2x^2+ad^2)^5}(dx)^{\frac{17}{2}}$$
$$+\frac{69615\,ad^{13}\sqrt{2}}{32768\,b^7}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)$$
$$+\frac{69615\,ad^{13}\sqrt{2}}{16384\,b^7}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)$$
$$+\frac{69615\,ad^{13}\sqrt{2}}{16384\,b^7}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int((d*x)^(27/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/5*d^11*(d*x)^(5/2)/b^6-12*a*d^13*(d*x)^(1/2)/b^7-20463/4096*d^2
3/b^7*a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(1/2)-56269/2560*d^21/b^6*a^5
/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)-75471/2048*d^19/b^5*a^4/(b*d^2*x
^2+a*d^2)^5*(d*x)^(9/2)-3597/128*d^17/b^4*a^3/(b*d^2*x^2+a*d^2)^5
*(d*x)^(13/2)-34139/4096*d^15/b^3*a^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(
17/2)+69615/32768*d^13/b^7*a*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+69615/16384*d^13/b^7*a
*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/
2)+1)+69615/16384*d^13/b^7*a*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(27/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293497, size = 664, normalized size = 1.58

$$-\frac{1392300 \left(-\frac{a^5 d^{54}}{b^{29}}\right)^{\frac{1}{4}} \left(b^{12} x^{10} + 5 \, a b^{11} x^{8} + 10 \, a^{2} b^{10} x^{6} + 10 \, a^{3} b^{9} x^{4} + 5 \, a^{4} b^{8} x^{2} + a^{5} b^{7}\right) \arctan\left(\frac{\left(-\frac{a^5 d^{54}}{b^{29}}\right)^{\frac{1}{4}} b^{7}}{\sqrt{d x} a d^{13} + \sqrt{a^{2} d^{27} x + \sqrt{-\frac{a^5 d^{54}}{b^{29}}} b^{14}}}\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(27/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1392300*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10 + 5*a*b^11*x^
8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^2 + a^5*b^7)*a
rctan((-a^5*d^54/b^29)^(1/4)*b^7/(sqrt(d*x)*a*d^13 + sqrt(a^2*d^2
7*x + sqrt(-a^5*d^54/b^29)*b^14))) - 348075*(-a^5*d^54/b^29)^(1/4
)*(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 +
5*a^4*b^8*x^2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 + 69615*(-a^5
*d^54/b^29)^(1/4)*b^7) + 348075*(-a^5*d^54/b^29)^(1/4)*(b^12*x^10
 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5*a^4*b^8*x^
2 + a^5*b^7)*log(69615*sqrt(d*x)*a*d^13 - 69615*(-a^5*d^54/b^29)^
(1/4)*b^7) - 4*(8192*b^6*d^13*x^12 - 204800*a*b^5*d^13*x^10 - 131
7575*a^2*b^4*d^13*x^8 - 2951200*a^3*b^3*d^13*x^6 - 3171350*a^4*b^
2*d^13*x^4 - 1670760*a^5*b*d^13*x^2 - 348075*a^6*d^13)*sqrt(d*x))
/(b^12*x^10 + 5*a*b^11*x^8 + 10*a^2*b^10*x^6 + 10*a^3*b^9*x^4 + 5
*a^4*b^8*x^2 + a^5*b^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(27/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279946, size = 510, normalized size = 1.21

$$\frac{1}{163840}\,d^{12}\left(\frac{696150\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}+\frac{696150\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(27/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^12*(696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^
8 + 696150*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^8 + 348075
*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*s
qrt(d*x) + sqrt(a*d^2/b))/b^8 - 348075*sqrt(2)*(a*b^3*d^2)^(1/4)*
a*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b
^8 - 8*(170695*sqrt(d*x)*a^2*b^4*d^11*x^8 + 575520*sqrt(d*x)*a^3*
b^3*d^11*x^6 + 754710*sqrt(d*x)*a^4*b^2*d^11*x^4 + 450152*sqrt(d*
x)*a^5*b*d^11*x^2 + 102315*sqrt(d*x)*a^6*d^11)/((b*d^2*x^2 + a*d^
2)^5*b^7) + 65536*(sqrt(d*x)*b^24*d^6*x^2 - 30*sqrt(d*x)*a*b^23*d
^6)/(b^30*d^5))
```

3.711 $\int \frac{(dx)^{25/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=402

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{27/4}}-\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}b^{27/4}}-\frac{4807d^9(dx)^{7/2}}{4096b^5(a+bx^2)}-\frac{437d^7(dx)^{11/2}}{1024b^4(a+bx^2)^2}-\frac{437d^5(dx)^{15/2}}{1920b^3(a+bx^2)^3}-\frac{23d^3(dx)^{19/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{23/2}}{10b(a+bx^2)^5}+\frac{33649d^{11}(dx)^{3/2}}{12288b^6}$$

```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a
+ b*x^2)^5) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (43
7*d^5*(d*x)^(15/2))/(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11
/2))/(1024*b^4*(a + b*x^2)^2) - (4807*d^9*(d*x)^(7/2))/(4096*b^5*
(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649
*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rubi [A] time = 1.01414, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{33649a^{3/4}d^{25/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}b^{27/4}}+\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{27/4}}-\frac{33649a^{3/4}d^{25/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}b^{27/4}}-\frac{4807d^9(dx)^{7/2}}{4096b^5(a+bx^2)}-\frac{437d^7(dx)^{11/2}}{1024b^4(a+bx^2)^2}-\frac{437d^5(dx)^{15/2}}{1920b^3(a+bx^2)^3}-\frac{23d^3(dx)^{19/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{23/2}}{10b(a+bx^2)^5}+\frac{33649d^{11}(dx)^{3/2}}{12288b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (33649*d^11*(d*x)^(3/2))/(12288*b^6) - (d*(d*x)^(23/2))/(10*b*(a
+ b*x^2)^5) - (23*d^3*(d*x)^(19/2))/(160*b^2*(a + b*x^2)^4) - (43
7*d^5*(d*x)^(15/2))/(1920*b^3*(a + b*x^2)^3) - (437*d^7*(d*x)^(11
/2))/(1024*b^4*(a + b*x^2)^2) - (4807*d^9*(d*x)^(7/2))/(4096*b^5*
(a + b*x^2)) + (33649*a^(3/4)*d^(25/2)*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649
*a^(3/4)*d^(25/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(8192*Sqrt[2]*b^(27/4)) - (33649*a^(3/4)*d^(25/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*b^(27/4)) + (33649*a^(3/4)*d^(25/2)*Log[
Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[d*x]])/(16384*Sqrt[2]*b^(27/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.388111, size = 323, normalized size = 0.8

$$\frac{d^{12}\sqrt{dx}\left(-504735\sqrt{2}a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+504735\sqrt{2}a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+1009470\sqrt{2}a^{3/}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(25/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^12*Sqrt[d*x]*(327680*b^(3/4)*x^(3/2) + (49152*a^5*b^(3/4)*x^(3
/2))/(a + b*x^2)^5 - (316416*a^4*b^(3/4)*x^(3/2))/(a + b*x^2)^4 +
 (886016*a^3*b^(3/4)*x^(3/2))/(a + b*x^2)^3 - (1460832*a^2*b^(3/4
)*x^(3/2))/(a + b*x^2)^2 + (1860360*a*b^(3/4)*x^(3/2))/(a + b*x^2
) + 1009470*Sqrt[2]*a^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)] - 1009470*Sqrt[2]*a^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[x])/a^(1/4)] - 504735*Sqrt[2]*a^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^
```

```
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 504735*Sqrt[2]*a^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(491520*b
^(27/4)*Sqrt[x])
```

Maple [A] time = 0.04, size = 354, normalized size = 0.9

$$\frac{2d^{11}}{3b^6}(dx)^{\frac{3}{2}}+\frac{25457d^{21}a^5}{12288b^6(bd^2x^2+ad^2)^5}(dx)^{\frac{3}{2}}+\frac{3527d^{19}a^4}{384b^5(bd^2x^2+ad^2)^5}(dx)^{\frac{7}{2}}+\frac{95821d^{17}a^3}{6144b^4(bd^2x^2+ad^2)^5}(dx)^{\frac{11}{2}}+\frac{31149d^{15}a^2}{2560b^3(bd^2x^2+ad^2)^5}(dx)^{\frac{15}{2}}+\frac{15503d^{13}a}{4096b^2(bd^2x^2+ad^2)^5}(dx)^{\frac{19}{2}}-\frac{33649d^{13}a\sqrt{2}}{32768b^7}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{33649d^{13}a\sqrt{2}}{16384b^7}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-\frac{33649d^{13}a\sqrt{2}}{16384b^7}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(25/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2/3*d^11*(d*x)^(3/2)/b^6+25457/12288*d^21*a^5/b^6/(b*d^2*x^2+a*d^
2)^5*(d*x)^(3/2)+3527/384*d^19*a^4/b^5/(b*d^2*x^2+a*d^2)^5*(d*x)^
(7/2)+95821/6144*d^17*a^3/b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)+31
149/2560*d^15*a^2/b^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)+15503/4096
*d^13*a/b^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)-33649/32768*d^13*a/b
^7/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2)))-33649/16384*d^13*a/b^7/(a*d^2/b)^(1/4)*2^(1/2)*a
rctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-33649/16384*d^13*a/b
^7/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(
1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(25/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293508, size = 664, normalized size = 1.65

$$-\frac{2018940 \left(-\frac{a^3 d^{50}}{b^{27}}\right)^{\frac{1}{4}} \left(b^{11}x^{10} + 5\,ab^{10}x^8 + 10\,a^2b^9x^6 + 10\,a^3b^8x^4 + 5\,a^4b^7x^2 + a^5b^6\right) \arctan\left(\frac{\left(-\frac{a^3d^{50}}{b^{27}}\right)^{\frac{3}{4}} b^{20}}{\sqrt{dx}a^2d^{37} + \sqrt{a^4d^{75}x - \sqrt{-\frac{a^3d^{50}}{b^{27}}}a^3b^{13}d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(25/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(2018940*(-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x
^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*a
rctan((-a^3*d^50/b^27)^(3/4)*b^20/(sqrt(d*x)*a^2*d^37 + sqrt(a^4*
d^75*x - sqrt(-a^3*d^50/b^27)*a^3*b^13*d^50))) + 504735*(-a^3*d^5
0/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3
*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sqrt(d*x)*
a^2*d^37 + 38099255258449*(-a^3*d^50/b^27)^(3/4)*b^20) - 504735*(
-a^3*d^50/b^27)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6
+ 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(38099255258449*sq
rt(d*x)*a^2*d^37 - 38099255258449*(-a^3*d^50/b^27)^(3/4)*b^20) -
4*(40960*b^5*d^12*x^11 + 437345*a*b^4*d^12*x^9 + 1157176*a^2*b^3*
d^12*x^7 + 1367810*a^3*b^2*d^12*x^5 + 769120*a^4*b*d^12*x^3 + 168
245*a^5*d^12*x)*sqrt(d*x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9
*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(25/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281474, size = 463, normalized size = 1.15

$$\frac{1}{491520}\,d^{11}\left(\frac{327680\,\sqrt{dx}dx}{b^6}-\frac{1009470\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^9}-\frac{1009470\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\right.\right.}{2}\right.}{b^9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(25/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^11*(327680*sqrt(d*x)*d*x/b^6 - 1009470*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d
*x))/(a*d^2/b)^(1/4))/b^9 - 1009470*sqrt(2)*(a*b^3*d^2)^(3/4)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b
)^(1/4))/b^9 + 504735*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*
(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^9 - 504735*sqrt(2)*(
a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/b^9 + 8*(232545*sqrt(d*x)*a*b^4*d^11*x^9 + 747576*sqr
t(d*x)*a^2*b^3*d^11*x^7 + 958210*sqrt(d*x)*a^3*b^2*d^11*x^5 + 564
320*sqrt(d*x)*a^4*b*d^11*x^3 + 127285*sqrt(d*x)*a^5*d^11*x)/((b*d
^2*x^2 + a*d^2)^5*b^6))
```

3.712 $\int \frac{(dx)^{23/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=402

$$\frac{13923\sqrt[4]{a}d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{25/4}} - \frac{13923\sqrt[4]{a}d^{23/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{25/4}} + \frac{13923\sqrt[4]{a}d^{23/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{25/4}} - \frac{13923\sqrt[4]{a}d^{23/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}b^{25/4}} - \frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} + \frac{13923d^{11}\sqrt{dx}}{4096b^6}$$

```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b
*x^2)^5) - (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d
^5*(d*x)^(13/2))/(640*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))
/(5120*b^4*(a + b*x^2)^2) - (13923*d^9*(d*x)^(5/2))/(20480*b^5*(a
 + b*x^2)) + (13923*a^(1/4)*d^(23/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) - (13923*a
^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rubi [A] time = 0.979152, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{13923\sqrt[4]{a}d^{23/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{25/4}} - \frac{13923\sqrt[4]{a}d^{23/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}b^{25/4}} + \frac{13923\sqrt[4]{a}d^{23/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}b^{25/4}} - \frac{13923\sqrt[4]{a}d^{23/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}b^{25/4}} - \frac{13923d^9(dx)^{5/2}}{20480b^5\left(a+bx^2\right)} - \frac{1547d^7(dx)^{9/2}}{5120b^4\left(a+bx^2\right)^2} - \frac{119d^5(dx)^{13/2}}{640b^3\left(a+bx^2\right)^3} - \frac{21d^3(dx)^{17/2}}{160b^2\left(a+bx^2\right)^4} - \frac{d(dx)^{21/2}}{10b\left(a+bx^2\right)^5} + \frac{13923d^{11}\sqrt{dx}}{4096b^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (13923*d^11*Sqrt[d*x])/(4096*b^6) - (d*(d*x)^(21/2))/(10*b*(a + b
*x^2)^5) - (21*d^3*(d*x)^(17/2))/(160*b^2*(a + b*x^2)^4) - (119*d
^5*(d*x)^(13/2))/(640*b^3*(a + b*x^2)^3) - (1547*d^7*(d*x)^(9/2))
/(5120*b^4*(a + b*x^2)^2) - (13923*d^9*(d*x)^(5/2))/(20480*b^5*(a
 + b*x^2)) + (13923*a^(1/4)*d^(23/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*b^(25/4)) - (13923*a
^(1/4)*d^(23/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*b^(25/4)) + (13923*a^(1/4)*d^(23/2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(16384*Sqrt[2]*b^(25/4)) - (13923*a^(1/4)*d^(23/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*b^(25/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.407543, size = 308, normalized size = 0.77

$$\frac{d^{11}\sqrt{dx}\left(\frac{16384a^5\sqrt[4]{b}}{(a+bx^2)^5}-\frac{103424a^4\sqrt[4]{b}}{(a+bx^2)^4}+\frac{280320a^3\sqrt[4]{b}}{(a+bx^2)^3}-\frac{433760a^2\sqrt[4]{b}}{(a+bx^2)^2}+\frac{469720a\sqrt[4]{b}}{a+bx^2}+\frac{69615\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt{x}}-\frac{69615\sqrt{2}\sqrt[4]{a}}{}\right.}{163840b^{25/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^11*Sqrt[d*x]*(327680*b^(1/4) + (16384*a^5*b^(1/4))/(a + b*x^2)
^5 - (103424*a^4*b^(1/4))/(a + b*x^2)^4 + (280320*a^3*b^(1/4))/(a
 + b*x^2)^3 - (433760*a^2*b^(1/4))/(a + b*x^2)^2 + (469720*a*b^(1
/4))/(a + b*x^2) + (139230*Sqrt[2]*a^(1/4)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x] - (139230*Sqrt[2]*a^(1/4)*ArcTan
```

```
[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/Sqrt[x] + (69615*Sqrt[2]
*a^(1/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/Sqrt[x] - (69615*Sqrt[2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/Sqrt[x]))/(163840*b^(25/4))
```

Maple [A] time = 0.038, size = 351, normalized size = 0.9

$$2\,{\frac{{d}^{11}\sqrt{dx}}{{b}^{6}}}+{\frac{5731\,{d}^{21}{a}^{5}}{4096\,{b}^{6}\left(b{d}^{2}{x}^{2}+a{d}^{2}\right)^{5}}\sqrt{dx}}+{\frac{16169\,{d}^{19}{a}^{4}}{2560\,{b}^{5}\left(b{d}^{2}{x}^{2}+a{d}^{2}\right)^{5}} \left(dx \right) ^{{\frac{5}{2}}}}+{\frac{22467\,{d}^{17}{a}^{3}}{2048\,{b}^{4}\left(b{d}^{2}{x}^{2}+a{d}^{2}\right)^{5}} \left(dx \right) ^{{\frac{9}{2}}}}+{\frac{1129\,{d}^{15}{a}^{2}}{128\,{b}^{3}\left(b{d}^{2}{x}^{2}+a{d}^{2}\right)^{5}} \left(dx \right) ^{{\frac{13}{2}}}}+{\frac{11743\,{d}^{13}a}{4096\,{b}^{2}\left(b{d}^{2}{x}^{2}+a{d}^{2}\right)^{5}} \left(dx \right) ^{{\frac{17}{2}}}}-{\frac{13923\,{d}^{11}\sqrt{2}}{32768\,{b}^{6}}\sqrt[4]{{\frac{a{d}^{2}}{b}}}\ln \left(1\left(dx+\sqrt[4]{{\frac{a{d}^{2}}{b}}}\sqrt{dx}\sqrt{2}+\sqrt{{\frac{a{d}^{2}}{b}}} \right) \left(dx-\sqrt[4]{{\frac{a{d}^{2}}{b}}}\sqrt{dx}\sqrt{2}+\sqrt{{\frac{a{d}^{2}}{b}}} \right) ^{-1} \right) }-{\frac{13923\,{d}^{11}\sqrt{2}}{16384\,{b}^{6}}\sqrt[4]{{\frac{a{d}^{2}}{b}}}\arctan \left(\sqrt{2}\sqrt{dx}{\frac{1}{\sqrt[4]{{\frac{a{d}^{2}}{b}}}}}+1 \right) }-{\frac{13923\,{d}^{11}\sqrt{2}}{16384\,{b}^{6}}\sqrt[4]{{\frac{a{d}^{2}}{b}}}\arctan \left(\sqrt{2}\sqrt{dx}{\frac{1}{\sqrt[4]{{\frac{a{d}^{2}}{b}}}}}-1 \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 2*d^11*(d*x)^(1/2)/b^6+5731/4096*d^21/b^6*a^5/(b*d^2*x^2+a*d^2)^5
*(d*x)^(1/2)+16169/2560*d^19/b^5*a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(5
/2)+22467/2048*d^17/b^4*a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)+1129/
128*d^15/b^3*a^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)+11743/4096*d^13
/b^2*a/(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-13923/32768*d^11/b^6*(a*d
^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))-13923/16384*d^11/b^6*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-13923/16384*d^11/b^6*(a*d^2/b
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292933, size = 618, normalized size = 1.54

$$\frac{278460 \left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}} \left(b^{11}x^{10} + 5\,ab^{10}x^{8} + 10\,a^{2}b^{9}x^{6} + 10\,a^{3}b^{8}x^{4} + 5\,a^{4}b^{7}x^{2} + a^{5}b^{6}\right) \arctan\left(\frac{\left(-\frac{ad^{46}}{b^{25}}\right)^{\frac{1}{4}} b^{6}}{\sqrt{dx}\,d^{11} + \sqrt{d^{23}x + \sqrt{-\frac{ad^{46}}{b^{25}}}\,b^{12}}}\right) - 69615}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(278460*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 +
10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*arctan
((-a*d^46/b^25)^(1/4)*b^6/(sqrt(d*x)*d^11 + sqrt(d^23*x + sqrt(-a
*d^46/b^25)*b^12))) - 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a
*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5
*b^6)*log(13923*sqrt(d*x)*d^11 + 13923*(-a*d^46/b^25)^(1/4)*b^6)
+ 69615*(-a*d^46/b^25)^(1/4)*(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b
^9*x^6 + 10*a^3*b^8*x^4 + 5*a^4*b^7*x^2 + a^5*b^6)*log(13923*sqrt
(d*x)*d^11 - 13923*(-a*d^46/b^25)^(1/4)*b^6) + 4*(40960*b^5*d^11*
x^10 + 263515*a*b^4*d^11*x^8 + 590240*a^2*b^3*d^11*x^6 + 634270*a
^3*b^2*d^11*x^4 + 334152*a^4*b*d^11*x^2 + 69615*a^5*d^11)*sqrt(d*
x))/(b^11*x^10 + 5*a*b^10*x^8 + 10*a^2*b^9*x^6 + 10*a^3*b^8*x^4 +
 5*a^4*b^7*x^2 + a^5*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278917, size = 466, normalized size = 1.16

$$-\frac{1}{163840}\,d^{10}\left(\frac{139230\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\frac{139230\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] -1/163840*d^10*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqr
t(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7
 + 139230*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^7 + 69615*sqr
t(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*
x) + sqrt(a*d^2/b))/b^7 - 69615*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^7 - 3276
80*sqrt(d*x)*d/b^6 - 8*(58715*sqrt(d*x)*a*b^4*d^11*x^8 + 180640*s
qrt(d*x)*a^2*b^3*d^11*x^6 + 224670*sqrt(d*x)*a^3*b^2*d^11*x^4 + 1
29352*sqrt(d*x)*a^4*b*d^11*x^2 + 28655*sqrt(d*x)*a^5*d^11)/((b*d^
2*x^2 + a*d^2)^5*b^6))
```

3.713 $\int \frac{(dx)^{21/2}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=385

$$\frac{4389 d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{4389 d^{21/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{4389 d^{21/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}+\frac{4389 d^{21/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{1463 d^9 (dx)^{3/2}}{4096 b^5\left(a+bx^2\right)}-\frac{209 d^7 (dx)^{7/2}}{1024 b^4\left(a+bx^2\right)^2}-\frac{19 d^5 (dx)^{11/2}}{128 b^3\left(a+bx^2\right)^3}-\frac{19 d^3 (dx)^{15/2}}{160 b^2\left(a+bx^2\right)^4}-\frac{d (dx)^{19/2}}{10 b\left(a+bx^2\right)^5}$$

```
[Out] -(d*(d*x)^(19/2))/(10*b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(1
60*b^2*(a + b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2
)^3) - (209*d^7*(d*x)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9
*(d*x)^(3/2))/(4096*b^5*(a + b*x^2)) - (4389*d^(21/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389
*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d
^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rubi [A] time = 0.923893, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$\frac{4389 d^{21/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{4389 d^{21/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{4389 d^{21/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}+\frac{4389 d^{21/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}\sqrt[4]{a}b^{23/4}}-\frac{1463 d^9 (dx)^{3/2}}{4096 b^5\left(a+bx^2\right)}-\frac{209 d^7 (dx)^{7/2}}{1024 b^4\left(a+bx^2\right)^2}-\frac{19 d^5 (dx)^{11/2}}{128 b^3\left(a+bx^2\right)^3}-\frac{19 d^3 (dx)^{15/2}}{160 b^2\left(a+bx^2\right)^4}-\frac{d (dx)^{19/2}}{10 b\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(19/2))/(10*b*(a + b*x^2)^5) - (19*d^3*(d*x)^(15/2))/(1
60*b^2*(a + b*x^2)^4) - (19*d^5*(d*x)^(11/2))/(128*b^3*(a + b*x^2
)^3) - (209*d^7*(d*x)^(7/2))/(1024*b^4*(a + b*x^2)^2) - (1463*d^9
*(d*x)^(3/2))/(4096*b^5*(a + b*x^2)) - (4389*d^(21/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
1/4)*b^(23/4)) + (4389*d^(21/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1/4)*b^(23/4)) + (4389
*d^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4)) - (4389*d
^(21/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(1/4)*b^(23/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.420735, size = 298, normalized size = 0.77

$$\frac{d^9(dx)^{3/2}\left(-\frac{16384a^4b^{3/4}}{(a+bx^2)^5}+\frac{84992a^3b^{3/4}}{(a+bx^2)^4}-\frac{180992a^2b^{3/4}}{(a+bx^2)^3}+\frac{205984ab^{3/4}}{(a+bx^2)^2}-\frac{152120b^{3/4}}{a+bx^2}+\frac{21945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{a}x^{3/2}}-\frac{21945\sqrt{2}\log\left(\sqrt{2}\cdots\right.}{\cdots}\cdots\right)}{163840b^{23/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*(d*x)^(3/2)*((-16384*a^4*b^(3/4))/(a + b*x^2)^5 + (84992*a^3
*b^(3/4))/(a + b*x^2)^4 - (180992*a^2*b^(3/4))/(a + b*x^2)^3 + (2
05984*a*b^(3/4))/(a + b*x^2)^2 - (152120*b^(3/4))/(a + b*x^2) - (
43890*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(
1/4)*x^(3/2)) + (43890*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/(a^(1/4)*x^(3/2)) + (21945*Sqrt[2]*Log[Sqrt[a] - Sqr
```

```
t[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(1/4)*x^(3/2)) - (2
1945*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/(a^(1/4)*x^(3/2))))/(163840*b^(23/4))
```

Maple [A] time = 0.034, size = 335, normalized size = 0.9

$$-\frac{1463\,d^{19}a^{4}}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{5}}\left(dx\right)^{\frac{3}{2}}-\frac{209\,d^{17}a^{3}}{128\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{4}}\left(dx\right)^{\frac{7}{2}}-\frac{5947\,d^{15}a^{2}}{2048\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{3}}\left(dx\right)^{\frac{11}{2}}-\frac{6289\,d^{13}a}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{2}}\left(dx\right)^{\frac{15}{2}}-\frac{3803\,d^{11}}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b}\left(dx\right)^{\frac{19}{2}}+\frac{4389\,d^{11}\sqrt{2}}{32768\,b^{6}}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{4389\,d^{11}\sqrt{2}}{16384\,b^{6}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{4389\,d^{11}\sqrt{2}}{16384\,b^{6}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -1463/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(3/2)-209/128*d
^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(7/2)-5947/2048*d^15/(b*d^2
*x^2+a*d^2)^5/b^3*a^2*(d*x)^(11/2)-6289/2560*d^13/(b*d^2*x^2+a*d^
2)^5/b^2*a*(d*x)^(15/2)-3803/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x
)^(19/2)+4389/32768*d^11/b^6/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+4389/16384*d^11/b^6/(a
*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+
1)+4389/16384*d^11/b^6/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*
d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29012, size = 630, normalized size = 1.64

$$\frac{87780\left(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5}\right)\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{42}}{ab^{23}}\right)^{\frac{3}{4}}ab^{17}}{\sqrt{dx}\,d^{31}+\sqrt{d^{63}x-\sqrt{-\frac{d^{42}}{ab^{23}}}\,ab^{11}d^{42}}}\right)+21945\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(87780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3
*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^42/(a*b^23))^(1/4)*arctan
((-d^42/(a*b^23))^(3/4)*a*b^17/(sqrt(d*x)*d^31 + sqrt(d^63*x - sq
rt(-d^42/(a*b^23))*a*b^11*d^42))) + 21945*(b^10*x^10 + 5*a*b^9*x^
8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-
d^42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31 + 84546715869
*(-d^42/(a*b^23))^(3/4)*a*b^17) - 21945*(b^10*x^10 + 5*a*b^9*x^8
+ 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^
42/(a*b^23))^(1/4)*log(84546715869*sqrt(d*x)*d^31 - 84546715869*(
-d^42/(a*b^23))^(3/4)*a*b^17) - 4*(19015*b^4*d^10*x^9 + 50312*a*b
^3*d^10*x^7 + 59470*a^2*b^2*d^10*x^5 + 33440*a^3*b*d^10*x^3 + 731
5*a^4*d^10*x)*sqrt(d*x))/(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^
6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.283535, size = 459, normalized size = 1.19

$$\frac{1}{163840}\,d^9\left(\frac{43890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8}+\frac{43890\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^8}-\frac{21945}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^9*(43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8)
+ 43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^8) - 21945*sqr
t(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
 + sqrt(a*d^2/b))/(a*b^8) + 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*
x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^8) -
8*(19015*sqrt(d*x)*b^4*d^11*x^9 + 50312*sqrt(d*x)*a*b^3*d^11*x^7
+ 59470*sqrt(d*x)*a^2*b^2*d^11*x^5 + 33440*sqrt(d*x)*a^3*b*d^11*x
^3 + 7315*sqrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

3.714 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=385

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^{19/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^9\sqrt{dx}}{4096b^5(a+bx^2)}-\frac{663d^7(dx)^{5/2}}{5120b^4(a+bx^2)^2}-\frac{221d^5(dx)^{9/2}}{1920b^3(a+bx^2)^3}-\frac{17d^3(dx)^{13/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{17/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(17/2))/(10*b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(1
60*b^2*(a + b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^
2)^3) - (663*d^7*(d*x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9
*Sqrt[d*x])/(4096*b^5*(a + b*x^2)) - (663*d^(19/2)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4
)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (663*d^(1
9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4))
```

Rubi [A] time = 0.894989, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{663d^{19/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^{19/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}+\frac{663d^{19/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{3/4}b^{21/4}}-\frac{663d^9\sqrt{dx}}{4096b^5(a+bx^2)}-\frac{663d^7(dx)^{5/2}}{5120b^4(a+bx^2)^2}-\frac{221d^5(dx)^{9/2}}{1920b^3(a+bx^2)^3}-\frac{17d^3(dx)^{13/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{17/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(17/2))/(10*b*(a + b*x^2)^5) - (17*d^3*(d*x)^(13/2))/(1
60*b^2*(a + b*x^2)^4) - (221*d^5*(d*x)^(9/2))/(1920*b^3*(a + b*x^
2)^3) - (663*d^7*(d*x)^(5/2))/(5120*b^4*(a + b*x^2)^2) - (663*d^9
*Sqrt[d*x])/(4096*b^5*(a + b*x^2)) - (663*d^(19/2)*ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4
)*b^(21/4)) + (663*d^(19/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(3/4)*b^(21/4)) - (663*d^(1
9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4)) + (663*d^(19/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(3/4)*b^(21/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.402204, size = 298, normalized size = 0.77

$$\frac{d^9\sqrt{dx}\left(-\frac{9945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}+\frac{9945\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{3/4}\sqrt{x}}-\frac{19890\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{3/4}\sqrt{x}}+\frac{19890\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+\cdots\right)}{a^{3/4}\sqrt{x}}\cdots\right)}{491520b^{21/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^9*Sqrt[d*x]*((-49152*a^4*b^(1/4))/(a + b*x^2)^5 + (248832*a^3*
b^(1/4))/(a + b*x^2)^4 - (508160*a^2*b^(1/4))/(a + b*x^2)^3 + (53
0080*a*b^(1/4))/(a + b*x^2)^2 - (301160*b^(1/4))/(a + b*x^2) - (1
9890*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(3
/4)*Sqrt[x]) + (19890*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)])/(a^(3/4)*Sqrt[x]) - (9945*Sqrt[2]*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(3/4)*Sqrt[x]) + (994
```

```
5*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x])/(a^(3/4)*Sqrt[x])))/(491520*b^(21/4))
```

Maple [A] time = 0.034, size = 344, normalized size = 0.9

$$-\frac{663\,d^{19}a^{4}}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{5}}\sqrt{dx}-\frac{1989\,d^{17}a^{3}}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{4}}\left(dx\right)^{\frac{5}{2}}-\frac{9061\,d^{15}a^{2}}{6144\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{3}}\left(dx\right)^{\frac{9}{2}}-\frac{527\,d^{13}a}{384\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{2}}\left(dx\right)^{\frac{13}{2}}-\frac{7529\,d^{11}}{12288\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b}\left(dx\right)^{\frac{17}{2}}+\frac{663\,d^{9}\sqrt{2}}{32768\,ab^{5}}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)+\frac{663\,d^{9}\sqrt{2}}{16384\,ab^{5}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)+\frac{663\,d^{9}\sqrt{2}}{16384\,ab^{5}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -663/4096*d^19/(b*d^2*x^2+a*d^2)^5/b^5*a^4*(d*x)^(1/2)-1989/2560*
d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(5/2)-9061/6144*d^15/(b*d^
2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(9/2)-527/384*d^13/(b*d^2*x^2+a*d^2)
^5/b^2*a*(d*x)^(13/2)-7529/12288*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)
^(17/2)+663/32768*d^9/b^5*(a*d^2/b)^(1/4)/a*2^(1/2)*ln((d*x+(a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/16384*d^9/b^5*(a*d^
2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1
)+663/16384*d^9/b^5*(a*d^2/b)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(a*d
^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292152, size = 626, normalized size = 1.63

$$-\frac{39780\left(b^{10}x^{10}+5\,ab^{9}x^{8}+10\,a^{2}b^{8}x^{6}+10\,a^{3}b^{7}x^{4}+5\,a^{4}b^{6}x^{2}+a^{5}b^{5}\right)\left(-\frac{d^{38}}{a^{3}b^{21}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{38}}{a^{3}b^{21}}\right)^{\frac{1}{4}}ab^{5}}{\sqrt{dx}\,d^{9}+\sqrt{d^{19}x+\sqrt{-\frac{d^{38}}{a^{3}b^{21}}}\,a^{2}b^{10}}}\right)-9945}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(39780*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a
^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21))^(1/4)*ar
ctan((-d^38/(a^3*b^21))^(1/4)*a*b^5/(sqrt(d*x)*d^9 + sqrt(d^19*x
+ sqrt(-d^38/(a^3*b^21))*a^2*b^10))) - 9945*(b^10*x^10 + 5*a*b^9*
x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*
(-d^38/(a^3*b^21))^(1/4)*log(663*sqrt(d*x)*d^9 + 663*(-d^38/(a^3*
b^21))^(1/4)*a*b^5) + 9945*(b^10*x^10 + 5*a*b^9*x^8 + 10*a^2*b^8*
x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2 + a^5*b^5)*(-d^38/(a^3*b^21)
)^(1/4)*log(663*sqrt(d*x)*d^9 - 663*(-d^38/(a^3*b^21))^(1/4)*a*b^
5) + 4*(37645*b^4*d^9*x^8 + 84320*a*b^3*d^9*x^6 + 90610*a^2*b^2*d
^9*x^4 + 47736*a^3*b*d^9*x^2 + 9945*a^4*d^9)*sqrt(d*x))/(b^10*x^1
0 + 5*a*b^9*x^8 + 10*a^2*b^8*x^6 + 10*a^3*b^7*x^4 + 5*a^4*b^6*x^2
 + a^5*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281548, size = 463, normalized size = 1.2

$$\frac{1}{491520}d^8\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^6}+\frac{994}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^8*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2
)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6
) + 19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^6) + 9945*
sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/(a*b^6) - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*d
*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b
^6) - 8*(37645*sqrt(d*x)*b^4*d^11*x^8 + 84320*sqrt(d*x)*a*b^3*d^1
1*x^6 + 90610*sqrt(d*x)*a^2*b^2*d^11*x^4 + 47736*sqrt(d*x)*a^3*b*
d^11*x^2 + 9945*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*b^5))
```

3.715 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=388

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}} - \frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}} - \frac{231d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}} + \frac{231d^{17/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{5/4}b^{19/4}} + \frac{231d^7(dx)^{3/2}}{4096ab^4(a+bx^2)} - \frac{77d^7(dx)^{3/2}}{1024b^4(a+bx^2)^2} - \frac{11d^5(dx)^{7/2}}{128b^3(a+bx^2)^3} - \frac{3d^3(dx)^{11/2}}{32b^2(a+bx^2)^4} - \frac{d(dx)^{15/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(15/2))/(10*b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32
*b^2*(a + b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3
) - (77*d^7*(d*x)^(3/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x
)^(3/2))/(4096*a*b^4*(a + b*x^2)) - (231*d^(17/2)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)
*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4))
```

Rubi [A] time = 0.922398, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{231d^{17/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}} - \frac{231d^{17/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{5/4}b^{19/4}} - \frac{231d^{17/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{5/4}b^{19/4}} + \frac{231d^{17/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{5/4}b^{19/4}} + \frac{231d^7(dx)^{3/2}}{4096ab^4(a+bx^2)} - \frac{77d^7(dx)^{3/2}}{1024b^4(a+bx^2)^2} - \frac{11d^5(dx)^{7/2}}{128b^3(a+bx^2)^3} - \frac{3d^3(dx)^{11/2}}{32b^2(a+bx^2)^4} - \frac{d(dx)^{15/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(15/2))/(10*b*(a + b*x^2)^5) - (3*d^3*(d*x)^(11/2))/(32
*b^2*(a + b*x^2)^4) - (11*d^5*(d*x)^(7/2))/(128*b^3*(a + b*x^2)^3
) - (77*d^7*(d*x)^(3/2))/(1024*b^4*(a + b*x^2)^2) + (231*d^7*(d*x
)^(3/2))/(4096*a*b^4*(a + b*x^2)) - (231*d^(17/2)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)
*b^(19/4)) + (231*d^(17/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(5/4)*b^(19/4)) + (231*d^(17
/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(
1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4)) - (231*d^(17/2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(5/4)*b^(19/4))
```

Rubi in Sympy [F(-1)]   time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A]   time = 0.42868, size = 308, normalized size = 0.79

$$\frac{d^8\sqrt{dx}\left(\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{5/4}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{5/4}}+\frac{2310\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{5/4}} -\right.}{163840b^{19/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^8*Sqrt[d*x]*((16384*a^3*b^(3/4)*x^(3/2))/(a + b*x^2)^5 - (6451
2*a^2*b^(3/4)*x^(3/2))/(a + b*x^2)^4 + (93952*a*b^(3/4)*x^(3/2))/
(a + b*x^2)^3 - (58144*b^(3/4)*x^(3/2))/(a + b*x^2)^2 + (9240*b^(
3/4)*x^(3/2))/(a^2 + a*b*x^2) - (2310*Sqrt[2]*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (2310*Sqrt[2]*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(5/4) + (1155*Sqrt[2]*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(5/4) - (1
```

```
155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x])/a^(5/4)))/(163840*b^(19/4)*Sqrt[x])
```

Maple [A] time = 0.033, size = 341, normalized size = 0.9

$$-\frac{77\,d^{17}a^3}{4096\,(bd^2x^2+ad^2)^5\,b^4}(dx)^{\frac{3}{2}}-\frac{11\,d^{15}a^2}{128\,(bd^2x^2+ad^2)^5\,b^3}(dx)^{\frac{7}{2}}-\frac{313\,d^{13}a}{2048\,(bd^2x^2+ad^2)^5\,b^2}(dx)^{\frac{11}{2}}-\frac{331\,d^{11}}{2560\,(bd^2x^2+ad^2)^5\,b}(dx)^{\frac{15}{2}}+\frac{231\,d^9}{4096\,(bd^2x^2+ad^2)^5\,a}(dx)^{\frac{19}{2}}+\frac{231\,d^9\sqrt{2}}{32768\,ab^5}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{231\,d^9\sqrt{2}}{16384\,ab^5}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{231\,d^9\sqrt{2}}{16384\,ab^5}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(3/2)-11/128*d^15
/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(7/2)-313/2048*d^13/(b*d^2*x^2
+a*d^2)^5/b^2*a*(d*x)^(11/2)-331/2560*d^11/(b*d^2*x^2+a*d^2)^5/b*
(d*x)^(15/2)+231/4096*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(19/2)+231/
32768*d^9/a/b^5/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a*d^2/b)^(1/2)))+231/16384*d^9/a/b^5/(a*d^2/b)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+231/16384*
d^9/a/b^5/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292578, size = 657, normalized size = 1.69

$$\frac{4620\left(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{34}}{a^5b^{19}}\right)^{\frac{3}{4}}a^4b^{14}}{\sqrt{dx}d^{25}+\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{a^5b^{19}}}a^3b^9d^{34}}}\right)+1155}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(4620*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a
^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*ar
ctan((-d^34/(a^5*b^19))^(3/4)*a^4*b^14/(sqrt(d*x)*d^25 + sqrt(d^5
1*x - sqrt(-d^34/(a^5*b^19))*a^3*b^9*d^34))) + 1155*(a*b^9*x^10 +
 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2
+ a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 +
 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) - 1155*(a*b^9*x^10 +
 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2
+ a^6*b^4)*(-d^34/(a^5*b^19))^(1/4)*log(12326391*sqrt(d*x)*d^25 -
 12326391*(-d^34/(a^5*b^19))^(3/4)*a^4*b^14) + 4*(1155*b^4*d^8*x^
9 - 2648*a*b^3*d^8*x^7 - 3130*a^2*b^2*d^8*x^5 - 1760*a^3*b*d^8*x^
3 - 385*a^4*d^8*x)*sqrt(d*x))/(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^
3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280274, size = 463, normalized size = 1.19

$$\frac{1}{163840}\,d^7\left(\frac{2310\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7}+\frac{2310\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^7}-\frac{1155\,\sqrt{2}\,(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^7*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7)
 + 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^7) - 1155*sq
rt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^2*b^7) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b^7
) + 8*(1155*sqrt(d*x)*b^4*d^11*x^9 - 2648*sqrt(d*x)*a*b^3*d^11*x^
7 - 3130*sqrt(d*x)*a^2*b^2*d^11*x^5 - 1760*sqrt(d*x)*a^3*b*d^11*x
^3 - 385*sqrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

3.716 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=388

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}-\frac{117d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}+\frac{39d^7\sqrt{dx}}{4096ab^4(a+bx^2)}-\frac{39d^7\sqrt{dx}}{1024b^4(a+bx^2)^2}-\frac{39d^5(dx)^{5/2}}{640b^3(a+bx^2)^3}-\frac{13d^3(dx)^{9/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{13/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(13/2))/(10*b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(16
0*b^2*(a + b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^
3) - (39*d^7*Sqrt[d*x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d
*x])/(4096*a*b^4*(a + b*x^2)) - (117*d^(15/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(
17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^(15/2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rubi [A] time = 0.947452, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{117d^{15/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{7/4}b^{17/4}}-\frac{117d^{15/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}+\frac{117d^{15/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{7/4}b^{17/4}}+\frac{39d^7\sqrt{dx}}{4096ab^4(a+bx^2)}-\frac{39d^7\sqrt{dx}}{1024b^4(a+bx^2)^2}-\frac{39d^5(dx)^{5/2}}{640b^3(a+bx^2)^3}-\frac{13d^3(dx)^{9/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{13/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(13/2))/(10*b*(a + b*x^2)^5) - (13*d^3*(d*x)^(9/2))/(16
0*b^2*(a + b*x^2)^4) - (39*d^5*(d*x)^(5/2))/(640*b^3*(a + b*x^2)^
3) - (39*d^7*Sqrt[d*x])/(1024*b^4*(a + b*x^2)^2) + (39*d^7*Sqrt[d
*x])/(4096*a*b^4*(a + b*x^2)) - (117*d^(15/2)*ArcTan[1 - (Sqrt[2]
*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(
17/4)) + (117*d^(15/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(7/4)*b^(17/4)) - (117*d^(15/2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4)) + (117*d^(15/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.397379, size = 298, normalized size = 0.77

$$\frac{d^7\sqrt{dx}\left(-\frac{585\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}+\frac{585\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{7/4}\sqrt{x}}-\frac{1170\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{7/4}\sqrt{x}}+\frac{1170\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{7/4}\sqrt{x}}+\right.}{163840b^{17/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^7*Sqrt[d*x]*((16384*a^3*b^(1/4))/(a + b*x^2)^5 - (62464*a^2*b^
(1/4))/(a + b*x^2)^4 + (85760*a*b^(1/4))/(a + b*x^2)^3 - (45920*b
^(1/4))/(a + b*x^2)^2 + (1560*b^(1/4))/(a^2 + a*b*x^2) - (1170*Sq
rt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(7/4)*Sqr
t[x]) + (1170*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4
)])/(a^(7/4)*Sqrt[x]) - (585*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7/4)*Sqrt[x]) + (585*Sqrt[2]*
```

```
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(7
/4)*Sqrt[x])))/(163840*b^(17/4))
```

Maple [A] time = 0.033, size = 341, normalized size = 0.9

$$-\frac{117\,d^{17}a^3}{4096\,\left(bd^2x^2+ad^2\right)^5 b^4}\sqrt{dx}-\frac{351\,d^{15}a^2}{2560\,\left(bd^2x^2+ad^2\right)^5 b^3}(dx)^{\frac{5}{2}}-\frac{533\,d^{13}a}{2048\,\left(bd^2x^2+ad^2\right)^5 b^2}(dx)^{\frac{9}{2}}-\frac{31\,d^{11}}{128\,\left(bd^2x^2+ad^2\right)^5 b}(dx)^{\frac{13}{2}}+\frac{39\,d^9}{4096\,\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{17}{2}}+\frac{117\,d^7\sqrt{2}}{32768\,a^2b^4}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{117\,d^7\sqrt{2}}{16384\,a^2b^4}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{117\,d^7\sqrt{2}}{16384\,a^2b^4}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -117/4096*d^17/(b*d^2*x^2+a*d^2)^5/b^4*a^3*(d*x)^(1/2)-351/2560*d
^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(5/2)-533/2048*d^13/(b*d^2*
x^2+a*d^2)^5/b^2*a*(d*x)^(9/2)-31/128*d^11/(b*d^2*x^2+a*d^2)^5/b*
(d*x)^(13/2)+39/4096*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(17/2)+117/3
2768*d^7/a^2/b^4*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+117/16384*d^7/a^2/b^4*(a*d^2/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+117/163
84*d^7/a^2/b^4*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287873, size = 651, normalized size = 1.68

$$-\frac{2340\left(ab^9x^{10}+5\,a^2b^8x^8+10\,a^3b^7x^6+10\,a^4b^6x^4+5\,a^5b^5x^2+a^6b^4\right)\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{30}}{a^7b^{17}}\right)^{\frac{1}{4}}a^2b^4}{\sqrt{dx}d^7+\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^7b^{17}}}a^4b^8}}\right)-585\,(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(2340*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*
a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a^7*b^17))^(1/4)*a
rctan((-d^30/(a^7*b^17))^(1/4)*a^2*b^4/(sqrt(d*x)*d^7 + sqrt(d^15
*x + sqrt(-d^30/(a^7*b^17))*a^4*b^8))) - 585*(a*b^9*x^10 + 5*a^2*
b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b
^4)*(-d^30/(a^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 + 117*(-d^30/(
a^7*b^17))^(1/4)*a^2*b^4) + 585*(a*b^9*x^10 + 5*a^2*b^8*x^8 + 10*
a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*b^5*x^2 + a^6*b^4)*(-d^30/(a
^7*b^17))^(1/4)*log(117*sqrt(d*x)*d^7 - 117*(-d^30/(a^7*b^17))^(1
/4)*a^2*b^4) - 4*(195*b^4*d^7*x^8 - 4960*a*b^3*d^7*x^6 - 5330*a^2
*b^2*d^7*x^4 - 2808*a^3*b*d^7*x^2 - 585*a^4*d^7)*sqrt(d*x))/(a*b^
9*x^10 + 5*a^2*b^8*x^8 + 10*a^3*b^7*x^6 + 10*a^4*b^6*x^4 + 5*a^5*
b^5*x^2 + a^6*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280145, size = 467, normalized size = 1.2

$$\frac{1}{163840}d^6\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^5}+\frac{585\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^
5) + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^5) + 585
*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a^2*b^5) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)
*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^2*b^5) + 8*(195*sqrt(d*x)*b^4*d^11*x^8 - 4960*sqrt(d*x)*a*b^3*d^
11*x^6 - 5330*sqrt(d*x)*a^2*b^2*d^11*x^4 - 2808*sqrt(d*x)*a^3*b*d
^11*x^2 - 585*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a*b^4))
```

3.717 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=391

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}+\frac{77d^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}+\frac{77d^5(dx)^{3/2}}{4096a^2b^3(a+bx^2)}+\frac{77d^5(dx)^{3/2}}{5120ab^3(a+bx^2)^2}-\frac{77d^5(dx)^{3/2}}{1920b^3(a+bx^2)^3}-\frac{11d^3(dx)^{7/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{11/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(11/2))/(10*b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(16
0*b^2*(a + b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)
^3) + (77*d^5*(d*x)^(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(
d*x)^(3/2))/(4096*a^2*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(
13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4))
```

Rubi [A] time = 0.934253, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{77d^{13/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{9/4}b^{15/4}}-\frac{77d^{13/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}+\frac{77d^{13/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{9/4}b^{15/4}}+\frac{77d^5(dx)^{3/2}}{4096a^2b^3(a+bx^2)}+\frac{77d^5(dx)^{3/2}}{5120ab^3(a+bx^2)^2}-\frac{77d^5(dx)^{3/2}}{1920b^3(a+bx^2)^3}-\frac{11d^3(dx)^{7/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{11/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```



```
[Out] -(d*(d*x)^(11/2))/(10*b*(a + b*x^2)^5) - (11*d^3*(d*x)^(7/2))/(16
0*b^2*(a + b*x^2)^4) - (77*d^5*(d*x)^(3/2))/(1920*b^3*(a + b*x^2)
^3) + (77*d^5*(d*x)^(3/2))/(5120*a*b^3*(a + b*x^2)^2) + (77*d^5*(
```

```
d*x)^(3/2))/(4096*a^2*b^3*(a + b*x^2)) - (77*d^(13/2)*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(
9/4)*b^(15/4)) + (77*d^(13/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(9/4)*b^(15/4)) + (77*d^(
13/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4)) - (77*d^(13/2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(9/4)*b^(15/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)

[Out] Timed out
```

Mathematica [A] time = 0.454961, size = 308, normalized size = 0.79

$$\frac{d^6\sqrt{dx}\left(\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}-\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{9/4}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{9/4}}+\frac{2310\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{9/4}} \cdots\right)}{491520b^{15/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]

[Out] (d^6*Sqrt[d*x]*((-49152*a^2*b^(3/4)*x^(3/2))/(a + b*x^2)^5 + (132
096*a*b^(3/4)*x^(3/2))/(a + b*x^2)^4 - (102656*b^(3/4)*x^(3/2))/(
a + b*x^2)^3 + (7392*b^(3/4)*x^(3/2))/(a*(a + b*x^2)^2) + (9240*b
^(3/4)*x^(3/2))/(a^2*(a + b*x^2)) - (2310*Sqrt[2]*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(9/4) + (2310*Sqrt[2]*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(9/4) + (1155*Sqrt[2]*Log
[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(9/4)
- (1155*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x])/a^(9/4)))/(491520*b^(15/4)*Sqrt[x])
```

Maple [A] time = 0.033, size = 339, normalized size = 0.9

$$-\frac{77\,d^{15}a^{2}}{12288\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{3}}(dx)^{\frac{3}{2}}-\frac{11\,d^{13}a}{384\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{2}}(dx)^{\frac{7}{2}}-\frac{313\,d^{11}}{6144\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b}(dx)^{\frac{11}{2}}+\frac{231\,d^{9}}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a}(dx)^{\frac{15}{2}}+\frac{77\,d^{7}b}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{2}}(dx)^{\frac{19}{2}}+\frac{77\,d^{7}\sqrt{2}}{32768\,a^{2}b^{4}}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{77\,d^{7}\sqrt{2}}{16384\,a^{2}b^{4}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{77\,d^{7}\sqrt{2}}{16384\,a^{2}b^{4}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/12288*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(3/2)-11/384*d^1
3/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(7/2)-313/6144*d^11/(b*d^2*x^2+
a*d^2)^5/b*(d*x)^(11/2)+231/2560*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^
(15/2)+77/4096*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(19/2)+77/3276
8*d^7/a^2/b^4/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2)))+77/16384*d^7/a^2/b^4/(a*d^2/b)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^
7/a^2/b^4/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*
(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292703, size = 668, normalized size = 1.71

$$4620\left(a^2b^8x^{10}+5a^3b^7x^8+10a^4b^6x^6+10a^5b^5x^4+5a^6b^4x^2+a^7b^3\right)\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{26}}{a^9b^{15}}\right)^{\frac{3}{4}}a^7b^{11}}{\sqrt{dx}d^{19}+\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^9b^{15}}}a^5b^7d^{26}}}\right)+115$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(4620*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 1
0*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)
*arctan((-d^26/(a^9*b^15))^(3/4)*a^7*b^11/(sqrt(d*x)*d^19 + sqrt(
d^39*x - sqrt(-d^26/(a^9*b^15))*a^5*b^7*d^26))) + 1155*(a^2*b^8*x
^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4
*x^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^1
9 + 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) - 1155*(a^2*b^8*x^1
0 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x
^2 + a^7*b^3)*(-d^26/(a^9*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19
- 456533*(-d^26/(a^9*b^15))^(3/4)*a^7*b^11) + 4*(1155*b^4*d^6*x^9
 + 5544*a*b^3*d^6*x^7 - 3130*a^2*b^2*d^6*x^5 - 1760*a^3*b*d^6*x^3
 - 385*a^4*d^6*x)*sqrt(d*x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a
^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282765, size = 463, normalized size = 1.18

$$\frac{1}{491520}d^5\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^6}-\frac{1155\sqrt{2}\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^5*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6)
 + 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^6) - 1155*sq
rt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x
) + sqrt(a*d^2/b))/(a^3*b^6) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^6
) + 8*(1155*sqrt(d*x)*b^4*d^11*x^9 + 5544*sqrt(d*x)*a*b^3*d^11*x^
7 - 3130*sqrt(d*x)*a^2*b^2*d^11*x^5 - 1760*sqrt(d*x)*a^3*b*d^11*x
^3 - 385*sqrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

3.718 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}-\frac{63d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}+\frac{21d^5\sqrt{dx}}{4096a^2b^3(a+bx^2)}+\frac{3d^5\sqrt{dx}}{1024ab^3(a+bx^2)^2}-\frac{3d^5\sqrt{dx}}{128b^3(a+bx^2)^3}-\frac{9d^3(dx)^{5/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{9/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(9/2))/(10*b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*
b^2*(a + b*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) +
(3*d^5*Sqrt[d*x])/(1024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])
/(4096*a^2*b^3*(a + b*x^2)) - (63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13
/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4))
```

Rubi [A] time = 0.92032, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{63d^{11/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{11/4}b^{13/4}}-\frac{63d^{11/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}+\frac{63d^{11/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{11/4}b^{13/4}}+\frac{21d^5\sqrt{dx}}{4096a^2b^3(a+bx^2)}+\frac{3d^5\sqrt{dx}}{1024ab^3(a+bx^2)^2}-\frac{3d^5\sqrt{dx}}{128b^3(a+bx^2)^3}-\frac{9d^3(dx)^{5/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{9/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(9/2))/(10*b*(a + b*x^2)^5) - (9*d^3*(d*x)^(5/2))/(160*
b^2*(a + b*x^2)^4) - (3*d^5*Sqrt[d*x])/(128*b^3*(a + b*x^2)^3) +
(3*d^5*Sqrt[d*x])/(1024*a*b^3*(a + b*x^2)^2) + (21*d^5*Sqrt[d*x])
```

```
/(4096*a^2*b^3*(a + b*x^2)) - (63*d^(11/2)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13
/4)) + (63*d^(11/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(8192*Sqrt[2]*a^(11/4)*b^(13/4)) - (63*d^(11/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4)) + (63*d^(11/2)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*a^(11/4)*b^(13/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.400915, size = 298, normalized size = 0.76

$$\frac{d^5\sqrt{dx}\left(-\frac{315\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}+\frac{315\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{11/4}\sqrt{x}}-\frac{630\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{11/4}\sqrt{x}}+\frac{630\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{11/4}\sqrt{x}}-\cdots\right)}{163840b^{13/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^5*Sqrt[d*x]*((-16384*a^2*b^(1/4))/(a + b*x^2)^5 + (41984*a*b^(
1/4))/(a + b*x^2)^4 - (29440*b^(1/4))/(a + b*x^2)^3 + (480*b^(1/4
))/(a*(a + b*x^2)^2) + (840*b^(1/4))/(a^2*(a + b*x^2)) - (630*Sqr
t[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(11/4)*Sqr
t[x]) + (630*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
])/(a^(11/4)*Sqrt[x]) - (315*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(11/4)*Sqrt[x]) + (315*Sqrt[2]
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(
11/4)*Sqrt[x])))/(163840*b^(13/4))
```

Maple [A] time = 0.033, size = 339, normalized size = 0.9

$$-\frac{63\,d^{15}a^{2}}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{3}}\sqrt{dx}-\frac{189\,d^{13}a}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{2}}\left(dx\right)^{\frac{5}{2}}-\frac{287\,d^{11}}{2048\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b}\left(dx\right)^{\frac{9}{2}}+\frac{3\,d^{9}}{128\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a}\left(dx\right)^{\frac{13}{2}}+\frac{21\,d^{7}b}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{2}}\left(dx\right)^{\frac{17}{2}}+\frac{63\,d^{5}\sqrt{2}}{32768\,a^{3}b^{3}}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)+\frac{63\,d^{5}\sqrt{2}}{16384\,a^{3}b^{3}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)+\frac{63\,d^{5}\sqrt{2}}{16384\,a^{3}b^{3}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -63/4096*d^15/(b*d^2*x^2+a*d^2)^5/b^3*a^2*(d*x)^(1/2)-189/2560*d^
13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(5/2)-287/2048*d^11/(b*d^2*x^2
+a*d^2)^5/b*(d*x)^(9/2)+3/128*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(13
/2)+21/4096*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(17/2)+63/32768*d
^5/a^3/b^3*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2)))+63/16384*d^5/a^3/b^3*(a*d^2/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a
^3/b^3*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290741, size = 662, normalized size = 1.69

$$-\frac{1260\left(a^{2}b^{8}x^{10}+5\,a^{3}b^{7}x^{8}+10\,a^{4}b^{6}x^{6}+10\,a^{5}b^{5}x^{4}+5\,a^{6}b^{4}x^{2}+a^{7}b^{3}\right)\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^{3}b^{3}\left(-\frac{d^{22}}{a^{11}b^{13}}\right)^{\frac{1}{4}}}{\sqrt{dx}d^{5}+\sqrt{a^{6}b^{6}\sqrt{-\frac{d^{22}}{a^{11}b^{13}}}+d^{11}x}}\right)-315}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(1260*(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 1
0*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*(-d^22/(a^11*b^13))^(1/4
)*arctan(a^3*b^3*(-d^22/(a^11*b^13))^(1/4)/(sqrt(d*x)*d^5 + sqrt(
a^6*b^6*sqrt(-d^22/(a^11*b^13)) + d^11*x))) - 315*(a^2*b^8*x^10 +
 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2
+ a^7*b^3)*(-d^22/(a^11*b^13))^(1/4)*log(63*a^3*b^3*(-d^22/(a^11*
b^13))^(1/4) + 63*sqrt(d*x)*d^5) + 315*(a^2*b^8*x^10 + 5*a^3*b^7*
x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x^4 + 5*a^6*b^4*x^2 + a^7*b^3)*
(-d^22/(a^11*b^13))^(1/4)*log(-63*a^3*b^3*(-d^22/(a^11*b^13))^(1/
4) + 63*sqrt(d*x)*d^5) - 4*(105*b^4*d^5*x^8 + 480*a*b^3*d^5*x^6 -
 2870*a^2*b^2*d^5*x^4 - 1512*a^3*b*d^5*x^2 - 315*a^4*d^5)*sqrt(d*
x))/(a^2*b^8*x^10 + 5*a^3*b^7*x^8 + 10*a^4*b^6*x^6 + 10*a^5*b^5*x
^4 + 5*a^6*b^4*x^2 + a^7*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279805, size = 467, normalized size = 1.19

$$\frac{1}{163840}\,d^{4}\left(\frac{630\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{4}}+\frac{630\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}b^{4}}+\frac{315\,\sqrt{2}\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^4*(630*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4
) + 630*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^4) + 315*s
qrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
d*x) + sqrt(a*d^2/b))/(a^3*b^4) - 315*sqrt(2)*(a*b^3*d^2)^(1/4)*d
*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3
*b^4) + 8*(105*sqrt(d*x)*b^4*d^11*x^8 + 480*sqrt(d*x)*a*b^3*d^11*
x^6 - 2870*sqrt(d*x)*a^2*b^2*d^11*x^4 - 1512*sqrt(d*x)*a^3*b*d^11
*x^2 - 315*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a^2*b^3))
```

3.719 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=394

$$\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{13/4}b^{11/4}}+\frac{63d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{13/4}b^{11/4}}+\frac{63d^3(dx)^{3/2}}{4096a^3b^2(a+bx^2)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2(a+bx^2)^2}+\frac{7d^3(dx)^{3/2}}{640ab^2(a+bx^2)^3}-\frac{7d^3(dx)^{3/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{7/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(7/2))/(10*b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*
b^2*(a + b*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3
) + (63*d^3*(d*x)^(3/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(
d*x)^(3/2))/(4096*a^3*b^2*(a + b*x^2)) - (63*d^(9/2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1
3/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(
9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4))
```

Rubi [A] time = 0.94411, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$\frac{63d^{9/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{13/4}b^{11/4}}-\frac{63d^{9/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{13/4}b^{11/4}}+\frac{63d^{9/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{13/4}b^{11/4}}+\frac{63d^3(dx)^{3/2}}{4096a^3b^2(a+bx^2)}+\frac{63d^3(dx)^{3/2}}{5120a^2b^2(a+bx^2)^2}+\frac{7d^3(dx)^{3/2}}{640ab^2(a+bx^2)^3}-\frac{7d^3(dx)^{3/2}}{160b^2(a+bx^2)^4}-\frac{d(dx)^{7/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(7/2))/(10*b*(a + b*x^2)^5) - (7*d^3*(d*x)^(3/2))/(160*
b^2*(a + b*x^2)^4) + (7*d^3*(d*x)^(3/2))/(640*a*b^2*(a + b*x^2)^3
) + (63*d^3*(d*x)^(3/2))/(5120*a^2*b^2*(a + b*x^2)^2) + (63*d^3*(
```

```
d*x)^(3/2))/(4096*a^3*b^2*(a + b*x^2)) - (63*d^(9/2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(1
3/4)*b^(11/4)) + (63*d^(9/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x
])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(13/4)*b^(11/4)) + (63*d^(
9/2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4)) - (63*d^(9/2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(13/4)*b^(11/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.441838, size = 308, normalized size = 0.78

$$\frac{d^4\sqrt{dx}\left(\frac{315\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}}-\frac{315\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{13/4}}-\frac{630\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{13/4}}+\frac{630\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{13/4}}+\frac{25}{a}\cdots\right)}{163840b^{11/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^4*Sqrt[d*x]*((16384*a*b^(3/4)*x^(3/2))/(a + b*x^2)^5 - (23552*
b^(3/4)*x^(3/2))/(a + b*x^2)^4 + (1792*b^(3/4)*x^(3/2))/(a*(a + b
*x^2)^3) + (2016*b^(3/4)*x^(3/2))/(a^2*(a + b*x^2)^2) + (2520*b^(
3/4)*x^(3/2))/(a^3*(a + b*x^2)) - (630*Sqrt[2]*ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(13/4) + (630*Sqrt[2]*ArcTan[1 + (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(13/4) + (315*Sqrt[2]*Log[Sq
rt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(13/4) -
(315*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x])/a^(13/4)))/(163840*b^(11/4)*Sqrt[x])
```

Maple [A] time = 0.033, size = 339, normalized size = 0.9

$$-\frac{21\,d^{13}a}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{2}}\left(dx\right)^{\frac{3}{2}}-\frac{3\,d^{11}}{128\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b}\left(dx\right)^{\frac{7}{2}}+\frac{287\,d^{9}}{2048\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a}\left(dx\right)^{\frac{11}{2}}+\frac{189\,d^{7}b}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{2}}\left(dx\right)^{\frac{15}{2}}+\frac{63\,d^{5}b^{2}}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{3}}\left(dx\right)^{\frac{19}{2}}+\frac{63\,d^{5}\sqrt{2}}{32768\,a^{3}b^{3}}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{63\,d^{5}\sqrt{2}}{16384\,a^{3}b^{3}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+\frac{63\,d^{5}\sqrt{2}}{16384\,a^{3}b^{3}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -21/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(3/2)-3/128*d^11/(b
*d^2*x^2+a*d^2)^5/b*(d*x)^(7/2)+287/2048*d^9/(b*d^2*x^2+a*d^2)^5/
a*(d*x)^(11/2)+189/2560*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(15/2
)+63/4096*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(19/2)+63/32768*d
^5/a^3/b^3/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2)))+63/16384*d^5/a^3/b^3/(a*d^2/b)^(1/4)*2^(
1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+63/16384*d^5/a
^3/b^3/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*
x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287692, size = 672, normalized size = 1.71

$$\frac{1260 \left(a^3 b^7 x^{10} + 5 a^4 b^6 x^8 + 10 a^5 b^5 x^6 + 10 a^6 b^4 x^4 + 5 a^7 b^3 x^2 + a^8 b^2\right) \left(-\frac{d^{18}}{a^{13} b^{11}}\right)^{\frac{1}{4}} \arctan\left(\frac{250047 a^{10} b^8 \left(-\frac{d^{18}}{a^{13} b^{1}}\right.}{250047 \sqrt{dx} d^{13} + \sqrt{-62523502209 a^7 b^5 d^{18} \sqrt{-}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(1260*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10
*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^13*b^11))^(1/4)
*arctan(250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4)/(250047*sqrt(d*
x)*d^13 + sqrt(-62523502209*a^7*b^5*d^18*sqrt(-d^18/(a^13*b^11))
+ 62523502209*d^27*x))) + 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*
a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a
^13*b^11))^(1/4)*log(250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) +
250047*sqrt(d*x)*d^13) - 315*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a
^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^18/(a^
13*b^11))^(1/4)*log(-250047*a^10*b^8*(-d^18/(a^13*b^11))^(3/4) +
250047*sqrt(d*x)*d^13) + 4*(315*b^4*d^4*x^9 + 1512*a*b^3*d^4*x^7
+ 2870*a^2*b^2*d^4*x^5 - 480*a^3*b*d^4*x^3 - 105*a^4*d^4*x)*sqrt(
d*x))/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4
*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.282487, size = 463, normalized size = 1.18

$$\frac{1}{163840}\,d^3\left(\frac{630\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5}+\frac{630\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^5}-\frac{315\,\sqrt{2}\left(ab^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d^3*(630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5)
+ 630*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^5) - 315*sqrt(
2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^4*b^5) + 315*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^5) +
8*(315*sqrt(d*x)*b^4*d^11*x^9 + 1512*sqrt(d*x)*a*b^3*d^11*x^7 + 2
870*sqrt(d*x)*a^2*b^2*d^11*x^5 - 480*sqrt(d*x)*a^3*b*d^11*x^3 - 1
05*sqrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^3*b^2))
```

3.720 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=394

$$-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}-\frac{77d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^3\sqrt{dx}}{12288a^3b^2(a+bx^2)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2(a+bx^2)^2}+\frac{d^3\sqrt{dx}}{384ab^2(a+bx^2)^3}-\frac{d^3\sqrt{dx}}{32b^2(a+bx^2)^4}-\frac{d(dx)^{5/2}}{10b(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(5/2))/(10*b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(
a + b*x^2)^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d
^3*Sqrt[d*x])/(3072*a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(
12288*a^3*b^2*(a + b*x^2)) - (77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)
) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Log[Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(15/4)*b^(9/4))
```

Rubi [A] time = 0.974453, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{77d^{7/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{15/4}b^{9/4}}-\frac{77d^{7/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^{7/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{15/4}b^{9/4}}+\frac{77d^3\sqrt{dx}}{12288a^3b^2(a+bx^2)}+\frac{11d^3\sqrt{dx}}{3072a^2b^2(a+bx^2)^2}+\frac{d^3\sqrt{dx}}{384ab^2(a+bx^2)^3}-\frac{d^3\sqrt{dx}}{32b^2(a+bx^2)^4}-\frac{d(dx)^{5/2}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*(d*x)^(5/2))/(10*b*(a + b*x^2)^5) - (d^3*Sqrt[d*x])/(32*b^2*(
a + b*x^2)^4) + (d^3*Sqrt[d*x])/(384*a*b^2*(a + b*x^2)^3) + (11*d
^3*Sqrt[d*x])/(3072*a^2*b^2*(a + b*x^2)^2) + (77*d^3*Sqrt[d*x])/(
```

```
12288*a^3*b^2*(a + b*x^2)) - (77*d^(7/2)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)
) + (77*d^(7/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*a^(15/4)*b^(9/4)) - (77*d^(7/2)*Log[Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(16384*Sqrt[2]*a^(15/4)*b^(9/4)) + (77*d^(7/2)*Log[Sqrt[a]*Sq
rt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(
16384*Sqrt[2]*a^(15/4)*b^(9/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.478005, size = 298, normalized size = 0.76

$$\frac{d^3\sqrt{dx}\left(-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{15/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{15/4}\sqrt{x}}+\frac{2310\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{15/4}\sqrt{x}}\right.}{491520b^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d^3*Sqrt[d*x]*((49152*a*b^(1/4))/(a + b*x^2)^5 - (64512*b^(1/4))
/(a + b*x^2)^4 + (1280*b^(1/4))/(a*(a + b*x^2)^3) + (1760*b^(1/4)
)/(a^2*(a + b*x^2)^2) + (3080*b^(1/4))/(a^3*(a + b*x^2)) - (2310*
Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(15/4)*
Sqrt[x]) + (2310*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)])/(a^(15/4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(15/4)*Sqrt[x]) + (1155*S
qrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
)/(a^(15/4)*Sqrt[x])))/(491520*b^(9/4))
```

Maple [A] time = 0.034, size = 339, normalized size = 0.9

$$-\frac{77\,d^{13}a}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b^{2}}\sqrt{dx}-\frac{231\,d^{11}}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}b}\left(dx\right)^{\frac{5}{2}}+\frac{313\,d^{9}}{6144\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a}\left(dx\right)^{\frac{9}{2}}+\frac{11\,d^{7}b}{384\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{2}}\left(dx\right)^{\frac{13}{2}}+\frac{77\,d^{5}b^{2}}{12288\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{3}}\left(dx\right)^{\frac{17}{2}}+\frac{77\,d^{3}\sqrt{2}}{32768\,a^{4}b^{2}}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)+\frac{77\,d^{3}\sqrt{2}}{16384\,a^{4}b^{2}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)+\frac{77\,d^{3}\sqrt{2}}{16384\,a^{4}b^{2}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -77/4096*d^13/(b*d^2*x^2+a*d^2)^5/b^2*a*(d*x)^(1/2)-231/2560*d^11
/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(5/2)+313/6144*d^9/(b*d^2*x^2+a*d^2)
^5/a*(d*x)^(9/2)+11/384*d^7/(b*d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(13/2
)+77/12288*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(17/2)+77/32768*
d^3/a^4/b^2*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))+77/16384*d^3/a^4/b^2*(a*d^2/b)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+77/16384*d^3/
a^4/b^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d
*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29248, size = 662, normalized size = 1.68

$$-\frac{4620\left(a^{3}b^{7}x^{10}+5\,a^{4}b^{6}x^{8}+10\,a^{5}b^{5}x^{6}+10\,a^{6}b^{4}x^{4}+5\,a^{7}b^{3}x^{2}+a^{8}b^{2}\right)\left(-\frac{d^{14}}{a^{15}b^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^{4}b^{2}\left(-\frac{d^{14}}{a^{15}b^{9}}\right)^{\frac{1}{4}}}{\sqrt{dx}d^{3}+\sqrt{a^{8}b^{4}\sqrt{-\frac{d^{14}}{a^{15}b^{9}}}+d^{7}x}}\right)-1155}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/245760*(4620*(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 +
10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d^14/(a^15*b^9))^(1/4
)*arctan(a^4*b^2*(-d^14/(a^15*b^9))^(1/4)/(sqrt(d*x)*d^3 + sqrt(a
^8*b^4*sqrt(-d^14/(a^15*b^9)) + d^7*x))) - 1155*(a^3*b^7*x^10 + 5
*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 +
a^8*b^2)*(-d^14/(a^15*b^9))^(1/4)*log(77*a^4*b^2*(-d^14/(a^15*b^9
))^(1/4) + 77*sqrt(d*x)*d^3) + 1155*(a^3*b^7*x^10 + 5*a^4*b^6*x^8
 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4 + 5*a^7*b^3*x^2 + a^8*b^2)*(-d
^14/(a^15*b^9))^(1/4)*log(-77*a^4*b^2*(-d^14/(a^15*b^9))^(1/4) +
77*sqrt(d*x)*d^3) - 4*(385*b^4*d^3*x^8 + 1760*a*b^3*d^3*x^6 + 313
0*a^2*b^2*d^3*x^4 - 5544*a^3*b*d^3*x^2 - 1155*a^4*d^3)*sqrt(d*x))
/(a^3*b^7*x^10 + 5*a^4*b^6*x^8 + 10*a^5*b^5*x^6 + 10*a^6*b^4*x^4
+ 5*a^7*b^3*x^2 + a^8*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280652, size = 467, normalized size = 1.19

$$\frac{1}{491520}d^{2}\left(\frac{2310\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{3}}+\frac{2310\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{a^{4}b^{3}}+\frac{1155}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/491520*d^2*(2310*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)
*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^
3) + 2310*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^3) + 115
5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sq
rt(d*x) + sqrt(a*d^2/b))/(a^4*b^3) - 1155*sqrt(2)*(a*b^3*d^2)^(1/
4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^4*b^3) + 8*(385*sqrt(d*x)*b^4*d^11*x^8 + 1760*sqrt(d*x)*a*b^3*
d^11*x^6 + 3130*sqrt(d*x)*a^2*b^2*d^11*x^4 - 5544*sqrt(d*x)*a^3*b
*d^11*x^2 - 1155*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*d^2)^5*a^3*b
^2))
```

3.721 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=389

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d(dx)^{3/2}}{4096a^4b\,(a+bx^2)}+\frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)^2}+\frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3}+\frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4}-\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5}$$

```
[Out] -(d*(d*x)^(3/2))/(10*b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*
b*(a + b*x^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) +
 (117*d*(d*x)^(3/2))/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3
/2))/(4096*a^4*b*(a + b*x^2)) - (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(
7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
*x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4))
```

Rubi [A] time = 0.963792, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{117d^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{17/4}b^{7/4}}-\frac{117d^{5/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d^{5/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{17/4}b^{7/4}}+\frac{117d(dx)^{3/2}}{4096a^4b\,(a+bx^2)}+\frac{117d(dx)^{3/2}}{5120a^3b\,(a+bx^2)^2}+\frac{13d(dx)^{3/2}}{640a^2b\,(a+bx^2)^3}+\frac{3d(dx)^{3/2}}{160ab\,(a+bx^2)^4}-\frac{d(dx)^{3/2}}{10b\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```



```
[Out] -(d*(d*x)^(3/2))/(10*b*(a + b*x^2)^5) + (3*d*(d*x)^(3/2))/(160*a*
b*(a + b*x^2)^4) + (13*d*(d*x)^(3/2))/(640*a^2*b*(a + b*x^2)^3) +
 (117*d*(d*x)^(3/2))/(5120*a^3*b*(a + b*x^2)^2) + (117*d*(d*x)^(3
```

```
/2))/(4096*a^4*b*(a + b*x^2)) - (117*d^(5/2)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(
7/4)) + (117*d^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(17/4)*b^(7/4)) + (117*d^(5/2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4)) - (117*d^(5/2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
*x]])/(16384*Sqrt[2]*a^(17/4)*b^(7/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.465504, size = 307, normalized size = 0.79

$$\frac{(dx)^{5/2}\left(\frac{585\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{17/4}}-\frac{585\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{17/4}}-\frac{1170\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{17/4}}+\frac{1170\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{17/4}}+\right.}{163840b^{7/4}x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(5/2)*((-16384*b^(3/4)*x^(3/2))/(a + b*x^2)^5 + (3072*b^(3
/4)*x^(3/2))/(a*(a + b*x^2)^4) + (3328*b^(3/4)*x^(3/2))/(a^2*(a +
 b*x^2)^3) + (3744*b^(3/4)*x^(3/2))/(a^3*(a + b*x^2)^2) + (4680*b
^(3/4)*x^(3/2))/(a^4*(a + b*x^2)) - (1170*Sqrt[2]*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(17/4) + (1170*Sqrt[2]*ArcTan[1
 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/a^(17/4) + (585*Sqrt[2]*Lo
g[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/a^(17/4
) - (585*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x])/a^(17/4)))/(163840*b^(7/4)*x^(5/2))
```

Maple [A] time = 0.033, size = 341, normalized size = 0.9

$$-\frac{39\,d^{11}}{4096\,\left(bd^2x^2+ad^2\right)^5 b}(dx)^{\frac{3}{2}}+\frac{31\,d^9}{128\,\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{7}{2}}+\frac{533\,d^7b}{2048\,\left(bd^2x^2+ad^2\right)^5 a^2}(dx)^{\frac{11}{2}}$$
$$+\frac{351\,d^5b^2}{2560\,\left(bd^2x^2+ad^2\right)^5 a^3}(dx)^{\frac{15}{2}}+\frac{117\,d^3b^3}{4096\,\left(bd^2x^2+ad^2\right)^5 a^4}(dx)^{\frac{19}{2}}$$
$$+\frac{117\,d^3\sqrt{2}}{32768\,a^4b^2}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$
$$+\frac{117\,d^3\sqrt{2}}{16384\,a^4b^2}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{117\,d^3\sqrt{2}}{16384\,a^4b^2}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -39/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(3/2)+31/128*d^9/(b*d^2
*x^2+a*d^2)^5/a*(d*x)^(7/2)+533/2048*d^7/(b*d^2*x^2+a*d^2)^5/a^2*
b*(d*x)^(11/2)+351/2560*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(15
/2)+117/4096*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(19/2)+117/327
68*d^3/a^4/b^2/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))+117/16384*d^3/a^4/b^2/(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+117/16384
*d^3/a^4/b^2/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/
4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293292, size = 662, normalized size = 1.7

$$2340\left(a^{4}b^{6}x^{10}+5\,a^{5}b^{5}x^{8}+10\,a^{6}b^{4}x^{6}+10\,a^{7}b^{3}x^{4}+5\,a^{8}b^{2}x^{2}+a^{9}b\right)\left(-\frac{d^{10}}{a^{17}b^{7}}\right)^{\frac{1}{4}}\arctan\left(\frac{1601613\,a^{13}b^{5}\left(-\frac{d^{10}}{a^{17}b^{7}}\right.}{1601613\,\sqrt{dx}\,d^{7}+\sqrt{-2565164201769\,a^{9}b^{3}d^{10}\sqrt{\cdots}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/81920*(2340*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10
*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^7))^(1/4)*ar
ctan(1601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4)/(1601613*sqrt(d*x)
*d^7 + sqrt(-2565164201769*a^9*b^3*d^10*sqrt(-d^10/(a^17*b^7)) +
2565164201769*d^15*x))) + 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*
a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^1
7*b^7))^(1/4)*log(1601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 160
1613*sqrt(d*x)*d^7) - 585*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*
b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^10/(a^17*b^
7))^(1/4)*log(-1601613*a^13*b^5*(-d^10/(a^17*b^7))^(3/4) + 160161
3*sqrt(d*x)*d^7) + 4*(585*b^4*d^2*x^9 + 2808*a*b^3*d^2*x^7 + 5330
*a^2*b^2*d^2*x^5 + 4960*a^3*b*d^2*x^3 - 195*a^4*d^2*x)*sqrt(d*x))
/(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4
+ 5*a^8*b^2*x^2 + a^9*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.281821, size = 460, normalized size = 1.18

$$\frac{1}{163840}d\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^5b^4}-\frac{585\sqrt{2}\left(ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 1/163840*d*(1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4) +
 1170*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^4) - 585*sqrt(
2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
 sqrt(a*d^2/b))/(a^5*b^4) + 585*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b^4) +
8*(585*sqrt(d*x)*b^4*d^11*x^9 + 2808*sqrt(d*x)*a*b^3*d^11*x^7 + 5
330*sqrt(d*x)*a^2*b^2*d^11*x^5 + 4960*sqrt(d*x)*a^3*b*d^11*x^3 -
195*sqrt(d*x)*a^4*d^11*x)/((b*d^2*x^2 + a*d^2)^5*a^4*b))
```

3.722 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=389

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}-\frac{231d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}+\frac{77d\sqrt{dx}}{4096a^4b(a+bx^2)}+\frac{11d\sqrt{dx}}{1024a^3b(a+bx^2)^2}+\frac{d\sqrt{dx}}{128a^2b(a+bx^2)^3}+\frac{d\sqrt{dx}}{160ab(a+bx^2)^4}-\frac{d\sqrt{dx}}{10b(a+bx^2)^5}$$

```
[Out] -(d*Sqrt[d*x])/(10*b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a +
 b*x^2)^4) + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt
[d*x])/(1024*a^3*b*(a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*
(a + b*x^2)) - (231*d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) + (231*d^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8
192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
Sqrt[2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(19/4)*b^(5/4))
```

Rubi [A] time = 0.957029, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$-\frac{231d^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{19/4}b^{5/4}}-\frac{231d^{3/2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}+\frac{231d^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{19/4}b^{5/4}}+\frac{77d\sqrt{dx}}{4096a^4b(a+bx^2)}+\frac{11d\sqrt{dx}}{1024a^3b(a+bx^2)^2}+\frac{d\sqrt{dx}}{128a^2b(a+bx^2)^3}+\frac{d\sqrt{dx}}{160ab(a+bx^2)^4}-\frac{d\sqrt{dx}}{10b(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] -(d*Sqrt[d*x])/(10*b*(a + b*x^2)^5) + (d*Sqrt[d*x])/(160*a*b*(a +
 b*x^2)^4) + (d*Sqrt[d*x])/(128*a^2*b*(a + b*x^2)^3) + (11*d*Sqrt
[d*x])/(1024*a^3*b*(a + b*x^2)^2) + (77*d*Sqrt[d*x])/(4096*a^4*b*
(a + b*x^2)) - (231*d^(3/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x]
)/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(19/4)*b^(5/4)) + (231*d^(3
/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8
192*Sqrt[2]*a^(19/4)*b^(5/4)) - (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d]
+ Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*
Sqrt[2]*a^(19/4)*b^(5/4)) + (231*d^(3/2)*Log[Sqrt[a]*Sqrt[d] + Sq
rt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt
[2]*a^(19/4)*b^(5/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.486686, size = 298, normalized size = 0.77

$$\frac{d\sqrt{dx}\left(-\frac{1155\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}+\frac{1155\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{a^{19/4}\sqrt{x}}-\frac{2310\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{a^{19/4}\sqrt{x}}+\frac{2310\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{a^{19/4}\sqrt{x}}\right.}{163840b^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*Sqrt[d*x]*((-16384*b^(1/4))/(a + b*x^2)^5 + (1024*b^(1/4))/(a*
(a + b*x^2)^4) + (1280*b^(1/4))/(a^2*(a + b*x^2)^3) + (1760*b^(1/
4))/(a^3*(a + b*x^2)^2) + (3080*b^(1/4))/(a^4*(a + b*x^2)) - (231
0*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/(a^(19/4
)*Sqrt[x]) + (2310*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)])/(a^(19/4)*Sqrt[x]) - (1155*Sqrt[2]*Log[Sqrt[a] - Sqrt[2]
```

```
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(a^(19/4)*Sqrt[x]) + (1155
*Sqrt[2]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(a^(19/4)*Sqrt[x])))/(163840*b^(5/4))
```

Maple [A] time = 0.031, size = 335, normalized size = 0.9

$$-\frac{231\,d^{11}}{4096\,\left(bd^2x^2+ad^2\right)^5 b}\sqrt{dx}+\frac{331\,d^9}{2560\,\left(bd^2x^2+ad^2\right)^5 a}(dx)^{\frac{5}{2}}+\frac{313\,d^7b}{2048\,\left(bd^2x^2+ad^2\right)^5 a^2}(dx)^{\frac{9}{2}}+\frac{11\,d^5b^2}{128\,\left(bd^2x^2+ad^2\right)^5 a^3}(dx)^{\frac{13}{2}}+\frac{77\,d^3b^3}{4096\,\left(bd^2x^2+ad^2\right)^5 a^4}(dx)^{\frac{17}{2}}+\frac{231\,d\sqrt{2}}{32768\,a^5b}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+\frac{231\,d\sqrt{2}}{16384\,a^5b}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)+\frac{231\,d\sqrt{2}}{16384\,a^5b}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -231/4096*d^11/(b*d^2*x^2+a*d^2)^5/b*(d*x)^(1/2)+331/2560*d^9/(b*
d^2*x^2+a*d^2)^5/a*(d*x)^(5/2)+313/2048*d^7/(b*d^2*x^2+a*d^2)^5/a
^2*b*(d*x)^(9/2)+11/128*d^5/(b*d^2*x^2+a*d^2)^5/a^3*b^2*(d*x)^(13
/2)+77/4096*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(d*x)^(17/2)+231/3276
8*d/a^5/b*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2)))+231/16384*d/a^5/b*(a*d^2/b)^(1/4)*2^(1/2)
*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+231/16384*d/a^5/b*
(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2
)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291097, size = 621, normalized size = 1.6

$$-\frac{4620\left(a^4b^6x^{10}+5\,a^5b^5x^8+10\,a^6b^4x^6+10\,a^7b^3x^4+5\,a^8b^2x^2+a^9b\right)\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^5b\left(-\frac{d^6}{a^{19}b^5}\right)^{\frac{1}{4}}}{\sqrt{dx}d+\sqrt{a^{10}b^2\sqrt{-\frac{d^6}{a^{19}b^5}}+d^3x}}\right)-1155\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] -1/81920*(4620*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x^6 + 1
0*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1/4)*ar
ctan(a^5*b*(-d^6/(a^19*b^5))^(1/4)/(sqrt(d*x)*d + sqrt(a^10*b^2*s
qrt(-d^6/(a^19*b^5)) + d^3*x))) - 1155*(a^4*b^6*x^10 + 5*a^5*b^5*
x^8 + 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-
d^6/(a^19*b^5))^(1/4)*log(231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231
*sqrt(d*x)*d) + 1155*(a^4*b^6*x^10 + 5*a^5*b^5*x^8 + 10*a^6*b^4*x
^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)*(-d^6/(a^19*b^5))^(1
/4)*log(-231*a^5*b*(-d^6/(a^19*b^5))^(1/4) + 231*sqrt(d*x)*d) - 4
*(385*b^4*d*x^8 + 1760*a*b^3*d*x^6 + 3130*a^2*b^2*d*x^4 + 2648*a^
3*b*d*x^2 - 1155*a^4*d)*sqrt(d*x))/(a^4*b^6*x^10 + 5*a^5*b^5*x^8
+ 10*a^6*b^4*x^6 + 10*a^7*b^3*x^4 + 5*a^8*b^2*x^2 + a^9*b)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279766, size = 460, normalized size = 1.18

$$\frac{231\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^5b^2}+\frac{231\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^5b^2}$$
$$+\frac{231\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^5b^2}$$
$$-\frac{231\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^5b^2}$$
$$+\frac{385\,\sqrt{dx}b^4d^{11}x^8+1760\,\sqrt{dx}ab^3d^{11}x^6+3130\,\sqrt{dx}a^2b^2d^{11}x^4+2648\,\sqrt{dx}a^3bd^{11}x^2-1155\,\sqrt{dx}a^4d^{11}}{20480\,(bd^2x^2+ad^2)^5a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 231/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 231/
16384*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^2) + 231/327
68*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*s
qrt(d*x) + sqrt(a*d^2/b))/(a^5*b^2) - 231/32768*sqrt(2)*(a*b^3*d^
2)^(1/4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/(a^5*b^2) + 1/20480*(385*sqrt(d*x)*b^4*d^11*x^8 + 1760*sqrt
(d*x)*a*b^3*d^11*x^6 + 3130*sqrt(d*x)*a^2*b^2*d^11*x^4 + 2648*sqr
t(d*x)*a^3*b*d^11*x^2 - 1155*sqrt(d*x)*a^4*d^11)/((b*d^2*x^2 + a*
d^2)^5*a^4*b)
```

3.723 $\int \frac{\sqrt{dx}}{\left(a^2+2abx^2+b^2x^4\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{663\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}+\frac{663\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}+\frac{663(dx)^{3/2}}{4096a^5d\left(a+bx^2\right)}+\frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}+\frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3}+\frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*
(a + b*x^2)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (
663*(d*x)^(3/2))/(5120*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(
4096*a^5*d*(a + b*x^2)) - (663*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4))
+ (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(16384*Sqrt[2]*a^(21/4)*b^(3/4))
```

Rubi [A] time = 0.952312, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321

$$\frac{663\sqrt{d}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{21/4}b^{3/4}}-\frac{663\sqrt{d}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}+\frac{663\sqrt{d}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{21/4}b^{3/4}}+\frac{663(dx)^{3/2}}{4096a^5d\left(a+bx^2\right)}+\frac{663(dx)^{3/2}}{5120a^4d\left(a+bx^2\right)^2}+\frac{221(dx)^{3/2}}{1920a^3d\left(a+bx^2\right)^3}+\frac{17(dx)^{3/2}}{160a^2d\left(a+bx^2\right)^4}+\frac{(dx)^{3/2}}{10ad\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```


```
[Out] (d*x)^(3/2)/(10*a*d*(a + b*x^2)^5) + (17*(d*x)^(3/2))/(160*a^2*d*
(a + b*x^2)^4) + (221*(d*x)^(3/2))/(1920*a^3*d*(a + b*x^2)^3) + (
663*(d*x)^(3/2))/(5120*a^4*d*(a + b*x^2)^2) + (663*(d*x)^(3/2))/(
```

```
4096*a^5*d*(a + b*x^2)) - (663*Sqrt[d]*ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4))
+ (663*Sqrt[d]*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(8192*Sqrt[2]*a^(21/4)*b^(3/4)) + (663*Sqrt[d]*Log[Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(16384*Sqrt[2]*a^(21/4)*b^(3/4)) - (663*Sqrt[d]*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(16384*Sqrt[2]*a^(21/4)*b^(3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.38709, size = 295, normalized size = 0.76

$$\frac{\sqrt{dx}\left(\frac{49152a^{17/4}x^{3/2}}{(a+bx^2)^5}+\frac{52224a^{13/4}x^{3/2}}{(a+bx^2)^4}+\frac{56576a^{9/4}x^{3/2}}{(a+bx^2)^3}+\frac{63648a^{5/4}x^{3/2}}{(a+bx^2)^2}+\frac{9945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}-\frac{9945\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\cdots\right.}{b^{3/4}}\cdots}{491520a^{21/4}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (Sqrt[d*x]*((49152*a^(17/4)*x^(3/2))/(a + b*x^2)^5 + (52224*a^(13
/4)*x^(3/2))/(a + b*x^2)^4 + (56576*a^(9/4)*x^(3/2))/(a + b*x^2)^
3 + (63648*a^(5/4)*x^(3/2))/(a + b*x^2)^2 + (79560*a^(1/4)*x^(3/2
))/(a + b*x^2) - (19890*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)])/b^(3/4) + (19890*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4
)*Sqrt[x])/a^(1/4)])/b^(3/4) + (9945*Sqrt[2]*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4) - (9945*Sqrt[2]*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4
)))/(491520*a^(21/4)*Sqrt[x])
```

Maple [A] time = 0.032, size = 336, normalized size = 0.9

$$\frac{7529\,d^9}{12288\,(bd^2x^2+ad^2)^5\,a}(dx)^{\frac{3}{2}}+\frac{527\,d^7b}{384\,(bd^2x^2+ad^2)^5\,a^2}(dx)^{\frac{7}{2}}+\frac{9061\,d^5b^2}{6144\,(bd^2x^2+ad^2)^5\,a^3}(dx)^{\frac{11}{2}}+\frac{1989\,d^3b^3}{2560\,(bd^2x^2+ad^2)^5\,a^4}(dx)^{\frac{15}{2}}+\frac{663\,b^4d}{4096\,(bd^2x^2+ad^2)^5\,a^5}(dx)^{\frac{19}{2}}+\frac{663\,d\sqrt{2}}{32768\,a^5b}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{663\,d\sqrt{2}}{16384\,a^5b}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{663\,d\sqrt{2}}{16384\,a^5b}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] 7529/12288*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(3/2)+527/384*d^7/(b*d
^2*x^2+a*d^2)^5/a^2*b*(d*x)^(7/2)+9061/6144*d^5/(b*d^2*x^2+a*d^2)
^5/a^3*b^2*(d*x)^(11/2)+1989/2560*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3
*(d*x)^(15/2)+663/4096*d/(b*d^2*x^2+a*d^2)^5/a^5*b^4*(d*x)^(19/2)
+663/32768*d/a^5/b/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+663/16384*d/a^5/b/(a*d^2/b)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+663/16384
*d/a^5/b/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(
d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288543, size = 609, normalized size = 1.57

$$\frac{39780\left(a^{5}b^{5}x^{10}+5\,a^{6}b^{4}x^{8}+10\,a^{7}b^{3}x^{6}+10\,a^{8}b^{2}x^{4}+5\,a^{9}bx^{2}+a^{10}\right)\left(-\frac{d^{2}}{a^{21}b^{3}}\right)^{\frac{1}{4}}\arctan\left(\frac{291434247\,a^{16}b^{2}\left(-\right.}{291434247\,\sqrt{dx}d+\sqrt{-84933920324457009\,a^{11}b}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] 1/245760*(39780*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 +
10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3))^(1/4)*arct
an(291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4)/(291434247*sqrt(d*x
)*d + sqrt(-84933920324457009*a^11*b*d^2*sqrt(-d^2/(a^21*b^3)) +
84933920324457009*d^3*x))) + 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 +
 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^2
1*b^3))^(1/4)*log(291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 29
1434247*sqrt(d*x)*d) - 9945*(a^5*b^5*x^10 + 5*a^6*b^4*x^8 + 10*a^
7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 + a^10)*(-d^2/(a^21*b^3)
)^(1/4)*log(-291434247*a^16*b^2*(-d^2/(a^21*b^3))^(3/4) + 2914342
47*sqrt(d*x)*d) + 4*(9945*b^4*x^9 + 47736*a*b^3*x^7 + 90610*a^2*b
^2*x^5 + 84320*a^3*b*x^3 + 37645*a^4*x)*sqrt(d*x))/(a^5*b^5*x^10
+ 5*a^6*b^4*x^8 + 10*a^7*b^3*x^6 + 10*a^8*b^2*x^4 + 5*a^9*b*x^2 +
 a^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.280483, size = 468, normalized size = 1.21

$$\frac{663\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{6}b^{3}d}+\frac{663\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{6}b^{3}d}$$
$$-\frac{663\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{6}b^{3}d}$$
$$+\frac{663\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{6}b^{3}d}$$
$$+\frac{9945\,\sqrt{dx}b^{4}d^{10}x^{9}+47736\,\sqrt{dx}ab^{3}d^{10}x^{7}+90610\,\sqrt{dx}a^{2}b^{2}d^{10}x^{5}+84320\,\sqrt{dx}a^{3}bd^{10}x^{3}+37645\,\sqrt{dx}a^{4}d^{10}x}{61440\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] 663/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3*d) + 663/
16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^3*d) - 663/327
68*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqr
t(d*x) + sqrt(a*d^2/b))/(a^6*b^3*d) + 663/32768*sqrt(2)*(a*b^3*d^
2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^6*b^3*d) + 1/61440*(9945*sqrt(d*x)*b^4*d^10*x^9 + 47736*sq
rt(d*x)*a*b^3*d^10*x^7 + 90610*sqrt(d*x)*a^2*b^2*d^10*x^5 + 84320
*sqrt(d*x)*a^3*b*d^10*x^3 + 37645*sqrt(d*x)*a^4*d^10*x)/((b*d^2*x
^2 + a*d^2)^5*a^5)
```

3.724 $\int \frac{1}{\sqrt{dx}(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=387

$$-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}-\frac{4389\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{1463\sqrt{dx}}{4096a^5d\,(a+bx^2)}+\frac{209\sqrt{dx}}{1024a^4d\,(a+bx^2)^2}+\frac{19\sqrt{dx}}{128a^3d\,(a+bx^2)^3}+\frac{19\sqrt{dx}}{160a^2d\,(a+bx^2)^4}+\frac{\sqrt{dx}}{10ad\,(a+bx^2)^5}$$

```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a +
 b*x^2)^4) + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt
[d*x])/(1024*a^4*d*(a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*
(a + b*x^2)) - (4389*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) + (4389*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sq
rt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[
2]*a^(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*
a^(23/4)*b^(1/4)*Sqrt[d])
```

Rubi [A] time = 0.960465, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$-\frac{4389\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}-\frac{4389\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{4389\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{23/4}\sqrt[4]{b}\sqrt{d}}+\frac{1463\sqrt{dx}}{4096a^5d\,(a+bx^2)}+\frac{209\sqrt{dx}}{1024a^4d\,(a+bx^2)^2}+\frac{19\sqrt{dx}}{128a^3d\,(a+bx^2)^3}+\frac{19\sqrt{dx}}{160a^2d\,(a+bx^2)^4}+\frac{\sqrt{dx}}{10ad\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```


```
[Out] Sqrt[d*x]/(10*a*d*(a + b*x^2)^5) + (19*Sqrt[d*x])/(160*a^2*d*(a +
 b*x^2)^4) + (19*Sqrt[d*x])/(128*a^3*d*(a + b*x^2)^3) + (209*Sqrt
[d*x])/(1024*a^4*d*(a + b*x^2)^2) + (1463*Sqrt[d*x])/(4096*a^5*d*
```

```
(a + b*x^2)) - (4389*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1
/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) + (4389*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sq
rt[2]*a^(23/4)*b^(1/4)*Sqrt[d]) - (4389*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[
2]*a^(23/4)*b^(1/4)*Sqrt[d]) + (4389*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*
a^(23/4)*b^(1/4)*Sqrt[d])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.474855, size = 295, normalized size = 0.76

$$\frac{\sqrt{x}\left(\frac{16384a^{19/4}\sqrt{x}}{(a+bx^2)^5}+\frac{19456a^{15/4}\sqrt{x}}{(a+bx^2)^4}+\frac{24320a^{11/4}\sqrt{x}}{(a+bx^2)^3}+\frac{33440a^{7/4}\sqrt{x}}{(a+bx^2)^2}+\frac{58520a^{3/4}\sqrt{x}}{a+bx^2}-\frac{21945\sqrt{2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{\sqrt[4]{b}}+\frac{21945\sqrt{2}\log\left(\sqrt{2}\sqrt[4]{a}\cdots\right.}{\sqrt[4]{\cdots}}\cdots\right.}{163840a^{23/4}\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[x]*((16384*a^(19/4)*Sqrt[x])/(a + b*x^2)^5 + (19456*a^(15/4
)*Sqrt[x])/(a + b*x^2)^4 + (24320*a^(11/4)*Sqrt[x])/(a + b*x^2)^3
 + (33440*a^(7/4)*Sqrt[x])/(a + b*x^2)^2 + (58520*a^(3/4)*Sqrt[x]
)/(a + b*x^2) - (43890*Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)])/b^(1/4) + (43890*Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)])/b^(1/4) - (21945*Sqrt[2]*Log[Sqrt[a] - Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (21945*Sqrt[2]*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/
4)))/(163840*a^(23/4)*Sqrt[d*x])
```

Maple [A] time = 0.032, size = 333, normalized size = 0.9

$$\frac{3803\,d^{9}}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a}\sqrt{dx}+\frac{6289\,d^{7}b}{2560\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{2}}(dx)^{\frac{5}{2}}+\frac{5947\,d^{5}b^{2}}{2048\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{3}}(dx)^{\frac{9}{2}}+\frac{209\,d^{3}b^{3}}{128\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{4}}(dx)^{\frac{13}{2}}+\frac{1463\,b^{4}d}{4096\,\left(bd^{2}x^{2}+ad^{2}\right)^{5}a^{5}}(dx)^{\frac{17}{2}}+\frac{4389\,\sqrt{2}}{32768\,da^{6}}\sqrt[4]{\frac{ad^{2}}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)+\frac{4389\,\sqrt{2}}{16384\,da^{6}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)+\frac{4389\,\sqrt{2}}{16384\,da^{6}}\sqrt[4]{\frac{ad^{2}}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^3/(d*x)^(1/2),x)
```

```
[Out] 3803/4096*d^9/(b*d^2*x^2+a*d^2)^5/a*(d*x)^(1/2)+6289/2560*d^7/(b*
d^2*x^2+a*d^2)^5/a^2*b*(d*x)^(5/2)+5947/2048*d^5/(b*d^2*x^2+a*d^2
)^5/a^3*b^2*(d*x)^(9/2)+209/128*d^3/(b*d^2*x^2+a*d^2)^5/a^4*b^3*(
d*x)^(13/2)+1463/4096*d/(b*d^2*x^2+a*d^2)^5/a^5*b^4*(d*x)^(17/2)+
4389/32768/d/a^6*(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+4389/16384/d/a^6*(a*d^2/b)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+4389/16384/d
/a^6*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)
^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289134, size = 616, normalized size = 1.59

$$-\frac{87780\left(a^5b^5dx^{10}+5\,a^6b^4dx^8+10\,a^7b^3dx^6+10\,a^8b^2dx^4+5\,a^9bdx^2+a^{10}d\right)\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}\arctan\left(\frac{a^6d\left(-\frac{1}{a^{23}bd^2}\right)^{\frac{1}{4}}}{\sqrt{a^{12}d^2\sqrt{-\frac{1}{a^{23}bd^2}}+dx}+\sqrt{dx}}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] -1/81920*(87780*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 10*a^7*b^3*d*
x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/(a^23*b*d^2)
)^(1/4)*arctan(a^6*d*(-1/(a^23*b*d^2))^(1/4)/(sqrt(a^12*d^2*sqrt(
-1/(a^23*b*d^2)) + d*x) + sqrt(d*x))) - 21945*(a^5*b^5*d*x^10 + 5
*a^6*b^4*d*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*
x^2 + a^10*d)*(-1/(a^23*b*d^2))^(1/4)*log(a^6*d*(-1/(a^23*b*d^2))
^(1/4) + sqrt(d*x)) + 21945*(a^5*b^5*d*x^10 + 5*a^6*b^4*d*x^8 + 1
0*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10*d)*(-1/
(a^23*b*d^2))^(1/4)*log(-a^6*d*(-1/(a^23*b*d^2))^(1/4) + sqrt(d*x
)) - 4*(7315*b^4*x^8 + 33440*a*b^3*x^6 + 59470*a^2*b^2*x^4 + 5031
2*a^3*b*x^2 + 19015*a^4)*sqrt(d*x))/(a^5*b^5*d*x^10 + 5*a^6*b^4*d
*x^8 + 10*a^7*b^3*d*x^6 + 10*a^8*b^2*d*x^4 + 5*a^9*b*d*x^2 + a^10
*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**3/(d*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274431, size = 467, normalized size = 1.21

$$\frac{4389\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^6bd}+\frac{4389\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^6bd}-\frac{4389\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^6bd}+\frac{7315\,\sqrt{dx}b^4d^9x^8+33440\,\sqrt{dx}ab^3d^9x^6+59470\,\sqrt{dx}a^2b^2d^9x^4+50312\,\sqrt{dx}a^3bd^9x^2+19015\,\sqrt{dx}a^4d^9}{20480\left(bd^2x^2+ad^2\right)^5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] 4389/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/
16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b*d) + 4389/3276
8*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/(a^6*b*d) - 4389/32768*sqrt(2)*(a*b^3*d^2)
^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^6*b*d) + 1/20480*(7315*sqrt(d*x)*b^4*d^9*x^8 + 33440*sqrt(d*
x)*a*b^3*d^9*x^6 + 59470*sqrt(d*x)*a^2*b^2*d^9*x^4 + 50312*sqrt(d
*x)*a^3*b*d^9*x^2 + 19015*sqrt(d*x)*a^4*d^9)/((b*d^2*x^2 + a*d^2)
^5*a^5)
```

3.725 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=404

$$-\frac{13923\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}-\frac{13923\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}-\frac{13923}{4096a^6d\sqrt{dx}}+\frac{13923}{20480a^5d\sqrt{dx}\,(a+bx^2)}+\frac{1547}{5120a^4d\sqrt{dx}\,(a+bx^2)^2}+\frac{119}{640a^3d\sqrt{dx}\,(a+bx^2)^3}+\frac{21}{160a^2d\sqrt{dx}\,(a+bx^2)^4}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5
) + 21/(160*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[
d*x]*(a + b*x^2)^3) + 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) +
 13923/(20480*a^5*d*Sqrt[d*x]*(a + b*x^2)) + (13923*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[
2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) -
 (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (
13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rubi [A] time = 1.08302, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357

$$-\frac{13923\sqrt[4]{b}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{25/4}d^{3/2}}+\frac{13923\sqrt[4]{b}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}-\frac{13923\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{25/4}d^{3/2}}-\frac{13923}{4096a^6d\sqrt{dx}}+\frac{13923}{20480a^5d\sqrt{dx}\,(a+bx^2)}+\frac{1547}{5120a^4d\sqrt{dx}\,(a+bx^2)^2}+\frac{119}{640a^3d\sqrt{dx}\,(a+bx^2)^3}+\frac{21}{160a^2d\sqrt{dx}\,(a+bx^2)^4}+\frac{1}{10ad\sqrt{dx}\,(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*Sqrt[d*x]) + 1/(10*a*d*Sqrt[d*x]*(a + b*x^2)^5
) + 21/(160*a^2*d*Sqrt[d*x]*(a + b*x^2)^4) + 119/(640*a^3*d*Sqrt[
d*x]*(a + b*x^2)^3) + 1547/(5120*a^4*d*Sqrt[d*x]*(a + b*x^2)^2) +
 13923/(20480*a^5*d*Sqrt[d*x]*(a + b*x^2)) + (13923*b^(1/4)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[
2]*a^(25/4)*d^(3/2)) - (13923*b^(1/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(25/4)*d^(3/2)) -
 (13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2)) + (
13923*b^(1/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(25/4)*d^(3/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.41306, size = 313, normalized size = 0.77

$$x\left(-\frac{16384a^{17/4}bx^2}{(a+bx^2)^5}-\frac{37888a^{13/4}bx^2}{(a+bx^2)^4}-\frac{68352a^{9/4}bx^2}{(a+bx^2)^3}-\frac{117856a^{5/4}bx^2}{(a+bx^2)^2}-\frac{229240\sqrt[4]{a}bx^2}{a+bx^2}-69615\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (x*(-327680*a^(1/4) - (16384*a^(17/4)*b*x^2)/(a + b*x^2)^5 - (378
88*a^(13/4)*b*x^2)/(a + b*x^2)^4 - (68352*a^(9/4)*b*x^2)/(a + b*x
^2)^3 - (117856*a^(5/4)*b*x^2)/(a + b*x^2)^2 - (229240*a^(1/4)*b*
x^2)/(a + b*x^2) + 139230*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 139230*Sqrt[2]*b^(1/4)*Sqrt[x]*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 69615*Sqrt[2]*b^(1
/4)*Sqrt[x]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x] + 69615*Sqrt[2]*b^(1/4)*Sqrt[x]*Log[Sqrt[a] + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(163840*a^(25/4)*(d*x)^(3/2))
```

Maple [A] time = 0.041, size = 349, normalized size = 0.9

$$-2\frac{1}{a^{6}d\sqrt{dx}}-\frac{11743\,d^{7}b}{4096\,a^{2}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}\left(dx\right)^{\frac{3}{2}}-\frac{1129\,d^{5}b^{2}}{128\,a^{3}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}\left(dx\right)^{\frac{7}{2}}-\frac{22467\,d^{3}b^{3}}{2048\,a^{4}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}\left(dx\right)^{\frac{11}{2}}-\frac{16169\,b^{4}d}{2560\,a^{5}\left(bd^{2}x^{2}+ad^{2}\right)^{5}}\left(dx\right)^{\frac{15}{2}}-\frac{5731\,b^{5}}{4096\,a^{6}d\left(bd^{2}x^{2}+ad^{2}\right)^{5}}\left(dx\right)^{\frac{19}{2}}-\frac{13923\,\sqrt{2}}{32768\,a^{6}d}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^{2}}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^{2}}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-\frac{13923\,\sqrt{2}}{16384\,a^{6}d}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-\frac{13923\,\sqrt{2}}{16384\,a^{6}d}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^{2}}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -2/a^6/d/(d*x)^(1/2)-11743/4096*d^7*b/a^2/(b*d^2*x^2+a*d^2)^5*(d*
x)^(3/2)-1129/128*d^5*b^2/a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(7/2)-224
67/2048*d^3*b^3/a^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(11/2)-16169/2560*d
*b^4/a^5/(b*d^2*x^2+a*d^2)^5*(d*x)^(15/2)-5731/4096/d*b^5/a^6/(b*
d^2*x^2+a*d^2)^5*(d*x)^(19/2)-13923/32768/d/a^6/(a*d^2/b)^(1/4)*2
^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))-139
23/16384/d/a^6/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)+1)-13923/16384/d/a^6/(a*d^2/b)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.326519, size = 659, normalized size = 1.63

$$-\frac{278460\,b^5x^{10} + 1336608\,ab^4x^8 + 2537080\,a^2b^3x^6 + 2360960\,a^3b^2x^4 + 1054060\,a^4bx^2 + 163840\,a^5 + 278460\,\left(a^6b^5dx^{10} + 5\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/81920*(278460*b^5*x^10 + 1336608*a*b^4*x^8 + 2537080*a^2*b^3*x
^6 + 2360960*a^3*b^2*x^4 + 1054060*a^4*b*x^2 + 163840*a^5 + 27846
0*(a^6*b^5*d*x^10 + 5*a^7*b^4*d*x^8 + 10*a^8*b^3*d*x^6 + 10*a^9*b
^2*d*x^4 + 5*a^10*b*d*x^2 + a^11*d)*sqrt(d*x)*(-b/(a^25*d^6))^(1/
4)*arctan(2698972561467*a^19*d^5*(-b/(a^25*d^6))^(3/4)/(269897256
1467*sqrt(d*x)*b + sqrt(-7284452887551739093192089*a^13*b*d^4*sqr
t(-b/(a^25*d^6)) + 7284452887551739093192089*b^2*d*x))) + 69615*(
a^6*b^5*d*x^10 + 5*a^7*b^4*d*x^8 + 10*a^8*b^3*d*x^6 + 10*a^9*b^2*
d*x^4 + 5*a^10*b*d*x^2 + a^11*d)*sqrt(d*x)*(-b/(a^25*d^6))^(1/4)*
log(2698972561467*a^19*d^5*(-b/(a^25*d^6))^(3/4) + 2698972561467*
sqrt(d*x)*b) - 69615*(a^6*b^5*d*x^10 + 5*a^7*b^4*d*x^8 + 10*a^8*b
^3*d*x^6 + 10*a^9*b^2*d*x^4 + 5*a^10*b*d*x^2 + a^11*d)*sqrt(d*x)*
(-b/(a^25*d^6))^(1/4)*log(-2698972561467*a^19*d^5*(-b/(a^25*d^6))
^(3/4) + 2698972561467*sqrt(d*x)*b))/((a^6*b^5*d*x^10 + 5*a^7*b^4
*d*x^8 + 10*a^8*b^3*d*x^6 + 10*a^9*b^2*d*x^4 + 5*a^10*b*d*x^2 + a
^11*d)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278729, size = 493, normalized size = 1.22

$$-\frac{\frac{327680}{\sqrt{dx}\,a^6} + \frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2} + \frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^7b^2d^2} - \frac{69615\,\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right.\right.}{a^7b^2d^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/163840*(327680/(sqrt(d*x)*a^6) + 139230*sqrt(2)*(a*b^3*d^2)^(3
/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a
*d^2/b)^(1/4))/(a^7*b^2*d^2) + 139230*sqrt(2)*(a*b^3*d^2)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2
/b)^(1/4))/(a^7*b^2*d^2) - 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x
 + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^
2) + 69615*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^7*b^2*d^2) + 8*(28655*sqrt(d*x
)*b^5*d^9*x^9 + 129352*sqrt(d*x)*a*b^4*d^9*x^7 + 224670*sqrt(d*x)
*a^2*b^3*d^9*x^5 + 180640*sqrt(d*x)*a^3*b^2*d^9*x^3 + 58715*sqrt(
d*x)*a^4*b*d^9*x)/((b*d^2*x^2 + a*d^2)^5*a^6))/d
```

3.726 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=404

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{27/4}d^{5/2}}+\frac{33649b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{27/4}d^{5/2}}-\frac{33649}{12288a^6d(dx)^{3/2}}+\frac{4807}{4096a^5d(dx)^{3/2}(a+bx^2)}+\frac{437}{1024a^4d(dx)^{3/2}(a+bx^2)^2}+\frac{437}{1920a^3d(dx)^{3/2}(a+bx^2)^3}+\frac{23}{160a^2d(dx)^{3/2}(a+bx^2)^4}+\frac{1}{10ad(dx)^{3/2}(a+bx^2)^5}$$

```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x
^2)^5) + 23/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3
*d*(d*x)^(3/2)*(a + b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a +
b*x^2)^2) + 4807/(4096*a^5*d*(d*x)^(3/2)*(a + b*x^2)) + (33649*b^
(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(27/4)
*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d
^(5/2)) - (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(
5/2))
```

Rubi [A] time = 1.07986, antiderivative size = 404, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{33649b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{27/4}d^{5/2}} - \frac{33649b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{16384\sqrt{2}a^{27/4}d^{5/2}} + \frac{33649b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{27/4}d^{5/2}} - \frac{33649b^{3/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{27/4}d^{5/2}} - \frac{33649}{12288a^6d(dx)^{3/2}} + \frac{4807}{4096a^5d(dx)^{3/2}(a+bx^2)} + \frac{437}{1024a^4d(dx)^{3/2}(a+bx^2)^2} + \frac{437}{1920a^3d(dx)^{3/2}(a+bx^2)^3} + \frac{23}{160a^2d(dx)^{3/2}(a+bx^2)^4} + \frac{1}{10ad(dx)^{3/2}(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -33649/(12288*a^6*d*(d*x)^(3/2)) + 1/(10*a*d*(d*x)^(3/2)*(a + b*x
^2)^5) + 23/(160*a^2*d*(d*x)^(3/2)*(a + b*x^2)^4) + 437/(1920*a^3
*d*(d*x)^(3/2)*(a + b*x^2)^3) + 437/(1024*a^4*d*(d*x)^(3/2)*(a +
b*x^2)^2) + 4807/(4096*a^5*d*(d*x)^(3/2)*(a + b*x^2)) + (33649*b^
(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(8192*Sqrt[2]*a^(27/4)*d^(5/2)) - (33649*b^(3/4)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(27/4)
*d^(5/2)) + (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d
^(5/2)) - (33649*b^(3/4)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(16384*Sqrt[2]*a^(27/4)*d^(
5/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.404546, size = 313, normalized size = 0.77

$$x\left(-\frac{49152a^{19/4}bx^2}{(a+bx^2)^5}-\frac{119808a^{15/4}bx^2}{(a+bx^2)^4}-\frac{231680a^{11/4}bx^2}{(a+bx^2)^3}-\frac{441440a^{7/4}bx^2}{(a+bx^2)^2}-\frac{1018280a^{3/4}bx^2}{a+bx^2}-327680a^{3/4}+504735\sqrt{2}b^{3/4}x^{3/2}\log\left(-\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (x*(-327680*a^(3/4) - (49152*a^(19/4)*b*x^2)/(a + b*x^2)^5 - (119
808*a^(15/4)*b*x^2)/(a + b*x^2)^4 - (231680*a^(11/4)*b*x^2)/(a +
b*x^2)^3 - (441440*a^(7/4)*b*x^2)/(a + b*x^2)^2 - (1018280*a^(3/4
)*b*x^2)/(a + b*x^2) + 1009470*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 1009470*Sqrt[2]*b^(3/4)*x^(
3/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 504735*Sqrt[
2]*b^(3/4)*x^(3/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x] - 504735*Sqrt[2]*b^(3/4)*x^(3/2)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(491520*a^(27/4)*(d*x)^
(5/2))
```

Maple [A] time = 0.039, size = 352, normalized size = 0.9

$$-\frac{2}{3\,a^6d}(dx)^{-\frac{3}{2}}-\frac{15503\,d^7b}{4096\,a^2\,(bd^2x^2+ad^2)^5}\sqrt{dx}-\frac{31149\,d^5b^2}{2560\,a^3\,(bd^2x^2+ad^2)^5}(dx)^{\frac{5}{2}}-\frac{95821\,d^3b^3}{6144\,a^4\,(bd^2x^2+ad^2)^5}(dx)^{\frac{9}{2}}-\frac{3527\,b^4d}{384\,a^5\,(bd^2x^2+ad^2)^5}(dx)^{\frac{13}{2}}-\frac{25457\,b^5}{12288\,a^6d\,(bd^2x^2+ad^2)^5}(dx)^{\frac{17}{2}}-\frac{33649\,b\sqrt{2}}{32768\,d^3a^7}\sqrt[4]{\frac{ad^2}{b}}\ln\left(1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)-\frac{33649\,b\sqrt{2}}{16384\,d^3a^7}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)-\frac{33649\,b\sqrt{2}}{16384\,d^3a^7}\sqrt[4]{\frac{ad^2}{b}}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -2/3/a^6/d/(d*x)^(3/2)-15503/4096*d^7/a^2*b/(b*d^2*x^2+a*d^2)^5*(
d*x)^(1/2)-31149/2560*d^5/a^3*b^2/(b*d^2*x^2+a*d^2)^5*(d*x)^(5/2)
-95821/6144*d^3/a^4*b^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(9/2)-3527/384*
d/a^5*b^4/(b*d^2*x^2+a*d^2)^5*(d*x)^(13/2)-25457/12288/d/a^6*b^5/
(b*d^2*x^2+a*d^2)^5*(d*x)^(17/2)-33649/32768/d^3/a^7*b*(a*d^2/b)^
```

```
(1/4)*2^(1/2)*ln((d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2))/(d*x-(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))-33649/16384/d^3/a^7*b*(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)-33649/16384/d^3/a^7*b*(a*d^2/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.327278, size = 741, normalized size = 1.83

$$-\frac{672980\,b^5x^{10} + 3076480\,ab^4x^8 + 5471240\,a^2b^3x^6 + 4628704\,a^3b^2x^4 + 1749380\,a^4bx^2 + 163840\,a^5 - 2018940\,\left(a^6b^5d^2x^{11} + \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] -1/245760*(672980*b^5*x^10 + 3076480*a*b^4*x^8 + 5471240*a^2*b^3*
x^6 + 4628704*a^3*b^2*x^4 + 1749380*a^4*b*x^2 + 163840*a^5 - 2018
940*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 +
10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*sqrt(d*x)*(-b
^3/(a^27*d^10))^(1/4)*arctan(a^7*d^3*(-b^3/(a^27*d^10))^(1/4)/(sq
rt(d*x)*b + sqrt(a^14*d^6*sqrt(-b^3/(a^27*d^10)) + b^2*d*x))) + 5
04735*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7
+ 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*sqrt(d*x)*(
-b^3/(a^27*d^10))^(1/4)*log(33649*a^7*d^3*(-b^3/(a^27*d^10))^(1/4
) + 33649*sqrt(d*x)*b) - 504735*(a^6*b^5*d^2*x^11 + 5*a^7*b^4*d^2
*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^5 + 5*a^10*b*d^2*x^3
 + a^11*d^2*x)*sqrt(d*x)*(-b^3/(a^27*d^10))^(1/4)*log(-33649*a^7*
d^3*(-b^3/(a^27*d^10))^(1/4) + 33649*sqrt(d*x)*b))/((a^6*b^5*d^2*
x^11 + 5*a^7*b^4*d^2*x^9 + 10*a^8*b^3*d^2*x^7 + 10*a^9*b^2*d^2*x^
5 + 5*a^10*b*d^2*x^3 + a^11*d^2*x)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.27606, size = 481, normalized size = 1.19

$$-\frac{33649\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^7d^3}-\frac{33649\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}+\frac{33649\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{32768\,a^7d^3}-\frac{2}{3\,\sqrt{dx}a^6d^2x}-\frac{127285\,\sqrt{dx}b^5d^8x^8+564320\,\sqrt{dx}ab^4d^8x^6+958210\,\sqrt{dx}a^2b^3d^8x^4+747576\,\sqrt{dx}a^3b^2d^8x^2+232545\,\sqrt{dx}a^4bd^8}{61440\left(bd^2x^2+ad^2\right)^5a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(5/2)),x, algorithm="giac")
```

```
[Out] -33649/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 336
49/16384*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*d^3) - 33649/
32768*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*
sqrt(d*x) + sqrt(a*d^2/b))/(a^7*d^3) + 33649/32768*sqrt(2)*(a*b^3
*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^7*d^3) - 2/3/(sqrt(d*x)*a^6*d^2*x) - 1/61440*(127285*sq
rt(d*x)*b^5*d^8*x^8 + 564320*sqrt(d*x)*a*b^4*d^8*x^6 + 958210*sqr
t(d*x)*a^2*b^3*d^8*x^4 + 747576*sqrt(d*x)*a^3*b^2*d^8*x^2 + 23254
5*sqrt(d*x)*a^4*b*d^8)/((b*d^2*x^2 + a*d^2)^5*a^6*d)
```

3.727 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=422

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}} - \frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}} - \frac{69615b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{29/4}d^{7/2}} + \frac{69615b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{29/4}d^{7/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} - \frac{13923}{4096a^6d(dx)^{5/2}} + \frac{7735}{4096a^5d(dx)^{5/2}(a+bx^2)} + \frac{595}{1024a^4d(dx)^{5/2}(a+bx^2)^2} + \frac{35}{128a^3d(dx)^{5/2}(a+bx^2)^3} + \frac{5}{32a^2d(dx)^{5/2}(a+bx^2)^4} + \frac{1}{10ad(dx)^{5/2}(a+bx^2)^5}$$

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*
x]) + 1/(10*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5
/2)*(a + b*x^2)^4) + 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 5
95/(1024*a^4*d*(d*x)^(5/2)*(a + b*x^2)^2) + 7735/(4096*a^5*d*(d*x
)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) +
 (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2))
```

Rubi [A] time = 1.1871, antiderivative size = 422, normalized size of antiderivative = 1., number of

steps used = 18, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{69615b^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}} - \frac{69615b^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{16384\sqrt{2}a^{29/4}d^{7/2}} - \frac{69615b^{5/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{8192\sqrt{2}a^{29/4}d^{7/2}} + \frac{69615b^{5/4}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{8192\sqrt{2}a^{29/4}d^{7/2}} + \frac{69615b}{4096a^7d^3\sqrt{dx}} - \frac{13923}{4096a^6d(dx)^{5/2}} + \frac{7735}{4096a^5d(dx)^{5/2}(a+bx^2)} + \frac{595}{1024a^4d(dx)^{5/2}(a+bx^2)^2} + \frac{35}{128a^3d(dx)^{5/2}(a+bx^2)^3} + \frac{5}{32a^2d(dx)^{5/2}(a+bx^2)^4} + \frac{1}{10ad(dx)^{5/2}(a+bx^2)^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] -13923/(4096*a^6*d*(d*x)^(5/2)) + (69615*b)/(4096*a^7*d^3*Sqrt[d*
x]) + 1/(10*a*d*(d*x)^(5/2)*(a + b*x^2)^5) + 5/(32*a^2*d*(d*x)^(5
/2)*(a + b*x^2)^4) + 35/(128*a^3*d*(d*x)^(5/2)*(a + b*x^2)^3) + 5
95/(1024*a^4*d*(d*x)^(5/2)*(a + b*x^2)^2) + 7735/(4096*a^5*d*(d*x
)^(5/2)*(a + b*x^2)) - (69615*b^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) +
 (69615*b^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(8192*Sqrt[2]*a^(29/4)*d^(7/2)) + (69615*b^(5/4)*Log[Sq
rt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
d*x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2)) - (69615*b^(5/4)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(16384*Sqrt[2]*a^(29/4)*d^(7/2))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 0.419401, size = 339, normalized size = 0.8

$$\sqrt{dx}\left(\frac{16384a^{17/4}b^2x^4}{(a+bx^2)^5}+\frac{58368a^{13/4}b^2x^4}{(a+bx^2)^4}+\frac{145152a^{9/4}b^2x^4}{(a+bx^2)^3}+\frac{327136a^{5/4}b^2x^4}{(a+bx^2)^2}-65536a^{5/4}+348075\sqrt{2}b^{5/4}x^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^3),x]
```

```
[Out] (Sqrt[d*x]*(-65536*a^(5/4) + 1966080*a^(1/4)*b*x^2 + (16384*a^(17
/4)*b^2*x^4)/(a + b*x^2)^5 + (58368*a^(13/4)*b^2*x^4)/(a + b*x^2)
^4 + (145152*a^(9/4)*b^2*x^4)/(a + b*x^2)^3 + (327136*a^(5/4)*b^2
*x^4)/(a + b*x^2)^2 + (818520*a^(1/4)*b^2*x^4)/(a + b*x^2) - 6961
50*Sqrt[2]*b^(5/4)*x^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)] + 696150*Sqrt[2]*b^(5/4)*x^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)] + 348075*Sqrt[2]*b^(5/4)*x^(5/2)*Log[Sqrt[a
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 348075*Sqrt[2]
*b^(5/4)*x^(5/2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x]))/(163840*a^(29/4)*d^4*x^3)
```

Maple [A] time = 0.043, size = 368, normalized size = 0.9

$$-\frac{2}{5\,a^6d}(dx)^{-\frac{5}{2}}+12\,\frac{b}{a^7d^3\sqrt{dx}}+\frac{34139\,d^5b^2}{4096\,a^3\,(bd^2x^2+ad^2)^5}(dx)^{\frac{3}{2}}+\frac{3597\,d^3b^3}{128\,a^4\,(bd^2x^2+ad^2)^5}(dx)^{\frac{7}{2}}$$
$$+\frac{75471\,b^4d}{2048\,a^5\,(bd^2x^2+ad^2)^5}(dx)^{\frac{11}{2}}+\frac{56269\,b^5}{2560\,a^6d\,(bd^2x^2+ad^2)^5}(dx)^{\frac{15}{2}}+\frac{20463\,b^6}{4096\,a^7d^3\,(bd^2x^2+ad^2)^5}(dx)^{\frac{19}{2}}$$
$$+\frac{69615\,b\sqrt{2}}{32768\,a^7d^3}\ln\left(1\left(dx-\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$
$$+\frac{69615\,b\sqrt{2}}{16384\,a^7d^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}+\frac{69615\,b\sqrt{2}}{16384\,a^7d^3}\arctan\left(\sqrt{2}\sqrt{dx}\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}-1\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] -2/5/a^6/d/(d*x)^(5/2)+12*b/a^7/d^3/(d*x)^(1/2)+34139/4096*d^5*b^
2/a^3/(b*d^2*x^2+a*d^2)^5*(d*x)^(3/2)+3597/128*d^3*b^3/a^4/(b*d^2
*x^2+a*d^2)^5*(d*x)^(7/2)+75471/2048*d*b^4/a^5/(b*d^2*x^2+a*d^2)^
5*(d*x)^(11/2)+56269/2560/d*b^5/a^6/(b*d^2*x^2+a*d^2)^5*(d*x)^(15
/2)+20463/4096/d^3*b^6/a^7/(b*d^2*x^2+a*d^2)^5*(d*x)^(19/2)+69615
```

```
/32768/d^3*b/a^7/(a*d^2/b)^(1/4)*2^(1/2)*ln((d*x-(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+69615/16384/d^3*b/a^7/(a*d^2/b)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(1/4)*(d*x)^(1/2)+1)+69615/1
6384/d^3*b/a^7/(a*d^2/b)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a*d^2/b)^(
1/4)*(d*x)^(1/2)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.342662, size = 784, normalized size = 1.86

$$\frac{1392300\, b^6 x^{12} + 6683040\, a b^5 x^{10} + 12685400\, a^2 b^4 x^8 + 11804800\, a^3 b^3 x^6 + 5270300\, a^4 b^2 x^4 + 819200\, a^5 b x^2 - 32768\, a^6 + 1392}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/81920*(1392300*b^6*x^12 + 6683040*a*b^5*x^10 + 12685400*a^2*b^4
*x^8 + 11804800*a^3*b^3*x^6 + 5270300*a^4*b^2*x^4 + 819200*a^5*b*
x^2 - 32768*a^6 + 1392300*(a^7*b^5*d^3*x^12 + 5*a^8*b^4*d^3*x^10
+ 10*a^9*b^3*d^3*x^8 + 10*a^10*b^2*d^3*x^6 + 5*a^11*b*d^3*x^4 + a
^12*d^3*x^2)*sqrt(d*x)*(-b^5/(a^29*d^14))^(1/4)*arctan(3373715701
83375*a^22*d^11*(-b^5/(a^29*d^14))^(3/4)/(337371570183375*sqrt(d*
x)*b^4 + sqrt(-113819576367995923331126390625*a^15*b^5*d^8*sqrt(-
b^5/(a^29*d^14)) + 113819576367995923331126390625*b^8*d*x))) + 34
8075*(a^7*b^5*d^3*x^12 + 5*a^8*b^4*d^3*x^10 + 10*a^9*b^3*d^3*x^8
+ 10*a^10*b^2*d^3*x^6 + 5*a^11*b*d^3*x^4 + a^12*d^3*x^2)*sqrt(d*x
)*(-b^5/(a^29*d^14))^(1/4)*log(337371570183375*a^22*d^11*(-b^5/(a
^29*d^14))^(3/4) + 337371570183375*sqrt(d*x)*b^4) - 348075*(a^7*b
^5*d^3*x^12 + 5*a^8*b^4*d^3*x^10 + 10*a^9*b^3*d^3*x^8 + 10*a^10*b
^2*d^3*x^6 + 5*a^11*b*d^3*x^4 + a^12*d^3*x^2)*sqrt(d*x)*(-b^5/(a^
29*d^14))^(1/4)*log(-337371570183375*a^22*d^11*(-b^5/(a^29*d^14))
^(3/4) + 337371570183375*sqrt(d*x)*b^4))/((a^7*b^5*d^3*x^12 + 5*a
```

```
^8*b^4*d^3*x^10 + 10*a^9*b^3*d^3*x^8 + 10*a^10*b^2*d^3*x^6 + 5*a^
11*b*d^3*x^4 + a^12*d^3*x^2)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.279109, size = 489, normalized size = 1.16

$$\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{8}bd^{5}}+\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{16384\,a^{8}bd^{5}}$$
$$-\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{8}bd^{5}}$$
$$+\frac{69615\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{32768\,a^{8}bd^{5}}$$
$$+\frac{348075\,b^{6}d^{12}x^{12}+1670760\,ab^{5}d^{12}x^{10}+3171350\,a^{2}b^{4}d^{12}x^{8}+2951200\,a^{3}b^{3}d^{12}x^{6}+1317575\,a^{4}b^{2}d^{12}x^{4}+204800\,a^{5}bd^{12}x^{2}-}{20480\left(\sqrt{dx}bd^{2}x^{2}+\sqrt{dx}ad^{2}\right)^{5}a^{7}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^(7/2)),x, algorithm="giac")
```

```
[Out] 69615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) + 69
615/16384*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^8*b*d^5) - 696
15/32768*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^8*b*d^5) + 69615/32768*sqrt(2)*(
a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqr
t(a*d^2/b))/(a^8*b*d^5) + 1/20480*(348075*b^6*d^12*x^12 + 1670760
*a*b^5*d^12*x^10 + 3171350*a^2*b^4*d^12*x^8 + 2951200*a^3*b^3*d^1
```

```
2*x^6 + 1317575*a^4*b^2*d^12*x^4 + 204800*a^5*b*d^12*x^2 - 8192*a
^6*d^12)/((sqrt(d*x)*b*d^2*x^2 + sqrt(d*x)*a*d^2)^5*a^7*d^3)
```

3.728 $\int (dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3(a+bx^2)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d(a+bx^2)}$$

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2
)) + (2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(
a + b*x^2))
```

Rubi [A] time = 0.086524, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3(a+bx^2)}+\frac{2a(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x^2
)) + (2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^3*(
a + b*x^2))
```

Rubi in Sympy [A] time = 23.5821, size = 75, normalized size = 0.81

$$\frac{8a(dx)^{\frac{7}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{77d(a+bx^2)}+\frac{2(dx)^{\frac{7}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{11d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 8*a*(d*x)**(7/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(77*d*(a + b
*x**2)) + 2*(d*x)**(7/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(11*
d)
```

Mathematica [A] time = 0.0349037, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{5/2}\sqrt{(a+bx^2)^2}\,(11a+7bx^2)}{77\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(11*a + 7*b*x^2))/(77*(a + b
*x^2))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.4

$$\frac{2\,\left(7\,bx^2+11\,a\right)x}{77\,bx^2+77\,a}(dx)^{\frac{5}{2}}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/77*x*(7*b*x^2+11*a)*(d*x)^(5/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.718629, size = 30, normalized size = 0.32

$$\frac{2}{77}\left(7\,bd^{\frac{5}{2}}x^3+11\,ad^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/77*(7*b*d^(5/2)*x^3 + 11*a*d^(5/2)*x)*x^(5/2)
```

Fricas [A] time = 0.258445, size = 35, normalized size = 0.38

$$\frac{2}{77}\left(7\,bd^2x^5+11\,ad^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b*d^2*x^5 + 11*a*d^2*x^3)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.264216, size = 61, normalized size = 0.66

$$\frac{2}{11}\sqrt{dx}bd^2x^5\mathrm{sign}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}ad^2x^3\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/11*sqrt(d*x)*b*d^2*x^5*sign(b*x^2 + a) + 2/7*sqrt(d*x)*a*d^2*x^
3*sign(b*x^2 + a)
```

3.729 $\int (dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3(a+bx^2)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d(a+bx^2)}$$

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2
)) + (2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a
+ b*x^2))
```

Rubi [A] time = 0.0834996, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3(a+bx^2)}+\frac{2a(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x^2
)) + (2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*d^3*(a
+ b*x^2))
```

Rubi in Sympy [A] time = 23.8377, size = 75, normalized size = 0.81

$$\frac{8a(dx)^{\frac{5}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{45d(a+bx^2)}+\frac{2(dx)^{\frac{5}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 8*a*(d*x)**(5/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(45*d*(a + b
*x**2)) + 2*(d*x)**(5/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(9*d
)
```

Mathematica [A] time = 0.0231847, size = 44, normalized size = 0.47

$$\frac{2x(dx)^{3/2}\sqrt{(a+bx^2)^2}\,(9a+5bx^2)}{45\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(9*a + 5*b*x^2))/(45*(a + b*
x^2))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.4

$$\frac{2\,\left(5\,bx^2+9\,a\right)x}{45\,bx^2+45\,a}(dx)^{\frac{3}{2}}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/45*x*(5*b*x^2+9*a)*(d*x)^(3/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.70954, size = 30, normalized size = 0.32

$$\frac{2}{45}\left(5\,bd^{\frac{3}{2}}x^3+9\,ad^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45*(5*b*d^(3/2)*x^3 + 9*a*d^(3/2)*x)*x^(3/2)
```

Fricas [A] time = 0.25838, size = 30, normalized size = 0.32

$$\frac{2}{45}\left(5\,bdx^4+9\,adx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b*d*x^4 + 9*a*d*x^2)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265489, size = 55, normalized size = 0.59

$$\frac{2}{9}\sqrt{dx}bdx^4\text{sign}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}adx^2\text{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(d*x)*b*d*x^4*sign(b*x^2 + a) + 2/5*sqrt(d*x)*a*d*x^2*sig
n(b*x^2 + a)
```

3.730 $\int \sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=93

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3(a+bx^2)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)}$$

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2
)) + (2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a
+ b*x^2))
```

Rubi [A] time = 0.0818645, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3(a+bx^2)}+\frac{2a(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2
)) + (2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d^3*(a
+ b*x^2))
```

Rubi in Sympy [A] time = 24.2543, size = 75, normalized size = 0.81

$$\frac{8a(dx)^{\frac{3}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{21d(a+bx^2)}+\frac{2(dx)^{\frac{3}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] 8*a*(d*x)**(3/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(21*d*(a + b
*x**2)) + 2*(d*x)**(3/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(7*d
)
```

Mathematica [A] time = 0.0222462, size = 44, normalized size = 0.47

$$\frac{2\sqrt{dx}\sqrt{(a+bx^2)^2}\,(7ax+3bx^3)}{21\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(7*a*x + 3*b*x^3))/(21*(a + b*x^
2))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.4

$$\frac{2\,(3\,bx^2+7\,a)\,x}{21\,bx^2+21\,a}\sqrt{dx}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)*((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 2/21*x*(3*b*x^2+7*a)*(d*x)^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)
```

Maxima [A] time = 0.713244, size = 30, normalized size = 0.32

$$\frac{2}{21}\left(3\,b\sqrt{d}x^3+7\,a\sqrt{d}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/21*(3*b*sqrt(d)*x^3 + 7*a*sqrt(d)*x)*sqrt(x)
```

Fricas [A] time = 0.26009, size = 24, normalized size = 0.26

$$\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b*x^3 + 7*a*x)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265323, size = 59, normalized size = 0.63

$$\frac{2\left(3\sqrt{dx}bdx^3\text{sign}\left(bx^2+a\right)+7\sqrt{dx}adx\text{sign}\left(bx^2+a\right)\right)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)*sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/21*(3*sqrt(d*x)*b*d*x^3*sign(b*x^2 + a) + 7*sqrt(d*x)*a*d*x*sig
n(b*x^2 + a))/d
```

3.731 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) +
 (2*b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*
x^2))
```

Rubi [A] time = 0.0808466, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3\left(a+bx^2\right)}+\frac{2a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2)) +
 (2*b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(a + b*
x^2))
```

Rubi in Sympy [A] time = 40.7226, size = 75, normalized size = 0.82

$$\frac{8a\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{2\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] 8*a*sqrt(d*x)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*d*(a + b*x**
2)) + 2*sqrt(d*x)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*d)
```

Mathematica [A] time = 0.0197705, size = 43, normalized size = 0.47

$$\frac{2\sqrt{(a+bx^2)^2}\,(5ax+bx^3)}{5\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(5*a*x + b*x^3))/(5*Sqrt[d*x]*(a + b*x^2))
```

Maple [A] time = 0.004, size = 38, normalized size = 0.4

$$\frac{2\,\left(bx^2+5\,a\right)x}{5\,bx^2+5\,a}\sqrt{\left(bx^2+a\right)^2}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/(d*x)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^2+5*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(1/2)
```

Maxima [A] time = 0.709548, size = 32, normalized size = 0.35

$$\frac{2\left(b\sqrt{d}x^3+5\,a\sqrt{d}x\right)}{5\,d\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/5*(b*sqrt(d)*x^3 + 5*a*sqrt(d)*x)/(d*sqrt(x))
```

Fricas [A] time = 0.256726, size = 26, normalized size = 0.29

$$\frac{2\left(bx^2+5\,a\right)\sqrt{dx}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/5*(b*x^2 + 5*a)*sqrt(d*x)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.262916, size = 54, normalized size = 0.59

$$\frac{2\left(\sqrt{dx}bx^2\mathrm{sign}\left(bx^2+a\right)+5\sqrt{dx}a\mathrm{sign}\left(bx^2+a\right)\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/5*(sqrt(d*x)*b*x^2*sign(b*x^2 + a) + 5*sqrt(d*x)*a*sign(b*x^2 +
 a))/d
```

3.732 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}$$

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2))
+ (2*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b
*x^2))
```

Rubi [A] time = 0.0808946, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2))
+ (2*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3*(a + b
*x^2))
```

Rubi in Sympy [A] time = 23.1746, size = 75, normalized size = 0.82

$$-\frac{8a\sqrt{a^2+2abx^2+b^2x^4}}{3d\sqrt{dx}\,(a+bx^2)}+\frac{2\sqrt{a^2+2abx^2+b^2x^4}}{3d\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] -8*a*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3*d*sqrt(d*x)*(a + b*x*
*2)) + 2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3*d*sqrt(d*x))
```

Mathematica [A] time = 0.0277656, size = 43, normalized size = 0.47

$$\frac{2x\left(bx^2-3a\right)\sqrt{(a+bx^2)^2}}{3(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-3*a + b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(3/2)*(a + b*x^
2))
```

Maple [A] time = 0.004, size = 39, normalized size = 0.4

$$-\frac{2\left(-bx^2+3a\right)x}{3bx^2+3a}\sqrt{\left(bx^2+a\right)^2}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] -2/3*x*(-b*x^2+3*a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(3/2)
```

Maxima [A] time = 0.720404, size = 32, normalized size = 0.35

$$\frac{2\left(b\sqrt{d}x^3-3a\sqrt{d}x\right)}{3d^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*sqrt(d)*x^3 - 3*a*sqrt(d)*x)/(d^2*x^(3/2))
```

Fricas [A] time = 0.258759, size = 26, normalized size = 0.29

$$\frac{2\left(bx^2-3a\right)}{3\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x^2 - 3*a)/(sqrt(d*x)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26651, size = 55, normalized size = 0.6

$$\frac{2\left(\frac{\sqrt{dx}bx\mathrm{sign}\left(bx^2+a\right)}{d}-\frac{3\,a\mathrm{sign}\left(bx^2+a\right)}{\sqrt{dx}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(sqrt(d*x)*b*x*sign(b*x^2 + a)/d - 3*a*sign(b*x^2 + a)/sqrt(d
*x))/d
```

3.733 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)} - \frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)}$$

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^
2)) + (2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b
*x^2))
```

Rubi [A] time = 0.0811995, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)} - \frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*x^
2)) + (2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a + b
*x^2))
```

Rubi in Sympy [A] time = 23.6093, size = 73, normalized size = 0.8

$$-\frac{8a\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(dx)^{\frac{3}{2}}\,(a+bx^2)} + \frac{2\sqrt{a^2+2abx^2+b^2x^4}}{d\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
```

```
[Out] -8*a*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3*d*(d*x)**(3/2)*(a + b
*x**2)) + 2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(d*x)**(3/2))
```

Mathematica [A] time = 0.0279089, size = 42, normalized size = 0.46

$$-\frac{2x\left(a-3bx^2\right)\sqrt{\left(a+bx^2\right)^2}}{3(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(5/2),x]
```

```
[Out] (-2*x*(a - 3*b*x^2)*Sqrt[(a + b*x^2)^2])/(3*(d*x)^(5/2)*(a + b*x^
2))
```

Maple [A] time = 0.004, size = 37, normalized size = 0.4

$$-\frac{2\left(-3\,bx^2+a\right)x}{3\,bx^2+3\,a}\sqrt{\left(bx^2+a\right)^2}\,(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/(d*x)^(5/2),x)
```

```
[Out] -2/3*x*(-3*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(5/2)
```

Maxima [A] time = 0.727984, size = 34, normalized size = 0.37

$$\frac{2\left(3\,b\sqrt{d}x^3-a\sqrt{d}x\right)}{3\,d^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*b*sqrt(d)*x^3 - a*sqrt(d)*x)/(d^3*x^(5/2))
```

Fricas [A] time = 0.260045, size = 31, normalized size = 0.34

$$\frac{2\left(3\,bx^2-a\right)}{3\,\sqrt{dx}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b*x^2 - a)/(sqrt(d*x)*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.265361, size = 57, normalized size = 0.63

$$\frac{2\left(3\sqrt{dx}b\mathrm{sign}\left(bx^2+a\right)-\frac{ad\mathrm{sign}\left(bx^2+a\right)}{\sqrt{dx}x}\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(d*x)*b*sign(b*x^2 + a) - a*d*sign(b*x^2 + a)/(sqrt(d*
x)*x))/d^3
```

3.734 $\int \frac{\sqrt{a^2+2abx^2+b^2x^4}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}$$

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^
2)) - (2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b
*x^2))
```

Rubi [A] time = 0.0856163, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\,(a+bx^2)}-\frac{2a\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*x^
2)) - (2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*(a + b
*x^2))
```

Rubi in Sympy [A] time = 23.5247, size = 73, normalized size = 0.8

$$\frac{8a\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(dx)^{\frac{5}{2}}\,(a+bx^2)}-\frac{2\sqrt{a^2+2abx^2+b^2x^4}}{d\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

```
[Out] 8*a*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(5*d*(d*x)**(5/2)*(a + b*
x**2)) - 2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(d*x)**(5/2))
```

Mathematica [A] time = 0.0278139, size = 42, normalized size = 0.46

$$-\frac{2x\sqrt{(a+bx^2)^2}\,(a+5bx^2)}{5(dx)^{7/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]/(d*x)^(7/2),x]
```

```
[Out] (-2*x*Sqrt[(a + b*x^2)^2]*(a + 5*b*x^2))/(5*(d*x)^(7/2)*(a + b*x^
2))
```

Maple [A] time = 0.005, size = 37, normalized size = 0.4

$$-\frac{2\,\left(5\,bx^2+a\right)\,x}{5\,bx^2+5\,a}\sqrt{\left(bx^2+a\right)^2}\,(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((b*x^2+a)^2)^(1/2)/(d*x)^(7/2),x)
```

```
[Out] -2/5*x*(5*b*x^2+a)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/(d*x)^(7/2)
```

Maxima [A] time = 0.726182, size = 32, normalized size = 0.35

$$-\frac{2\left(5\,b\sqrt{d}x^3+a\sqrt{d}x\right)}{5\,d^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] -2/5*(5*b*sqrt(d)*x^3 + a*sqrt(d)*x)/(d^4*x^(7/2))
```

Fricas [A] time = 0.261584, size = 28, normalized size = 0.31

$$-\frac{2\left(5\,bx^2+a\right)}{5\,\sqrt{dx}d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(5*b*x^2 + a)/(sqrt(d*x)*d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((b*x**2+a)**2)**(1/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.266895, size = 59, normalized size = 0.65

$$-\frac{2\left(5\,bd^3x^2\mathrm{sign}\left(bx^2+a\right)+ad^3\mathrm{sign}\left(bx^2+a\right)\right)}{5\,\sqrt{dx}d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((b*x^2 + a)^2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/5*(5*b*d^3*x^2*sign(b*x^2 + a) + a*d^3*sign(b*x^2 + a))/(sqrt(
d*x)*d^6*x^2)
```

3.735 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x
^2)) + (6*a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11
*d^3*(a + b*x^2)) + (2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rubi [A] time = 0.164974, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2ab^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x
^2)) + (6*a^2*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11
*d^3*(a + b*x^2)) + (2*a*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])/(5*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(19/2)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.121, size = 156, normalized size = 0.8

$$\frac{256a^3\left(dx\right)^{\frac{7}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{7315d\left(a+bx^2\right)}+\frac{64a^2\left(dx\right)^{\frac{7}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{1045d}+\frac{8a\left(dx\right)^{\frac{7}{2}}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{95d}+\frac{2\left(dx\right)^{\frac{7}{2}}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{19d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 256*a**3*(d*x)**(7/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(7315*d
*(a + b*x**2)) + 64*a**2*(d*x)**(7/2)*sqrt(a**2 + 2*a*b*x**2 + b*
*2*x**4)/(1045*d) + 8*a*(d*x)**(7/2)*(a + b*x**2)*sqrt(a**2 + 2*a
*b*x**2 + b**2*x**4)/(95*d) + 2*(d*x)**(7/2)*(a**2 + 2*a*b*x**2 +
 b**2*x**4)**(3/2)/(19*d)
```

Mathematica [A] time = 0.0453032, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{5/2}\sqrt{(a+bx^2)^2}\left(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6\right)}{7315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(1045*a^3 + 1995*a^2*b*x^2 +
 1463*a*b^2*x^4 + 385*b^3*x^6))/(7315*(a + b*x^2))
```

Maple [A] time = 0.009, size = 61, normalized size = 0.3

$$\frac{2\,x\left(385\,b^3x^6+1463\,ax^4b^2+1995\,a^2bx^2+1045\,a^3\right)}{7315\,\left(bx^2+a\right)^3}(dx)^{\frac{5}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/7315*x*(385*b^3*x^6+1463*a*b^2*x^4+1995*a^2*b*x^2+1045*a^3)*(d*
x)^(5/2)*((b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.719992, size = 112, normalized size = 0.57

$$\frac{2}{285}\left(15\,b^3d^{\frac{5}{2}}x^3+19\,ab^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{4}{165}\left(11\,ab^2d^{\frac{5}{2}}x^3+15\,a^2bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^2bd^{\frac{5}{2}}x^3+11\,a^3d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/285*(15*b^3*d^(5/2)*x^3 + 19*a*b^2*d^(5/2)*x)*x^(13/2) + 4/165*
(11*a*b^2*d^(5/2)*x^3 + 15*a^2*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^2
*b*d^(5/2)*x^3 + 11*a^3*d^(5/2)*x)*x^(5/2)
```

Fricas [A] time = 0.267976, size = 73, normalized size = 0.37

$$\frac{2}{7315}\left(385\,b^{3}d^{2}x^{9} + 1463\,ab^{2}d^{2}x^{7} + 1995\,a^{2}bd^{2}x^{5} + 1045\,a^{3}d^{2}x^{3}\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*b^3*d^2*x^9 + 1463*a*b^2*d^2*x^7 + 1995*a^2*b*d^2*x^5
 + 1045*a^3*d^2*x^3)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.26796, size = 134, normalized size = 0.69

$$\frac{2}{19}\sqrt{dx}b^{3}d^{2}x^{9}\mathrm{sign}\left(bx^{2}+a\right) + \frac{2}{5}\sqrt{dx}ab^{2}d^{2}x^{7}\mathrm{sign}\left(bx^{2}+a\right) + \frac{6}{11}\sqrt{dx}a^{2}bd^{2}x^{5}\mathrm{sign}\left(bx^{2}+a\right) + \frac{2}{7}\sqrt{dx}a^{3}d^{2}x^{3}\mathrm{sign}\left(bx^{2}+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/19*sqrt(d*x)*b^3*d^2*x^9*sign(b*x^2 + a) + 2/5*sqrt(d*x)*a*b^2*
d^2*x^7*sign(b*x^2 + a) + 6/11*sqrt(d*x)*a^2*b*d^2*x^5*sign(b*x^2
 + a) + 2/7*sqrt(d*x)*a^3*d^2*x^3*sign(b*x^2 + a)
```

3.736 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{3/2} \, dx$

Optimal. Leaf size=195

$$\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\,(a+bx^2)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}+\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\,(a+bx^2)}+\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}$$

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x
^2)) + (2*a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d
^3*(a + b*x^2)) + (6*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(13*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rubi [A] time = 0.158058, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{6ab^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\,(a+bx^2)}+\frac{2a^2b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3\,(a+bx^2)}+\frac{2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\,(a+bx^2)}+\frac{2a^3(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x
^2)) + (2*a^2*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d
^3*(a + b*x^2)) + (6*a*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(13*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(17/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.2194, size = 156, normalized size = 0.8

$$\frac{256a^3\,(dx)^{\frac{5}{2}}\,\sqrt{a^2+2abx^2+b^2x^4}}{3315d\,(a+bx^2)}+\frac{64a^2\,(dx)^{\frac{5}{2}}\,\sqrt{a^2+2abx^2+b^2x^4}}{663d}+\frac{24a\,(dx)^{\frac{5}{2}}\,\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{221d}+\frac{2\,(dx)^{\frac{5}{2}}\,\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{17d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 256*a**3*(d*x)**(5/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3315*d
*(a + b*x**2)) + 64*a**2*(d*x)**(5/2)*sqrt(a**2 + 2*a*b*x**2 + b*
*2*x**4)/(663*d) + 24*a*(d*x)**(5/2)*(a + b*x**2)*sqrt(a**2 + 2*a
*b*x**2 + b**2*x**4)/(221*d) + 2*(d*x)**(5/2)*(a**2 + 2*a*b*x**2
+ b**2*x**4)**(3/2)/(17*d)
```

Mathematica [A] time = 0.0402871, size = 66, normalized size = 0.34

$$\frac{2x(dx)^{3/2}\sqrt{(a+bx^2)^2}\left(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6\right)}{3315\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(663*a^3 + 1105*a^2*b*x^2 +
765*a*b^2*x^4 + 195*b^3*x^6))/(3315*(a + b*x^2))
```

Maple [A] time = 0.009, size = 61, normalized size = 0.3

$$\frac{2\,x\left(195\,b^3x^6+765\,ax^4b^2+1105\,a^2bx^2+663\,a^3\right)}{3315\,\left(bx^2+a\right)^3}(dx)^{\frac{3}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 2/3315*x*(195*b^3*x^6+765*a*b^2*x^4+1105*a^2*b*x^2+663*a^3)*(d*x)
^(3/2)*((b*x^2+a)^2)^(3/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.712017, size = 112, normalized size = 0.57

$$\frac{2}{221}\left(13\,b^3d^{\frac{3}{2}}x^3+17\,ab^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{4}{117}\left(9\,ab^2d^{\frac{3}{2}}x^3+13\,a^2bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^2bd^{\frac{3}{2}}x^3+9\,a^3d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/221*(13*b^3*d^(3/2)*x^3 + 17*a*b^2*d^(3/2)*x)*x^(11/2) + 4/117*
(9*a*b^2*d^(3/2)*x^3 + 13*a^2*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^2*
b*d^(3/2)*x^3 + 9*a^3*d^(3/2)*x)*x^(3/2)
```

Fricas [A] time = 0.27291, size = 62, normalized size = 0.32

$$\frac{2}{3315}\left(195\,b^3dx^8 + 765\,ab^2dx^6 + 1105\,a^2bdx^4 + 663\,a^3dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3315*(195*b^3*d*x^8 + 765*a*b^2*d*x^6 + 1105*a^2*b*d*x^4 + 663*
a^3*d*x^2)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267341, size = 123, normalized size = 0.63

$$\frac{2}{17}\sqrt{dx}b^3dx^8\mathrm{sign}\left(bx^2+a\right) + \frac{6}{13}\sqrt{dx}ab^2dx^6\mathrm{sign}\left(bx^2+a\right) + \frac{2}{3}\sqrt{dx}a^2bdx^4\mathrm{sign}\left(bx^2+a\right) + \frac{2}{5}\sqrt{dx}a^3dx^2\mathrm{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/17*sqrt(d*x)*b^3*d*x^8*sign(b*x^2 + a) + 6/13*sqrt(d*x)*a*b^2*d
*x^6*sign(b*x^2 + a) + 2/3*sqrt(d*x)*a^2*b*d*x^4*sign(b*x^2 + a)
+ 2/5*sqrt(d*x)*a^3*d*x^2*sign(b*x^2 + a)
```

3.737 $\int \sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x
^2)) + (6*a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d
^3*(a + b*x^2)) + (6*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(11*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rubi [A] time = 0.156643, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{6ab^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5\left(a+bx^2\right)}+\frac{6a^2b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^7\left(a+bx^2\right)}+\frac{2a^3(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*a^3*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x
^2)) + (6*a^2*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d
^3*(a + b*x^2)) + (6*a*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(11*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(15/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(15*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.1618, size = 156, normalized size = 0.8

$$\frac{256a^3\left(dx\right)^{\frac{3}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{1155d\left(a+bx^2\right)}+\frac{64a^2\left(dx\right)^{\frac{3}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{385d}+\frac{8a\left(dx\right)^{\frac{3}{2}}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{55d}+\frac{2\left(dx\right)^{\frac{3}{2}}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)*(d*x)**(1/2),x)
```

```
[Out] 256*a**3*(d*x)**(3/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1155*d
*(a + b*x**2)) + 64*a**2*(d*x)**(3/2)*sqrt(a**2 + 2*a*b*x**2 + b*
*2*x**4)/(385*d) + 8*a*(d*x)**(3/2)*(a + b*x**2)*sqrt(a**2 + 2*a*
b*x**2 + b**2*x**4)/(55*d) + 2*(d*x)**(3/2)*(a**2 + 2*a*b*x**2 +
b**2*x**4)**(3/2)/(15*d)
```

Mathematica [A] time = 0.0352154, size = 66, normalized size = 0.34

$$\frac{2\sqrt{dx}\sqrt{(a+bx^2)^2}\left(385a^3x+495a^2bx^3+315ab^2x^5+77b^3x^7\right)}{1155\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(385*a^3*x + 495*a^2*b*x^3 + 315
*a*b^2*x^5 + 77*b^3*x^7))/(1155*(a + b*x^2))
```

Maple [A] time = 0.008, size = 61, normalized size = 0.3

$$\frac{2\,x\left(77\,b^3x^6+315\,ax^4b^2+495\,a^2bx^2+385\,a^3\right)}{1155\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)*(d*x)^(1/2),x)
```

```
[Out] 2/1155*x*(77*b^3*x^6+315*a*b^2*x^4+495*a^2*b*x^2+385*a^3)*((b*x^2
+a)^2)^(3/2)*(d*x)^(1/2)/(b*x^2+a)^3
```

Maxima [A] time = 0.709566, size = 112, normalized size = 0.57

$$\frac{2}{165}\left(11\,b^3\sqrt{d}x^3+15\,ab^2\sqrt{d}x\right)x^{\frac{9}{2}}+\frac{4}{77}\left(7\,ab^2\sqrt{d}x^3+11\,a^2b\sqrt{d}x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^2b\sqrt{d}x^3+7\,a^3\sqrt{d}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/165*(11*b^3*sqrt(d)*x^3 + 15*a*b^2*sqrt(d)*x)*x^(9/2) + 4/77*(7
*a*b^2*sqrt(d)*x^3 + 11*a^2*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^2*b*
sqrt(d)*x^3 + 7*a^3*sqrt(d)*x)*sqrt(x)
```

Fricas [A] time = 0.270706, size = 54, normalized size = 0.28

$$\frac{2}{1155}\left(77\,b^3x^7 + 315\,ab^2x^5 + 495\,a^2bx^3 + 385\,a^3x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*s
qrt(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)*(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.266292, size = 127, normalized size = 0.65

$$\frac{2\left(77\,\sqrt{dx}b^3dx^7\mathrm{sign}\left(bx^2 + a\right) + 315\,\sqrt{dx}ab^2dx^5\mathrm{sign}\left(bx^2 + a\right) + 495\,\sqrt{dx}a^2bdx^3\mathrm{sign}\left(bx^2 + a\right) + 385\,\sqrt{dx}a^3dx\mathrm{sign}\left(bx^2 +\right.\right.}{1155\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/1155*(77*sqrt(d*x)*b^3*d*x^7*sign(b*x^2 + a) + 315*sqrt(d*x)*a*
b^2*d*x^5*sign(b*x^2 + a) + 495*sqrt(d*x)*a^2*b*d*x^3*sign(b*x^2
```

```
+ a) + 385*sqrt(d*x)*a^3*d*x*sign(b*x^2 + a))/d
```

3.738 $\int \frac{(a^2+2abx^2+b^2x^4)^{3/2}}{\sqrt{dx}} dx$

Optimal. Leaf size=193

$$\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5(a+bx^2)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3(a+bx^2)}+\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7(a+bx^2)}+\frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)}$$

```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2))
 + (6*a^2*b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(
a + b*x^2)) + (2*a*b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(3*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(13*d^7*(a + b*x^2))
```

Rubi [A] time = 0.159151, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2ab^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5(a+bx^2)}+\frac{6a^2b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^3(a+bx^2)}+\frac{2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7(a+bx^2)}+\frac{2a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2))
 + (6*a^2*b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^3*(
a + b*x^2)) + (2*a*b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])/(3*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(13*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.2877, size = 156, normalized size = 0.81

$$\frac{256a^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{195d(a+bx^2)}+\frac{64a^2\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{195d}+\frac{8a\sqrt{dx}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{39d}+\frac{2\sqrt{dx}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{13d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] 256*a**3*sqrt(d*x)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(195*d*(a
+ b*x**2)) + 64*a**2*sqrt(d*x)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4
)/(195*d) + 8*a*sqrt(d*x)*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b
**2*x**4)/(39*d) + 2*sqrt(d*x)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(
3/2)/(13*d)
```

Mathematica [A] time = 0.0322495, size = 66, normalized size = 0.34

$$\frac{2\sqrt{(a+bx^2)^2}\left(195a^3x+117a^2bx^3+65ab^2x^5+15b^3x^7\right)}{195\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(195*a^3*x + 117*a^2*b*x^3 + 65*a*b^2*x^5
+ 15*b^3*x^7))/(195*Sqrt[d*x]*(a + b*x^2))
```

Maple [A] time = 0.009, size = 61, normalized size = 0.3

$$\frac{2\,\left(15\,b^3x^6+65\,ax^4b^2+117\,a^2bx^2+195\,a^3\right)x}{195\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 2/195*x*(15*b^3*x^6+65*a*b^2*x^4+117*a^2*b*x^2+195*a^3)*((b*x^2+a
)^2)^(3/2)/(b*x^2+a)^3/(d*x)^(1/2)
```

Maxima [A] time = 0.711173, size = 117, normalized size = 0.61

$$\frac{2\left(5\left(9\,b^3\sqrt{d}x^3+13\,ab^2\sqrt{d}x\right)x^{\frac{7}{2}}+26\left(5\,ab^2\sqrt{d}x^3+9\,a^2b\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{117\left(a^2b\sqrt{d}x^3+5\,a^3\sqrt{d}x\right)}{\sqrt{x}}\right)}{585\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/585*(5*(9*b^3*sqrt(d)*x^3 + 13*a*b^2*sqrt(d)*x)*x^(7/2) + 26*(5
*a*b^2*sqrt(d)*x^3 + 9*a^2*b*sqrt(d)*x)*x^(3/2) + 117*(a^2*b*sqrt
(d)*x^3 + 5*a^3*sqrt(d)*x)/sqrt(x))/d
```

Fricas [A] time = 0.271389, size = 57, normalized size = 0.3

$$\frac{2\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{dx}}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(
d*x)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/sqrt(d*x), x)
```

GIAC/XCAS [A] time = 0.267712, size = 120, normalized size = 0.62

$$\frac{2\left(15\,\sqrt{dx}b^3x^6\mathrm{sign}\left(bx^2+a\right)+65\,\sqrt{dx}ab^2x^4\mathrm{sign}\left(bx^2+a\right)+117\,\sqrt{dx}a^2bx^2\mathrm{sign}\left(bx^2+a\right)+195\,\sqrt{dx}a^3\mathrm{sign}\left(bx^2+a\right)\right)}{195\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/195*(15*sqrt(d*x)*b^3*x^6*sign(b*x^2 + a) + 65*sqrt(d*x)*a*b^2*
x^4*sign(b*x^2 + a) + 117*sqrt(d*x)*a^2*b*x^2*sign(b*x^2 + a) + 1
95*sqrt(d*x)*a^3*sign(b*x^2 + a))/d
```

3.739 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=191

$$\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)
) + (2*a^2*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a
 + b*x^2)) + (6*a*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(7*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(11*d^7*(a + b*x^2))
```

Rubi [A] time = 0.159989, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{6ab^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5\left(a+bx^2\right)}+\frac{2a^2b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)
) + (2*a^2*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a
 + b*x^2)) + (6*a*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(7*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(11*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.2303, size = 156, normalized size = 0.82

$$-\frac{256a^3\sqrt{a^2+2abx^2+b^2x^4}}{77d\sqrt{dx}\left(a+bx^2\right)}+\frac{64a^2\sqrt{a^2+2abx^2+b^2x^4}}{77d\sqrt{dx}}+\frac{24a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{77d\sqrt{dx}}+\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{11d\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] -256*a**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(77*d*sqrt(d*x)*(a
+ b*x**2)) + 64*a**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(77*d*sq
rt(d*x)) + 24*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/
(77*d*sqrt(d*x)) + 2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(11*d
*sqrt(d*x))
```

Mathematica [A] time = 0.0414621, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{(a+bx^2)^2}\left(-77a^3+77a^2bx^2+33ab^2x^4+7b^3x^6\right)}{77(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-77*a^3 + 77*a^2*b*x^2 + 33*a*b^2*x^4 +
 7*b^3*x^6))/(77*(d*x)^(3/2)*(a + b*x^2))
```

Maple [A] time = 0.009, size = 61, normalized size = 0.3

$$-\frac{2\left(-7b^3x^6-33ax^4b^2-77a^2bx^2+77a^3\right)x}{77\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(3/2),x)
```

```
[Out] -2/77*x*(-7*b^3*x^6-33*a*b^2*x^4-77*a^2*b*x^2+77*a^3)*((b*x^2+a)^
2)^(3/2)/(b*x^2+a)^3/(d*x)^(3/2)
```

Maxima [A] time = 0.721661, size = 117, normalized size = 0.61

$$\frac{2\left(3\left(7b^3\sqrt{d}x^3+11ab^2\sqrt{d}x\right)x^{\frac{5}{2}}+22\left(3ab^2\sqrt{d}x^3+7a^2b\sqrt{d}x\right)\sqrt{x}+\frac{77\left(a^2b\sqrt{d}x^3-3a^3\sqrt{d}x\right)}{x^{\frac{3}{2}}}\right)}{231d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/231*(3*(7*b^3*sqrt(d)*x^3 + 11*a*b^2*sqrt(d)*x)*x^(5/2) + 22*(3
*a*b^2*sqrt(d)*x^3 + 7*a^2*b*sqrt(d)*x)*sqrt(x) + 77*(a^2*b*sqrt(
d)*x^3 - 3*a^3*sqrt(d)*x)/x^(3/2))/d^2
```

Fricas [A] time = 0.274854, size = 57, normalized size = 0.3

$$\frac{2\left(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3\right)}{77\,\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/(sqrt(d*x
)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(3/2), x)
```

GIAC/XCAS [A] time = 0.267531, size = 138, normalized size = 0.72

$$-\frac{2\left(\frac{77\,a^3\mathrm{sign}\left(bx^2+a\right)}{\sqrt{dx}}-\frac{7\,\sqrt{dx}b^3d^{65}x^5\mathrm{sign}\left(bx^2+a\right)+33\,\sqrt{dx}ab^2d^{65}x^3\mathrm{sign}\left(bx^2+a\right)+77\,\sqrt{dx}a^2bd^{65}x\mathrm{sign}\left(bx^2+a\right)}{d^{66}}\right)}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/77*(77*a^3*sign(b*x^2 + a)/sqrt(d*x) - (7*sqrt(d*x)*b^3*d^65*x
^5*sign(b*x^2 + a) + 33*sqrt(d*x)*a*b^2*d^65*x^3*sign(b*x^2 + a)
+ 77*sqrt(d*x)*a^2*b*d^65*x*sign(b*x^2 + a))/d^66)/d
```

3.740 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=193

$$\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*
x^2)) + (6*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*
(a + b*x^2)) + (6*a*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(5*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(9*d^7*(a + b*x^2))
```

Rubi [A] time = 0.159123, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{6ab^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d^5\left(a+bx^2\right)}+\frac{6a^2b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```


```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*
x^2)) + (6*a^2*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*
(a + b*x^2)) + (6*a*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(5*d^5*(a + b*x^2)) + (2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(9*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.1344, size = 156, normalized size = 0.81

$$-\frac{256a^3\sqrt{a^2+2abx^2+b^2x^4}}{45d\,(dx)^{\frac{3}{2}}\left(a+bx^2\right)}+\frac{64a^2\sqrt{a^2+2abx^2+b^2x^4}}{15d\,(dx)^{\frac{3}{2}}}+\frac{8a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{15d\,(dx)^{\frac{3}{2}}}+\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{9d\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
```

```
[Out] -256*a**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(45*d*(d*x)**(3/2)*
(a + b*x**2)) + 64*a**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(15*d
*(d*x)**(3/2)) + 8*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x
**4)/(15*d*(d*x)**(3/2)) + 2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/
2)/(9*d*(d*x)**(3/2))
```

Mathematica [A] time = 0.0422326, size = 66, normalized size = 0.34

$$\frac{2x\sqrt{(a+bx^2)^2}\left(-15a^3+135a^2bx^2+27ab^2x^4+5b^3x^6\right)}{45(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-15*a^3 + 135*a^2*b*x^2 + 27*a*b^2*x^4
+ 5*b^3*x^6))/(45*(d*x)^(5/2)*(a + b*x^2))
```

Maple [A] time = 0.009, size = 61, normalized size = 0.3

$$-\frac{2\left(-5\,b^3x^6-27\,ax^4b^2-135\,a^2bx^2+15\,a^3\right)x}{45\,\left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(5/2),x)
```

```
[Out] -2/45*x*(-5*b^3*x^6-27*a*b^2*x^4-135*a^2*b*x^2+15*a^3)*((b*x^2+a)
^2)^(3/2)/(b*x^2+a)^3/(d*x)^(5/2)
```

Maxima [A] time = 0.712538, size = 116, normalized size = 0.6

$$\frac{2\left(\left(5\,b^3\sqrt{d}x^3+9\,ab^2\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{18\left(ab^2\sqrt{d}x^3+5\,a^2b\sqrt{d}x\right)}{\sqrt{x}}+\frac{15\left(3\,a^2b\sqrt{d}x^3-a^3\sqrt{d}x\right)}{x^{\frac{5}{2}}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45*((5*b^3*sqrt(d)*x^3 + 9*a*b^2*sqrt(d)*x)*x^(3/2) + 18*(a*b^2
*sqrt(d)*x^3 + 5*a^2*b*sqrt(d)*x)/sqrt(x) + 15*(3*a^2*b*sqrt(d)*x
^3 - a^3*sqrt(d)*x)/x^(5/2))/d^3
```

Fricas [A] time = 0.271187, size = 61, normalized size = 0.32

$$\frac{2\left(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3\right)}{45\,\sqrt{dx}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/(sqrt(d*
x)*d^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(5/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**(3/2)/(d*x)**(5/2), x)
```

GIAC/XCAS [A] time = 0.26696, size = 142, normalized size = 0.74

$$-\frac{2\left(\frac{15\,a^3d\mathrm{sign}\left(bx^2+a\right)}{\sqrt{dx}x}-\frac{5\,\sqrt{dx}b^3d^{36}x^4\mathrm{sign}\left(bx^2+a\right)+27\,\sqrt{dx}ab^2d^{36}x^2\mathrm{sign}\left(bx^2+a\right)+135\,\sqrt{dx}a^2bd^{36}\mathrm{sign}\left(bx^2+a\right)}{d^{36}}\right)}{45\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(5/2),x, algorithm="giac")

[Out] -2/45*(15*a^3*d*sign(b*x^2 + a)/(sqrt(d*x)*x) - (5*sqrt(d*x)*b^3*
d^36*x^4*sign(b*x^2 + a) + 27*sqrt(d*x)*a*b^2*d^36*x^2*sign(b*x^2
 + a) + 135*sqrt(d*x)*a^2*b*d^36*sign(b*x^2 + a))/d^36)/d^3
```

3.741 $\int \frac{\left(a^2+2abx^2+b^2x^4\right)^{3/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=191

$$\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*
x^2)) - (6*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*
(a + b*x^2)) + (2*a*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(d^5*(a + b*x^2)) + (2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rubi [A] time = 0.157869, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2ab^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5\left(a+bx^2\right)}-\frac{6a^2b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}\left(a+bx^2\right)}+\frac{2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7\left(a+bx^2\right)}-\frac{2a^3\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*
x^2)) - (6*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]*
(a + b*x^2)) + (2*a*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(d^5*(a + b*x^2)) + (2*b^3*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(7*d^7*(a + b*x^2))
```

Rubi in Sympy [A] time = 18.3959, size = 156, normalized size = 0.82

$$\frac{256a^3\sqrt{a^2+2abx^2+b^2x^4}}{35d\,(dx)^{\frac{5}{2}}\left(a+bx^2\right)}-\frac{64a^2\sqrt{a^2+2abx^2+b^2x^4}}{7d\,(dx)^{\frac{5}{2}}}+\frac{8a\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{7d\,(dx)^{\frac{5}{2}}}+\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{7d\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
```

```
[Out] 256*a**3*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(35*d*(d*x)**(5/2)*(
a + b*x**2)) - 64*a**2*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(7*d*(
d*x)**(5/2)) + 8*a*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**
4)/(7*d*(d*x)**(5/2)) + 2*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/
(7*d*(d*x)**(5/2))
```

Mathematica [A] time = 0.0405466, size = 66, normalized size = 0.35

$$\frac{2x\sqrt{(a+bx^2)^2}\left(-7a^3-105a^2bx^2+35ab^2x^4+5b^3x^6\right)}{35(dx)^{7/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-7*a^3 - 105*a^2*b*x^2 + 35*a*b^2*x^4 +
 5*b^3*x^6))/(35*(d*x)^(7/2)*(a + b*x^2))
```

Maple [A] time = 0.009, size = 61, normalized size = 0.3

$$-\frac{2\ \left(-5\,b^3x^6-35\,ax^4b^2+105\,a^2bx^2+7\,a^3\right)x}{35\ \left(bx^2+a\right)^3}\left(\left(bx^2+a\right)^2\right)^{\frac{3}{2}}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(7/2),x)
```

```
[Out] -2/35*x*(-5*b^3*x^6-35*a*b^2*x^4+105*a^2*b*x^2+7*a^3)*((b*x^2+a)^
2)^(3/2)/(b*x^2+a)^3/(d*x)^(7/2)
```

Maxima [A] time = 0.72334, size = 116, normalized size = 0.61

$$\frac{2\left(5\left(3\,b^3\sqrt{d}x^3+7\,ab^2\sqrt{d}x\right)\sqrt{x}+\frac{70\left(ab^2\sqrt{d}x^3-3\,a^2b\sqrt{d}x\right)}{x^{\frac{3}{2}}}-\frac{21\left(5\,a^2b\sqrt{d}x^3+a^3\sqrt{d}x\right)}{x^{\frac{7}{2}}}\right)}{105\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/105*(5*(3*b^3*sqrt(d)*x^3 + 7*a*b^2*sqrt(d)*x)*sqrt(x) + 70*(a*
b^2*sqrt(d)*x^3 - 3*a^2*b*sqrt(d)*x)/x^(3/2) - 21*(5*a^2*b*sqrt(d
)*x^3 + a^3*sqrt(d)*x)/x^(7/2))/d^4
```

Fricas [A] time = 0.270475, size = 61, normalized size = 0.32

$$\frac{2\left(5\,b^{3}x^{6}+35\,ab^{2}x^{4}-105\,a^{2}bx^{2}-7\,a^{3}\right)}{35\,\sqrt{dx}d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/(sqrt(d*x
)*d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269321, size = 144, normalized size = 0.75

$$-\frac{2\left(\frac{7\left(15\,a^{2}bd^{3}x^{2}\mathrm{sign}\left(bx^{2}+a\right)+a^{3}d^{3}\mathrm{sign}\left(bx^{2}+a\right)\right)}{\sqrt{dx}d^{2}x^{2}}-\frac{5\left(\sqrt{dx}b^{3}d^{21}x^{3}\mathrm{sign}\left(bx^{2}+a\right)+7\,\sqrt{dx}ab^{2}d^{21}x\mathrm{sign}\left(bx^{2}+a\right)\right)}{d^{21}}\right)}{35\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/35*(7*(15*a^2*b*d^3*x^2*sign(b*x^2 + a) + a^3*d^3*sign(b*x^2 +
 a))/(sqrt(d*x)*d^2*x^2) - 5*(sqrt(d*x)*b^3*d^21*x^3*sign(b*x^2 +
 a) + 7*sqrt(d*x)*a*b^2*d^21*x*sign(b*x^2 + a))/d^21)/d^4
```

3.742 $\int (dx)^{5/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}$$

```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x
^2)) + (10*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(1
1*d^3*(a + b*x^2)) + (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2)) + (10*a*b^4*(d*x
)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*(a + b*x^2)) +
(2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^11*(a
+ b*x^2))
```

Rubi [A] time = 0.238752, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{27/2}\sqrt{a^2+2abx^2+b^2x^4}}{27d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^7\left(a+bx^2\right)}+\frac{2a^5(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^3\left(a+bx^2\right)}+\frac{4a^3b^2(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (2*a^5*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*d*(a + b*x
^2)) + (10*a^4*b*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(1
1*d^3*(a + b*x^2)) + (4*a^3*b^2*(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])/(3*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(19/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^7*(a + b*x^2)) + (10*a*b^4*(d*x
)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^9*(a + b*x^2)) +
(2*b^5*(d*x)^(27/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(27*d^11*(a
+ b*x^2))
```

Rubi in Sympy [A] time = 29.2781, size = 238, normalized size = 0.8

$$\frac{16384a^5\left(dx\right)^{\frac{7}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{908523d\left(a+bx^2\right)}+\frac{4096a^4\left(dx\right)^{\frac{7}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{129789d}+\frac{512a^3\left(dx\right)^{\frac{7}{2}}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{11799d}+\frac{640a^2\left(dx\right)^{\frac{7}{2}}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{11799d}+\frac{40a\left(dx\right)^{\frac{7}{2}}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{621d}+\frac{2\left(dx\right)^{\frac{7}{2}}\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{27d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 16384*a**5*(d*x)**(7/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(9085
23*d*(a + b*x**2)) + 4096*a**4*(d*x)**(7/2)*sqrt(a**2 + 2*a*b*x**
2 + b**2*x**4)/(129789*d) + 512*a**3*(d*x)**(7/2)*(a + b*x**2)*sq
rt(a**2 + 2*a*b*x**2 + b**2*x**4)/(11799*d) + 640*a**2*(d*x)**(7/
2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(11799*d) + 40*a*(d*x)*
*(7/2)*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(621*d
) + 2*(d*x)**(7/2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(27*d)
```

Mathematica [A] time = 0.0587774, size = 88, normalized size = 0.3

$$\frac{2x(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}\left(129789a^5+412965a^4bx^2+605682a^3b^2x^4+478170a^2b^3x^6+197505ab^4x^8+33649b^5x^{10}\right)}{908523\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(5/2)*Sqrt[(a + b*x^2)^2]*(129789*a^5 + 412965*a^4*b*x
^2 + 605682*a^3*b^2*x^4 + 478170*a^2*b^3*x^6 + 197505*a*b^4*x^8 +
 33649*b^5*x^10))/(908523*(a + b*x^2))
```

Maple [A] time = 0.008, size = 83, normalized size = 0.3

$$\frac{2\,x\left(33649\,b^5x^{10}+197505\,ab^4x^8+478170\,a^2b^3x^6+605682\,a^3b^2x^4+412965\,a^4bx^2+129789\,a^5\right)}{908523\,\left(bx^2+a\right)^5}\left(dx\right)^{\frac{5}{2}}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/908523*x*(33649*b^5*x^10+197505*a*b^4*x^8+478170*a^2*b^3*x^6+60
5682*a^3*b^2*x^4+412965*a^4*b*x^2+129789*a^5)*(d*x)^(5/2)*((b*x^2
+a)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.7288, size = 198, normalized size = 0.67

$$\frac{2}{621}\left(23\,b^5d^{\frac{5}{2}}x^3+27\,ab^4d^{\frac{5}{2}}x\right)x^{\frac{21}{2}}+\frac{8}{437}\left(19\,ab^4d^{\frac{5}{2}}x^3+23\,a^2b^3d^{\frac{5}{2}}x\right)x^{\frac{17}{2}}+\frac{4}{95}\left(15\,a^2b^3d^{\frac{5}{2}}x^3+19\,a^3b^2d^{\frac{5}{2}}x\right)x^{\frac{13}{2}}+\frac{8}{165}\left(11\,a^3b^2d^{\frac{5}{2}}x^3+15\,a^4bd^{\frac{5}{2}}x\right)x^{\frac{9}{2}}+\frac{2}{77}\left(7\,a^4bd^{\frac{5}{2}}x^3+11\,a^5d^{\frac{5}{2}}x\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/621*(23*b^5*d^(5/2)*x^3 + 27*a*b^4*d^(5/2)*x)*x^(21/2) + 8/437*
(19*a*b^4*d^(5/2)*x^3 + 23*a^2*b^3*d^(5/2)*x)*x^(17/2) + 4/95*(15
*a^2*b^3*d^(5/2)*x^3 + 19*a^3*b^2*d^(5/2)*x)*x^(13/2) + 8/165*(11
*a^3*b^2*d^(5/2)*x^3 + 15*a^4*b*d^(5/2)*x)*x^(9/2) + 2/77*(7*a^4*
b*d^(5/2)*x^3 + 11*a^5*d^(5/2)*x)*x^(5/2)
```

Fricas [A] time = 0.274293, size = 111, normalized size = 0.37

$$\frac{2}{908523}\left(33649\,b^5d^2x^{13}+197505\,ab^4d^2x^{11}+478170\,a^2b^3d^2x^9+605682\,a^3b^2d^2x^7+412965\,a^4bd^2x^5+129789\,a^5d^2x^3\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/908523*(33649*b^5*d^2*x^13 + 197505*a*b^4*d^2*x^11 + 478170*a^2
*b^3*d^2*x^9 + 605682*a^3*b^2*d^2*x^7 + 412965*a^4*b*d^2*x^5 + 12
9789*a^5*d^2*x^3)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.268335, size = 207, normalized size = 0.7

$$\frac{2}{27}\sqrt{dx}b^5d^2x^{13}\operatorname{sign}\left(bx^2+a\right)+\frac{10}{23}\sqrt{dx}ab^4d^2x^{11}\operatorname{sign}\left(bx^2+a\right)+\frac{20}{19}\sqrt{dx}a^2b^3d^2x^9\operatorname{sign}\left(bx^2+a\right)+\frac{4}{3}\sqrt{dx}a^3b^2d^2x^7\operatorname{sign}\left(bx^2+a\right)+\frac{10}{11}\sqrt{dx}a^4bd^2x^5\operatorname{sign}\left(bx^2+a\right)+\frac{2}{7}\sqrt{dx}a^5d^2x^3\operatorname{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(d*x)*b^5*d^2*x^13*sign(b*x^2 + a) + 10/23*sqrt(d*x)*a*b
^4*d^2*x^11*sign(b*x^2 + a) + 20/19*sqrt(d*x)*a^2*b^3*d^2*x^9*sig
n(b*x^2 + a) + 4/3*sqrt(d*x)*a^3*b^2*d^2*x^7*sign(b*x^2 + a) + 10
/11*sqrt(d*x)*a^4*b*d^2*x^5*sign(b*x^2 + a) + 2/7*sqrt(d*x)*a^5*d
^2*x^3*sign(b*x^2 + a)
```

3.743 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}$$

```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x
^2)) + (10*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*
d^3*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(13*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2)) + (10*a*b^4*(d*x
)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*(a + b*x^2)) +
(2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^11*(a
+ b*x^2))
```

Rubi [A] time = 0.226831, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{25/2}\sqrt{a^2+2abx^2+b^2x^4}}{25d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^7\left(a+bx^2\right)}+\frac{2a^5(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{5d\left(a+bx^2\right)}+\frac{10a^4b(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (2*a^5*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(a + b*x
^2)) + (10*a^4*b*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(9*
d^3*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(13/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(13*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(17/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^7*(a + b*x^2)) + (10*a*b^4*(d*x
)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^9*(a + b*x^2)) +
(2*b^5*(d*x)^(25/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(25*d^11*(a
+ b*x^2))
```

Rubi in Sympy [A] time = 29.1006, size = 238, normalized size = 0.8

$$\frac{16384a^5(dx)^{\frac{5}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{348075d(a+bx^2)}+\frac{4096a^4(dx)^{\frac{5}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{69615d}+\frac{512a^3(dx)^{\frac{5}{2}}(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{7735d}+\frac{128a^2(dx)^{\frac{5}{2}}(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{1785d}+\frac{8a(dx)^{\frac{5}{2}}(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{105d}+\frac{2(dx)^{\frac{5}{2}}(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{25d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 16384*a**5*(d*x)**(5/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(3480
75*d*(a + b*x**2)) + 4096*a**4*(d*x)**(5/2)*sqrt(a**2 + 2*a*b*x**
2 + b**2*x**4)/(69615*d) + 512*a**3*(d*x)**(5/2)*(a + b*x**2)*sqr
t(a**2 + 2*a*b*x**2 + b**2*x**4)/(7735*d) + 128*a**2*(d*x)**(5/2)
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(1785*d) + 8*a*(d*x)**(5/
2)*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(105*d) +
2*(d*x)**(5/2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(25*d)
```

Mathematica [A] time = 0.0463342, size = 88, normalized size = 0.3

$$\frac{2x(dx)^{3/2}\sqrt{(a+bx^2)^2}\left(69615a^5+193375a^4bx^2+267750a^3b^2x^4+204750a^2b^3x^6+82875ab^4x^8+13923b^5x^{10}\right)}{348075(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*x*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2]*(69615*a^5 + 193375*a^4*b*x^
2 + 267750*a^3*b^2*x^4 + 204750*a^2*b^3*x^6 + 82875*a*b^4*x^8 + 1
3923*b^5*x^10))/(348075*(a + b*x^2))
```

Maple [A] time = 0.008, size = 83, normalized size = 0.3

$$\frac{2x\left(13923b^5x^{10}+82875ab^4x^8+204750a^2b^3x^6+267750a^3b^2x^4+193375a^4bx^2+69615a^5\right)}{348075(bx^2+a)^5}(dx)^{\frac{3}{2}}\left((bx^2+a)^2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 2/348075*x*(13923*b^5*x^10+82875*a*b^4*x^8+204750*a^2*b^3*x^6+267
750*a^3*b^2*x^4+193375*a^4*b*x^2+69615*a^5)*(d*x)^(3/2)*((b*x^2+a
)^2)^(5/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.721859, size = 198, normalized size = 0.67

$$\frac{2}{525}\left(21\,b^5d^{\frac{3}{2}}x^3+25\,ab^4d^{\frac{3}{2}}x\right)x^{\frac{19}{2}}+\frac{8}{357}\left(17\,ab^4d^{\frac{3}{2}}x^3+21\,a^2b^3d^{\frac{3}{2}}x\right)x^{\frac{15}{2}}+\frac{12}{221}\left(13\,a^2b^3d^{\frac{3}{2}}x^3+17\,a^3b^2d^{\frac{3}{2}}x\right)x^{\frac{11}{2}}+\frac{8}{117}\left(9\,a^3b^2d^{\frac{3}{2}}x^3+13\,a^4bd^{\frac{3}{2}}x\right)x^{\frac{7}{2}}+\frac{2}{45}\left(5\,a^4bd^{\frac{3}{2}}x^3+9\,a^5d^{\frac{3}{2}}x\right)x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/525*(21*b^5*d^(3/2)*x^3 + 25*a*b^4*d^(3/2)*x)*x^(19/2) + 8/357*
(17*a*b^4*d^(3/2)*x^3 + 21*a^2*b^3*d^(3/2)*x)*x^(15/2) + 12/221*(
13*a^2*b^3*d^(3/2)*x^3 + 17*a^3*b^2*d^(3/2)*x)*x^(11/2) + 8/117*(
9*a^3*b^2*d^(3/2)*x^3 + 13*a^4*b*d^(3/2)*x)*x^(7/2) + 2/45*(5*a^4
*b*d^(3/2)*x^3 + 9*a^5*d^(3/2)*x)*x^(3/2)
```

Fricas [A] time = 0.272575, size = 95, normalized size = 0.32

$$\frac{2}{348075}\left(13923\,b^5dx^{12}+82875\,ab^4dx^{10}+204750\,a^2b^3dx^8+267750\,a^3b^2dx^6+193375\,a^4bdx^4+69615\,a^5dx^2\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/348075*(13923*b^5*d*x^12 + 82875*a*b^4*d*x^10 + 204750*a^2*b^3*
d*x^8 + 267750*a^3*b^2*d*x^6 + 193375*a^4*b*d*x^4 + 69615*a^5*d*x
^2)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269821, size = 190, normalized size = 0.64

$$\frac{2}{25}\sqrt{dx}b^5dx^{12}\operatorname{sign}\left(bx^2+a\right)+\frac{10}{21}\sqrt{dx}ab^4dx^{10}\operatorname{sign}\left(bx^2+a\right)+\frac{20}{17}\sqrt{dx}a^2b^3dx^8\operatorname{sign}\left(bx^2+a\right)+\frac{20}{13}\sqrt{dx}a^3b^2dx^6\operatorname{sign}\left(bx^2+a\right)+\frac{10}{9}\sqrt{dx}a^4bdx^4\operatorname{sign}\left(bx^2+a\right)+\frac{2}{5}\sqrt{dx}a^5dx^2\operatorname{sign}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/25*sqrt(d*x)*b^5*d*x^12*sign(b*x^2 + a) + 10/21*sqrt(d*x)*a*b^4
*d*x^10*sign(b*x^2 + a) + 20/17*sqrt(d*x)*a^2*b^3*d*x^8*sign(b*x^
2 + a) + 20/13*sqrt(d*x)*a^3*b^2*d*x^6*sign(b*x^2 + a) + 10/9*sqr
t(d*x)*a^4*b*d*x^4*sign(b*x^2 + a) + 2/5*sqrt(d*x)*a^5*d*x^2*sign
(b*x^2 + a)
```

3.744 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx$

Optimal. Leaf size=297

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}(a+bx^2)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9(a+bx^2)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7(a+bx^2)}+\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3(a+bx^2)}+\frac{20a^3b^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5(a+bx^2)}$$

```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x
^2)) + (10*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*
d^3*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(11*d^5*(a + b*x^2)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^
(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a + b*x^2)) + (2
*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*(a +
b*x^2))
```

Rubi [A] time = 0.22648, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{23/2}\sqrt{a^2+2abx^2+b^2x^4}}{23d^{11}(a+bx^2)}+\frac{10ab^4(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^9(a+bx^2)}+\frac{4a^2b^3(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^7(a+bx^2)}+\frac{2a^5(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d(a+bx^2)}+\frac{10a^4b(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^3(a+bx^2)}+\frac{20a^3b^2(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (2*a^5*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x
^2)) + (10*a^4*b*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(7*
d^3*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(11/2)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(11*d^5*(a + b*x^2)) + (4*a^2*b^3*(d*x)^(15/2)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^
(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^9*(a + b*x^2)) + (2
*b^5*(d*x)^(23/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(23*d^11*(a +
b*x^2))
```

Rubi in Sympy [A] time = 29.5441, size = 238, normalized size = 0.8

$$\frac{16384a^5(dx)^{\frac{3}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{100947d(a+bx^2)}+\frac{4096a^4(dx)^{\frac{3}{2}}\sqrt{a^2+2abx^2+b^2x^4}}{33649d}+\frac{512a^3(dx)^{\frac{3}{2}}(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}{4807d}+\frac{128a^2(dx)^{\frac{3}{2}}(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{1311d}+\frac{40a(dx)^{\frac{3}{2}}(a+bx^2)(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{437d}+\frac{2(dx)^{\frac{3}{2}}(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{23d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)*(d*x)**(1/2),x)
```

```
[Out] 16384*a**5*(d*x)**(3/2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1009
47*d*(a + b*x**2)) + 4096*a**4*(d*x)**(3/2)*sqrt(a**2 + 2*a*b*x**
2 + b**2*x**4)/(33649*d) + 512*a**3*(d*x)**(3/2)*(a + b*x**2)*sqr
t(a**2 + 2*a*b*x**2 + b**2*x**4)/(4807*d) + 128*a**2*(d*x)**(3/2)
*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(1311*d) + 40*a*(d*x)**(3
/2)*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(437*d) +
 2*(d*x)**(3/2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(23*d)
```

Mathematica [A] time = 0.0467687, size = 88, normalized size = 0.3

$$\frac{2\sqrt{dx}\sqrt{(a+bx^2)^2}\left(33649a^5x+72105a^4bx^3+91770a^3b^2x^5+67298a^2b^3x^7+26565ab^4x^9+4389b^5x^{11}\right)}{100947(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[(a + b*x^2)^2]*(33649*a^5*x + 72105*a^4*b*x^3 +
 91770*a^3*b^2*x^5 + 67298*a^2*b^3*x^7 + 26565*a*b^4*x^9 + 4389*b
^5*x^11))/(100947*(a + b*x^2))
```

Maple [A] time = 0.009, size = 83, normalized size = 0.3

$$\frac{2x\left(4389\,b^5x^{10}+26565\,ab^4x^8+67298\,a^2b^3x^6+91770\,a^3b^2x^4+72105\,a^4bx^2+33649\,a^5\right)}{100947\,(bx^2+a)^5}\left((bx^2+a)^2\right)^{\frac{5}{2}}\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)*(d*x)^(1/2),x)
```

```
[Out] 2/100947*x*(4389*b^5*x^10+26565*a*b^4*x^8+67298*a^2*b^3*x^6+91770
*a^3*b^2*x^4+72105*a^4*b*x^2+33649*a^5)*((b*x^2+a)^2)^(5/2)*(d*x)
^(1/2)/(b*x^2+a)^5
```

Maxima [A] time = 0.717625, size = 198, normalized size = 0.67

$$\frac{2}{437}\left(19\,b^5\sqrt{d}x^3+23\,ab^4\sqrt{d}x\right)x^{\frac{17}{2}}+\frac{8}{285}\left(15\,ab^4\sqrt{d}x^3+19\,a^2b^3\sqrt{d}x\right)x^{\frac{13}{2}}+\frac{4}{55}\left(11\,a^2b^3\sqrt{d}x^3+15\,a^3b^2\sqrt{d}x\right)x^{\frac{9}{2}}+\frac{8}{77}\left(7\,a^3b^2\sqrt{d}x^3+11\,a^4b\sqrt{d}x\right)x^{\frac{5}{2}}+\frac{2}{21}\left(3\,a^4b\sqrt{d}x^3+7\,a^5\sqrt{d}x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/437*(19*b^5*sqrt(d)*x^3 + 23*a*b^4*sqrt(d)*x)*x^(17/2) + 8/285*
(15*a*b^4*sqrt(d)*x^3 + 19*a^2*b^3*sqrt(d)*x)*x^(13/2) + 4/55*(11
*a^2*b^3*sqrt(d)*x^3 + 15*a^3*b^2*sqrt(d)*x)*x^(9/2) + 8/77*(7*a^
3*b^2*sqrt(d)*x^3 + 11*a^4*b*sqrt(d)*x)*x^(5/2) + 2/21*(3*a^4*b*s
qrt(d)*x^3 + 7*a^5*sqrt(d)*x)*sqrt(x)
```

Fricas [A] time = 0.269025, size = 84, normalized size = 0.28

$$\frac{2}{100947}\left(4389\,b^5x^{11}+26565\,ab^4x^9+67298\,a^2b^3x^7+91770\,a^3b^2x^5+72105\,a^4bx^3+33649\,a^5x\right)\sqrt{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/100947*(4389*b^5*x^11 + 26565*a*b^4*x^9 + 67298*a^2*b^3*x^7 + 9
1770*a^3*b^2*x^5 + 72105*a^4*b*x^3 + 33649*a^5*x)*sqrt(d*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)*(d*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.267834, size = 194, normalized size = 0.65

$$\frac{2\left(4389\sqrt{dx}b^5dx^{11}\text{sign}\left(bx^2+a\right)+26565\sqrt{dx}ab^4dx^9\text{sign}\left(bx^2+a\right)+67298\sqrt{dx}a^2b^3dx^7\text{sign}\left(bx^2+a\right)+91770\sqrt{dx}a^3b^2\right.}{100947\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/100947*(4389*sqrt(d*x)*b^5*d*x^11*sign(b*x^2 + a) + 26565*sqrt(
d*x)*a*b^4*d*x^9*sign(b*x^2 + a) + 67298*sqrt(d*x)*a^2*b^3*d*x^7*
sign(b*x^2 + a) + 91770*sqrt(d*x)*a^3*b^2*d*x^5*sign(b*x^2 + a) +
 72105*sqrt(d*x)*a^4*b*d*x^3*sign(b*x^2 + a) + 33649*sqrt(d*x)*a^
5*d*x*sign(b*x^2 + a))/d
```

3.745 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^5\left(a+bx^2\right)}$$

```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2))
 + (2*a^4*b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a
+ b*x^2)) + (20*a^3*b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(9*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^(17/2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^2)) + (2*b^5*(d
*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*x^2))
```

Rubi [A] time = 0.229649, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{21/2}\sqrt{a^2+2abx^2+b^2x^4}}{21d^{11}\left(a+bx^2\right)}+\frac{10ab^4(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^9\left(a+bx^2\right)}+\frac{20a^2b^3(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^7\left(a+bx^2\right)}+\frac{2a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d\left(a+bx^2\right)}+\frac{2a^4b(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^3\left(a+bx^2\right)}+\frac{20a^3b^2(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^5\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```


```
[Out] (2*a^5*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(a + b*x^2))
 + (2*a^4*b*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*(a
+ b*x^2)) + (20*a^3*b^2*(d*x)^(9/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(9*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(13/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(13*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^(17/2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^9*(a + b*x^2)) + (2*b^5*(d
*x)^(21/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(21*d^11*(a + b*x^2))
```

Rubi in Sympy [A] time = 29.3908, size = 238, normalized size = 0.81

$$\frac{16384a^5\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{13923d\,(a+bx^2)}+\frac{4096a^4\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{13923d}+\frac{2560a^3\sqrt{dx}\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}{13923d}+\frac{640a^2\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{4641d}+\frac{40a\sqrt{dx}\,(a+bx^2)\,(a^2+2abx^2+b^2x^4)^{\frac{3}{2}}}{357d}+\frac{2\sqrt{dx}\,(a^2+2abx^2+b^2x^4)^{\frac{5}{2}}}{21d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] 16384*a**5*sqrt(d*x)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(13923*d
*(a + b*x**2)) + 4096*a**4*sqrt(d*x)*sqrt(a**2 + 2*a*b*x**2 + b**
2*x**4)/(13923*d) + 2560*a**3*sqrt(d*x)*(a + b*x**2)*sqrt(a**2 +
2*a*b*x**2 + b**2*x**4)/(13923*d) + 640*a**2*sqrt(d*x)*(a**2 + 2*
a*b*x**2 + b**2*x**4)**(3/2)/(4641*d) + 40*a*sqrt(d*x)*(a + b*x**
2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(357*d) + 2*sqrt(d*x)*(
a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(21*d)
```

Mathematica [A] time = 0.0453342, size = 88, normalized size = 0.3

$$\frac{2\sqrt{(a+bx^2)^2}\,\left(13923a^5x+13923a^4bx^3+15470a^3b^2x^5+10710a^2b^3x^7+4095ab^4x^9+663b^5x^{11}\right)}{13923\sqrt{dx}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[(a + b*x^2)^2]*(13923*a^5*x + 13923*a^4*b*x^3 + 15470*a^3
*b^2*x^5 + 10710*a^2*b^3*x^7 + 4095*a*b^4*x^9 + 663*b^5*x^11))/(1
3923*Sqrt[d*x]*(a + b*x^2))
```

Maple [A] time = 0.008, size = 83, normalized size = 0.3

$$\frac{2\,\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+13923\,a^5\right)x}{13923\,(bx^2+a)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 2/13923*x*(663*b^5*x^10+4095*a*b^4*x^8+10710*a^2*b^3*x^6+15470*a^
3*b^2*x^4+13923*a^4*b*x^2+13923*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a
)^5/(d*x)^(1/2)
```

Maxima [A] time = 0.723626, size = 204, normalized size = 0.7

$$\frac{2\left(195\left(17\,b^5\sqrt{d}x^3+21\,ab^4\sqrt{d}x\right)x^{\frac{15}{2}}+1260\left(13\,ab^4\sqrt{d}x^3+17\,a^2b^3\sqrt{d}x\right)x^{\frac{11}{2}}+3570\left(9\,a^2b^3\sqrt{d}x^3+13\,a^3b^2\sqrt{d}x\right)x^{\frac{7}{2}}+6188\right.}{69615\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] 2/69615*(195*(17*b^5*sqrt(d)*x^3 + 21*a*b^4*sqrt(d)*x)*x^(15/2) +
 1260*(13*a*b^4*sqrt(d)*x^3 + 17*a^2*b^3*sqrt(d)*x)*x^(11/2) + 35
70*(9*a^2*b^3*sqrt(d)*x^3 + 13*a^3*b^2*sqrt(d)*x)*x^(7/2) + 6188*
(5*a^3*b^2*sqrt(d)*x^3 + 9*a^4*b*sqrt(d)*x)*x^(3/2) + 13923*(a^4*
b*sqrt(d)*x^3 + 5*a^5*sqrt(d)*x)/sqrt(x))/d
```

Fricas [A] time = 0.270794, size = 86, normalized size = 0.29

$$\frac{2\left(663\,b^5x^{10}+4095\,ab^4x^8+10710\,a^2b^3x^6+15470\,a^3b^2x^4+13923\,a^4bx^2+13923\,a^5\right)\sqrt{dx}}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] 2/13923*(663*b^5*x^10 + 4095*a*b^4*x^8 + 10710*a^2*b^3*x^6 + 1547
0*a^3*b^2*x^4 + 13923*a^4*b*x^2 + 13923*a^5)*sqrt(d*x)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269203, size = 185, normalized size = 0.63

$$\frac{2\left(663\,\sqrt{dx}b^{5}x^{10}\mathrm{sign}\left(bx^{2}+a\right)+4095\,\sqrt{dx}ab^{4}x^{8}\mathrm{sign}\left(bx^{2}+a\right)+10710\,\sqrt{dx}a^{2}b^{3}x^{6}\mathrm{sign}\left(bx^{2}+a\right)+15470\,\sqrt{dx}a^{3}b^{2}x^{4}\mathrm{sign}\right.}{13923\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/sqrt(d*x),x, algorithm="giac")
```

```
[Out] 2/13923*(663*sqrt(d*x)*b^5*x^10*sign(b*x^2 + a) + 4095*sqrt(d*x)*
a*b^4*x^8*sign(b*x^2 + a) + 10710*sqrt(d*x)*a^2*b^3*x^6*sign(b*x^
2 + a) + 15470*sqrt(d*x)*a^3*b^2*x^4*sign(b*x^2 + a) + 13923*sqrt
(d*x)*a^4*b*x^2*sign(b*x^2 + a) + 13923*sqrt(d*x)*a^5*sign(b*x^2
+ a))/d
```

3.746 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{3/2}} dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}(a+bx^2)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}(a+bx^2)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3(a+bx^2)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5(a+bx^2)}$$

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)
) + (10*a^4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3
*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(7*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2)) + (2*a*b^4*(d*x)^(15/
2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*x^2)) + (2*b^5*
(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a + b*x^2
))
```

Rubi [A] time = 0.224748, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{19/2}\sqrt{a^2+2abx^2+b^2x^4}}{19d^{11}(a+bx^2)}+\frac{2ab^4(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^7(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{d\sqrt{dx}(a+bx^2)}+\frac{10a^4b(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^3(a+bx^2)}+\frac{20a^3b^2(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```


```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*Sqrt[d*x]*(a + b*x^2)
) + (10*a^4*b*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^3
*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(7*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(11/2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(11*d^7*(a + b*x^2)) + (2*a*b^4*(d*x)^(15/
2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d^9*(a + b*x^2)) + (2*b^5*
(d*x)^(19/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(19*d^11*(a + b*x^2
))
```

Rubi in Sympy [A] time = 29.1336, size = 238, normalized size = 0.81

$$-\frac{16384a^5\sqrt{a^2+2abx^2+b^2x^4}}{4389d\sqrt{dx}\,(a+bx^2)}+\frac{4096a^4\sqrt{a^2+2abx^2+b^2x^4}}{4389d\sqrt{dx}}+\frac{512a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{1463d\sqrt{dx}}+\frac{128a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{627d\sqrt{dx}}+\frac{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{57d\sqrt{dx}}+\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{19d\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

```
[Out] -16384*a**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4389*d*sqrt(d*x)
*(a + b*x**2)) + 4096*a**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(4
389*d*sqrt(d*x)) + 512*a**3*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 +
 b**2*x**4)/(1463*d*sqrt(d*x)) + 128*a**2*(a**2 + 2*a*b*x**2 + b*
*2*x**4)**(3/2)/(627*d*sqrt(d*x)) + 8*a*(a + b*x**2)*(a**2 + 2*a*
b*x**2 + b**2*x**4)**(3/2)/(57*d*sqrt(d*x)) + 2*(a**2 + 2*a*b*x**
2 + b**2*x**4)**(5/2)/(19*d*sqrt(d*x))
```

Mathematica [A] time = 0.0525559, size = 88, normalized size = 0.3

$$\frac{2x\sqrt{(a+bx^2)^2}\left(-4389a^5+7315a^4bx^2+6270a^3b^2x^4+3990a^2b^3x^6+1463ab^4x^8+231b^5x^{10}\right)}{4389(dx)^{3/2}\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-4389*a^5 + 7315*a^4*b*x^2 + 6270*a^3*b
^2*x^4 + 3990*a^2*b^3*x^6 + 1463*a*b^4*x^8 + 231*b^5*x^10))/(4389
*(d*x)^(3/2)*(a + b*x^2))
```

Maple [A] time = 0.009, size = 83, normalized size = 0.3

$$-\frac{2\left(-231\,b^5x^{10}-1463\,ab^4x^8-3990\,a^2b^3x^6-6270\,a^3b^2x^4-7315\,a^4bx^2+4389\,a^5\right)x}{4389\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(3/2),x)
```

```
[Out] -2/4389*x*(-231*b^5*x^10-1463*a*b^4*x^8-3990*a^2*b^3*x^6-6270*a^3
*b^2*x^4-7315*a^4*b*x^2+4389*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5
/(d*x)^(3/2)
```

Maxima [A] time = 0.721828, size = 204, normalized size = 0.69

$$\frac{2\left(77\left(15\,b^5\sqrt{d}x^3+19\,ab^4\sqrt{d}x\right)x^{\frac{13}{2}}+532\left(11\,ab^4\sqrt{d}x^3+15\,a^2b^3\sqrt{d}x\right)x^{\frac{9}{2}}+1710\left(7\,a^2b^3\sqrt{d}x^3+11\,a^3b^2\sqrt{d}x\right)x^{\frac{5}{2}}+4180\left(3\right.\right.}{21945\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21945*(77*(15*b^5*sqrt(d)*x^3 + 19*a*b^4*sqrt(d)*x)*x^(13/2) +
532*(11*a*b^4*sqrt(d)*x^3 + 15*a^2*b^3*sqrt(d)*x)*x^(9/2) + 1710*
(7*a^2*b^3*sqrt(d)*x^3 + 11*a^3*b^2*sqrt(d)*x)*x^(5/2) + 4180*(3*
a^3*b^2*sqrt(d)*x^3 + 7*a^4*b*sqrt(d)*x)*sqrt(x) + 7315*(a^4*b*sq
rt(d)*x^3 - 3*a^5*sqrt(d)*x)/x^(3/2))/d^2
```

Fricas [A] time = 0.270393, size = 86, normalized size = 0.29

$$\frac{2\left(231\,b^5x^{10}+1463\,ab^4x^8+3990\,a^2b^3x^6+6270\,a^3b^2x^4+7315\,a^4bx^2-4389\,a^5\right)}{4389\,\sqrt{dx}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/4389*(231*b^5*x^10 + 1463*a*b^4*x^8 + 3990*a^2*b^3*x^6 + 6270*a
^3*b^2*x^4 + 7315*a^4*b*x^2 - 4389*a^5)/(sqrt(d*x)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.269237, size = 211, normalized size = 0.72

$$-\frac{2\left(\frac{4389\,a^{5}\operatorname{sign}\left(bx^{2}+a\right)}{\sqrt{dx}}-\frac{231\,\sqrt{dx}b^{5}d^{189}x^{9}\operatorname{sign}\left(bx^{2}+a\right)+1463\,\sqrt{dx}ab^{4}d^{189}x^{7}\operatorname{sign}\left(bx^{2}+a\right)+3990\,\sqrt{dx}a^{2}b^{3}d^{189}x^{5}\operatorname{sign}\left(bx^{2}+a\right)+6270\,\sqrt{dx}a^{3}b^{2}d^{189}x^{3}\operatorname{sign}(}{d^{190}}\right.}{4389\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/4389*(4389*a^5*sign(b*x^2 + a)/sqrt(d*x) - (231*sqrt(d*x)*b^5*
d^189*x^9*sign(b*x^2 + a) + 1463*sqrt(d*x)*a*b^4*d^189*x^7*sign(b
*x^2 + a) + 3990*sqrt(d*x)*a^2*b^3*d^189*x^5*sign(b*x^2 + a) + 62
70*sqrt(d*x)*a^3*b^2*d^189*x^3*sign(b*x^2 + a) + 7315*sqrt(d*x)*a
^4*b*d^189*x*sign(b*x^2 + a))/d^190)/d
```

3.747 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=293

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}(a+bx^2)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(a+bx^2)}$$

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*
x^2)) + (10*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3
*(a + b*x^2)) + (4*a^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^(13/2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2)) + (2*b^5*(d*
x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x^2))
```

Rubi [A] time = 0.22698, antiderivative size = 293, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{17/2}\sqrt{a^2+2abx^2+b^2x^4}}{17d^{11}(a+bx^2)}+\frac{10ab^4(dx)^{13/2}\sqrt{a^2+2abx^2+b^2x^4}}{13d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{9/2}\sqrt{a^2+2abx^2+b^2x^4}}{9d^7(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{3d(dx)^{3/2}(a+bx^2)}+\frac{10a^4b\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}{d^3(a+bx^2)}+\frac{4a^3b^2(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}{d^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```


```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(d*x)^(3/2)*(a + b*
x^2)) + (10*a^4*b*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3
*(a + b*x^2)) + (4*a^3*b^2*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])/(d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(9/2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(9*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^(13/2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(13*d^9*(a + b*x^2)) + (2*b^5*(d*
x)^(17/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(17*d^11*(a + b*x^2))
```

Rubi in Sympy [A] time = 28.9581, size = 238, normalized size = 0.81

$$-\frac{16384a^5\sqrt{a^2+2abx^2+b^2x^4}}{1989d\,(dx)^{\frac{3}{2}}\,(a+bx^2)}+\frac{4096a^4\sqrt{a^2+2abx^2+b^2x^4}}{663d\,(dx)^{\frac{3}{2}}}+\frac{512a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{663d\,(dx)^{\frac{3}{2}}}+\frac{640a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{1989d\,(dx)^{\frac{3}{2}}}+\frac{40a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{221d\,(dx)^{\frac{3}{2}}}+\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{17d\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

```
[Out] -16384*a**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1989*d*(d*x)**(3
/2)*(a + b*x**2)) + 4096*a**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)
/(663*d*(d*x)**(3/2)) + 512*a**3*(a + b*x**2)*sqrt(a**2 + 2*a*b*x
**2 + b**2*x**4)/(663*d*(d*x)**(3/2)) + 640*a**2*(a**2 + 2*a*b*x*
*2 + b**2*x**4)**(3/2)/(1989*d*(d*x)**(3/2)) + 40*a*(a + b*x**2)*
(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(221*d*(d*x)**(3/2)) + 2*(
a**2 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(17*d*(d*x)**(3/2))
```

Mathematica [A] time = 0.0519995, size = 88, normalized size = 0.3

$$\frac{2x\sqrt{(a+bx^2)^2}\left(-663a^5+9945a^4bx^2+3978a^3b^2x^4+2210a^2b^3x^6+765ab^4x^8+117b^5x^{10}\right)}{1989(dx)^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(5/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-663*a^5 + 9945*a^4*b*x^2 + 3978*a^3*b^
2*x^4 + 2210*a^2*b^3*x^6 + 765*a*b^4*x^8 + 117*b^5*x^10))/(1989*(
d*x)^(5/2)*(a + b*x^2))
```

Maple [A] time = 0.009, size = 83, normalized size = 0.3

$$-\frac{2\left(-117\,b^5x^{10}-765\,ab^4x^8-2210\,a^2b^3x^6-3978\,a^3b^2x^4-9945\,a^4bx^2+663\,a^5\right)x}{1989\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(5/2),x)
```

```
[Out] -2/1989*x*(-117*b^5*x^10-765*a*b^4*x^8-2210*a^2*b^3*x^6-3978*a^3*
b^2*x^4-9945*a^4*b*x^2+663*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(
d*x)^(5/2)
```

Maxima [A] time = 0.712217, size = 204, normalized size = 0.7

$$\frac{2\left(45\left(13\,b^{5}\sqrt{d}x^{3}+17\,ab^{4}\sqrt{d}x\right)x^{\frac{11}{2}}+340\left(9\,ab^{4}\sqrt{d}x^{3}+13\,a^{2}b^{3}\sqrt{d}x\right)x^{\frac{7}{2}}+1326\left(5\,a^{2}b^{3}\sqrt{d}x^{3}+9\,a^{3}b^{2}\sqrt{d}x\right)x^{\frac{3}{2}}+\frac{7956\left(a^{3}b^{2}\sqrt{d}\ldots\right.}{\ldots}\ldots\right.}{9945\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9945*(45*(13*b^5*sqrt(d)*x^3 + 17*a*b^4*sqrt(d)*x)*x^(11/2) + 3
40*(9*a*b^4*sqrt(d)*x^3 + 13*a^2*b^3*sqrt(d)*x)*x^(7/2) + 1326*(5
*a^2*b^3*sqrt(d)*x^3 + 9*a^3*b^2*sqrt(d)*x)*x^(3/2) + 7956*(a^3*b
^2*sqrt(d)*x^3 + 5*a^4*b*sqrt(d)*x)/sqrt(x) + 3315*(3*a^4*b*sqrt(
d)*x^3 - a^5*sqrt(d)*x)/x^(5/2))/d^3
```

Fricas [A] time = 0.271358, size = 90, normalized size = 0.31

$$\frac{2\left(117\,b^{5}x^{10}+765\,ab^{4}x^{8}+2210\,a^{2}b^{3}x^{6}+3978\,a^{3}b^{2}x^{4}+9945\,a^{4}bx^{2}-663\,a^{5}\right)}{1989\,\sqrt{dx}d^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1989*(117*b^5*x^10 + 765*a*b^4*x^8 + 2210*a^2*b^3*x^6 + 3978*a^
3*b^2*x^4 + 9945*a^4*b*x^2 - 663*a^5)/(sqrt(d*x)*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(5/2),x)
```

[Out] Timed out

GIAC/XCAS [A] time = 0.27225, size = 215, normalized size = 0.73

$$-\frac{2\left(\frac{663\,a^5d\mathrm{sign}\left(bx^2+a\right)}{\sqrt{dx}x}-\frac{117\,\sqrt{dx}b^5d^{136}x^8\mathrm{sign}\left(bx^2+a\right)+765\,\sqrt{dx}ab^4d^{136}x^6\mathrm{sign}\left(bx^2+a\right)+2210\,\sqrt{dx}a^2b^3d^{136}x^4\mathrm{sign}\left(bx^2+a\right)+3978\,\sqrt{dx}a^3b^2d^{136}x^2\mathrm{sign}\left(b\right.}{d^{136}}\right.}{1989\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] -2/1989*(663*a^5*d*sign(b*x^2 + a)/(sqrt(d*x)*x) - (117*sqrt(d*x)
*b^5*d^136*x^8*sign(b*x^2 + a) + 765*sqrt(d*x)*a*b^4*d^136*x^6*si
gn(b*x^2 + a) + 2210*sqrt(d*x)*a^2*b^3*d^136*x^4*sign(b*x^2 + a)
+ 3978*sqrt(d*x)*a^3*b^2*d^136*x^2*sign(b*x^2 + a) + 9945*sqrt(d*
x)*a^4*b*d^136*sign(b*x^2 + a))/d^136)/d^3
```

3.748 $\int \frac{(a^2+2abx^2+b^2x^4)^{5/2}}{(dx)^{7/2}}\,dx$

Optimal. Leaf size=295

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}(a+bx^2)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}(a+bx^2)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5(a+bx^2)}$$

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*
x^2)) - (10*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]
*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(3*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^(11/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*x^2)) + (2*b^5*
(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a + b*x^2
))
```

Rubi [A] time = 0.229594, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{2b^5(dx)^{15/2}\sqrt{a^2+2abx^2+b^2x^4}}{15d^{11}(a+bx^2)}+\frac{10ab^4(dx)^{11/2}\sqrt{a^2+2abx^2+b^2x^4}}{11d^9(a+bx^2)}+\frac{20a^2b^3(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}{7d^7(a+bx^2)}-\frac{2a^5\sqrt{a^2+2abx^2+b^2x^4}}{5d(dx)^{5/2}(a+bx^2)}-\frac{10a^4b\sqrt{a^2+2abx^2+b^2x^4}}{d^3\sqrt{dx}(a+bx^2)}+\frac{20a^3b^2(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^5(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (-2*a^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d*(d*x)^(5/2)*(a + b*
x^2)) - (10*a^4*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3*Sqrt[d*x]
*(a + b*x^2)) + (20*a^3*b^2*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4])/(3*d^5*(a + b*x^2)) + (20*a^2*b^3*(d*x)^(7/2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])/(7*d^7*(a + b*x^2)) + (10*a*b^4*(d*x)^(11/2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(11*d^9*(a + b*x^2)) + (2*b^5*
(d*x)^(15/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(15*d^11*(a + b*x^2
))
```

Rubi in Sympy [A] time = 28.897, size = 238, normalized size = 0.81

$$\frac{16384a^5\sqrt{a^2+2abx^2+b^2x^4}}{1155d\,(dx)^{\frac{5}{2}}\,(a+bx^2)}-\frac{4096a^4\sqrt{a^2+2abx^2+b^2x^4}}{231d\,(dx)^{\frac{5}{2}}}+\frac{512a^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{231d\,(dx)^{\frac{5}{2}}}+\frac{128a^2\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{231d\,(dx)^{\frac{5}{2}}}+\frac{8a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{33d\,(dx)^{\frac{5}{2}}}+\frac{2\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{15d\,(dx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

```
[Out] 16384*a**5*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(1155*d*(d*x)**(5/
2)*(a + b*x**2)) - 4096*a**4*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/
(231*d*(d*x)**(5/2)) + 512*a**3*(a + b*x**2)*sqrt(a**2 + 2*a*b*x*
*2 + b**2*x**4)/(231*d*(d*x)**(5/2)) + 128*a**2*(a**2 + 2*a*b*x**
2 + b**2*x**4)**(3/2)/(231*d*(d*x)**(5/2)) + 8*a*(a + b*x**2)*(a*
*2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(33*d*(d*x)**(5/2)) + 2*(a**2
 + 2*a*b*x**2 + b**2*x**4)**(5/2)/(15*d*(d*x)**(5/2))
```

Mathematica [A] time = 0.0504978, size = 88, normalized size = 0.3

$$\frac{2x\sqrt{(a+bx^2)^2}\left(-231a^5-5775a^4bx^2+3850a^3b^2x^4+1650a^2b^3x^6+525ab^4x^8+77b^5x^{10}\right)}{1155(dx)^{7/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)/(d*x)^(7/2),x]
```

```
[Out] (2*x*Sqrt[(a + b*x^2)^2]*(-231*a^5 - 5775*a^4*b*x^2 + 3850*a^3*b^
2*x^4 + 1650*a^2*b^3*x^6 + 525*a*b^4*x^8 + 77*b^5*x^10))/(1155*(d
*x)^(7/2)*(a + b*x^2))
```

Maple [A] time = 0.009, size = 83, normalized size = 0.3

$$-\frac{2\left(-77\,b^5x^{10}-525\,ab^4x^8-1650\,a^2b^3x^6-3850\,a^3b^2x^4+5775\,a^4bx^2+231\,a^5\right)x}{1155\,\left(bx^2+a\right)^5}\left(\left(bx^2+a\right)^2\right)^{\frac{5}{2}}(dx)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(7/2),x)
```

```
[Out] -2/1155*x*(-77*b^5*x^10-525*a*b^4*x^8-1650*a^2*b^3*x^6-3850*a^3*b
^2*x^4+5775*a^4*b*x^2+231*a^5)*((b*x^2+a)^2)^(5/2)/(b*x^2+a)^5/(d
*x)^(7/2)
```

Maxima [A] time = 0.724518, size = 203, normalized size = 0.69

$$\frac{2\left(7\left(11\,b^{5}\sqrt{d}x^{3}+15\,ab^{4}\sqrt{d}x\right)x^{\frac{9}{2}}+60\left(7\,ab^{4}\sqrt{d}x^{3}+11\,a^{2}b^{3}\sqrt{d}x\right)x^{\frac{5}{2}}+330\left(3\,a^{2}b^{3}\sqrt{d}x^{3}+7\,a^{3}b^{2}\sqrt{d}x\right)\sqrt{x}+\frac{1540\left(a^{3}b^{2}\sqrt{d}x^{3}-\right.}{x^{\frac{3}{2}}}\right.}{1155\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(7*(11*b^5*sqrt(d)*x^3 + 15*a*b^4*sqrt(d)*x)*x^(9/2) + 60*
(7*a*b^4*sqrt(d)*x^3 + 11*a^2*b^3*sqrt(d)*x)*x^(5/2) + 330*(3*a^2
*b^3*sqrt(d)*x^3 + 7*a^3*b^2*sqrt(d)*x)*sqrt(x) + 1540*(a^3*b^2*s
qrt(d)*x^3 - 3*a^4*b*sqrt(d)*x)/x^(3/2) - 231*(5*a^4*b*sqrt(d)*x^
3 + a^5*sqrt(d)*x)/x^(7/2))/d^4
```

Fricas [A] time = 0.273514, size = 90, normalized size = 0.31

$$\frac{2\left(77\,b^{5}x^{10}+525\,ab^{4}x^{8}+1650\,a^{2}b^{3}x^{6}+3850\,a^{3}b^{2}x^{4}-5775\,a^{4}bx^{2}-231\,a^{5}\right)}{1155\,\sqrt{dx}d^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*b^5*x^10 + 525*a*b^4*x^8 + 1650*a^2*b^3*x^6 + 3850*a^3
*b^2*x^4 - 5775*a^4*b*x^2 - 231*a^5)/(sqrt(d*x)*d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(7/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271414, size = 219, normalized size = 0.74

$$-\frac{2\left(\frac{231\left(25\,a^4bd^3x^2\mathrm{sign}\left(bx^2+a\right)+a^5d^3\mathrm{sign}\left(bx^2+a\right)\right)}{\sqrt{dx}d^2x^2}-\frac{77\,\sqrt{dx}b^5d^{105}x^7\,\mathrm{sign}\left(bx^2+a\right)+525\,\sqrt{dx}ab^4d^{105}x^5\mathrm{sign}\left(bx^2+a\right)+1650\,\sqrt{dx}a^2b^3d^{105}x^3\mathrm{sign}\left(bx^2+a\right)+}{d^{105}}}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)/(d*x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/1155*(231*(25*a^4*b*d^3*x^2*sign(b*x^2 + a) + a^5*d^3*sign(b*x
^2 + a))/(sqrt(d*x)*d^2*x^2) - (77*sqrt(d*x)*b^5*d^105*x^7*sign(b
*x^2 + a) + 525*sqrt(d*x)*a*b^4*d^105*x^5*sign(b*x^2 + a) + 1650*
sqrt(d*x)*a^2*b^3*d^105*x^3*sign(b*x^2 + a) + 3850*sqrt(d*x)*a^3*
b^2*d^105*x*sign(b*x^2 + a))/d^105)/d^4
```

3.749 $\int \frac{(dx)^{7/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=457

$$-\frac{2ad^3\sqrt{dx}\,(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\,(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (2*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*(a + b*x^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(9/4)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
a^(5/4)*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.778857, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{2ad^3\sqrt{dx}\,(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{2d(dx)^{5/2}\,(a+bx^2)}{5b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{5/4}d^{7/2}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{5/4}d^{7/2}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (-2*a*d^3*Sqrt[d*x]*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (2*d*(d*x)^(5/2)*(a + b*x^2))/(5*b*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (a^(5/4)*d^(7/2)*(a + b*x^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(9/4)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(5/4)*d^(7/2)*(a + b*x^2)*Log
[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sq
rt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
a^(5/4)*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(9/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.184803, size = 238, normalized size = 0.52

$$\frac{d^3\sqrt{dx}\left(a+bx^2\right)\left(-5\sqrt{2}a^{5/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}a^{5/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-10\sqrt{2}a^{5/4}\tan^{-1}\left(\cdots\right.\right.}{20b^{9/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(7/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (d^3*Sqrt[d*x]*(a + b*x^2)*(-40*a*b^(1/4)*Sqrt[x] + 8*b^(5/4)*x^(
5/2) - 10*Sqrt[2]*a^(5/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)] + 10*Sqrt[2]*a^(5/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)] - 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] + 5*Sqrt[2]*a^(5/4)*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(20*b^(9/4)*Sqrt[x]*Sqrt[(
```

```
a + b*x^2)^2])
```

Maple [A] time = 0.016, size = 241, normalized size = 0.5

$$\frac{(bx^2+a)\,d}{20\,b^2}\left(5\,ad^2\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\ln\left(-1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+10\,ad^2\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)*d*(5*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))+10*a*d^2*(a*d^2/b)^(1/4)
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))-10*a*d^2*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/
2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))+8*(d*x)^(5/2)*b-40*a*d^2*(d*
x)^(1/2))/((b*x^2+a)^2)^(1/2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(7/2)/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293704, size = 270, normalized size = 0.59

$$-\frac{20\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\arctan\left(\frac{\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2}{\sqrt{dx}\,ad^3+\sqrt{a^2d^7x+\sqrt{-\frac{a^5d^{14}}{b^9}}\,b^4}}\right)-5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log\left(\sqrt{dx}\,ad^3+\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\right)+5\left(-\frac{a^5d^{14}}{b^9}\right)^{\frac{1}{4}}b^2\log}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(7/2)/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] -1/10*(20*(-a^5*d^14/b^9)^(1/4)*b^2*arctan((-a^5*d^14/b^9)^(1/4)*
b^2/(sqrt(d*x)*a*d^3 + sqrt(a^2*d^7*x + sqrt(-a^5*d^14/b^9)*b^4))
) - 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3 + (-a^5*d^14/
b^9)^(1/4)*b^2) + 5*(-a^5*d^14/b^9)^(1/4)*b^2*log(sqrt(d*x)*a*d^3
 - (-a^5*d^14/b^9)^(1/4)*b^2) - 4*(b*d^3*x^2 - 5*a*d^3)*sqrt(d*x)
)/b^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276292, size = 374, normalized size = 0.82

$$\frac{1}{20}\,d^2\left(\frac{10\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,ad\,\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{10\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,ad\,\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^3}+\frac{5\,\sqrt{2}\,\left(ab^3d^2\right)}{}\right.$$

$$\left.+\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(7/2)/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/20*d^2*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 10*sq
rt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^3 + 5*sqrt(2)*(a*b^3*d^
2)^(1/4)*a*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/b^3 - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*ln(d*x - sqrt(2)*(a
*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^3 + 8*(sqrt(d*x)*b^4*d
^6*x^2 - 5*sqrt(d*x)*a*b^3*d^6)/(b^5*d^5))*sign(b*x^2 + a)
```

3.750 $\int \frac{(dx)^{5/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$\frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (a^(3/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*
d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.663373, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{2d(dx)^{3/2}\left(a+bx^2\right)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{a^{3/4}d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*(d*x)^(3/2)*(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (a^(3/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
```

```
t[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(7/4)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (a^(3/4)*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (a^(3/4)*
d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(7/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.144548, size = 222, normalized size = 0.54

$$\frac{(dx)^{5/2}\left(a+bx^2\right)\left(-3\sqrt{2}a^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+3\sqrt{2}a^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+6\sqrt{2}a^{3/4}\tan^{-1}\left(1\right.\right.}{12b^{7/4}x^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(8*b^(3/4)*x^(3/2) + 6*Sqrt[2]*a^(3/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 6*Sqrt[2]*a^(3/4)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 3*Sqrt[2]*a^(3/4)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 3*S
qrt[2]*a^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sq
rt[b]*x]))/(12*b^(7/4)*x^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.011, size = 220, normalized size = 0.5

$$-\frac{(bx^2+a)\,d}{12\,b^2}\left(3\,ad^2\sqrt{2}\ln\left(-1\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+6\,ad^2\sqrt{2}\arctan\left(1\left(\sqrt{2}\sqrt{\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)*d*(3*a*d^2*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2)))+6*a*d^2*2^(1/2)*arctan((2^(1/2)*(d*x)^(
1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-6*a*d^2*2^(1/2)*arctan((-2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-8*(d*x)^(3/2
)*b*(a*d^2/b)^(1/4))/((b*x^2+a)^2)^(1/2)/b^2/(a*d^2/b)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(5/2)/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292289, size = 265, normalized size = 0.64

$$\frac{4\sqrt{dx}d^2x-12\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\arctan\left(\frac{\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}}b^5}{\sqrt{dx}a^2d^7+\sqrt{a^4d^{15}x-\sqrt{-\frac{a^3d^{10}}{b^7}}a^3b^3d^{10}}}\right)-3\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}a^2d^7+\left(-\frac{a^3d^{10}}{b^7}\right)^{\frac{3}{4}}b^5\right)+3\left(\right.}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(5/2)/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(d*x)*d^2*x - 12*(-a^3*d^10/b^7)^(1/4)*b*arctan((-a^3*
d^10/b^7)^(3/4)*b^5/(sqrt(d*x)*a^2*d^7 + sqrt(a^4*d^15*x - sqrt(-
```

```
a^3*d^10/b^7)*a^3*b^3*d^10))) - 3*(-a^3*d^10/b^7)^(1/4)*b*log(sqr
t(d*x)*a^2*d^7 + (-a^3*d^10/b^7)^(3/4)*b^5) + 3*(-a^3*d^10/b^7)^(
1/4)*b*log(sqrt(d*x)*a^2*d^7 - (-a^3*d^10/b^7)^(3/4)*b^5))/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.277938, size = 325, normalized size = 0.79

$$\frac{1}{12}\left(\frac{8\sqrt{dx}dx}{b} - \frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4} - \frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4} + \frac{3\sqrt{2}\left(ab^3d^2\right.}{}\right.$$
$$\left.+ a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(5/2)/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/12*(8*sqrt(d*x)*d*x/b - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*
sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
b^4 - 6*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^4 + 3*sqrt(2)*(a*
b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/b^4 - 3*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^4)*d*sign(b*x^2 + a)
```

3.751 $\int \frac{(dx)^{3/2}}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=410

$$\frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
 (a^(1/4)*d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*
x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) - (a^(1/4)*d^(3/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2
*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

Rubi [A] time = 0.631327, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{2d\sqrt{dx}\left(a+bx^2\right)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{a}d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (2*d*Sqrt[d*x]*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
 (a^(1/4)*d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*
```

```
x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) - (a^(1/4)*d^(3/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) + (a^(1/4)*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2
*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (a^(1/4)*d^(3
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*b^(5/4)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.126342, size = 221, normalized size = 0.54

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(\sqrt{2}\sqrt[4]{a}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\sqrt{2}\sqrt[4]{a}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\sqrt{2}\sqrt[4]{a}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{\cdots}}\right.\cdots}{4b^{5/4}x^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(8*b^(1/4)*Sqrt[x] + 2*Sqrt[2]*a^(1/4)*A
rcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*a^(1/4)*
ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Sqrt[2]*a^(1/4)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Sqrt[
2]*a^(1/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x]))/(4*b^(5/4)*x^(3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.01, size = 216, normalized size = 0.5

$$-\frac{(bx^2+a)\,d}{4\,b}\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\ln\left(-1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\arctan\left(1\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*d*((a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))+2*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-2*(a*d^2/
b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))-8*(d*x)^(1/2))/((b*x^2+a)^2)^(1/2)/b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291391, size = 198, normalized size = 0.48

$$\frac{4\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\arctan\left(\frac{\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b}{\sqrt{dx}d+\sqrt{d^3x+\sqrt{-\frac{ad^6}{b^5}}b^2}}\right)-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}d+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)+\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\log\left(\sqrt{dx}d-\left(-\frac{ad^6}{b^5}\right)^{\frac{1}{4}}b\right)+}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(-a*d^6/b^5)^(1/4)*b*arctan((-a*d^6/b^5)^(1/4)*b/(sqrt(d*x
)*d + sqrt(d^3*x + sqrt(-a*d^6/b^5)*b^2))) - (-a*d^6/b^5)^(1/4)*b
```

```
*log(sqrt(d*x)*d + (-a*d^6/b^5)^(1/4)*b) + (-a*d^6/b^5)^(1/4)*b*l
og(sqrt(d*x)*d - (-a*d^6/b^5)^(1/4)*b) + 4*sqrt(d*x)*d)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276139, size = 327, normalized size = 0.8

$$-\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^2}+\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(d\right.}{}\right.$$

$$\left.+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] -1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + 2*sqrt(2)*(a
*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
2*sqrt(d*x))/(a*d^2/b)^(1/4))/b^2 + sqrt(2)*(a*b^3*d^2)^(1/4)*d*l
n(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/b^2 -
sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/b^2 - 8*sqrt(d*x)*d/b)*sign(b*x^2 + a)
```

3.752 $\int \frac{\sqrt{dx}}{\sqrt{a^2+2abx^2+b^2x^4}} dx$

Optimal. Leaf size=368

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])) + (Sqrt[d]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[
2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[d]*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.549211, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$

$$\frac{\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -((Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 +
```

```
b^2*x^4])) + (Sqrt[d]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] +
 Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[
2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[d]*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(1/4)*b^(3/4)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0900819, size = 178, normalized size = 0.48

$$\frac{\sqrt{dx}\left(a+bx^2\right)\left(\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{}}\right.\right.}{2\sqrt{2}\sqrt[4]{a}b^{3/4}\sqrt{x}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[d*x]*(a + b*x^2)*(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^
(1/4)*b^(3/4)*Sqrt[x]*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.009, size = 182, normalized size = 0.5

$$\frac{d\left(bx^2+a\right)\sqrt{2}}{4\,b}\left(\ln\left(-1\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\,\arctan\left(1\left(\sqrt{2}\sqrt{dx}+\sqrt[4]{\frac{ad^2}{b}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)*d/b/(a*d^2/b)^(1/4)*2^(1/2)*(ln
(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*arctan((2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-2*arctan((-2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/sqrt((b*x^2 + a)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292185, size = 213, normalized size = 0.58

$$2\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\arctan\left(\frac{ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}}{\sqrt{dx}d+\sqrt{-abd^2\sqrt{-\frac{d^2}{ab^3}}+d^3x}}\right)+\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right)-\frac{1}{2}\left(-\frac{d^2}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{d^2}{ab^3}\right)^{\frac{3}{4}}+\sqrt{dx}d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/sqrt((b*x^2 + a)^2),x, algorithm="fricas")
```

```
[Out] 2*(-d^2/(a*b^3))^(1/4)*arctan(a*b^2*(-d^2/(a*b^3))^(3/4)/(sqrt(d*
x)*d + sqrt(-a*b*d^2*sqrt(-d^2/(a*b^3)) + d^3*x))) + 1/2*(-d^2/(a
*b^3))^(1/4)*log(a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*d) - 1/2*
(-d^2/(a*b^3))^(1/4)*log(-a*b^2*(-d^2/(a*b^3))^(3/4) + sqrt(d*x)*
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.276468, size = 339, normalized size = 0.92

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3d}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3d}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\right.}{ab^3}\right.$$

$$+ a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/sqrt((b*x^2 + a)^2),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*d) + 2*sqrt(2)
*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) -
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*d) - sqrt(2)*(a*b^3*d^2)^(3
/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(
a*b^3*d) + sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(
1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*d))*sign(b*x^2 + a)
```

3.753 $\int \frac{1}{\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=368

$$-\frac{(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])) + ((a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(3/
4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.533048, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267

$$-\frac{(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -(((a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sq
rt[d])])/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])) + ((a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
```

```
(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - ((a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(3/
4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(3/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.0706142, size = 178, normalized size = 0.48

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(-\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}\ldots}{}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{b}\sqrt{dx}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(-2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)] + 2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + Log[Sqrt[a]
 + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2*Sqrt[2]*a^(3
/4)*b^(1/4)*Sqrt[d*x]*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.008, size = 184, normalized size = 0.5

$$\frac{\left(bx^2+a\right)\sqrt{2}}{4\,ad}\sqrt[4]{\frac{ad^2}{b}}\left(\ln\left(-1\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\,\arctan\left(1\left(\sqrt{2}\sqrt{dx}+\sqrt[4]{\ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(1/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/4/((b*x^2+a)^2)^(1/2)*(b*x^2+a)/d*(a*d^2/b)^(1/4)/a*2^(1/2)*(ln
(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))+2*arctan((2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-2*arctan((-2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.289282, size = 194, normalized size = 0.53

$$-2\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\arctan\left(\frac{ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}}{\sqrt{a^2d^2\sqrt{-\frac{1}{a^3bd^2}}+dx}+\sqrt{dx}}\right)+\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)-\frac{1}{2}\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}\log\left(-ad\left(-\frac{1}{a^3bd^2}\right)^{\frac{1}{4}}+\sqrt{dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] -2*(-1/(a^3*b*d^2))^(1/4)*arctan(a*d*(-1/(a^3*b*d^2))^(1/4)/(sqrt
(a^2*d^2*sqrt(-1/(a^3*b*d^2)) + d*x) + sqrt(d*x))) + 1/2*(-1/(a^3
*b*d^2))^(1/4)*log(a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x)) - 1/2*
(-1/(a^3*b*d^2))^(1/4)*log(-a*d*(-1/(a^3*b*d^2))^(1/4) + sqrt(d*x
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(1/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.271688, size = 339, normalized size = 0.92

$$\frac{1}{4}\left(\frac{2\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{2\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{abd}+\frac{\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\right.}{ab}\right.$$

$+ a)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] 1/4*(2*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + 2*sqrt(2)*(
a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b*d) + sqrt(2)*(a*b^3*d^2)^(1/4)*
ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b*
d) - sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*s
qrt(d*x) + sqrt(a*d^2/b))/(a*b*d))*sign(b*x^2 + a)
```

3.754 $\int \frac{1}{(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=412

$$-\frac{2(a+bx^2)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) - (b^(1/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2
]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi [A] time = 0.637513, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{2(a+bx^2)}{ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt[4]{b}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt[4]{b}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{5/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(a*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
```

```
(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) - (b^(1/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) - (b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2
]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(1/4)*(a
+ b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(5/4)*d^(3/2)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.15467, size = 232, normalized size = 0.56

$$\frac{x\left(a+bx^2\right)\left(-\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+\sqrt{2}\sqrt[4]{b}\sqrt{x}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+2\sqrt{2}\sqrt[4]{b}\sqrt{x}\tan^{-1}\left(1 -\right.\right.}{4a^{5/4}(dx)^{3/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2)*(-8*a^(1/4) + 2*Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 2*Sqrt[2]*b^(1/4)*Sqrt[x]*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - Sqrt[2]*b^(1/4)*Sq
rt[x]*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
+ Sqrt[2]*b^(1/4)*Sqrt[x]*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[x] + Sqrt[b]*x]))/(4*a^(5/4)*(d*x)^(3/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.013, size = 223, normalized size = 0.5

$$-\frac{bx^2+a}{4\,ad}\left(\sqrt{2}\sqrt{dx}\ln\left(-1\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+2\,\sqrt{2}\sqrt{dx}\arctan\left(1\left(\sqrt{2}\sqrt{dx}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)/d*(2^(1/2)*(d*x)^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2)))+2*2^(1/2)*(d*x)^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-2*2^(1/2)*(d*x)^(
1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))+8*(a*d^2/b)^(1/4))/((b*x^2+a)^2)^(1/2)/a/(d*x)^(1/2)/(a*d^2/b)
^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295164, size = 252, normalized size = 0.61

$$-\frac{4\,\sqrt{dx}ad\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\arctan\left(\frac{a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}}{\sqrt{dx}b+\sqrt{-a^3bd^4\sqrt{-\frac{b}{a^5d^6}}+b^2dx}}\right)+\sqrt{dx}ad\left(-\frac{b}{a^5d^6}\right)^{\frac{1}{4}}\log\left(a^4d^5\left(-\frac{b}{a^5d^6}\right)^{\frac{3}{4}}+\sqrt{dx}b\right)-\sqrt{dx}ad\left(-\frac{b}{a^5d^6}\right.}{2\,\sqrt{dx}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(d*x)*a*d*(-b/(a^5*d^6))^(1/4)*arctan(a^4*d^5*(-b/(a^
5*d^6))^(3/4)/(sqrt(d*x)*b + sqrt(-a^3*b*d^4*sqrt(-b/(a^5*d^6)) +
```

```
 b^2*d*x))) + sqrt(d*x)*a*d*(-b/(a^5*d^6))^(1/4)*log(a^4*d^5*(-b/
(a^5*d^6))^(3/4) + sqrt(d*x)*b) - sqrt(d*x)*a*d*(-b/(a^5*d^6))^(1
/4)*log(-a^4*d^5*(-b/(a^5*d^6))^(3/4) + sqrt(d*x)*b) + 4)/(sqrt(d
*x)*a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(3/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.278023, size = 356, normalized size = 0.86

$$-\frac{\left(\frac{8}{\sqrt{dx}a}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}+\frac{2\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^2d^2}-\frac{\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{a^2b^2d^2}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/4*(8/(sqrt(d*x)*a) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sq
rt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a
^2*b^2*d^2) + 2*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*d^
2) - sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*s
qrt(d*x) + sqrt(a*d^2/b))/(a^2*b^2*d^2) + sqrt(2)*(a*b^3*d^2)^(3/
4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^2*b^2*d^2))*sign(b*x^2 + a)/d
```

3.755 $\int \frac{1}{(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=414

$$-\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (b^(3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (b^(3/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sq
rt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.632493, antiderivative size = 414, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{2(a+bx^2)}{3ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{2\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{3/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{3/4}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{7/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(3*a*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (b^(3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])
```

```
/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) - (b^(3/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4]) + (b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d
] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sq
rt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b^(3/4)
*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(7/4)*d^(5/2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.162379, size = 233, normalized size = 0.56

$$\frac{x\left(a+bx^2\right)\left(-8a^{3/4}+3\sqrt{2}b^{3/4}x^{3/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-3\sqrt{2}b^{3/4}x^{3/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+6\sqrt{2}b^{3}\right.}{12a^{7/4}(dx)^{5/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2)*(-8*a^(3/4) + 6*Sqrt[2]*b^(3/4)*x^(3/2)*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 6*Sqrt[2]*b^(3/4)*x^(3/2)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 3*Sqrt[2]*b^(3/4)*
x^(3/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x
] - 3*Sqrt[2]*b^(3/4)*x^(3/2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x]))/(12*a^(7/4)*(d*x)^(5/2)*Sqrt[(a + b*x^2
)^2])
```

Maple [A] time = 0.013, size = 241, normalized size = 0.6

$$-\frac{bx^2+a}{12\,d^3a^2}\left(6\,b\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\,(dx)^{3/2}\arctan\left(1\left(\sqrt{2}\sqrt{dx}+\sqrt[4]{\frac{ad^2}{b}}\right)\frac{1}{\sqrt[4]{\frac{ad^2}{b}}}\right)-6\,b\sqrt[4]{\frac{ad^2}{b}}\sqrt{2}\,(dx)^{3/2}\arctan\left(1\left(-\sqrt{2}\sqrt{dx}+\sqrt[4]{\frac{ad^2}{b}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(5/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] -1/12*(b*x^2+a)/d^3*(6*b*(a*d^2/b)^(1/4)*2^(1/2)*(d*x)^(3/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-6*b*(a*
d^2/b)^(1/4)*2^(1/2)*(d*x)^(3/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))+3*b*(a*d^2/b)^(1/4)*2^(1/2)*(d*x)^
(3/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))+8*a
*d^2)/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(5/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29593, size = 286, normalized size = 0.69

$$\frac{12\,\sqrt{dx}ad^2x\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac14}\arctan\left(\frac{a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac14}}{\sqrt{dx}b+\sqrt{a^4d^6\sqrt{-\frac{b^3}{a^7d^{10}}}+b^2dx}}\right)-3\,\sqrt{dx}ad^2x\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac14}\log\left(a^2d^3\left(-\frac{b^3}{a^7d^{10}}\right)^{\frac14}+\sqrt{dx}b\right)+3\,\sqrt{dx}a}{6\,\sqrt{dx}ad^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(5/2)),x, algorithm="fricas")
```

```
[Out] 1/6*(12*sqrt(d*x)*a*d^2*x*(-b^3/(a^7*d^10))^(1/4)*arctan(a^2*d^3*
(-b^3/(a^7*d^10))^(1/4)/(sqrt(d*x)*b + sqrt(a^4*d^6*sqrt(-b^3/(a^
```

```
7*d^10)) + b^2*d*x))) - 3*sqrt(d*x)*a*d^2*x*(-b^3/(a^7*d^10))^(1/
4)*log(a^2*d^3*(-b^3/(a^7*d^10))^(1/4) + sqrt(d*x)*b) + 3*sqrt(d*
x)*a*d^2*x*(-b^3/(a^7*d^10))^(1/4)*log(-a^2*d^3*(-b^3/(a^7*d^10))
^(1/4) + sqrt(d*x)*b) - 4)/(sqrt(d*x)*a*d^2*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(5/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.273039, size = 346, normalized size = 0.84

$$-\frac{1}{12}\left(\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2d^3}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx\right.}{}\right.$$
$$\left.+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(5/2)),x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 6*sqrt(
2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*d^3) + 3*sqrt(2)*(a*b^3*d^2
)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^2*d^3) - 3*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*d^3) + 8/(sqrt(d*x)*a*
d^2*x))*sign(b*x^2 + a)
```

3.756 $\int \frac{1}{(dx)^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=459

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (2*b*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (b^(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(9/4)*d^(7/2)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.739714, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{2b\left(a+bx^2\right)}{a^2d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2\left(a+bx^2\right)}{5ad(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{2\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{\sqrt{2}a^{9/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (-2*(a + b*x^2))/(5*a*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (2*b*(a + b*x^2))/(a^2*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (b^(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqr
t[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(Sqrt[2]*a^(9/4)*d^(7/2)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b^(5/4)*(a + b*x^2)*Log[Sqrt[a]
*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]]
)/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqr
t[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(2*Sqrt[2]*a^(9/4)*d^(7/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.195032, size = 244, normalized size = 0.53

$$\frac{x\left(a+bx^2\right)\left(-8a^{5/4}+5\sqrt{2}b^{5/4}x^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-5\sqrt{2}b^{5/4}x^{5/2}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-10\sqrt{2}\cdots\right.}{20a^{9/4}(dx)^{7/2}\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (x*(a + b*x^2)*(-8*a^(5/4) + 40*a^(1/4)*b*x^2 - 10*Sqrt[2]*b^(5/4
)*x^(5/2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt
[2]*b^(5/4)*x^(5/2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + 5*Sqrt[2]*b^(5/4)*x^(5/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] - 5*Sqrt[2]*b^(5/4)*x^(5/2)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(20*a^(9/4)*(d*x)^
(7/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.017, size = 250, normalized size = 0.5

$$\frac{bx^2+a}{20\,d^3a^2}\left(5\,b\sqrt{2}\,(dx)^{5/2}\ln\left(-1\left(\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}-dx-\sqrt{\frac{ad^2}{b}}\right)\left(dx+\sqrt[4]{\frac{ad^2}{b}}\sqrt{dx}\sqrt{2}+\sqrt{\frac{ad^2}{b}}\right)^{-1}\right)+10\,b\sqrt{2}\,(dx)^{5/2}\arctan\left(1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(7/2)/((b*x^2+a)^2)^(1/2),x)
```

```
[Out] 1/20*(b*x^2+a)/d^3*(5*b*2^(1/2)*(d*x)^(5/2)*ln(-((a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))+10*b*2^(1/2)*(d*x)^(5/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))-10*b*2^(
1/2)*(d*x)^(5/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))+40*b*x^2*d^2*(a*d^2/b)^(1/4)-8*a*d^2*(a*d^2/b)^(1/
4))/((b*x^2+a)^2)^(1/2)/a^2/(d*x)^(5/2)/(a*d^2/b)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(7/2)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296786, size = 327, normalized size = 0.71

$$\frac{20\sqrt{dx}a^2d^3x^2\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^7d^{11}\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{3}{4}}}{\sqrt{dx}b^4+\sqrt{-a^5b^5d^8\sqrt{-\frac{b^5}{a^9d^{14}}}+b^8dx}}\right)+5\sqrt{dx}a^2d^3x^2\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{1}{4}}\log\left(a^7d^{11}\left(-\frac{b^5}{a^9d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}b^4\right)}{10\sqrt{dx}a^2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(20*sqrt(d*x)*a^2*d^3*x^2*(-b^5/(a^9*d^14))^(1/4)*arctan(a^7
*d^11*(-b^5/(a^9*d^14))^(3/4)/(sqrt(d*x)*b^4 + sqrt(-a^5*b^5*d^8*
sqrt(-b^5/(a^9*d^14)) + b^8*d*x))) + 5*sqrt(d*x)*a^2*d^3*x^2*(-b^
5/(a^9*d^14))^(1/4)*log(a^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d
*x)*b^4) - 5*sqrt(d*x)*a^2*d^3*x^2*(-b^5/(a^9*d^14))^(1/4)*log(-a
^7*d^11*(-b^5/(a^9*d^14))^(3/4) + sqrt(d*x)*b^4) + 20*b*x^2 - 4*a
)/(sqrt(d*x)*a^2*d^3*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(7/2)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.274727, size = 383, normalized size = 0.83

$$\frac{1}{20}\left(\frac{10\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}+\frac{10\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3bd^5}-\frac{5\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx\right.}{}\right.$$

$$\left.+\,a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt((b*x^2 + a)^2)*(d*x)^(7/2)),x, algorithm="giac")
```

```
[Out] 1/20*(10*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) + 10*sq
rt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d^5) - 5*sqrt(2)*(a*b^
3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*
d^2/b))/(a^3*b*d^5) + 5*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2
)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d^5) + 8*(5*b
*d^2*x^2 - a*d^2)/(sqrt(d*x)*a^2*d^5*x^2))*sign(b*x^2 + a)
```

3.757 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=551

$$-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117ad^7\sqrt{dx}\left(a+bx^2\right)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}\left(a+bx^2\right)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
(d*(d*x)^(13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (117*a*d^7*Sqrt[d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) + (117*d^5*(d*x)^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[
2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*d^(15
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17
/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*d^(15/2)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rubi [A] time = 0.9165, antiderivative size = 551, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{13d^3(dx)^{9/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117ad^7\sqrt{dx}(a+bx^2)}{16b^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117d^5(dx)^{5/2}(a+bx^2)}{80b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117a^{5/4}d^{15/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117a^{5/4}d^{15/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-13*d^3*(d*x)^(9/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
(d*(d*x)^(13/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (117*a*d^7*Sqrt[d*x]*(a + b*x^2))/(16*b^4*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4]) + (117*d^5*(d*x)^(5/2)*(a + b*x^2))/(80*b^3*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Arc
Tan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[
2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*d^(15
/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(32*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (117*a^(5/4)*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17
/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*a^(5/4)*d^(15/2)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.521081, size = 498, normalized size = 0.9

$$\frac{d^7\sqrt{dx}\left(-585\sqrt{2}a^{5/4}b^2x^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+585\sqrt{2}a^{5/4}b^2x^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-1170\sqrt{2}a^{9/4}b}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d^7*Sqrt[d*x]*(-4680*a^3*b^(1/4)*Sqrt[x] - 8424*a^2*b^(5/4)*x^(5
/2) - 3328*a*b^(9/4)*x^(9/2) + 256*b^(13/4)*x^(13/2) - 1170*Sqrt[
2]*a^(5/4)*(a + b*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(
1/4)] + 1170*Sqrt[2]*a^(5/4)*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*a^(13/4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1170*Sqrt[2]*a^(9/4)*b
*x^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] -
 585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*a^(13/4)*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 1170*Sqrt[2]*a^(9/4)*b*
x^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] +
585*Sqrt[2]*a^(5/4)*b^2*x^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x]))/(640*b^(17/4)*Sqrt[x]*(a + b*x^2)*Sqrt[(a
 + b*x^2)^2])
```

Maple [B] time = 0.028, size = 743, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640*(585*(a*d^2/b)^(1/4)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2
^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-
(a*d^2/b)^(1/2)))*2^(1/2)*x^4*a*b^2*d^2+1170*(a*d^2/b)^(1/4)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)
*x^4*a*b^2*d^2-1170*(a*d^2/b)^(1/4)*arctan((-2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^4*a*b^2*d^2+256*(d*x)
^(5/2)*x^4*b^3+1170*(a*d^2/b)^(1/4)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x
)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)-d*x-(a*d^2/b)^(1/2)))*2^(1/2)*x^2*a^2*b*d^2+2340*(a*d^2/b)^(
1/4)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*2^(1/2)*x^2*a^2*b*d^2-2340*(a*d^2/b)^(1/4)*arctan((-2^(1/2)*(d*
x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*x^2*a^2*b*d^2+
512*(d*x)^(5/2)*x^2*a*b^2-3840*(d*x)^(1/2)*x^4*a*b^2*d^2+585*(a*d
```

```
^2/b)^(1/4)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)
))*2^(1/2)*a^3*d^2+1170*(a*d^2/b)^(1/4)*arctan((2^(1/2)*(d*x)^(1/
2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*a^3*d^2-1170*(a*d^2/
b)^(1/4)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*2^(1/2)*a^3*d^2-744*(d*x)^(5/2)*a^2*b-7680*(d*x)^(1/2)*x^2
*a^2*b*d^2-4680*(d*x)^(1/2)*a^3*d^2)*d^5*(b*x^2+a)/b^4/((b*x^2+a)
^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298641, size = 429, normalized size = 0.78

$$-\frac{2340\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)\arctan\left(\frac{\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{1}{4}}b^{4}}{\sqrt{dx}\,ad^{7}+\sqrt{a^{2}d^{15}x+\sqrt{-\frac{a^{5}d^{30}}{b^{17}}}\,b^{8}}}\right)-585\left(-\frac{a^{5}d^{30}}{b^{17}}\right)^{\frac{1}{4}}\left(b^{6}x^{4}+2\,ab^{5}x^{2}+a^{2}b^{4}\right)\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/320*(2340*(-a^5*d^30/b^17)^(1/4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*
b^4)*arctan((-a^5*d^30/b^17)^(1/4)*b^4/(sqrt(d*x)*a*d^7 + sqrt(a^
2*d^15*x + sqrt(-a^5*d^30/b^17)*b^8))) - 585*(-a^5*d^30/b^17)^(1/
4)*(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 + 11
7*(-a^5*d^30/b^17)^(1/4)*b^4) + 585*(-a^5*d^30/b^17)^(1/4)*(b^6*x
^4 + 2*a*b^5*x^2 + a^2*b^4)*log(117*sqrt(d*x)*a*d^7 - 117*(-a^5*d
^30/b^17)^(1/4)*b^4) - 4*(32*b^3*d^7*x^6 - 416*a*b^2*d^7*x^4 - 10
53*a^2*b*d^7*x^2 - 585*a^3*d^7)*sqrt(d*x))/(b^6*x^4 + 2*a*b^5*x^2
 + a^2*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291305, size = 571, normalized size = 1.04

$$\frac{1}{640}d^6\left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^5\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{585\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/640*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^5*sig
n(b*d^4*x^2 + a*d^4)) + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(b^5*sign(b*d^4*x^2 + a*d^4)) + 585*sqrt(2)*(a*b^3*d^2)^(1/
4)*a*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(b^5*sign(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)*a
*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b
^5*sign(b*d^4*x^2 + a*d^4)) - 40*(25*sqrt(d*x)*a^2*b*d^5*x^2 + 21
*sqrt(d*x)*a^3*d^5)/((b*d^2*x^2 + a*d^2)^2*b^4*sign(b*d^4*x^2 + a
*d^4)) + 256*(sqrt(d*x)*b^12*d^6*x^2 - 15*sqrt(d*x)*a*b^11*d^6)/(
b^15*d^5*sign(b*d^4*x^2 + a*d^4)))
```

3.758 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
(d*(d*x)^(11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (77*d^5*(d*x)^(3/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) + (77*a^(3/4)*d^(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*b^(15/4)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*S
qrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.832414, antiderivative size = 504, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{11d^3(dx)^{7/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}\left(a+bx^2\right)}{48b^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77a^{3/4}d^{13/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-11*d^3*(d*x)^(7/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
(d*(d*x)^(11/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + (77*d^5*(d*x)^(3/2)*(a + b*x^2))/(48*b^3*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) + (77*a^(3/4)*d^(13/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*b^(15/4)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^(13/2)*(a + b*x^2)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*S
qrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*a^(3/4)*d^
(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) + (77*a^(3/4)*d^(13/2)*(a + b*x^2)*Log[Sqrt[
a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x
]])/(64*Sqrt[2]*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.365914, size = 476, normalized size = 0.94

$$-\frac{77a^{3/4}(dx)^{13/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}b^{15/4}x^{13/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{77a^{3/4}(dx)^{13/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}b^{15/4}x^{13/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{77a^{3/4}(dx)^{13/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}b^{15/4}x^{13/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{77a^{3/4}(dx)^{13/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}b^{15/4}x^{13/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{a^2(dx)^{13/2}\left(a+bx^2\right)}{4b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{2(dx)^{13/2}\left(a+bx^2\right)^3}{3b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{19a(dx)^{13/2}\left(a+bx^2\right)^2}{16b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a^2*(d*x)^(13/2)*(a + b*x^2))/(4*b^3*x^5*((a + b*x^2)^2)^(3/2))
 + (19*a*(d*x)^(13/2)*(a + b*x^2)^2)/(16*b^3*x^5*((a + b*x^2)^2)^
(3/2)) + (2*(d*x)^(13/2)*(a + b*x^2)^3)/(3*b^3*x^5*((a + b*x^2)^2
)^(3/2)) - (77*a^(3/4)*(d*x)^(13/2)*(a + b*x^2)^3*ArcTan[(-(Sqrt[
2]*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*
b^(15/4)*x^(13/2)*((a + b*x^2)^2)^(3/2)) - (77*a^(3/4)*(d*x)^(13/
2)*(a + b*x^2)^3*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sq
rt[2]*a^(1/4))])/(32*Sqrt[2]*b^(15/4)*x^(13/2)*((a + b*x^2)^2)^(3
/2)) - (77*a^(3/4)*(d*x)^(13/2)*(a + b*x^2)^3*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(15/4)*x^(
13/2)*((a + b*x^2)^2)^(3/2)) + (77*a^(3/4)*(d*x)^(13/2)*(a + b*x^
2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/
(64*Sqrt[2]*b^(15/4)*x^(13/2)*((a + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.028, size = 676, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/384*(256*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^4*b^3*d^2-231*2^(1/2)*ln
(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a*b^2*d
^4-462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^4*a*b^2*d^4+462*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a*b^2*d^4+456*(a*d^2/b)^(
1/4)*(d*x)^(7/2)*a*b^2+512*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^2*a*b^2*
d^2-462*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d
^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^2*a^2*b*d^4-924*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^2*b*d^4+924*2^(1/2)*arctan((-
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^2*b*d
^4+616*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^2*b*d^2-231*2^(1/2)*ln(-((a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^3*d^4-462*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a^3*d^4+462*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^3*d^4)*d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^4/((b*
x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298538, size = 429, normalized size = 0.85

$$-\frac{924\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\arctan\left(\frac{\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{3}{4}}b^{11}}{\sqrt{dx}\,a^2d^{19}+\sqrt{a^4d^{39}x-\sqrt{-\frac{a^3d^{26}}{b^{15}}}\,a^3b^7d^{26}}}\right)+231\left(-\frac{a^3d^{26}}{b^{15}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/192*(924*(-a^3*d^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b
^3)*arctan((-a^3*d^26/b^15)^(3/4)*b^11/(sqrt(d*x)*a^2*d^19 + sqrt
```

```
(a^4*d^39*x - sqrt(-a^3*d^26/b^15)*a^3*b^7*d^26))) + 231*(-a^3*d^
26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt(
d*x)*a^2*d^19 + 456533*(-a^3*d^26/b^15)^(3/4)*b^11) - 231*(-a^3*d
^26/b^15)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*log(456533*sqrt
(d*x)*a^2*d^19 - 456533*(-a^3*d^26/b^15)^(3/4)*b^11) - 4*(32*b^2*
d^6*x^5 + 121*a*b*d^6*x^3 + 77*a^2*d^6*x)*sqrt(d*x))/(b^5*x^4 + 2
*a*b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.293264, size = 524, normalized size = 1.04

$$\frac{1}{384}d^5\left(\frac{256\sqrt{dx}dx}{b^3\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{b^6\mathrm{sign}\left(bd^4x^2+ad^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*d^5*(256*sqrt(d*x)*d*x/(b^3*sign(b*d^4*x^2 + a*d^4)) - 462*
sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(
1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sign(b*d^4*x^2 + a*d^4)
) - 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*sign(b*d^4*x^2
 + a*d^4)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sign(b*d^4*x^2 + a*d^4
)) - 231*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^6*sign(b*d^4*x^2 + a*d^4)) + 24*
(19*sqrt(d*x)*a*b*d^5*x^3 + 15*sqrt(d*x)*a^2*d^5*x)/((b*d^2*x^2 +
 a*d^2)^2*b^3*sign(b*d^4*x^2 + a*d^4)))
```

3.759 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
d*(d*x)^(9/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (45*d^5*Sqrt[d*x]*(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*b^(13/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[
2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.819311, antiderivative size = 504, normalized size of antiderivative = 1., number

of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{9d^3(dx)^{5/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{a}d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^5\sqrt{dx}\left(a+bx^2\right)}{16b^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-9*d^3*(d*x)^(5/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
d*(d*x)^(9/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (45*d^5*Sqrt[d*x]*(a + b*x^2))/(16*b^3*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (45*a^(1/4)*d^(11/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*b^(13/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*(a + b*x^2)*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[
2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*a^(1/4)*d^(11/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) - (45*a^(1/4)*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(64*Sqrt[2]*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.350338, size = 474, normalized size = 0.94

$$-\frac{a^2(dx)^{11/2}\left(a+bx^2\right)}{4b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{45\sqrt[4]{a}(dx)^{11/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{45\sqrt[4]{a}(dx)^{11/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{45\sqrt[4]{a}(dx)^{11/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{45\sqrt[4]{a}(dx)^{11/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}b^{13/4}x^{11/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{2(dx)^{11/2}\left(a+bx^2\right)^3}{b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{17a(dx)^{11/2}\left(a+bx^2\right)^2}{16b^3x^5\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -(a^2*(d*x)^(11/2)*(a + b*x^2))/(4*b^3*x^5*((a + b*x^2)^2)^(3/2))
 + (17*a*(d*x)^(11/2)*(a + b*x^2)^2)/(16*b^3*x^5*((a + b*x^2)^2)^
(3/2)) + (2*(d*x)^(11/2)*(a + b*x^2)^3)/(b^3*x^5*((a + b*x^2)^2)^
(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*ArcTan[(-(Sqrt[2]
*a^(1/4)) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*b^
(13/4)*x^(11/2)*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)
*(a + b*x^2)^3*ArcTan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt
[2]*a^(1/4))])/(32*Sqrt[2]*b^(13/4)*x^(11/2)*((a + b*x^2)^2)^(3/2
)) + (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)^3*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*b^(13/4)*x^(11
/2)*((a + b*x^2)^2)^(3/2)) - (45*a^(1/4)*(d*x)^(11/2)*(a + b*x^2)
^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(6
4*Sqrt[2]*b^(13/4)*x^(11/2)*((a + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.027, size = 702, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(45*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)-d*x-(a*d^2/b)^(1/2)))*x^4*b^2*d^2+90*(a*d^2/b)^(1/4)*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4
*b^2*d^2-90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+90*(a*d^2/b)^(1/4)*
2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x
^2*a*b*d^2+180*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2-180*(a*d^2/b)^(1/
4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*x^2*a*b*d^2-256*(d*x)^(1/2)*x^4*b^2*d^2+45*(a*d^2/b)^(1/
4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))
)*a^2*d^2+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2-90*(a*d^2/b)^(1/4)*2^(1
/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*a^2*d^2-136*(d*x)^(5/2)*a*b-512*(d*x)^(1/2)*x^2*a*b*d^2-360*(d*
x)^(1/2)*a^2*d^2)*d^3*(b*x^2+a)/b^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295295, size = 383, normalized size = 0.76

$$\frac{180\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\arctan\left(\frac{\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}b^3}{\sqrt{dx}\,d^5+\sqrt{d^{11}x+\sqrt{-\frac{ad^{22}}{b^{13}}}\,b^6}}\right)-45\left(-\frac{ad^{22}}{b^{13}}\right)^{\frac{1}{4}}\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)\log\left(45\sqrt{dx}\right.}{64\,(b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(180*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)*
arctan((-a*d^22/b^13)^(1/4)*b^3/(sqrt(d*x)*d^5 + sqrt(d^11*x + sq
rt(-a*d^22/b^13)*b^6))) - 45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*
```

```
b^4*x^2 + a^2*b^3)*log(45*sqrt(d*x)*d^5 + 45*(-a*d^22/b^13)^(1/4)
*b^3) + 45*(-a*d^22/b^13)^(1/4)*(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
*log(45*sqrt(d*x)*d^5 - 45*(-a*d^22/b^13)^(1/4)*b^3) + 4*(32*b^2*
d^5*x^4 + 81*a*b*d^5*x^2 + 45*a^2*d^5)*sqrt(d*x))/(b^5*x^4 + 2*a*
b^4*x^2 + a^2*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.287295, size = 527, normalized size = 1.05

$$-\frac{1}{128}\,d^4\left(\frac{90\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{90\,\sqrt{2}\,\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^4\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{45\,\sqrt{2}\,\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^4*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^4*sign(b
*d^4*x^2 + a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*s
qrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(
b^4*sign(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(
d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sig
n(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x - s
qrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^4*sign(b*d^4
*x^2 + a*d^4)) - 256*sqrt(d*x)*d/(b^3*sign(b*d^4*x^2 + a*d^4)) -
8*(17*sqrt(d*x)*a*b*d^5*x^2 + 13*sqrt(d*x)*a^2*d^5)/((b*d^2*x^2 +
 a*d^2)^2*b^3*sign(b*d^4*x^2 + a*d^4)))
```

3.760 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
d*(d*x)^(7/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
- (21*d^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(1/4)*b^(11/4)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) - (21*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b
^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.72581, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{7d^3(dx)^{3/2}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21d^{9/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21d^{9/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-7*d^3*(d*x)^(3/2))/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
d*(d*x)^(7/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
- (21*d^(9/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^
(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(1/4)*b^(11/4)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (21*d^(9/2)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) - (21*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt
[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1/4)*b
^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.408292, size = 434, normalized size = 0.95

$$\frac{21(dx)^{9/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}x^{9/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{21(dx)^{9/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}\sqrt[4]{a}b^{11/4}x^{9/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{21(dx)^{9/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}x^{9/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{21(dx)^{9/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}\sqrt[4]{a}b^{11/4}x^{9/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{11(dx)^{9/2}\left(a+bx^2\right)^2}{16b^2x^3\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{a(dx)^{9/2}\left(a+bx^2\right)}{4b^2x^3\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a*(d*x)^(9/2)*(a + b*x^2))/(4*b^2*x^3*((a + b*x^2)^2)^(3/2)) - (
11*(d*x)^(9/2)*(a + b*x^2)^2)/(16*b^2*x^3*((a + b*x^2)^2)^(3/2))
+ (21*(d*x)^(9/2)*(a + b*x^2)^3*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^
(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*a^(1/4)*b^(11/4)*x
^(9/2)*((a + b*x^2)^2)^(3/2)) + (21*(d*x)^(9/2)*(a + b*x^2)^3*Arc
Tan[(Sqrt[2]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(32
*Sqrt[2]*a^(1/4)*b^(11/4)*x^(9/2)*((a + b*x^2)^2)^(3/2)) + (21*(d
*x)^(9/2)*(a + b*x^2)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(1/4)*b^(11/4)*x^(9/2)*((a + b*x
^2)^2)^(3/2)) - (21*(d*x)^(9/2)*(a + b*x^2)^3*Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(1/4)*b^(1
1/4)*x^(9/2)*((a + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.025, size = 609, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128*(-21*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-
(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2)))*x^4*b^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^4+42*2^(1/2)*arctan((-2^
(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^4+8
8*(a*d^2/b)^(1/4)*(d*x)^(7/2)*b^2-42*2^(1/2)*ln(-((a*d^2/b)^(1/4)
*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^4-84*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d
^4+84*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^2*a*b*d^4+56*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a*b*d^2-21
*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(
1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*
a^2*d^4-42*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*a^2*d^4+42*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^4)*d*(b*x^2+a)/(a*d^2/b)^(
1/4)/b^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298409, size = 396, normalized size = 0.86

$$\frac{84\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{3}{4}}ab^8}{\sqrt{dx}d^{13}+\sqrt{d^{27}x-\sqrt{-\frac{d^{18}}{ab^{11}}}ab^5d^{18}}}\right)+21\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{18}}{ab^{11}}\right)^{\frac{1}{4}}\log\left(9261\right.}{64\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(84*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)
*arctan((-d^18/(a*b^11))^(3/4)*a*b^8/(sqrt(d*x)*d^13 + sqrt(d^27*
x - sqrt(-d^18/(a*b^11))*a*b^5*d^18))) + 21*(b^4*x^4 + 2*a*b^3*x^
2 + a^2*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 + 926
1*(-d^18/(a*b^11))^(3/4)*a*b^8) - 21*(b^4*x^4 + 2*a*b^3*x^2 + a^2
*b^2)*(-d^18/(a*b^11))^(1/4)*log(9261*sqrt(d*x)*d^13 - 9261*(-d^1
8/(a*b^11))^(3/4)*a*b^8) - 4*(11*b*d^4*x^3 + 7*a*d^4*x)*sqrt(d*x)
)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291303, size = 497, normalized size = 1.09

$$-\frac{1}{128}\,d^{3}\left(\frac{8\left(11\,\sqrt{dx}bd^{5}x^{3}+7\,\sqrt{dx}ad^{5}x\right)}{(bd^{2}x^{2}+ad^{2})^{2}b^{2}\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{42\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{ab^{5}\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{42\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{3}{4}}\arctan\left(-\cdots\right.}{ab^{5}\mathrm{sign}\left(bd^{4}x^{2}\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/128*d^3*(8*(11*sqrt(d*x)*b*d^5*x^3 + 7*sqrt(d*x)*a*d^5*x)/((b*
d^2*x^2 + a*d^2)^2*b^2*sign(b*d^4*x^2 + a*d^4)) - 42*sqrt(2)*(a*b
^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sign(b*d^4*x^2 + a*d^4)) - 42*sqr
t(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/
4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*sign(b*d^4*x^2 + a*d^4)
) + 21*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)
*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^5*sign(b*d^4*x^2 + a*d^4)) - 21*
sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a*b^5*sign(b*d^4*x^2 + a*d^4)))
```

3.761 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=458

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*
(d*x)^(5/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
(5*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
5*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.727506, antiderivative size = 458, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{5d^3\sqrt{dx}}{16b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^{7/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{3/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-5*d^3*Sqrt[d*x])/(16*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*
(d*x)^(5/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
(5*d^(7/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^
(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) + (5*d^(7/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
5*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(3/4)*b^(9/4)*Sq
rt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.407838, size = 272, normalized size = 0.59

$$\frac{(dx)^{7/2}\left(a+bx^2\right)\left(-72a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+32a^{7/4}\sqrt[4]{b}\sqrt{x}-5\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+5\sqrt{2}\left(a+bx\right.\right.}{128a^{3/4}b^{9/4}x^{7/2}\left(\left(\right.\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(7/2)*(a + b*x^2)*(32*a^(7/4)*b^(1/4)*Sqrt[x] - 72*a^(3/4)
*b^(1/4)*Sqrt[x]*(a + b*x^2) - 10*Sqrt[2]*(a + b*x^2)^2*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*(a + b*x^2)^2*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 5*Sqrt[2]*(a + b*x
^2)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]
+ 5*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*S
```

```
qrt[x] + Sqrt[b]*x]))/(128*a^(3/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2
)^(3/2))
```

Maple [B] time = 0.024, size = 672, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2)))*x^4*b^2*d^2+10*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b
^2*d^2-10*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+10*(a*d^2/b)^(1/4)*2^
(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^2
*a*b*d^2+20*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2-20*(a*d^2/b)^(1/4)*2
^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*x^2*a*b*d^2+5*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1
/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*a^2*d^2+10*(a*d^2/b)^(1/4)*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*a^2*d^2-10*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2-72*(d*x)^(5/2)*a*b-40*(
d*x)^(1/2)*a^2*d^2)*d*(b*x^2+a)/a/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300424, size = 392, normalized size = 0.86

$$-\frac{20\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}ab^2}{\sqrt{dx}d^3+\sqrt{d^7x+\sqrt{-\frac{d^{14}}{a^3b^9}}a^2b^4}}\right)-5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{d^{14}}{a^3b^9}\right)^{\frac{1}{4}}\log\left(5\sqrt{dx}d\right.}{64\left(b^4x^4+2\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(20*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-d^14/(a^3*b^9))^(1/
4)*arctan((-d^14/(a^3*b^9))^(1/4)*a*b^2/(sqrt(d*x)*d^3 + sqrt(d^7
*x + sqrt(-d^14/(a^3*b^9))*a^2*b^4))) - 5*(b^4*x^4 + 2*a*b^3*x^2
+ a^2*b^2)*(-d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 + 5*(-d^14
/(a^3*b^9))^(1/4)*a*b^2) + 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-
d^14/(a^3*b^9))^(1/4)*log(5*sqrt(d*x)*d^3 - 5*(-d^14/(a^3*b^9))^(
1/4)*a*b^2) + 4*(9*b*d^3*x^2 + 5*a*d^3)*sqrt(d*x))/(b^4*x^4 + 2*a
*b^3*x^2 + a^2*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.288672, size = 501, normalized size = 1.09

$$\frac{1}{128}\,d^2\left(\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{10\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\,d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^3\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{5\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d^2*(10*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqr
t(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^3*sign(
b*d^4*x^2 + a*d^4)) + 10*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*
sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/
(a*b^3*sign(b*d^4*x^2 + a*d^4)) + 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*l
n(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3
*sign(b*d^4*x^2 + a*d^4)) - 5*sqrt(2)*(a*b^3*d^2)^(1/4)*d*ln(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^3*sign(
b*d^4*x^2 + a*d^4)) - 8*(9*sqrt(d*x)*b*d^5*x^2 + 5*sqrt(d*x)*a*d^
5)/((b*d^2*x^2 + a*d^2)^2*b^2*sign(b*d^4*x^2 + a*d^4)))
```

3.762 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(
d*x)^(3/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
3*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3
*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.738773, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3d(dx)^{3/2}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{4b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{5/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (3*d*(d*x)^(3/2))/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(
d*x)^(3/2))/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
3*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*
Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*S
qrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/
(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3
*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.782528, size = 436, normalized size = 0.95

$$\frac{3(dx)^{5/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}x^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{3(dx)^{5/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{5/4}b^{7/4}x^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{3(dx)^{5/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}x^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{3(dx)^{5/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}x^{5/2}\left(\left(a+bx^2\right)^2\right)^{3/2}} + \frac{3(dx)^{5/2}\left(a+bx^2\right)^2}{16abx\left(\left(a+bx^2\right)^2\right)^{3/2}} - \frac{(dx)^{5/2}\left(a+bx^2\right)}{4bx\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -((d*x)^(5/2)*(a + b*x^2))/(4*b*x*((a + b*x^2)^2)^(3/2)) + (3*(d*
x)^(5/2)*(a + b*x^2)^2)/(16*a*b*x*((a + b*x^2)^2)^(3/2)) + (3*(d*
x)^(5/2)*(a + b*x^2)^3*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqr
t[x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*a^(5/4)*b^(7/4)*x^(5/2)*((a
 + b*x^2)^2)^(3/2)) + (3*(d*x)^(5/2)*(a + b*x^2)^3*ArcTan[(Sqrt[2
]*a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*a^
(5/4)*b^(7/4)*x^(5/2)*((a + b*x^2)^2)^(3/2)) + (3*(d*x)^(5/2)*(a
+ b*x^2)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*x^(5/2)*((a + b*x^2)^2)^(3/2))
- (3*(d*x)^(5/2)*(a + b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(5/4)*b^(7/4)*x^(5/2)*((a
 + b*x^2)^2)^(3/2))
```

Maple [B] time = 0.024, size = 614, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*
d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*x^4*b^2*d^4+6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^4-6*2^(1/2)*arctan((-2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^4+24*(a*
d^2/b)^(1/4)*(d*x)^(7/2)*b^2+6*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1
/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^4+12*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^4-12*
2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*x^2*a*b*d^4-8*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a*b*d^2+3*2^(1/2)
*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*d^4+
6*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*a^2*d^4-6*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*a^2*d^4)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^2/a
/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298193, size = 416, normalized size = 0.91

$$\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}\arctan\left(\frac{27\,a^4b^5\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{3}{4}}}{27\,\sqrt{dx}d^7+\sqrt{-729\,a^3b^3d^{10}\sqrt{-\frac{d^{10}}{a^5b^7}}+729\,d^{15}x}}\right)+3\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^{10}}{a^5b^7}\right)^{\frac{1}{4}}}{64\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1
/4)*arctan(27*a^4*b^5*(-d^10/(a^5*b^7))^(3/4)/(27*sqrt(d*x)*d^7 +
 sqrt(-729*a^3*b^3*d^10*sqrt(-d^10/(a^5*b^7)) + 729*d^15*x))) + 3
*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(
27*a^4*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) - 3*(a*b^3
*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^10/(a^5*b^7))^(1/4)*log(-27*a^4
*b^5*(-d^10/(a^5*b^7))^(3/4) + 27*sqrt(d*x)*d^7) + 4*(3*b*d^2*x^3
 - a*d^2*x)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.290692, size = 498, normalized size = 1.08

$$\frac{1}{128}\,d\left(\frac{8\left(3\sqrt{dx}bd^5x^3-\sqrt{dx}ad^5x\right)}{(bd^2x^2+ad^2)^2ab\operatorname{sign}(bd^4x^2+ad^4)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\operatorname{sign}(bd^4x^2+ad^4)}+\frac{6\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\right.}{2}\right.}{a^2b^4\operatorname{sign}(bd^4x^2+a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/128*d*(8*(3*sqrt(d*x)*b*d^5*x^3 - sqrt(d*x)*a*d^5*x)/((b*d^2*x^
2 + a*d^2)^2*a*b*sign(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(a*b^3*d^2)
^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))
/(a*d^2/b)^(1/4))/(a^2*b^4*sign(b*d^4*x^2 + a*d^4)) + 6*sqrt(2)*(
a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2
*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*sign(b*d^4*x^2 + a*d^4)) -
3*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt
(d*x) + sqrt(a*d^2/b))/(a^2*b^4*sign(b*d^4*x^2 + a*d^4)) + 3*sqrt
(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x)
+ sqrt(a*d^2/b))/(a^2*b^4*sign(b*d^4*x^2 + a*d^4)))
```

3.763 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=459

$$\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[
d*x])/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3
/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sq
rt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.756855, antiderivative size = 459, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{d\sqrt{dx}}{16ab\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{4b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3d^{3/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3d^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{7/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x])/(16*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*Sqrt[
d*x])/(4*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*d^(3
/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (3*d^(3/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (3*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d]
 + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sq
rt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3*d^(3/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.406402, size = 436, normalized size = 0.95

$$-\frac{3(dx)^{3/2}\left(a+bx^2\right)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{3(dx)^{3/2}\left(a+bx^2\right)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{64\sqrt{2}a^{7/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{3(dx)^{3/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{2\sqrt[4]{b}\sqrt{x}-\sqrt{2}\sqrt[4]{a}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{3(dx)^{3/2}\left(a+bx^2\right)^3\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{b}\sqrt{x}}{\sqrt{2}\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}x^{3/2}\left(\left(a+bx^2\right)^2\right)^{3/2}}+\frac{(dx)^{3/2}\left(a+bx^2\right)^2}{16abx\left(\left(a+bx^2\right)^2\right)^{3/2}}-\frac{(dx)^{3/2}\left(a+bx^2\right)}{4bx\left(\left(a+bx^2\right)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -((d*x)^(3/2)*(a + b*x^2))/(4*b*x*((a + b*x^2)^2)^(3/2)) + ((d*x)
^(3/2)*(a + b*x^2)^2)/(16*a*b*x*((a + b*x^2)^2)^(3/2)) + (3*(d*x)
^(3/2)*(a + b*x^2)^3*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*b^(1/4)*Sqrt[
x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*a^(7/4)*b^(5/4)*x^(3/2)*((a +
 b*x^2)^2)^(3/2)) + (3*(d*x)^(3/2)*(a + b*x^2)^3*ArcTan[(Sqrt[2]*
a^(1/4) + 2*b^(1/4)*Sqrt[x])/(Sqrt[2]*a^(1/4))])/(32*Sqrt[2]*a^(7
/4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(3/2)) - (3*(d*x)^(3/2)*(a +
b*x^2)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(3/2)) +
(3*(d*x)^(3/2)*(a + b*x^2)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/(64*Sqrt[2]*a^(7/4)*b^(5/4)*x^(3/2)*((a +
 b*x^2)^2)^(3/2))
```

Maple [B] time = 0.024, size = 674, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(3*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2)))*x^4*b^2*d^2+6*(a*d^2/b)^(1/4)*2^(1/2)*arc
tan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^
2*d^2-6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+6*(a*d^2/b)^(1/4)*2^(1/
2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/
((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^2*a*
b*d^2+12*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*d^2-12*(a*d^2/b)^(1/4)*2^(1
/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*x^2*a*b*d^2+3*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*a^2*d^2+6*(a*d^2/b)^(1/4)*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2
*d^2-6*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2+8*(d*x)^(5/2)*a*b-24*(d*x)^(
1/2)*a^2*d^2)/d*(b*x^2+a)/b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294974, size = 382, normalized size = 0.83

$$-\frac{12\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}}{\sqrt{dx}d+\sqrt{a^4b^2\sqrt{-\frac{d^6}{a^7b^5}}+d^3x}}\right)-3\left(ab^3x^4+2\,a^2b^2x^2+a^3b\right)\left(-\frac{d^6}{a^7b^5}\right)^{\frac{1}{4}}\log\left(3\,a^2b\right.}{64\,(ab^3x^4+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(12*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6/(a^7*b^5))^(1
/4)*arctan(a^2*b*(-d^6/(a^7*b^5))^(1/4)/(sqrt(d*x)*d + sqrt(a^4*b
^2*sqrt(-d^6/(a^7*b^5)) + d^3*x))) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2
 + a^3*b)*(-d^6/(a^7*b^5))^(1/4)*log(3*a^2*b*(-d^6/(a^7*b^5))^(1/
4) + 3*sqrt(d*x)*d) + 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-d^6
/(a^7*b^5))^(1/4)*log(-3*a^2*b*(-d^6/(a^7*b^5))^(1/4) + 3*sqrt(d*
x)*d) - 4*(b*d*x^2 - 3*a*d)*sqrt(d*x))/(a*b^3*x^4 + 2*a^2*b^2*x^2
 + a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.286759, size = 497, normalized size = 1.08

$$\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^2b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{3\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^2b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{\sqrt{dx}bd^5x^2-3\sqrt{dx}ad^5}{16\left(bd^2x^2+ad^2\right)^2ab\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 3/64*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d
^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^2*sign(b*d^4*x
^2 + a*d^4)) + 3/64*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2*sqrt(
2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*
b^2*sign(b*d^4*x^2 + a*d^4)) + 3/128*sqrt(2)*(a*b^3*d^2)^(1/4)*d*
ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*
b^2*sign(b*d^4*x^2 + a*d^4)) - 3/128*sqrt(2)*(a*b^3*d^2)^(1/4)*d*
ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*
b^2*sign(b*d^4*x^2 + a*d^4)) + 1/16*(sqrt(d*x)*b*d^5*x^2 - 3*sqrt
(d*x)*a*d^5)/((b*d^2*x^2 + a*d^2)^2*a*b*sign(b*d^4*x^2 + a*d^4))
```

3.764 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x
)^(3/2)/(4*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*
Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
4*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*S
qrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.764969, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{5(dx)^{3/2}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{4ad\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5\sqrt{d}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{9/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (5*(d*x)^(3/2))/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*x
)^(3/2)/(4*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*
Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/
4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) + (5*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(6
4*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (5*S
qrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(9/4)*b^(3/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.382944, size = 272, normalized size = 0.59

$$\frac{\sqrt{dx}\left(a+bx^2\right)\left(32a^{5/4}b^{3/4}x^{3/2}+40\sqrt[4]{a}b^{3/4}x^{3/2}\left(a+bx^2\right)+5\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-5\sqrt{2}\left(a+bx^2\right)\right.}{128a^{9/4}b^{3/4}\sqrt{x}\left(\left(a\right.\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[d*x]*(a + b*x^2)*(32*a^(5/4)*b^(3/4)*x^(3/2) + 40*a^(1/4)*b
^(3/4)*x^(3/2)*(a + b*x^2) - 10*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 10*Sqrt[2]*(a + b*x^2)^2*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 5*Sqrt[2]*(a + b*x^2
)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] -
5*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[x] + Sqrt[b]*x]))/(128*a^(9/4)*b^(3/4)*Sqrt[x]*((a + b*x^2)^2)^
(3/2))
```

Maple [B] time = 0.015, size = 614, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/128*(5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*
d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*x^4*b^2*d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2-10*2^(1/2)*arctan((-2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2*d^2+40*(
a*d^2/b)^(1/4)*(d*x)^(3/2)*x^2*b^2+10*2^(1/2)*ln(-((a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a*b*d^2+20*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b*
d^2-20*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*x^2*a*b*d^2+72*(d*x)^(3/2)*a*b*(a*d^2/b)^(1/4)+5*2^(
1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)
)/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^2*
d^2+10*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a^2*d^2-10*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^
2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2*d^2)/d*(b*x^2+a)/(a*d^2/b)^(1/4)
/b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301567, size = 386, normalized size = 0.84

$$\frac{20\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{125\,a^7b^2\left(-\frac{d^2}{a^9b^3}\right)^{\frac{3}{4}}}{125\,\sqrt{dx}d+\sqrt{-15625\,a^5bd^2\sqrt{-\frac{d^2}{a^9b^3}}+15625\,d^3x}}\right)+5\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\left(-\frac{d^2}{a^9b^3}\right)^{\frac{1}{4}}}{64\,(a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(20*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)
*arctan(125*a^7*b^2*(-d^2/(a^9*b^3))^(3/4)/(125*sqrt(d*x)*d + sqr
t(-15625*a^5*b*d^2*sqrt(-d^2/(a^9*b^3)) + 15625*d^3*x))) + 5*(a^2
*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(125*a^7*
b^2*(-d^2/(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) - 5*(a^2*b^2*x^4 +
2*a^3*b*x^2 + a^4)*(-d^2/(a^9*b^3))^(1/4)*log(-125*a^7*b^2*(-d^2/
(a^9*b^3))^(3/4) + 125*sqrt(d*x)*d) + 4*(5*b*x^3 + 9*a*x)*sqrt(d*
x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{dx}}{\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.291987, size = 506, normalized size = 1.1

$$\frac{5\sqrt{dx}bd^4x^3+9\sqrt{dx}ad^4x}{16\,(bd^2x^2+ad^2)^2a^2\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3d\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3d\mathrm{sign}\,(bd^4x^2+ad^4)}-\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3b^3d\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{5\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3b^3d\mathrm{sign}\,(bd^4x^2+ad^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(5*sqrt(d*x)*b*d^4*x^3 + 9*sqrt(d*x)*a*d^4*x)/((b*d^2*x^2 +
a*d^2)^2*a^2*sign(b*d^4*x^2 + a*d^4)) + 5/64*sqrt(2)*(a*b^3*d^2)^
(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^3*b^3*d*sign(b*d^4*x^2 + a*d^4)) + 5/64*sqrt(
2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
 - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*d*sign(b*d^4*x^2 + a*d^
4)) - 5/128*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*d*sign(b*d^4*x^2 + a*d^
4)) + 5/128*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^
(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b^3*d*sign(b*d^4*x^2 + a*d^
4))
```

3.765 $\int \frac{1}{\sqrt{dx}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=460

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d
*x]/(4*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
1*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.767655, antiderivative size = 460, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{7\sqrt{dx}}{16a^2d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{4ad(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{21(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{21(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (7*Sqrt[d*x])/(16*a^2*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sqrt[d
*x]/(4*a*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (21*(a
+ b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])
])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) + (21*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/
(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2
 + 2*a*b*x^2 + b^2*x^4]) - (21*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] +
Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[
2]*a^(11/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (2
1*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(11/4)*b^(1/4)*Sqrt[d]*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.38884, size = 272, normalized size = 0.59

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(56a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+32a^{7/4}\sqrt[4]{b}\sqrt{x}-21\sqrt{2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+21\sqrt{2}\left(a+bx^2\right)\right.}{128a^{11/4}\sqrt[4]{b}\sqrt{dx}\left((a\right.}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(32*a^(7/4)*b^(1/4)*Sqrt[x] + 56*a^(3/4)*b^(
1/4)*Sqrt[x]*(a + b*x^2) - 42*Sqrt[2]*(a + b*x^2)^2*ArcTan[1 - (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 42*Sqrt[2]*(a + b*x^2)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 21*Sqrt[2]*(a + b*x^2)
^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 2
1*Sqrt[2]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
```

```
t[x] + Sqrt[b]*x]))/(128*a^(11/4)*b^(1/4)*Sqrt[d*x]*((a + b*x^2)^
2)^(3/2))
```

Maple [B] time = 0.014, size = 644, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] 1/128*(21*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^
(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2)))*x^4*b^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2-
42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*x^4*b^2+42*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^2*a*b+84*(a*
d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*x^2*a*b-84*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a*b+21*(a*
d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2
/b)^(1/2)))*a^2+42*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^2-42*(a*d^2/b)^(1/4)*2^
(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*a^2+56*(d*x)^(1/2)*x^2*a*b+88*(d*x)^(1/2)*a^2)/d*(b*x^2+a)/a^
3/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295579, size = 377, normalized size = 0.82

$$-\frac{84\left(a^2b^2dx^4+2\,a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3d\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}}{\sqrt{a^6d^2\sqrt{-\frac{1}{a^{11}bd^2}}+dx}+\sqrt{dx}}\right)-21\left(a^2b^2dx^4+2\,a^3bdx^2+a^4d\right)\left(-\frac{1}{a^{11}bd^2}\right)^{\frac{1}{4}}\,\mathrm{l}\cdots}{64\left(a^2b^2dx^4\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] -1/64*(84*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2
))^(1/4)*arctan(a^3*d*(-1/(a^11*b*d^2))^(1/4)/(sqrt(a^6*d^2*sqrt(
-1/(a^11*b*d^2)) + d*x) + sqrt(d*x))) - 21*(a^2*b^2*d*x^4 + 2*a^3
*b*d*x^2 + a^4*d)*(-1/(a^11*b*d^2))^(1/4)*log(a^3*d*(-1/(a^11*b*d
^2))^(1/4) + sqrt(d*x)) + 21*(a^2*b^2*d*x^4 + 2*a^3*b*d*x^2 + a^4
*d)*(-1/(a^11*b*d^2))^(1/4)*log(-a^3*d*(-1/(a^11*b*d^2))^(1/4) +
sqrt(d*x)) - 4*(7*b*x^2 + 11*a)*sqrt(d*x))/(a^2*b^2*d*x^4 + 2*a^3
*b*d*x^2 + a^4*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.282072, size = 505, normalized size = 1.1

$$\frac{7\sqrt{dx}bd^3x^2+11\sqrt{dx}ad^3}{16\,(bd^2x^2+ad^2)^2a^2\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3bd\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3bd\mathrm{sign}\,(bd^4x^2+ad^4)}-\frac{21\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^3bd\mathrm{sign}\,(bd^4x^2+ad^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] 1/16*(7*sqrt(d*x)*b*d^3*x^2 + 11*sqrt(d*x)*a*d^3)/((b*d^2*x^2 + a
*d^2)^2*a^2*sign(b*d^4*x^2 + a*d^4)) + 21/64*sqrt(2)*(a*b^3*d^2)^
(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/
(a*d^2/b)^(1/4))/(a^3*b*d*sign(b*d^4*x^2 + a*d^4)) + 21/64*sqrt(2
)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4)
- 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b*d*sign(b*d^4*x^2 + a*d^4))
 + 21/128*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1
/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sign(b*d^4*x^2 + a*d^4))
- 21/128*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/
4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b*d*sign(b*d^4*x^2 + a*d^4))
```

3.766 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{45\sqrt[4]{b}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\,(a+bx^2)}{16a^3d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d
*Sqrt[d*x]*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a
+ b*x^2))/(16*a^3*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
(45*b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3/2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(13/4)*d^
(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.864768, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{9}{16a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}-\frac{45\sqrt[4]{b}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{13/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45\,(a+bx^2)}{16a^3d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 9/(16*a^2*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*a*d
*Sqrt[d*x]*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*(a
+ b*x^2))/(16*a^3*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
(45*b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a
^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(13/4)*d^(3/2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*b^(1/4)*(a + b*x^2)*Log[Sqr
t[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d
*x]])/(64*Sqrt[2]*a^(13/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) + (45*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d
]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(13/4)*d^
(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.385175, size = 307, normalized size = 0.61

$$\frac{x\left(a+bx^2\right)\left(-32a^{5/4}bx^2-104\sqrt[4]{a}bx^2\left(a+bx^2\right)-256\sqrt[4]{a}\left(a+bx^2\right)^2-45\sqrt{2}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (x*(a + b*x^2)*(-32*a^(5/4)*b*x^2 - 104*a^(1/4)*b*x^2*(a + b*x^2)
 - 256*a^(1/4)*(a + b*x^2)^2 + 90*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*
x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 90*Sqrt[2]
*b^(1/4)*Sqrt[x]*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x
])/a^(1/4)] - 45*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^2*Log[Sqrt[a
```

```
] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]*b^(
1/4)*Sqrt[x]*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x] + Sqrt[b]*x]))/(128*a^(13/4)*(d*x)^(3/2)*((a + b*x^2)^2)^
(3/2))
```

Maple [A] time = 0.03, size = 642, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/128/d*(45*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))*2^(1/2)*(d*x)^(1/2)*x^4*b^2+90*arctan((2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(1/2)*x^4*b^2-90*ar
ctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1
/2)*(d*x)^(1/2)*x^4*b^2+90*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*2^(1/2)*(d*x)^(1/2)*x^2*a*b+180*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(1/2
)*x^2*a*b-180*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*2^(1/2)*(d*x)^(1/2)*x^2*a*b+360*(a*d^2/b)^(1/4)*x^4*b
^2+45*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2
))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*2^(
1/2)*(d*x)^(1/2)*a^2+90*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/
4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(1/2)*a^2-90*arctan((-2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(1/2)
*a^2+648*(a*d^2/b)^(1/4)*x^2*a*b+256*(a*d^2/b)^(1/4)*a^2)*(b*x^2+
a)/(a*d^2/b)^(1/4)/(d*x)^(1/2)/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(3/2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302468, size = 420, normalized size = 0.83

$$-\frac{180\,b^2x^4+324\,abx^2+180\,\left(a^3b^2dx^4+2\,a^4bdx^2+a^5d\right)\sqrt{dx}\left(-\frac{b}{a^{13}d^6}\right)^{\frac{1}{4}}\arctan\left(\frac{91125\,a^{10}d^5\left(-\frac{b}{a^{13}d^6}\right)^{\frac{3}{4}}}{91125\,\sqrt{dx}b+\sqrt{-8303765625\,a^7bd^4\sqrt{-\frac{b}{a^{13}d^6}}+83037656}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(3/2)),x, algorithm="fricas
```

```
[Out] -1/64*(180*b^2*x^4 + 324*a*b*x^2 + 180*(a^3*b^2*d*x^4 + 2*a^4*b*d
*x^2 + a^5*d)*sqrt(d*x)*(-b/(a^13*d^6))^(1/4)*arctan(91125*a^10*d
^5*(-b/(a^13*d^6))^(3/4)/(91125*sqrt(d*x)*b + sqrt(-8303765625*a^
7*b*d^4*sqrt(-b/(a^13*d^6)) + 8303765625*b^2*d*x))) + 45*(a^3*b^2
*d*x^4 + 2*a^4*b*d*x^2 + a^5*d)*sqrt(d*x)*(-b/(a^13*d^6))^(1/4)*l
og(91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*x)*b) - 45
*(a^3*b^2*d*x^4 + 2*a^4*b*d*x^2 + a^5*d)*sqrt(d*x)*(-b/(a^13*d^6)
)^(1/4)*log(-91125*a^10*d^5*(-b/(a^13*d^6))^(3/4) + 91125*sqrt(d*
x)*b) + 128*a^2)/((a^3*b^2*d*x^4 + 2*a^4*b*d*x^2 + a^5*d)*sqrt(d*
x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.289165, size = 554, normalized size = 1.09

$$-\frac{\frac{256}{\sqrt{dx}a^3\mathrm{sign}(bd^4x^2+ad^4)}+\frac{8\left(13\,\sqrt{dx}b^2d^3x^3+17\,\sqrt{dx}abd^3x\right)}{(bd^2x^2+ad^2)^2a^3\mathrm{sign}(bd^4x^2+ad^4)}+\frac{90\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^2d^2\mathrm{sign}(bd^4x^2+ad^4)}+\frac{90\,\sqrt{2}(ab^3d^2)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\right.}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right.}{a^4b^2d^2\mathrm{sign}(bd^4x^2+ad^4)}}{128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/128*(256/(sqrt(d*x)*a^3*sign(b*d^4*x^2 + a*d^4)) + 8*(13*sqrt(
d*x)*b^2*d^3*x^3 + 17*sqrt(d*x)*a*b*d^3*x)/((b*d^2*x^2 + a*d^2)^2
*a^3*sign(b*d^4*x^2 + a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arct
an(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(a^4*b^2*d^2*sign(b*d^4*x^2 + a*d^4)) + 90*sqrt(2)*(a*b^3*
d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(
d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*d^2*sign(b*d^4*x^2 + a*d^4)) - 45
*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(
d*x) + sqrt(a*d^2/b))/(a^4*b^2*d^2*sign(b*d^4*x^2 + a*d^4)) + 45*
sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d
*x) + sqrt(a*d^2/b))/(a^4*b^2*d^2*sign(b*d^4*x^2 + a*d^4)))/d
```

3.767 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=506

$$\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}+\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77\left(a+bx^2\right)}{48a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*
a*d*(d*x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7
7*(a + b*x^2))/(48*a^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (77*b^(3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/4)*d^
(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1
5/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.843464, antiderivative size = 506, normalized size of antiderivative = 1., number

of steps used = 14, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{11}{16a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{4ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}+\frac{77b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{64\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{15/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77\left(a+bx^2\right)}{48a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 11/(16*a^2*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*
a*d*(d*x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7
7*(a + b*x^2))/(48*a^3*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (77*b^(3/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d
*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[
2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(15/4)*d^
(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*b^(3/4)*(a + b*x^2)*
Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[d*x]])/(64*Sqrt[2]*a^(15/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) - (77*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]
*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(1
5/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.384614, size = 307, normalized size = 0.61

$$\frac{x\left(a+bx^2\right)\left(-96a^{7/4}bx^2-360a^{3/4}bx^2\left(a+bx^2\right)-256a^{3/4}\left(a+bx^2\right)^2+231\sqrt{2}b^{3/4}x^{3/2}\left(a+bx^2\right)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (x*(a + b*x^2)*(-96*a^(7/4)*b*x^2 - 360*a^(3/4)*b*x^2*(a + b*x^2)
 - 256*a^(3/4)*(a + b*x^2)^2 + 462*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b
*x^2)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 462*Sqrt[
2]*b^(3/4)*x^(3/2)*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)] + 231*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^2*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 231*Sqrt[2]
*b^(3/4)*x^(3/2)*(a + b*x^2)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x] + Sqrt[b]*x]))/(384*a^(15/4)*(d*x)^(5/2)*((a + b*x^2)
^2)^(3/2))
```

Maple [B] time = 0.029, size = 713, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] -1/384/d^3*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/2)*x^4*b^3+462*(a*d^2/b)^
(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*(d*x)^(3/2)*x^4*b^3-462*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3
/2)*x^4*b^3+462*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/2)*x^2*a*b^2+924*(a*d^2/b
)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*(d*x)^(3/2)*x^2*a*b^2-924*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x
)^(3/2)*x^2*a*b^2+231*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/2)*a^2*b+462*(a*d^2
/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
```

```
*d^2/b)^(1/4))*(d*x)^(3/2)*a^2*b-462*(a*d^2/b)^(1/4)*2^(1/2)*arct
an((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^
(3/2)*a^2*b+616*a*d^2*b^2*x^4+968*x^2*a^2*b*d^2+256*a^3*d^2)*(b*x
^2+a)/(d*x)^(3/2)/a^4/((b*x^2+a)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(5/2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.306007, size = 470, normalized size = 0.93

$$-\frac{308\,b^2x^4 + 484\,abx^2 - 924\left(a^3b^2d^2x^5 + 2\,a^4bd^2x^3 + a^5d^2x\right)\sqrt{dx}\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}\arctan\left(\frac{a^4d^3\left(-\frac{b^3}{a^{15}d^{10}}\right)^{\frac{1}{4}}}{\sqrt{dx}b+\sqrt{a^8d^6\sqrt{-\frac{b^3}{a^{15}d^{10}}}+b^2dx}}\right) + 231\left(a^3b^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(5/2)),x, algorithm="fricas
```

```
[Out] -1/192*(308*b^2*x^4 + 484*a*b*x^2 - 924*(a^3*b^2*d^2*x^5 + 2*a^4*
b*d^2*x^3 + a^5*d^2*x)*sqrt(d*x)*(-b^3/(a^15*d^10))^(1/4)*arctan(
a^4*d^3*(-b^3/(a^15*d^10))^(1/4)/(sqrt(d*x)*b + sqrt(a^8*d^6*sqrt
(-b^3/(a^15*d^10)) + b^2*d*x))) + 231*(a^3*b^2*d^2*x^5 + 2*a^4*b*
d^2*x^3 + a^5*d^2*x)*sqrt(d*x)*(-b^3/(a^15*d^10))^(1/4)*log(77*a^
4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b) - 231*(a^3*b^2*d
^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*sqrt(d*x)*(-b^3/(a^15*d^10)
)^(1/4)*log(-77*a^4*d^3*(-b^3/(a^15*d^10))^(1/4) + 77*sqrt(d*x)*b
) + 128*a^2)/((a^3*b^2*d^2*x^5 + 2*a^4*b*d^2*x^3 + a^5*d^2*x)*sqr
t(d*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{5}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(5/2)*((a + b*x**2)**2)**(3/2)), x)
```

GIAC/XCAS [A] time = 0.286566, size = 541, normalized size = 1.07

$$-\frac{15\sqrt{dx}b^2d^2x^2+19\sqrt{dx}abd^2}{16\left(bd^2x^2+ad^2\right)^2a^3d\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4d^3\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^4d^3\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^4d^3\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{2}{3\sqrt{dx}a^3d^2x\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(5/2)),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(d*x)*b^2*d^2*x^2 + 19*sqrt(d*x)*a*b*d^2)/((b*d^2*x
^2 + a*d^2)^2*a^3*d*sign(b*d^4*x^2 + a*d^4)) - 77/64*sqrt(2)*(a*b
^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqr
t(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sign(b*d^4*x^2 + a*d^4)) - 77/6
4*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b
)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*d^3*sign(b*d^4*x^2 +
 a*d^4)) - 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x + sqrt(2)*(a*d
^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sign(b*d^4*x^2 +
a*d^4)) + 77/128*sqrt(2)*(a*b^3*d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^
2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*d^3*sign(b*d^4*x^2 + a
*d^4)) - 2/3/(sqrt(d*x)*a^3*d^2*x*sign(b*d^4*x^2 + a*d^4))
```

3.768 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=553

$$\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b\left(a+bx^2\right)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117\left(a+bx^2\right)}{80a^3d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*
a*d*(d*x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1
17*(a + b*x^2))/(80*a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (117*b*(a + b*x^2))/(16*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(17/4)*d^(7/
2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt
[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5
/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.953435, antiderivative size = 553, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{1}{4ad(dx)^{5/2}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13}{16a^2d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{64\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b^{5/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{32\sqrt{2}a^{17/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{117b\left(a+bx^2\right)}{16a^4d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{117\left(a+bx^2\right)}{80a^3d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] 13/(16*a^2*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(4*
a*d*(d*x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1
17*(a + b*x^2))/(80*a^3*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (117*b*(a + b*x^2))/(16*a^4*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt[2]*a^(17/4)*d^(7/
2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5/4)*(a + b*x^2)*Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(32*Sqrt
[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (117*b^(5
/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (117*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(64*Sqrt[2]*a^(17/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.398619, size = 508, normalized size = 0.92

$$x\left(8424a^{5/4}b^2x^4+3328a^{9/4}bx^2-256a^{13/4}+585\sqrt{2}a^2b^{5/4}x^{5/2}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)-585\sqrt{2}a^2b^{5/4}x^{5/2}\log\left(\sqrt{2}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (x*(-256*a^(13/4) + 3328*a^(9/4)*b*x^2 + 8424*a^(5/4)*b^2*x^4 + 4
680*a^(1/4)*b^3*x^6 - 1170*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^2*
ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 1170*Sqrt[2]*b^(5
/4)*x^(5/2)*(a + b*x^2)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)] + 585*Sqrt[2]*a^2*b^(5/4)*x^(5/2)*Log[Sqrt[a] - Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 1170*Sqrt[2]*a*b^(9/4)*x^(9/
2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 5
85*Sqrt[2]*b^(13/4)*x^(13/2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] - 585*Sqrt[2]*a^2*b^(5/4)*x^(5/2)*Log[Sqrt
[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1170*Sqrt[2]
*a*b^(9/4)*x^(9/2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x] - 585*Sqrt[2]*b^(13/4)*x^(13/2)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(640*a^(17/4)*(d*x)^(7/2
)*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A] time = 0.033, size = 684, normalized size = 1.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] 1/640/d^3*(1170*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*2^(1/2)*(d*x)^(5/2)*x^4*b^3-1170*arctan((-2^(1/2)*(d
*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(5/2)*x
^4*b^3+585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(
a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*(d*x)^(5/2)*x^4*b^3+2340*arctan((2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(5/2)*x^2*a*b^2-234
0*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
2^(1/2)*(d*x)^(5/2)*x^2*a*b^2+4680*(a*d^2/b)^(1/4)*x^6*b^3*d^2+11
70*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))
```

```
)*(d*x)^(5/2)*x^2*a*b^2+1170*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d*x)^(5/2)*a^2*b-1170*arctan((
-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*2^(1/2)*(d
*x)^(5/2)*a^2*b+8424*(a*d^2/b)^(1/4)*x^4*a*b^2*d^2+585*2^(1/2)*ln
(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(5/2)
*a^2*b+3328*(a*d^2/b)^(1/4)*x^2*a^2*b*d^2-256*(a*d^2/b)^(1/4)*a^3
*d^2)*(b*x^2+a)/(a*d^2/b)^(1/4)/(d*x)^(5/2)/a^4/((b*x^2+a)^2)^(3/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(7/2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300936, size = 513, normalized size = 0.93

$$\frac{2340\,b^3x^6 + 4212\,ab^2x^4 + 1664\,a^2bx^2 - 128\,a^3 + 2340\,\left(a^4b^2d^3x^6 + 2\,a^5bd^3x^4 + a^6d^3x^2\right)\sqrt{dx}\left(-\frac{b^5}{a^{17}d^{14}}\right)^{\frac{1}{4}}\arctan\left(\frac{}{1601613\,\sqrt{dx}b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(7/2)),x, algorithm="fricas
```

```
[Out] 1/320*(2340*b^3*x^6 + 4212*a*b^2*x^4 + 1664*a^2*b*x^2 - 128*a^3 +
 2340*(a^4*b^2*d^3*x^6 + 2*a^5*b*d^3*x^4 + a^6*d^3*x^2)*sqrt(d*x)
*(-b^5/(a^17*d^14))^(1/4)*arctan(1601613*a^13*d^11*(-b^5/(a^17*d^
14))^(3/4)/(1601613*sqrt(d*x)*b^4 + sqrt(-2565164201769*a^9*b^5*d
^8*sqrt(-b^5/(a^17*d^14)) + 2565164201769*b^8*d*x))) + 585*(a^4*b
^2*d^3*x^6 + 2*a^5*b*d^3*x^4 + a^6*d^3*x^2)*sqrt(d*x)*(-b^5/(a^17
*d^14))^(1/4)*log(1601613*a^13*d^11*(-b^5/(a^17*d^14))^(3/4) + 16
01613*sqrt(d*x)*b^4) - 585*(a^4*b^2*d^3*x^6 + 2*a^5*b*d^3*x^4 + a
^6*d^3*x^2)*sqrt(d*x)*(-b^5/(a^17*d^14))^(1/4)*log(-1601613*a^13*
d^11*(-b^5/(a^17*d^14))^(3/4) + 1601613*sqrt(d*x)*b^4))/((a^4*b^2
*d^3*x^6 + 2*a^5*b*d^3*x^4 + a^6*d^3*x^2)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.28811, size = 583, normalized size = 1.05

$$\frac{21\sqrt{dx}b^3d^3x^3+25\sqrt{dx}ab^2d^3x}{16\,(bd^2x^2+ad^2)^2a^4d^3\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5bd^5\mathrm{sign}\,(bd^4x^2+ad^4)}-\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^5bd^5\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{117\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{128\,a^5bd^5\mathrm{sign}\,(bd^4x^2+ad^4)}+\frac{2\left(15\,bd^2x^2-ad^2\right)}{5\sqrt{dx}a^4d^5x^2\mathrm{sign}\,(bd^4x^2+ad^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^(7/2)),x, algorithm="giac")
```

```
[Out] 1/16*(21*sqrt(d*x)*b^3*d^3*x^3 + 25*sqrt(d*x)*a*b^2*d^3*x)/((b*d^
2*x^2 + a*d^2)^2*a^4*d^3*sign(b*d^4*x^2 + a*d^4)) + 117/64*sqrt(2
)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) +
 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d^5*sign(b*d^4*x^2 + a*d^4)
) + 117/64*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)
*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d^5*sign(
b*d^4*x^2 + a*d^4)) - 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x +
sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*sig
n(b*d^4*x^2 + a*d^4)) + 117/128*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x
- sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^5*b*d^5*s
ign(b*d^4*x^2 + a*d^4)) + 2/5*(15*b*d^2*x^2 - a*d^2)/(sqrt(d*x)*a
```

```
^4*d^5*x^2*sign(b*d^4*x^2 + a*d^4))
```

3.769 $\int \frac{(dx)^{23/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=647

$$-\frac{7d^3(dx)^{17/2}}{32b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923ad^{11}\sqrt{dx}\,(a+bx^2)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\,(a+bx^2)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (d*(d*x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (7*d^3*(d*x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (119*d^5*(d*x)^(13/2))/(256*b^3*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*a*d^11*Sqrt[d*x]*(a +
 b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*d^9*
(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*a^(5/4)
*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 1.15504, antiderivative size = 647, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{7d^3(dx)^{17/2}}{32b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{21/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923ad^{11}\sqrt{dx}\,(a+bx^2)}{1024b^6\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923d^9(dx)^{5/2}\,(a+bx^2)}{5120b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1547d^7(dx)^{9/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{119d^5(dx)^{13/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923a^{5/4}d^{23/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923a^{5/4}d^{23/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{25/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1547*d^7*(d*x)^(9/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (d*(d*x)^(21/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (7*d^3*(d*x)^(17/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (119*d^5*(d*x)^(13/2))/(256*b^3*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*a*d^11*Sqrt[d*x]*(a +
 b*x^2))/(1024*b^6*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*d^9*
(d*x)^(5/2)*(a + b*x^2))/(5120*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) - (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(25/4)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*S
qrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*a^(5/4)
*d^(23/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + (13923*a^(5/4)*d^(23/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*b^(25/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.656709, size = 374, normalized size = 0.58

$$\frac{(dx)^{23/2}\left(a+bx^2\right)\left(-69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+69615\sqrt{2}a^{5/4}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(23/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(23/2)*(a + b*x^2)*(5120*a^5*b^(1/4)*Sqrt[x] - 34560*a^4*b
^(1/4)*Sqrt[x]*(a + b*x^2) + 106080*a^3*b^(1/4)*Sqrt[x]*(a + b*x^
2)^2 - 223960*a^2*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 409600*a*b^(1/4
)*Sqrt[x]*(a + b*x^2)^4 + 16384*b^(5/4)*x^(5/2)*(a + b*x^2)^4 - 1
39230*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*S
qrt[x])/a^(1/4)] + 139230*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*ArcTan[1
+ (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 69615*Sqrt[2]*a^(5/4)*(a +
 b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x] + 69615*Sqrt[2]*a^(5/4)*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(40960*b^(25/4)*x^(23/2)*((
a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.035, size = 1297, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(23/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/40960*(16384*(d*x)^(5/2)*x^8*b^5*d^4-565800*(d*x)^(9/2)*a^3*b^2
*d^2-477896*(d*x)^(5/2)*a^4*b*d^4+556920*(a*d^2/b)^(1/4)*2^(1/2)*
```

```
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2
*a^4*b*d^6-556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^4*b*d^6+835380*(a*d
^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*x^4*a^3*b^2*d^6-835380*(a*d^2/b)^(1/4)*2^(1/2)*a
rctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4
*a^3*b^2*d^6-139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x
)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*a*b^4*d^6+278460*(a
*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^
2/b)^(1/2)))*x^6*a^2*b^3*d^6+556920*(a*d^2/b)^(1/4)*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a^2*
b^3*d^6-556920*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/
2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a^2*b^3*d^6+417690*(a*d^
2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b
)^(1/2)))*x^4*a^3*b^2*d^6-223960*(d*x)^(13/2)*a^2*b^3-556920*(d*x
)^(1/2)*a^5*d^6+139230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*a*b^4*d^6+69615*(
a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d
^2/b)^(1/2)))*x^8*a*b^4*d^6+278460*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d
*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^2*a^4*b*d^6-2
457600*(d*x)^(1/2)*x^4*a^3*b^2*d^6+69615*(a*d^2/b)^(1/4)*2^(1/2)*
ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*a^5*d^6+1
39230*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^6-139230*(a*d^2/b)^(1/4)*2^(1/2)
*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a
^5*d^6-1638400*(d*x)^(1/2)*x^2*a^4*b*d^6+65536*(d*x)^(5/2)*x^6*a*
b^4*d^4-409600*(d*x)^(1/2)*x^8*a*b^4*d^6+98304*(d*x)^(5/2)*x^4*a^
2*b^3*d^4-1638400*(d*x)^(1/2)*x^6*a^2*b^3*d^6+65536*(d*x)^(5/2)*x
^2*a^3*b^2*d^4)*d^5*(b*x^2+a)/b^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304628, size = 586, normalized size = 0.91

$$-\frac{278460 \left(-\frac{a^5 d^{46}}{b^{25}}\right)^{\frac{1}{4}} \left(b^{10}x^8 + 4\,ab^9x^6 + 6\,a^2b^8x^4 + 4\,a^3b^7x^2 + a^4b^6\right) \arctan\left(\frac{\left(-\frac{a^5d^{46}}{b^{25}}\right)^{\frac{1}{4}} b^6}{\sqrt{dx}ad^{11} + \sqrt{a^2d^{23}x + \sqrt{-\frac{a^5d^{46}}{b^{25}}}b^{12}}}\right) - 69615 \left(-\frac{a^5d^{46}}{b^{25}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(23/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/20480*(278460*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 +
 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*arctan((-a^5*d^46/b^25)
^(1/4)*b^6/(sqrt(d*x)*a*d^11 + sqrt(a^2*d^23*x + sqrt(-a^5*d^46/b
^25)*b^12))) - 69615*(-a^5*d^46/b^25)^(1/4)*(b^10*x^8 + 4*a*b^9*x
^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)*log(13923*sqrt(d*x)
*a*d^11 + 13923*(-a^5*d^46/b^25)^(1/4)*b^6) + 69615*(-a^5*d^46/b^
25)^(1/4)*(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2
 + a^4*b^6)*log(13923*sqrt(d*x)*a*d^11 - 13923*(-a^5*d^46/b^25)^(
1/4)*b^6) - 4*(2048*b^5*d^11*x^10 - 43008*a*b^4*d^11*x^8 - 220507
*a^2*b^3*d^11*x^6 - 369733*a^3*b^2*d^11*x^4 - 264537*a^4*b*d^11*x
^2 - 69615*a^5*d^11)*sqrt(d*x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b
^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(23/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.295822, size = 622, normalized size = 0.96

$$\frac{1}{40960}\,d^{10}\left(\frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\,\mathrm{sign}\left(bd^4x^2+ad^4\right)} + \frac{139230\,\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}ad\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^7\,\mathrm{sign}\left(bd^4x^2+ad^4\right)} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(23/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/40960*d^10*(139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d*arctan(1/2*sqr
t(2)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^
7*sign(b*d^4*x^2 + a*d^4)) + 139230*sqrt(2)*(a*b^3*d^2)^(1/4)*a*d
*arctan(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d
^2/b)^(1/4))/(b^7*sign(b*d^4*x^2 + a*d^4)) + 69615*sqrt(2)*(a*b^3
*d^2)^(1/4)*a*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt
(a*d^2/b))/(b^7*sign(b*d^4*x^2 + a*d^4)) - 69615*sqrt(2)*(a*b^3*d
^2)^(1/4)*a*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a
*d^2/b))/(b^7*sign(b*d^4*x^2 + a*d^4)) - 40*(5599*sqrt(d*x)*a^2*b
^3*d^9*x^6 + 14145*sqrt(d*x)*a^3*b^2*d^9*x^4 + 12357*sqrt(d*x)*a^
4*b*d^9*x^2 + 3683*sqrt(d*x)*a^5*d^9)/((b*d^2*x^2 + a*d^2)^4*b^6*
sign(b*d^4*x^2 + a*d^4)) + 16384*(sqrt(d*x)*b^20*d^6*x^2 - 25*sqr
t(d*x)*a*b^19*d^6)/(b^25*d^5*sign(b*d^4*x^2 + a*d^4)))
```

3.770 $\int \frac{(dx)^{21/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{19d^3(dx)^{15/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315d^9(dx)^{3/2}(a+bx^2)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (d*(d*x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (19*d^3*(d*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (95*d^5*(d*x)^(11/2))/(256*b^3*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*d^9*(d*x)^(3/2)*(a + b
*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)
*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 1.07388, antiderivative size = 600, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{19d^3(dx)^{15/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{19/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315d^9(dx)^{3/2}(a+bx^2)}{3072b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1045d^7(dx)^{7/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{95d^5(dx)^{11/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315a^{3/4}d^{21/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315a^{3/4}d^{21/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{23/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-1045*d^7*(d*x)^(7/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (d*(d*x)^(19/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (19*d^3*(d*x)^(15/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (95*d^5*(d*x)^(11/2))/(256*b^3*(a + b*x^2
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*d^9*(d*x)^(3/2)*(a + b
*x^2))/(3072*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*a^(3/4)
*d^(21/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(
1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b
^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(23/4)*Sqrt
[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*a^(3/4)*d^(21/2)*(a + b*x^2)
*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[d*x]])/(4096*Sqrt[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (7315*a^(3/4)*d^(21/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + S
qrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt
[2]*b^(23/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.326894, size = 350, normalized size = 0.58

$$(dx)^{21/2}\left(a+bx^2\right)\left(-21945\sqrt{2}a^{3/4}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)+21945\sqrt{2}a^{3/4}\left(a+bx^2\right)^4\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(21/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(21/2)*(a + b*x^2)*(-3072*a^4*b^(3/4)*x^(3/2) + 17152*a^3*
b^(3/4)*x^(3/2)*(a + b*x^2) - 42144*a^2*b^(3/4)*x^(3/2)*(a + b*x^
2)^2 + 70200*a*b^(3/4)*x^(3/2)*(a + b*x^2)^3 + 16384*b^(3/4)*x^(3
/2)*(a + b*x^2)^4 + 43890*Sqrt[2]*a^(3/4)*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 43890*Sqrt[2]*a^(3/4)*(a +
 b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 21945*S
qrt[2]*a^(3/4)*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x] + 21945*Sqrt[2]*a^(3/4)*(a + b*x^2)^4*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*b
^(23/4)*x^(21/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.035, size = 1166, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(21/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(16384*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^8*b^5*d^6-21945*2^(1
/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))
/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^8*a
*b^4*d^8-43890*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*x^8*a*b^4*d^8+43890*2^(1/2)*arctan((-2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*a*b^4*d^8+70200
*(a*d^2/b)^(1/4)*(d*x)^(15/2)*a*b^4+65536*(a*d^2/b)^(1/4)*(d*x)^(
3/2)*x^6*a*b^4*d^6-87780*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2)))*x^6*a^2*b^3*d^8-175560*2^(1/2)*arctan((2^
(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a^2*b^3*d
^8+175560*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(
```

```
a*d^2/b)^(1/4))*x^6*a^2*b^3*d^8+168456*(a*d^2/b)^(1/4)*(d*x)^(11/
2)*a^2*b^3*d^2+98304*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^4*a^2*b^3*d^6-
131670*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^
2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2)))*x^4*a^3*b^2*d^8-263340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^3*b^2*d^8+263340*2^(1/2)*
arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^
4*a^3*b^2*d^8+143464*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^3*b^2*d^4+6553
6*(a*d^2/b)^(1/4)*(d*x)^(3/2)*x^2*a^3*b^2*d^6-87780*2^(1/2)*ln(-(
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^2*a^4*b*d^8-
175560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^2*a^4*b*d^8+175560*2^(1/2)*arctan((-2^(1/2)*(d*x)^(
1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^4*b*d^8+58520*(a*d^2
/b)^(1/4)*(d*x)^(3/2)*a^4*b*d^6-21945*2^(1/2)*ln(-((a*d^2/b)^(1/4
)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^5*d^8-43890*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^5*d^8+4
3890*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2
/b)^(1/4))*a^5*d^8)*d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^6/((b*x^2+a)^
2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.305634, size = 586, normalized size = 0.98

$$-\frac{87780\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{1}{4}}\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)\arctan\left(\frac{\left(-\frac{a^3d^{42}}{b^{23}}\right)^{\frac{3}{4}}b^{17}}{\sqrt{dx}\,a^2d^{31}+\sqrt{a^4d^{63}x-\sqrt{-\frac{a^3d^{42}}{b^{23}}}\,a^3b^{11}d^{42}}}\right)+21945\left(-\frac{a^3d}{b^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(87780*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6
*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan((-a^3*d^42/b^23)^(
```

```
3/4)*b^17/(sqrt(d*x)*a^2*d^31 + sqrt(a^4*d^63*x - sqrt(-a^3*d^42/
b^23)*a^3*b^11*d^42))) + 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 +
4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419
980875*sqrt(d*x)*a^2*d^31 + 391419980875*(-a^3*d^42/b^23)^(3/4)*b
^17) - 21945*(-a^3*d^42/b^23)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^
2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(391419980875*sqrt(d*x)*a
^2*d^31 - 391419980875*(-a^3*d^42/b^23)^(3/4)*b^17) - 4*(2048*b^4
*d^10*x^9 + 16967*a*b^3*d^10*x^7 + 33345*a^2*b^2*d^10*x^5 + 26125
*a^3*b*d^10*x^3 + 7315*a^4*d^10*x)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*
x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(21/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296215, size = 575, normalized size = 0.96

$$\frac{1}{24576}d^9\left(\frac{16384\sqrt{dx}dx}{b^5\operatorname{sign}(bd^4x^2+ad^4)} - \frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^8\operatorname{sign}(bd^4x^2+ad^4)} - \frac{43890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{\ }\right.\right.}{2\left(\right.}\right.}{b^8\operatorname{sign}(bd^4x^2+ad^4)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(21/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^9*(16384*sqrt(d*x)*d*x/(b^5*sign(b*d^4*x^2 + a*d^4)) -
43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^
2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^8*sign(b*d^4*x^2 +
a*d^4)) - 43890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^8*sign(b
*d^4*x^2 + a*d^4)) + 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x + sqr
t(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*sign(b*d^4*x
^2 + a*d^4)) - 21945*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(
```

```
a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^8*sign(b*d^4*x^2 + a
*d^4)) + 8*(8775*sqrt(d*x)*a*b^3*d^9*x^7 + 21057*sqrt(d*x)*a^2*b^
2*d^9*x^5 + 17933*sqrt(d*x)*a^3*b*d^9*x^3 + 5267*sqrt(d*x)*a^4*d^
9*x)/((b*d^2*x^2 + a*d^2)^4*b^5*sign(b*d^4*x^2 + a*d^4)))
```

3.771 $\int \frac{(dx)^{19/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=600

$$-\frac{17d^3(dx)^{13/2}}{96b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{3315\sqrt[4]{a}d^{19/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{3315\sqrt[4]{a}d^{19/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{3315\sqrt[4]{a}d^{19/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}d^{19/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{3315d^9\sqrt{dx}\left(a+bx^2\right)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (17*d^3*(d*x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (221*d^5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*d^9*Sqrt[d*x]*(a + b*x^
2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*a^(1/4)*d^
(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 1.04916, antiderivative size = 600, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{17d^3(dx)^{13/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{17/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{a}d^{19/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}b^{21/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315d^9\sqrt{dx}(a+bx^2)}{1024b^5\sqrt{a^2+2abx^2+b^2x^4}}-\frac{663d^7(dx)^{5/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{221d^5(dx)^{9/2}}{768b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-663*d^7*(d*x)^(5/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(17/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (17*d^3*(d*x)^(13/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (221*d^5*(d*x)^(9/2))/(768*b^3*(a + b*x^2)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*d^9*Sqrt[d*x]*(a + b*x^
2))/(1024*b^5*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*a^(1/4)*d^
(19/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4
)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*b^(21/4)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) + (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(4096*Sqrt[2]*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) - (3315*a^(1/4)*d^(19/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt
[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]
*b^(21/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.303498, size = 350, normalized size = 0.58

$$\frac{(dx)^{19/2}\left(a+bx^2\right)\left(-3072a^4\sqrt[4]{b}\sqrt{x}+16640a^3\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-38560a^2\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+55400a\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+491\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(19/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(19/2)*(a + b*x^2)*(-3072*a^4*b^(1/4)*Sqrt[x] + 16640*a^3*
b^(1/4)*Sqrt[x]*(a + b*x^2) - 38560*a^2*b^(1/4)*Sqrt[x]*(a + b*x^
2)^2 + 55400*a*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 + 49152*b^(1/4)*Sqrt
[x]*(a + b*x^2)^4 + 19890*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*ArcTan[1
- (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 19890*Sqrt[2]*a^(1/4)*(a +
 b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 9945*Sq
rt[2]*a^(1/4)*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)
*Sqrt[x] + Sqrt[b]*x] - 9945*Sqrt[2]*a^(1/4)*(a + b*x^2)^4*Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*b^(
21/4)*x^(19/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.035, size = 1212, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(19/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576*(9945*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^8*b^4*d^6+19890*(a*d^2/b)^(1/4)*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*x^8*b^4*d^6-19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d
*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+39780*(a*
d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2
/b)^(1/2)))*x^6*a*b^3*d^6+79560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2
^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^
6-79560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6-49152*(d*x)^(1/2)*x^8
```

```
*b^4*d^6+59670*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^6+119340*(a*d^2/b)^(1
/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*x^4*a^2*b^2*d^6-119340*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2
*d^6-55400*(d*x)^(13/2)*a*b^3-196608*(d*x)^(1/2)*x^6*a*b^3*d^6+39
780*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*
2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x
-(a*d^2/b)^(1/2)))*x^2*a^3*b*d^6+79560*(a*d^2/b)^(1/4)*2^(1/2)*ar
ctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a
^3*b*d^6-79560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/
2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-127640*(d*x)^(
9/2)*a^2*b^2*d^2-294912*(d*x)^(1/2)*x^4*a^2*b^2*d^6+9945*(a*d^2/b
)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(
1/2)))*a^4*d^6+19890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-19890*(a*d^2/b)
^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*a^4*d^6-105720*(d*x)^(5/2)*a^3*b*d^4-196608*(d*x)^(1
/2)*x^2*a^3*b*d^6-79560*(d*x)^(1/2)*a^4*d^6)*d^3*(b*x^2+a)/b^5/((
b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.303305, size = 540, normalized size = 0.9

$$39780 \left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}} \left(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5\right) \arctan\left(\frac{\left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}} b^5}{\sqrt{dx}\,d^9 + \sqrt{d^{19}x + \sqrt{-\frac{ad^{38}}{b^{21}}}\,b^{10}}}\right) - 9945 \left(-\frac{ad^{38}}{b^{21}}\right)^{\frac{1}{4}} \left(b^9x^8 + 4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(39780*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^
2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*arctan((-a*d^38/b^21)^(1/4)*
b^5/(sqrt(d*x)*d^9 + sqrt(d^19*x + sqrt(-a*d^38/b^21)*b^10))) - 9
945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 +
 4*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*d^9 + 3315*(-a*d^38/
b^21)^(1/4)*b^5) + 9945*(-a*d^38/b^21)^(1/4)*(b^9*x^8 + 4*a*b^8*x
^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)*log(3315*sqrt(d*x)*
d^9 - 3315*(-a*d^38/b^21)^(1/4)*b^5) + 4*(6144*b^4*d^9*x^8 + 3150
1*a*b^3*d^9*x^6 + 52819*a^2*b^2*d^9*x^4 + 37791*a^3*b*d^9*x^2 + 9
945*a^4*d^9)*sqrt(d*x))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 +
4*a^3*b^6*x^2 + a^4*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(19/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.293601, size = 578, normalized size = 0.96

$$-\frac{1}{24576}d^8\left(\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{b^6\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{994}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(19/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24576*d^8*(19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2
)*(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(b^6*s
ign(b*d^4*x^2 + a*d^4)) + 19890*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arcta
n(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^
(1/4))/(b^6*sign(b*d^4*x^2 + a*d^4)) + 9945*sqrt(2)*(a*b^3*d^2)^(
1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(b^6*sign(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2)^(1/4)*
```

```
d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(b^
6*sign(b*d^4*x^2 + a*d^4)) - 49152*sqrt(d*x)*d/(b^5*sign(b*d^4*x^
2 + a*d^4)) - 8*(6925*sqrt(d*x)*a*b^3*d^9*x^6 + 15955*sqrt(d*x)*a
^2*b^2*d^9*x^4 + 13215*sqrt(d*x)*a^3*b*d^9*x^2 + 3801*sqrt(d*x)*a
^4*d^9)/((b*d^2*x^2 + a*d^2)^4*b^5*sign(b*d^4*x^2 + a*d^4)))
```

3.772 $\int \frac{(dx)^{17/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{5d^3(dx)^{11/2}}{32b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{1155d^{17/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{1155d^{17/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{1155d^{17/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (5*d^3*(d*x)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) - (55*d^5*(d*x)^(7/2))/(256*b^3*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt
[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*d^(
17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2
]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*d^(17
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.967651, antiderivative size = 554, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{5d^3(dx)^{11/2}}{32b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{15/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155d^{17/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155d^{17/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155d^{17/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}\sqrt[4]{a}b^{19/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{385d^7(dx)^{3/2}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{55d^5(dx)^{7/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-385*d^7*(d*x)^(3/2))/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(15/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (5*d^3*(d*x)^(11/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) - (55*d^5*(d*x)^(7/2))/(256*b^3*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*d^(17/2)*(a + b*x^2)*ArcT
an[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt
[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*d^(
17/2)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)
*Sqrt[d])])/(2048*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + (1155*d^(17/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqr
t[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2
]*a^(1/4)*b^(19/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*d^(17
/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(1/4)*b^(19/4)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.338408, size = 324, normalized size = 0.58

$$(dx)^{17/2}\left(a+bx^2\right)\left(1024a^{13/4}b^{3/4}x^{3/2}+7392a^{5/4}b^{3/4}x^{3/2}\left(a+bx^2\right)^2-4352a^{9/4}b^{3/4}x^{3/2}\left(a+bx^2\right)-7144\sqrt[4]{a}b^{3/4}x^{3/2}\left(a+b\right.\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(17/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(17/2)*(a + b*x^2)*(1024*a^(13/4)*b^(3/4)*x^(3/2) - 4352*a
^(9/4)*b^(3/4)*x^(3/2)*(a + b*x^2) + 7392*a^(5/4)*b^(3/4)*x^(3/2)
*(a + b*x^2)^2 - 7144*a^(1/4)*b^(3/4)*x^(3/2)*(a + b*x^2)^3 - 231
0*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)] + 2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqr
t[x])/a^(1/4)] + 1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 1155*Sqrt[2]*(a + b*x^2)^
4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(8
192*a^(1/4)*b^(19/4)*x^(17/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.031, size = 1041, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(17/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/8192*(-1155*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d
*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2)))*x^8*b^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2
)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^8+2310*2^(1/2)*arct
an((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^
4*d^8+7144*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4-4620*2^(1/2)*ln(-((a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8-924
0*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*x^6*a*b^3*d^8+9240*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+14040*(a*d^2/b)^(1/
4)*(d*x)^(11/2)*a*b^3*d^2-6930*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1
```

```
/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8-13860*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*
b^2*d^8+13860*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8+11000*(a*d^2/b)^(1/4)*(d*x)^(
7/2)*a^2*b^2*d^4-4620*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)+(a*d^2/b)^(1/2)))*x^2*a^3*b*d^8-9240*2^(1/2)*arctan((2^(1/2)*(
d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8+9240*2
^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*x^2*a^3*b*d^8+3080*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6-115
5*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
*a^4*d^8-2310*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4)
)/(a*d^2/b)^(1/4))*a^4*d^8+2310*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1
/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)*d*(b*x^2+a)/(a*d^2
/b)^(1/4)/b^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304403, size = 552, normalized size = 1.

$$4620\left(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4\right)\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{34}}{ab^{19}}\right)^{\frac{3}{4}}ab^{14}}{\sqrt{dx}\,d^{25}+\sqrt{d^{51}x-\sqrt{-\frac{d^{34}}{ab^{19}}}\,ab^9d^{34}}}\right)+1155\left(b^8x^8+4\,ab^7x^6+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/4096*(4620*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x
^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*arctan((-d^34/(a*b^19))^(3/4
)*a*b^14/(sqrt(d*x)*d^25 + sqrt(d^51*x - sqrt(-d^34/(a*b^19))*a*b
^9*d^34))) + 1155*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*
b^5*x^2 + a^4*b^4)*(-d^34/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x
)*d^25 + 1540798875*(-d^34/(a*b^19))^(3/4)*a*b^14) - 1155*(b^8*x^
```

```
8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^34
/(a*b^19))^(1/4)*log(1540798875*sqrt(d*x)*d^25 - 1540798875*(-d^3
4/(a*b^19))^(3/4)*a*b^14) - 4*(893*b^3*d^8*x^7 + 1755*a*b^2*d^8*x
^5 + 1375*a^2*b*d^8*x^3 + 385*a^3*d^8*x)*sqrt(d*x))/(b^8*x^8 + 4*
a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(17/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294687, size = 548, normalized size = 0.99

$$\frac{1}{8192}d^7\left(\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7\operatorname{sign}\left(bd^4x^2+ad^4\right)}+\frac{2310\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^7\operatorname{sign}\left(bd^4x^2+ad^4\right)}-\frac{1155\sqrt{2}\left(ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(17/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^7*(2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^7*sign
(b*d^4*x^2 + a*d^4)) + 2310*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a*b^7*sign(b*d^4*x^2 + a*d^4)) - 1155*sqrt(2)*(a*b^3*d^2)^(3/4)
*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b
^7*sign(b*d^4*x^2 + a*d^4)) + 1155*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d
*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a*b^7*si
gn(b*d^4*x^2 + a*d^4)) - 8*(893*sqrt(d*x)*b^3*d^9*x^7 + 1755*sqrt
(d*x)*a*b^2*d^9*x^5 + 1375*sqrt(d*x)*a^2*b*d^9*x^3 + 385*sqrt(d*x
)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)^4*b^4*sign(b*d^4*x^2 + a*d^4)))
```

3.773 $\int \frac{(dx)^{15/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=554

$$-\frac{13d^3(dx)^{9/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (d*(d*x)^(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) - (13*d^3*(d*x)^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (39*d^5*(d*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(15/2)*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]
*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^(15/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(
3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^(15/2)*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.951387, antiderivative size = 554, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{13d^3(dx)^{9/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{13/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^7\sqrt{dx}}{1024b^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{39d^5(dx)^{5/2}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195d^{15/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195d^{15/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{3/4}b^{17/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (-195*d^7*Sqrt[d*x])/(1024*b^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (d*(d*x)^(13/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) - (13*d^3*(d*x)^(9/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4]) - (39*d^5*(d*x)^(5/2))/(256*b^3*(a + b*x^2)*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*d^(15/2)*(a + b*x^2)*ArcTan[
1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]
*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^(15/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(2048*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (195*d^(15/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*
Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(
3/4)*b^(17/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*d^(15/2)*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(3/4)*b^(17/4)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.343842, size = 324, normalized size = 0.58

$$(dx)^{15/2}\left(a+bx^2\right)\left(-14824a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+19616a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-12544a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+3072a^{15/4}\sqrt[4]{b}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(15/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(15/2)*(a + b*x^2)*(3072*a^(15/4)*b^(1/4)*Sqrt[x] - 12544*
a^(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) + 19616*a^(7/4)*b^(1/4)*Sqrt
[x]*(a + b*x^2)^2 - 14824*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 -
 1170*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/
a^(1/4)] + 1170*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)] - 585*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 585*Sqrt[2]*(a + b*x^2
)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/
(24576*a^(3/4)*b^(17/4)*x^(15/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.032, size = 1144, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(15/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)-d*x-(a*d^2/b)^(1/2)))*x^8*b^4*d^6+1170*(a*d^2/b)^(1/4)*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*x^8*b^4*d^6-1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+2340*(a*d^2/b
)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(
1/2)))*x^6*a*b^3*d^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6-4680
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+3510*(a*d^2/b)^(1/4)*2^(1/2)
*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((
```

```
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^4*a^2*
b^2*d^6+7020*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6-7020*(a*d^2/b)^
(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*x^4*a^2*b^2*d^6-14824*(d*x)^(13/2)*a*b^3+2340*(a*d^2/
b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2)))*x^2*a^3*b*d^6+4680*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-468
0*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-24856*(d*x)^(9/2)*a^2*b^2*d
^2+585*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/
2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-
d*x-(a*d^2/b)^(1/2)))*a^4*d^6+1170*(a*d^2/b)^(1/4)*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-1
170*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-17784*(d*x)^(5/2)*a^3*b*d^4-468
0*(d*x)^(1/2)*a^4*d^6)*d*(b*x^2+a)/a/b^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.30072, size = 548, normalized size = 0.99

$$-\frac{2340\left(b^{8}x^{8}+4\,ab^{7}x^{6}+6\,a^{2}b^{6}x^{4}+4\,a^{3}b^{5}x^{2}+a^{4}b^{4}\right)\left(-\frac{d^{30}}{a^{3}b^{17}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{30}}{a^{3}b^{17}}\right)^{\frac{1}{4}}ab^{4}}{\sqrt{dx}\,d^{7}+\sqrt{d^{15}x+\sqrt{-\frac{d^{30}}{a^{3}b^{17}}}\,a^{2}b^{8}}}\right)-585\left(b^{8}x^{8}+4\,ab^{7}x^{6}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(2340*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5
*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*arctan((-d^30/(a^3*b^17)
)^(1/4)*a*b^4/(sqrt(d*x)*d^7 + sqrt(d^15*x + sqrt(-d^30/(a^3*b^17
))*a^2*b^8))) - 585*(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^
```

```
3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17))^(1/4)*log(195*sqrt(d*x)*d
^7 + 195*(-d^30/(a^3*b^17))^(1/4)*a*b^4) + 585*(b^8*x^8 + 4*a*b^7
*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)*(-d^30/(a^3*b^17)
)^(1/4)*log(195*sqrt(d*x)*d^7 - 195*(-d^30/(a^3*b^17))^(1/4)*a*b^
4) + 4*(1853*b^3*d^7*x^6 + 3107*a*b^2*d^7*x^4 + 2223*a^2*b*d^7*x^
2 + 585*a^3*d^7)*sqrt(d*x))/(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^
4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(15/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.293368, size = 552, normalized size = 1.

$$\frac{1}{24576} d^6 \left(\frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5 \operatorname{sign}\left(bd^4x^2+ad^4\right)} + \frac{1170\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}} d \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{ab^5 \operatorname{sign}\left(bd^4x^2+ad^4\right)} + 585\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(15/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^6*(1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*
(sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a*b^5*s
ign(b*d^4*x^2 + a*d^4)) + 1170*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a*b^5*sign(b*d^4*x^2 + a*d^4)) + 585*sqrt(2)*(a*b^3*d^2)^(
1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a*b^5*sign(b*d^4*x^2 + a*d^4)) - 585*sqrt(2)*(a*b^3*d^2)^(1/4)
*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
*b^5*sign(b*d^4*x^2 + a*d^4)) - 8*(1853*sqrt(d*x)*b^3*d^9*x^6 + 3
107*sqrt(d*x)*a*b^2*d^9*x^4 + 2223*sqrt(d*x)*a^2*b*d^9*x^2 + 585*
sqrt(d*x)*a^3*d^9)/((b*d^2*x^2 + a*d^2)^4*b^4*sign(b*d^4*x^2 + a*
```

```
d^4)))
```

3.774 $\int \frac{(dx)^{13/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{11d^3(dx)^{7/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (11*d^3*(d*x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) - (77*d^5*(d*x)^(3/2))/(768*b^3*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2
]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(13/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5
/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(13/2)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.955171, antiderivative size = 557, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{11d^3(dx)^{7/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{11/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^5(dx)^{3/2}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^5(dx)^{3/2}}{768b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{13/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{13/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{5/4}b^{15/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d^5*(d*x)^(3/2))/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(11/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) - (11*d^3*(d*x)^(7/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) - (77*d^5*(d*x)^(3/2))/(768*b^3*(a + b*x^2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(13/2)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2
]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(13/2
)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqr
t[d])])/(2048*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + (77*d^(13/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*S
qrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5
/4)*b^(15/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(13/2)*(a +
 b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(5/4)*b^(15/4)*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.353759, size = 324, normalized size = 0.58

$$\frac{(dx)^{13/2}\left(a+bx^2\right)\left(-3072a^{13/4}b^{3/4}x^{3/2}-8352a^{5/4}b^{3/4}x^{3/2}\left(a+bx^2\right)^2+8960a^{9/4}b^{3/4}x^{3/2}\left(a+bx^2\right)+1848\sqrt[4]{a}b^{3/4}x^{3/2}\left(a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(13/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(13/2)*(a + b*x^2)*(-3072*a^(13/4)*b^(3/4)*x^(3/2) + 8960*
a^(9/4)*b^(3/4)*x^(3/2)*(a + b*x^2) - 8352*a^(5/4)*b^(3/4)*x^(3/2
)*(a + b*x^2)^2 + 1848*a^(1/4)*b^(3/4)*x^(3/2)*(a + b*x^2)^3 - 46
2*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)] + 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)] + 231*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a
^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 231*Sqrt[2]*(a + b*x^2)^4*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(2457
6*a^(5/4)*b^(15/4)*x^(13/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.031, size = 1046, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(13/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x
-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2)))*x^8*b^4*d^8+462*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^8-462*2^(1/2)*arctan((
-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^
8+1848*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+924*2^(1/2)*ln(-((a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(
1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+1848*2^(
1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*x^6*a*b^3*d^8-1848*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/
b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8-2808*(a*d^2/b)^(1/4)*(d*
x)^(11/2)*a*b^3*d^2+1386*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+2772*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8
-2772*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
```

```
2/b)^(1/4))*x^4*a^2*b^2*d^8-2200*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*
b^2*d^4+924*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-
(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2)))*x^2*a^3*b*d^8+1848*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8-1848*2^(1/2)*arc
tan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a
^3*b*d^8-616*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+231*2^(1/2)*ln
(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+462
*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*a^4*d^8-462*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d*(b*x^2+a)/(a*d^2/b)^(1/4)/b^4/
a/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304049, size = 579, normalized size = 1.04

$$\frac{924\left(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{26}}{a^5b^{15}}\right)^{\frac{3}{4}}a^4b^{11}}{\sqrt{dx}\,d^{19}+\sqrt{d^{39}x-\sqrt{-\frac{d^{26}}{a^5b^{15}}}\,a^3b^7d^{26}}}\right)+231\left(ab^7x^8+4\,a^2b^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")

[Out] 1/12288*(924*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b
^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*arctan((-d^26/(a^5*b^1
5))^(3/4)*a^4*b^11/(sqrt(d*x)*d^19 + sqrt(d^39*x - sqrt(-d^26/(a^
5*b^15))*a^3*b^7*d^26))) + 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3
*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(
456533*sqrt(d*x)*d^19 + 456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11)
 - 231*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2
 + a^5*b^3)*(-d^26/(a^5*b^15))^(1/4)*log(456533*sqrt(d*x)*d^19 -
456533*(-d^26/(a^5*b^15))^(3/4)*a^4*b^11) + 4*(231*b^3*d^6*x^7 -
```

```
351*a*b^2*d^6*x^5 - 275*a^2*b*d^6*x^3 - 77*a^3*d^6*x)*sqrt(d*x))/
(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*
b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(13/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296778, size = 552, normalized size = 0.99

$$\frac{1}{24576}d^5\left(\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{462\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^6\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{231\sqrt{2}\left(ab^3 \cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(13/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^5*(462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^6*si
gn(b*d^4*x^2 + a*d^4)) + 462*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(a^2*b^6*sign(b*d^4*x^2 + a*d^4)) - 231*sqrt(2)*(a*b^3*d^2)^(3/
4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^2*b^6*sign(b*d^4*x^2 + a*d^4)) + 231*sqrt(2)*(a*b^3*d^2)^(3/4)*l
n(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^2*b
^6*sign(b*d^4*x^2 + a*d^4)) + 8*(231*sqrt(d*x)*b^3*d^9*x^7 - 351*
sqrt(d*x)*a*b^2*d^9*x^5 - 275*sqrt(d*x)*a^2*b*d^9*x^3 - 77*sqrt(d
*x)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)^4*a*b^3*sign(b*d^4*x^2 + a*d^
4)))
```

3.775 $\int \frac{(dx)^{11/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=557

$$-\frac{3d^3(dx)^{5/2}}{32b^2\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (d*(d*x)^(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (3*d^3*(d*x)^(5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]) - (15*d^5*Sqrt[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)*(a + b*x^2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(7
/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^
(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rubi [A] time = 0.977092, antiderivative size = 557, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{3d^3(dx)^{5/2}}{32b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{9/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{15d^5\sqrt{dx}}{1024ab^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{15d^5\sqrt{dx}}{256b^3(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{11/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{11/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{7/4}b^{13/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (15*d^5*Sqrt[d*x])/(1024*a*b^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (d*(d*x)^(9/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (3*d^3*(d*x)^(5/2))/(32*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]) - (15*d^5*Sqrt[d*x])/(256*b^3*(a + b*x^2)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(11/2)*(a + b*x^2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(7
/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)*(a +
 b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])]
)/(2048*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (45*d^(11/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]
*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^
(13/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(11/2)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(7/4)*b^(13/4)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.762663, size = 325, normalized size = 0.58

$$d(dx)^{9/2}\left(a+bx^2\right)\left(120a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3-2272a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+2816a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-1024a^{15/4}\sqrt[4]{b}\sqrt{x}-\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(11/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (d*(d*x)^(9/2)*(a + b*x^2)*(-1024*a^(15/4)*b^(1/4)*Sqrt[x] + 2816
*a^(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) - 2272*a^(7/4)*b^(1/4)*Sqrt
[x]*(a + b*x^2)^2 + 120*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 9
0*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)] + 90*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[
x])/a^(1/4)] - 45*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 45*Sqrt[2]*(a + b*x^2)^4*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(8192*a^
(7/4)*b^(13/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.03, size = 1146, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(11/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(45*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)-d*x-(a*d^2/b)^(1/2)))*x^8*b^4*d^6+90*(a*d^2/b)^(1/4)*2^(1/2)*
arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8
*b^4*d^6-90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+180*(a*d^2/b)^(1/4)
*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*
x^6*a*b^3*d^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(
1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6-360*(a*d^2/b
)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^6*a*b^3*d^6+270*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(
```

```
1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^6+54
0*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6-540*(a*d^2/b)^(1/4)*2^(1/2
)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^4*a^2*b^2*d^6+120*(d*x)^(13/2)*a*b^3+180*(a*d^2/b)^(1/4)*2^(1/2
)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/(
(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^2*a^3
*b*d^6+360*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a
*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-360*(a*d^2/b)^(1/4)
*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^
(1/4))*x^2*a^3*b*d^6-1912*(d*x)^(9/2)*a^2*b^2*d^2+45*(a*d^2/b)^(1
/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)
))*a^4*d^6+90*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-90*(a*d^2/b)^(1/4)*2^(
1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4
))*a^4*d^6-1368*(d*x)^(5/2)*a^3*b*d^4-360*(d*x)^(1/2)*a^4*d^6)/d*
(b*x^2+a)/b^3/a^2/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302369, size = 572, normalized size = 1.03

$$-\frac{180\,\left(ab^7x^8+4\,a^2b^6x^6+6\,a^3b^5x^4+4\,a^4b^4x^2+a^5b^3\right)\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(-\frac{d^{22}}{a^7b^{13}}\right)^{\frac{1}{4}}a^2b^3}{\sqrt{dx}\,d^5+\sqrt{d^{11}x+\sqrt{-\frac{d^{22}}{a^7b^{13}}}\,a^4b^6}}\right)-45\,\left(ab^7x^8+4\,a^2b^6x^6\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/4096*(180*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b
^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*arctan((-d^22/(a^7*b^1
3))^(1/4)*a^2*b^3/(sqrt(d*x)*d^5 + sqrt(d^11*x + sqrt(-d^22/(a^7*
b^13))*a^4*b^6))) - 45*(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4
```

```
 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/(a^7*b^13))^(1/4)*log(45*sqrt(
d*x)*d^5 + 45*(-d^22/(a^7*b^13))^(1/4)*a^2*b^3) + 45*(a*b^7*x^8 +
 4*a^2*b^6*x^6 + 6*a^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)*(-d^22/
(a^7*b^13))^(1/4)*log(45*sqrt(d*x)*d^5 - 45*(-d^22/(a^7*b^13))^(1
/4)*a^2*b^3) - 4*(15*b^3*d^5*x^6 - 239*a*b^2*d^5*x^4 - 171*a^2*b*
d^5*x^2 - 45*a^3*d^5)*sqrt(d*x))/(a*b^7*x^8 + 4*a^2*b^6*x^6 + 6*a
^3*b^5*x^4 + 4*a^4*b^4*x^2 + a^5*b^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(11/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.292394, size = 556, normalized size = 1.

$$\frac{1}{8192}d^4\left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^2b^4\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{45\sqrt{2}\left(ab^3d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(11/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d^4*(90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^2*b^4*si
gn(b*d^4*x^2 + a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1
/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4
))/(a^2*b^4*sign(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(1/
4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/
(a^2*b^4*sign(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(1/4)*
d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^
2*b^4*sign(b*d^4*x^2 + a*d^4)) + 8*(15*sqrt(d*x)*b^3*d^9*x^6 - 23
9*sqrt(d*x)*a*b^2*d^9*x^4 - 171*sqrt(d*x)*a^2*b*d^9*x^2 - 45*sqrt
(d*x)*a^3*d^9)/((b*d^2*x^2 + a*d^2)^4*a*b^3*sign(b*d^4*x^2 + a*d^
```

4)))

3.776 $\int \frac{(dx)^{9/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7d^3(dx)^{3/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{9/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{9/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) - (d*(d*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (7*d^3*(d*x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) + (7*d^3*(d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2
]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(9/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4
)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(9/2)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi [A] time = 0.970671, antiderivative size = 560, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{35d^3(dx)^{3/2}}{1024a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7d^3(dx)^{3/2}}{256ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{7d^3(dx)^{3/2}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{7/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}} + \frac{35d^{9/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35d^{9/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}} - \frac{35d^{9/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{9/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{9/4}b^{11/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*(d*x)^(3/2))/(1024*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
]) - (d*(d*x)^(7/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^
2*x^4]) - (7*d^3*(d*x)^(3/2))/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*
a*b*x^2 + b^2*x^4]) + (7*d^3*(d*x)^(3/2))/(256*a*b^2*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(9/2)*(a + b*x^2)*ArcTan
[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2
]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(9/2)
*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (35*d^(9/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqr
t[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4
)*b^(11/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(9/2)*(a + b*
x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(9/4)*b^(11/4)*Sqrt[a^2 + 2*a*b
*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.353007, size = 324, normalized size = 0.58

$$\frac{(dx)^{9/2}\left(a+bx^2\right)\left(3072a^{13/4}b^{3/4}x^{3/2}+672a^{5/4}b^{3/4}x^{3/2}\left(a+bx^2\right)^2-4864a^{9/4}b^{3/4}x^{3/2}\left(a+bx^2\right)+840\sqrt[4]{a}b^{3/4}x^{3/2}\left(a+bx^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(9/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(9/2)*(a + b*x^2)*(3072*a^(13/4)*b^(3/4)*x^(3/2) - 4864*a^
(9/4)*b^(3/4)*x^(3/2)*(a + b*x^2) + 672*a^(5/4)*b^(3/4)*x^(3/2)*(
a + b*x^2)^2 + 840*a^(1/4)*b^(3/4)*x^(3/2)*(a + b*x^2)^3 - 210*Sq
rt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
 + 210*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)] + 105*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 105*Sqrt[2]*(a + b*x^2)^4*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a^
(9/4)*b^(11/4)*x^(9/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.031, size = 1046, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(9/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x
-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2)))*x^8*b^4*d^8+210*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(
a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^8-210*2^(1/2)*arctan((
-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^
8+840*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+420*2^(1/2)*ln(-((a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+840*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^6*a*b^3*d^8-840*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+3192*(a*d^2/b)^(1/4)*(d*x)^
(11/2)*a*b^3*d^2+630*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+1260*2^(1/2)*arctan((2^(1/2)*
(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8-126
0*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)
```

```
^(1/4))*x^4*a^2*b^2*d^8-1000*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*
d^4+420*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d
^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(
1/2)))*x^2*a^3*b*d^8+840*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8-840*2^(1/2)*arctan((-
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d
^8-280*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+105*2^(1/2)*ln(-((a*
d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+210*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
a^4*d^8-210*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*a^4*d^8)/d^3*(b*x^2+a)/(a*d^2/b)^(1/4)/b^3/a^2/
((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301604, size = 594, normalized size = 1.06

$$\frac{420\left(a^{2} b^{6} x^{8}+4 a^{3} b^{5} x^{6}+6 a^{4} b^{4} x^{4}+4 a^{5} b^{3} x^{2}+a^{6} b^{2}\right)\left(-\frac{d^{18}}{a^{9} b^{11}}\right)^{\frac{1}{4}} \arctan\left(\frac{42875\, a^{7} b^{8}\left(-\frac{d^{18}}{a^{9} b^{11}}\right)^{\frac{3}{4}}}{42875 \sqrt{d x}\, d^{13}+\sqrt{-1838265625\, a^{5} b^{5} d^{18} \sqrt{-\frac{d^{18}}{a^{9} b^{11}}}+1838265625\, d^{27}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5
*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*arctan(42875*a^7*b^8
*(-d^18/(a^9*b^11))^(3/4)/(42875*sqrt(d*x)*d^13 + sqrt(-183826562
5*a^5*b^5*d^18*sqrt(-d^18/(a^9*b^11)) + 1838265625*d^27*x))) + 10
5*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 +
a^6*b^2)*(-d^18/(a^9*b^11))^(1/4)*log(42875*a^7*b^8*(-d^18/(a^9*b
^11))^(3/4) + 42875*sqrt(d*x)*d^13) - 105*(a^2*b^6*x^8 + 4*a^3*b^
5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^18/(a^9*b^11
))^(1/4)*log(-42875*a^7*b^8*(-d^18/(a^9*b^11))^(3/4) + 42875*sqrt
```

```
(d*x)*d^13) + 4*(105*b^3*d^4*x^7 + 399*a*b^2*d^4*x^5 - 125*a^2*b*
d^4*x^3 - 35*a^3*d^4*x)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b^5*x^6 +
 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(9/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.296657, size = 552, normalized size = 0.99

$$\frac{1}{24576}d^3\left(\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^5\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{105\sqrt{2}\left(ab^3d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(9/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^3*(210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sq
rt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^5*si
gn(b*d^4*x^2 + a*d^4)) + 210*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(a^3*b^5*sign(b*d^4*x^2 + a*d^4)) - 105*sqrt(2)*(a*b^3*d^2)^(3/
4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a
^3*b^5*sign(b*d^4*x^2 + a*d^4)) + 105*sqrt(2)*(a*b^3*d^2)^(3/4)*l
n(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^3*b
^5*sign(b*d^4*x^2 + a*d^4)) + 8*(105*sqrt(d*x)*b^3*d^9*x^7 + 399*
sqrt(d*x)*a*b^2*d^9*x^5 - 125*sqrt(d*x)*a^2*b*d^9*x^3 - 35*sqrt(d
*x)*a^3*d^9*x)/((b*d^2*x^2 + a*d^2)^4*a^2*b^2*sign(b*d^4*x^2 + a*
d^4)))
```

3.777 $\int \frac{(dx)^{7/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=560

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) - (5*d^3*Sqrt[d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]) + (5*d^3*Sqrt[d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(a + b*x^2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(1
1/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(2048*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
- (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])
```

Rubi [A] time = 0.98409, antiderivative size = 560, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{35d^3\sqrt{dx}}{3072a^2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{5d^3\sqrt{dx}}{768ab^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{5d^3\sqrt{dx}}{96b^2(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{5/2}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{35d^{7/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{35d^{7/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{11/4}b^{9/4}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (35*d^3*Sqrt[d*x])/(3072*a^2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 - (d*(d*x)^(5/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) - (5*d^3*Sqrt[d*x])/(96*b^2*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]) + (5*d^3*Sqrt[d*x])/(768*a*b^2*(a + b*x^2)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) - (35*d^(7/2)*(a + b*x^2)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(1
1/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*(a +
b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])
/(2048*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
- (35*d^(7/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x
 - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(
9/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (35*d^(7/2)*(a + b*x^2)*L
og[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[d*x]])/(4096*Sqrt[2]*a^(11/4)*b^(9/4)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])
```

Rubi in Sympy [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.323296, size = 324, normalized size = 0.58

$$(dx)^{7/2}\left(a+bx^2\right)\left(280a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+160a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2-4352a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)+3072a^{15/4}\sqrt[4]{b}\sqrt{x}-10$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(7/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(7/2)*(a + b*x^2)*(3072*a^(15/4)*b^(1/4)*Sqrt[x] - 4352*a^
(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) + 160*a^(7/4)*b^(1/4)*Sqrt[x]*
(a + b*x^2)^2 + 280*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 210*S
qrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
] + 210*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 105*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 105*Sqrt[2]*(a + b*x^2)^4*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a
^(11/4)*b^(9/4)*x^(7/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.03, size = 1146, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(105*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)-d*x-(a*d^2/b)^(1/2)))*x^8*b^4*d^6+210*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^8*b^4*d^6-210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1
/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+420*(a*d^2/b)^(
1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2
)))*x^6*a*b^3*d^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6-840*(a*d
^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+630*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(
d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/
```

```
b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^
6+1260*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6-1260*(a*d^2/b)^(1/4)*
2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*x^4*a^2*b^2*d^6+280*(d*x)^(13/2)*a*b^3+420*(a*d^2/b)^(1/4)*
2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1
/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x
^2*a^3*b*d^6+840*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1
/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-840*(a*d^2/b)
^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d
^2/b)^(1/4))*x^2*a^3*b*d^6+1000*(d*x)^(9/2)*a^2*b^2*d^2+105*(a*d^
2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b
)^(1/2)))*a^4*d^6+210*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-210*(a*d^2/b)^
(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*a^4*d^6-3192*(d*x)^(5/2)*a^3*b*d^4-840*(d*x)^(1/2)*a^
4*d^6)/d^3*(b*x^2+a)/b^2/a^3/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301979, size = 583, normalized size = 1.04

$$-\frac{420\left(a^2b^6x^8+4\,a^3b^5x^6+6\,a^4b^4x^4+4\,a^5b^3x^2+a^6b^2\right)\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}\arctan\left(\frac{a^3b^2\left(-\frac{d^{14}}{a^{11}b^9}\right)^{\frac{1}{4}}}{\sqrt{dx}d^3+\sqrt{a^6b^4\sqrt{-\frac{d^{14}}{a^{11}b^9}}+d^7x}}\right)-105\left(a^2b^6x^8+4\,a^3b^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(420*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^
5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*arctan(a^3*b^2*(-d^
14/(a^11*b^9))^(1/4)/(sqrt(d*x)*d^3 + sqrt(a^6*b^4*sqrt(-d^14/(a^
11*b^9)) + d^7*x))) - 105*(a^2*b^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^
```

```
4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)*(-d^14/(a^11*b^9))^(1/4)*log(35*
a^3*b^2*(-d^14/(a^11*b^9))^(1/4) + 35*sqrt(d*x)*d^3) + 105*(a^2*b
^6*x^8 + 4*a^3*b^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
*(-d^14/(a^11*b^9))^(1/4)*log(-35*a^3*b^2*(-d^14/(a^11*b^9))^(1/4
) + 35*sqrt(d*x)*d^3) - 4*(35*b^3*d^3*x^6 + 125*a*b^2*d^3*x^4 - 3
99*a^2*b*d^3*x^2 - 105*a^3*d^3)*sqrt(d*x))/(a^2*b^6*x^8 + 4*a^3*b
^5*x^6 + 6*a^4*b^4*x^4 + 4*a^5*b^3*x^2 + a^6*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.292966, size = 556, normalized size = 0.99

$$\frac{1}{24576}d^2\left(\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{210\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^3b^3\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{105\sqrt{2}\left(a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(7/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24576*d^2*(210*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(
sqrt(2)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^3*b^3*
sign(b*d^4*x^2 + a*d^4)) + 210*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan
(-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(
1/4))/(a^3*b^3*sign(b*d^4*x^2 + a*d^4)) + 105*sqrt(2)*(a*b^3*d^2)
^(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^3*b^3*sign(b*d^4*x^2 + a*d^4)) - 105*sqrt(2)*(a*b^3*d^2)^(
1/4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b)
)/(a^3*b^3*sign(b*d^4*x^2 + a*d^4)) + 8*(35*sqrt(d*x)*b^3*d^9*x^6
 + 125*sqrt(d*x)*a*b^2*d^9*x^4 - 399*sqrt(d*x)*a^2*b*d^9*x^2 - 10
5*sqrt(d*x)*a^3*d^9)/((b*d^2*x^2 + a*d^2)^4*a^2*b^2*sign(b*d^4*x^
```

```
2 + a*d^4)))
```

3.778 $\int \frac{(dx)^{5/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{9d(dx)^{3/2}}{256a^2b\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (d*(d*x)^(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (d*(d*x)^(3/2))/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (9*d*(d*x)^(3/2))/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(13/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (4
5*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rubi [A] time = 0.973764, antiderivative size = 557, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{9d(dx)^{3/2}}{256a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(dx)^{3/2}}{32ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(dx)^{3/2}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{45d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d^{5/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{13/4}b^{7/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{45d(dx)^{3/2}}{1024a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (45*d*(d*x)^(3/2))/(1024*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (d*(d*x)^(3/2))/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + (d*(d*x)^(3/2))/(32*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (9*d*(d*x)^(3/2))/(256*a^2*b*(a + b*x^2)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a + b*x^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(13/4)
*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (45*d^(5/2)*(a + b*x^
2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(20
48*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (4
5*d^(5/2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (45*d^(5/2)*(a + b*x^2)*Log[S
qrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt
[d*x]])/(4096*Sqrt[2]*a^(13/4)*b^(7/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.36038, size = 324, normalized size = 0.58

$$\frac{(dx)^{5/2}\left(a+bx^2\right)\left(-1024a^{13/4}b^{3/4}x^{3/2}+288a^{5/4}b^{3/4}x^{3/2}\left(a+bx^2\right)^2+256a^{9/4}b^{3/4}x^{3/2}\left(a+bx^2\right)+360\sqrt[4]{a}b^{3/4}x^{3/2}\left(a+bx^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(5/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(5/2)*(a + b*x^2)*(-1024*a^(13/4)*b^(3/4)*x^(3/2) + 256*a^
(9/4)*b^(3/4)*x^(3/2)*(a + b*x^2) + 288*a^(5/4)*b^(3/4)*x^(3/2)*(
a + b*x^2)^2 + 360*a^(1/4)*b^(3/4)*x^(3/2)*(a + b*x^2)^3 - 90*Sqr
t[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
+ 90*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)] + 45*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 45*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(8192*a^(13/4
)*b^(7/4)*x^(5/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.029, size = 1046, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/8192*(45*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(
a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b
)^(1/2)))*x^8*b^4*d^8+90*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^8-90*2^(1/2)*arctan((-2^(
1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^8+36
0*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+180*2^(1/2)*ln(-((a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+360*2^(1/2)*a
rctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*
a*b^3*d^8-360*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+1368*(a*d^2/b)^(1/4)*(d*x)^(11/
2)*a*b^3*d^2+270*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
```

```
*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+540*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8-540*2^(1
/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4)
)*x^4*a^2*b^2*d^8+1912*(a*d^2/b)^(1/4)*(d*x)^(7/2)*a^2*b^2*d^4+18
0*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
*x^2*a^3*b*d^8+360*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^
(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8-360*2^(1/2)*arctan((-2^(1/2
)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8-120
*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+45*2^(1/2)*ln(-((a*d^2/b)^
(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4*d^8+90*2^(1/2)*arcta
n((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8-
90*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*a^4*d^8)/d^5*(b*x^2+a)/(a*d^2/b)^(1/4)/b^2/a^3/((b*x^2+a
)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.301588, size = 583, normalized size = 1.05

$$180\left(a^3b^5x^8+4a^4b^4x^6+6a^5b^3x^4+4a^6b^2x^2+a^7b\right)\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{1}{4}}\arctan\left(\frac{91125\,a^{10}b^5\left(-\frac{d^{10}}{a^{13}b^7}\right)^{\frac{3}{4}}}{91125\sqrt{dx}\,d^7+\sqrt{-8303765625\,a^7b^3d^{10}\sqrt{-\frac{d^{10}}{a^{13}b^7}}+8303765625\,d^{15}x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")

[Out] 1/4096*(180*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*
b^2*x^2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*arctan(91125*a^10*b^5*(
-d^10/(a^13*b^7))^(3/4)/(91125*sqrt(d*x)*d^7 + sqrt(-8303765625*a
^7*b^3*d^10*sqrt(-d^10/(a^13*b^7)) + 8303765625*d^15*x))) + 45*(a
^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*
b)*(-d^10/(a^13*b^7))^(1/4)*log(91125*a^10*b^5*(-d^10/(a^13*b^7))
```

```
^(3/4) + 91125*sqrt(d*x)*d^7) - 45*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 +
 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^10/(a^13*b^7))^(1/4)*
log(-91125*a^10*b^5*(-d^10/(a^13*b^7))^(3/4) + 91125*sqrt(d*x)*d^
7) + 4*(45*b^3*d^2*x^7 + 171*a*b^2*d^2*x^5 + 239*a^2*b*d^2*x^3 -
15*a^3*d^2*x)*sqrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3
*x^4 + 4*a^6*b^2*x^2 + a^7*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{5}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(5/2)/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.296103, size = 549, normalized size = 0.99

$$\frac{1}{8192} d \left(\frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4\operatorname{sign}\left(bd^4x^2+ad^4\right)} + \frac{90\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^4b^4\operatorname{sign}\left(bd^4x^2+ad^4\right)} - \frac{45\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln}{a^4b} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(5/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8192*d*(90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2
)*(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^4*sign(b
*d^4*x^2 + a*d^4)) + 90*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqr
t(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^
4*b^4*sign(b*d^4*x^2 + a*d^4)) - 45*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(
d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4
*sign(b*d^4*x^2 + a*d^4)) + 45*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d*x -
 sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^4*b^4*sign
(b*d^4*x^2 + a*d^4)) + 8*(45*sqrt(d*x)*b^3*d^9*x^7 + 171*sqrt(d*x
```

```
)*a*b^2*d^9*x^5 + 239*sqrt(d*x)*a^2*b*d^9*x^3 - 15*sqrt(d*x)*a^3*
d^9*x)/((b*d^2*x^2 + a*d^2)^4*a^3*b*sign(b*d^4*x^2 + a*d^4)))
```

3.779 $\int \frac{(dx)^{3/2}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=557

$$\frac{11d\sqrt{dx}}{768a^2b\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
d*Sqrt[d*x])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (d*Sqrt[d*x])/(96*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (11*d*Sqrt[d*x])/(768*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(15/4)*b^(5/4
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt
[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.999639, antiderivative size = 557, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{11d\sqrt{dx}}{768a^2b(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\sqrt{dx}}{96ab(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\sqrt{dx}}{8b(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{77d^{3/2}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d^{3/2}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{15/4}b^{5/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{77d\sqrt{dx}}{3072a^3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (77*d*Sqrt[d*x])/(3072*a^3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (
d*Sqrt[d*x])/(8*b*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
+ (d*Sqrt[d*x])/(96*a*b*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (11*d*Sqrt[d*x])/(768*a^2*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*
x^2 + b^2*x^4]) - (77*d^(3/2)*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(
1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(15/4)*b^(5/4
)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)*ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt
[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (77*d^(3/
2)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a
^2 + 2*a*b*x^2 + b^2*x^4]) + (77*d^(3/2)*(a + b*x^2)*Log[Sqrt[a]*
Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])
/(4096*Sqrt[2]*a^(15/4)*b^(5/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.341839, size = 324, normalized size = 0.58

$$\frac{(dx)^{3/2}\left(a+bx^2\right)\left(616a^{3/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^3+352a^{7/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)^2+256a^{11/4}\sqrt[4]{b}\sqrt{x}\left(a+bx^2\right)-3072a^{15/4}\sqrt[4]{b}\sqrt{x}-231\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(3/2)*(a + b*x^2)*(-3072*a^(15/4)*b^(1/4)*Sqrt[x] + 256*a^
(11/4)*b^(1/4)*Sqrt[x]*(a + b*x^2) + 352*a^(7/4)*b^(1/4)*Sqrt[x]*
(a + b*x^2)^2 + 616*a^(3/4)*b^(1/4)*Sqrt[x]*(a + b*x^2)^3 - 462*S
qrt[2]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
] + 462*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)] - 231*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] + 231*Sqrt[2]*(a + b*x^2)^4*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a
^(15/4)*b^(5/4)*x^(3/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.03, size = 1146, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(231*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)-d*x-(a*d^2/b)^(1/2)))*x^8*b^4*d^6+462*(a*d^2/b)^(1/4)*2^(1/
2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
x^8*b^4*d^6-462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1
/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+924*(a*d^2/b)^(
1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/
b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2
)))*x^6*a*b^3*d^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d
*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6-1848*(a
*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4
))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+1386*(a*d^2/b)^(1/4)*2^(1/2)*ln
(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d
```

```
^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^4*a^2*b^2
*d^6+2772*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6-2772*(a*d^2/b)^(1/
4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*x^4*a^2*b^2*d^6+616*(d*x)^(13/2)*a*b^3+924*(a*d^2/b)^(1/
4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)
^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))
)*x^2*a^3*b*d^6+1848*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x
)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-1848*(a*d
^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+2200*(d*x)^(9/2)*a^2*b^2*d^2+231*
(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1
/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*
d^2/b)^(1/2)))*a^4*d^6+462*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2
)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6-462*(a*d^
2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*a^4*d^6+2808*(d*x)^(5/2)*a^3*b*d^4-1848*(d*x)^(1
/2)*a^4*d^6)/d^5*(b*x^2+a)/b/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304074, size = 545, normalized size = 0.98

$$-\frac{924\left(a^3b^5x^8+4\,a^4b^4x^6+6\,a^5b^3x^4+4\,a^6b^2x^2+a^7b\right)\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^4b\left(-\frac{d^6}{a^{15}b^5}\right)^{\frac{1}{4}}}{\sqrt{dx}\,d+\sqrt{a^8b^2\sqrt{-\frac{d^6}{a^{15}b^5}}+d^3x}}\right)-231\left(a^3b^5x^8+4\,a^4b^4x^6\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12288*(924*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^
6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*arctan(a^4*b*(-d^6/(a^
15*b^5))^(1/4)/(sqrt(d*x)*d + sqrt(a^8*b^2*sqrt(-d^6/(a^15*b^5))
+ d^3*x))) - 231*(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4
```

```
*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^(1/4)*log(77*a^4*b*(-d^6/
(a^15*b^5))^(1/4) + 77*sqrt(d*x)*d) + 231*(a^3*b^5*x^8 + 4*a^4*b^
4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^2*x^2 + a^7*b)*(-d^6/(a^15*b^5))^
(1/4)*log(-77*a^4*b*(-d^6/(a^15*b^5))^(1/4) + 77*sqrt(d*x)*d) - 4
*(77*b^3*d*x^6 + 275*a*b^2*d*x^4 + 351*a^2*b*d*x^2 - 231*a^3*d)*s
qrt(d*x))/(a^3*b^5*x^8 + 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 + 4*a^6*b^
2*x^2 + a^7*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**(3/2)/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [A] time = 0.291851, size = 549, normalized size = 0.99

$$\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^4b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^4b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$
$$+\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^4b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$
$$-\frac{77\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}d\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^4b^2\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$
$$+\frac{77\sqrt{dx}b^3d^9x^6+275\sqrt{dx}ab^2d^9x^4+351\sqrt{dx}a^2bd^9x^2-231\sqrt{dx}a^3d^9}{3072\left(bd^2x^2+ad^2\right)^4a^3b\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 77/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^4*b^2*sign(b*d^
```

```
4*x^2 + a*d^4)) + 77/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*d*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a^4*b^2*sign(b*d^4*x^2 + a*d^4)) + 77/8192*sqrt(2)*(a*b^3*d^2)^
(1/4)*d*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^4*b^2*sign(b*d^4*x^2 + a*d^4)) - 77/8192*sqrt(2)*(a*b^3*d^2
)^(1/4)*d*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^4*b^2*sign(b*d^4*x^2 + a*d^4)) + 1/3072*(77*sqrt(d*x)*b^3
*d^9*x^6 + 275*sqrt(d*x)*a*b^2*d^9*x^4 + 351*sqrt(d*x)*a^2*b*d^9*
x^2 - 231*sqrt(d*x)*a^3*d^9)/((b*d^2*x^2 + a*d^2)^4*a^3*b*sign(b*
d^4*x^2 + a*d^4))
```

3.780 $\int \frac{\sqrt{dx}}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{13(dx)^{3/2}}{96a^2d(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
(d*x)^(3/2)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (13*(d*x)^(3/2))/(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (39*(d*x)^(3/2))/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(17/4)
*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2
048*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a + b*x^2)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rubi [A] time = 0.984167, antiderivative size = 556, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{13(dx)^{3/2}}{96a^2d\,(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(dx)^{3/2}}{8ad\,(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{195\sqrt{d}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195\sqrt{d}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{17/4}b^{3/4}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{195(dx)^{3/2}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{39(dx)^{3/2}}{256a^3d\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (195*(d*x)^(3/2))/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) +
(d*x)^(3/2)/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + (13*(d*x)^(3/2))/(96*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]) + (39*(d*x)^(3/2))/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a + b*x^2)*ArcTan[1 - (Sqrt
[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(17/4)
*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (195*Sqrt[d]*(a + b*x
^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2
048*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
195*Sqrt[d]*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x -
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/
4)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (195*Sqrt[d]*(a + b*x^2)*Lo
g[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*S
qrt[d*x]])/(4096*Sqrt[2]*a^(17/4)*b^(3/4)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.310446, size = 319, normalized size = 0.57

$$\frac{\sqrt{dx}\left(a+bx^2\right)\left(3744a^{5/4}x^{3/2}\left(a+bx^2\right)^2+3328a^{9/4}x^{3/2}\left(a+bx^2\right)+3072a^{13/4}x^{3/2}+\frac{585\sqrt{2}\left(a+bx^2\right)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{b^{3/4}}\right.}{24576a^{17/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (Sqrt[d*x]*(a + b*x^2)*(3072*a^(13/4)*x^(3/2) + 3328*a^(9/4)*x^(3
/2)*(a + b*x^2) + 3744*a^(5/4)*x^(3/2)*(a + b*x^2)^2 + 4680*a^(1/
4)*x^(3/2)*(a + b*x^2)^3 - (1170*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 -
 (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4) + (1170*Sqrt[2]*(a +
 b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(3/4)
+ (585*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/b^(3/4) - (585*Sqrt[2]*(a + b*x^2)^4*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(3/4)))
/(24576*a^(17/4)*Sqrt[x]*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.029, size = 1046, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/24576*(585*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x
-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2
/b)^(1/2)))*x^8*b^4*d^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^8-1170*2^(1/2)*arctan
((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*
d^8+4680*(a*d^2/b)^(1/4)*(d*x)^(15/2)*b^4+2340*2^(1/2)*ln(-((a*d^
2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b
)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*x^6*a*b^3*d^8+4680*
2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1
/4))*x^6*a*b^3*d^8-4680*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d
^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^8+17784*(a*d^2/b)^(1/4)
*(d*x)^(11/2)*a*b^3*d^2+3510*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2
)*2^(1/2)+(a*d^2/b)^(1/2)))*x^4*a^2*b^2*d^8+7020*2^(1/2)*arctan((
```

```
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2
*d^8-7020*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(
a*d^2/b)^(1/4))*x^4*a^2*b^2*d^8+24856*(a*d^2/b)^(1/4)*(d*x)^(7/2)
*a^2*b^2*d^4+2340*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
a*d^2/b)^(1/2)))*x^2*a^3*b*d^8+4680*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^8-4680*2^(1/
2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*x^2*a^3*b*d^8+14824*(a*d^2/b)^(1/4)*(d*x)^(3/2)*a^3*b*d^6+585*2^
(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2
))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*a^4
*d^8+1170*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a
*d^2/b)^(1/4))*a^4*d^8-1170*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+
(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^8)/d^7*(b*x^2+a)/(a*d^2/b
)^(1/4)/b/a^4/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304031, size = 535, normalized size = 0.96

$$\frac{2340\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{1}{4}}\arctan\left(\frac{7414875\,a^{13}b^2\left(-\frac{d^2}{a^{17}b^3}\right)^{\frac{3}{4}}}{7414875\,\sqrt{dx}d+\sqrt{-54980371265625\,a^9bd^2\sqrt{-\frac{d^2}{a^{17}b^3}}+5498037126562}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12288*(2340*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^
7*b*x^2 + a^8)*(-d^2/(a^17*b^3))^(1/4)*arctan(7414875*a^13*b^2*(-
d^2/(a^17*b^3))^(3/4)/(7414875*sqrt(d*x)*d + sqrt(-54980371265625
*a^9*b*d^2*sqrt(-d^2/(a^17*b^3)) + 54980371265625*d^3*x))) + 585*
(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
*(-d^2/(a^17*b^3))^(1/4)*log(7414875*a^13*b^2*(-d^2/(a^17*b^3))^(
3/4) + 7414875*sqrt(d*x)*d) - 585*(a^4*b^4*x^8 + 4*a^5*b^3*x^6 +
```

```
6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)*(-d^2/(a^17*b^3))^(1/4)*log(-7
414875*a^13*b^2*(-d^2/(a^17*b^3))^(3/4) + 7414875*sqrt(d*x)*d) +
4*(585*b^3*x^7 + 2223*a*b^2*x^5 + 3107*a^2*b*x^3 + 1853*a^3*x)*sq
rt(d*x))/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x
^2 + a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.297449, size = 558, normalized size = 1.

$$\frac{195\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5b^3d\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{195\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5b^3d\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}-\frac{195\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5b^3d\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{195\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5b^3d\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{585\sqrt{dx}b^3d^8x^7+2223\sqrt{dx}ab^2d^8x^5+3107\sqrt{dx}a^2bd^8x^3+1853\sqrt{dx}a^3d^8x}{3072\left(bd^2x^2+ad^2\right)^4a^4\,\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] 195/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a
*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b^3*d*sign(b*d
^4*x^2 + a*d^4)) + 195/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a^5*b^3*d*sign(b*d^4*x^2 + a*d^4)) - 195/8192*sqrt(2)*(a*b^3*d^
2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
```

```
b))/(a^5*b^3*d*sign(b*d^4*x^2 + a*d^4)) + 195/8192*sqrt(2)*(a*b^3
*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d
^2/b))/(a^5*b^3*d*sign(b*d^4*x^2 + a*d^4)) + 1/3072*(585*sqrt(d*x
)*b^3*d^8*x^7 + 2223*sqrt(d*x)*a*b^2*d^8*x^5 + 3107*sqrt(d*x)*a^2
*b*d^8*x^3 + 1853*sqrt(d*x)*a^3*d^8*x)/((b*d^2*x^2 + a*d^2)^4*a^4
*sign(b*d^4*x^2 + a*d^4))
```

3.781 $\int \frac{1}{\sqrt{dx}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=556

$$\frac{5\sqrt{dx}}{32a^2d\left(a+bx^2\right)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad\left(a+bx^2\right)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sq
rt[d*x]/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
5*Sqrt[d*x])/(32*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (55*Sqrt[d*x])/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(
19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])
```

Rubi [A] time = 0.992613, antiderivative size = 556, normalized size of antiderivative = 1., number

of steps used = 15, number of rules used = 9, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{5\sqrt{dx}}{32a^2d(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\sqrt{dx}}{8ad(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{1155(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1155(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{19/4}\sqrt[4]{b}\sqrt{d}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{385\sqrt{dx}}{1024a^4d\sqrt{a^2+2abx^2+b^2x^4}}+\frac{55\sqrt{dx}}{256a^3d(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (385*Sqrt[d*x])/(1024*a^4*d*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + Sq
rt[d*x]/(8*a*d*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (
5*Sqrt[d*x])/(32*a^2*d*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4]) + (55*Sqrt[d*x])/(256*a^3*d*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4]) - (1155*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt
[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(
19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (1155*(a
 + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqr
t[a^2 + 2*a*b*x^2 + b^2*x^4]) + (1155*(a + b*x^2)*Log[Sqrt[a]*Sqr
t[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4
096*Sqrt[2]*a^(19/4)*b^(1/4)*Sqrt[d]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x
^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.315586, size = 319, normalized size = 0.57

$$\frac{\sqrt{x}\left(a+bx^2\right)\left(3080a^{3/4}\sqrt{x}\left(a+bx^2\right)^3+1760a^{7/4}\sqrt{x}\left(a+bx^2\right)^2+1280a^{11/4}\sqrt{x}\left(a+bx^2\right)+1024a^{15/4}\sqrt{x}-\frac{1155\sqrt{2}\left(a+bx^2\right)^4\log}{}\right.}{8192a^{19/4}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(a + b*x^2)*(1024*a^(15/4)*Sqrt[x] + 1280*a^(11/4)*Sqrt[
x]*(a + b*x^2) + 1760*a^(7/4)*Sqrt[x]*(a + b*x^2)^2 + 3080*a^(3/4
)*Sqrt[x]*(a + b*x^2)^3 - (2310*Sqrt[2]*(a + b*x^2)^4*ArcTan[1 -
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) + (2310*Sqrt[2]*(a +
b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)])/b^(1/4) -
 (1155*Sqrt[2]*(a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x] + Sqrt[b]*x])/b^(1/4) + (1155*Sqrt[2]*(a + b*x^2)^4*Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x])/b^(1/4))
)/(8192*a^(19/4)*Sqrt[d*x]*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.029, size = 1143, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^4+2*a*b*x^2+a^2)^(5/2)/(d*x)^(1/2),x)
```

```
[Out] 1/8192*(1155*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*
x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^
(1/2)-d*x-(a*d^2/b)^(1/2)))*x^8*b^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1
/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))
*x^8*b^4*d^6-2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^
(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^8*b^4*d^6+4620*(a*d^2/b
)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d
^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(
1/2)))*x^6*a*b^3*d^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6-9240
*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(
```

```
1/4))/(a*d^2/b)^(1/4))*x^6*a*b^3*d^6+6930*(a*d^2/b)^(1/4)*2^(1/2)
*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((
a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*x^4*a^2*
b^2*d^6+13860*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^4*a^2*b^2*d^6-13860*(a*d^2/b
)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*x^4*a^2*b^2*d^6+3080*(d*x)^(13/2)*a*b^3+4620*(a*d^2
/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2)))*x^2*a^3*b*d^6+9240*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6-92
40*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)
^(1/4))/(a*d^2/b)^(1/4))*x^2*a^3*b*d^6+11000*(d*x)^(9/2)*a^2*b^2*
d^2+1155*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)-d*x-(a*d^2/b)^(1/2)))*a^4*d^6+2310*(a*d^2/b)^(1/4)*2^(1/2)*arct
an((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6
-2310*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*a^4*d^6+14040*(d*x)^(5/2)*a^3*b*d^4+7
144*(d*x)^(1/2)*a^4*d^6)/d^7*(b*x^2+a)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299105, size = 536, normalized size = 0.96

$$-\frac{4620\left(a^4b^4dx^8+4a^5b^3dx^6+6a^6b^2dx^4+4a^7bdx^2+a^8d\right)\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}\arctan\left(\frac{a^5d\left(-\frac{1}{a^{19}bd^2}\right)^{\frac{1}{4}}}{\sqrt{a^{10}d^2\sqrt{-\frac{1}{a^{19}bd^2}}+dx}+\sqrt{dx}}\right)-1155\left(a^4b^4dx^8+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] -1/4096*(4620*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4
+ 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*arctan(a^5*d*(-1
```

```
/(a^19*b*d^2))^(1/4)/(sqrt(a^10*d^2*sqrt(-1/(a^19*b*d^2)) + d*x)
+ sqrt(d*x))) - 1155*(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2
*d*x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19*b*d^2))^(1/4)*log(a^5*d
*(-1/(a^19*b*d^2))^(1/4) + sqrt(d*x)) + 1155*(a^4*b^4*d*x^8 + 4*a
^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*b*d*x^2 + a^8*d)*(-1/(a^19
*b*d^2))^(1/4)*log(-a^5*d*(-1/(a^19*b*d^2))^(1/4) + sqrt(d*x)) -
4*(385*b^3*x^6 + 1375*a*b^2*x^4 + 1755*a^2*b*x^2 + 893*a^3)*sqrt(
d*x))/(a^4*b^4*d*x^8 + 4*a^5*b^3*d*x^6 + 6*a^6*b^2*d*x^4 + 4*a^7*
b*d*x^2 + a^8*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**4+2*a*b*x**2+a**2)**(5/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*((a + b*x**2)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.291159, size = 556, normalized size = 1.

$$\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sign}\left(bd^4x^2+ad^4\right)}+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^5bd\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$
$$+\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$
$$-\frac{1155\sqrt{2}\left(ab^3d^2\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^2}{b}}\right)}{8192\,a^5bd\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$
$$+\frac{385\sqrt{dx}b^3d^7x^6+1375\sqrt{dx}ab^2d^7x^4+1755\sqrt{dx}a^2bd^7x^2+893\sqrt{dx}a^3d^7}{1024\left(bd^2x^2+ad^2\right)^4a^4\mathrm{sign}\left(bd^4x^2+ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] 1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(
a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^5*b*d*sign(b*d^
4*x^2 + a*d^4)) + 1155/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/2
*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))
/(a^5*b*d*sign(b*d^4*x^2 + a*d^4)) + 1155/8192*sqrt(2)*(a*b^3*d^2
)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b
))/(a^5*b*d*sign(b*d^4*x^2 + a*d^4)) - 1155/8192*sqrt(2)*(a*b^3*d
^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2
/b))/(a^5*b*d*sign(b*d^4*x^2 + a*d^4)) + 1/1024*(385*sqrt(d*x)*b^
3*d^7*x^6 + 1375*sqrt(d*x)*a*b^2*d^7*x^4 + 1755*sqrt(d*x)*a^2*b*d
^7*x^2 + 893*sqrt(d*x)*a^3*d^7)/((b*d^2*x^2 + a*d^2)^4*a^4*sign(b
*d^4*x^2 + a*d^4))
```

3.782 $\int \frac{1}{(dx)^{3/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{17}{96a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}-\frac{3315\sqrt[4]{b}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\,(a+bx^2)}{1024a^5d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{221}{768a^3d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

```
[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8
*a*d*Sqrt[d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1
7/(96*a^2*d*Sqrt[d*x]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + 221/(768*a^3*d*Sqrt[d*x]*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^5*d*Sqrt[d*x]*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(21/4
)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(2048*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^
(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rubi [A] time = 1.10559, antiderivative size = 602, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{17}{96a^2d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ad\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}-\frac{3315\sqrt[4]{b}\,(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{3315\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\sqrt[4]{b}\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{21/4}d^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3315\,(a+bx^2)}{1024a^5d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{663}{1024a^4d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{221}{768a^3d\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 663/(1024*a^4*d*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1/(8
*a*d*Sqrt[d*x]*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1
7/(96*a^2*d*Sqrt[d*x]*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) + 221/(768*a^3*d*Sqrt[d*x]*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) - (3315*(a + b*x^2))/(1024*a^5*d*Sqrt[d*x]*Sqrt[a^2 +
2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(21/4
)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3315*b^(1/4)*(a + b
*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/
(2048*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) -
 (3315*b^(1/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*
x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^
(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (3315*b^(1/4)*(a + b*x^2
)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(21/4)*d^(3/2)*Sqrt[a^2 + 2*a*b*x^
2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.32584, size = 347, normalized size = 0.58

$$\frac{x\left(a+bx^2\right)\left(-14496a^{5/4}bx^2\left(a+bx^2\right)^2-7424a^{9/4}bx^2\left(a+bx^2\right)-3072a^{13/4}bx^2-49152\sqrt[4]{a}\left(a+bx^2\right)^4-30408\sqrt[4]{a}bx^2\left(a+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (x*(a + b*x^2)*(-3072*a^(13/4)*b*x^2 - 7424*a^(9/4)*b*x^2*(a + b*
x^2) - 14496*a^(5/4)*b*x^2*(a + b*x^2)^2 - 30408*a^(1/4)*b*x^2*(a
 + b*x^2)^3 - 49152*a^(1/4)*(a + b*x^2)^4 + 19890*Sqrt[2]*b^(1/4)
*Sqrt[x]*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)] - 19890*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^4*ArcTan[1 + (Sqr
t[2]*b^(1/4)*Sqrt[x])/a^(1/4)] - 9945*Sqrt[2]*b^(1/4)*Sqrt[x]*(a
+ b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b
]*x] + 9945*Sqrt[2]*b^(1/4)*Sqrt[x]*(a + b*x^2)^4*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a^(21/4)*(d*
x)^(3/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.036, size = 1076, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d*(9945*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)
-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2)))*(d*x)^(1/2)*x^8*b^4+19890*2^(1/2)*arctan((2^(1/2)
*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*x^8*b^
4-19890*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*
d^2/b)^(1/4))*(d*x)^(1/2)*x^8*b^4+79560*(a*d^2/b)^(1/4)*x^8*b^4+3
9780*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))*(d*x)^(1/2)*x^6*a*b^3+79560*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/
```

```
2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*x^6*a*b^3-79560*
2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*(d*x)^(1/2)*x^6*a*b^3+302328*(a*d^2/b)^(1/4)*x^6*a*b^3+5967
0*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^
(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))
*(d*x)^(1/2)*x^4*a^2*b^2+119340*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/
2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*x^4*a^2*b^2-1193
40*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b
)^(1/4))*(d*x)^(1/2)*x^4*a^2*b^2+422552*(a*d^2/b)^(1/4)*x^4*a^2*b
^2+39780*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*
d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^
(1/2)))*(d*x)^(1/2)*x^2*a^3*b+79560*2^(1/2)*arctan((2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*x^2*a^3*b-79
560*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*(d*x)^(1/2)*x^2*a^3*b+252008*(a*d^2/b)^(1/4)*x^2*a^3*b+
9945*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/
b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2
)))*(d*x)^(1/2)*a^4+19890*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(1/2)*a^4-19890*2^(1/2)*arct
an((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^
(1/2)*a^4+49152*(a*d^2/b)^(1/4)*a^4)*(b*x^2+a)/(a*d^2/b)^(1/4)/(d
*x)^(1/2)/a^5/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(3/2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.320028, size = 579, normalized size = 0.96

$$-\frac{39780\,b^4x^8 + 151164\,ab^3x^6 + 211276\,a^2b^2x^4 + 126004\,a^3bx^2 + 24576\,a^4 + 39780\,\left(a^5b^4dx^8 + 4\,a^6b^3dx^6 + 6\,a^7b^2dx^4 + 4\,a^8b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(3/2)),x, algorithm="fricas
```

```
[Out] -1/12288*(39780*b^4*x^8 + 151164*a*b^3*x^6 + 211276*a^2*b^2*x^4 +
 126004*a^3*b*x^2 + 24576*a^4 + 39780*(a^5*b^4*d*x^8 + 4*a^6*b^3*
d*x^6 + 6*a^7*b^2*d*x^4 + 4*a^8*b*d*x^2 + a^9*d)*sqrt(d*x)*(-b/(a
^21*d^6))^(1/4)*arctan(36429280875*a^16*d^5*(-b/(a^21*d^6))^(3/4)
/(36429280875*sqrt(d*x)*b + sqrt(-1327092505069640765625*a^11*b*d
^4*sqrt(-b/(a^21*d^6)) + 1327092505069640765625*b^2*d*x))) + 9945
*(a^5*b^4*d*x^8 + 4*a^6*b^3*d*x^6 + 6*a^7*b^2*d*x^4 + 4*a^8*b*d*x
^2 + a^9*d)*sqrt(d*x)*(-b/(a^21*d^6))^(1/4)*log(36429280875*a^16*
d^5*(-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b) - 9945*(a^5*
b^4*d*x^8 + 4*a^6*b^3*d*x^6 + 6*a^7*b^2*d*x^4 + 4*a^8*b*d*x^2 + a
^9*d)*sqrt(d*x)*(-b/(a^21*d^6))^(1/4)*log(-36429280875*a^16*d^5*(
-b/(a^21*d^6))^(3/4) + 36429280875*sqrt(d*x)*b))/((a^5*b^4*d*x^8
+ 4*a^6*b^3*d*x^6 + 6*a^7*b^2*d*x^4 + 4*a^8*b*d*x^2 + a^9*d)*sqrt
(d*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(\left(a+bx^2\right)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(3/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*((a + b*x**2)**2)**(5/2)), x)
```

GIAC/XCAS [A] time = 0.29172, size = 605, normalized size = 1.

$$-\frac{\frac{49152}{\sqrt{dx}\,a^5\,\mathrm{sign}(bd^4x^2+ad^4)}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\,\mathrm{sign}(bd^4x^2+ad^4)}+\frac{19890\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{a^6b^2d^2\,\mathrm{sign}(bd^4x^2+ad^4)}-\frac{9945\sqrt{2}\left(ab^3d^2\right)^{\frac{3}{4}}\ln\Big(\cdots}{a^6b^2d^2\mathrm{s}\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] -1/24576*(49152/(sqrt(d*x)*a^5*sign(b*d^4*x^2 + a*d^4)) + 19890*s
qrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1
/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^2*d^2*sign(b*d^4*x^2 +
 a*d^4)) + 19890*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*b^2*d
```

```
^2*sign(b*d^4*x^2 + a*d^4)) - 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(d
*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2*
d^2*sign(b*d^4*x^2 + a*d^4)) + 9945*sqrt(2)*(a*b^3*d^2)^(3/4)*ln(
d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/b))/(a^6*b^2
*d^2*sign(b*d^4*x^2 + a*d^4)) + 8*(3801*sqrt(d*x)*b^4*d^7*x^7 + 1
3215*sqrt(d*x)*a*b^3*d^7*x^5 + 15955*sqrt(d*x)*a^2*b^2*d^7*x^3 +
6925*sqrt(d*x)*a^3*b*d^7*x)/((b*d^2*x^2 + a*d^2)^4*a^5*sign(b*d^4
*x^2 + a*d^4)))/d
```

3.783 $\int \frac{1}{(dx)^{5/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=602

$$\frac{19}{96a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}+\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{7315b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315\left(a+bx^2\right)}{3072a^5d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{95}{256a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1
/(8*a*d*(d*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + 19/(96*a^2*d*(d*x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + 95/(256*a^3*d*(d*x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/(3072*a^5*d*(d*x)^(3/2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a + b*x^2)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[
2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3
/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(2048*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a
^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 1.11195, antiderivative size = 602, normalized size of antiderivative = 1., number of

steps used = 16, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{19}{96a^2d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^2}+\frac{1}{8ad(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)^3}$$
$$+\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{7315b^{3/4}\left(a+bx^2\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{7315b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{7315b^{3/4}\left(a+bx^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{23/4}d^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{7315\left(a+bx^2\right)}{3072a^5d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1045}{1024a^4d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{95}{256a^3d(dx)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1045/(1024*a^4*d*(d*x)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1
/(8*a*d*(d*x)^(3/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + 19/(96*a^2*d*(d*x)^(3/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4]) + 95/(256*a^3*d*(d*x)^(3/2)*(a + b*x^2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) - (7315*(a + b*x^2))/(3072*a^5*d*(d*x)^(3/2)*S
qrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (7315*b^(3/4)*(a + b*x^2)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[
2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3
/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*S
qrt[d])])/(2048*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b
^2*x^4]) + (7315*b^(3/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b
]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a
^(23/4)*d^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (7315*b^(3/4)*
(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x + Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(23/4)*d^(5/2)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.333684, size = 347, normalized size = 0.58

$$\frac{x\left(a+bx^2\right)\left(-16384a^{3/4}\left(a+bx^2\right)^4-42136a^{3/4}bx^2\left(a+bx^2\right)^3-17056a^{7/4}bx^2\left(a+bx^2\right)^2-7936a^{11/4}bx^2\left(a+bx^2\right)-307\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (x*(a + b*x^2)*(-3072*a^(15/4)*b*x^2 - 7936*a^(11/4)*b*x^2*(a + b
*x^2) - 17056*a^(7/4)*b*x^2*(a + b*x^2)^2 - 42136*a^(3/4)*b*x^2*(
a + b*x^2)^3 - 16384*a^(3/4)*(a + b*x^2)^4 + 43890*Sqrt[2]*b^(3/4
)*x^(3/2)*(a + b*x^2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)] - 43890*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^4*ArcTan[1 + (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 21945*Sqrt[2]*b^(3/4)*x^(3/2)*(
a + b*x^2)^4*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt
[b]*x] - 21945*Sqrt[2]*b^(3/4)*x^(3/2)*(a + b*x^2)^4*Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(24576*a^(23/4)*
(d*x)^(5/2)*((a + b*x^2)^2)^(5/2))
```

Maple [B] time = 0.035, size = 1193, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(5/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] -1/24576/d^3*(21945*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1
/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(
1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/2)*x^8*b^5+43890*(a*d
^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/
(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^8*b^5-43890*(a*d^2/b)^(1/4)*2^(1/2
)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*
(d*x)^(3/2)*x^8*b^5+87780*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(
d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/2)*x^6*a*b^4+17
```

```
5560*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b
)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^6*a*b^4-175560*(a*d^2/b)^
(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^
2/b)^(1/4))*(d*x)^(3/2)*x^6*a*b^4+131670*(a*d^2/b)^(1/4)*2^(1/2)*
ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a
*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/
2)*x^4*a^2*b^3+263340*(a*d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*
x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^4*a^2*b^
3-263340*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*
d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^4*a^2*b^3+58520*x^8*
a*b^4*d^2+87780*(a*d^2/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*
(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)
*2^(1/2)-d*x-(a*d^2/b)^(1/2)))*(d*x)^(3/2)*x^2*a^3*b^2+175560*(a*
d^2/b)^(1/4)*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*(d*x)^(3/2)*x^2*a^3*b^2-175560*(a*d^2/b)^(1/4)*
2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(
1/4))*(d*x)^(3/2)*x^2*a^3*b^2+209000*x^6*a^2*b^3*d^2+21945*(a*d^2
/b)^(1/4)*2^(1/2)*ln(-(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a
*d^2/b)^(1/2))/((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)
^(1/2)))*(d*x)^(3/2)*a^4*b+43890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((
2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)
*a^4*b-43890*(a*d^2/b)^(1/4)*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)
+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(3/2)*a^4*b+266760*x^4*a
^3*b^2*d^2+135736*x^2*a^4*b*d^2+16384*a^5*d^2)*(b*x^2+a)/(d*x)^(3
/2)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(5/2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.324128, size = 651, normalized size = 1.08

$$-\frac{29260\, b^4x^8 + 104500\, ab^3x^6 + 133380\, a^2b^2x^4 + 67868\, a^3bx^2 + 8192\, a^4 - 87780\, \left(a^5b^4d^2x^9 + 4\, a^6b^3d^2x^7 + 6\, a^7b^2d^2x^5 + 4\, a^8 b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(5/2)),x, algorithm="fricas
```

```
[Out] -1/12288*(29260*b^4*x^8 + 104500*a*b^3*x^6 + 133380*a^2*b^2*x^4 +
 67868*a^3*b*x^2 + 8192*a^4 - 87780*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*
d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*sqrt(d
*x)*(-b^3/(a^23*d^10))^(1/4)*arctan(a^6*d^3*(-b^3/(a^23*d^10))^(1
/4)/(sqrt(d*x)*b + sqrt(a^12*d^6*sqrt(-b^3/(a^23*d^10)) + b^2*d*x
))) + 21945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*
x^5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*sqrt(d*x)*(-b^3/(a^23*d^10))^(
1/4)*log(7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b
) - 21945*(a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^
5 + 4*a^8*b*d^2*x^3 + a^9*d^2*x)*sqrt(d*x)*(-b^3/(a^23*d^10))^(1/
4)*log(-7315*a^6*d^3*(-b^3/(a^23*d^10))^(1/4) + 7315*sqrt(d*x)*b)
)/((a^5*b^4*d^2*x^9 + 4*a^6*b^3*d^2*x^7 + 6*a^7*b^2*d^2*x^5 + 4*a
^8*b*d^2*x^3 + a^9*d^2*x)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(5/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.289074, size = 593, normalized size = 0.99

$$-\frac{7315\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}+2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^{6}d^{3}\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{7315\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}-2\,\sqrt{dx}\right)}{2\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}}\right)}{4096\,a^{6}d^{3}\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{7315\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\ln\left(dx+\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{8192\,a^{6}d^{3}\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}+\frac{7315\,\sqrt{2}\left(ab^{3}d^{2}\right)^{\frac{1}{4}}\ln\left(dx-\sqrt{2}\left(\frac{ad^{2}}{b}\right)^{\frac{1}{4}}\sqrt{dx}+\sqrt{\frac{ad^{2}}{b}}\right)}{8192\,a^{6}d^{3}\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{2}{3\,\sqrt{dx}a^{5}d^{2}x\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}-\frac{5267\,\sqrt{dx}b^{4}d^{6}x^{6}+17933\,\sqrt{dx}ab^{3}d^{6}x^{4}+21057\,\sqrt{dx}a^{2}b^{2}d^{6}x^{2}+8775\,\sqrt{dx}a^{3}bd^{6}}{3072\left(bd^{2}x^{2}+ad^{2}\right)^{4}a^{5}d\mathrm{sign}\left(bd^{4}x^{2}+ad^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(5/2)),x, algorithm="giac")
```

```
[Out] -7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^6*d^3*sign(b*d
^4*x^2 + a*d^4)) - 7315/4096*sqrt(2)*(a*b^3*d^2)^(1/4)*arctan(-1/
2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1/4)
)/(a^6*d^3*sign(b*d^4*x^2 + a*d^4)) - 7315/8192*sqrt(2)*(a*b^3*d^
2)^(1/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^2/
b))/(a^6*d^3*sign(b*d^4*x^2 + a*d^4)) + 7315/8192*sqrt(2)*(a*b^3*
d^2)^(1/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(a*d^
2/b))/(a^6*d^3*sign(b*d^4*x^2 + a*d^4)) - 2/3/(sqrt(d*x)*a^5*d^2*
x*sign(b*d^4*x^2 + a*d^4)) - 1/3072*(5267*sqrt(d*x)*b^4*d^6*x^6 +
 17933*sqrt(d*x)*a*b^3*d^6*x^4 + 21057*sqrt(d*x)*a^2*b^2*d^6*x^2
+ 8775*sqrt(d*x)*a^3*b*d^6)/((b*d^2*x^2 + a*d^2)^4*a^5*d*sign(b*d
^4*x^2 + a*d^4))
```

3.784 $\int \frac{1}{(dx)^{7/2}(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=649

$$\frac{7}{32a^2d(dx)^{5/2}(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{13923b^{5/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{13923b^{5/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{d}x\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$-\frac{13923b^{5/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{13923b(a+bx^2)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923(a+bx^2)}{5120a^5d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$
$$+\frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{119}{256a^3d(dx)^{5/2}(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1
/(8*a*d*(d*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + 7/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) + 119/(256*a^3*d*(d*x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))/(5120*a^5*d*(d*x)^(5/2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b*x^2))/(1024*a^
6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^(5/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (13923*b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25
/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 1.21121, antiderivative size = 649, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{7}{32a^2d(dx)^{5/2}(a+bx^2)^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1}{8ad(dx)^{5/2}(a+bx^2)^3\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}(a+bx^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}(a+bx^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{dx}+\sqrt{a}\sqrt{d}+\sqrt{b}\sqrt{dx}\right)}{4096\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923b^{5/4}(a+bx^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b^{5/4}(a+bx^2)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{dx}}{\sqrt[4]{a}\sqrt{d}}+1\right)}{2048\sqrt{2}a^{25/4}d^{7/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{13923b(a+bx^2)}{1024a^6d^3\sqrt{dx}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{13923(a+bx^2)}{5120a^5d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{1547}{1024a^4d(dx)^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{119}{256a^3d(dx)^{5/2}(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] 1547/(1024*a^4*d*(d*x)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + 1
/(8*a*d*(d*x)^(5/2)*(a + b*x^2)^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
) + 7/(32*a^2*d*(d*x)^(5/2)*(a + b*x^2)^2*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4]) + 119/(256*a^3*d*(d*x)^(5/2)*(a + b*x^2)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4]) - (13923*(a + b*x^2))/(5120*a^5*d*(d*x)^(5/2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b*(a + b*x^2))/(1024*a^
6*d^3*Sqrt[d*x]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (13923*b^(5/4)
*(a + b*x^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[d*x])/(a^(1/4)*Sqrt
[d])])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*
x^4]) + (13923*b^(5/4)*(a + b*x^2)*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sq
rt[d*x])/(a^(1/4)*Sqrt[d])])/(2048*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[
a^2 + 2*a*b*x^2 + b^2*x^4]) + (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt
[a]*Sqrt[d] + Sqrt[b]*Sqrt[d]*x - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*
x]])/(4096*Sqrt[2]*a^(25/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4]) - (13923*b^(5/4)*(a + b*x^2)*Log[Sqrt[a]*Sqrt[d] + Sqrt[b]*Sq
rt[d]*x + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[d*x]])/(4096*Sqrt[2]*a^(25
/4)*d^(7/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Exception raised: RecursionError
```

Mathematica [A] time = 0.33842, size = 375, normalized size = 0.58

$$\frac{x\left(a+bx^2\right)\left(5120a^{13/4}b^2x^4+52320a^{5/4}b^2x^4\left(a+bx^2\right)^2+19200a^{9/4}b^2x^4\left(a+bx^2\right)-16384a^{5/4}\left(a+bx^2\right)^4+69615\sqrt{2}b^{5/4}x}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/((d*x)^(7/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2)),x]
```

```
[Out] (x*(a + b*x^2)*(5120*a^(13/4)*b^2*x^4 + 19200*a^(9/4)*b^2*x^4*(a
+ b*x^2) + 52320*a^(5/4)*b^2*x^4*(a + b*x^2)^2 + 147320*a^(1/4)*b
^2*x^4*(a + b*x^2)^3 - 16384*a^(5/4)*(a + b*x^2)^4 + 409600*a^(1/
4)*b*x^2*(a + b*x^2)^4 - 139230*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^
2)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)] + 139230*Sqrt[
2]*b^(5/4)*x^(5/2)*(a + b*x^2)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt
[x])/a^(1/4)] + 69615*Sqrt[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^4*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x] - 69615*Sqr
t[2]*b^(5/4)*x^(5/2)*(a + b*x^2)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x] + Sqrt[b]*x]))/(40960*a^(25/4)*(d*x)^(7/2)*((a +
b*x^2)^2)^(5/2))
```

Maple [B] time = 0.04, size = 1124, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(7/2)/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] 1/40960/d^3*(69615*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/
2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+
(a*d^2/b)^(1/2)))*(d*x)^(5/2)*x^8*b^5+139230*2^(1/2)*arctan((2^(1
/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^8
*b^5-139230*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))
/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^8*b^5+556920*(a*d^2/b)^(1/4)*x^10
*b^5*d^2+278460*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-
d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*
```

```
d^2/b)^(1/2)))*(d*x)^(5/2)*x^6*a*b^4+556920*2^(1/2)*arctan((2^(1/
2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^6*
a*b^4-556920*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4)
)/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^6*a*b^4+2116296*(a*d^2/b)^(1/4)*
x^8*a*b^4*d^2+417690*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(
1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)^(1/2)*2^(1/2
)+(a*d^2/b)^(1/2)))*(d*x)^(5/2)*x^4*a^2*b^3+835380*2^(1/2)*arctan
((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/
2)*x^4*a^2*b^3-835380*2^(1/2)*arctan((-2^(1/2)*(d*x)^(1/2)+(a*d^2
/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^4*a^2*b^3+2957864*(a*d^
2/b)^(1/4)*x^6*a^2*b^3*d^2+278460*2^(1/2)*ln(-((a*d^2/b)^(1/4)*(d
*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)^(1/4)*(d*x)
^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(5/2)*x^2*a^3*b^2+556920*2
^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/
4))*(d*x)^(5/2)*x^2*a^3*b^2-556920*2^(1/2)*arctan((-2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*x^2*a^3*b^2+
1764056*(a*d^2/b)^(1/4)*x^4*a^3*b^2*d^2+69615*2^(1/2)*ln(-((a*d^2
/b)^(1/4)*(d*x)^(1/2)*2^(1/2)-d*x-(a*d^2/b)^(1/2))/(d*x+(a*d^2/b)
^(1/4)*(d*x)^(1/2)*2^(1/2)+(a*d^2/b)^(1/2)))*(d*x)^(5/2)*a^4*b+13
9230*2^(1/2)*arctan((2^(1/2)*(d*x)^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/
b)^(1/4))*(d*x)^(5/2)*a^4*b-139230*2^(1/2)*arctan((-2^(1/2)*(d*x)
^(1/2)+(a*d^2/b)^(1/4))/(a*d^2/b)^(1/4))*(d*x)^(5/2)*a^4*b+344064
*(a*d^2/b)^(1/4)*x^2*a^4*b*d^2-16384*(a*d^2/b)^(1/4)*a^5*d^2)*(b*
x^2+a)/(a*d^2/b)^(1/4)/(d*x)^(5/2)/a^6/((b*x^2+a)^2)^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(7/2)),x, algorithm="maxima
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.321289, size = 694, normalized size = 1.07

$$\frac{278460\, b^5 x^{10} + 1058148\, ab^4 x^8 + 1478932\, a^2 b^3 x^6 + 882028\, a^3 b^2 x^4 + 172032\, a^4 b x^2 - 8192\, a^5 + 278460\, \left(a^6 b^4 d^3 x^{10} + 4\, a^7 b^3 d^3 \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(7/2)),x, algorithm="fricas
```

```
[Out] 1/20480*(278460*b^5*x^10 + 1058148*a*b^4*x^8 + 1478932*a^2*b^3*x^
6 + 882028*a^3*b^2*x^4 + 172032*a^4*b*x^2 - 8192*a^5 + 278460*(a^
6*b^4*d^3*x^10 + 4*a^7*b^3*d^3*x^8 + 6*a^8*b^2*d^3*x^6 + 4*a^9*b*
d^3*x^4 + a^10*d^3*x^2)*sqrt(d*x)*(-b^5/(a^25*d^14))^(1/4)*arctan
(2698972561467*a^19*d^11*(-b^5/(a^25*d^14))^(3/4)/(2698972561467*
sqrt(d*x)*b^4 + sqrt(-7284452887551739093192089*a^13*b^5*d^8*sqrt
(-b^5/(a^25*d^14)) + 7284452887551739093192089*b^8*d*x))) + 69615
*(a^6*b^4*d^3*x^10 + 4*a^7*b^3*d^3*x^8 + 6*a^8*b^2*d^3*x^6 + 4*a^
9*b*d^3*x^4 + a^10*d^3*x^2)*sqrt(d*x)*(-b^5/(a^25*d^14))^(1/4)*lo
g(2698972561467*a^19*d^11*(-b^5/(a^25*d^14))^(3/4) + 269897256146
7*sqrt(d*x)*b^4) - 69615*(a^6*b^4*d^3*x^10 + 4*a^7*b^3*d^3*x^8 +
6*a^8*b^2*d^3*x^6 + 4*a^9*b*d^3*x^4 + a^10*d^3*x^2)*sqrt(d*x)*(-b
^5/(a^25*d^14))^(1/4)*log(-2698972561467*a^19*d^11*(-b^5/(a^25*d^
14))^(3/4) + 2698972561467*sqrt(d*x)*b^4))/((a^6*b^4*d^3*x^10 + 4
*a^7*b^3*d^3*x^8 + 6*a^8*b^2*d^3*x^6 + 4*a^9*b*d^3*x^4 + a^10*d^3
*x^2)*sqrt(d*x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(7/2)/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.294315, size = 635, normalized size = 0.98

$$\frac{13923 \sqrt{2} \left(ab^3d^2\right)^{\frac{3}{4}} \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}+2 \sqrt{dx}\right)}{2 \left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096 \, a^7bd^5\mathrm{sign}\left(bd^4x^2 + ad^4\right)} + \frac{13923 \sqrt{2} \left(ab^3d^2\right)^{\frac{3}{4}} \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{ad^2}{b}\right)^{\frac{1}{4}}-2 \sqrt{dx}\right)}{2 \left(\frac{ad^2}{b}\right)^{\frac{1}{4}}}\right)}{4096 \, a^7bd^5\mathrm{sign}\left(bd^4x^2 + ad^4\right)} - \frac{13923 \sqrt{2} \left(ab^3d^2\right)^{\frac{3}{4}} \ln\left(dx + \sqrt{2} \left(\frac{ad^2}{b}\right)^{\frac{1}{4}} \sqrt{dx} + \sqrt{\frac{ad^2}{b}}\right)}{8192 \, a^7bd^5\mathrm{sign}\left(bd^4x^2 + ad^4\right)} + \frac{13923 \sqrt{2} \left(ab^3d^2\right)^{\frac{3}{4}} \ln\left(dx - \sqrt{2} \left(\frac{ad^2}{b}\right)^{\frac{1}{4}} \sqrt{dx} + \sqrt{\frac{ad^2}{b}}\right)}{8192 \, a^7bd^5\mathrm{sign}\left(bd^4x^2 + ad^4\right)} + \frac{3683 \sqrt{dx}b^5d^7x^7 + 12357 \sqrt{dx}ab^4d^7x^5 + 14145 \sqrt{dx}a^2b^3d^7x^3 + 5599 \sqrt{dx}a^3b^2d^7x}{1024 \left(bd^2x^2 + ad^2\right)^4a^6d^3\mathrm{sign}\left(bd^4x^2 + ad^4\right)} + \frac{2 \left(25 \, bd^2x^2 - ad^2\right)}{5 \sqrt{dx}a^6d^5x^2\mathrm{sign}\left(bd^4x^2 + ad^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^(7/2)),x, algorithm="giac")
```

```
[Out] 13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*
(a*d^2/b)^(1/4) + 2*sqrt(d*x))/(a*d^2/b)^(1/4))/(a^7*b*d^5*sign(b
*d^4*x^2 + a*d^4)) + 13923/4096*sqrt(2)*(a*b^3*d^2)^(3/4)*arctan(
-1/2*sqrt(2)*(sqrt(2)*(a*d^2/b)^(1/4) - 2*sqrt(d*x))/(a*d^2/b)^(1
/4))/(a^7*b*d^5*sign(b*d^4*x^2 + a*d^4)) - 13923/8192*sqrt(2)*(a*
b^3*d^2)^(3/4)*ln(d*x + sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) + sqrt(
a*d^2/b))/(a^7*b*d^5*sign(b*d^4*x^2 + a*d^4)) + 13923/8192*sqrt(2
)*(a*b^3*d^2)^(3/4)*ln(d*x - sqrt(2)*(a*d^2/b)^(1/4)*sqrt(d*x) +
sqrt(a*d^2/b))/(a^7*b*d^5*sign(b*d^4*x^2 + a*d^4)) + 1/1024*(3683
*sqrt(d*x)*b^5*d^7*x^7 + 12357*sqrt(d*x)*a*b^4*d^7*x^5 + 14145*sq
rt(d*x)*a^2*b^3*d^7*x^3 + 5599*sqrt(d*x)*a^3*b^2*d^7*x)/((b*d^2*x
^2 + a*d^2)^4*a^6*d^3*sign(b*d^4*x^2 + a*d^4)) + 2/5*(25*b*d^2*x^
2 - a*d^2)/(sqrt(d*x)*a^6*d^5*x^2*sign(b*d^4*x^2 + a*d^4))
```

3.785 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^3 \, dx$

Optimal. Leaf size=150

$$\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3
 + m)) + (15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(
d*x)^(7 + m))/(d^7*(7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9
+ m)) + (6*a*b^5*(d*x)^(11 + m))/(d^11*(11 + m)) + (b^6*(d*x)^(13
 + m))/(d^13*(13 + m))
```

Rubi [A] time = 0.292102, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{a^6(dx)^{m+1}}{d(m+1)}+\frac{6a^5b(dx)^{m+3}}{d^3(m+3)}+\frac{15a^4b^2(dx)^{m+5}}{d^5(m+5)}+\frac{20a^3b^3(dx)^{m+7}}{d^7(m+7)}+\frac{15a^2b^4(dx)^{m+9}}{d^9(m+9)}+\frac{6ab^5(dx)^{m+11}}{d^{11}(m+11)}+\frac{b^6(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (a^6*(d*x)^(1 + m))/(d*(1 + m)) + (6*a^5*b*(d*x)^(3 + m))/(d^3*(3
 + m)) + (15*a^4*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (20*a^3*b^3*(
d*x)^(7 + m))/(d^7*(7 + m)) + (15*a^2*b^4*(d*x)^(9 + m))/(d^9*(9
+ m)) + (6*a*b^5*(d*x)^(11 + m))/(d^11*(11 + m)) + (b^6*(d*x)^(13
 + m))/(d^13*(13 + m))
```

Rubi in Sympy [A] time = 56.6513, size = 138, normalized size = 0.92

$$\frac{a^6\,(dx)^{m+1}}{d\,(m+1)}+\frac{6a^5b\,(dx)^{m+3}}{d^3\,(m+3)}+\frac{15a^4b^2\,(dx)^{m+5}}{d^5\,(m+5)}+\frac{20a^3b^3\,(dx)^{m+7}}{d^7\,(m+7)}+\frac{15a^2b^4\,(dx)^{m+9}}{d^9\,(m+9)}+\frac{6ab^5\,(dx)^{m+11}}{d^{11}\,(m+11)}+\frac{b^6\,(dx)^{m+13}}{d^{13}\,(m+13)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] a**6*(d*x)**(m + 1)/(d*(m + 1)) + 6*a**5*b*(d*x)**(m + 3)/(d**3*(
m + 3)) + 15*a**4*b**2*(d*x)**(m + 5)/(d**5*(m + 5)) + 20*a**3*b*
```

```
*3*(d*x)**(m + 7)/(d**7*(m + 7)) + 15*a**2*b**4*(d*x)**(m + 9)/(d
**9*(m + 9)) + 6*a*b**5*(d*x)**(m + 11)/(d**11*(m + 11)) + b**6*(
d*x)**(m + 13)/(d**13*(m + 13))
```

Mathematica [A] time = 0.0751618, size = 105, normalized size = 0.7

$$(dx)^m\left(\frac{a^6x}{m+1}+\frac{6a^5bx^3}{m+3}+\frac{15a^4b^2x^5}{m+5}+\frac{20a^3b^3x^7}{m+7}+\frac{15a^2b^4x^9}{m+9}+\frac{6ab^5x^{11}}{m+11}+\frac{b^6x^{13}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (d*x)^m*((a^6*x)/(1 + m) + (6*a^5*b*x^3)/(3 + m) + (15*a^4*b^2*x^
5)/(5 + m) + (20*a^3*b^3*x^7)/(7 + m) + (15*a^2*b^4*x^9)/(9 + m)
+ (6*a*b^5*x^11)/(11 + m) + (b^6*x^13)/(13 + m))
```

Maple [B] time = 0.013, size = 602, normalized size = 4.

$$(dx)^m\left(b^6m^6x^{12}+36\,b^6m^5x^{12}+6\,ab^5m^6x^{10}+505\,b^6m^4x^{12}+228\,ab^5m^5x^{10}+3480\,b^6m^3x^{12}+15\,a^2b^4m^6x^8+3330\,ab^5m^4x^{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] (d*x)^m*(b^6*m^6*x^12+36*b^6*m^5*x^12+6*a*b^5*m^6*x^10+505*b^6*m^
4*x^12+228*a*b^5*m^5*x^10+3480*b^6*m^3*x^12+15*a^2*b^4*m^6*x^8+33
30*a*b^5*m^4*x^10+12139*b^6*m^2*x^12+600*a^2*b^4*m^5*x^8+23640*a*
b^5*m^3*x^10+19524*b^6*m*x^12+20*a^3*b^3*m^6*x^6+9195*a^2*b^4*m^4
*x^8+84234*a*b^5*m^2*x^10+10395*b^6*x^12+840*a^3*b^3*m^5*x^6+6792
0*a^2*b^4*m^3*x^8+137412*a*b^5*m*x^10+15*a^4*b^2*m^6*x^4+13580*a^
3*b^3*m^4*x^6+249405*a^2*b^4*m^2*x^8+73710*a*b^5*x^10+660*a^4*b^2
*m^5*x^4+105840*a^3*b^3*m^3*x^6+415320*a^2*b^4*m*x^8+6*a^5*b*m^6*
x^2+11295*a^4*b^2*m^4*x^4+406700*a^3*b^3*m^2*x^6+225225*a^2*b^4*x
^8+276*a^5*b*m^5*x^2+94200*a^4*b^2*m^3*x^4+699720*a^3*b^3*m*x^6+a
^6*m^6+5010*a^5*b*m^4*x^2+389685*a^4*b^2*m^2*x^4+386100*a^3*b^3*x
^6+48*a^6*m^5+45240*a^5*b*m^3*x^2+711540*a^4*b^2*m*x^4+925*a^6*m^
4+208554*a^5*b*m^2*x^2+405405*a^4*b^2*x^4+9120*a^6*m^3+438324*a^5
*b*m*x^2+48259*a^6*m^2+270270*a^5*b*x^2+129072*a^6*m+135135*a^6)*
x/(13+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291417, size = 684, normalized size = 4.56

$$\frac{\left(\left(b^{6} m^{6} + 36\, b^{6} m^{5} + 505\, b^{6} m^{4} + 3480\, b^{6} m^{3} + 12139\, b^{6} m^{2} + 19524\, b^{6} m + 10395\, b^{6}\right) x^{13} + 6 \left(a b^{5} m^{6} + 38\, a b^{5} m^{5} + 555\, a b^{5} m^{4} + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^6*m^6 + 36*b^6*m^5 + 505*b^6*m^4 + 3480*b^6*m^3 + 12139*b^6*m
^2 + 19524*b^6*m + 10395*b^6)*x^13 + 6*(a*b^5*m^6 + 38*a*b^5*m^5
+ 555*a*b^5*m^4 + 3940*a*b^5*m^3 + 14039*a*b^5*m^2 + 22902*a*b^5*
m + 12285*a*b^5)*x^11 + 15*(a^2*b^4*m^6 + 40*a^2*b^4*m^5 + 613*a^
2*b^4*m^4 + 4528*a^2*b^4*m^3 + 16627*a^2*b^4*m^2 + 27688*a^2*b^4*
m + 15015*a^2*b^4)*x^9 + 20*(a^3*b^3*m^6 + 42*a^3*b^3*m^5 + 679*a
^3*b^3*m^4 + 5292*a^3*b^3*m^3 + 20335*a^3*b^3*m^2 + 34986*a^3*b^3
*m + 19305*a^3*b^3)*x^7 + 15*(a^4*b^2*m^6 + 44*a^4*b^2*m^5 + 753*
a^4*b^2*m^4 + 6280*a^4*b^2*m^3 + 25979*a^4*b^2*m^2 + 47436*a^4*b^
2*m + 27027*a^4*b^2)*x^5 + 6*(a^5*b*m^6 + 46*a^5*b*m^5 + 835*a^5*
b*m^4 + 7540*a^5*b*m^3 + 34759*a^5*b*m^2 + 73054*a^5*b*m + 45045*
a^5*b)*x^3 + (a^6*m^6 + 48*a^6*m^5 + 925*a^6*m^4 + 9120*a^6*m^3 +
 48259*a^6*m^2 + 129072*a^6*m + 135135*a^6)*x)*(d*x)^m/(m^7 + 49*
m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 1
35135)
```

Sympy [A] time = 19.4705, size = 3188, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

[Out] Piecewise(((-a**6/(12*x**12) - 3*a**5*b/(5*x**10) - 15*a**4*b**2/(8*x**8) - 10*a**3*b**3/(3*x**6) - 15*a**2*b**4/(4*x**4) - 3*a*b**5/x**2 + b**6*log(x))/d**13, Eq(m, -13)), ((-a**6/(10*x**10) - 3*a**5*b/(4*x**8) - 5*a**4*b**2/(2*x**6) - 5*a**3*b**3/x**4 - 15*a**2*b**4/(2*x**2) + 6*a*b**5*log(x) + b**6*x**2/2)/d**11, Eq(m, -11)), ((-a**6/(8*x**8) - a**5*b/x**6 - 15*a**4*b**2/(4*x**4) - 10*a**3*b**3/x**2 + 15*a**2*b**4*log(x) + 3*a*b**5*x**2 + b**6*x**4/4)/d**9, Eq(m, -9)), ((-a**6/(6*x**6) - 3*a**5*b/(2*x**4) - 15*a**4*b**2/(2*x**2) + 20*a**3*b**3*log(x) + 15*a**2*b**4*x**2/2 + 3*a*b**5*x**4/2 + b**6*x**6/6)/d**7, Eq(m, -7)), ((-a**6/(4*x**4) - 3*a**5*b/x**2 + 15*a**4*b**2*log(x) + 10*a**3*b**3*x**2 + 15*a**2*b**4*x**4/4 + a*b**5*x**6 + b**6*x**8/8)/d**5, Eq(m, -5)), ((-a**6/(2*x**2) + 6*a**5*b*log(x) + 15*a**4*b**2*x**2/2 + 5*a**3*b**3*x**4 + 5*a**2*b**4*x**6/2 + 3*a*b**5*x**8/4 + b**6*x**10/10)/d**3, Eq(m, -3)), ((a**6*log(x) + 3*a**5*b*x**2 + 15*a**4*b**2*x**4/4 + 10*a**3*b**3*x**6/3 + 15*a**2*b**4*x**8/8 + 3*a*b**5*x**10/5 + b**6*x**12/12)/d, Eq(m, -1)), (a**6*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**6*d**m*m**5*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 925*a**6*d**m*m**4*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 9120*a**6*d**m*m**3*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 48259*a**6*d**m*m**2*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a**6*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**6*d**m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 6*a**5*b*d**m*m**6*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 276*a**5*b*d**m*m**5*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 5010*a**5*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 45240*a**5*b*d**m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 208554*a**5*b*d**m*m**2*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 438324*a**5*b*d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 270270*a**5*b*d**m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 15*a**4*b**2*d**m*m**6*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 660*a**4*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 11295*a**4*b**2*d**m*m**4*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 94200*a**4*b**2*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 389685*a**4*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 711540*a**4*b**2*d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 405405*a**4*b**2*d**m*x**5*x

```
**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331
*m**2 + 264207*m + 135135) + 20*a**3*b**3*d**m*m**6*x**7*x**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
 264207*m + 135135) + 840*a**3*b**3*d**m*m**5*x**7*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
7*m + 135135) + 13580*a**3*b**3*d**m*m**4*x**7*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 105840*a**3*b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6
+ 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1
35135) + 406700*a**3*b**3*d**m*m**2*x**7*x**m/(m**7 + 49*m**6 + 9
73*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 699720*a**3*b**3*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) +
386100*a**3*b**3*d**m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 15*a**2*
b**4*d**m*m**6*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 600*a**2*b**4*d
**m*m**5*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 9195*a**2*b**4*d**m*m
**4*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**
3 + 177331*m**2 + 264207*m + 135135) + 67920*a**2*b**4*d**m*m**3*
x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 249405*a**2*b**4*d**m*m**2*x**
9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 415320*a**2*b**4*d**m*m*x**9*x**m
/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m*
*2 + 264207*m + 135135) + 225225*a**2*b**4*d**m*x**9*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264
207*m + 135135) + 6*a*b**5*d**m*m**6*x**11*x**m/(m**7 + 49*m**6 +
 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13
5135) + 228*a*b**5*d**m*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*m**
5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) +
3330*a*b**5*d**m*m**4*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 100
45*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 23640*a
*b**5*d**m*m**3*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 84234*a*b**5*
d**m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57
379*m**3 + 177331*m**2 + 264207*m + 135135) + 137412*a*b**5*d**m*
m*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
 + 177331*m**2 + 264207*m + 135135) + 73710*a*b**5*d**m*x**11*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m
**2 + 264207*m + 135135) + b**6*d**m*m**6*x**13*x**m/(m**7 + 49*m
**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135) + 36*b**6*d**m*m**5*x**13*x**m/(m**7 + 49*m**6 + 973*m
**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
+ 505*b**6*d**m*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 3480*b**
6*d**m*m**3*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 +
57379*m**3 + 177331*m**2 + 264207*m + 135135) + 12139*b**6*d**m*m
**2*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m*
*3 + 177331*m**2 + 264207*m + 135135) + 19524*b**6*d**m*m*x**13*x
**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331
*m**2 + 264207*m + 135135) + 10395*b**6*d**m*x**13*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
```

```
7*m + 135135), True))
```

GIAC/XCAS [A] time = 0.278666, size = 1276, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^3*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^6*m^6*x^13*e^(m*ln(d*x)) + 36*b^6*m^5*x^13*e^(m*ln(d*x)) + 6*a
*b^5*m^6*x^11*e^(m*ln(d*x)) + 505*b^6*m^4*x^13*e^(m*ln(d*x)) + 22
8*a*b^5*m^5*x^11*e^(m*ln(d*x)) + 3480*b^6*m^3*x^13*e^(m*ln(d*x))
+ 15*a^2*b^4*m^6*x^9*e^(m*ln(d*x)) + 3330*a*b^5*m^4*x^11*e^(m*ln(
d*x)) + 12139*b^6*m^2*x^13*e^(m*ln(d*x)) + 600*a^2*b^4*m^5*x^9*e^
(m*ln(d*x)) + 23640*a*b^5*m^3*x^11*e^(m*ln(d*x)) + 19524*b^6*m*x^
13*e^(m*ln(d*x)) + 20*a^3*b^3*m^6*x^7*e^(m*ln(d*x)) + 9195*a^2*b^
4*m^4*x^9*e^(m*ln(d*x)) + 84234*a*b^5*m^2*x^11*e^(m*ln(d*x)) + 10
395*b^6*x^13*e^(m*ln(d*x)) + 840*a^3*b^3*m^5*x^7*e^(m*ln(d*x)) +
67920*a^2*b^4*m^3*x^9*e^(m*ln(d*x)) + 137412*a*b^5*m*x^11*e^(m*ln
(d*x)) + 15*a^4*b^2*m^6*x^5*e^(m*ln(d*x)) + 13580*a^3*b^3*m^4*x^7
*e^(m*ln(d*x)) + 249405*a^2*b^4*m^2*x^9*e^(m*ln(d*x)) + 73710*a*b
^5*x^11*e^(m*ln(d*x)) + 660*a^4*b^2*m^5*x^5*e^(m*ln(d*x)) + 10584
0*a^3*b^3*m^3*x^7*e^(m*ln(d*x)) + 415320*a^2*b^4*m*x^9*e^(m*ln(d*
x)) + 6*a^5*b*m^6*x^3*e^(m*ln(d*x)) + 11295*a^4*b^2*m^4*x^5*e^(m*
ln(d*x)) + 406700*a^3*b^3*m^2*x^7*e^(m*ln(d*x)) + 225225*a^2*b^4*
x^9*e^(m*ln(d*x)) + 276*a^5*b*m^5*x^3*e^(m*ln(d*x)) + 94200*a^4*b
^2*m^3*x^5*e^(m*ln(d*x)) + 699720*a^3*b^3*m*x^7*e^(m*ln(d*x)) + a
^6*m^6*x*e^(m*ln(d*x)) + 5010*a^5*b*m^4*x^3*e^(m*ln(d*x)) + 38968
5*a^4*b^2*m^2*x^5*e^(m*ln(d*x)) + 386100*a^3*b^3*x^7*e^(m*ln(d*x)
) + 48*a^6*m^5*x*e^(m*ln(d*x)) + 45240*a^5*b*m^3*x^3*e^(m*ln(d*x)
) + 711540*a^4*b^2*m*x^5*e^(m*ln(d*x)) + 925*a^6*m^4*x*e^(m*ln(d*
x)) + 208554*a^5*b*m^2*x^3*e^(m*ln(d*x)) + 405405*a^4*b^2*x^5*e^(
m*ln(d*x)) + 9120*a^6*m^3*x*e^(m*ln(d*x)) + 438324*a^5*b*m*x^3*e^
(m*ln(d*x)) + 48259*a^6*m^2*x*e^(m*ln(d*x)) + 270270*a^5*b*x^3*e^
(m*ln(d*x)) + 129072*a^6*m*x*e^(m*ln(d*x)) + 135135*a^6*x*e^(m*ln
(d*x)))/(m^7 + 49*m^6 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*
m^2 + 264207*m + 135135)
```

3.786 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^2 \, dx$

Optimal. Leaf size=104

$$\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3
 + m)) + (6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)
^(7 + m))/(d^7*(7 + m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rubi [A] time = 0.193038, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{a^4(dx)^{m+1}}{d(m+1)}+\frac{4a^3b(dx)^{m+3}}{d^3(m+3)}+\frac{6a^2b^2(dx)^{m+5}}{d^5(m+5)}+\frac{4ab^3(dx)^{m+7}}{d^7(m+7)}+\frac{b^4(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (a^4*(d*x)^(1 + m))/(d*(1 + m)) + (4*a^3*b*(d*x)^(3 + m))/(d^3*(3
 + m)) + (6*a^2*b^2*(d*x)^(5 + m))/(d^5*(5 + m)) + (4*a*b^3*(d*x)
^(7 + m))/(d^7*(7 + m)) + (b^4*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rubi in Sympy [A] time = 39.2428, size = 94, normalized size = 0.9

$$\frac{a^4\,(dx)^{m+1}}{d\,(m+1)}+\frac{4a^3b\,(dx)^{m+3}}{d^3\,(m+3)}+\frac{6a^2b^2\,(dx)^{m+5}}{d^5\,(m+5)}+\frac{4ab^3\,(dx)^{m+7}}{d^7\,(m+7)}+\frac{b^4\,(dx)^{m+9}}{d^9\,(m+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] a**4*(d*x)**(m + 1)/(d*(m + 1)) + 4*a**3*b*(d*x)**(m + 3)/(d**3*(
m + 3)) + 6*a**2*b**2*(d*x)**(m + 5)/(d**5*(m + 5)) + 4*a*b**3*(d
*x)**(m + 7)/(d**7*(m + 7)) + b**4*(d*x)**(m + 9)/(d**9*(m + 9))
```

Mathematica [A] time = 0.0469924, size = 73, normalized size = 0.7

$$(dx)^m\left(\frac{a^4x}{m+1}+\frac{4a^3bx^3}{m+3}+\frac{6a^2b^2x^5}{m+5}+\frac{4ab^3x^7}{m+7}+\frac{b^4x^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (d*x)^m*((a^4*x)/(1 + m) + (4*a^3*b*x^3)/(3 + m) + (6*a^2*b^2*x^5
)/(5 + m) + (4*a*b^3*x^7)/(7 + m) + (b^4*x^9)/(9 + m))
```

Maple [B] time = 0.011, size = 292, normalized size = 2.8

$$\frac{(dx)^m\left(b^4m^4x^8+16\,b^4m^3x^8+4\,ab^3m^4x^6+86\,b^4m^2x^8+72\,ab^3m^3x^6+176\,b^4mx^8+6\,a^2b^2m^4x^4+416\,ab^3m^2x^6+105\,b^4x^8+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] (d*x)^m*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^
8+72*a*b^3*m^3*x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*
x^6+105*b^4*x^8+120*a^2*b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x
^2+780*a^2*b^2*m^2*x^4+540*a*b^3*x^6+88*a^3*b*m^3*x^2+1800*a^2*b^
2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a^2*b^2*x^4+24*a^4*m^3+183
2*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m+945*a^4)*x/(9+
m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287238, size = 342, normalized size = 3.29

$$\frac{\left(\left(b^4m^4 + 16\,b^4m^3 + 86\,b^4m^2 + 176\,b^4m + 105\,b^4\right)x^9 + 4\,\left(ab^3m^4 + 18\,ab^3m^3 + 104\,ab^3m^2 + 222\,ab^3m + 135\,ab^3\right)x^7 + 6\,\left(a^2b^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 +
4*(a*b^3*m^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a
*b^3)*x^7 + 6*(a^2*b^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 3
00*a^2*b^2*m + 189*a^2*b^2)*x^5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 1
64*a^3*b*m^2 + 458*a^3*b*m + 315*a^3*b)*x^3 + (a^4*m^4 + 24*a^4*m
^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*(d*x)^m/(m^5 + 25*m^4
+ 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 8.4627, size = 1321, normalized size = 12.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Piecewise(((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x
**4) - 2*a*b**3/x**2 + b**4*log(x))/d**9, Eq(m, -9)), ((-a**4/(6*
x**6) - a**3*b/x**4 - 3*a**2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x
**2/2)/d**7, Eq(m, -7)), ((-a**4/(4*x**4) - 2*a**3*b/x**2 + 6*a**
2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4)/d**5, Eq(m, -5)), ((
-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4
 + b**4*x**6/6)/d**3, Eq(m, -3)), ((a**4*log(x) + 2*a**3*b*x**2 +
 3*a**2*b**2*x**4/2 + 2*a*b**3*x**6/3 + b**4*x**8/8)/d, Eq(m, -1)
), (a**4*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 24*a**4*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m
**3 + 950*m**2 + 1689*m + 945) + 206*a**4*d**m*m**2*x*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**4*d**m*m
*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 9
45*a**4*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*
m + 945) + 4*a**3*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*d**m*m**3*x**3*x**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*d*
*m*m**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 1832*a**3*b*d**m*m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
+ 950*m**2 + 1689*m + 945) + 1260*a**3*b*d**m*x**3*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 6*a**2*b**2*d**m*m
**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 94
```

```
5) + 120*a**2*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 780*a**2*b**2*d**m*m**2*x**5*x**m/(
m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 1800*a**2*
b**2*d**m*m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 168
9*m + 945) + 1134*a**2*b**2*d**m*x**5*x**m/(m**5 + 25*m**4 + 230*
m**3 + 950*m**2 + 1689*m + 945) + 4*a*b**3*d**m*m**4*x**7*x**m/(m
**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 72*a*b**3*d
**m*m**3*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
 + 945) + 416*a*b**3*d**m*m**2*x**7*x**m/(m**5 + 25*m**4 + 230*m*
*3 + 950*m**2 + 1689*m + 945) + 888*a*b**3*d**m*m*x**7*x**m/(m**5
 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 540*a*b**3*d**
m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
 + b**4*d**m*m**4*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 16*b**4*d**m*m**3*x**9*x**m/(m**5 + 25*m**4 +
230*m**3 + 950*m**2 + 1689*m + 945) + 86*b**4*d**m*m**2*x**9*x**m
/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 176*b**4
*d**m*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m
+ 945) + 105*b**4*d**m*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950
*m**2 + 1689*m + 945), True))
```

GIAC/XCAS [A] time = 0.273163, size = 628, normalized size = 6.04

$$\frac{b^4m^4x^9e^{(m\ln(dx))} + 16\,b^4m^3x^9e^{(m\ln(dx))} + 4\,ab^3m^4x^7e^{(m\ln(dx))} + 86\,b^4m^2x^9e^{(m\ln(dx))} + 72\,ab^3m^3x^7e^{(m\ln(dx))} + 176\,b^4mx^9e^{(m\ln(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^2*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^4*m^4*x^9*e^(m*ln(d*x)) + 16*b^4*m^3*x^9*e^(m*ln(d*x)) + 4*a*b
^3*m^4*x^7*e^(m*ln(d*x)) + 86*b^4*m^2*x^9*e^(m*ln(d*x)) + 72*a*b^
3*m^3*x^7*e^(m*ln(d*x)) + 176*b^4*m*x^9*e^(m*ln(d*x)) + 6*a^2*b^2
*m^4*x^5*e^(m*ln(d*x)) + 416*a*b^3*m^2*x^7*e^(m*ln(d*x)) + 105*b^
4*x^9*e^(m*ln(d*x)) + 120*a^2*b^2*m^3*x^5*e^(m*ln(d*x)) + 888*a*b
^3*m*x^7*e^(m*ln(d*x)) + 4*a^3*b*m^4*x^3*e^(m*ln(d*x)) + 780*a^2*
b^2*m^2*x^5*e^(m*ln(d*x)) + 540*a*b^3*x^7*e^(m*ln(d*x)) + 88*a^3*
b*m^3*x^3*e^(m*ln(d*x)) + 1800*a^2*b^2*m*x^5*e^(m*ln(d*x)) + a^4*
m^4*x*e^(m*ln(d*x)) + 656*a^3*b*m^2*x^3*e^(m*ln(d*x)) + 1134*a^2*
b^2*x^5*e^(m*ln(d*x)) + 24*a^4*m^3*x*e^(m*ln(d*x)) + 1832*a^3*b*m
*x^3*e^(m*ln(d*x)) + 206*a^4*m^2*x*e^(m*ln(d*x)) + 1260*a^3*b*x^3
*e^(m*ln(d*x)) + 744*a^4*m*x*e^(m*ln(d*x)) + 945*a^4*x*e^(m*ln(d*
x)))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

3.787 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right) \, dx$

Optimal. Leaf size=58

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 +
 m)) + (b^2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rubi [A] time = 0.062182, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{b^2(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 +
 m)) + (b^2*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rubi in Sympy [A] time = 22.5281, size = 49, normalized size = 0.84

$$\frac{a^2\left(dx\right)^{m+1}}{d\left(m+1\right)}+\frac{2ab\left(dx\right)^{m+3}}{d^3\left(m+3\right)}+\frac{b^2\left(dx\right)^{m+5}}{d^5\left(m+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] a**2*(d*x)**(m + 1)/(d*(m + 1)) + 2*a*b*(d*x)**(m + 3)/(d**3*(m +
 3)) + b**2*(d*x)**(m + 5)/(d**5*(m + 5))
```

Mathematica [A] time = 0.0304457, size = 41, normalized size = 0.71

$$(dx)^m\left(\frac{a^2x}{m+1}+\frac{2abx^3}{m+3}+\frac{b^2x^5}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (d*x)^m*((a^2*x)/(1 + m) + (2*a*b*x^3)/(3 + m) + (b^2*x^5)/(5 + m
))
```

Maple [A] time = 0.01, size = 94, normalized size = 1.6

$$\frac{(dx)^m\left(b^2m^2x^4+4\,b^2mx^4+2\,abm^2x^2+3\,b^2x^4+12\,abmx^2+a^2m^2+10\,abx^2+8\,a^2m+15\,a^2\right)x}{(5+m)\,(3+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] (d*x)^m*(b^2*m^2*x^4+4*b^2*m*x^4+2*a*b*m^2*x^2+3*b^2*x^4+12*a*b*m
*x^2+a^2*m^2+10*a*b*x^2+8*a^2*m+15*a^2)*x/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285003, size = 117, normalized size = 2.02

$$\frac{\left(\left(b^2m^2+4\,b^2m+3\,b^2\right)x^5+2\left(abm^2+6\,abm+5\,ab\right)x^3+\left(a^2m^2+8\,a^2m+15\,a^2\right)x\right)(dx)^m}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*
x^3 + (a^2*m^2 + 8*a^2*m + 15*a^2)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m
```

```
+ 15)
```

Sympy [A] time = 2.53314, size = 345, normalized size = 5.95

$$\begin{cases} \frac{-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\log{(x)}}{d^5} \\ \frac{-\frac{a^2}{2x^2}+2ab\log{(x)}+\frac{b^2x^2}{2}}{d^3} \\ \frac{a^2\log{(x)}+abx^2+\frac{b^2x^4}{4}}{d} \\ \frac{a^2d^mm^2xx^m}{m^3+9m^2+23m+15}+\frac{8a^2d^mmxx^m}{m^3+9m^2+23m+15}+\frac{15a^2d^mxx^m}{m^3+9m^2+23m+15}+\frac{2abd^mm^2x^3x^m}{m^3+9m^2+23m+15}+\frac{12abd^mmx^3x^m}{m^3+9m^2+23m+15}+\frac{10abd^mx^3x^m}{m^3+9m^2+23m+15}+\frac{b^2d^mm^2x^5x^m}{m^3+9m^2+23m+15}+\frac{4\ldots}{m^3\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Piecewise(((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x))/d**5, Eq(m,
-5)), ((-a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2)/d**3, Eq(m,
-3)), ((a**2*log(x) + a*b*x**2 + b**2*x**4/4)/d, Eq(m, -1)), (a**
2*d**m*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8*a**2*d**m*m*x*
x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*d**m*x*x**m/(m**3 + 9*
m**2 + 23*m + 15) + 2*a*b*d**m*m**2*x**3*x**m/(m**3 + 9*m**2 + 23
*m + 15) + 12*a*b*d**m*m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) +
10*a*b*d**m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + b**2*d**m*m**
2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*d**m*m*x**5*x**m
/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*d**m*x**5*x**m/(m**3 + 9*m*
*2 + 23*m + 15), True))
```

GIAC/XCAS [A] time = 0.266189, size = 207, normalized size = 3.57

$$\frac{b^2m^2x^5e^{(m\ln(dx))}+4\,b^2mx^5e^{(m\ln(dx))}+2\,abm^2x^3e^{(m\ln(dx))}+3\,b^2x^5e^{(m\ln(dx))}+12\,abmx^3e^{(m\ln(dx))}+a^2m^2xe^{(m\ln(dx))}+10\,abx^{3}\ldots}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^5*e^(m*ln(d*x)) + 4*b^2*m*x^5*e^(m*ln(d*x)) + 2*a*b*m^
2*x^3*e^(m*ln(d*x)) + 3*b^2*x^5*e^(m*ln(d*x)) + 12*a*b*m*x^3*e^(m
*ln(d*x)) + a^2*m^2*x*e^(m*ln(d*x)) + 10*a*b*x^3*e^(m*ln(d*x)) +
8*a^2*m*x*e^(m*ln(d*x)) + 15*a^2*x*e^(m*ln(d*x)))/(m^3 + 9*m^2 +
23*m + 15)
```

3.788 $\int \frac{(dx)^m}{a^2+2abx^2+b^2x^4} \, dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1} {}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/(a^2*d*(1 + m))
```

Rubi [A] time = 0.051882, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{(dx)^{m+1} {}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^2d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/(a^2*d*(1 + m))
```

Rubi in Sympy [A] time = 10.8576, size = 34, normalized size = 0.77

$$\frac{(dx)^{m+1} {}_2F_1\left(\begin{matrix} 2, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| -\frac{bx^2}{a}\right)}{a^2d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] (d*x)**(m + 1)*hyper((2, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*
*2*d*(m + 1))
```

Mathematica [A] time = 0.0268667, size = 42, normalized size = 0.95

$$\frac{x(dx)^m \, _2F_1\left(2,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4),x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^
2)/a)])/(a^2*(1 + m))
```

Maple [F] time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2)**2, x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b^2x^4+2\,abx^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.789 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^2}\,dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1}\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/(a^4*d*(1 + m))
```

Rubi [A] time = 0.0519988, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$

$$\frac{(dx)^{m+1}\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^4d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[4, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/(a^4*d*(1 + m))
```

Rubi in Sympy [A] time = 10.6726, size = 34, normalized size = 0.77

$$\frac{(dx)^{m+1}\,{}_2F_1\left(\begin{matrix} 4, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle|{-\frac{bx^2}{a}}\right)}{a^4d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] (d*x)**(m + 1)*hyper((4, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*
*4*d*(m + 1))
```

Mathematica [A] time = 0.0365097, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(4,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^4(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[4, (1 + m)/2, 1 + (1 + m)/2, -((b*x^
2)/a)])/(a^4*(1 + m))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b
*x^2 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^2, x)
```

3.790 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^3}\,dx$

Optimal. Leaf size=44

$$\frac{(dx)^{m+1}\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/(a^6*d*(1 + m))
```

Rubi [A] time = 0.0502732, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{(dx)^{m+1}\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^6d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] ((d*x)^(1 + m)*Hypergeometric2F1[6, (1 + m)/2, (3 + m)/2, -((b*x^
2)/a)])/(a^6*d*(1 + m))
```

Rubi in Sympy [A] time = 10.6674, size = 34, normalized size = 0.77

$$\frac{(dx)^{m+1}\,{}_2F_1\left(\begin{matrix}6,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{a^6d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] (d*x)**(m + 1)*hyper((6, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*
*6*d*(m + 1))
```

Mathematica [A] time = 0.0439317, size = 42, normalized size = 0.95

$$\frac{x(dx)^m\,{}_2F_1\left(6,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a^6(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^3,x]
```

```
[Out] (x*(d*x)^m*Hypergeometric2F1[6, (1 + m)/2, 1 + (1 + m)/2, -((b*x^
2)/a)])/(a^6*(1 + m))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{b^6x^{12}+6\,ab^5x^{10}+15\,a^2b^4x^8+20\,a^3b^3x^6+15\,a^4b^2x^4+6\,a^5bx^2+a^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^6*x^12 + 6*a*b^5*x^10 + 15*a^2*b^4*x^8 + 20*a
^3*b^3*x^6 + 15*a^4*b^2*x^4 + 6*a^5*b*x^2 + a^6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4 + 2\,abx^2 + a^2)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^3, x)
```

3.791 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{5/2}\,dx$

Optimal. Leaf size=313

$$\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+11}}{d^{11}(m+11)(a+bx^2)}+\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+9}}{d^9(m+9)(a+bx^2)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)(a+bx^2)}+\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)(a+bx^2)}+\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)(a+bx^2)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)(a+bx^2)}$$

```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a
 + b*x^2)) + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(d^3*(3 + m)*(a + b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^3
*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^7*(7 + m)*(a +
 b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rubi [A] time = 0.33743, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{b^5\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+11}}{d^{11}(m+11)(a+bx^2)}+\frac{5ab^4\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+9}}{d^9(m+9)(a+bx^2)}+\frac{10a^2b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)(a+bx^2)}+\frac{a^5\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)(a+bx^2)}+\frac{5a^4b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)(a+bx^2)}+\frac{10a^3b^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```


```
[Out] (a^5*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a
 + b*x^2)) + (5*a^4*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(d^3*(3 + m)*(a + b*x^2)) + (10*a^3*b^2*(d*x)^(5 + m)*Sqrt[a^
2 + 2*a*b*x^2 + b^2*x^4])/(d^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^3
*(d*x)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^7*(7 + m)*(a +
 b*x^2)) + (5*a*b^4*(d*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]
)/(d^9*(9 + m)*(a + b*x^2)) + (b^5*(d*x)^(11 + m)*Sqrt[a^2 + 2*a*
b*x^2 + b^2*x^4])/(d^11*(11 + m)*(a + b*x^2))
```

Rubi in Sympy [A] time = 64.5371, size = 298, normalized size = 0.95

$$\frac{3840a^5(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)(m+1)(m+3)(m+5)(m+7)(m+9)(m+11)} + \frac{1920a^4(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{d(m+3)(m+5)(m+7)(m+9)(m+11)} + \frac{480a^3(dx)^{m+1}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{d(m+5)(m+7)(m+9)(m+11)} + \frac{80a^2(dx)^{m+1}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{d(m+7)(m+9)(m+11)} + \frac{10a(dx)^{m+1}\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{d(m+9)(m+11)} + \frac{(dx)^{m+1}\left(a^2+2abx^2+b^2x^4\right)^{\frac{5}{2}}}{d(m+11)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] 3840*a**5*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(
a + b*x**2)*(m + 1)*(m + 3)*(m + 5)*(m + 7)*(m + 9)*(m + 11)) + 1
920*a**4*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(m
 + 3)*(m + 5)*(m + 7)*(m + 9)*(m + 11)) + 480*a**3*(d*x)**(m + 1)
*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(m + 5)*(m +
 7)*(m + 9)*(m + 11)) + 80*a**2*(d*x)**(m + 1)*(a**2 + 2*a*b*x**2
 + b**2*x**4)**(3/2)/(d*(m + 7)*(m + 9)*(m + 11)) + 10*a*(d*x)**(
m + 1)*(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**(3/2)/(d*(m
+ 9)*(m + 11)) + (d*x)**(m + 1)*(a**2 + 2*a*b*x**2 + b**2*x**4)**
(5/2)/(d*(m + 11))
```

Mathematica [A] time = 0.11613, size = 111, normalized size = 0.35

$$\frac{\left(\left(a+bx^2\right)^2\right)^{5/2}(dx)^m\left(\frac{a^5x}{m+1}+\frac{5a^4bx^3}{m+3}+\frac{10a^3b^2x^5}{m+5}+\frac{10a^2b^3x^7}{m+7}+\frac{5ab^4x^9}{m+9}+\frac{b^5x^{11}}{m+11}\right)}{\left(a+bx^2\right)^5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^m*((a + b*x^2)^2)^(5/2)*((a^5*x)/(1 + m) + (5*a^4*b*x^3)/(
3 + m) + (10*a^3*b^2*x^5)/(5 + m) + (10*a^2*b^3*x^7)/(7 + m) + (5
*a*b^4*x^9)/(9 + m) + (b^5*x^11)/(11 + m)))/(a + b*x^2)^5
```

Maple [A] time = 0.01, size = 453, normalized size = 1.5

$$\frac{\left(b^5m^5x^{10} + 25\,b^5m^4x^{10} + 5\,ab^4m^5x^8 + 230\,b^5m^3x^{10} + 135\,ab^4m^4x^8 + 950\,b^5m^2x^{10} + 10\,a^2b^3m^5x^6 + 1310\,ab^4m^3x^8 + 1689\,b^5 \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+
135*a*b^4*m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*
m^3*x^8+1689*b^5*m*x^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+94
5*b^5*x^10+10*a^3*b^2*m^5*x^4+3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*
x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^3*m^2*x^6+5775*a*b^4*x^8+5*a^
4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^3*m*x^6+165*a^4*b*m^
4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+2030*a^4*b*
m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+20790*
a^3*b^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*
b*x^2+9129*a^5*m+10395*a^5)*(d*x)^m*((b*x^2+a)^2)^(5/2)/(11+m)/(9
+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^5
```

Maxima [A] time = 0.702545, size = 328, normalized size = 1.05

$$\frac{\left(\left(m^5 + 25\,m^4 + 230\,m^3 + 950\,m^2 + 1689\,m + 945\right)b^5d^mx^{11} + 5\,\left(m^5 + 27\,m^4 + 262\,m^3 + 1122\,m^2 + 2041\,m + 1155\right)ab^4d^mx^9 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*d^m*x^11 +
 5*(m^5 + 27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*d^m*
x^9 + 10*(m^5 + 29*m^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*
b^3*d^m*x^7 + 10*(m^5 + 31*m^4 + 350*m^3 + 1730*m^2 + 3489*m + 20
79)*a^3*b^2*d^m*x^5 + 5*(m^5 + 33*m^4 + 406*m^3 + 2262*m^2 + 5353
*m + 3465)*a^4*b*d^m*x^3 + (m^5 + 35*m^4 + 470*m^3 + 3010*m^2 + 9
129*m + 10395)*a^5*d^m*x)*x^m/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3
+ 12139*m^2 + 19524*m + 10395)
```

Fricas [A] time = 0.290448, size = 498, normalized size = 1.59

$$\frac{\left(\left(b^5m^5 + 25\,b^5m^4 + 230\,b^5m^3 + 950\,b^5m^2 + 1689\,b^5m + 945\,b^5\right)x^{11} + 5\,\left(ab^4m^5 + 27\,ab^4m^4 + 262\,ab^4m^3 + 1122\,ab^4m^2 + 204\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m +
 945*b^5)*x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 11
22*a*b^4*m^2 + 2041*a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 +
 29*a^2*b^3*m^4 + 302*a^2*b^3*m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b
^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b^2*m^5 + 31*a^3*b^2*m^4 + 350
*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^2*m + 2079*a^3*b^2)*
x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 2262*a^4*b*m^
2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 470*
a^5*m^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*(d*x)^m/(m^6
+ 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 0.291179, size = 1312, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^5*m^5*x^11*e^(m*ln(d*x))*sign(b*x^2 + a) + 25*b^5*m^4*x^11*e^(
m*ln(d*x))*sign(b*x^2 + a) + 5*a*b^4*m^5*x^9*e^(m*ln(d*x))*sign(b
*x^2 + a) + 230*b^5*m^3*x^11*e^(m*ln(d*x))*sign(b*x^2 + a) + 135*
a*b^4*m^4*x^9*e^(m*ln(d*x))*sign(b*x^2 + a) + 950*b^5*m^2*x^11*e^
(m*ln(d*x))*sign(b*x^2 + a) + 10*a^2*b^3*m^5*x^7*e^(m*ln(d*x))*si
gn(b*x^2 + a) + 1310*a*b^4*m^3*x^9*e^(m*ln(d*x))*sign(b*x^2 + a)
+ 1689*b^5*m*x^11*e^(m*ln(d*x))*sign(b*x^2 + a) + 290*a^2*b^3*m^4
*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + 5610*a*b^4*m^2*x^9*e^(m*ln(d
*x))*sign(b*x^2 + a) + 945*b^5*x^11*e^(m*ln(d*x))*sign(b*x^2 + a)
 + 10*a^3*b^2*m^5*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 3020*a^2*b^
```

3*m^3*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + 10205*a*b^4*m*x^9*e^(m*ln(d*x))*sign(b*x^2 + a) + 310*a^3*b^2*m^4*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 13660*a^2*b^3*m^2*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + 5775*a*b^4*x^9*e^(m*ln(d*x))*sign(b*x^2 + a) + 5*a^4*b*m^5*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 3500*a^3*b^2*m^3*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 25770*a^2*b^3*m*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + 165*a^4*b*m^4*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 17300*a^3*b^2*m^2*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 14850*a^2*b^3*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + a^5*m^5*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 2030*a^4*b*m^3*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 34890*a^3*b^2*m*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 35*a^5*m^4*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 11310*a^4*b*m^2*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 20790*a^3*b^2*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 470*a^5*m^3*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 26765*a^4*b*m*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 3010*a^5*m^2*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 17325*a^4*b*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 9129*a^5*m*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 10395*a^5*x*e^(m*ln(d*x))*sign(b*x^2 + a))/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)

3.792 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx$

Optimal. Leaf size=205

$$\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)(a+bx^2)}+\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)(a+bx^2)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)(a+bx^2)}+\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)(a+bx^2)}$$

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a
 + b*x^2)) + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(d^3*(3 + m)*(a + b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^5*(5 + m)*(a + b*x^2)) + (b^3*(d*x)^(7
+ m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^7*(7 + m)*(a + b*x^2))
```

Rubi [A] time = 0.219708, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{3ab^2\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+5}}{d^5(m+5)(a+bx^2)}+\frac{3a^2b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)(a+bx^2)}+\frac{b^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+7}}{d^7(m+7)(a+bx^2)}+\frac{a^3\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a
 + b*x^2)) + (3*a^2*b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^
4])/(d^3*(3 + m)*(a + b*x^2)) + (3*a*b^2*(d*x)^(5 + m)*Sqrt[a^2 +
 2*a*b*x^2 + b^2*x^4])/(d^5*(5 + m)*(a + b*x^2)) + (b^3*(d*x)^(7
+ m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^7*(7 + m)*(a + b*x^2))
```

Rubi in Sympy [A] time = 29.9014, size = 182, normalized size = 0.89

$$\frac{48a^3(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{d(a+bx^2)(m+1)(m+3)(m+5)(m+7)}+\frac{24a^2(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{d(m+3)(m+5)(m+7)}+\frac{6a(dx)^{m+1}\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}{d(m+5)(m+7)}+\frac{(dx)^{m+1}\left(a^2+2abx^2+b^2x^4\right)^{\frac{3}{2}}}{d(m+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] 48*a**3*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(a
+ b*x**2)*(m + 1)*(m + 3)*(m + 5)*(m + 7)) + 24*a**2*(d*x)**(m +
1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(m + 3)*(m + 5)*(m + 7)
) + 6*a*(d*x)**(m + 1)*(a + b*x**2)*sqrt(a**2 + 2*a*b*x**2 + b**2
*x**4)/(d*(m + 5)*(m + 7)) + (d*x)**(m + 1)*(a**2 + 2*a*b*x**2 +
b**2*x**4)**(3/2)/(d*(m + 7))
```

Mathematica [A] time = 0.0721606, size = 79, normalized size = 0.39

$$\frac{\left(\left(a+bx^2\right)^2\right)^{3/2}(dx)^m\left(\frac{a^3x}{m+1}+\frac{3a^2bx^3}{m+3}+\frac{3ab^2x^5}{m+5}+\frac{b^3x^7}{m+7}\right)}{\left(a+bx^2\right)^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*x)/(1 + m) + (3*a^2*b*x^3)/(
3 + m) + (3*a*b^2*x^5)/(5 + m) + (b^3*x^7)/(7 + m)))/(a + b*x^2)^
3
```

Maple [A] time = 0.009, size = 199, normalized size = 1.

$$\frac{\left(b^3m^3x^6+9\,b^3m^2x^6+3\,ab^2m^3x^4+23\,b^3mx^6+33\,ab^2m^2x^4+15\,b^3x^6+3\,a^2bm^3x^2+93\,ab^2mx^4+39\,a^2bm^2x^2+63\,ax^4b^2+\ldots\right.}{(7+m)(5+m)(3+m)(1+m)\left(bx^2+a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] x*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^
2*m^2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*
x^2+63*a*b^2*x^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2
+71*a^3*m+105*a^3)*(d*x)^m*((b*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/
(1+m)/(b*x^2+a)^3
```

Maxima [A] time = 0.691084, size = 161, normalized size = 0.79

$$\frac{\left(\left(m^3+9\,m^2+23\,m+15\right)b^3d^mx^7+3\left(m^3+11\,m^2+31\,m+21\right)ab^2d^mx^5+3\left(m^3+13\,m^2+47\,m+35\right)a^2bd^mx^3+\left(m^3+15\,\ldots\right.\right.}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*d^m*x^7 + 3*(m^3 + 11*m^2 + 31*m +
 21)*a*b^2*d^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*d^m*x^3 +
 (m^3 + 15*m^2 + 71*m + 105)*a^3*d^m*x)*x^m/(m^4 + 16*m^3 + 86*m^
2 + 176*m + 105)
```

Fricas [A] time = 0.287, size = 215, normalized size = 1.05

$$\frac{\left(\left(b^3m^3 + 9\,b^3m^2 + 23\,b^3m + 15\,b^3\right)x^7 + 3\left(ab^2m^3 + 11\,ab^2m^2 + 31\,ab^2m + 21\,ab^2\right)x^5 + 3\left(a^2bm^3 + 13\,a^2bm^2 + 47\,a^2bm + 35\right.}{m^4 + 16\,m^3 + 86\,m^2 + 176\,m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 1
1*a*b^2*m^2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*
b*m^2 + 47*a^2*b*m + 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a
^3*m + 105*a^3)*x)*(d*x)^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [A] time = 0.27468, size = 562, normalized size = 2.74

$$\frac{b^3m^3x^7e^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right) + 9\,b^3m^2x^7e^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right) + 3\,ab^2m^3x^5e^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right) + 23\,b^3mx^7e^{(m\ln(dx))}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + 9*b^3*m^2*x^7*e^(m*l
n(d*x))*sign(b*x^2 + a) + 3*a*b^2*m^3*x^5*e^(m*ln(d*x))*sign(b*x^
2 + a) + 23*b^3*m*x^7*e^(m*ln(d*x))*sign(b*x^2 + a) + 33*a*b^2*m^
2*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 15*b^3*x^7*e^(m*ln(d*x))*si
gn(b*x^2 + a) + 3*a^2*b*m^3*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 9
3*a*b^2*m*x^5*e^(m*ln(d*x))*sign(b*x^2 + a) + 39*a^2*b*m^2*x^3*e^
(m*ln(d*x))*sign(b*x^2 + a) + 63*a*b^2*x^5*e^(m*ln(d*x))*sign(b*x
^2 + a) + a^3*m^3*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 141*a^2*b*m*x
^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 15*a^3*m^2*x*e^(m*ln(d*x))*sig
n(b*x^2 + a) + 105*a^2*b*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + 71*a
^3*m*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 105*a^3*x*e^(m*ln(d*x))*si
gn(b*x^2 + a))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

3.793 $\int (dx)^m \sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=97

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)(a+bx^2)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)(a+bx^2)}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a +
 b*x^2)) + (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3
*(3 + m)*(a + b*x^2))
```

Rubi [A] time = 0.103666, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+3}}{d^3(m+3)(a+bx^2)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}(dx)^{m+1}}{d(m+1)(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(1 + m)*(a +
 b*x^2)) + (b*(d*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^3
*(3 + m)*(a + b*x^2))
```

Rubi in Sympy [A] time = 10.6697, size = 80, normalized size = 0.82

$$\frac{2a\,(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{d\,(a+bx^2)\,(m+1)\,(m+3)}+\frac{(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}}{d\,(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] 2*a*(d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)/(d*(a + b*
x**2)*(m + 1)*(m + 3)) + (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 +
b**2*x**4)/(d*(m + 3))
```

Mathematica [A] time = 0.0434534, size = 47, normalized size = 0.48

$$\frac{\sqrt{(a+bx^2)^2}(dx)^m\left(\frac{ax}{m+1}+\frac{bx^3}{m+3}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^m*Sqrt[(a + b*x^2)^2]*((a*x)/(1 + m) + (b*x^3)/(3 + m)))/(
a + b*x^2)
```

Maple [A] time = 0.005, size = 56, normalized size = 0.6

$$\frac{(bmx^2+bx^2+am+3\,a)\,x\,(dx)^m}{(3+m)\,(1+m)\,(bx^2+a)}\sqrt{(bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] x*(b*m*x^2+b*x^2+a*m+3*a)*(d*x)^m*((b*x^2+a)^2)^(1/2)/(3+m)/(1+m)
/(b*x^2+a)
```

Maxima [A] time = 0.698239, size = 47, normalized size = 0.48

$$\frac{(bd^m(m+1)x^3+ad^m(m+3)x)\,x^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] (b*d^m*(m + 1)*x^3 + a*d^m*(m + 3)*x)*x^m/(m^2 + 4*m + 3)
```

Fricas [A] time = 0.285496, size = 47, normalized size = 0.48

$$\frac{((bm+b)x^3+(am+3\,a)x)\,(dx)^m}{m^2+4\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((b*m + b)*x^3 + (a*m + 3*a)*x)*(d*x)^m/(m^2 + 4*m + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{(a + bx^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m*sqrt((a + b*x**2)**2), x)
```

GIAC/XCAS [A] time = 0.267416, size = 123, normalized size = 1.27

$$\frac{bmx^3e^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right) + bx^3e^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right) + amxe^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right) + 3\,axe^{(m\ln(dx))}\mathrm{sign}\left(bx^2 + a\right)}{m^2 + 4\,m + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (b*m*x^3*e^(m*ln(d*x))*sign(b*x^2 + a) + b*x^3*e^(m*ln(d*x))*sign
(b*x^2 + a) + a*m*x*e^(m*ln(d*x))*sign(b*x^2 + a) + 3*a*x*e^(m*ln
(d*x))*sign(b*x^2 + a))/(m^2 + 4*m + 3)
```

3.794 $\int \frac{(dx)^m}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^2)\,(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi [A] time = 0.0957069, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{(a+bx^2)\,(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(a*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rubi in Sympy [A] time = 15.725, size = 61, normalized size = 0.84

$$\frac{(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{ad\,(a+bx^2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*hyper((1, m/2
+ 1/2), (m/2 + 3/2,), -b*x**2/a)/(a*d*(a + b*x**2)*(m + 1))
```

Mathematica [A] time = 0.0471476, size = 62, normalized size = 0.85

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{a(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/(a*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{b^2x^4+2\,abx^2+a^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{(a + bx^2)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt((a + b*x**2)**2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{b^2x^4 + 2\,abx^2 + a^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

3.795 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^2)\,(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 0.0955504, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{(a+bx^2)\,(dx)^{m+1}\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(a^3*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi in Sympy [A] time = 15.4142, size = 63, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}\,{}_2F_1\left(\begin{matrix}3,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{a^3d\,(a+bx^2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*hyper((3, m/2
+ 1/2), (m/2 + 3/2,), -b*x**2/a)/(a**3*d*(a + b*x**2)*(m + 1))
```

Mathematica [A] time = 0.0577592, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(3,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^3(m+1)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a^3*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{3}{2}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

3.796 $\int \frac{(dx)^m}{(a^2+2abx^2+b^2x^4)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{(a+bx^2)\,(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi [A] time = 0.0973308, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{(a+bx^2)\,(dx)^{m+1}\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5d(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(a^5*d*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rubi in Sympy [A] time = 15.4337, size = 63, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a^2+2abx^2+b^2x^4}\,{}_2F_1\left(\begin{matrix}5,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{a^5d\,(a+bx^2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a**2 + 2*a*b*x**2 + b**2*x**4)*hyper((5, m/2
+ 1/2), (m/2 + 3/2,), -b*x**2/a)/(a**5*d*(a + b*x**2)*(m + 1))
```

Mathematica [A] time = 0.0691074, size = 60, normalized size = 0.82

$$\frac{x\left(a+bx^2\right)(dx)^m\,{}_2F_1\left(5,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{a^5(m+1)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m/(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a + b*x^2)*Hypergeometric2F1[5, (1 + m)/2, (3 + m)/2,
 -((b*x^2)/a)])/(a^5*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int (dx)^m\left(b^2x^4+2\,abx^2+a^2\right)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

```
[Out] int((d*x)^m/(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(b^2x^4+2\,abx^2+a^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b^2x^4+2\,abx^2+a^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/((b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*
b*x^2 + a^4)*sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left((a+bx^2)^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((d*x)**m/((a + b*x**2)**2)**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b^2x^4+2\,abx^2+a^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(5/2), x)
```

3.797 $\int (dx)^m \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=74

$$\frac{(a+bx^2)\,(dx)^{m+1}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(1,\frac{1}{2}(m+4p+3);\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ad(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeo
metric2F1[1, (3 + m + 4*p)/2, (3 + m)/2, -((b*x^2)/a)])/(a*d*(1 +
 m))
```

Rubi [A] time = 0.0726291, antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$\frac{(dx)^{m+1}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+3}{2};-\frac{bx^2}{a}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[(1
 + m)/2, -2*p, (3 + m)/2, -((b*x^2)/a)])/(d*(1 + m)*(1 + (b*x^2)/
a)^(2*p))
```

Rubi in Sympy [A] time = 18.4921, size = 66, normalized size = 0.89

$$\frac{(dx)^{m+1}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\left.\begin{matrix}-2p,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\right|-\frac{bx^2}{a}\right)}{d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] (d*x)**(m + 1)*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x
**4)**p*hyper((-2*p, m/2 + 1/2), (m/2 + 3/2,), -b*x**2/a)/(d*(m +
 1))
```

Mathematica [A] time = 0.0612803, size = 66, normalized size = 0.89

$$\frac{x(dx)^m \left(\left(a+bx^2\right)^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{m+1}{2},-2p;\frac{m+1}{2}+1;-\frac{bx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*(d*x)^m*((a + b*x^2)^2)^p*Hypergeometric2F1[(1 + m)/2, -2*p, 1
 + (1 + m)/2, -((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.155, size = 0, normalized size = 0.

$$\int (dx)^m \left(b^2x^4+2\,abx^2+a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^m*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p (dx)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4+2\,abx^2+a^2\right)^p (dx)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(\left(a + bx^2\right)^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral((d*x)**m*((a + b*x**2)**2)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*(d*x)^m, x)
```

3.798 $\int x^7 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=174

$$\frac{(a+bx^2)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a(a+bx^2)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2(a+bx^2)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+1)}$$

```
[Out] -(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(1 + 2*p)
) + (3*a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1
 + p)) - (3*a*(a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4
*(3 + 2*p)) + ((a + b*x^2)^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^
4*(2 + p))
```

Rubi [A] time = 0.263751, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{(a+bx^2)^4 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+2)}-\frac{3a(a+bx^2)^3 \left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+3)}+\frac{3a^2(a+bx^2)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^4(p+1)}-\frac{a^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^4(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4*(1 + 2*p)
) + (3*a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^4*(1
 + p)) - (3*a*(a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^4
*(3 + 2*p)) + ((a + b*x^2)^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^
4*(2 + p))
```

Rubi in Sympy [A] time = 43.4795, size = 189, normalized size = 1.09

$$-\frac{3a^3\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{4b^4\left(p+2\right)\left(2p+1\right)\left(2p+3\right)}+\frac{3a^2\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b^4\left(p+1\right)\left(p+2\right)\left(2p+3\right)}-\frac{3ax^4\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{8b^2\left(p+2\right)\left(2p+3\right)}+\frac{x^6\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{8b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] -3*a**3*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*
*4*(p + 2)*(2*p + 1)*(2*p + 3)) + 3*a**2*(a**2 + 2*a*b*x**2 + b**
2*x**4)**(p + 1)/(4*b**4*(p + 1)*(p + 2)*(2*p + 3)) - 3*a*x**4*(2
*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*(p + 2)
*(2*p + 3)) + x**6*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**
4)**p/(8*b*(p + 2))
```

Mathematica [A] time = 0.0776516, size = 110, normalized size = 0.63

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^p\left(-3a^3+3a^2b(2p+1)x^2-3ab^2\left(2p^2+3p+1\right)x^4+b^3\left(4p^3+12p^2+11p+3\right)x^6\right)}{4b^4(p+1)(p+2)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-3*a^3 + 3*a^2*b*(1 + 2*p)*x^2 -
3*a*b^2*(1 + 3*p + 2*p^2)*x^4 + b^3*(3 + 11*p + 12*p^2 + 4*p^3)*x
^6))/(4*b^4*(1 + p)*(2 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.011, size = 150, normalized size = 0.9

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p\left(-4\,b^3p^3x^6-12\,b^3p^2x^6-11\,b^3px^6+6\,ab^2p^2x^4-3\,b^3x^6+9\,ab^2px^4+3\,ax^4b^2-6\,a^2bpx^2-3\,a^2bx^2\right.}{4\,b^4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-4*b^3*p^3*x^6-12*b^3*p^2*x^6-11*
b^3*p*x^6+6*a*b^2*p^2*x^4-3*b^3*x^6+9*a*b^2*p*x^4+3*a*b^2*x^4-6*a
^2*b*p*x^2-3*a^2*b*x^2+3*a^3)*(b*x^2+a)/b^4/(4*p^4+20*p^3+35*p^2+
25*p+6)
```

Maxima [A] time = 0.692871, size = 155, normalized size = 0.89

$$\frac{\left(\left(4\,p^3+12\,p^2+11\,p+3\right)b^4x^8+2\left(2\,p^3+3\,p^2+p\right)ab^3x^6-3\left(2\,p^2+p\right)a^2b^2x^4+6\,a^3bpx^2-3\,a^4\right)\left(bx^2+a\right)^{2\,p}}{4\left(4\,p^4+20\,p^3+35\,p^2+25\,p+6\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^7,x, algorithm="maxima")
```

```
[Out] 1/4*((4*p^3 + 12*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*
a*b^3*x^6 - 3*(2*p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b
*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 35*p^2 + 25*p + 6)*b^4)
```

Fricas [A] time = 0.289315, size = 220, normalized size = 1.26

$$\frac{\left(\left(4 b^4 p^3 + 12 b^4 p^2 + 11 b^4 p + 3 b^4\right) x^8 + 6 a^3 b p x^2 + 2 \left(2 a b^3 p^3 + 3 a b^3 p^2 + a b^3 p\right) x^6 - 3 \left(2 a^2 b^2 p^2 + a^2 b^2 p\right) x^4 - 3 a^4\right) \left(b^2 x^4 + \right.}{4 \left(4 b^4 p^4 + 20 b^4 p^3 + 35 b^4 p^2 + 25 b^4 p + 6 b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^7,x, algorithm="fricas")
```

```
[Out] 1/4*((4*b^4*p^3 + 12*b^4*p^2 + 11*b^4*p + 3*b^4)*x^8 + 6*a^3*b*p*
x^2 + 2*(2*a*b^3*p^3 + 3*a*b^3*p^2 + a*b^3*p)*x^6 - 3*(2*a^2*b^2*
p^2 + a^2*b^2*p)*x^4 - 3*a^4)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^
4*p^4 + 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.272281, size = 536, normalized size = 3.08

$$\frac{4 b^4 p^3 x^8 e^{\left(p \ln\left(b^2 x^4 + 2 a b x^2 + a^2\right)\right)} + 12 b^4 p^2 x^8 e^{\left(p \ln\left(b^2 x^4 + 2 a b x^2 + a^2\right)\right)} + 4 a b^3 p^3 x^6 e^{\left(p \ln\left(b^2 x^4 + 2 a b x^2 + a^2\right)\right)} + 11 b^4 p x^8 e^{\left(p \ln\left(b^2 x^4 + 2 a b x^2 + a^2\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^7,x, algorithm="giac")
```

```
[Out] 1/4*(4*b^4*p^3*x^8*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 12*b^4*p
^2*x^8*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 4*a*b^3*p^3*x^6*e^(p
*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 11*b^4*p*x^8*e^(p*ln(b^2*x^4 +
2*a*b*x^2 + a^2)) + 6*a*b^3*p^2*x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 +
 a^2)) + 3*b^4*x^8*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*a*b^3*
p*x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) - 6*a^2*b^2*p^2*x^4*e^(
p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) - 3*a^2*b^2*p*x^4*e^(p*ln(b^2*x^
4 + 2*a*b*x^2 + a^2)) + 6*a^3*b*p*x^2*e^(p*ln(b^2*x^4 + 2*a*b*x^2
 + a^2)) - 3*a^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)))/(4*b^4*p^4
+ 20*b^4*p^3 + 35*b^4*p^2 + 25*b^4*p + 6*b^4)
```

3.799 $\int x^5 \left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=130

$$\frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p))
 - (a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)
) + ((a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p
))
```

Rubi [A] time = 0.193112, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+3)}-\frac{a\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(p+1)}+\frac{a^2\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + 2*p))
 - (a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(1 + p)
) + ((a + b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^3*(3 + 2*p
))
```

Rubi in Sympy [A] time = 27.5382, size = 128, normalized size = 0.98

$$\frac{a^2\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^3\left(2p+1\right)\left(2p+3\right)}-\frac{a\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{2b^3\left(p+1\right)\left(2p+3\right)}+\frac{x^4\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{4b\left(2p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] a**2*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(2*b**3*
(2*p + 1)*(2*p + 3)) - a*(a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)
/(2*b**3*(p + 1)*(2*p + 3)) + x**4*(2*a + 2*b*x**2)*(a**2 + 2*a*b
*x**2 + b**2*x**4)**p/(4*b*(2*p + 3))
```

Mathematica [A] time = 0.0501561, size = 77, normalized size = 0.59

$$\frac{(a+bx^2)\left((a+bx^2)^2\right)^p\left(a^2-ab(2p+1)x^2+b^2\left(2p^2+3p+1\right)x^4\right)}{2b^3(p+1)(2p+1)(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(a^2 - a*b*(1 + 2*p)*x^2 + b^2*(1
+ 3*p + 2*p^2)*x^4))/(2*b^3*(1 + p)*(1 + 2*p)*(3 + 2*p))
```

Maple [A] time = 0.01, size = 96, normalized size = 0.7

$$\frac{\left(2\,b^2p^2x^4+3\,b^2px^4+b^2x^4-2\,abpx^2-abx^2+a^2\right)\left(bx^2+a\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\,b^3\left(4\,p^3+12\,p^2+11\,p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(2*b^2*p^2*x^4+3*b^2*p*x^4+b^2*x^4-2*a*b*p*x^2-a*b*
x^2+a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(4*p^3+12*p^2+11*p+3)
```

Maxima [A] time = 0.69588, size = 107, normalized size = 0.82

$$\frac{\left(\left(2\,p^2+3\,p+1\right)b^3x^6+\left(2\,p^2+p\right)ab^2x^4-2\,a^2bpx^2+a^3\right)\left(bx^2+a\right)^{2\,p}}{2\left(4\,p^3+12\,p^2+11\,p+3\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^5,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*
p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^3)
```

Fricas [A] time = 0.289216, size = 146, normalized size = 1.12

$$\frac{\left(\left(2\,b^3p^2+3\,b^3p+b^3\right)x^6-2\,a^2bpx^2+\left(2\,ab^2p^2+ab^2p\right)x^4+a^3\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{2\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^5,x, algorithm="fricas")
```

```
[Out] 1/2*((2*b^3*p^2 + 3*b^3*p + b^3)*x^6 - 2*a^2*b*p*x^2 + (2*a*b^2*p
^2 + a*b^2*p)*x^4 + a^3)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(4*b^3*p^3
 + 12*b^3*p^2 + 11*b^3*p + 3*b^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.268915, size = 336, normalized size = 2.58

$$\frac{2\,b^3p^2x^6e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+3\,b^3px^6e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+2\,ab^2p^2x^4e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+b^3x^6e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+ab}{2\,(4\,b^3p^3+12\,b^3p^2+11\,b^3p+3\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^5,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*p^2*x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 3*b^3*p*
x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*a*b^2*p^2*x^4*e^(p*ln
(b^2*x^4 + 2*a*b*x^2 + a^2)) + b^3*x^6*e^(p*ln(b^2*x^4 + 2*a*b*x^
2 + a^2)) + a*b^2*p*x^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) - 2*a
^2*b*p*x^2*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + a^3*e^(p*ln(b^2*
x^4 + 2*a*b*x^2 + a^2)))/(4*b^3*p^3 + 12*b^3*p^2 + 11*b^3*p + 3*b
^3)
```

3.800 $\int x^3 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=84

$$\frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p))
+ ((a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rubi [A] time = 0.140804, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\left(a+bx^2\right)^2 \left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}-\frac{a\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -(a*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(1 + 2*p))
+ ((a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^2*(1 + p))
```

Rubi in Sympy [A] time = 15.8737, size = 71, normalized size = 0.85

$$-\frac{a\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2\left(2p+1\right)}+\frac{\left(a^2+2abx^2+b^2x^4\right)^{p+1}}{4b^2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] -a*(2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b**2*(2
*p + 1)) + (a**2 + 2*a*b*x**2 + b**2*x**4)**(p + 1)/(4*b**2*(p +
1))
```

Mathematica [A] time = 0.0309718, size = 51, normalized size = 0.61

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p\left(b(2p+1)x^2-a\right)}{4b^2(p+1)(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p*(-a + b*(1 + 2*p)*x^2))/(4*b^2*(1
+ p)*(1 + 2*p))
```

Maple [A] time = 0.009, size = 60, normalized size = 0.7

$$-\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p\left(-2\,x^2pb-bx^2+a\right)\left(bx^2+a\right)}{4\,b^2\left(2\,p^2+3\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-b*x^2+a)*(b*x^2+a)/b^2
/(2*p^2+3*p+1)
```

Maxima [A] time = 0.70667, size = 73, normalized size = 0.87

$$\frac{\left(b^2(2\,p+1)x^4+2\,abpx^2-a^2\right)\left(bx^2+a\right)^{2\,p}}{4\,(2\,p^2+3\,p+1)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2
*p^2 + 3*p + 1)*b^2)
```

Fricas [A] time = 0.286955, size = 95, normalized size = 1.13

$$\frac{\left(2\,abpx^2+\left(2\,b^2p+b^2\right)x^4-a^2\right)\left(b^2x^4+2\,abx^2+a^2\right)^p}{4\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*p*x^2 + (2*b^2*p + b^2)*x^4 - a^2)*(b^2*x^4 + 2*a*b*x^
2 + a^2)^p/(2*b^2*p^2 + 3*b^2*p + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.268182, size = 189, normalized size = 2.25

$$\frac{2\,b^2px^4e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+b^2x^4e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+2\,abpx^2e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}-a^2e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}}{4\left(2\,b^2p^2+3\,b^2p+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^3,x, algorithm="giac")
```

```
[Out] 1/4*(2*b^2*p*x^4*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + b^2*x^4*e^
(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + 2*a*b*p*x^2*e^(p*ln(b^2*x^4 +
 2*a*b*x^2 + a^2)) - a^2*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)))/(2*
b^2*p^2 + 3*b^2*p + b^2)
```

3.801 $\int x \left(a^2 + 2abx^2 + b^2x^4\right)^p \, dx$

Optimal. Leaf size=41

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rubi [A] time = 0.0641963, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{2b(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b*(1 + 2*p))
```

Rubi in Sympy [A] time = 9.11644, size = 37, normalized size = 0.9

$$\frac{\left(2a+2bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p}{4b\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] (2*a + 2*b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*(2*p + 1
))
```

Mathematica [A] time = 0.00761304, size = 29, normalized size = 0.71

$$\frac{\left(a+bx^2\right)\left(\left(a+bx^2\right)^2\right)^p}{4bp+2b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)*((a + b*x^2)^2)^p)/(2*b + 4*b*p)
```

Maple [A] time = 0.005, size = 40, normalized size = 1.

$$\frac{(bx^2 + a)\,(b^2x^4 + 2\,abx^2 + a^2)^p}{2\,b\,(1 + 2\,p)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] 1/2*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p/b/(1+2*p)
```

Maxima [A] time = 0.70515, size = 41, normalized size = 1.

$$\frac{(bx^2 + a)\,(bx^2 + a)^{2\,p}}{2\,b(2\,p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)/(b*(2*p + 1))
```

Fricas [A] time = 0.290706, size = 50, normalized size = 1.22

$$\frac{(bx^2 + a)\,(b^2x^4 + 2\,abx^2 + a^2)^p}{2\,(2\,bp + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + a)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b*p + b)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Exception raised: TypeError
```

GIAC/XCAS [A] time = 0.270583, size = 84, normalized size = 2.05

$$\frac{bx^2e^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}+ae^{\left(p\ln\left(b^2x^4+2\,abx^2+a^2\right)\right)}}{2\,(2\,bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2*e^(p*ln(b^2*x^4 + 2*a*b*x^2 + a^2)) + a*e^(p*ln(b^2*x^
4 + 2*a*b*x^2 + a^2)))/(2*b*p + b)
```

3.802 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x} \, dx$

Optimal. Leaf size=63

$$-\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1,
1 + 2*p, 2*(1 + p), 1 + (b*x^2)/a])/(2*a*(1 + 2*p))
```

Rubi [A] time = 0.0946251, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{(a+bx^2)\,(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(1,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] -((a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1,
1 + 2*p, 2*(1 + p), 1 + (b*x^2)/a])/(2*a*(1 + 2*p))
```

Rubi in Sympy [A] time = 17.6159, size = 76, normalized size = 1.21

$$-\frac{(ab+b^2x^2)^{-2p}\,(ab+b^2x^2)^{2p+1}\,(a^2+2abx^2+b^2x^4)^p\,{}_2F_1\left(\begin{matrix}1,2p+1\\2p+2\end{matrix}\middle|1+\frac{bx^2}{a}\right)}{2ab\,(2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x,x)
```

```
[Out] -(a*b + b**2*x**2)**(-2*p)*(a*b + b**2*x**2)**(2*p + 1)*(a**2 + 2
*a*b*x**2 + b**2*x**4)**p*hyper((1, 2*p + 1), (2*p + 2,), 1 + b*x
**2/a)/(2*a*b*(2*p + 1))
```

Mathematica [A] time = 0.0241706, size = 53, normalized size = 0.84

$$\frac{\left(\frac{a}{bx^2}+1\right)^{-2p}\left(\left(a+bx^2\right)^2\right)^p {}_2F_1\left(-2p,-2p;1-2p;-\frac{a}{bx^2}\right)}{4p}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x,x]
```

```
[Out] (((a + b*x^2)^2)^p*Hypergeometric2F1[-2*p, -2*p, 1 - 2*p, -(a/(b*
x^2))])/(4*p*(1 + a/(b*x^2))^(2*p))
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x, x)
```

3.803 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^3}\,dx$

Optimal. Leaf size=64

$$\frac{b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

```
[Out] (b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2,
 1 + 2*p, 2*(1 + p), 1 + (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rubi [A] time = 0.104259, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{b\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(2,2p+1;2(p+1);\frac{bx^2}{a}+1\right)}{2a^2(2p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[2,
 1 + 2*p, 2*(1 + p), 1 + (b*x^2)/a])/(2*a^2*(1 + 2*p))
```

Rubi in Sympy [A] time = 17.3873, size = 75, normalized size = 1.17

$$\frac{\left(ab+b^2x^2\right)^{-2p}\left(ab+b^2x^2\right)^{2p+1}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}2,2p+1\\2p+2\end{matrix}\middle|1+\frac{bx^2}{a}\right)}{2a^2\left(2p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**3,x)
```

```
[Out] (a*b + b**2*x**2)**(-2*p)*(a*b + b**2*x**2)**(2*p + 1)*(a**2 + 2*
a*b*x**2 + b**2*x**4)**p*hyper((2, 2*p + 1), (2*p + 2,), 1 + b*x*
*2/a)/(2*a**2*(2*p + 1))
```

Mathematica [A] time = 0.0305529, size = 62, normalized size = 0.97

$$\frac{\left(\frac{a}{bx^2}+1\right)^{-2p}\left((a+bx^2)^2\right)^p\,{}_2F_1\left(1-2p,-2p;2-2p;-\frac{a}{bx^2}\right)}{2(2p-1)x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^3,x]
```

```
[Out] (((a + b*x^2)^2)^p*Hypergeometric2F1[1 - 2*p, -2*p, 2 - 2*p, -(a/
(b*x^2))])/(2*(-1 + 2*p)*(1 + a/(b*x^2))^(2*p)*x^2)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**3,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^3, x)
```

3.804 $\int x^4 \left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7
/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0615657, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{5}x^5\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/2, -2*p, 7
/2, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.2961, size = 53, normalized size = 0.88

$$\frac{x^5\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\begin{matrix}-2p,\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] x**5*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hy
per((-2*p, 5/2), (7/2,), -b*x**2/a)/5
```

Mathematica [A] time = 0.027795, size = 51, normalized size = 0.85

$$\frac{1}{5}x^5\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}\,{}_2F_1\left(\frac{5}{2},-2p;\frac{7}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^5*((a + b*x^2)^2)^p*Hypergeometric2F1[5/2, -2*p, 7/2, -((b*x^2
)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.071, size = 0, normalized size = 0.

$$\int x^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^4*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)^{p} x^{4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left({\left(b^{2} x^{4}+2 \, a b x^{2}+a^{2}\right)}^{p} x^{4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(\left(a + b x^2 \right)^2 \right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(b**2*x**4+2*a*b*x**2+a**2)**p,x)

[Out] Integral(x**4*((a + b*x**2)**2)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^4 + 2 \, a b x^2 + a^2 \right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4,x, algorithm="giac")

[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^4, x)
```

3.805 $\int x^2 \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=60

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5
/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.061046, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{1}{3}x^3\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/2, -2*p, 5
/2, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.754, size = 53, normalized size = 0.88

$$\frac{x^3\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}-2p,\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] x**3*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hy
per((-2*p, 3/2), (5/2,), -b*x**2/a)/3
```

Mathematica [A] time = 0.0192236, size = 51, normalized size = 0.85

$$\frac{1}{3}x^3\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{3}{2},-2p;\frac{5}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x^3*((a + b*x^2)^2)^p*Hypergeometric2F1[3/2, -2*p, 5/2, -((b*x^2
)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int x^{2} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int(x^2*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(b^{2} x^{4} + 2 \, a b x^{2} + a^{2}\right)}^{p} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(b^{2} x^{4} + 2 \, a b x^{2} + a^{2}\right)}^{p} x^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(\left(a+bx^2\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(x**2*((a + b*x**2)**2)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p*x^2, x)
```

3.806 $\int \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=55

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2
, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(2*p)
```

Rubi [A] time = 0.0347242, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$x\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/2, -2*p, 3/2
, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(2*p)
```

Rubi in Sympy [A] time = 21.5735, size = 49, normalized size = 0.89

$$x\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}-2p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] x*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hyper
((-2*p, 1/2), (3/2,), -b*x**2/a)
```

Mathematica [A] time = 0.0167089, size = 46, normalized size = 0.84

$$x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(\frac{1}{2},-2p;\frac{3}{2};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/2, -2*p, 3/2, -((b*x^2)/
a)])/(1 + (b*x^2)/a)^(2*p)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4 + 2\,abx^2 + a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^4 + 2\,abx^2 + a^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^2+2abx^2+b^2x^4\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p,x)

[Out] Integral((a**2 + 2*a*b*x**2 + b**2*x**4)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="giac")

[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

3.807 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^2}\,dx$

Optimal. Leaf size=58

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

```
[Out] -(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/
2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Rubi [A] time = 0.0546614, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -(((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/2, -2*p, 1/
2, -((b*x^2)/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Rubi in Sympy [A] time = 16.6059, size = 53, normalized size = 0.91

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\begin{matrix}-2p,-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**2,x)
```

```
[Out] -(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hyper(
(-2*p, -1/2), (1/2,), -b*x**2/a)/x
```

Mathematica [A] time = 0.0189638, size = 49, normalized size = 0.84

$$-\frac{\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{1}{2},-2p;\frac{1}{2};-\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^2,x]
```

```
[Out] -((((a + b*x^2)^2)^p*Hypergeometric2F1[-1/2, -2*p, 1/2, -((b*x^2)
/a)])/(x*(1 + (b*x^2)/a)^(2*p)))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int\frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)^p}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**2,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^2, x)
```

3.808 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

```
[Out] -((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/
2, -((b*x^2)/a)])/(3*x^3*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0559551, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -((a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/2, -2*p, -1/
2, -((b*x^2)/a)])/(3*x^3*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 16.5107, size = 58, normalized size = 0.97

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}-2p,-\frac{3}{2}\\-\frac{1}{2}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**4,x)
```

```
[Out] -(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hyper(
(-2*p, -3/2), (-1/2,), -b*x**2/a)/(3*x**3)
```

Mathematica [A] time = 0.0258837, size = 51, normalized size = 0.85

$$-\frac{\left(\left(a+bx^2\right)^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-2p} {}_2F_1\left(-\frac{3}{2},-2p;-\frac{1}{2};-\frac{bx^2}{a}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/x^4,x]
```

```
[Out] -(((a + b*x^2)^2)^p*Hypergeometric2F1[-3/2, -2*p, -1/2, -((b*x^2)
/a)])/(3*x^3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.066, size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a + bx^2\right)^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p/x**4,x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/x^4, x)
```

3.809 $\int (dx)^{3/2} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{5/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

```
[Out] (2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/
4, -2*p, 9/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0629362, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2(dx)^{5/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(5/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[5/
4, -2*p, 9/4, -((b*x^2)/a)])/(5*d*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.0761, size = 60, normalized size = 0.9

$$\frac{2\,(dx)^{\frac{5}{2}}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}-2p,\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] 2*(d*x)**(5/2)*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x
**4)**p*hyper((-2*p, 5/4), (9/4,), -b*x**2/a)/(5*d)
```

Mathematica [A] time = 0.0256822, size = 56, normalized size = 0.84

$$\frac{2}{5}x(dx)^{3/2}\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{5}{4},-2p;\frac{9}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*(d*x)^(3/2)*((a + b*x^2)^2)^p*Hypergeometric2F1[5/4, -2*p, 9
/4, -((b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(3/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p dx,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*d*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(b^2x^4 + 2\,abx^2 + a^2\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

3.810 $\int \sqrt{dx} \left(a^2+2abx^2+b^2x^4\right)^p \, dx$

Optimal. Leaf size=67

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

```
[Out] (2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/
4, -2*p, 7/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0630344, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2(dx)^{3/2}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*(d*x)^(3/2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[3/
4, -2*p, 7/4, -((b*x^2)/a)])/(3*d*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.1153, size = 60, normalized size = 0.9

$$\frac{2\left(dx\right)^{\frac{3}{2}}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}-2p,\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] 2*(d*x)**(3/2)*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x
**4)**p*hyper((-2*p, 3/4), (7/4,), -b*x**2/a)/(3*d)
```

Mathematica [A] time = 0.024855, size = 56, normalized size = 0.84

$$\frac{2}{3}x\sqrt{dx}\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{3}{4},-2p;\frac{7}{4};-\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (2*x*Sqrt[d*x]*((a + b*x^2)^2)^p*Hypergeometric2F1[3/4, -2*p, 7/4
, -((b*x^2)/a)])/(3*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

```
[Out] int((d*x)^(1/2)*(b^2*x^4+2*a*b*x^2+a^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{dx}\left(b^2x^4+2\,abx^2+a^2\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \left(\left(a + bx^2\right)^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Integral(sqrt(d*x)*((a + b*x**2)**2)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx} \left(b^2x^4 + 2\,abx^2 + a^2\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p, x)
```

3.811 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{\sqrt{dx}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

```
[Out] (2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4,
 -2*p, 5/4, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0622319, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{2\sqrt{dx}\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{d}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[1/4,
 -2*p, 5/4, -((b*x^2)/a)])/(d*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.3414, size = 58, normalized size = 0.89

$$\frac{2\sqrt{dx}\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\begin{matrix}-2p,\frac{1}{4}\\\frac{5}{4}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(1/2),x)
```

```
[Out] 2*sqrt(d*x)*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4
)**p*hyper((-2*p, 1/4), (5/4,), -b*x**2/a)/d
```

Mathematica [A] time = 0.0237895, size = 54, normalized size = 0.83

$$\frac{2x\left((a+bx^2)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(\frac{1}{4},-2p;\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/Sqrt[d*x],x]
```

```
[Out] (2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[1/4, -2*p, 5/4, -((b*x^2
)/a)])/(Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (b^2x^4+2\,abx^2+a^2)^p\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^4+2\,abx^2+a^2)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b^2x^4+2\,abx^2+a^2)^p}{\sqrt{dx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(a+bx^2\right)^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(1/2),x)
```

```
[Out] Integral(((a + b*x**2)**2)**p/sqrt(d*x), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/sqrt(d*x), x)
```

3.812 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3
/4, -((b*x^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0630066, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-1/4, -2*p, 3
/4, -((b*x^2)/a)])/(d*Sqrt[d*x]*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.2758, size = 61, normalized size = 0.94

$$-\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\,{}_2F_1\left(\begin{matrix}-2p,-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|-\frac{bx^2}{a}\right)}{d\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(3/2),x)
```

```
[Out] -2*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hype
r((-2*p, -1/4), (3/4,), -b*x**2/a)/(d*sqrt(d*x))
```

Mathematica [A] time = 0.0261215, size = 54, normalized size = 0.83

$$-\frac{2x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{1}{4},-2p;\frac{3}{4};-\frac{bx^2}{a}\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(3/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-1/4, -2*p, 3/4, -((b*x
^2)/a)])/((d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.016, size = 0, normalized size = 0.

$$\int\left(b^2x^4+2\,abx^2+a^2\right)^p(dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{\sqrt{dx}dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(sqrt(d*x)*d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4 + 2\,abx^2 + a^2\right)^p}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(3/2), x)
```

3.813 $\int \frac{(a^2+2abx^2+b^2x^4)^p}{(dx)^{5/2}} \, dx$

Optimal. Leaf size=67

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1
/4, -((b*x^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rubi [A] time = 0.0625362, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{2\left(\frac{bx^2}{a}+1\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p*Hypergeometric2F1[-3/4, -2*p, 1
/4, -((b*x^2)/a)])/(3*d*(d*x)^(3/2)*(1 + (b*x^2)/a)^(2*p))
```

Rubi in Sympy [A] time = 17.316, size = 63, normalized size = 0.94

$$-\frac{2\left(1+\frac{bx^2}{a}\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p {}_2F_1\left(\begin{matrix}-2p,-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle| -\frac{bx^2}{a}\right)}{3d\,(dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(5/2),x)
```

```
[Out] -2*(1 + b*x**2/a)**(-2*p)*(a**2 + 2*a*b*x**2 + b**2*x**4)**p*hype
r((-2*p, -3/4), (1/4,), -b*x**2/a)/(3*d*(d*x)**(3/2))
```

Mathematica [A] time = 0.0302982, size = 56, normalized size = 0.84

$$-\frac{2x\left(\left(a+bx^2\right)^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-2p}{}_2F_1\left(-\frac{3}{4},-2p;\frac{1}{4};-\frac{bx^2}{a}\right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a^2 + 2*a*b*x^2 + b^2*x^4)^p/(d*x)^(5/2),x]
```

```
[Out] (-2*x*((a + b*x^2)^2)^p*Hypergeometric2F1[-3/4, -2*p, 1/4, -((b*x
^2)/a)])/(3*(d*x)^(5/2)*(1 + (b*x^2)/a)^(2*p))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \left(b^2x^4+2\,abx^2+a^2\right)^p(dx)^{-\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

```
[Out] int((b^2*x^4+2*a*b*x^2+a^2)^p/(d*x)^(5/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^4+2\,abx^2+a^2\right)^p}{\sqrt{dx}d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(sqrt(d*x)*d^2*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**2*x**4+2*a*b*x**2+a**2)**p/(d*x)**(5/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b^2x^4 + 2\,abx^2 + a^2)^p}{(dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b^2*x^4 + 2*a*b*x^2 + a^2)^p/(d*x)^(5/2), x)
```

3.814 $\int x^2 \left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi [A] time = 0.0189504, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4),x]

[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Rubi in Sympy [A] time = 5.38413, size = 19, normalized size = 0.76

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(c*x**4+b*x**2+a),x)

[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

Mathematica [A] time = 0.00362829, size = 25, normalized size = 1.

$$\frac{ax^3}{3}+\frac{bx^5}{5}+\frac{cx^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5 + (c*x^7)/7
```

Maple [A] time = 0., size = 20, normalized size = 0.8

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5+1/7*c*x^7
```

Maxima [A] time = 0.690021, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7 + \frac{1}{5}\,bx^5 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

Fricas [A] time = 0.230326, size = 1, normalized size = 0.04

$$\frac{1}{7}x^7c + \frac{1}{5}x^5b + \frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c + 1/5*x^5*b + 1/3*x^3*a
```

Sympy [A] time = 0.064272, size = 19, normalized size = 0.76

$$\frac{ax^3}{3} + \frac{bx^5}{5} + \frac{cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a),x)

[Out] a*x**3/3 + b*x**5/5 + c*x**7/7
```

GIAC/XCAS [A] time = 0.263795, size = 26, normalized size = 1.04

$$\frac{1}{7}\,cx^7+\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^2,x, algorithm="giac")

[Out] 1/7*c*x^7 + 1/5*b*x^5 + 1/3*a*x^3
```

3.815 $\int x\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=25

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

```
[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi [A] time = 0.0196812, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2 + c*x^4),x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\int^{x^2}x\,dx}{2}+\frac{cx^6}{6}+\frac{\int^{x^2}a\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a),x)

[Out] b*Integral(x, (x, x**2))/2 + c*x**6/6 + Integral(a, (x, x**2))/2
```

Mathematica [A] time = 0.0019599, size = 25, normalized size = 1.

$$\frac{ax^2}{2}+\frac{bx^4}{4}+\frac{cx^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2 + c*x^4),x]

[Out] (a*x^2)/2 + (b*x^4)/4 + (c*x^6)/6
```

Maple [A] time = 0.001, size = 20, normalized size = 0.8

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a),x)

[Out] 1/2*a*x^2+1/4*b*x^4+1/6*c*x^6
```

Maxima [A] time = 0.688813, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x,x, algorithm="maxima")

[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

Fricas [A] time = 0.233277, size = 1, normalized size = 0.04

$$\frac{1}{6}x^6c + \frac{1}{4}x^4b + \frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x,x, algorithm="fricas")

[Out] 1/6*x^6*c + 1/4*x^4*b + 1/2*x^2*a
```

Sympy [A] time = 0.064729, size = 19, normalized size = 0.76

$$\frac{ax^2}{2} + \frac{bx^4}{4} + \frac{cx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a),x)

[Out] a*x**2/2 + b*x**4/4 + c*x**6/6
```

GIAC/XCAS [A] time = 0.260769, size = 26, normalized size = 1.04

$$\frac{1}{6}\,cx^6 + \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x,x, algorithm="giac")

[Out] 1/6*c*x^6 + 1/4*b*x^4 + 1/2*a*x^2
```

3.816 $\int (a + bx^2 + cx^4)\ dx$

Optimal. Leaf size=20

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi [A] time = 0.0121312, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^3}{3} + \frac{cx^5}{5} + \int a\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(c*x**4+b*x**2+a,x)
```

```
[Out] b*x**3/3 + c*x**5/5 + Integral(a, x)
```

Mathematica [A] time = 0.0000787158, size = 20, normalized size = 1.

$$ax + \frac{bx^3}{3} + \frac{cx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x^2 + c*x^4,x]
```

```
[Out] a*x + (b*x^3)/3 + (c*x^5)/5
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c*x^4+b*x^2+a,x)
```

```
[Out] a*x+1/3*b*x^3+1/5*c*x^5
```

Maxima [A] time = 0.681658, size = 22, normalized size = 1.1

$$\frac{1}{5}\,cx^5+\frac{1}{3}\,bx^3+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^2 + a,x, algorithm="maxima")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

Fricas [A] time = 0.233018, size = 1, normalized size = 0.05

$$\frac{1}{5}x^5c+\frac{1}{3}x^3b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^2 + a,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c + 1/3*x^3*b + x*a
```

Sympy [A] time = 0.054634, size = 15, normalized size = 0.75

$$ax+\frac{bx^3}{3}+\frac{cx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**4+b*x**2+a,x)
```

```
[Out] a*x + b*x**3/3 + c*x**5/5
```

GIAC/XCAS [A] time = 0.261482, size = 22, normalized size = 1.1

$$\frac{1}{5}\,cx^5 + \frac{1}{3}\,bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^4 + b*x^2 + a,x, algorithm="giac")
```

```
[Out] 1/5*c*x^5 + 1/3*b*x^3 + a*x
```

3.817 $\int \frac{a+bx^2+cx^4}{x} \, dx$

Optimal. Leaf size=21

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi [A] time = 0.0163444, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a\log\left(x^2\right)}{2}+\frac{c\int^{x^2}x\,dx}{2}+\frac{\int^{x^2}b\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*log(x**2)/2 + c*Integral(x, (x, x**2))/2 + Integral(b, (x, x**2
))/2
```

Mathematica [A] time = 0.00269426, size = 21, normalized size = 1.

$$a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x,x]
```

```
[Out] (b*x^2)/2 + (c*x^4)/4 + a*Log[x]
```

Maple [A] time = 0.003, size = 18, normalized size = 0.9

$$\frac{bx^2}{2} + \frac{cx^4}{4} + a\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/2*b*x^2+1/4*c*x^4+a*ln(x)
```

Maxima [A] time = 0.675504, size = 27, normalized size = 1.29

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + \frac{1}{2}a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x,x, algorithm="maxima")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*log(x^2)
```

Fricas [A] time = 0.255565, size = 23, normalized size = 1.1

$$\frac{1}{4}cx^4 + \frac{1}{2}bx^2 + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + a*log(x)
```

Sympy [A] time = 0.144223, size = 17, normalized size = 0.81

$$a\log(x) + \frac{bx^2}{2} + \frac{cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*log(x) + b*x**2/2 + c*x**4/4
```

GIAC/XCAS [A] time = 0.261725, size = 27, normalized size = 1.29

$$\frac{1}{4}\,cx^4 + \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x,x, algorithm="giac")
```

```
[Out] 1/4*c*x^4 + 1/2*b*x^2 + 1/2*a*ln(x^2)
```

3.818 $\int \frac{a+bx^2+cx^4}{x^2} \, dx$

Optimal. Leaf size=18

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

```
[Out] -(a/x) + b*x + (c*x^3)/3
```

Rubi [A] time = 0.0178563, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^2,x]

[Out] -(a/x) + b*x + (c*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a}{x}+\frac{cx^3}{3}+\int b\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**2,x)

[Out] -a/x + c*x**3/3 + Integral(b, x)
```

Mathematica [A] time = 0.00335182, size = 18, normalized size = 1.

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^2,x]

[Out] -(a/x) + b*x + (c*x^3)/3
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$-\frac{a}{x}+bx+\frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^2,x)

[Out] -a/x+b*x+1/3*c*x^3
```

Maxima [A] time = 0.69748, size = 22, normalized size = 1.22

$$\frac{1}{3}\,cx^3+bx-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^2,x, algorithm="maxima")

[Out] 1/3*c*x^3 + b*x - a/x
```

Fricas [A] time = 0.248854, size = 27, normalized size = 1.5

$$\frac{cx^4+3\,bx^2-3\,a}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^2,x, algorithm="fricas")

[Out] 1/3*(c*x^4 + 3*b*x^2 - 3*a)/x
```

Sympy [A] time = 0.928187, size = 12, normalized size = 0.67

$$-\frac{a}{x} + bx + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a/x + b*x + c*x**3/3
```

GIAC/XCAS [A] time = 0.262397, size = 22, normalized size = 1.22

$$\frac{1}{3}\,cx^3 + bx - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + b*x - a/x
```

3.819 $\int \frac{a+bx^2+cx^4}{x^3} \, dx$

Optimal. Leaf size=21

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

```
[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Rubi [A] time = 0.0189481, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^3,x]
```

```
[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a}{2x^2}+\frac{b\log\left(x^2\right)}{2}+\frac{\int^{x^2} c\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x**2)/2 + Integral(c, (x, x**2))/2
```

Mathematica [A] time = 0.00349613, size = 21, normalized size = 1.

$$-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^3,x]

[Out] -a/(2*x^2) + (c*x^2)/2 + b*Log[x]
```

Maple [A] time = 0.005, size = 18, normalized size = 0.9

$$-\frac{a}{2\,x^{2}}+\frac{cx^{2}}{2}+b\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^3,x)

[Out] -1/2*a/x^2+1/2*c*x^2+b*ln(x)
```

Maxima [A] time = 0.698306, size = 27, normalized size = 1.29

$$\frac{1}{2}\,cx^{2}+\frac{1}{2}\,b\log\left(x^{2}\right)-\frac{a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^3,x, algorithm="maxima")

[Out] 1/2*c*x^2 + 1/2*b*log(x^2) - 1/2*a/x^2
```

Fricas [A] time = 0.254102, size = 30, normalized size = 1.43

$$\frac{cx^{4}+2\,bx^{2}\log\left(x\right)-a}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^3,x, algorithm="fricas")

[Out] 1/2*(c*x^4 + 2*b*x^2*log(x) - a)/x^2
```

Sympy [A] time = 1.01701, size = 17, normalized size = 0.81

$$-\frac{a}{2x^2} + b\log(x) + \frac{cx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a/(2*x**2) + b*log(x) + c*x**2/2
```

GIAC/XCAS [A] time = 0.263205, size = 35, normalized size = 1.67

$$\frac{1}{2}cx^2 + \frac{1}{2}b\ln\left(x^2\right) - \frac{bx^2 + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*c*x^2 + 1/2*b*ln(x^2) - 1/2*(b*x^2 + a)/x^2
```

3.820 $\int \frac{a+bx^2+cx^4}{x^4} dx$

Optimal. Leaf size=18

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

```
[Out] -a/(3*x^3) - b/x + c*x
```

Rubi [A] time = 0.0183997, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^4,x]

[Out] -a/(3*x^3) - b/x + c*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a}{3x^3} - \frac{b}{x} + \int c\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)/x**4,x)

[Out] -a/(3*x**3) - b/x + Integral(c, x)
```

Mathematica [A] time = 0.00665181, size = 18, normalized size = 1.

$$-\frac{a}{3x^3} - \frac{b}{x} + cx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/x + c*x
```

Maple [A] time = 0.008, size = 17, normalized size = 0.9

$$-\frac{a}{3\,x^3}-\frac{b}{x}+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4,x)

[Out] -1/3*a/x^3-b/x+c*x
```

Maxima [A] time = 0.684414, size = 23, normalized size = 1.28

$$cx-\frac{3\,bx^2+a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/x^4,x, algorithm="maxima")

[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

Fricas [A] time = 0.246355, size = 28, normalized size = 1.56

$$\frac{3\,cx^4-3\,bx^2-a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/x^4,x, algorithm="fricas")

[Out] 1/3*(3*c*x^4 - 3*b*x^2 - a)/x^3
```

Sympy [A] time = 1.03216, size = 15, normalized size = 0.83

$$cx-\frac{a+3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] c*x - (a + 3*b*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.261663, size = 23, normalized size = 1.28

$$cx - \frac{3\,bx^2 + a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^4,x, algorithm="giac")
```

```
[Out] c*x - 1/3*(3*b*x^2 + a)/x^3
```

3.821 $\int \frac{a+bx^2+cx^4}{x^5} \, dx$

Optimal. Leaf size=21

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Rubi [A] time = 0.0192931, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Rubi in Sympy [A] time = 6.7375, size = 20, normalized size = 0.95

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + \frac{c\log\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] -a/(4*x**4) - b/(2*x**2) + c*log(x**2)/2
```

Mathematica [A] time = 0.00479111, size = 21, normalized size = 1.

$$-\frac{a}{4x^4} - \frac{b}{2x^2} + c\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/x^5,x]
```

```
[Out] -a/(4*x^4) - b/(2*x^2) + c*Log[x]
```

Maple [A] time = 0.008, size = 18, normalized size = 0.9

$$-\frac{a}{4\,x^4}-\frac{b}{2\,x^2}+c\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5,x)
```

```
[Out] -1/4*a/x^4-1/2*b/x^2+c*ln(x)
```

Maxima [A] time = 0.683969, size = 28, normalized size = 1.33

$$\frac{1}{2}\,c\log\left(x^2\right)-\frac{2\,bx^2+a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*c*log(x^2) - 1/4*(2*b*x^2 + a)/x^4
```

Fricas [A] time = 0.254707, size = 31, normalized size = 1.48

$$\frac{4\,cx^4\log(x)-2\,bx^2-a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(4*c*x^4*log(x) - 2*b*x^2 - a)/x^4
```

Sympy [A] time = 1.32537, size = 17, normalized size = 0.81

$$c\log(x)-\frac{a+2bx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] c*log(x) - (a + 2*b*x**2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.261654, size = 36, normalized size = 1.71

$$\frac{1}{2}\,c\ln\left(x^2\right) - \frac{3\,cx^4 + 2\,bx^2 + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*c*ln(x^2) - 1/4*(3*c*x^4 + 2*b*x^2 + a)/x^4
```

3.822 $\int \frac{a+bx^2+cx^4}{x^6}\,dx$

Optimal. Leaf size=23

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Rubi [A] time = 0.019615, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^6,x]

[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Rubi in Sympy [A] time = 4.84748, size = 17, normalized size = 0.74

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**6,x)

[Out] -a/(5*x**5) - b/(3*x**3) - c/x
```

Mathematica [A] time = 0.00433609, size = 23, normalized size = 1.

$$-\frac{a}{5x^5}-\frac{b}{3x^3}-\frac{c}{x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^6,x]
```

```
[Out] -a/(5*x^5) - b/(3*x^3) - c/x
```

Maple [A] time = 0.007, size = 20, normalized size = 0.9

$$-\frac{a}{5\,x^5}-\frac{b}{3\,x^3}-\frac{c}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^6,x)

[Out] -1/5*a/x^5-1/3*b/x^3-c/x
```

Maxima [A] time = 0.687557, size = 28, normalized size = 1.22

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^6,x, algorithm="maxima")

[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Fricas [A] time = 0.247243, size = 28, normalized size = 1.22

$$-\frac{15\,cx^4+5\,bx^2+3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^6,x, algorithm="fricas")

[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

Sympy [A] time = 1.42502, size = 22, normalized size = 0.96

$$-\frac{3a+5bx^2+15cx^4}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] -(3*a + 5*b*x**2 + 15*c*x**4)/(15*x**5)
```

GIAC/XCAS [A] time = 0.260849, size = 28, normalized size = 1.22

$$-\frac{15\,cx^4 + 5\,bx^2 + 3\,a}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*c*x^4 + 5*b*x^2 + 3*a)/x^5
```

3.823 $\int \frac{a+bx^2+cx^4}{x^7}\,dx$

Optimal. Leaf size=25

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Rubi [A] time = 0.0202527, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^7,x]

[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Rubi in Sympy [A] time = 6.94023, size = 20, normalized size = 0.8

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**7,x)

[Out] -a/(6*x**6) - b/(4*x**4) - c/(2*x**2)
```

Mathematica [A] time = 0.00435337, size = 25, normalized size = 1.

$$-\frac{a}{6x^6}-\frac{b}{4x^4}-\frac{c}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^7,x]
```

```
[Out] -a/(6*x^6) - b/(4*x^4) - c/(2*x^2)
```

Maple [A] time = 0.007, size = 20, normalized size = 0.8

$$-\frac{a}{6\,x^{6}}-\frac{b}{4\,x^{4}}-\frac{c}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^7,x)
```

```
[Out] -1/6*a/x^6-1/4*b/x^4-1/2*c/x^2
```

Maxima [A] time = 0.685433, size = 28, normalized size = 1.12

$$-\frac{6\,cx^{4}+3\,bx^{2}+2\,a}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^7,x, algorithm="maxima")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Fricas [A] time = 0.247331, size = 28, normalized size = 1.12

$$-\frac{6\,cx^{4}+3\,bx^{2}+2\,a}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

Sympy [A] time = 1.64384, size = 22, normalized size = 0.88

$$-\frac{2a+3bx^{2}+6cx^{4}}{12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] -(2*a + 3*b*x**2 + 6*c*x**4)/(12*x**6)
```

GIAC/XCAS [A] time = 0.263818, size = 28, normalized size = 1.12

$$-\frac{6\,cx^4 + 3\,bx^2 + 2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^7,x, algorithm="giac")
```

```
[Out] -1/12*(6*c*x^4 + 3*b*x^2 + 2*a)/x^6
```

3.824 $\int \frac{a+bx^2+cx^4}{x^8} dx$

Optimal. Leaf size=25

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

```
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Rubi [A] time = 0.0193401, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^8,x]

[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Rubi in Sympy [A] time = 4.97606, size = 20, normalized size = 0.8

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**8,x)

[Out] -a/(7*x**7) - b/(5*x**5) - c/(3*x**3)
```

Mathematica [A] time = 0.00430089, size = 25, normalized size = 1.

$$-\frac{a}{7x^7}-\frac{b}{5x^5}-\frac{c}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^8,x]
```

```
[Out] -a/(7*x^7) - b/(5*x^5) - c/(3*x^3)
```

Maple [A] time = 0.007, size = 20, normalized size = 0.8

$$-\frac{a}{7\,x^7}-\frac{b}{5\,x^5}-\frac{c}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^8,x)
```

```
[Out] -1/7*a/x^7-1/5*b/x^5-1/3*c/x^3
```

Maxima [A] time = 0.681514, size = 28, normalized size = 1.12

$$-\frac{35\,cx^4+21\,bx^2+15\,a}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Fricas [A] time = 0.246825, size = 28, normalized size = 1.12

$$-\frac{35\,cx^4+21\,bx^2+15\,a}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

Sympy [A] time = 1.61393, size = 22, normalized size = 0.88

$$-\frac{15a+21bx^2+35cx^4}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**8,x)
```

```
[Out] -(15*a + 21*b*x**2 + 35*c*x**4)/(105*x**7)
```

GIAC/XCAS [A] time = 0.262192, size = 28, normalized size = 1.12

$$-\frac{35\,cx^4 + 21\,bx^2 + 15\,a}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*c*x^4 + 21*b*x^2 + 15*a)/x^7
```

3.825 $\int x^2 \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)
/9 + (c^2*x^11)/11
```

Rubi [A] time = 0.0795987, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)
/9 + (c^2*x^11)/11
```

Rubi in Sympy [A] time = 11.388, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{2bcx^9}{9}+\frac{c^2x^{11}}{11}+x^7\left(\frac{2ac}{7}+\frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*
(2*a*c/7 + b**2/7)
```

Mathematica [A] time = 0.0114176, size = 54, normalized size = 1.

$$\frac{a^2x^3}{3}+\frac{1}{7}x^7\left(2ac+b^2\right)+\frac{2}{5}abx^5+\frac{2}{9}bcx^9+\frac{c^2x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + ((b^2 + 2*a*c)*x^7)/7 + (2*b*c*x^9)
/9 + (c^2*x^11)/11
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^3}{3}+\frac{2\,abx^5}{5}+\frac{\left(2\,ac+b^2\right)x^7}{7}+\frac{2\,bcx^9}{9}+\frac{c^2x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*(2*a*c+b^2)*x^7+2/9*b*c*x^9+1/11*c^2*
x^11
```

Maxima [A] time = 0.689692, size = 59, normalized size = 1.09

$$\frac{1}{11}\,c^2x^{11}+\frac{2}{9}\,bcx^9+\frac{1}{7}\left(b^2+2\,ac\right)x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*(b^2 + 2*a*c)*x^7 + 2/5*a*b*x^5
 + 1/3*a^2*x^3
```

Fricas [A] time = 0.233502, size = 1, normalized size = 0.02

$$\frac{1}{11}x^{11}c^2+\frac{2}{9}x^9cb+\frac{1}{7}x^7b^2+\frac{2}{7}x^7ca+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2 + 2/9*x^9*c*b + 1/7*x^7*b^2 + 2/7*x^7*c*a + 2/5*x^5
*b*a + 1/3*x^3*a^2
```

Sympy [A] time = 0.104222, size = 51, normalized size = 0.94

$$\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{2bcx^9}{9} + \frac{c^2x^{11}}{11} + x^7\left(\frac{2ac}{7} + \frac{b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + 2*b*c*x**9/9 + c**2*x**11/11 + x**7*
(2*a*c/7 + b**2/7)
```

GIAC/XCAS [A] time = 0.263262, size = 62, normalized size = 1.15

$$\frac{1}{11}\,c^2x^{11} + \frac{2}{9}\,bcx^9 + \frac{1}{7}\,b^2x^7 + \frac{2}{7}\,acx^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^2*x^11 + 2/9*b*c*x^9 + 1/7*b^2*x^7 + 2/7*a*c*x^7 + 2/5*a*b
*x^5 + 1/3*a^2*x^3
```

3.826 $\int x \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=54

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 +
 (c^2*x^10)/10
```

Rubi [A] time = 0.0953351, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^2x^2}{2}+\frac{1}{6}x^6\left(2ac+b^2\right)+\frac{1}{2}abx^4+\frac{1}{4}bcx^8+\frac{c^2x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*x^2)/2 + (a*b*x^4)/2 + ((b^2 + 2*a*c)*x^6)/6 + (b*c*x^8)/4 +
 (c^2*x^10)/10
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$ab\int^{x^2} x\,dx+\frac{bcx^8}{4}+\frac{c^2x^{10}}{10}+x^6\left(\frac{ac}{3}+\frac{b^2}{6}\right)+\frac{\int^{x^2} a^2\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a*b*Integral(x, (x, x**2)) + b*c*x**8/4 + c**2*x**10/10 + x**6*(a
*c/3 + b**2/6) + Integral(a**2, (x, x**2))/2
```

Mathematica [A] time = 0.0128176, size = 48, normalized size = 0.89

$$\frac{1}{60}x^2\left(30a^2+10x^4\left(2ac+b^2\right)+30abx^2+15bcx^6+6c^2x^8\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(30*a^2 + 30*a*b*x^2 + 10*(b^2 + 2*a*c)*x^4 + 15*b*c*x^6 + 6
*c^2*x^8))/60
```

Maple [A] time = 0.001, size = 45, normalized size = 0.8

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{(2\,ac + b^2)\,x^6}{6} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*a^2*x^2+1/2*a*b*x^4+1/6*(2*a*c+b^2)*x^6+1/4*b*c*x^8+1/10*c^2*
x^10
```

Maxima [A] time = 0.691685, size = 59, normalized size = 1.09

$$\frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,(b^2 + 2\,ac)\,x^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x,x, algorithm="maxima")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*(b^2 + 2*a*c)*x^6 + 1/2*a*b*x^4
 + 1/2*a^2*x^2
```

Fricas [A] time = 0.233344, size = 1, normalized size = 0.02

$$\frac{1}{10}x^{10}c^2 + \frac{1}{4}x^8cb + \frac{1}{6}x^6b^2 + \frac{1}{3}x^6ca + \frac{1}{2}x^4ba + \frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c^2 + 1/4*x^8*c*b + 1/6*x^6*b^2 + 1/3*x^6*c*a + 1/2*x^4
*b*a + 1/2*x^2*a^2
```

Sympy [A] time = 0.100798, size = 46, normalized size = 0.85

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{bcx^8}{4} + \frac{c^2x^{10}}{10} + x^6\left(\frac{ac}{3} + \frac{b^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b*c*x**8/4 + c**2*x**10/10 + x**6*(a*c
/3 + b**2/6)
```

GIAC/XCAS [A] time = 0.260759, size = 62, normalized size = 1.15

$$\frac{1}{10}\,c^2x^{10} + \frac{1}{4}\,bcx^8 + \frac{1}{6}\,b^2x^6 + \frac{1}{3}\,acx^6 + \frac{1}{2}\,abx^4 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x,x, algorithm="giac")
```

```
[Out] 1/10*c^2*x^10 + 1/4*b*c*x^8 + 1/6*b^2*x^6 + 1/3*a*c*x^6 + 1/2*a*b
*x^4 + 1/2*a^2*x^2
```

3.827 $\int \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=49

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Rubi [A] time = 0.0502748, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2abx^3}{3}+\frac{2bcx^7}{7}+\frac{c^2x^9}{9}+x^5\left(\frac{2ac}{5}+\frac{b^2}{5}\right)+\int a^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/5 + b**2/
5) + Integral(a**2, x)
```

Mathematica [A] time = 0.00829876, size = 49, normalized size = 1.

$$a^2x+\frac{1}{5}x^5\left(2ac+b^2\right)+\frac{2}{3}abx^3+\frac{2}{7}bcx^7+\frac{c^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + ((b^2 + 2*a*c)*x^5)/5 + (2*b*c*x^7)/7 + (
c^2*x^9)/9
```

Maple [A] time = 0.002, size = 42, normalized size = 0.9

$$a^{2}x+\frac{2\,ab{x}^{3}}{3}+\frac{\left(2\,ac+{b}^{2}\right){x}^{5}}{5}+\frac{2\,bc{x}^{7}}{7}+\frac{{c}^{2}{x}^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*(2*a*c+b^2)*x^5+2/7*b*c*x^7+1/9*c^2*x^9
```

Maxima [A] time = 0.69558, size = 61, normalized size = 1.24

$$\frac{1}{9}\,c^{2}x^{9}+\frac{2}{7}\,bcx^{7}+\frac{1}{5}\,b^{2}x^{5}+a^{2}x+\frac{2}{15}\,\left(3\,cx^{5}+5\,bx^{3}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + a^2*x + 2/15*(3*c*x^5 +
 5*b*x^3)*a
```

Fricas [A] time = 0.230447, size = 1, normalized size = 0.02

$$\frac{1}{9}x^{9}c^{2}+\frac{2}{7}x^{7}cb+\frac{1}{5}x^{5}b^{2}+\frac{2}{5}x^{5}ca+\frac{2}{3}x^{3}ba+xa^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^2 + 2/7*x^7*c*b + 1/5*x^5*b^2 + 2/5*x^5*c*a + 2/3*x^3*b
*a + x*a^2
```

Sympy [A] time = 0.096348, size = 48, normalized size = 0.98

$$a^2x + \frac{2abx^3}{3} + \frac{2bcx^7}{7} + \frac{c^2x^9}{9} + x^5\left(\frac{2ac}{5} + \frac{b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + 2*b*c*x**7/7 + c**2*x**9/9 + x**5*(2*a*c/
5 + b**2/5)
```

GIAC/XCAS [A] time = 0.261574, size = 58, normalized size = 1.18

$$\frac{1}{9}\,c^2x^9 + \frac{2}{7}\,bcx^7 + \frac{1}{5}\,b^2x^5 + \frac{2}{5}\,acx^5 + \frac{2}{3}\,abx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] 1/9*c^2*x^9 + 2/7*b*c*x^7 + 1/5*b^2*x^5 + 2/5*a*c*x^5 + 2/3*a*b*x
^3 + a^2*x
```

3.828 $\int \frac{(a+bx^2+cx^4)^2}{x} dx$

Optimal. Leaf size=47

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2
*Log[x]
```

Rubi [A] time = 0.117743, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2
*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^2\log\left(x^2\right)}{2}+abx^2+\frac{bcx^6}{3}+\frac{c^2x^8}{8}+\left(ac+\frac{b^2}{2}\right)\int^{x^2} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x**2)/2 + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + (a*c + b
**2/2)*Integral(x, (x, x**2))
```

Mathematica [A] time = 0.0193983, size = 47, normalized size = 1.

$$a^2\log(x)+\frac{1}{4}x^4\left(2ac+b^2\right)+abx^2+\frac{1}{3}bcx^6+\frac{c^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x,x]
```

```
[Out] a*b*x^2 + ((b^2 + 2*a*c)*x^4)/4 + (b*c*x^6)/3 + (c^2*x^8)/8 + a^2
*Log[x]
```

Maple [A] time = 0.003, size = 44, normalized size = 0.9

$$\frac{c^2x^8}{8} + \frac{bcx^6}{3} + \frac{x^4ac}{2} + \frac{b^2x^4}{4} + abx^2 + a^2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/8*c^2*x^8+1/3*b*c*x^6+1/2*x^4*a*c+1/4*b^2*x^4+a*b*x^2+a^2*ln(x)
```

Maxima [A] time = 0.686006, size = 59, normalized size = 1.26

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}\left(b^2 + 2ac\right)x^4 + abx^2 + \frac{1}{2}a^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + 1/2
*a^2*log(x^2)
```

Fricas [A] time = 0.254806, size = 55, normalized size = 1.17

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}\left(b^2 + 2ac\right)x^4 + abx^2 + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*(b^2 + 2*a*c)*x^4 + a*b*x^2 + a^2
*log(x)
```

Sympy [A] time = 1.05899, size = 42, normalized size = 0.89

$$a^2\log(x) + abx^2 + \frac{bcx^6}{3} + \frac{c^2x^8}{8} + x^4\left(\frac{ac}{2} + \frac{b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] a**2*log(x) + a*b*x**2 + b*c*x**6/3 + c**2*x**8/8 + x**4*(a*c/2 +
 b**2/4)
```

GIAC/XCAS [A] time = 0.265541, size = 62, normalized size = 1.32

$$\frac{1}{8}c^2x^8 + \frac{1}{3}bcx^6 + \frac{1}{4}b^2x^4 + \frac{1}{2}acx^4 + abx^2 + \frac{1}{2}a^2\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x,x, algorithm="giac")
```

```
[Out] 1/8*c^2*x^8 + 1/3*b*c*x^6 + 1/4*b^2*x^4 + 1/2*a*c*x^4 + a*b*x^2 +
 1/2*a^2*ln(x^2)
```

3.829 $\int \frac{(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=48

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2
*x^7)/7
```

Rubi [A] time = 0.0612371, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2
*x^7)/7
```

Rubi in Sympy [A] time = 10.975, size = 44, normalized size = 0.92

$$-\frac{a^2}{x}+2abx+\frac{2bcx^5}{5}+\frac{c^2x^7}{7}+x^3\left(\frac{2ac}{3}+\frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 +
b**2/3)
```

Mathematica [A] time = 0.0296499, size = 48, normalized size = 1.

$$-\frac{a^2}{x}+\frac{1}{3}x^3\left(2ac+b^2\right)+2abx+\frac{2}{5}bcx^5+\frac{c^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + ((b^2 + 2*a*c)*x^3)/3 + (2*b*c*x^5)/5 + (c^2
*x^7)/7
```

Maple [A] time = 0.005, size = 45, normalized size = 0.9

$$\frac{c^2x^7}{7} + \frac{2\,bcx^5}{5} + \frac{2\,x^3ac}{3} + \frac{b^2x^3}{3} + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^2,x)
```

```
[Out] 1/7*c^2*x^7+2/5*b*c*x^5+2/3*x^3*a*c+1/3*b^2*x^3+2*a*b*x-a^2/x
```

Maxima [A] time = 0.690635, size = 57, normalized size = 1.19

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,\left(b^2 + 2\,ac\right)x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*(b^2 + 2*a*c)*x^3 + 2*a*b*x - a^2
/x
```

Fricas [A] time = 0.248162, size = 62, normalized size = 1.29

$$\frac{15\,c^2x^8 + 42\,bcx^6 + 35\,\left(b^2 + 2\,ac\right)x^4 + 210\,abx^2 - 105\,a^2}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(15*c^2*x^8 + 42*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 210*a*b*x
^2 - 105*a^2)/x
```

Sympy [A] time = 1.05135, size = 44, normalized size = 0.92

$$-\frac{a^2}{x} + 2abx + \frac{2bcx^5}{5} + \frac{c^2x^7}{7} + x^3\left(\frac{2ac}{3} + \frac{b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -a**2/x + 2*a*b*x + 2*b*c*x**5/5 + c**2*x**7/7 + x**3*(2*a*c/3 +
b**2/3)
```

GIAC/XCAS [A] time = 0.262427, size = 59, normalized size = 1.23

$$\frac{1}{7}\,c^2x^7 + \frac{2}{5}\,bcx^5 + \frac{1}{3}\,b^2x^3 + \frac{2}{3}\,acx^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c^2*x^7 + 2/5*b*c*x^5 + 1/3*b^2*x^3 + 2/3*a*c*x^3 + 2*a*b*x -
 a^2/x
```

3.830 $\int \frac{(a+bx^2+cx^4)^2}{x^3} dx$

Optimal. Leaf size=51

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

```
[Out] -a^2/(2*x^2) + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6
+ 2*a*b*Log[x]
```

Rubi [A] time = 0.0963491, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{a^2}{2x^2}+\frac{1}{2}x^2\left(2ac+b^2\right)+2ab\log(x)+\frac{1}{2}bcx^4+\frac{c^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) + ((b^2 + 2*a*c)*x^2)/2 + (b*c*x^4)/2 + (c^2*x^6)/6
+ 2*a*b*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{2x^2}+ab\log\left(x^2\right)+bc\int^{x^2}x\,dx+\frac{c^2x^6}{6}+\frac{\left(2ac+b^2\right)\int^{x^2}b^2\,dx}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + a*b*log(x**2) + b*c*Integral(x, (x, x**2)) + c**
2*x**6/6 + (2*a*c + b**2)*Integral(b**2, (x, x**2))/(2*b**2)
```

Mathematica [A] time = 0.0253631, size = 46, normalized size = 0.9

$$\frac{1}{6}\left(-\frac{3a^2}{x^2}+3x^2\left(2ac+b^2\right)+12ab\log(x)+3bcx^4+c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^3,x]
```

```
[Out] ((-3*a^2)/x^2 + 3*(b^2 + 2*a*c)*x^2 + 3*b*c*x^4 + c^2*x^6 + 12*a*
b*Log[x])/6
```

Maple [A] time = 0.009, size = 45, normalized size = 0.9

$$\frac{c^2x^6}{6}+\frac{bcx^4}{2}+x^2ac+\frac{b^2x^2}{2}+2\,ab\ln\left(x\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/6*c^2*x^6+1/2*b*c*x^4+x^2*a*c+1/2*b^2*x^2+2*a*b*ln(x)-1/2*a^2/x
^2
```

Maxima [A] time = 0.687854, size = 59, normalized size = 1.16

$$\frac{1}{6}\,c^2x^6+\frac{1}{2}\,bcx^4+\frac{1}{2}\,\left(b^2+2\,ac\right)x^2+ab\log\left(x^2\right)-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*(b^2 + 2*a*c)*x^2 + a*b*log(x^2)
- 1/2*a^2/x^2
```

Fricas [A] time = 0.253825, size = 63, normalized size = 1.24

$$\frac{c^2x^8+3\,bcx^6+3\,\left(b^2+2\,ac\right)x^4+12\,abx^2\log\left(x\right)-3\,a^2}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(c^2*x^8 + 3*b*c*x^6 + 3*(b^2 + 2*a*c)*x^4 + 12*a*b*x^2*log(x
) - 3*a^2)/x^2
```

Sympy [A] time = 1.156, size = 44, normalized size = 0.86

$$-\frac{a^2}{2x^2} + 2ab\log(x) + \frac{bcx^4}{2} + \frac{c^2x^6}{6} + x^2\left(ac + \frac{b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] -a**2/(2*x**2) + 2*a*b*log(x) + b*c*x**4/2 + c**2*x**6/6 + x**2*(
a*c + b**2/2)
```

GIAC/XCAS [A] time = 0.265301, size = 72, normalized size = 1.41

$$\frac{1}{6}c^2x^6 + \frac{1}{2}bcx^4 + \frac{1}{2}b^2x^2 + acx^2 + ab\ln\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c^2*x^6 + 1/2*b*c*x^4 + 1/2*b^2*x^2 + a*c*x^2 + a*b*ln(x^2) -
 1/2*(2*a*b*x^2 + a^2)/x^2
```

3.831 $\int \frac{(a+bx^2+cx^4)^2}{x^4} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2
*x^5)/5
```

Rubi [A] time = 0.0633064, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2
*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+\frac{2bcx^3}{3}+\frac{c^2x^5}{5}+\frac{\left(2ac+b^2\right)\int b^2\,dx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] -a**2/(3*x**3) - 2*a*b/x + 2*b*c*x**3/3 + c**2*x**5/5 + (2*a*c +
b**2)*Integral(b**2, x)/b**2
```

Mathematica [A] time = 0.0329877, size = 47, normalized size = 1.

$$-\frac{a^2}{3x^3}+x\left(2ac+b^2\right)-\frac{2ab}{x}+\frac{2}{3}bcx^3+\frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^4,x]
```

```
[Out] -a^2/(3*x^3) - (2*a*b)/x + (b^2 + 2*a*c)*x + (2*b*c*x^3)/3 + (c^2
*x^5)/5
```

Maple [A] time = 0.008, size = 42, normalized size = 0.9

$$\frac{c^2x^5}{5}+\frac{2\,bcx^3}{3}+2\,xac+b^2x-\frac{a^2}{3\,x^3}-2\,\frac{ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^4,x)
```

```
[Out] 1/5*c^2*x^5+2/3*b*c*x^3+2*x*a*c+b^2*x-1/3*a^2/x^3-2*a*b/x
```

Maxima [A] time = 0.682364, size = 57, normalized size = 1.21

$$\frac{1}{5}\,c^2x^5+\frac{2}{3}\,bcx^3+\left(b^2+2\,ac\right)x-\frac{6\,abx^2+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + (b^2 + 2*a*c)*x - 1/3*(6*a*b*x^2 + a^
2)/x^3
```

Fricas [A] time = 0.24771, size = 62, normalized size = 1.32

$$\frac{3\,c^2x^8+10\,bcx^6+15\left(b^2+2\,ac\right)x^4-30\,abx^2-5\,a^2}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*c^2*x^8 + 10*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 - 30*a*b*x^2
- 5*a^2)/x^3
```

Sympy [A] time = 1.17521, size = 44, normalized size = 0.94

$$\frac{2bcx^3}{3} + \frac{c^2x^5}{5} + x\left(2ac + b^2\right) - \frac{a^2 + 6abx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*b*c*x**3/3 + c**2*x**5/5 + x*(2*a*c + b**2) - (a**2 + 6*a*b*x**
2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.262874, size = 57, normalized size = 1.21

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,bcx^3 + b^2x + 2\,acx - \frac{6\,abx^2 + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/5*c^2*x^5 + 2/3*b*c*x^3 + b^2*x + 2*a*c*x - 1/3*(6*a*b*x^2 + a^
2)/x^3
```

3.832 $\int \frac{(a+bx^2+cx^4)^2}{x^5}\,dx$

Optimal. Leaf size=45

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*
Log[x]
```

Rubi [A] time = 0.111425, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{a^2}{4x^4}+\log(x)\left(2ac+b^2\right)-\frac{ab}{x^2}+bcx^2+\frac{c^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (a*b)/x^2 + b*c*x^2 + (c^2*x^4)/4 + (b^2 + 2*a*c)*
Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+bcx^2+\frac{c^2\int^{x^2}x\,dx}{2}+\left(ac+\frac{b^2}{2}\right)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] -a**2/(4*x**4) - a*b/x**2 + b*c*x**2 + c**2*Integral(x, (x, x**2)
)/2 + (a*c + b**2/2)*log(x**2)
```

Mathematica [A] time = 0.0324623, size = 41, normalized size = 0.91

$$\log(x)\left(2ac+b^2\right)+\frac{\left(cx^4-a\right)\left(a+4bx^2+cx^4\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^5,x]
```

```
[Out] ((-a + c*x^4)*(a + 4*b*x^2 + c*x^4))/(4*x^4) + (b^2 + 2*a*c)*Log[
x]
```

Maple [A] time = 0.009, size = 43, normalized size = 1.

$$\frac{c^2x^4}{4} + bcx^2 + 2\ \ln(x)\,ac + b^2\ln(x) - \frac{ab}{x^2} - \frac{a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/4*c^2*x^4+b*c*x^2+2*ln(x)*a*c+b^2*ln(x)-a*b/x^2-1/4*a^2/x^4
```

Maxima [A] time = 0.691901, size = 61, normalized size = 1.36

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\left(b^2 + 2\,ac\right)\,\log\left(x^2\right) - \frac{4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*log(x^2) - 1/4*(4*a*b*x
^2 + a^2)/x^4
```

Fricas [A] time = 0.260574, size = 63, normalized size = 1.4

$$\frac{c^2x^8 + 4\,bcx^6 + 4\left(b^2 + 2\,ac\right)x^4\log\left(x\right) - 4\,abx^2 - a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(c^2*x^8 + 4*b*c*x^6 + 4*(b^2 + 2*a*c)*x^4*log(x) - 4*a*b*x^2
 - a^2)/x^4
```

Sympy [A] time = 1.78518, size = 42, normalized size = 0.93

$$bcx^2 + \frac{c^2x^4}{4} + \left(2ac + b^2\right)\log\left(x\right) - \frac{a^2 + 4abx^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] b*c*x**2 + c**2*x**4/4 + (2*a*c + b**2)*log(x) - (a**2 + 4*a*b*x*
*2)/(4*x**4)
```

GIAC/XCAS [A] time = 0.262837, size = 81, normalized size = 1.8

$$\frac{1}{4}\,c^2x^4 + bcx^2 + \frac{1}{2}\left(b^2 + 2\,ac\right)\ln\left(x^2\right) - \frac{3\,b^2x^4 + 6\,acx^4 + 4\,abx^2 + a^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/4*c^2*x^4 + b*c*x^2 + 1/2*(b^2 + 2*a*c)*ln(x^2) - 1/4*(3*b^2*x^
4 + 6*a*c*x^4 + 4*a*b*x^2 + a^2)/x^4
```

3.833 $\int \frac{(a+bx^2+cx^4)^2}{x^6} dx$

Optimal. Leaf size=48

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2
*x^3)/3
```

Rubi [A] time = 0.0631938, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^2}{5x^5}-\frac{2ac+b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) - (b^2 + 2*a*c)/x + 2*b*c*x + (c^2
*x^3)/3
```

Rubi in Sympy [A] time = 11.0716, size = 42, normalized size = 0.88

$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}-\frac{2ac+b^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] -a**2/(5*x**5) - 2*a*b/(3*x**3) + 2*b*c*x + c**2*x**3/3 - (2*a*c
+ b**2)/x
```

Mathematica [A] time = 0.0378178, size = 49, normalized size = 1.02

$$-\frac{a^2}{5x^5}+\frac{-2ac-b^2}{x}-\frac{2ab}{3x^3}+2bcx+\frac{c^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (2*a*b)/(3*x^3) + (-b^2 - 2*a*c)/x + 2*b*c*x + (c^
2*x^3)/3
```

Maple [A] time = 0.008, size = 43, normalized size = 0.9

$$\frac{c^2x^3}{3} + 2\,bcx - \frac{2\,ab}{3\,x^3} - \frac{2\,ac + b^2}{x} - \frac{a^2}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/3*c^2*x^3+2*b*c*x-2/3*a*b/x^3-(2*a*c+b^2)/x-1/5*a^2/x^5
```

Maxima [A] time = 0.685536, size = 61, normalized size = 1.27

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\left(b^2 + 2\,ac\right)x^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*(b^2 + 2*a*c)*x^4 + 10*a*b*x^2 +
 3*a^2)/x^5
```

Fricas [A] time = 0.247846, size = 62, normalized size = 1.29

$$\frac{5\,c^2x^8 + 30\,bcx^6 - 15\left(b^2 + 2\,ac\right)x^4 - 10\,abx^2 - 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/15*(5*c^2*x^8 + 30*b*c*x^6 - 15*(b^2 + 2*a*c)*x^4 - 10*a*b*x^2
- 3*a^2)/x^5
```

Sympy [A] time = 1.9813, size = 46, normalized size = 0.96

$$2bcx + \frac{c^2x^3}{3} - \frac{3a^2 + 10abx^2 + x^4\left(30ac + 15b^2\right)}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] 2*b*c*x + c**2*x**3/3 - (3*a**2 + 10*a*b*x**2 + x**4*(30*a*c + 15
*b**2))/(15*x**5)
```

GIAC/XCAS [A] time = 0.26334, size = 63, normalized size = 1.31

$$\frac{1}{3}\,c^2x^3 + 2\,bcx - \frac{15\,b^2x^4 + 30\,acx^4 + 10\,abx^2 + 3\,a^2}{15\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/3*c^2*x^3 + 2*b*c*x - 1/15*(15*b^2*x^4 + 30*a*c*x^4 + 10*a*b*x^
2 + 3*a^2)/x^5
```

3.834 $\int \frac{(a+bx^2+cx^4)^2}{x^7} dx$

Optimal. Leaf size=51

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/
2 + 2*b*c*Log[x]
```

Rubi [A] time = 0.0895239, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{a^2}{6x^6}-\frac{2ac+b^2}{2x^2}-\frac{ab}{2x^4}+2bc\log(x)+\frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -a^2/(6*x^6) - (a*b)/(2*x^4) - (b^2 + 2*a*c)/(2*x^2) + (c^2*x^2)/
2 + 2*b*c*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{6x^6}-\frac{ab}{2x^4}+bc\log\left(x^2\right)+\frac{\int^{x^2}c^2\,dx}{2}-\frac{ac+\frac{b^2}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] -a**2/(6*x**6) - a*b/(2*x**4) + b*c*log(x**2) + Integral(c**2, (x
, x**2))/2 - (a*c + b**2/2)/x**2
```

Mathematica [A] time = 0.0300371, size = 50, normalized size = 0.98

$$-\frac{a^2+3abx^2+6acx^4+3b^2x^4-12bcx^6\log(x)-3c^2x^8}{6x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^7,x]
```

```
[Out] -(a^2 + 3*a*b*x^2 + 3*b^2*x^4 + 6*a*c*x^4 - 3*c^2*x^8 - 12*b*c*x^
6*Log[x])/(6*x^6)
```

Maple [A]   time = 0.009, size = 46, normalized size = 0.9

$$\frac{c^2x^2}{2} - \frac{a^2}{6\,x^6} + 2\,bc\ln(x) - \frac{ac}{x^2} - \frac{b^2}{2\,x^2} - \frac{ab}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/2*c^2*x^2-1/6*a^2/x^6+2*b*c*ln(x)-1/x^2*a*c-1/2*b^2/x^2-1/2*a*b
/x^4
```

Maxima [A]   time = 0.69197, size = 61, normalized size = 1.2

$$\frac{1}{2}\,c^2x^2 + bc\log\left(x^2\right) - \frac{3\,\left(b^2 + 2\,ac\right)x^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*x^2 + b*c*log(x^2) - 1/6*(3*(b^2 + 2*a*c)*x^4 + 3*a*b*x^2
 + a^2)/x^6
```

Fricas [A]   time = 0.253649, size = 65, normalized size = 1.27

$$\frac{3\,c^2x^8 + 12\,bcx^6\log(x) - 3\,\left(b^2 + 2\,ac\right)x^4 - 3\,abx^2 - a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/6*(3*c^2*x^8 + 12*b*c*x^6*log(x) - 3*(b^2 + 2*a*c)*x^4 - 3*a*b*
x^2 - a^2)/x^6
```

Sympy [A] time = 2.93551, size = 46, normalized size = 0.9

$$2bc\log(x) + \frac{c^2x^2}{2} - \frac{a^2 + 3abx^2 + x^4\left(6ac + 3b^2\right)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*b*c*log(x) + c**2*x**2/2 - (a**2 + 3*a*b*x**2 + x**4*(6*a*c + 3
*b**2))/(6*x**6)
```

GIAC/XCAS [A] time = 0.263137, size = 73, normalized size = 1.43

$$\frac{1}{2}c^2x^2 + bc\ln\left(x^2\right) - \frac{11\,bcx^6 + 3\,b^2x^4 + 6\,acx^4 + 3\,abx^2 + a^2}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/2*c^2*x^2 + b*c*ln(x^2) - 1/6*(11*b*c*x^6 + 3*b^2*x^4 + 6*a*c*x
^4 + 3*a*b*x^2 + a^2)/x^6
```

3.835 $\int \frac{(a+bx^2+cx^4)^2}{x^8}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/
x + c^2*x
```

Rubi [A] time = 0.0622405, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$-\frac{a^2}{7x^7}-\frac{2ac+b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) - (b^2 + 2*a*c)/(3*x^3) - (2*b*c)/
x + c^2*x
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{2bc}{x}+\int c^2\,dx-\frac{\frac{2ac}{3}+\frac{b^2}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

```
[Out] -a**2/(7*x**7) - 2*a*b/(5*x**5) - 2*b*c/x + Integral(c**2, x) - (
2*a*c/3 + b**2/3)/x**3
```

Mathematica [A] time = 0.042591, size = 49, normalized size = 1.04

$$-\frac{a^2}{7x^7}+\frac{-2ac-b^2}{3x^3}-\frac{2ab}{5x^5}-\frac{2bc}{x}+c^2x$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (2*a*b)/(5*x^5) + (-b^2 - 2*a*c)/(3*x^3) - (2*b*c)
/x + c^2*x
```

Maple [A] time = 0.008, size = 42, normalized size = 0.9

$$c^2x - \frac{2\,ac + b^2}{3\,x^3} - 2\,\frac{bc}{x} - \frac{2\,ab}{5\,x^5} - \frac{a^2}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^8,x)
```

```
[Out] c^2*x-1/3*(2*a*c+b^2)/x^3-2*b*c/x-2/5*a*b/x^5-1/7*a^2/x^7
```

Maxima [A] time = 0.691059, size = 59, normalized size = 1.26

$$c^2x - \frac{210\,bcx^6 + 35\,(b^2 + 2\,ac)\,x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^8,x, algorithm="maxima")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*(b^2 + 2*a*c)*x^4 + 42*a*b*x^2 +
15*a^2)/x^7
```

Fricas [A] time = 0.248692, size = 62, normalized size = 1.32

$$\frac{105\,c^2x^8 - 210\,bcx^6 - 35\,(b^2 + 2\,ac)\,x^4 - 42\,abx^2 - 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*c^2*x^8 - 210*b*c*x^6 - 35*(b^2 + 2*a*c)*x^4 - 42*a*b*
x^2 - 15*a^2)/x^7
```

Sympy [A] time = 2.89466, size = 44, normalized size = 0.94

$$c^2x - \frac{15a^2 + 42abx^2 + 210bcx^6 + x^4\left(70ac + 35b^2\right)}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**8,x)
```

```
[Out] c**2*x - (15*a**2 + 42*a*b*x**2 + 210*b*c*x**6 + x**4*(70*a*c + 3
5*b**2))/(105*x**7)
```

GIAC/XCAS [A] time = 0.261715, size = 62, normalized size = 1.32

$$c^2x - \frac{210\,bcx^6 + 35\,b^2x^4 + 70\,acx^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^8,x, algorithm="giac")
```

```
[Out] c^2*x - 1/105*(210*b*c*x^6 + 35*b^2*x^4 + 70*a*c*x^4 + 42*a*b*x^2
 + 15*a^2)/x^7
```

3.836 $\int \frac{(a+bx^2+cx^4)^2}{x^9} dx$

Optimal. Leaf size=48

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2
+ c^2*Log[x]
```

Rubi [A] time = 0.0866348, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{a^2}{8x^8}-\frac{2ac+b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) - (b^2 + 2*a*c)/(4*x^4) - (b*c)/x^2
+ c^2*Log[x]
```

Rubi in Sympy [A] time = 14.98, size = 46, normalized size = 0.96

$$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{bc}{x^2}+\frac{c^2\log\left(x^2\right)}{2}-\frac{\frac{ac}{2}+\frac{b^2}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

```
[Out] -a**2/(8*x**8) - a*b/(3*x**6) - b*c/x**2 + c**2*log(x**2)/2 - (a*
c/2 + b**2/4)/x**4
```

Mathematica [A] time = 0.046921, size = 50, normalized size = 1.04

$$-\frac{a^2}{8x^8}+\frac{-2ac-b^2}{4x^4}-\frac{ab}{3x^6}-\frac{bc}{x^2}+c^2\log(x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^9,x]
```

```
[Out] -a^2/(8*x^8) - (a*b)/(3*x^6) + (-b^2 - 2*a*c)/(4*x^4) - (b*c)/x^2
 + c^2*Log[x]
```

Maple [A] time = 0.009, size = 45, normalized size = 0.9

$$-\frac{a^2}{8\,x^8}-\frac{ab}{3\,x^6}+c^2\ln(x)-\frac{bc}{x^2}-\frac{ac}{2\,x^4}-\frac{b^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^9,x)
```

```
[Out] -1/8*a^2/x^8-1/3*a*b/x^6+c^2*ln(x)-b*c/x^2-1/2/x^4*a*c-1/4*b^2/x^
4
```

Maxima [A] time = 0.684689, size = 65, normalized size = 1.35

$$\frac{1}{2}\,c^2\log\left(x^2\right)-\frac{24\,bcx^6+6\left(b^2+2\,ac\right)x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*c^2*log(x^2) - 1/24*(24*b*c*x^6 + 6*(b^2 + 2*a*c)*x^4 + 8*a*b
*x^2 + 3*a^2)/x^8
```

Fricas [A] time = 0.254708, size = 65, normalized size = 1.35

$$\frac{24\,c^2x^8\log(x)-24\,bcx^6-6\left(b^2+2\,ac\right)x^4-8\,abx^2-3\,a^2}{24\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^9,x, algorithm="fricas")
```

```
[Out] 1/24*(24*c^2*x^8*log(x) - 24*b*c*x^6 - 6*(b^2 + 2*a*c)*x^4 - 8*a*
b*x^2 - 3*a^2)/x^8
```

Sympy [A] time = 4.65832, size = 46, normalized size = 0.96

$$c^2\log(x) - \frac{3a^2 + 8abx^2 + 24bcx^6 + x^4(12ac + 6b^2)}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**9,x)
```

```
[Out] c**2*log(x) - (3*a**2 + 8*a*b*x**2 + 24*b*c*x**6 + x**4*(12*a*c +
 6*b**2))/(24*x**8)
```

GIAC/XCAS [A] time = 0.262694, size = 78, normalized size = 1.62

$$\frac{1}{2}c^2\ln(x^2) - \frac{25c^2x^8 + 24bcx^6 + 6b^2x^4 + 12acx^4 + 8abx^2 + 3a^2}{24x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^9,x, algorithm="giac")
```

```
[Out] 1/2*c^2*ln(x^2) - 1/24*(25*c^2*x^8 + 24*b*c*x^6 + 6*b^2*x^4 + 12*
a*c*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

3.837 $\int \frac{(a+bx^2+cx^4)^2}{x^{10}}\,dx$

Optimal. Leaf size=52

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/
(3*x^3) - c^2/x
```

Rubi [A] time = 0.0625679, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^2}{9x^9}-\frac{2ac+b^2}{5x^5}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -a^2/(9*x^9) - (2*a*b)/(7*x^7) - (b^2 + 2*a*c)/(5*x^5) - (2*b*c)/
(3*x^3) - c^2/x
```

Rubi in Sympy [A] time = 11.2097, size = 49, normalized size = 0.94

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{2bc}{3x^3}-\frac{c^2}{x}-\frac{\frac{2ac}{5}+\frac{b^2}{5}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

```
[Out] -a**2/(9*x**9) - 2*a*b/(7*x**7) - 2*b*c/(3*x**3) - c**2/x - (2*a*
c/5 + b**2/5)/x**5
```

Mathematica [A] time = 0.0351783, size = 50, normalized size = 0.96

$$-\frac{35a^2+90abx^2+126acx^4+63b^2x^4+210bcx^6+315c^2x^8}{315x^9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^10,x]
```

```
[Out] -(35*a^2 + 90*a*b*x^2 + 63*b^2*x^4 + 126*a*c*x^4 + 210*b*c*x^6 +
315*c^2*x^8)/(315*x^9)
```

Maple [A] time = 0.008, size = 45, normalized size = 0.9

$$-\frac{2\,bc}{3\,x^3}-\frac{a^2}{9\,x^9}-\frac{c^2}{x}-\frac{2\,ac+b^2}{5\,x^5}-\frac{2\,ab}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^10,x)
```

```
[Out] -2/3*b*c/x^3-1/9*a^2/x^9-c^2/x-1/5*(2*a*c+b^2)/x^5-2/7*a*b/x^7
```

Maxima [A] time = 0.691133, size = 62, normalized size = 1.19

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,\left(b^2+2\,ac\right)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^10,x, algorithm="maxima")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b
*x^2 + 35*a^2)/x^9
```

Fricas [A] time = 0.247836, size = 62, normalized size = 1.19

$$-\frac{315\,c^2x^8+210\,bcx^6+63\,\left(b^2+2\,ac\right)x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^10,x, algorithm="fricas")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*(b^2 + 2*a*c)*x^4 + 90*a*b
*x^2 + 35*a^2)/x^9
```

Sympy [A] time = 4.93467, size = 49, normalized size = 0.94

$$-\frac{35a^2 + 90abx^2 + 210bcx^6 + 315c^2x^8 + x^4\left(126ac + 63b^2\right)}{315x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**10,x)
```

```
[Out] -(35*a**2 + 90*a*b*x**2 + 210*b*c*x**6 + 315*c**2*x**8 + x**4*(12
6*a*c + 63*b**2))/(315*x**9)
```

GIAC/XCAS [A] time = 0.261027, size = 65, normalized size = 1.25

$$-\frac{315\,c^2x^8 + 210\,bcx^6 + 63\,b^2x^4 + 126\,acx^4 + 90\,abx^2 + 35\,a^2}{315\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^10,x, algorithm="giac")
```

```
[Out] -1/315*(315*c^2*x^8 + 210*b*c*x^6 + 63*b^2*x^4 + 126*a*c*x^4 + 90
*a*b*x^2 + 35*a^2)/x^9
```

3.838 $\int \frac{(a+bx^2+cx^4)^2}{x^{11}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

```
[Out] -a^2/(10*x^10) - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2
*x^4) - c^2/(2*x^2)
```

Rubi [A] time = 0.088051, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{a^2}{10x^{10}}-\frac{2ac+b^2}{6x^6}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -a^2/(10*x^10) - (a*b)/(4*x^8) - (b^2 + 2*a*c)/(6*x^6) - (b*c)/(2
*x^4) - c^2/(2*x^2)
```

Rubi in Sympy [A] time = 15.3318, size = 48, normalized size = 0.89

$$-\frac{a^2}{10x^{10}}-\frac{ab}{4x^8}-\frac{bc}{2x^4}-\frac{c^2}{2x^2}-\frac{\frac{ac}{3}+\frac{b^2}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

```
[Out] -a**2/(10*x**10) - a*b/(4*x**8) - b*c/(2*x**4) - c**2/(2*x**2) -
(a*c/3 + b**2/6)/x**6
```

Mathematica [A] time = 0.0290196, size = 53, normalized size = 0.98

$$-\frac{6a^2+5a\left(3bx^2+4cx^4\right)+10x^4\left(b^2+3bcx^2+3c^2x^4\right)}{60x^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^11,x]
```

```
[Out] -(6*a^2 + 5*a*(3*b*x^2 + 4*c*x^4) + 10*x^4*(b^2 + 3*b*c*x^2 + 3*c
^2*x^4))/(60*x^10)
```

Maple [A] time = 0.007, size = 45, normalized size = 0.8

$$-\frac{ab}{4\,x^8}-\frac{2\,ac+b^2}{6\,x^6}-\frac{c^2}{2\,x^2}-\frac{a^2}{10\,x^{10}}-\frac{bc}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^11,x)
```

```
[Out] -1/4*a*b/x^8-1/6*(2*a*c+b^2)/x^6-1/2*c^2/x^2-1/10*a^2/x^10-1/2*b*
c/x^4
```

Maxima [A] time = 0.685209, size = 62, normalized size = 1.15

$$-\frac{30\,c^2x^8+30\,bcx^6+10\left(b^2+2\,ac\right)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^11,x, algorithm="maxima")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^
2 + 6*a^2)/x^10
```

Fricas [A] time = 0.246775, size = 62, normalized size = 1.15

$$-\frac{30\,c^2x^8+30\,bcx^6+10\left(b^2+2\,ac\right)x^4+15\,abx^2+6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^11,x, algorithm="fricas")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*(b^2 + 2*a*c)*x^4 + 15*a*b*x^
2 + 6*a^2)/x^10
```

Sympy [A] time = 6.52052, size = 49, normalized size = 0.91

$$-\frac{6a^2 + 15abx^2 + 30bcx^6 + 30c^2x^8 + x^4\left(20ac + 10b^2\right)}{60x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**11,x)
```

```
[Out] -(6*a**2 + 15*a*b*x**2 + 30*b*c*x**6 + 30*c**2*x**8 + x**4*(20*a*
c + 10*b**2))/(60*x**10)
```

GIAC/XCAS [A] time = 0.262013, size = 65, normalized size = 1.2

$$-\frac{30\,c^2x^8 + 30\,bcx^6 + 10\,b^2x^4 + 20\,acx^4 + 15\,abx^2 + 6\,a^2}{60\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^11,x, algorithm="giac")
```

```
[Out] -1/60*(30*c^2*x^8 + 30*b*c*x^6 + 10*b^2*x^4 + 20*a*c*x^4 + 15*a*b
*x^2 + 6*a^2)/x^10
```

3.839 $\int \frac{(a+bx^2+cx^4)^2}{x^{12}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c
)/(5*x^5) - c^2/(3*x^3)
```

Rubi [A] time = 0.0616367, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^2}{11x^{11}}-\frac{2ac+b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) - (b^2 + 2*a*c)/(7*x^7) - (2*b*c
)/(5*x^5) - c^2/(3*x^3)
```

Rubi in Sympy [A] time = 11.1524, size = 53, normalized size = 0.98

$$-\frac{a^2}{11x^{11}}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}-\frac{\frac{2ac}{7}+\frac{b^2}{7}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

```
[Out] -a**2/(11*x**11) - 2*a*b/(9*x**9) - 2*b*c/(5*x**5) - c**2/(3*x**3
) - (2*a*c/7 + b**2/7)/x**7
```

Mathematica [A] time = 0.0430966, size = 56, normalized size = 1.04

$$-\frac{a^2}{11x^{11}}+\frac{-2ac-b^2}{7x^7}-\frac{2ab}{9x^9}-\frac{2bc}{5x^5}-\frac{c^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^12,x]
```

```
[Out] -a^2/(11*x^11) - (2*a*b)/(9*x^9) + (-b^2 - 2*a*c)/(7*x^7) - (2*b*
c)/(5*x^5) - c^2/(3*x^3)
```

Maple [A] time = 0.008, size = 45, normalized size = 0.8

$$-\frac{c^2}{3x^3}-\frac{2ab}{9x^9}-\frac{a^2}{11x^{11}}-\frac{2bc}{5x^5}-\frac{2ac+b^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^12,x)
```

```
[Out] -1/3*c^2/x^3-2/9*a*b/x^9-1/11*a^2/x^11-2/5*b*c/x^5-1/7*(2*a*c+b^2
)/x^7
```

Maxima [A] time = 0.700302, size = 62, normalized size = 1.15

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,\left(b^2+2\,ac\right)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^12,x, algorithm="maxima")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 77
0*a*b*x^2 + 315*a^2)/x^11
```

Fricas [A] time = 0.250001, size = 62, normalized size = 1.15

$$-\frac{1155\,c^2x^8+1386\,bcx^6+495\,\left(b^2+2\,ac\right)x^4+770\,abx^2+315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^12,x, algorithm="fricas")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*(b^2 + 2*a*c)*x^4 + 77
0*a*b*x^2 + 315*a^2)/x^11
```

Sympy [A] time = 6.35452, size = 49, normalized size = 0.91

$$-\frac{315a^2 + 770abx^2 + 1386bcx^6 + 1155c^2x^8 + x^4\left(990ac + 495b^2\right)}{3465x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**12,x)
```

```
[Out] -(315*a**2 + 770*a*b*x**2 + 1386*b*c*x**6 + 1155*c**2*x**8 + x**4
*(990*a*c + 495*b**2))/(3465*x**11)
```

GIAC/XCAS [A] time = 0.260907, size = 65, normalized size = 1.2

$$-\frac{1155\,c^2x^8 + 1386\,bcx^6 + 495\,b^2x^4 + 990\,acx^4 + 770\,abx^2 + 315\,a^2}{3465\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^12,x, algorithm="giac")
```

```
[Out] -1/3465*(1155*c^2*x^8 + 1386*b*c*x^6 + 495*b^2*x^4 + 990*a*c*x^4
+ 770*a*b*x^2 + 315*a^2)/x^11
```

3.840 $\int \frac{(a+bx^2+cx^4)^2}{x^{13}}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

```
[Out] -a^2/(12*x^12) - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(
3*x^6) - c^2/(4*x^4)
```

Rubi [A] time = 0.0892001, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{a^2}{12x^{12}}-\frac{2ac+b^2}{8x^8}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -a^2/(12*x^12) - (a*b)/(5*x^10) - (b^2 + 2*a*c)/(8*x^8) - (b*c)/(
3*x^6) - c^2/(4*x^4)
```

Rubi in Sympy [A] time = 15.3126, size = 48, normalized size = 0.89

$$-\frac{a^2}{12x^{12}}-\frac{ab}{5x^{10}}-\frac{bc}{3x^6}-\frac{c^2}{4x^4}-\frac{\frac{ac}{4}+\frac{b^2}{8}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

```
[Out] -a**2/(12*x**12) - a*b/(5*x**10) - b*c/(3*x**6) - c**2/(4*x**4) -
 (a*c/4 + b**2/8)/x**8
```

Mathematica [A] time = 0.0275253, size = 50, normalized size = 0.93

$$-\frac{10a^2+24abx^2+30acx^4+15b^2x^4+40bcx^6+30c^2x^8}{120x^{12}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^13,x]
```

```
[Out] -(10*a^2 + 24*a*b*x^2 + 15*b^2*x^4 + 30*a*c*x^4 + 40*b*c*x^6 + 30
*c^2*x^8)/(120*x^12)
```

Maple [A] time = 0.009, size = 45, normalized size = 0.8

$$-\frac{a^2}{12\,x^{12}}-\frac{2\,ac+b^2}{8\,x^8}-\frac{bc}{3\,x^6}-\frac{ab}{5\,x^{10}}-\frac{c^2}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^13,x)
```

```
[Out] -1/12*a^2/x^12-1/8*(2*a*c+b^2)/x^8-1/3*b*c/x^6-1/5*a*b/x^10-1/4*c
^2/x^4
```

Maxima [A] time = 0.692199, size = 62, normalized size = 1.15

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,\left(b^2+2\,ac\right)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^13,x, algorithm="maxima")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x
^2 + 10*a^2)/x^12
```

Fricas [A] time = 0.248639, size = 62, normalized size = 1.15

$$-\frac{30\,c^2x^8+40\,bcx^6+15\,\left(b^2+2\,ac\right)x^4+24\,abx^2+10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^13,x, algorithm="fricas")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*(b^2 + 2*a*c)*x^4 + 24*a*b*x
^2 + 10*a^2)/x^12
```

Sympy [A] time = 8.50504, size = 49, normalized size = 0.91

$$-\frac{10a^2 + 24abx^2 + 40bcx^6 + 30c^2x^8 + x^4\left(30ac + 15b^2\right)}{120x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**13,x)
```

```
[Out] -(10*a**2 + 24*a*b*x**2 + 40*b*c*x**6 + 30*c**2*x**8 + x**4*(30*a
*c + 15*b**2))/(120*x**12)
```

GIAC/XCAS [A] time = 0.262737, size = 65, normalized size = 1.2

$$-\frac{30\,c^2x^8 + 40\,bcx^6 + 15\,b^2x^4 + 30\,acx^4 + 24\,abx^2 + 10\,a^2}{120\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^13,x, algorithm="giac")
```

```
[Out] -1/120*(30*c^2*x^8 + 40*b*c*x^6 + 15*b^2*x^4 + 30*a*c*x^4 + 24*a*
b*x^2 + 10*a^2)/x^12
```

3.841 $\int x^2 \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=89

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2
 + 6*a*c)*x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13
+ (c^3*x^15)/15
```

Rubi [A] time = 0.146462, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2
 + 6*a*c)*x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13
+ (c^3*x^15)/15
```

Rubi in Sympy [A] time = 16.4892, size = 85, normalized size = 0.96

$$\frac{a^3x^3}{3}+\frac{3a^2bx^5}{5}+\frac{3ax^7\left(ac+b^2\right)}{7}+\frac{3bc^2x^{13}}{13}+\frac{bx^9\left(6ac+b^2\right)}{9}+\frac{c^3x^{15}}{15}+\frac{3cx^{11}\left(ac+b^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*x**7*(a*c + b**2)/7 + 3*b*c**
2*x**13/13 + b*x**9*(6*a*c + b**2)/9 + c**3*x**15/15 + 3*c*x**11*
(a*c + b**2)/11
```

Mathematica [A] time = 0.0229847, size = 89, normalized size = 1.

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{11}cx^{11}\left(ac+b^2\right)+\frac{1}{9}bx^9\left(6ac+b^2\right)+\frac{3}{7}ax^7\left(ac+b^2\right)+\frac{3}{13}bc^2x^{13}+\frac{c^3x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*(b^2 + a*c)*x^7)/7 + (b*(b^2
 + 6*a*c)*x^9)/9 + (3*c*(b^2 + a*c)*x^11)/11 + (3*b*c^2*x^13)/13
+ (c^3*x^15)/15
```

Maple [A] time = 0.001, size = 111, normalized size = 1.3

$$\frac{c^3x^{15}}{15}+\frac{3\,bc^2x^{13}}{13}+\frac{\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)x^{11}}{11}+\frac{\left(4\,abc+b\left(2\,ac+b^2\right)\right)x^9}{9}+\frac{\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)x^7}{7}+\frac{3\,a^2bx^5}{5}+\frac{a^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/15*c^3*x^15+3/13*b*c^2*x^13+1/11*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*
x^11+1/9*(4*a*b*c+b*(2*a*c+b^2))*x^9+1/7*(a*(2*a*c+b^2)+2*a*b^2+a
^2*c)*x^7+3/5*a^2*b*x^5+1/3*a^3*x^3
```

Maxima [A] time = 0.699202, size = 109, normalized size = 1.22

$$\frac{1}{15}\,c^3x^{15}+\frac{3}{13}\,bc^2x^{13}+\frac{3}{11}\left(b^2c+ac^2\right)x^{11}+\frac{1}{9}\left(b^3+6\,abc\right)x^9+\frac{3}{5}\,a^2bx^5+\frac{3}{7}\left(ab^2+a^2c\right)x^7+\frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^2,x, algorithm="maxima")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*(b^2*c + a*c^2)*x^11 + 1/9
*(b^3 + 6*a*b*c)*x^9 + 3/5*a^2*b*x^5 + 3/7*(a*b^2 + a^2*c)*x^7 +
1/3*a^3*x^3
```

Fricas [A] time = 0.231861, size = 1, normalized size = 0.01

$$\frac{1}{15}x^{15}c^3+\frac{3}{13}x^{13}c^2b+\frac{3}{11}x^{11}cb^2+\frac{3}{11}x^{11}c^2a+\frac{1}{9}x^9b^3+\frac{2}{3}x^9cba+\frac{3}{7}x^7b^2a+\frac{3}{7}x^7ca^2+\frac{3}{5}x^5ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*c^3 + 3/13*x^13*c^2*b + 3/11*x^11*c*b^2 + 3/11*x^11*c^2
*a + 1/9*x^9*b^3 + 2/3*x^9*c*b*a + 3/7*x^7*b^2*a + 3/7*x^7*c*a^2
+ 3/5*x^5*b*a^2 + 1/3*x^3*a^3
```

Sympy [A] time = 0.133557, size = 97, normalized size = 1.09

$$\frac{a^3x^3}{3} + \frac{3a^2bx^5}{5} + \frac{3bc^2x^{13}}{13} + \frac{c^3x^{15}}{15} + x^{11}\left(\frac{3ac^2}{11} + \frac{3b^2c}{11}\right) + x^9\left(\frac{2abc}{3} + \frac{b^3}{9}\right) + x^7\left(\frac{3a^2c}{7} + \frac{3ab^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*b*c**2*x**13/13 + c**3*x**15/15
 + x**11*(3*a*c**2/11 + 3*b**2*c/11) + x**9*(2*a*b*c/3 + b**3/9)
+ x**7*(3*a**2*c/7 + 3*a*b**2/7)
```

GIAC/XCAS [A] time = 0.261876, size = 117, normalized size = 1.31

$$\frac{1}{15}c^3x^{15} + \frac{3}{13}bc^2x^{13} + \frac{3}{11}b^2cx^{11} + \frac{3}{11}ac^2x^{11} + \frac{1}{9}b^3x^9 + \frac{2}{3}abcx^9 + \frac{3}{7}ab^2x^7 + \frac{3}{7}a^2cx^7 + \frac{3}{5}a^2bx^5 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^2,x, algorithm="giac")
```

```
[Out] 1/15*c^3*x^15 + 3/13*b*c^2*x^13 + 3/11*b^2*c*x^11 + 3/11*a*c^2*x^
11 + 1/9*b^3*x^9 + 2/3*a*b*c*x^9 + 3/7*a*b^2*x^7 + 3/7*a^2*c*x^7
+ 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

3.842 $\int x\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=89

$$\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 +
 6*a*c)*x^8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^
3*x^14)/14
```

Rubi [A] time = 0.192922, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{a^3x^2}{2}+\frac{3}{4}a^2bx^4+\frac{3}{10}cx^{10}\left(ac+b^2\right)+\frac{1}{8}bx^8\left(6ac+b^2\right)+\frac{1}{2}ax^6\left(ac+b^2\right)+\frac{1}{4}bc^2x^{12}+\frac{c^3x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*x^2)/2 + (3*a^2*b*x^4)/4 + (a*(b^2 + a*c)*x^6)/2 + (b*(b^2 +
 6*a*c)*x^8)/8 + (3*c*(b^2 + a*c)*x^10)/10 + (b*c^2*x^12)/4 + (c^
3*x^14)/14
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{3a^2b\int^{x^2}x\,dx}{2}+\frac{ax^6\left(ac+b^2\right)}{2}+\frac{bc^2x^{12}}{4}+\frac{bx^8\left(6ac+b^2\right)}{8}+\frac{c^3x^{14}}{14}+\frac{3cx^{10}\left(ac+b^2\right)}{10}+\frac{\int^{x^2}a^3\,dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a**2*b*Integral(x, (x, x**2))/2 + a*x**6*(a*c + b**2)/2 + b*c**
2*x**12/4 + b*x**8*(6*a*c + b**2)/8 + c**3*x**14/14 + 3*c*x**10*(
a*c + b**2)/10 + Integral(a**3, (x, x**2))/2
```

Mathematica [A] time = 0.0291661, size = 79, normalized size = 0.89

$$\frac{1}{280}x^2\left(140a^3+210a^2bx^2+84cx^8\left(ac+b^2\right)+35bx^6\left(6ac+b^2\right)+140ax^4\left(ac+b^2\right)+70bc^2x^{10}+20c^3x^{12}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(140*a^3 + 210*a^2*b*x^2 + 140*a*(b^2 + a*c)*x^4 + 35*b*(b^2
 + 6*a*c)*x^6 + 84*c*(b^2 + a*c)*x^8 + 70*b*c^2*x^10 + 20*c^3*x^1
2))/280
```

Maple [A] time = 0.001, size = 111, normalized size = 1.3

$$\frac{c^3x^{14}}{14}+\frac{bc^2x^{12}}{4}+\frac{\left(ac^2+2\,b^2c+c\left(2\,ac+b^2\right)\right)x^{10}}{10}+\frac{\left(4\,abc+b\left(2\,ac+b^2\right)\right)x^8}{8}+\frac{\left(a\left(2\,ac+b^2\right)+2\,ab^2+a^2c\right)x^6}{6}+\frac{3\,a^2bx^4}{4}+\frac{a^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/14*c^3*x^14+1/4*b*c^2*x^12+1/10*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*x
^10+1/8*(4*a*b*c+b*(2*a*c+b^2))*x^8+1/6*(a*(2*a*c+b^2)+2*a*b^2+a^
2*c)*x^6+3/4*a^2*b*x^4+1/2*a^3*x^2
```

Maxima [A] time = 0.692803, size = 109, normalized size = 1.22

$$\frac{1}{14}\,c^3x^{14}+\frac{1}{4}\,bc^2x^{12}+\frac{3}{10}\left(b^2c+ac^2\right)x^{10}+\frac{1}{8}\left(b^3+6\,abc\right)x^8+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\left(ab^2+a^2c\right)x^6+\frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x,x, algorithm="maxima")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*(b^2*c + a*c^2)*x^10 + 1/8*
(b^3 + 6*a*b*c)*x^8 + 3/4*a^2*b*x^4 + 1/2*(a*b^2 + a^2*c)*x^6 + 1
/2*a^3*x^2
```

Fricas [A] time = 0.232941, size = 1, normalized size = 0.01

$$\frac{1}{14}x^{14}c^3+\frac{1}{4}x^{12}c^2b+\frac{3}{10}x^{10}cb^2+\frac{3}{10}x^{10}c^2a+\frac{1}{8}x^8b^3+\frac{3}{4}x^8cba+\frac{1}{2}x^6b^2a+\frac{1}{2}x^6ca^2+\frac{3}{4}x^4ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*c^3 + 1/4*x^12*c^2*b + 3/10*x^10*c*b^2 + 3/10*x^10*c^2*
a + 1/8*x^8*b^3 + 3/4*x^8*c*b*a + 1/2*x^6*b^2*a + 1/2*x^6*c*a^2 +
 3/4*x^4*b*a^2 + 1/2*x^2*a^3
```

Sympy [A] time = 0.133142, size = 92, normalized size = 1.03

$$\frac{a^3x^2}{2}+\frac{3a^2bx^4}{4}+\frac{bc^2x^{12}}{4}+\frac{c^3x^{14}}{14}+x^{10}\left(\frac{3ac^2}{10}+\frac{3b^2c}{10}\right)+x^8\left(\frac{3abc}{4}+\frac{b^3}{8}\right)+x^6\left(\frac{a^2c}{2}+\frac{ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x**2/2 + 3*a**2*b*x**4/4 + b*c**2*x**12/4 + c**3*x**14/14 +
x**10*(3*a*c**2/10 + 3*b**2*c/10) + x**8*(3*a*b*c/4 + b**3/8) + x
**6*(a**2*c/2 + a*b**2/2)
```

GIAC/XCAS [A] time = 0.262272, size = 117, normalized size = 1.31

$$\frac{1}{14}c^3x^{14}+\frac{1}{4}bc^2x^{12}+\frac{3}{10}b^2cx^{10}+\frac{3}{10}ac^2x^{10}+\frac{1}{8}b^3x^8+\frac{3}{4}abcx^8+\frac{1}{2}ab^2x^6+\frac{1}{2}a^2cx^6+\frac{3}{4}a^2bx^4+\frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x,x, algorithm="giac")
```

```
[Out] 1/14*c^3*x^14 + 1/4*b*c^2*x^12 + 3/10*b^2*c*x^10 + 3/10*a*c^2*x^1
0 + 1/8*b^3*x^8 + 3/4*a*b*c*x^8 + 1/2*a*b^2*x^6 + 1/2*a^2*c*x^6 +
 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

3.843 $\int (a+bx^2+cx^4)^3\,dx$

Optimal. Leaf size=81

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^
7)/7 + (c*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rubi [A] time = 0.110665, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^
7)/7 + (c*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2bx^3+\frac{3ax^5\left(ac+b^2\right)}{5}+\frac{3bc^2x^{11}}{11}+\frac{bx^7\left(6ac+b^2\right)}{7}+\frac{c^3x^{13}}{13}+\frac{cx^9\left(ac+b^2\right)}{3}+\int a^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**2*b*x**3 + 3*a*x**5*(a*c + b**2)/5 + 3*b*c**2*x**11/11 + b*x**
7*(6*a*c + b**2)/7 + c**3*x**13/13 + c*x**9*(a*c + b**2)/3 + Inte
gral(a**3, x)
```

Mathematica [A] time = 0.0189248, size = 81, normalized size = 1.

$$a^3x+a^2bx^3+\frac{1}{3}cx^9\left(ac+b^2\right)+\frac{1}{7}bx^7\left(6ac+b^2\right)+\frac{3}{5}ax^5\left(ac+b^2\right)+\frac{3}{11}bc^2x^{11}+\frac{c^3x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*x + a^2*b*x^3 + (3*a*(b^2 + a*c)*x^5)/5 + (b*(b^2 + 6*a*c)*x^
7)/7 + (c*(b^2 + a*c)*x^9)/3 + (3*b*c^2*x^11)/11 + (c^3*x^13)/13
```

Maple [A] time = 0.001, size = 107, normalized size = 1.3

$$\frac{c^3x^{13}}{13} + \frac{3\,bc^2x^{11}}{11} + \frac{\left(ac^2 + 2\,b^2c + c\left(2\,ac + b^2\right)\right)x^9}{9} + \frac{\left(4\,abc + b\left(2\,ac + b^2\right)\right)x^7}{7} + \frac{\left(a\left(2\,ac + b^2\right) + 2\,ab^2 + a^2c\right)x^5}{5} + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/13*c^3*x^13+3/11*b*c^2*x^11+1/9*(a*c^2+2*b^2*c+c*(2*a*c+b^2))*x
^9+1/7*(4*a*b*c+b*(2*a*c+b^2))*x^7+1/5*(a*(2*a*c+b^2)+2*a*b^2+a^2
*c)*x^5+a^2*b*x^3+a^3*x
```

Maxima [A] time = 0.68973, size = 115, normalized size = 1.42

$$\frac{1}{13}\,c^3x^{13} + \frac{3}{11}\,bc^2x^{11} + \frac{1}{3}\,b^2cx^9 + \frac{1}{7}\,b^3x^7 + a^3x + \frac{1}{5}\left(3\,cx^5 + 5\,bx^3\right)a^2 + \frac{1}{105}\left(35\,c^2x^9 + 90\,bcx^7 + 63\,b^2x^5\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/7*b^3*x^7 + a
^3*x + 1/5*(3*c*x^5 + 5*b*x^3)*a^2 + 1/105*(35*c^2*x^9 + 90*b*c*x
^7 + 63*b^2*x^5)*a
```

Fricas [A] time = 0.234606, size = 1, normalized size = 0.01

$$\frac{1}{13}x^{13}c^3 + \frac{3}{11}x^{11}c^2b + \frac{1}{3}x^9cb^2 + \frac{1}{3}x^9c^2a + \frac{1}{7}x^7b^3 + \frac{6}{7}x^7cba + \frac{3}{5}x^5b^2a + \frac{3}{5}x^5ca^2 + x^3ba^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^3 + 3/11*x^11*c^2*b + 1/3*x^9*c*b^2 + 1/3*x^9*c^2*a +
 1/7*x^7*b^3 + 6/7*x^7*c*b*a + 3/5*x^5*b^2*a + 3/5*x^5*c*a^2 + x^
3*b*a^2 + x*a^3
```

Sympy [A] time = 0.120644, size = 87, normalized size = 1.07

$$a^3x + a^2bx^3 + \frac{3bc^2x^{11}}{11} + \frac{c^3x^{13}}{13} + x^9\left(\frac{ac^2}{3} + \frac{b^2c}{3}\right) + x^7\left(\frac{6abc}{7} + \frac{b^3}{7}\right) + x^5\left(\frac{3a^2c}{5} + \frac{3ab^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*b*x**3 + 3*b*c**2*x**11/11 + c**3*x**13/13 + x**9*(
a*c**2/3 + b**2*c/3) + x**7*(6*a*b*c/7 + b**3/7) + x**5*(3*a**2*c
/5 + 3*a*b**2/5)
```

GIAC/XCAS [A] time = 0.260852, size = 112, normalized size = 1.38

$$\frac{1}{13}c^3x^{13} + \frac{3}{11}bc^2x^{11} + \frac{1}{3}b^2cx^9 + \frac{1}{3}ac^2x^9 + \frac{1}{7}b^3x^7 + \frac{6}{7}abcx^7 + \frac{3}{5}ab^2x^5 + \frac{3}{5}a^2cx^5 + a^2bx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/13*c^3*x^13 + 3/11*b*c^2*x^11 + 1/3*b^2*c*x^9 + 1/3*a*c^2*x^9 +
 1/7*b^3*x^7 + 6/7*a*b*c*x^7 + 3/5*a*b^2*x^5 + 3/5*a^2*c*x^5 + a^
2*b*x^3 + a^3*x
```

3.844 $\int \frac{(a+bx^2+cx^4)^3}{x} dx$

Optimal. Leaf size=85

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{8}cx^8\left(ac+b^2\right) + \frac{1}{6}bx^6\left(6ac+b^2\right) + \frac{3}{4}ax^4\left(ac+b^2\right) + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)
/6 + (3*c*(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12
+ a^3*Log[x]
```

Rubi [A] time = 0.181391, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^3 \log(x) + \frac{3}{2}a^2bx^2 + \frac{3}{8}cx^8\left(ac+b^2\right) + \frac{1}{6}bx^6\left(6ac+b^2\right) + \frac{3}{4}ax^4\left(ac+b^2\right) + \frac{3}{10}bc^2x^{10} + \frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)
/6 + (3*c*(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12
+ a^3*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^3 \log\left(x^2\right)}{2} + \frac{3a^2bx^2}{2} + \frac{3a\left(ac+b^2\right)\int^{x^2} x\,dx}{2} + \frac{3bc^2x^{10}}{10} + \frac{bx^6\left(6ac+b^2\right)}{6} + \frac{c^3x^{12}}{12} + \frac{3cx^8\left(ac+b^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x**2)/2 + 3*a**2*b*x**2/2 + 3*a*(a*c + b**2)*Integral(x,
 (x, x**2))/2 + 3*b*c**2*x**10/10 + b*x**6*(6*a*c + b**2)/6 + c**
3*x**12/12 + 3*c*x**8*(a*c + b**2)/8
```

Mathematica [A] time = 0.0363254, size = 85, normalized size = 1.

$$a^3\log(x)+\frac{3}{2}a^2bx^2+\frac{3}{8}cx^8\left(ac+b^2\right)+\frac{1}{6}bx^6\left(6ac+b^2\right)+\frac{3}{4}ax^4\left(ac+b^2\right)+\frac{3}{10}bc^2x^{10}+\frac{c^3x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x,x]
```

```
[Out] (3*a^2*b*x^2)/2 + (3*a*(b^2 + a*c)*x^4)/4 + (b*(b^2 + 6*a*c)*x^6)
/6 + (3*c*(b^2 + a*c)*x^8)/8 + (3*b*c^2*x^10)/10 + (c^3*x^12)/12
+ a^3*Log[x]
```

Maple [A] time = 0.004, size = 85, normalized size = 1.

$$\frac{c^3x^{12}}{12}+\frac{3\,bc^2x^{10}}{10}+\frac{3\,x^8ac^2}{8}+\frac{3\,b^2cx^8}{8}+x^6abc+\frac{b^3x^6}{6}+\frac{3\,x^4a^2c}{4}+\frac{3\,ax^4b^2}{4}+\frac{3\,a^2bx^2}{2}+a^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x,x)
```

```
[Out] 1/12*c^3*x^12+3/10*b*c^2*x^10+3/8*x^8*a*c^2+3/8*b^2*c*x^8+x^6*a*b
*c+1/6*b^3*x^6+3/4*x^4*a^2*c+3/4*a*x^4*b^2+3/2*a^2*b*x^2+a^3*ln(x
)
```

Maxima [A] time = 0.686222, size = 111, normalized size = 1.31

$$\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}\,a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+\frac{1}{2}\,a^3\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(
b^3 + 6*a*b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + 1/
2*a^3*log(x^2)
```

Fricas [A] time = 0.254026, size = 107, normalized size = 1.26

$$\frac{1}{12}\,c^3x^{12}+\frac{3}{10}\,bc^2x^{10}+\frac{3}{8}\left(b^2c+ac^2\right)x^8+\frac{1}{6}\left(b^3+6\,abc\right)x^6+\frac{3}{2}\,a^2bx^2+\frac{3}{4}\left(ab^2+a^2c\right)x^4+a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*(b^2*c + a*c^2)*x^8 + 1/6*(
b^3 + 6*a*b*c)*x^6 + 3/2*a^2*b*x^2 + 3/4*(a*b^2 + a^2*c)*x^4 + a^
3*log(x)
```

Sympy [A] time = 1.23174, size = 92, normalized size = 1.08

$$a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3bc^2x^{10}}{10} + \frac{c^3x^{12}}{12} + x^8\left(\frac{3ac^2}{8} + \frac{3b^2c}{8}\right) + x^6\left(abc + \frac{b^3}{6}\right) + x^4\left(\frac{3a^2c}{4} + \frac{3ab^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] a**3*log(x) + 3*a**2*b*x**2/2 + 3*b*c**2*x**10/10 + c**3*x**12/12
 + x**8*(3*a*c**2/8 + 3*b**2*c/8) + x**6*(a*b*c + b**3/6) + x**4*
(3*a**2*c/4 + 3*a*b**2/4)
```

GIAC/XCAS [A] time = 0.262871, size = 117, normalized size = 1.38

$$\frac{1}{12}c^3x^{12} + \frac{3}{10}bc^2x^{10} + \frac{3}{8}b^2cx^8 + \frac{3}{8}ac^2x^8 + \frac{1}{6}b^3x^6 + abcx^6 + \frac{3}{4}ab^2x^4 + \frac{3}{4}a^2cx^4 + \frac{3}{2}a^2bx^2 + \frac{1}{2}a^3\ln\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x,x, algorithm="giac")
```

```
[Out] 1/12*c^3*x^12 + 3/10*b*c^2*x^10 + 3/8*b^2*c*x^8 + 3/8*a*c^2*x^8 +
 1/6*b^3*x^6 + a*b*c*x^6 + 3/4*a*b^2*x^4 + 3/4*a^2*c*x^4 + 3/2*a^
2*b*x^2 + 1/2*a^3*ln(x^2)
```

3.845 $\int \frac{(a+bx^2+cx^4)^3}{x^2} \, dx$

Optimal. Leaf size=80

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/
5 + (3*c*(b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi [A] time = 0.118057, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/
5 + (3*c*(b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Rubi in Sympy [A] time = 15.6835, size = 73, normalized size = 0.91

$$-\frac{a^3}{x}+3a^2bx+ax^3\left(ac+b^2\right)+\frac{bc^2x^9}{3}+\frac{bx^5\left(6ac+b^2\right)}{5}+\frac{c^3x^{11}}{11}+\frac{3cx^7\left(ac+b^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + a*x**3*(a*c + b**2) + b*c**2*x**9/3 + b*x*
*5*(6*a*c + b**2)/5 + c**3*x**11/11 + 3*c*x**7*(a*c + b**2)/7
```

Mathematica [A] time = 0.0420576, size = 80, normalized size = 1.

$$-\frac{a^3}{x}+3a^2bx+\frac{3}{7}cx^7\left(ac+b^2\right)+\frac{1}{5}bx^5\left(6ac+b^2\right)+ax^3\left(ac+b^2\right)+\frac{1}{3}bc^2x^9+\frac{c^3x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a^2*b*x + a*(b^2 + a*c)*x^3 + (b*(b^2 + 6*a*c)*x^5)/
5 + (3*c*(b^2 + a*c)*x^7)/7 + (b*c^2*x^9)/3 + (c^3*x^11)/11
```

Maple [A] time = 0.005, size = 84, normalized size = 1.1

$$\frac{c^3x^{11}}{11}+\frac{bc^2x^9}{3}+\frac{3\,x^7ac^2}{7}+\frac{3\,b^2cx^7}{7}+\frac{6\,x^5abc}{5}+\frac{b^3x^5}{5}+x^3a^2c+x^3ab^2+3\,a^2bx-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^2,x)
```

```
[Out] 1/11*c^3*x^11+1/3*b*c^2*x^9+3/7*x^7*a*c^2+3/7*b^2*c*x^7+6/5*x^5*a
*b*c+1/5*b^3*x^5+x^3*a^2*c+x^3*a*b^2+3*a^2*b*x-a^3/x
```

Maxima [A] time = 0.683549, size = 105, normalized size = 1.31

$$\frac{1}{11}\,c^3x^{11}+\frac{1}{3}\,bc^2x^9+\frac{3}{7}\left(b^2c+ac^2\right)x^7+\frac{1}{5}\left(b^3+6\,abc\right)x^5+3\,a^2bx+\left(ab^2+a^2c\right)x^3-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*(b^2*c + a*c^2)*x^7 + 1/5*(b^
3 + 6*a*b*c)*x^5 + 3*a^2*b*x + (a*b^2 + a^2*c)*x^3 - a^3/x
```

Fricas [A] time = 0.244943, size = 112, normalized size = 1.4

$$\frac{105\,c^3x^{12}+385\,bc^2x^{10}+495\left(b^2c+ac^2\right)x^8+231\left(b^3+6\,abc\right)x^6+3465\,a^2bx^2+1155\left(ab^2+a^2c\right)x^4-1155\,a^3}{1155\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/1155*(105*c^3*x^12 + 385*b*c^2*x^10 + 495*(b^2*c + a*c^2)*x^8 +
 231*(b^3 + 6*a*b*c)*x^6 + 3465*a^2*b*x^2 + 1155*(a*b^2 + a^2*c)*
x^4 - 1155*a^3)/x
```

Sympy [A] time = 1.22686, size = 82, normalized size = 1.02

$$-\frac{a^3}{x} + 3a^2bx + \frac{bc^2x^9}{3} + \frac{c^3x^{11}}{11} + x^7\left(\frac{3ac^2}{7} + \frac{3b^2c}{7}\right) + x^5\left(\frac{6abc}{5} + \frac{b^3}{5}\right) + x^3\left(a^2c + ab^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -a**3/x + 3*a**2*b*x + b*c**2*x**9/3 + c**3*x**11/11 + x**7*(3*a*
c**2/7 + 3*b**2*c/7) + x**5*(6*a*b*c/5 + b**3/5) + x**3*(a**2*c +
 a*b**2)
```

GIAC/XCAS [A] time = 0.264618, size = 112, normalized size = 1.4

$$\frac{1}{11}\,c^3x^{11} + \frac{1}{3}\,bc^2x^9 + \frac{3}{7}\,b^2cx^7 + \frac{3}{7}\,ac^2x^7 + \frac{1}{5}\,b^3x^5 + \frac{6}{5}\,abcx^5 + ab^2x^3 + a^2cx^3 + 3\,a^2bx - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/11*c^3*x^11 + 1/3*b*c^2*x^9 + 3/7*b^2*c*x^7 + 3/7*a*c^2*x^7 + 1
/5*b^3*x^5 + 6/5*a*b*c*x^5 + a*b^2*x^3 + a^2*c*x^3 + 3*a^2*b*x -
a^3/x
```

3.846 $\int \frac{(a+bx^2+cx^4)^3}{x^3}\,dx$

Optimal. Leaf size=86

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

```
[Out] -a^3/(2*x^2) + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4
+ (c*(b^2 + a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2
*b*Log[x]
```

Rubi [A] time = 0.201523, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{1}{2}cx^6\left(ac+b^2\right)+\frac{1}{4}bx^4\left(6ac+b^2\right)+\frac{3}{2}ax^2\left(ac+b^2\right)+\frac{3}{8}bc^2x^8+\frac{c^3x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*(b^2 + a*c)*x^2)/2 + (b*(b^2 + 6*a*c)*x^4)/4
+ (c*(b^2 + a*c)*x^6)/2 + (3*b*c^2*x^8)/8 + (c^3*x^10)/10 + 3*a^2
*b*Log[x]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a^3}{2x^2}+\frac{3a^2b\log\left(x^2\right)}{2}+\frac{3ax^2\left(ac+b^2\right)}{2}+\frac{3bc^2x^8}{8}+\frac{b\left(6ac+b^2\right)\int^{x^2}x\,dx}{2}+\frac{c^3x^{10}}{10}+\frac{cx^6\left(ac+b^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x**2)/2 + 3*a*x**2*(a*c + b**2)/2 +
 3*b*c**2*x**8/8 + b*(6*a*c + b**2)*Integral(x, (x, x**2))/2 + c*
*3*x**10/10 + c*x**6*(a*c + b**2)/2
```

Mathematica [A] time = 0.0569752, size = 78, normalized size = 0.91

$$\frac{1}{40}\left(-\frac{20a^3}{x^2}+120a^2b\log(x)+20cx^6\left(ac+b^2\right)+10bx^4\left(6ac+b^2\right)+60ax^2\left(ac+b^2\right)+15bc^2x^8+4c^3x^{10}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x^3,x]
```

```
[Out] ((-20*a^3)/x^2 + 60*a*(b^2 + a*c)*x^2 + 10*b*(b^2 + 6*a*c)*x^4 +
20*c*(b^2 + a*c)*x^6 + 15*b*c^2*x^8 + 4*c^3*x^10 + 120*a^2*b*Log[
x])/40
```

Maple [A] time = 0.009, size = 87, normalized size = 1.

$$\frac{c^3x^{10}}{10}+\frac{3\,bc^2x^8}{8}+\frac{x^6ac^2}{2}+\frac{b^2cx^6}{2}+\frac{3\,x^4abc}{2}+\frac{b^3x^4}{4}+\frac{3\,x^2a^2c}{2}+\frac{3\,ab^2x^2}{2}+3\,a^2b\ln(x)-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^3,x)
```

```
[Out] 1/10*c^3*x^10+3/8*b*c^2*x^8+1/2*x^6*a*c^2+1/2*b^2*c*x^6+3/2*x^4*a
*b*c+1/4*b^3*x^4+3/2*x^2*a^2*c+3/2*a*b^2*x^2+3*a^2*b*ln(x)-1/2*a^
3/x^2
```

Maxima [A] time = 0.69536, size = 111, normalized size = 1.29

$$\frac{1}{10}\,c^3x^{10}+\frac{3}{8}\,bc^2x^8+\frac{1}{2}\left(b^2c+ac^2\right)x^6+\frac{1}{4}\left(b^3+6\,abc\right)x^4+\frac{3}{2}\,a^2b\log\left(x^2\right)+\frac{3}{2}\left(ab^2+a^2c\right)x^2-\frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*(b^2*c + a*c^2)*x^6 + 1/4*(b^
3 + 6*a*b*c)*x^4 + 3/2*a^2*b*log(x^2) + 3/2*(a*b^2 + a^2*c)*x^2 -
 1/2*a^3/x^2
```

Fricas [A] time = 0.253357, size = 115, normalized size = 1.34

$$\frac{4\,c^3x^{12} + 15\,bc^2x^{10} + 20\,\left(b^2c + ac^2\right)x^8 + 10\,\left(b^3 + 6\,abc\right)x^6 + 120\,a^2bx^2\log\left(x\right) + 60\,\left(ab^2 + a^2c\right)x^4 - 20\,a^3}{40\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/40*(4*c^3*x^12 + 15*b*c^2*x^10 + 20*(b^2*c + a*c^2)*x^8 + 10*(b
^3 + 6*a*b*c)*x^6 + 120*a^2*b*x^2*log(x) + 60*(a*b^2 + a^2*c)*x^4
 - 20*a^3)/x^2
```

Sympy [A] time = 1.38243, size = 92, normalized size = 1.07

$$-\frac{a^3}{2x^2} + 3a^2b\log\left(x\right) + \frac{3bc^2x^8}{8} + \frac{c^3x^{10}}{10} + x^6\left(\frac{ac^2}{2} + \frac{b^2c}{2}\right) + x^4\left(\frac{3abc}{2} + \frac{b^3}{4}\right) + x^2\left(\frac{3a^2c}{2} + \frac{3ab^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*b*c**2*x**8/8 + c**3*x**10/1
0 + x**6*(a*c**2/2 + b**2*c/2) + x**4*(3*a*b*c/2 + b**3/4) + x**2
*(3*a**2*c/2 + 3*a*b**2/2)
```

GIAC/XCAS [A] time = 0.264958, size = 132, normalized size = 1.53

$$\frac{1}{10}\,c^3x^{10} + \frac{3}{8}\,bc^2x^8 + \frac{1}{2}\,b^2cx^6 + \frac{1}{2}\,ac^2x^6 + \frac{1}{4}\,b^3x^4 + \frac{3}{2}\,abcx^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2cx^2 + \frac{3}{2}\,a^2b\ln\left(x^2\right) - \frac{3\,a^2bx^2 + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/10*c^3*x^10 + 3/8*b*c^2*x^8 + 1/2*b^2*c*x^6 + 1/2*a*c^2*x^6 + 1
/4*b^3*x^4 + 3/2*a*b*c*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*c*x^2 + 3/2*
a^2*b*ln(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/x^2
```

3.847 $\int \frac{(a+bx^2+cx^4)^3}{x^4}\,dx$

Optimal. Leaf size=83

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)
*x^3)/3 + (3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi [A] time = 0.116133, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)
*x^3)/3 + (3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Rubi in Sympy [A] time = 18.7193, size = 78, normalized size = 0.94

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ax\left(ac+b^2\right)+\frac{3bc^2x^7}{7}+\frac{bx^3\left(6ac+b^2\right)}{3}+\frac{c^3x^9}{9}+\frac{3cx^5\left(ac+b^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

```
[Out] -a**3/(3*x**3) - 3*a**2*b/x + 3*a*x*(a*c + b**2) + 3*b*c**2*x**7/
7 + b*x**3*(6*a*c + b**2)/3 + c**3*x**9/9 + 3*c*x**5*(a*c + b**2)
/5
```

Mathematica [A] time = 0.0468228, size = 83, normalized size = 1.

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+\frac{3}{5}cx^5\left(ac+b^2\right)+\frac{1}{3}bx^3\left(6ac+b^2\right)+3ax\left(ac+b^2\right)+\frac{3}{7}bc^2x^7+\frac{c^3x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/x + 3*a*(b^2 + a*c)*x + (b*(b^2 + 6*a*c)
*x^3)/3 + (3*c*(b^2 + a*c)*x^5)/5 + (3*b*c^2*x^7)/7 + (c^3*x^9)/9
```

Maple [A] time = 0.009, size = 84, normalized size = 1.

$$\frac{c^3x^9}{9}+\frac{3\,bc^2x^7}{7}+\frac{3\,x^5ac^2}{5}+\frac{3\,b^2cx^5}{5}+2\,x^3abc+\frac{b^3x^3}{3}+3\,xa^2c+3\,ab^2x-\frac{a^3}{3\,x^3}-3\,\frac{a^2b}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^4,x)
```

```
[Out] 1/9*c^3*x^9+3/7*b*c^2*x^7+3/5*x^5*a*c^2+3/5*b^2*c*x^5+2*x^3*a*b*c
+1/3*b^3*x^3+3*x*a^2*c+3*a*b^2*x-1/3*a^3/x^3-3*a^2*b/x
```

Maxima [A] time = 0.694771, size = 108, normalized size = 1.3

$$\frac{1}{9}\,c^3x^9+\frac{3}{7}\,bc^2x^7+\frac{3}{5}\left(b^2c+ac^2\right)x^5+\frac{1}{3}\left(b^3+6\,abc\right)x^3+3\left(ab^2+a^2c\right)x-\frac{9\,a^2bx^2+a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*(b^2*c + a*c^2)*x^5 + 1/3*(b^3
+ 6*a*b*c)*x^3 + 3*(a*b^2 + a^2*c)*x - 1/3*(9*a^2*b*x^2 + a^3)/x^
3
```

Fricas [A] time = 0.248899, size = 112, normalized size = 1.35

$$\frac{35\,c^3x^{12}+135\,bc^2x^{10}+189\left(b^2c+ac^2\right)x^8+105\left(b^3+6\,abc\right)x^6-945\,a^2bx^2+945\left(ab^2+a^2c\right)x^4-105\,a^3}{315\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/315*(35*c^3*x^12 + 135*b*c^2*x^10 + 189*(b^2*c + a*c^2)*x^8 + 1
05*(b^3 + 6*a*b*c)*x^6 - 945*a^2*b*x^2 + 945*(a*b^2 + a^2*c)*x^4
- 105*a^3)/x^3
```

Sympy [A] time = 1.38167, size = 88, normalized size = 1.06

$$\frac{3bc^2x^7}{7} + \frac{c^3x^9}{9} + x^5\left(\frac{3ac^2}{5} + \frac{3b^2c}{5}\right) + x^3\left(2abc + \frac{b^3}{3}\right) + x\left(3a^2c + 3ab^2\right) - \frac{a^3 + 9a^2bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**4,x)
```

```
[Out] 3*b*c**2*x**7/7 + c**3*x**9/9 + x**5*(3*a*c**2/5 + 3*b**2*c/5) +
x**3*(2*a*b*c + b**3/3) + x*(3*a**2*c + 3*a*b**2) - (a**3 + 9*a**
2*b*x**2)/(3*x**3)
```

GIAC/XCAS [A] time = 0.264661, size = 113, normalized size = 1.36

$$\frac{1}{9}c^3x^9 + \frac{3}{7}bc^2x^7 + \frac{3}{5}b^2cx^5 + \frac{3}{5}ac^2x^5 + \frac{1}{3}b^3x^3 + 2\,abcx^3 + 3\,ab^2x + 3\,a^2cx - \frac{9\,a^2bx^2 + a^3}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^4,x, algorithm="giac")
```

```
[Out] 1/9*c^3*x^9 + 3/7*b*c^2*x^7 + 3/5*b^2*c*x^5 + 3/5*a*c^2*x^5 + 1/3
*b^3*x^3 + 2*a*b*c*x^3 + 3*a*b^2*x + 3*a^2*c*x - 1/3*(9*a^2*b*x^2
 + a^3)/x^3
```

3.848 $\int \frac{x^7}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=100

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)
*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi [A] time = 0.282588, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{bx^2}{2c^2}+\frac{x^4}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(b*x^2)/(2*c^2) + x^4/(4*c) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)
*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(-3ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\sqrt{-4ac+b^2}}+\frac{\int^{x^2} x\,dx}{2c}-\frac{\int^{x^2} b\,dx}{2c^2}+\frac{\left(-ac+b^2\right)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] b*(-3*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*
*3*sqrt(-4*a*c + b**2)) + Integral(x, (x, x**2))/(2*c) - Integral
(b, (x, x**2))/(2*c**2) + (-a*c + b**2)*log(a + b*x**2 + c*x**4)/
(4*c**3)
```

Mathematica [A] time = 0.180034, size = 93, normalized size = 0.93

$$\frac{-\frac{2b(b^2-3ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)+cx^2\left(cx^2-2b\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*x^2*(-2*b + c*x^2) - (2*b*(b^2 - 3*a*c)*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (b^2 - a*c)*Log[a + b*x^
2 + c*x^4])/(4*c^3)
```

Maple [A] time = 0.007, size = 142, normalized size = 1.4

$$\frac{x^4}{4\,c}-\frac{bx^2}{2\,c^2}-\frac{\ln\left(cx^4+bx^2+a\right)a}{4\,c^2}+\frac{\ln\left(cx^4+bx^2+a\right)b^2}{4\,c^3}+\frac{3\,ab}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{b^3}{2\,c^3}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*x^4/c-1/2*b*x^2/c^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a+1/4/c^3*ln(c*
x^4+b*x^2+a)*b^2+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*
a*c-b^2)^(1/2))*a*b-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/
(4*a*c-b^2)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.270734, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^3-3\,abc\right)\log\left(-\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(c^2x^4-2\,bcx^2+\left(b^2-ac\right)\log\left(cx^4+bx^2+a\right)\right)\sqrt{b^2-4\,ac}}{4\,\sqrt{b^2-4\,ac}c^3},\right.$$
$$\left.-\frac{2\,\left(b^3-3\,abc\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(c^2x^4-2\,bcx^2+\left(b^2-ac\right)\log\left(cx^4+bx^2+a\right)\right)\sqrt{-b^2+4\,ac}}{4\,\sqrt{-b^2+4\,ac}c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3 - 3*a*b*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*
x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c
*x^4 + b*x^2 + a)) - (c^2*x^4 - 2*b*c*x^2 + (b^2 - a*c)*log(c*x^4
 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*c^3), -1/4*(
2*(b^3 - 3*a*b*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) - (c^2*x^4 - 2*b*c*x^2 + (b^2 - a*c)*log(c*x^4 + b*x^2 +
 a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)]
```

Sympy [A] time = 6.64268, size = 391, normalized size = 3.91

$$-\frac{bx^2}{2c^2}+\left(-\frac{b\sqrt{-4ac+b^2}\,\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\,\left(3ac-b^2\right)}{4c^3\left(4ac-b^2\right)}-\frac{ac-b^2}{4c^3}\right)\log\left(x^2+\frac{2a^2c-ab^2+8ac^3\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)-2b^2c^2\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4c^3(4ac-b^2)}-\frac{ac-b^2}{4c^3}\right)}{3abc-b^3}\right)$$
$$+\frac{x^4}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x**2/(2*c**2) + (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3
*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(x**2 + (2*a**2*c -
a*b**2 + 8*a*c**3*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*
(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c**2*(-b*sqrt(-
4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**
2)/(4*c**3)))/(3*a*b*c - b**3)) + (b*sqrt(-4*a*c + b**2)*(3*a*c -
 b**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3))*log(x**2
+ (2*a**2*c - a*b**2 + 8*a*c**3*(b*sqrt(-4*a*c + b**2)*(3*a*c - b
**2)/(4*c**3*(4*a*c - b**2)) - (a*c - b**2)/(4*c**3)) - 2*b**2*c*
*2*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*c**3*(4*a*c - b**2))
- (a*c - b**2)/(4*c**3)))/(3*a*b*c - b**3)) + x**4/(4*c)
```

GIAC/XCAS [A] time = 0.290605, size = 124, normalized size = 1.24

$$\frac{cx^4 - 2\,bx^2}{4\,c^2} + \frac{\left(b^2 - ac\right)\ln\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(b^3 - 3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*(c*x^4 - 2*b*x^2)/c^2 + 1/4*(b^2 - a*c)*ln(c*x^4 + b*x^2 + a)
/c^3 - 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.849 $\int \frac{x^5}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=81

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Rubi [A] time = 0.185071, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Rubi in Sympy [A] time = 26.5892, size = 73, normalized size = 0.9

$$-\frac{b\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*log(a + b*x**2 + c*x**4)/(4*c**2) + x**2/(2*c) - (-2*a*c + b**
2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*sqrt(-4*a*c
+ b**2))
```

Mathematica [A] time = 0.075212, size = 78, normalized size = 0.96

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-b\log\left(a+bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*
c]])/Sqrt[-b^2 + 4*a*c] - b*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.004, size = 111, normalized size = 1.4

$$\frac{x^2}{2\,c}-\frac{b\ln\left(cx^4+bx^2+a\right)}{4\,c^2}-\frac{a}{c}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,c^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*x^2/c-1/4*b*ln(c*x^4+b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268004, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^{2}-2\,ac\right)\log\left(\frac{b^{3}-4\,abc+2\,\left(b^{2}c-4\,ac^{2}\right)x^{2}+\left(2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-\left(2\,cx^{2}-b\log\left(cx^{4}+bx^{2}+a\right)\right)\sqrt{b^{2}-4\,ac}}{4\,\sqrt{b^{2}-4\,ac}c^{2}},\ \frac{2\,(b^{2}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2
 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
4 + b*x^2 + a)) - (2*c*x^2 - b*log(c*x^4 + b*x^2 + a))*sqrt(b^2 -
 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), 1/4*(2*(b^2 - 2*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^2 - b*log(
c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)]
```

Sympy [A] time = 5.3122, size = 316, normalized size = 3.9

$$\left(-\frac{b}{4c^{2}}-\frac{\sqrt{-4ac+b^{2}}\left(2ac-b^{2}\right)}{4c^{2}\left(4ac-b^{2}\right)}\right)\log\left(x^{2}+\frac{-ab-8ac^{2}\left(-\frac{b}{4c^{2}}-\frac{\sqrt{-4ac+b^{2}}(2ac-b^{2})}{4c^{2}(4ac-b^{2})}\right)+2b^{2}c\left(-\frac{b}{4c^{2}}-\frac{\sqrt{-4ac+b^{2}}(2ac-b^{2})}{4c^{2}(4ac-b^{2})}\right)}{2ac-b^{2}}\right)+\left(-\frac{b}{4c^{2}}+\frac{\sqrt{-4ac+b^{2}}\left(2ac-b^{2}\right)}{4c^{2}\left(4ac-b^{2}\right)}\right)\log\left(x^{2}+\frac{-ab-8ac^{2}\left(-\frac{b}{4c^{2}}+\frac{\sqrt{-4ac+b^{2}}(2ac-b^{2})}{4c^{2}(4ac-b^{2})}\right)+2b^{2}c\left(-\frac{b}{4c^{2}}+\frac{\sqrt{-4ac+b^{2}}(2ac-b^{2})}{4c^{2}(4ac-b^{2})}\right)}{2ac-b^{2}}\right)+\frac{x^{2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c
- b**2)))*log(x**2 + (-a*b - 8*a*c**2*(-b/(4*c**2) - sqrt(-4*a*c
+ b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-b/(4
```

```
*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2
))))/(2*a*c - b**2)) + (-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c
- b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (-a*b - 8*a*c**2*(-b/
(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b*
*2))) + 2*b**2*c*(-b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2
)/(4*c**2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

GIAC/XCAS [A] time = 0.291988, size = 101, normalized size = 1.25

$$\frac{x^2}{2\,c} - \frac{b\ln\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c - 1/4*b*ln(c*x^4 + b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.850 $\int \frac{x^3}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^2 + c*x^4]/(4*c)
```

Rubi [A] time = 0.130779, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*
c]) + Log[a + b*x^2 + c*x^4]/(4*c)
```

Rubi in Sympy [A] time = 17.272, size = 54, normalized size = 0.86

$$\frac{b\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}+\frac{\log\left(a+bx^2+cx^4\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c + b*
*2)) + log(a + b*x**2 + c*x**4)/(4*c)
```

Mathematica [A] time = 0.0404382, size = 62, normalized size = 0.98

$$\frac{\log\left(a+bx^2+cx^4\right)-\frac{2b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] + Log[a + b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.004, size = 60, normalized size = 1.

$$\frac{\ln\left(cx^4+bx^2+a\right)}{4c}-\frac{b}{2c}\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4+b*x^2+a)/c-1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269309, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^4+bx^2+a\right)}{4\,\sqrt{b^2-4\,ac}c},\right.$$
$$\left.-\frac{2\,b\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\sqrt{-b^2+4\,ac}\log\left(cx^4+bx^2+a\right)}{4\,\sqrt{-b^2+4\,ac}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4
 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) + sqrt(b^2 - 4*a*c)*log(c*x^4 + b*x^2 + a))/(sqrt(b^2 - 4*a*c)
*c), -1/4*(2*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*
a*c)) - sqrt(-b^2 + 4*a*c)*log(c*x^4 + b*x^2 + a))/(sqrt(-b^2 + 4
*a*c)*c)]
```

Sympy [A] time = 2.53546, size = 223, normalized size = 3.54

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{-8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)+2a+2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2
+ (-8*a*c*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))
 + 2*a + 2*b**2*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/
(4*c)))/b) + (b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c
))*log(x**2 + (-8*a*c*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))
 + 1/(4*c)) + 2*a + 2*b**2*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b
**2)) + 1/(4*c)))/b)
```

GIAC/XCAS [A] time = 0.290682, size = 80, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{2 \, c x^{2}+b}{\sqrt{-b^{2}+4 \, a c}}\right)}{2 \, \sqrt{-b^{2} + 4 \, a c} c} + \frac{\ln\left(c x^{4} + b x^{2} + a\right)}{4 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*c) + 1/4*ln(c*x^4 + b*x^2 + a)/c
```

3.851 $\int \frac{x}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.075036, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^2 + c*x^4),x]

[Out] -(ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 9.02773, size = 34, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2+a),x)

[Out] -atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.0152462, size = 39, normalized size = 1.08

$$\frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^2 + c*x^4),x]

[Out] ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.003, size = 36, normalized size = 1.

$$1\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2+a),x)

[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264245, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2-(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{2\,\sqrt{b^2-4\,ac}},\ \frac{\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
/sqrt(b^2 - 4*a*c), arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c))/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 1.32798, size = 131, normalized size = 3.64

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*c))/2 + sqrt(-1/(4*a*c
- b**2))*log(x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1
/(4*a*c - b**2)) + b)/(2*c))/2
```

GIAC/XCAS [A] time = 0.290665, size = 47, normalized size = 1.31

$$\frac{\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.852 $\int \frac{1}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=69

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rubi [A] time = 0.158266, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a + b*x^2 + c*x^4]/(4*a)
```

Rubi in Sympy [A] time = 23.4247, size = 63, normalized size = 0.91

$$\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}}+\frac{\log\left(x^2\right)}{2a}-\frac{\log\left(a+bx^2+cx^4\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2+a),x)
```

```
[Out] b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a*sqrt(-4*a*c + b*
*2)) + log(x**2)/(2*a) - log(a + b*x**2 + c*x**4)/(4*a)
```

Mathematica [A] time = 0.127611, size = 113, normalized size = 1.64

$$\frac{-\left(\sqrt{b^2-4ac}+b\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)+\left(b-\sqrt{b^2-4ac}\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] - (b + Sqrt[b^2 - 4*a*c])*Log[b - Sqr
t[b^2 - 4*a*c] + 2*c*x^2] + (b - Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.009, size = 66, normalized size = 1.

$$-\frac{\ln\left(cx^4+bx^2+a\right)}{4\,a}-\frac{b}{2\,a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*ln(c*x^4+b*x^2+a)/a-1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))+ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27021, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^4+bx^2+a\right)-4\,\log\left(x\right)\right)}{4\,\sqrt{b^2-4\,ac}a}, -\frac{2\,b\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\sqrt{-b^2+4\,ac}\left(\log\left(cx^4+bx^2+a\right)-4\,\log\left(x\right)\right)}{4\,\sqrt{-b^2+4\,ac}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4
 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - sqrt(b^2 - 4*a*c)*(log(c*x^4 + b*x^2 + a) - 4*log(x)))/(sqrt
(b^2 - 4*a*c)*a), -1/4*(2*b*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)) + sqrt(-b^2 + 4*a*c)*(log(c*x^4 + b*x^2 + a) -
 4*log(x)))/(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 8.84093, size = 253, normalized size = 3.67

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{4a\,(4ac-b^2)}-\frac{1}{4a}\right)\log\left(x^2+\frac{-8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ac+b^2}{bc}\right)$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2
+ (-8*a**2*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*
a)) + 2*a*b**2*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(
4*a)) - 2*a*c + b**2)/(b*c)) + (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c
 - b**2)) - 1/(4*a))*log(x**2 + (-8*a**2*c*(b*sqrt(-4*a*c + b**2)
/(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*b**2*(b*sqrt(-4*a*c + b**2
)/(4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*c + b**2)/(b*c)) + log(x)
```

```
/a
```

GIAC/XCAS [A] time = 0.289687, size = 92, normalized size = 1.33

$$-\frac{b \arctan\left(\frac{2 cx^2+b}{\sqrt{-b^2+4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac}a} - \frac{\ln\left(cx^4 + bx^2 + a\right)}{4 a} + \frac{\ln\left(x^2\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a) - 1/4*ln(c*x^4 + b*x^2 + a)/a + 1/2*ln(x^2)/a
```

3.853 $\int \frac{1}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.294826, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$-\frac{\left(b^2-2ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) - ((b^2 - 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - (b*Log[x])/a^2 + (b*Log[a + b*
x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 37.337, size = 87, normalized size = 0.98

$$-\frac{1}{2ax^2}-\frac{b\log\left(x^2\right)}{2a^2}+\frac{b\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x**2)/(2*a**2) + b*log(a + b*x**2 + c*x**4)
/(4*a**2) - (-2*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b*
*2))/(2*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.255415, size = 135, normalized size = 1.52

$$\frac{\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}-4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 - 4*b*Log[x] + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*L
og[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-b^2 +
 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.012, size = 119, normalized size = 1.3

$$\frac{b\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{c}{a}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,a^2}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{1}{2\,ax^2}-\frac{b\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*b*ln(c*x^4+b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b^2-1/2/a/x^2-b*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282309, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)x^2\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(bx^2\log\left(cx^4+bx^2+a\right)-4\,bx^2\log\left(x\right)-2\,a\right)\cdot}{4\,\sqrt{b^2-4\,ac}a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*x^2*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)
*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(
c*x^4 + b*x^2 + a)) - (b*x^2*log(c*x^4 + b*x^2 + a) - 4*b*x^2*log
(x) - 2*a)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x^2), 1/4*(2
*(b^2 - 2*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c)) + (b*x^2*log(c*x^4 + b*x^2 + a) - 4*b*x^2*log(x) - 2*a)
*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^2)]
```

Sympy [A] time = 21.1476, size = 345, normalized size = 3.88

$$\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+3abc-b^3}{2ac^2-b^2c}\right)$$
$$+\left(\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+3abc-b^3}{2ac^2-b^2c}\right)$$
$$-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] (b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c -
 b**2)))*log(x**2 + (-8*a**3*c*(b/(4*a**2) - sqrt(-4*a*c + b**2)*
(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2)
```

```
 - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) +
3*a*b*c - b**3)/(2*a*c**2 - b**2*c)) + (b/(4*a**2) + sqrt(-4*a*c
+ b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-8*a*
*3*c*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*
a*c - b**2))) + 2*a**2*b**2*(b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*
a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 3*a*b*c - b**3)/(2*a*c**2
- b**2*c)) - 1/(2*a*x**2) - b*log(x)/a**2
```

GIAC/XCAS [A] time = 0.292446, size = 127, normalized size = 1.43

$$\frac{b\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{b\ln\left(x^2\right)}{2\,a^2}+\frac{\left(b^2-2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*b*ln(c*x^4 + b*x^2 + a)/a^2 - 1/2*b*ln(x^2)/a^2 + 1/2*(b^2 -
2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*
c)*a^2) + 1/2*(b*x^2 - a)/(a^2*x^2)
```

3.854 $\int \frac{1}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=114

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}-\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)
*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi [A] time = 0.427216, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\left(b^2-3ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}-\frac{\left(b^2-ac\right)\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)\left(b^2-ac\right)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b*(b^2 - 3*a*c)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2 - 4*a*c]) + ((b^2 - a*c)
*Log[x])/a^3 - ((b^2 - a*c)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rubi in Sympy [A] time = 44.8793, size = 109, normalized size = 0.96

$$-\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b\left(-3ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^3\sqrt{-4ac+b^2}}+\frac{\left(-ac+b^2\right)\log\left(x^2\right)}{2a^3}-\frac{\left(-ac+b^2\right)\log\left(a+bx^2+cx^4\right)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] -1/(4*a*x**4) + b/(2*a**2*x**2) + b*(-3*a*c + b**2)*atanh((b + 2*
c*x**2)/sqrt(-4*a*c + b**2))/(2*a**3*sqrt(-4*a*c + b**2)) + (-a*c
 + b**2)*log(x**2)/(2*a**3) - (-a*c + b**2)*log(a + b*x**2 + c*x*
*4)/(4*a**3)
```

Mathematica [A] time = 0.559158, size = 188, normalized size = 1.65

$$\frac{-\frac{a^2}{x^4}+4\log(x)\left(b^2-ac\right)-\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(-b^2\sqrt{b^2-4ac}+ac\sqrt{b^2-4ac}-3abc+b^3\right)\log\left(\sqrt{b^2-4ac}+\right.}{\sqrt{b^2-4ac}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(a^2/x^4) + (2*a*b)/x^2 + 4*(b^2 - a*c)*Log[x] - ((b^3 - 3*a*b*
c + b^2*Sqrt[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((b^3 - 3*a*b*c - b^2
*Sqrt[b^2 - 4*a*c] + a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^3)
```

Maple [A] time = 0.013, size = 159, normalized size = 1.4

$$\frac{c\ln\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{\ln\left(cx^4+bx^2+a\right)b^2}{4\,a^3}+\frac{3\,bc}{2\,a^2}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{b^3}{2\,a^3}\arctan\left(\left(2\,cx^2+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{1}{4\,ax^4}-\frac{c\ln(x)}{a^2}+\frac{b^2\ln(x)}{a^3}+\frac{b}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4/a^2*c*ln(c*x^4+b*x^2+a)-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2+3/2/a^2
/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c-1/2/
a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3-1
/4/a/x^4-1/a^2*ln(x)*c+1/a^3*ln(x)*b^2+1/2*b/a^2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299061, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^3-3\,abc\right)x^4\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(\left(b^2-ac\right)x^4\log\left(cx^4+bx^2+a\right)-4\left(b^2-a\right.\right.}{4\sqrt{b^2-4\,ac}a^3x^4}\right.$$

$$-\frac{2\left(b^3-3\,abc\right)x^4\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(\left(b^2-ac\right)x^4\log\left(cx^4+bx^2+a\right)-4\left(b^2-ac\right)x^4\log\left(x\right)-2\,abx^2+a^2\right)\sqrt{-}}{4\sqrt{-b^2+4\,ac}a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^3 - 3*a*b*c)*x^4*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c
^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c)
)/(c*x^4 + b*x^2 + a)) + ((b^2 - a*c)*x^4*log(c*x^4 + b*x^2 + a)
- 4*(b^2 - a*c)*x^4*log(x) - 2*a*b*x^2 + a^2)*sqrt(b^2 - 4*a*c))/
(sqrt(b^2 - 4*a*c)*a^3*x^4), -1/4*(2*(b^3 - 3*a*b*c)*x^4*arctan(-
(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2 - a*c)*x^
4*log(c*x^4 + b*x^2 + a) - 4*(b^2 - a*c)*x^4*log(x) - 2*a*b*x^2 +
 a^2)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^3*x^4)]
```

Sympy [A] time = 26.4883, size = 423, normalized size = 3.71

$$\left(-\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4a^3\left(4ac-b^2\right)}+\frac{ac-b^2}{4a^3}\right)\log\left(x^2+\frac{8a^4c\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4a^3(4ac-b^2)}+\frac{ac-b^2}{4a^3}\right)-2a^3b^2\left(-\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4a^3(4ac-b^2)}+\frac{ac-b^2}{4a^3}\right)-2a^2c^2+4ab^2c-b^4}{3abc^2-b^3c}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}\left(3ac-b^2\right)}{4a^3\left(4ac-b^2\right)}+\frac{ac-b^2}{4a^3}\right)\log\left(x^2+\frac{8a^4c\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4a^3(4ac-b^2)}+\frac{ac-b^2}{4a^3}\right)-2a^3b^2\left(\frac{b\sqrt{-4ac+b^2}(3ac-b^2)}{4a^3(4ac-b^2)}+\frac{ac-b^2}{4a^3}\right)-2a^2c^2+4ab^2c-b^4}{3abc^2-b^3c}\right)$$
$$+\frac{-a+2bx^2}{4a^2x^4}-\frac{\left(ac-b^2\right)\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) +
(a*c - b**2)/(4*a**3))*log(x**2 + (8*a**4*c*(-b*sqrt(-4*a*c + b**
2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3)
) - 2*a**3*b**2*(-b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4
*a*c - b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**2*c**2 + 4*a*b**2*c
 - b**4)/(3*a*b*c**2 - b**3*c)) + (b*sqrt(-4*a*c + b**2)*(3*a*c -
 b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a**3))*log(x**2
+ (8*a**4*c*(b*sqrt(-4*a*c + b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c
- b**2)) + (a*c - b**2)/(4*a**3)) - 2*a**3*b**2*(b*sqrt(-4*a*c +
b**2)*(3*a*c - b**2)/(4*a**3*(4*a*c - b**2)) + (a*c - b**2)/(4*a*
*3)) - 2*a**2*c**2 + 4*a*b**2*c - b**4)/(3*a*b*c**2 - b**3*c)) +
(-a + 2*b*x**2)/(4*a**2*x**4) - (a*c - b**2)*log(x)/a**3
```

GIAC/XCAS [A] time = 0.291943, size = 170, normalized size = 1.49

$$-\frac{\left(b^2-ac\right)\ln\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{\left(b^2-ac\right)\ln\left(x^2\right)}{2\,a^3}-\frac{\left(b^3-3\,abc\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}a^3}-\frac{3\,b^2x^4-3\,acx^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] -1/4*(b^2 - a*c)*ln(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2 - a*c)*ln(x
^2)/a^3 - 1/2*(b^3 - 3*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*
a*c))/(sqrt(-b^2 + 4*a*c)*a^3) - 1/4*(3*b^2*x^4 - 3*a*c*x^4 - 2*a
*b*x^2 + a^2)/(a^3*x^4)
```

3.855 $\int \frac{x^6}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=203

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c
+ (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi [A] time = 1.30672, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c
+ (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi in Sympy [A] time = 67.7667, size = 212, normalized size = 1.04

$$-\frac{bx}{c^2}+\frac{x^3}{3c}+\frac{\sqrt{2}\left(b\left(-3ac+b^2\right)+\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\left(b\left(-3ac+b^2\right)-\sqrt{-4ac+b^2}\left(-ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x/c**2 + x**3/(3*c) + sqrt(2)*(b*(-3*a*c + b**2) + sqrt(-4*a*c
 + b**2)*(-a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a
*c + b**2)))/(2*c**(5/2)*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*
c + b**2)) - sqrt(2)*(b*(-3*a*c + b**2) - sqrt(-4*a*c + b**2)*(-a
*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))
/(2*c**(5/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.284146, size = 250, normalized size = 1.23

$$\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{5/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{bx}{c^2}+\frac{x^3}{3c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*x)/c^2) + x^3/(3*c) + ((-b^3 + 3*a*b*c + b^2*Sqrt[b^2 - 4*a*
c] - a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((b^3 - 3*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - a*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Maple [B] time = 0.053, size = 467, normalized size = 2.3

$$\frac{x^3}{3c} - \frac{bx}{c^2} - \frac{\sqrt{2}a}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b^2}{2c^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{3\sqrt{2}ab}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b^3}{2c^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}a}{2c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b^2}{2c^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{3\sqrt{2}ab}{2c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b^3}{2c^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*x^3/c-b*x/c^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a+1/2/c^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
```

```
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^3+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c^
2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-3/2/c/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*a*b+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3 - 3\,bx}{3\,c^2} - \frac{-\int \frac{(b^2 - ac)\,x^2 + ab}{cx^4 + bx^2 + a}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^3 - 3*b*x)/c^2 - integrate(-((b^2 - a*c)*x^2 + a*b)/(c*x
^4 + b*x^2 + a), x)/c^2
```

Fricas [A] time = 0.289796, size = 2111, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*x^3 - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c
^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*
c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sqrt(1/2)*(b^7
- 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b^2*c
^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*
c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2
*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 -
```

```
4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqrt(1/2)*(
b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 - (b^4*c^5 - 6*a*b
^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a*b^3
*c + 5*a^2*b*c^2 + (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(
b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^
5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x + sqrt(1/
2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5 - 6
*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 - 5*a
*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)
))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b^5 - 5*a*b^3*c
 + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))/(b^
2*c^5 - 4*a*c^6))*log(2*(a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*x - sqr
t(1/2)*(b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3 + (b^4*c^5
 - 6*a*b^2*c^6 + 8*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^
2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^2*c^10 - 4*a*c
^11)))/(b^2*c^5 - 4*a*c^6))) - 6*b*x)/c^2
```

Sympy [A] time = 7.62957, size = 194, normalized size = 0.96

$$-\frac{bx}{c^2} + \mathrm{RootSum}\left(t^4\left(256a^2c^7 - 128ab^2c^6 + 16b^4c^5\right) + t^2\left(-80a^3bc^3 + 100a^2b^3c^2 - 36ab^5c + 4b^7\right) + a^5, \left(t \mapsto t\log\left(x + \frac{-64t^3a^2c^7}{}\right.\right.\right. + \frac{x^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2+a),x)
```

```
[Out] -b*x/c**2 + RootSum(_t**4*(256*a**2*c**7 - 128*a*b**2*c**6 + 16*b
**4*c**5) + _t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b*
*5*c + 4*b**7) + a**5, Lambda(_t, _t*log(x + (-64*_t**3*a**2*c**7
 + 48*_t**3*a*b**2*c**6 - 8*_t**3*b**4*c**5 + 14*_t*a**3*b*c**3 -
 28*_t*a**2*b**3*c**2 + 14*_t*a*b**5*c - 2*_t*b**7)/(a**4*c**2 -
3*a**3*b**2*c + a**2*b**4)))) + x**3/(3*c)
```

GIAC/XCAS [A] time = 0.856911, size = 1, normalized size = 0.

$$\text{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.856 $\int \frac{x^4}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.594199, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 48.2508, size = 189, normalized size = 1.06

$$\frac{x}{c}-\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] x/c - sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2
)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b + s
qrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-2*a*c + b**2
 - b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4
*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*
a*c + b**2))
```

Mathematica [A] time = 0.195517, size = 202, normalized size = 1.13

$$-\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4
*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2 - 2*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [B] time = 0.026, size = 343, normalized size = 1.9

$$\frac{x}{c}-\frac{\sqrt{2}\,b}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2}{2\,c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{\sqrt{2}\,b}{2\,c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\sqrt{2}aArtanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{\sqrt{2}\,b^2}{2\,c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] x/c-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))*b^2+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c} - \frac{\int \frac{bx^2+a}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] x/c - integrate((b*x^2 + a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [A] time = 0.274532, size = 1430, normalized size = 7.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a
*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*
a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/
(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 -
 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^
2*c + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b
^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c - (b^2*c^3
 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7))
)/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4
- 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*c - (b^2
*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(b^3 - 3*a*b*c -
(b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/
2)*(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4
 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(b^3 - 3*a*b*
c - (b^2*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6
 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 5.6177, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(48a^2bc^2-28ab^3c+4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{32t^3abc^4-8t^3b^3c^3-4ta}{a^2c-ab}\right.\right.\right.$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) +
_t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + a**3, Lambda(_t,
_t*log(x + (32*_t**3*a*b*c**4 - 8*_t**3*b**3*c**3 - 4*_t*a**2*c**
2 + 8*_t*a*b**2*c - 2*_t*b**4)/(a**2*c - a*b**2)))) + x/c
```

GIAC/XCAS [A] time = 0.790652, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.857 $\int \frac{x^2}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=150

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sq
rt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.242353, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (Sq
rt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 21.6519, size = 141, normalized size = 0.94

$$-\frac{\sqrt{2}\sqrt{b-\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqr
t(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2)) + sqr
```

```
t(2)*sqrt(b + sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
+ sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.170069, size = 165, normalized size = 1.1

$$\frac{\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2
 - 4*a*c])
```

Maple [A] time = 0.021, size = 208, normalized size = 1.4

$$\frac{\sqrt{2}}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}}{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{\sqrt{2}b}{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.268537, size = 755, normalized size = 5.03

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b + \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b + \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right) - \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b - \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right) + \frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b - \frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c - 4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c - 4\,ac^2\right)\sqrt{-\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2 - 4\,ac^3}} + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b +
(b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqr
t(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt(1/2)*sqrt(-(b + (b^2*c - 4*a
*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*
(b^2*c - 4*a*c^2)*sqrt(-(b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a
*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) - 1/2*sqrt
(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*
c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt(-(b - (b^2*c -
 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^
2 - 4*a*c^3) + x) + 1/2*sqrt(1/2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sq
rt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c -
 4*a*c^2)*sqrt(-(b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] time = 2.40416, size = 75, normalized size = 0.5

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3 - 128ab^2c^2 + 16b^4c\right) + t^2\left(-16abc + 4b^3\right) + a, \left(t \mapsto t\log\left(64t^3ac^2 - 16t^3b^2c - 2tb + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t*
*2*(-16*a*b*c + 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 -
16*_t**3*b**2*c - 2*_t*b + x)))
```

GIAC/XCAS [A] time = 0.740355, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.858 $\int \frac{1}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.234976, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[
2]*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]
])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 19.6759, size = 138, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b*
*2)))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(
```

```
2)*sqrt(c)*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/
(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.140534, size = 129, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]]/Sqrt[b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]]/Sqrt[b + Sqrt[b^2 - 4*a*c]]))/Sqrt[
b^2 - 4*a*c]
```

Maple [A] time = 0.017, size = 116, normalized size = 0.8

$$-c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}-c\sqrt{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.2665, size = 828, normalized size = 5.52

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac-\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx+\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(2cx-\sqrt{\frac{1}{2}}\left(b^2-4ac+\frac{ab^3-4a^2bc}{\sqrt{a^2b^2-4a^3c}}\right)\sqrt{-\frac{b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*
c))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^
3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2
*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*
sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*
a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/
sqrt(a^2*b^2 - 4*a^3*c))*sqrt(-(b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^
2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt(-(b - (a*b
^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c
*x + sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 -
4*a^3*c))*sqrt(-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt(-(b - (a*b^2 - 4*a^2*c)/s
qrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*
(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt(
```

```
-(b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c
)))
```

Sympy [A] time = 2.85539, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^3c^2-128a^2b^2c+16ab^4\right)+t^2\left(-16abc+4b^3\right)+c,\left(t\mapsto t\log\left(x+\frac{32t^3a^2bc-8t^3ab^3+4tac-2tb^2}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t*
*2*(-16*a*b*c + 4*b**3) + c, Lambda(_t, _t*log(x + (32*_t**3*a**2
*b*c - 8*_t**3*a*b**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

GIAC/XCAS [A] time = 0.366756, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.859 $\int \frac{1}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=174

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.495297, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 40.6179, size = 177, normalized size = 1.02

$$\frac{\sqrt{2}\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/
sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(b + sqrt(-4*a*c + b**2))
*sqrt(-4*a*c + b**2)) - sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**2))
*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a*sqrt(
b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - 1/(a*x)
```

Mathematica [A] time = 0.765641, size = 191, normalized size = 1.1

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(2/x + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [A] time = 0.025, size = 232, normalized size = 1.3

$$-\frac{c\sqrt{2}}{2\,a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}b}{2\,a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}}{2\,a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}b}{2\,a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*c/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*c/a/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))+1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{cx^2+b}{cx^4+bx^2+a}\,dx}{a} - \frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 + b)/(c*x^4 + b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [A] time = 0.275968, size = 1507, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4
*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2
*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c +
(a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4
*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*
b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b
^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x -
 sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*
c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*
c)))*sqrt(-(b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b
^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqr
t(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 -
```

```
 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*
log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b
*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^2
- 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))
/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(b^3 - 3*a*b*c - (a^
3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2
)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5
*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^
2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + 2)/(a*x)
```

Sympy [A] time = 6.19716, size = 148, normalized size = 0.85

$$\text{RootSum}\left(t^4\left(256a^5c^2 - 128a^4b^2c + 16a^3b^4\right) + t^2\left(48a^2bc^2 - 28ab^3c + 4b^5\right) + c^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^5c^2 + 48t^3a^4b^2c - \right.\right.\right.$$

$$-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) +
_t**2*(48*a**2*b*c**2 - 28*a*b**3*c + 4*b**5) + c**3, Lambda(_t,
_t*log(x + (-64*_t**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3*
a**3*b**4 - 10*_t*a**2*b*c**2 + 10*_t*a*b**3*c - 2*_t*b**5)/(a*c*
*3 - b**2*c**2)))) - 1/(a*x)
```

GIAC/XCAS [A] time = 0.795577, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.860 $\int \frac{1}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.987949, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\sqrt{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (Sqrt[c]*(b + (b^2 - 2*a*c)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(b - (b^2 -
 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 80.5926, size = 207, normalized size = 1.06

$$-\frac{1}{3ax^3}+\frac{b}{a^2x}-\frac{\sqrt{2}\sqrt{c}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a^2\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] -1/(3*a*x**3) + b/(a**2*x) - sqrt(2)*sqrt(c)*(-2*a*c + b**2 - b*s
qrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c +
 b**2)))/(2*a**2*sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2
)) + sqrt(2)*sqrt(c)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan
(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a**2*sqrt(b
- sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.245223, size = 216, normalized size = 1.1

$$\frac{\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{2a}{x^3}+\frac{6b}{x}}{6a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^3 + (6*b)/x + (3*Sqrt[2]*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2
]*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [B] time = 0.027, size = 368, normalized size = 1.9

$$\frac{c\sqrt{2}b}{2a^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{c^2\sqrt{2}}{a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{c\sqrt{2}b^2}{2a^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{c\sqrt{2}b}{2a^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{c^2\sqrt{2}}{a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{c\sqrt{2}b^2}{2a^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{1}{3ax^3}+\frac{b}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/a*c^2/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b^2-1/2/a^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^
2-1/3/a/x^3+b/a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{bcx^2+b^2-ac}{cx^4+bx^2+a}\,dx}{a^2} + \frac{3\,bx^2 - a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate((b*c*x^2 + b^2 - a*c)/(c*x^4 + b*x^2 + a), x)/a^2 + 1/3
*(3*b*x^2 - a)/(a^2*x^3)
```

Fricas [A] time = 0.286602, size = 2190, normalized size = 11.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 +
(a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))
*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a
*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 - (a^5*b^5 -
 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^
11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6
*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11
*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5
)*x - sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^
3 + 4*a^4*c^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2
 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^2 -
4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) + 3*sqrt(
1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*
a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(b^4*c
^3 - 3*a*b^2*c^4 + a^2*c^5)*x + sqrt(1/2)*(b^8 - 8*a*b^6*c + 20*a
^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4 + (a^5*b^5 - 7*a^6*b^3*c
+ 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
```

```
2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*
b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqrt(-(b^5 - 5*a*b^3*c + 5
*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^
2 - 4*a^6*c))*log(2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x - sqrt(1/
2)*(b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4
 + (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^2 - 4*a^11*c))
)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^2 - 4*a^6*c)*sqrt
((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^
10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*b*x^2 + 2*a)/(a^2*
x^3)
```

Sympy [A] time = 8.52191, size = 211, normalized size = 1.08

$$\mathrm{RootSum}\left(t^4\left(256a^7c^2-128a^6b^2c+16a^5b^4\right)+t^2\left(-80a^3bc^3+100a^2b^3c^2-36ab^5c+4b^7\right)+c^5,\left(t\mapsto t\log\left(x+\frac{-96t^3a^7bc^2\right.\right.\right.$$
$$+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**7*c**2 - 128*a**6*b**2*c + 16*a**5*b**4) +
_t**2*(-80*a**3*b*c**3 + 100*a**2*b**3*c**2 - 36*a*b**5*c + 4*b**
7) + c**5, Lambda(_t, _t*log(x + (-96*_t**3*a**7*b*c**2 + 56*_t**
3*a**6*b**3*c - 8*_t**3*a**5*b**5 - 4*_t*a**4*c**4 + 32*_t*a**3*b
**2*c**3 - 40*_t*a**2*b**4*c**2 + 16*_t*a*b**6*c - 2*_t*b**8)/(a*
*2*c**5 - 3*a*b**2*c**4 + b**4*c**3)))) + (-a + 3*b*x**2)/(3*a**2
*x**3)
```

GIAC/XCAS [A] time = 0.796963, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] Done
```

3.861 $\int \frac{x^7}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=132

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}-\frac{bx^2}{2c(b^2-4ac)}+\frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 +
 c*x^4]/(4*c^2)
```

Rubi [A] time = 0.372598, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2(b^2-4ac)^{3/2}}-\frac{bx^2}{2c(b^2-4ac)}+\frac{x^4(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^2)/(2*c*(b^2 - 4*a*c)) + (x^4*(2*a + b*x^2))/(2*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + Log[a + b*x^2 +
 c*x^4]/(4*c^2)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b(-6ac+b^2)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2(-4ac+b^2)^{\frac{3}{2}}}+\frac{x^4(2a+bx^2)}{2(-4ac+b^2)(a+bx^2+cx^4)}-\frac{\int^{x^2} b\,dx}{2c(-4ac+b^2)}+\frac{\log(a+bx^2+cx^4)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] b*(-6*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*
*2*(-4*a*c + b**2)**(3/2)) + x**4*(2*a + b*x**2)/(2*(-4*a*c + b**
2)*(a + b*x**2 + c*x**4)) - Integral(b, (x, x**2))/(2*c*(-4*a*c +
```

```
b**2)) + log(a + b*x**2 + c*x**4)/(4*c**2)
```

Mathematica [A] time = 0.338068, size = 121, normalized size = 0.92

$$\frac{\frac{2\left(-2a^2c+ab\left(b-3cx^2\right)+b^3x^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{2b\left(b^2-6ac\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c + b^3*x^2 + a*b*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (2*b*(b^2 - 6*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + Log[a + b*x^2 + c*x^4])/(4*
c^2)
```

Maple [B] time = 0.021, size = 342, normalized size = 2.6

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(\frac{b\left(3\,ac-b^2\right)x^2}{c^2\left(4\,ac-b^2\right)}+\frac{a\left(2\,ac-b^2\right)}{c^2\left(4\,ac-b^2\right)}\right)+\frac{\ln\left(c\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)}{4\,c^2}$$
$$-3\,\frac{ab}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\arctan\left(\frac{2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)$$
$$+\frac{b^3}{2\,c}\arctan\left(\left(2\,c^2\left(4\,ac-b^2\right)x^2+c\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3-b^6c^2}}\right)\frac{1}{\sqrt{64\,a^3c^5-48\,a^2b^2c^4+12\,ab^4c^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(b*(3*a*c-b^2)/c^2/(4*a*c-b^2)*x^2+a*(2*a*c-b^2)/(4*a*c-b^2)/
c^2)/(c*x^4+b*x^2+a)+1/4/c^2*ln(c*(4*a*c-b^2)*(c*x^4+b*x^2+a))-3/
(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*
c^2*(4*a*c-b^2)*x^2+c*(4*a*c-b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+1
2*a*b^4*c^3-b^6*c^2)^(1/2))*a*b+1/2/(64*a^3*c^5-48*a^2*b^2*c^4+12
*a*b^4*c^3-b^6*c^2)^(1/2)*arctan((2*c^2*(4*a*c-b^2)*x^2+c*(4*a*c-
b^2)*b)/(64*a^3*c^5-48*a^2*b^2*c^4+12*a*b^4*c^3-b^6*c^2)^(1/2))*b
^3/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276471, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(2\,ab^2-4\right.}{4\left(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c\right.}\right.$$

$$-\frac{2\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(2\,ab^2-4\,a^2c+2\left(b^3-3\,abc\right)x^2+\right.}{4\left(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b
^2*c)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + (2*a*b^2 - 4*a^2*c + 2*(b^3 - 3*a*b*c)*x^2 + ((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b
*x^2 + a))*sqrt(b^2 - 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3
- 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)), -
1/4*(2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*
b^2*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - (2*a*b^2 - 4*a^2*c + 2*(b^3 - 3*a*b*c)*x^2 + ((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b*x^
2 + a))*sqrt(-b^2 + 4*a*c))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 -
4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 12.2503, size = 745, normalized size = 5.64

$$\left(-\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\left(-\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)-ab}{6abc-b^3}\right.$$

$$+\left(\frac{b\sqrt{-\left(4ac-b^2\right)^3}\left(6ac-b^2\right)}{4c^2\left(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6\right)}+\frac{1}{4c^2}\right)\log\left(x^2+\frac{-32a^2c^3\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)+8a^2c+16ab^2c^2\left(\frac{b\sqrt{-(4ac-b^2)^3}(6ac-b^2)}{4c^2(64a^3c^3-48a^2b^2c^2+12ab^4c-b^6)}+\frac{1}{4c^2}\right)-ab^2-}{6abc-b^3}\right.$$

$$+\frac{2a^2c-ab^2+x^2\left(3abc-b^3\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x**
2 + (-32*a**2*c**3*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4
*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(-b*sqrt(-(4*a*c - b**2)*
*3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12
*a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(-b*sqrt(-(4
*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b
**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3))
+ (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**
3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2))*log(x*
*2 + (-32*a**2*c**3*(b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4
*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) +
1/(4*c**2)) + 8*a**2*c + 16*a*b**2*c**2*(b*sqrt(-(4*a*c - b**2)**
3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*
a*b**4*c - b**6)) + 1/(4*c**2)) - a*b**2 - 2*b**4*c*(b*sqrt(-(4*a
*c - b**2)**3)*(6*a*c - b**2)/(4*c**2*(64*a**3*c**3 - 48*a**2*b**
2*c**2 + 12*a*b**4*c - b**6)) + 1/(4*c**2)))/(6*a*b*c - b**3)) +
(2*a**2*c - a*b**2 + x**2*(3*a*b*c - b**3))/(8*a**2*c**3 - 2*a*b*
*2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 - 2*b*
*3*c**2))
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.862 $\int \frac{x^5}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=78

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.137891, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{2a\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\,(2a+bx^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*a*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 17.1153, size = 70, normalized size = 0.9

$$\frac{2a\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{x^2\,(2a+bx^2)}{2\,(-4ac+b^2)\,(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3
/2) + x**2*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4
))
```

Mathematica [A] time = 0.156662, size = 93, normalized size = 1.19

$$\frac{2a\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}+\frac{a\left(b-2cx^2\right)+b^2x^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (b^2*x^2 + a*(b - 2*c*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^
4)) + (2*a*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*
c)^(3/2)
```

Maple [A] time = 0.014, size = 104, normalized size = 1.3

$$\frac{1}{2\,cx^4+2\,bx^2+2\,a}\left(-\frac{\left(2\,ac-b^2\right)x^2}{c\left(4\,ac-b^2\right)}+\frac{ab}{c\left(4\,ac-b^2\right)}\right)+2\,\frac{a}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c-b^2)/c/(4*a*c-b^2)*x^2+a*b/c/(4*a*c-b^2))/(c*x^4+b*x
^2+a)+2*a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264563, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(ac^2x^4+abcx^2+a^2c\right)\log\left(-\frac{b^3-4\,abc+2\,(b^2c-4\,ac^2)\,x^2-(2\,c^2x^4+2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left((b^2-2\,ac)\,x^2+ab\right)\sqrt{b^2-4\,ac}}{2\,((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2)\sqrt{b^2-4\,ac}},\right.$$
$$\left.-\frac{4\left(ac^2x^4+abcx^2+a^2c\right)\arctan\left(-\frac{(2\,cx^2+b)\,\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left((b^2-2\,ac)\,x^2+ab\right)\sqrt{-b^2+4\,ac}}{2\,((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2)\sqrt{-b^2+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*c^2*x^4 + a*b*c*x^2 + a^2*c)*log(-(b^3 - 4*a*b*c + 2*
(b^2*c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqr
t(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2 - 2*a*c)*x^2 + a*b)*
sqrt(b^2 - 4*a*c))/(((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^
2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)), -1/2*(4*(a*c^2*x
^4 + a*b*c*x^2 + a^2*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/
(b^2 - 4*a*c)) + ((b^2 - 2*a*c)*x^2 + a*b)*sqrt(-b^2 + 4*a*c))/((
(b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^
2)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 6.05842, size = 282, normalized size = 3.62

$$-a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}-ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)$$
$$+a\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^3c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8a^2b^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab^4\sqrt{-\frac{1}{(4ac-b^2)^3}}+ab}{2ac}\right)$$
$$-\frac{-ab+x^2\left(2ac-b^2\right)}{8a^2c^2-2ab^2c+x^4\left(8ac^3-2b^2c^2\right)+x^2\left(8abc^2-2b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -a*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**3*c**2*sqrt(-1/(
4*a*c - b**2)**3) + 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) - a*
b**4*sqrt(-1/(4*a*c - b**2)**3) + a*b)/(2*a*c)) + a*sqrt(-1/(4*a*
c - b**2)**3)*log(x**2 + (16*a**3*c**2*sqrt(-1/(4*a*c - b**2)**3)
```

```
 - 8*a**2*b**2*c*sqrt(-1/(4*a*c - b**2)**3) + a*b**4*sqrt(-1/(4*a
*c - b**2)**3) + a*b)/(2*a*c)) - (-a*b + x**2*(2*a*c - b**2))/(8*
a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*
a*b*c**2 - 2*b**3*c))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.863 $\int \frac{x^3}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=75

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.13212, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTanh[
(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 15.6058, size = 65, normalized size = 0.87

$$-\frac{b\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}+\frac{2a+bx^2}{2(-4ac+b^2)(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3/
2) + (2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.115049, size = 79, normalized size = 1.05

$$\frac{2a+bx^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a + b*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (b*ArcTan[(
b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

Maple [A] time = 0.008, size = 77, normalized size = 1.

$$\frac{-bx^2-2a}{(8ac-2b^2)(cx^4+bx^2+a)}-b\arctan\left((2cx^2+b)\frac{1}{\sqrt{4ac-b^2}}\right)(4ac-b^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)-b/(4*a*c-b^2)^(3/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.26668, size = 1, normalized size = 0.01

$$\left[-\frac{(bcx^4+b^2x^2+ab)\log\left(\frac{b^3-4abc+2(b^2c-4ac^2)x^2+(2c^2x^4+2bcx^2+b^2-2ac)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-(bx^2+2a)\sqrt{b^2-4ac}}{2((b^2c-4ac^2)x^4+ab^2-4a^2c+(b^3-4abc)x^2)\sqrt{b^2-4ac}}, \frac{2(bcx^4+b^2x^2+a}{2((b^2c-4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*x^4 + b^2*x^2 + a*b)*log((b^3 - 4*a*b*c + 2*(b^2*c -
4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - (b*x^2 + 2*a)*sqrt(b^2 - 4*a*c))/((
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sq
rt(b^2 - 4*a*c)), 1/2*(2*(b*c*x^4 + b^2*x^2 + a*b)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b*x^2 + 2*a)*sqrt(-b
^2 + 4*a*c))/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4
*a*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.56716, size = 267, normalized size = 3.56

$$\frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{-16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}+8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}-b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{b\sqrt{-\frac{1}{(4ac-b^2)^3}}\log\left(x^2+\frac{16a^2bc^2\sqrt{-\frac{1}{(4ac-b^2)^3}}-8ab^3c\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^5\sqrt{-\frac{1}{(4ac-b^2)^3}}+b^2}{2bc}\right)}{2} - \frac{2a+bx^2}{8a^2c-2ab^2+x^4(8ac^2-2b^2c)+x^2(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] b*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*b*c**2*sqrt(-1/
(4*a*c - b**2)**3) + 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) - b**5
*sqrt(-1/(4*a*c - b**2)**3) + b**2)/(2*b*c))/2 - b*sqrt(-1/(4*a*c
 - b**2)**3)*log(x**2 + (16*a**2*b*c**2*sqrt(-1/(4*a*c - b**2)**3
) - 8*a*b**3*c*sqrt(-1/(4*a*c - b**2)**3) + b**5*sqrt(-1/(4*a*c -
 b**2)**3) + b**2)/(2*b*c))/2 - (2*a + b*x**2)/(8*a**2*c - 2*a*b*
*2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.864 $\int \frac{x}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=74

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi [A] time = 0.11887, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2c\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx^2}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b + 2*c*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*c*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rubi in Sympy [A] time = 12.4287, size = 66, normalized size = 0.89

$$\frac{2c\,\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{b+2cx^2}{2\,(-4ac+b^2)\,(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*c*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**(3
/2) - (b + 2*c*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.140603, size = 79, normalized size = 1.07

$$-\frac{\frac{4c\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{b+2cx^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((b + 2*c*x^2)/(a + b*x^2 + c*x^4) + (4*c*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c))
```

Maple [A] time = 0.008, size = 75, normalized size = 1.

$$\frac{2\,cx^2+b}{(8\,ac-2\,b^2)\,(cx^4+bx^2+a)}+2\,\frac{c}{(4\,ac-b^2)^{3/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+2*c/(4*a*c-b^2)^(3/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265106, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(c^{2}x^{4}+bcx^{2}+ac\right)\log\left(-\frac{b^{3}-4\,abc+2\left(b^{2}c-4\,ac^{2}\right)x^{2}-\left(2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac\right)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)+\left(2\,cx^{2}+b\right)\sqrt{b^{2}-4\,ac}}{2\left((b^{2}c-4\,ac^{2})x^{4}+ab^{2}-4\,a^{2}c+(b^{3}-4\,abc)x^{2}\right)\sqrt{b^{2}-4\,ac}},\right.$$
$$\left.-\frac{4\left(c^{2}x^{4}+bcx^{2}+ac\right)\arctan\left(-\frac{\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)+\left(2\,cx^{2}+b\right)\sqrt{-b^{2}+4\,ac}}{2\left((b^{2}c-4\,ac^{2})x^{4}+ab^{2}-4\,a^{2}c+(b^{3}-4\,abc)x^{2}\right)\sqrt{-b^{2}+4\,ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(c^2*x^4 + b*c*x^2 + a*c)*log(-(b^3 - 4*a*b*c + 2*(b^2*c
 - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2
- 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
*sqrt(b^2 - 4*a*c)), -1/2*(4*(c^2*x^4 + b*c*x^2 + a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^2 + b)*sqr
t(-b^2 + 4*a*c))/(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
 - 4*a*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 5.40924, size = 267, normalized size = 3.61

$$-c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x^{2}+\frac{-16a^{2}c^{3}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+8ab^{2}c^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-b^{4}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+bc}{2c^{2}}\right)$$
$$+c\sqrt{-\frac{1}{\left(4ac-b^{2}\right)^{3}}}\log\left(x^{2}+\frac{16a^{2}c^{3}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}-8ab^{2}c^{2}\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+b^{4}c\sqrt{-\frac{1}{(4ac-b^{2})^{3}}}+bc}{2c^{2}}\right)$$
$$+\frac{b+2cx^{2}}{8a^{2}c-2ab^{2}+x^{4}\left(8ac^{2}-2b^{2}c\right)+x^{2}\left(8abc-2b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -c*sqrt(-1/(4*a*c - b**2)**3)*log(x**2 + (-16*a**2*c**3*sqrt(-1/(
4*a*c - b**2)**3) + 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) - b*
*4*c*sqrt(-1/(4*a*c - b**2)**3) + b*c)/(2*c**2)) + c*sqrt(-1/(4*a
*c - b**2)**3)*log(x**2 + (16*a**2*c**3*sqrt(-1/(4*a*c - b**2)**3
) - 8*a*b**2*c**2*sqrt(-1/(4*a*c - b**2)**3) + b**4*c*sqrt(-1/(4*
```

```
a*c - b**2)**3) + b*c)/(2*c**2)) + (b + 2*c*x**2)/(8*a**2*c - 2*a
*b**2 + x**4*(8*a*c**2 - 2*b**2*c) + x**2*(8*a*b*c - 2*b**3))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.865 $\int \frac{1}{x(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=122

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^
2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a
^2)
```

Rubi [A] time = 0.428213, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b(b^2-6ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2(b^2-4ac)^{3/2}}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{\log(x)}{a^2}+\frac{-2ac+b^2+bcx^2}{2a(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
 (b*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^
2*(b^2 - 4*a*c)^(3/2)) + Log[x]/a^2 - Log[a + b*x^2 + c*x^4]/(4*a
^2)
```

Rubi in Sympy [A] time = 44.701, size = 116, normalized size = 0.95

$$\frac{-2ac+b^2+bcx^2}{2a(-4ac+b^2)(a+bx^2+cx^4)}+\frac{b(-6ac+b^2)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2(-4ac+b^2)^{\frac{3}{2}}}+\frac{\log\left(x^2\right)}{2a^2}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*(-4*a*c + b**2)*(a + b*x**2 + c*x
**4)) + b*(-6*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2
))/(2*a**2*(-4*a*c + b**2)**(3/2)) + log(x**2)/(2*a**2) - log(a +
```

```
b*x**2 + c*x**4)/(4*a**2)
```

Mathematica [A] time = 0.727698, size = 207, normalized size = 1.7

$$\frac{\frac{2a(-2ac+b^2+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-6abc+b^3\right)\log\left(\sqrt{b^2-4ac}+b+2cx\right.}{(b^2-4ac)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((2*a*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + 4*Log[x] - ((b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sq
rt[b^2 - 4*a*c])*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a
*c)^(3/2) + ((b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[
b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)
^(3/2))/(4*a^2)
```

Maple [B] time = 0.028, size = 405, normalized size = 3.3

$$-\frac{bcx^2}{2\,a\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+\frac{c}{(4\,ac-b^2)\,(cx^4+bx^2+a)}-\frac{b^2}{2\,a\,(cx^4+bx^2+a)\,(4\,ac-b^2)}$$
$$-\frac{c\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)}{(4\,ac-b^2)\,a}+\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^2}{4\,a^2\,(4\,ac-b^2)}$$
$$-3\,\frac{bc}{a\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)$$
$$+\frac{b^3}{2\,a^2}\arctan\left(\left(2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}$$
$$+\frac{\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x^2+1/(c*x^4+b*x^2+a)/(4*a
*c-b^2)*c-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2-1/a/(4*a*c-b^2)*c
*ln((4*a*c-b^2)*(c*x^4+b*x^2+a))+1/4/a^2/(4*a*c-b^2)*ln((4*a*c-b^
2)*(c*x^4+b*x^2+a))*b^2-3/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c
-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^
3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b*c+1/2/a^2/(64*a^3*c^3-4
8*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(
```

```
4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b
^3+ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.346798, size = 1, normalized size = 0.01

$$\left[\frac{\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\log\left(\frac{b^3-4\,abc+2\,\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(2\,abcx^2+\cdots\right.}{4\left(a^3b^2-4\,a^4c+\cdots\right.}\right.$$

$$-\frac{2\left(\left(b^3c-6\,abc^2\right)x^4+ab^3-6\,a^2bc+\left(b^4-6\,ab^2c\right)x^2\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-\left(2\,abcx^2+2\,ab^2-4\,a^2c-\left(\left(b^2c-4\cdots\right.\right.\right.}{4\left(a^3b^2-4\,a^4c+\left(a^2b^2c-4\,\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*b
^2*c)*x^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*
x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) + (2*a*b*c*x^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*x^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b*x^2 + a)
+ 4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*log(x))*sqrt(b^2 - 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c -
4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(b^2 - 4*a*c)), -
1/4*(2*((b^3*c - 6*a*b*c^2)*x^4 + a*b^3 - 6*a^2*b*c + (b^4 - 6*a*
b^2*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c
)) - (2*a*b*c*x^2 + 2*a*b^2 - 4*a^2*c - ((b^2*c - 4*a*c^2)*x^4 +
a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*log(c*x^4 + b*x^2 + a) + 4
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
log(x))*sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*
a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 104.571, size = 772, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3
 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2))*log(x**
2 + (-32*a**4*c**2*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4
*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)) -
1/(4*a**2)) + 16*a**3*b**2*c*(-b*sqrt(-(4*a*c - b**2)**3)*(6*a*c
- b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c -
 b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(-b*sqrt(-(4*a*c - b**2)**3)*
(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b
**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 + 7*a*b**2*c - b**4)/(
6*a*b*c**2 - b**3*c)) + (b*sqrt(-(4*a*c - b**2)**3)*(6*a*c - b**2
)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 12*a*b**4*c - b**6)
) - 1/(4*a**2))*log(x**2 + (-32*a**4*c**2*(b*sqrt(-(4*a*c - b**2)
**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2*c**2 + 1
2*a*b**4*c - b**6)) - 1/(4*a**2)) + 16*a**3*b**2*c*(b*sqrt(-(4*a*
c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**2*b**2
*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 2*a**2*b**4*(b*sqrt(
-(4*a*c - b**2)**3)*(6*a*c - b**2)/(4*a**2*(64*a**3*c**3 - 48*a**
2*b**2*c**2 + 12*a*b**4*c - b**6)) - 1/(4*a**2)) - 8*a**2*c**2 +
7*a*b**2*c - b**4)/(6*a*b*c**2 - b**3*c)) - (-2*a*c + b**2 + b*c*
x**2)/(8*a**3*c - 2*a**2*b**2 + x**4*(8*a**2*c**2 - 2*a*b**2*c) +
 x**2*(8*a**2*b*c - 2*a*b**3)) + log(x)/a**2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.866 $\int \frac{1}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2
)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
 + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2
 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2 + c*x^4])/
(2*a^3)
```

Rubi [A] time = 0.550584, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389

$$\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{2b\log(x)}{a^3}-\frac{b^2-3ac}{a^2x^2\left(b^2-4ac\right)}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{a^3\left(b^2-4ac\right)^{3/2}}+\frac{-2ac+b^2+bcx^2}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -((b^2 - 3*a*c)/(a^2*(b^2 - 4*a*c)*x^2)) + (b^2 - 2*a*c + b*c*x^2
)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - ((b^4 - 6*a*b^2*c
 + 6*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(a^3*(b^2
 - 4*a*c)^(3/2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^2 + c*x^4])/
(2*a^3)
```

Rubi in Sympy [A] time = 76.9038, size = 153, normalized size = 0.94

$$\frac{-2ac+b^2+bcx^2}{2ax^2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{-3ac+b^2}{a^2x^2\left(-4ac+b^2\right)}-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(a+bx^2+cx^4\right)}{2a^3}-\frac{\left(6a^2c^2-6ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{a^3\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*x**2*(-4*a*c + b**2)*(a + b*x**2
+ c*x**4)) - (-3*a*c + b**2)/(a**2*x**2*(-4*a*c + b**2)) - b*log(
x**2)/a**3 + b*log(a + b*x**2 + c*x**4)/(2*a**3) - (6*a**2*c**2 -
 6*a*b**2*c + b**4)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(a*
*3*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.497072, size = 248, normalized size = 1.53

$$\frac{\frac{\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}+\frac{\left(-6a^2c^2+6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{(b^2-4ac)^{3/2}}-}{2a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-(a/x^2) - (a*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 -
 4*a*c)*(a + b*x^2 + c*x^4)) - 4*b*Log[x] + ((b^4 - 6*a*b^2*c + 6
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c])*Log
[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-b^4 +
 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2
 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3
/2))/(2*a^3)
```

Maple [B] time = 0.026, size = 569, normalized size = 3.5

$$-\frac{c^2x^2}{a\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+\frac{b^2cx^2}{2\,a^2\,(cx^4+bx^2+a)\,(4\,ac-b^2)}-\frac{3\,bc}{2\,a\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+\frac{b^3}{2\,a^2\,(cx^4+bx^2+a)\,(4\,ac-b^2)}+2\,\frac{c\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b}{a^2\,(4\,ac-b^2)}-\frac{\ln\left(\left(4\,ac-b^2\right)\left(cx^4+bx^2+a\right)\right)b^3}{2\,a^3\,(4\,ac-b^2)}-6\,\frac{c^2}{a\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)+6\,\frac{b^2c}{a^2\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\arctan\left(\frac{2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)-\frac{b^4}{a^3}\arctan\left(\left(2\,\left(4\,ac-b^2\right)cx^2+\left(4\,ac-b^2\right)b\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}\right)\frac{1}{\sqrt{64\,a^3c^3-48\,a^2b^2c^2+12\,ab^4c-b^6}}-\frac{1}{2\,a^2x^2}-2\,\frac{b\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2+1/2/a^2/(c*x^4+b*x^2+a)*
c/(4*a*c-b^2)*x^2*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*c+1/2/a
^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)+2/a^2/(4*a*c-b^2)*c*ln((4*a*c-
b^2)*(c*x^4+b*x^2+a))*b-1/2/a^3/(4*a*c-b^2)*ln((4*a*c-b^2)*(c*x^4
+b*x^2+a))*b^3-6/a/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/
2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*
b^2*c^2+12*a*b^4*c-b^6)^(1/2))*c^2+6/a^2/(64*a^3*c^3-48*a^2*b^2*c
^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c-b^2)*c*x^2+(4*a*c-b^2)*
b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2))*b^2*c-1/a^3/
(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-b^6)^(1/2)*arctan((2*(4*a*c
-b^2)*c*x^2+(4*a*c-b^2)*b)/(64*a^3*c^3-48*a^2*b^2*c^2+12*a*b^4*c-
b^6)^(1/2))*b^4-1/2/a^2/x^2-2*b*ln(x)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.411242, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="fricas")
```

```
[Out] [-1/2*(((b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c
+ 6*a^2*b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*log((
b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2
+ b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*(a*b^
2*c - 3*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*
x^2 - ((b^3*c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 -
 4*a^2*b*c)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^3*c - 4*a*b*c^2)*
x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*x^2)*log(x))*sq
rt(b^2 - 4*a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4
*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(b^2 - 4*a*c)), 1/2*(2*(
(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*x^6 + (b^5 - 6*a*b^3*c + 6*a^2*
b*c^2)*x^4 + (a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*x^2)*arctan(-(2*c*
x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(a*b^2*c - 3*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (2*a*b^3 - 7*a^2*b*c)*x^2 - ((b^3*
c - 4*a*b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^3*c - 4*a*b*c^2)*x^6 + (b^4 -
 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*x^2)*log(x))*sqrt(-b^2 + 4*
a*c))/(((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 +
 (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^3),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.867 $\int \frac{x^8}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=331

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2-10ac\right)}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{2c\left(b^2-4ac\right)}+\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 -
4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rubi [A] time = 1.62853, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{\left(-\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2c^2-19ab^2c+3b^4}{\sqrt{b^2-4ac}}-13abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{5/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(3b^2-10ac\right)}{2c^2\left(b^2-4ac\right)}-\frac{bx^3}{2c\left(b^2-4ac\right)}+\frac{x^5\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2 - 10*a*c)*x)/(2*c^2*(b^2 - 4*a*c)) - (b*x^3)/(2*c*(b^2 -
4*a*c)) + (x^5*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) - ((3*b^3 - 13*a*b*c - (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
```

```
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) - ((3*b^3 - 13*a*b*c + (3*b^4 - 19*a*b^2*c + 20*a^2*c^2)/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*
c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.17643, size = 327, normalized size = 0.99

$$\frac{-\frac{2\sqrt{c}x\left(2a^2c-ab\left(b-3cx^2\right)+b^3\left(-x^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\sqrt{2}\left(-20a^2c^2+19ab^2c-13abc\sqrt{b^2-4ac}+3b^3\sqrt{b^2-4ac}-3b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(20a^2c^2-19ab^2c-13abc\sqrt{b^2}\cdots\right.}{(b^2-\cdots}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*x - (2*Sqrt[c]*x*(2*a^2*c - b^3*x^2 - a*b*(b - 3*c*x^2
)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4 + 19*a
*b^2*c - 20*a^2*c^2 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*b*c*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(
3*b^4 - 19*a*b^2*c + 20*a^2*c^2 + 3*b^3*Sqrt[b^2 - 4*a*c] - 13*a*
b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))
/(4*c^(5/2))
```

Maple [B] time = 0.112, size = 2280, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x/c^2+3/2/c/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)*x^3*a-1/2/c^2/(c*x^4+b*
x^2+a)*b^3/(4*a*c-b^2)*x^3+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x-
1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2-80*c^2/(-(4*a*c-b^2)^
3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2
))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/
((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*a^4+
116*c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(
-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2
+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c
-b^2)*c)^(1/2))*a^3*b^2-55/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(
1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*
arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*a^2*b^4+43/4/c/(-(4*a*c-b^2)
^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)
/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*a*b
^6-3/4/c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+
b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*
a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(
4*a*c-b^2)*c)^(1/2))*b^8-13/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-
(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+
2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2)*c)^(1/2))*a^2*b+25/4/c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-
(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+
2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2)*c)^(1/2))*a*b^3-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(
-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2
+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c
-b^2)*c)^(1/2))*b^5+80*c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(
1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*a
rctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^4-116*c/(-(4*a*c-b^2)^3)^(1/2)
/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2
)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^3*b^2+55/(-(4*
a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3
+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^
(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))
*a^2*b^4-43/4/c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*
c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(
8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b
^2)^3)^(1/2)))^(1/2))*a*b^6+3/4/c^2/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c
-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*
a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^8-13/(4*a*c-b^2)*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+
(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^2*b+25/4/c/(4*a*c-b^2)*2^(1/2)/
((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan
```

```
(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4
*a*c-b^2)^3)^(1/2)))^(1/2))*a*b^3-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*
c-b^2)^3)^(1/2)))^(1/2))*b^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^3-3\,abc\right)x^3+\left(ab^2-2\,a^2c\right)x}{2\,(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3)x^2)}+\frac{-\int\frac{3\,ab^2-10\,a^2c+\left(3\,b^3-13\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{2\,(b^2c^2-4\,ac^3)}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b^3 - 3*a*b*c)*x^3 + (a*b^2 - 2*a^2*c)*x)/(a*b^2*c^2 - 4*a^
2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + 1/
2*integrate(-(3*a*b^2 - 10*a^2*c + (3*b^3 - 13*a*b*c)*x^2)/(c*x^4
 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3) + x/c^2
```

Fricas [A] time = 0.382643, size = 3856, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*x^5 + 2*(3*b^3 - 11*a*b*c)*x^3 + sqrt(1/
2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 42
0*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c
^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*
c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*
c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 250
0*a^5*c^3)*x + 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^
6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 - (3*
b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024
*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*
a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c
^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a
```

```
^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 -
64*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a
*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c
 + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*
b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^
11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 4
8*a^2*b^2*c^7 - 64*a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c +
 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 45
9*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c
^4 - 4000*a^5*c^5 - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 -
 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^6*c + 3
051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*
a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*
a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (b^6*c^5 - 12*a*b^4*c
^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a
*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*
c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a
^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqr
t(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (b^6*
c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*b^8 -
918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/
(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6
*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(-(189*a^2
*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*x + 1/2*
sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^
4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 + (3*b^9*c^5 - 52*a*b^7*
c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*sqrt((8
1*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13
)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3
- (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)
))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) - sqr
t(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^
2 - 4*a*b*c^3)*x^2)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2
- 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a
^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*
b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^1
2 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8))*log(-(189*a^2*b^6 - 1971*a^3*b^4*c + 5625*a^4*b^2*c^2 -
 2500*a^5*c^3)*x - 1/2*sqrt(1/2)*(27*b^10 - 459*a*b^8*c + 2961*a^
2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5 +
 (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 +
1024*a^4*b*c^9)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2
550*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2
*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b
^3*c^2 - 420*a^3*b*c^3 - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7
 - 64*a^3*c^8)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 25
50*a^3*b^2*c^3 + 625*a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*
b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
```

```
7 - 64*a^3*c^8))) + 2*(3*a*b^2 - 10*a^2*c)*x)/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

Sympy [A] time = 21.2623, size = 450, normalized size = 1.36

$$\frac{x^3\left(3abc-b^3\right)+x\left(2a^2c-ab^2\right)}{8a^2c^3-2ab^2c^2+x^4\left(8ac^4-2b^2c^3\right)+x^2\left(8abc^3-2b^3c^2\right)}$$
$$+\mathrm{RootSum}\left(t^4\left(1048576a^6c^{11}-1572864a^5b^2c^{10}+983040a^4b^4c^9-327680a^3b^6c^8+61440a^2b^8c^7-6144ab^{10}c^6+256b^{12}c^5\right)+\right.$$
$$+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (x**3*(3*a*b*c - b**3) + x*(2*a**2*c - a*b**2))/(8*a**2*c**3 - 2*
a*b**2*c**2 + x**4*(8*a*c**4 - 2*b**2*c**3) + x**2*(8*a*b*c**3 -
2*b**3*c**2)) + RootSum(_t**4*(1048576*a**6*c**11 - 1572864*a**5*
b**2*c**10 + 983040*a**4*b**4*c**9 - 327680*a**3*b**6*c**8 + 6144
0*a**2*b**8*c**7 - 6144*a*b**10*c**6 + 256*b**12*c**5) + _t**2*(4
30080*a**6*b*c**6 - 716800*a**5*b**3*c**5 + 483840*a**4*b**5*c**4
 - 170496*a**3*b**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**11*c
+ 144*b**13) + 10000*a**7*c**2 - 4200*a**6*b**2*c + 441*a**5*b**4
, Lambda(_t, _t*log(x + (65536*_t**3*a**4*b*c**9 - 61440*_t**3*a*
*3*b**3*c**8 + 21504*_t**3*a**2*b**5*c**7 - 3328*_t**3*a*b**7*c**
6 + 192*_t**3*b**9*c**5 - 8000*_t*a**5*c**5 + 36160*_t*a**4*b**2*
c**4 - 32476*_t*a**3*b**4*c**3 + 11592*_t*a**2*b**6*c**2 - 1836*_
t*a*b**8*c + 108*_t*b**10)/(2500*a**5*c**3 - 5625*a**4*b**2*c**2
+ 1971*a**3*b**4*c - 189*a**2*b**6)))) + x/c**2
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.868 $\int \frac{x^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=271

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bx}{2c(b^2-4ac)}$$

```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2
)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.18703, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\left(-\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{b(b^2-8ac)}{\sqrt{b^2-4ac}}-6ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}c^{3/2}(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3\left(2a+bx^2\right)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{bx}{2c(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a + b*x^2 + c*x^4)^2,x]
```


```
[Out] -(b*x)/(2*c*(b^2 - 4*a*c)) + (x^3*(2*a + b*x^2))/(2*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)) + ((b^2 - 6*a*c - (b*(b^2 - 8*a*c))/Sqrt[b^
2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + ((b^2 - 6*a*c + (b*(b^2 - 8*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2
)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 96.2992, size = 267, normalized size = 0.99

$$-\frac{bx}{2c(-4ac+b^2)}+\frac{x^3(2a+bx^2)}{2(-4ac+b^2)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-2abc+b(-6ac+b^2)+(-6ac+b^2)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}(-4ac+b^2)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(-abc+\frac{b(-6ac+b^2)}{2}-\frac{(-6ac+b^2)\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}(-4ac+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -b*x/(2*c*(-4*a*c + b**2)) + x**3*(2*a + b*x**2)/(2*(-4*a*c + b**
2)*(a + b*x**2 + c*x**4)) + sqrt(2)*(-2*a*b*c + b*(-6*a*c + b**2)
 + (-6*a*c + b**2)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sq
rt(b + sqrt(-4*a*c + b**2)))/(4*c**(3/2)*sqrt(b + sqrt(-4*a*c + b
**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(-a*b*c + b*(-6*a*c + b**
2)/2 - (-6*a*c + b**2)*sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c
)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a
*c + b**2))*(-4*a*c + b**2)**(3/2))
```

Mathematica [A] time = 0.988531, size = 282, normalized size = 1.04

$$\frac{-\frac{2\sqrt{c}x\left(a\left(b-2cx^2\right)+b^2x^2\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+8abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(b^2*x^2 + a*(b - 2*c*x^2)))/((b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) + (Sqrt[2]*(-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c]
 - 6*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - S
qrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (Sqrt[2]*(b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c*Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^
(3/2))
```

Maple [B] time = 0.116, size = 2158, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(2*a*c-b^2)/c/(4*a*c-b^2)*x^3+1/2*a*b/c/(4*a*c-b^2)*x)/(c*x
^4+b*x^2+a)+32*c^3/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)
/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)
*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c
^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^3*b-20*c^2/(-c^2*(
4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b
^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2
*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)
^3)^(1/2)))^(1/2))*a^2*b^3+4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-
b^2)*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(
1/2)))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b
^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a*b^5-1/
4/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*(4*
a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*
c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4
*a*c-b^2)^3)^(1/2)))^(1/2))*b^7+6*c/(4*a*c-b^2)*2^(1/2)/((4*a*c-b
^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(
-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a^2-5/2/(4*a*c-b^2)*2^(1/2)/((
4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2)*ar
ctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*(4*a*b*c^2-
b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*a*b^2+1/4/c/(4*a*c-b^2)
*2^(1/2)/((4*a*c-b^2)*(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)
))^(1/2)*arctan(1/2*(8*a*c^3-2*b^2*c^2)*x*2^(1/2)/c/((4*a*c-b^2)*
(4*a*b*c^2-b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4-32*c^3/(
-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+
(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*
c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3
)^(1/2))*(4*a*c-b^2))^(1/2))*a^3*b+20*c^2/(-c^2*(4*a*c-b^2)^3)^(1
/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(
1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/
2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^
(1/2))*a^2*b^3-4*c/(-c^2*(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)
/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2
)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c
+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*a*b^5+1/4/(-c^2*
(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2
*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2
*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2))^(1/2))*b^7+6*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c^2+
b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*arctanh(1/2*
(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-
```

```
b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*a^2-5/2/(4*a*c-b^2)*2^(1/2)/((
-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2)*a
rctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a*b*c^2+b^3*c+(-
c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*a*b^2+1/4/c/(4*a*c-
b^2)*2^(1/2)/((-4*a*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*
c-b^2))^(1/2)*arctanh(1/2*(-8*a*c^3+2*b^2*c^2)*x*2^(1/2)/c/((-4*a
*b*c^2+b^3*c+(-c^2*(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2))^(1/2))*b^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(b^2-2\,ac\right)x^3+abx}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{-\int\frac{\left(b^2-6\,ac\right)x^2+ab}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^3 + a*b*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2
*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) - 1/2*integrate(-((b^2
- 6*a*c)*x^2 + a*b)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

Fricas [A] time = 0.307, size = 3047, normalized size = 11.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(b^2 - 2*a*c)*x^3 + 2*a*b*x - sqrt(1/2)*((b^2*c^2 - 4*a*c
^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b
^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2
*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12
*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*
b^2*c + 324*a^3*c^2)*x + 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2
*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*
c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))
*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a*b^4*c^4
 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6
*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*
((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c
```

```
^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c^3 - 12*a
*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log((
5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17
*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 - (b^8*c^3 - 24*a*b^6*c
^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 -
 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^
8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (b^6*c
^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a
*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^
6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2
- (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((b^
4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2
*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 6
4*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^3*c^2)*x + 1/2*sq
rt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144*a^3*b*c^3 + (b^8
*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*b^2*c^6 + 768*a^4
*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^
7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a
^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)
*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 4
8*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^
2*c^5 - 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^
2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(b^5 - 15*a*b^3*
c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*
a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 4
8*a^2*b^2*c^5 - 64*a^3*c^6))*log((5*a*b^4 - 81*a^2*b^2*c + 324*a^
3*c^2)*x - 1/2*sqrt(1/2)*(b^7 - 17*a*b^5*c + 88*a^2*b^3*c^2 - 144
*a^3*b*c^3 + (b^8*c^3 - 24*a*b^6*c^4 + 192*a^2*b^4*c^5 - 640*a^3*
b^2*c^6 + 768*a^4*c^7)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^5 -
15*a*b^3*c + 60*a^2*b*c^2 - (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*
c^5 - 64*a^3*c^6)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(b^6*c^6 -
 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^
4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))))/((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

Sympy [A] time = 15.2759, size = 379, normalized size = 1.4

$$-\frac{-abx + x^3\left(2ac - b^2\right)}{8a^2c^2 - 2ab^2c + x^4\left(8ac^3 - 2b^2c^2\right) + x^2\left(8abc^2 - 2b^3c\right)}$$
$$+\mathrm{RootSum}\left(t^4\left(1048576a^6c^9 - 1572864a^5b^2c^8 + 983040a^4b^4c^7 - 327680a^3b^6c^6 + 61440a^2b^8c^5 - 6144ab^{10}c^4 + 256b^{12}c^3\right) + t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(-a*b*x + x**3*(2*a*c - b**2))/(8*a**2*c**2 - 2*a*b**2*c + x**4*
(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c)) + RootSu
m(_t**4*(1048576*a**6*c**9 - 1572864*a**5*b**2*c**8 + 983040*a**4
*b**4*c**7 - 327680*a**3*b**6*c**6 + 61440*a**2*b**8*c**5 - 6144*
a*b**10*c**4 + 256*b**12*c**3) + _t**2*(-61440*a**5*b*c**5 + 6144
0*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 4
32*a*b**9*c + 16*b**11) + 1296*a**5*c**2 - 360*a**4*b**2*c + 25*a
**3*b**4, Lambda(_t, _t*log(x + (49152*_t**3*a**4*c**7 - 40960*_t
**3*a**3*b**2*c**6 + 12288*_t**3*a**2*b**4*c**5 - 1536*_t**3*a*b*
*6*c**4 + 64*_t**3*b**8*c**3 - 1728*_t*a**3*b*c**3 + 656*_t*a**2*
b**3*c**2 - 88*_t*a*b**5*c + 4*_t*b**7)/(324*a**3*c**2 - 81*a**2*
b**2*c + 5*a*b**4))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.869 $\int \frac{x^4}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=237

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (
b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sq
rt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.757968, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{x\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(b-\frac{4ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b - (
b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) + ((b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sq
rt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 55.512, size = 218, normalized size = 0.92

$$\frac{x\left(2a+bx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(4ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt{2}\left(4ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] x*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)) + sqrt
(2)*(4*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x
/sqrt(b + sqrt(-4*a*c + b**2)))/(4*sqrt(c)*sqrt(b + sqrt(-4*a*c +
 b**2))*(-4*a*c + b**2)**(3/2)) - sqrt(2)*(4*a*c + b**2 - b*sqrt(
-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**
2)))/(4*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3
/2))
```

Mathematica [A] time = 0.767157, size = 235, normalized size = 0.99

$$\frac{1}{4}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}-4ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b\sqrt{b^2-4ac}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
2]*(-b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^2 + 4*a*c + b*Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

Maple [B] time = 0.055, size = 1641, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*b/(4*a*c-b^2)*x^3-a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+16/(-(4*
a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^3*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c
-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*
c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c
)^(1/2))*a^3-4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^2*2^(1/2)/((-
4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(
1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^
(1/2))*(4*a*c-b^2)*c)^(1/2))*a^2*b^2-c/(-(4*a*c-b^2)^3)^(1/2)/(4*
a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^
2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b
^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^4*a+1/4/(-(4*a
*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)
^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*
2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/
2))*b^6-c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/
2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)
/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b*a
+1/4/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(
4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4
*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^3-16/(
-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^3*2^(1/2)/((4*a*c-b^2)*c*(4*a
*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2
*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2))
)^(1/2))*a^3+4/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*c^2*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/
2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*
c-b^2)^3)^(1/2)))^(1/2))*a^2*b^2+c/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-
b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))
^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a
*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4*a-1/4/(-(4*a*c-b^2)^
3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c
-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4
*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^6-c/(4
*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1
/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c
*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b*a+1/4/(4*a*c-b^2)
*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/
2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c
-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^3 + 2\,ax}{2\,((b^2c - 4\,ac^2)x^4 + ab^2 - 4\,a^2c + (b^3 - 4\,abc)x^2)} + \frac{\int \frac{bx^2 - 2\,a}{cx^4 + bx^2 + a}\,dx}{2\,(b^2 - 4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^3 + 2*a*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b
^3 - 4*a*b*c)*x^2) + 1/2*integrate((b*x^2 - 2*a)/(c*x^4 + b*x^2 +
 a), x)/(b^2 - 4*a*c)
```

Fricas [A] time = 0.280536, size = 2252, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4
*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 +
 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2
*c^3 - 64*a^3*c^4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*
b^2*c + 16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^
3*c^5))*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^
4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3
- 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 4
8*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 6
4*a^3*c^4))*log((3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c +
16*a^2*c^2 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*
c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*
sqrt(-(b^3 + 12*a*b*c + (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + s
qrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*
c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^
2*c^3 - 64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4
- 64*a^3*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^
4))*log((3*b^2 + 4*a*c)*x + sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c
^2 - 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqr
```

```
t(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b
^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c
^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(
b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) - sqrt(1/2)
*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
sqrt(-(b^3 + 12*a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 -
64*a^3*c^4)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3
*c^5))/(b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))*log(
(3*b^2 + 4*a*c)*x - sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - 2*(
b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)/sqrt(b^6*c^
2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))*sqrt(-(b^3 + 12*
a*b*c - (b^6*c - 12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4)/sqrt
(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^6*c -
12*a*b^4*c^2 + 48*a^2*b^2*c^3 - 64*a^3*c^4))) + 4*a*x)/((b^2*c -
4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 11.2805, size = 294, normalized size = 1.24

$$-\frac{2ax+bx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}$$
$$+\operatorname{RootSum}\left(t^4\left(1048576a^6c^7-1572864a^5b^2c^6+983040a^4b^4c^5-327680a^3b^6c^4+61440a^2b^8c^3-6144ab^{10}c^2+256b^{12}c\right)+t^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(2*a*x + b*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*
c) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**6*c**7
- 1572864*a**5*b**2*c**6 + 983040*a**4*b**4*c**5 - 327680*a**3*b*
*6*c**4 + 61440*a**2*b**8*c**3 - 6144*a*b**10*c**2 + 256*b**12*c)
 + _t**2*(-12288*a**4*b*c**4 + 8192*a**3*b**3*c**3 - 1536*a**2*b*
*5*c**2 + 16*b**9) + 16*a**3*c**2 + 24*a**2*b**2*c + 9*a*b**4, La
mbda(_t, _t*log(x + (16384*_t**3*a**3*b*c**4 - 12288*_t**3*a**2*b
**3*c**3 + 3072*_t**3*a*b**5*c**2 - 256*_t**3*b**7*c + 64*_t*a**2
*c**2 - 128*_t*a*b**2*c - 4*_t*b**4)/(4*a*c + 3*b**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.870 $\int \frac{x^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.565784, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{x\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(2b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[
c]*(2*b - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 44.2488, size = 201, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{c}\left(b+\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(b-\frac{\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{x\left(b+2cx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c)
*x/sqrt(b + sqrt(-4*a*c + b**2)))/(sqrt(b + sqrt(-4*a*c + b**2))*
(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2
)/2)*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(sqrt(
b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) - x*(b + 2*c*x**
2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.855564, size = 222, normalized size = 1.

$$\frac{-bx-2cx^3}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}-2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (-(b*x) - 2*c*x^3)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[
c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*(b^2 - 4*a*c
)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A] time = 0.106, size = 342, normalized size = 1.6

$$\frac{x}{8ac-2b^2}\left(x^2+\frac{1}{2c}\sqrt{-4ac+b^2}+\frac{b}{2c}\right)^{-1}$$
$$+\frac{c\sqrt{2}b}{4ac-b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{c\sqrt{2}}{8ac-2b^2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\frac{x}{8ac-2b^2}\left(x^2+\frac{b}{2c}-\frac{1}{2c}\sqrt{-4ac+b^2}\right)^{-1}$$
$$+\frac{c\sqrt{2}b}{4ac-b^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{c\sqrt{2}}{8ac-2b^2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)+c/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2/(4*a*c-b^2)*x/(x^2+1/2*
b/c-1/2/c*(-4*a*c+b^2)^(1/2))+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,cx^3+bx}{2\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)}-\frac{\int\frac{2\,cx^2-b}{cx^4+bx^2+a}\,dx}{2\,(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^3 + b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (
b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*c*x^2 - b)/(c*x^4 + b*x^2
+ a), x)/(b^2 - 4*a*c)
```

Fricas [A] time = 0.283975, size = 2268, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*x^3 + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*
c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*
b^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^
2*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b
^5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^
2*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 4
8*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*
b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 6
4*a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
 + (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b
^4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2
*c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^
5 - 8*a*b^3*c + 16*a^2*b*c^2 - (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2
*c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
- 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c + (a*b^6 - 12*a^2*b^4*c + 48
*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64
*a^4*c^3))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c
+ (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^
4*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c +
48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*
c^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x + 1/2*sqrt(1/2)*(b^5
 - 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*
c^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*
a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^
2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*
a^4*c^3))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
 (b^3 - 4*a*b*c)*x^2)*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4
*c + 48*a^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 4
```

```
8*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c
^2 - 64*a^4*c^3))*log((3*b^2*c + 4*a*c^2)*x - 1/2*sqrt(1/2)*(b^5
- 8*a*b^3*c + 16*a^2*b*c^2 + (a*b^8 - 8*a^2*b^6*c + 128*a^4*b^2*c
^3 - 256*a^5*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
64*a^5*c^3))*sqrt(-(b^3 + 12*a*b*c - (a*b^6 - 12*a^2*b^4*c + 48*a
^3*b^2*c^2 - 64*a^4*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2
*c^2 - 64*a^5*c^3))/(a*b^6 - 12*a^2*b^4*c + 48*a^3*b^2*c^2 - 64*a
^4*c^3))) + 2*b*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^
3 - 4*a*b*c)*x^2)
```

Sympy [A] time = 11.8416, size = 298, normalized size = 1.35

$$\frac{bx+2cx^3}{8a^2c-2ab^2+x^4\left(8ac^2-2b^2c\right)+x^2\left(8abc-2b^3\right)}$$
$$+\text{RootSum}\left(t^4\left(1048576a^7c^6-1572864a^6b^2c^5+983040a^5b^4c^4-327680a^4b^6c^3+61440a^3b^8c^2-6144a^2b^{10}c+256ab^{12}\right)+t^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (b*x + 2*c*x**3)/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b**2*c
) + x**2*(8*a*b*c - 2*b**3)) + RootSum(_t**4*(1048576*a**7*c**6 -
 1572864*a**6*b**2*c**5 + 983040*a**5*b**4*c**4 - 327680*a**4*b**
6*c**3 + 61440*a**3*b**8*c**2 - 6144*a**2*b**10*c + 256*a*b**12)
+ _t**2*(-12288*a**4*b*c**4 + 8192*a**3*b**3*c**3 - 1536*a**2*b**
5*c**2 + 16*b**9) + 16*a**2*c**3 + 24*a*b**2*c**2 + 9*b**4*c, Lam
bda(_t, _t*log(x + (16384*_t**3*a**5*c**4 - 8192*_t**3*a**4*b**2*
c**3 + 512*_t**3*a**2*b**6*c - 64*_t**3*a*b**8 - 128*_t*a**2*b*c*
*2 - 16*_t*a*b**3*c - 4*_t*b**5)/(4*a*c**2 + 3*b**2*c))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.871 $\int \frac{1}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=252

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c
- b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi [A] time = 0.968768, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(-b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)) + (Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*
a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^2 - 12*a*c
- b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
 - 4*a*c]])
```

Rubi in Sympy [A] time = 64.2297, size = 230, normalized size = 0.91

$$-\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(-12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{x\left(-2ac+b^2+bcx^2\right)}{2a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(2)*sqrt(c)*(-12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(sq
rt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(4*a*sqrt(b + sqrt
(-4*a*c + b**2))*(-4*a*c + b**2)**(3/2)) + sqrt(2)*sqrt(c)*(-12*a
*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b
- sqrt(-4*a*c + b**2)))/(4*a*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*
c + b**2)**(3/2)) + x*(-2*a*c + b**2 + b*c*x**2)/(2*a*(-4*a*c + b
**2)*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.793494, size = 243, normalized size = 0.96

$$\frac{\frac{2x\left(-2ac+b^2+bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}-12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\sqrt{b^2-4ac}+12ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}}{4a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(-2),x]
```

```
[Out] ((2*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + (Sqrt[2]*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^2 + 12*
a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c
]]))/(4*a)
```

Maple [B] time = 0.072, size = 733, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)*b-c/(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2
*b/c)+1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2+1/2/c*(-4*a*c+b
^2)^(1/2)+1/2*b/c)*b^2-1/4*c/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^2-1/4/(4*a*c-b^2)/a*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b
^2)^(1/2))*b+c/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*x/(x^2+1/2*b/c-1/2/
c*(-4*a*c+b^2)^(1/2))-1/4/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a*x/(x^2
+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))*b^2+1/4*c/(4*a*c-b^2)/a*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2))*b-3*c^2/(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/4*c/(-4*a*c+b^2)^(1/2)/(4*a*c-b
^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^3 + (b^2 - 2\,ac)\,x}{2\,((ab^2c - 4\,a^2c^2)x^4 + a^2b^2 - 4\,a^3c + (ab^3 - 4\,a^2bc)x^2)} + \frac{\int \frac{bcx^2+b^2-6\,ac}{cx^4+bx^2+a}\,dx}{2\,(ab^2 - 4\,a^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^3 + (b^2 - 2*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*
b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate((b*c*x^2
 + b^2 - 6*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

Fricas [A] time = 0.316047, size = 3117, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 -
 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*
a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3
)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b
^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4
)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3
*b^2*c^3 + 864*a^4*c^4 - (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^
2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))
*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c
^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) - sqrt(1/2)*
((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b
*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 + (a^3*b^6 - 12*a
^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c +
81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3
)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((
5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1/2)*(b^8 -
23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 - (a
^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a
^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c +
60*a^2*b*c^2 + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c
 + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*
b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 -
 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a
^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c
 + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 +
324*a^2*c^4)*x + 1/2*sqrt(1/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^
2 - 672*a^3*b^2*c^3 + 864*a^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144
*a^5*b^5*c^2 - 448*a^6*b^3*c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*
b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64
*a^9*c^3)))*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 1
2*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c
 + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*
c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) -
 sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(b^5 - 15*a*b^3*c + 60*a^2*b*c^2 - (a^3
*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((b^4 - 18
*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 -
 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*
c^3))*log((5*b^4*c^2 - 81*a*b^2*c^3 + 324*a^2*c^4)*x - 1/2*sqrt(1
/2)*(b^8 - 23*a*b^6*c + 190*a^2*b^4*c^2 - 672*a^3*b^2*c^3 + 864*a
^4*c^4 + (a^3*b^9 - 20*a^4*b^7*c + 144*a^5*b^5*c^2 - 448*a^6*b^3*
c^3 + 512*a^7*b*c^4)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^5 - 15
*a*b^3*c + 60*a^2*b*c^2 - (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2 - 64*a^6*c^3)*sqrt((b^4 - 18*a*b^2*c + 81*a^2*c^2)/(a^6*b^6 - 1
2*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^
```

```
4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + 2*(b^2 - 2*a*c)*x)/((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2
)
```

Sympy [A] time = 14.9236, size = 394, normalized size = 1.56

$$-\frac{bcx^3 + x\left(-2ac + b^2\right)}{8a^3c - 2a^2b^2 + x^4\left(8a^2c^2 - 2ab^2c\right) + x^2\left(8a^2bc - 2ab^3\right)} + \text{RootSum}\left(t^4\left(1048576a^9c^6 - 1572864a^8b^2c^5 + 983040a^7b^4c^4 - 327680a^6b^6c^3 + 61440a^5b^8c^2 - 6144a^4b^{10}c + 256a^3b^{12}\right) + t\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -(b*c*x**3 + x*(-2*a*c + b**2))/(8*a**3*c - 2*a**2*b**2 + x**4*(8
*a**2*c**2 - 2*a*b**2*c) + x**2*(8*a**2*b*c - 2*a*b**3)) + RootSu
m(_t**4*(1048576*a**9*c**6 - 1572864*a**8*b**2*c**5 + 983040*a**7
*b**4*c**4 - 327680*a**6*b**6*c**3 + 61440*a**5*b**8*c**2 - 6144*
a**4*b**10*c + 256*a**3*b**12) + _t**2*(-61440*a**5*b*c**5 + 6144
0*a**4*b**3*c**4 - 24064*a**3*b**5*c**3 + 4608*a**2*b**7*c**2 - 4
32*a*b**9*c + 16*b**11) + 1296*a**2*c**5 - 360*a*b**2*c**4 + 25*b
**4*c**3, Lambda(_t, _t*log(x + (32768*_t**3*a**7*b*c**4 - 28672*
_t**3*a**6*b**3*c**3 + 9216*_t**3*a**5*b**5*c**2 - 1280*_t**3*a**
4*b**7*c + 64*_t**3*a**3*b**9 + 1728*_t*a**4*c**4 - 2304*_t*a**3*
b**2*c**3 + 740*_t*a**2*b**4*c**2 - 92*_t*a*b**6*c + 4*_t*b**8)/(
324*a**2*c**4 - 81*a*b**2*c**3 + 5*b**4*c**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.872 $\int \frac{1}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=308

$$-\frac{3b^2-10ac}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^
2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 -
16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3 - 16*a*b*
c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 2.86047, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{3b^2-10ac}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\left(3b^2-10ac\right)\sqrt{b^2-4ac}-16abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{2ax(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```


```
[Out] -(3*b^2 - 10*a*c)/(2*a^2*(b^2 - 4*a*c)*x) + (b^2 - 2*a*c + b*c*x^
2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c]*(3*b^3 -
16*a*b*c + (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*
c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(3*b^3 - 16*a*b*
c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 137.02, size = 282, normalized size = 0.92

$$\frac{-2ac+b^2+bcx^2}{2ax\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3-\left(-10ac+3b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{4a^2\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{\sqrt{2}\sqrt{c}\left(-16abc+3b^3+\left(-10ac+3b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{4a^2\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{-10ac+3b^2}{2a^2x\left(-4ac+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(2*a*x*(-4*a*c + b**2)*(a + b*x**2 + c
*x**4)) + sqrt(2)*sqrt(c)*(-16*a*b*c + 3*b**3 - (-10*a*c + 3*b**2
)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*
c + b**2)))/(4*a**2*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)
**(3/2)) - sqrt(2)*sqrt(c)*(-16*a*b*c + 3*b**3 + (-10*a*c + 3*b**
2)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a
*c + b**2)))/(4*a**2*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2
)**(3/2)) - (-10*a*c + 3*b**2)/(2*a**2*x*(-4*a*c + b**2))
```

Mathematica [A] time = 1.14168, size = 302, normalized size = 0.98

$$\frac{-\frac{2x\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc \cdots\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4a \cdots}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (-4/x - (2*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3 + 16*a*b*c
- 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^3 - 16*a*b
*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [B] time = 0.063, size = 2012, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3+1/2/a^2/(c*x^4+b*x^2+a)*
c/(4*a*c-b^2)*x^3*b^2-3/2/a/(c*x^4+b*x^2+a)*b*c/(4*a*c-b^2)*x+1/2
/a^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x-64*a/(-(4*a*c-b^2)^3)^(1/2
)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a
*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*
b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b*c^4+44/(-
(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-
b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c
)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)
^(1/2))*b^3*c^3-10/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/(
(-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2)*arctan
h(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3
)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^5*c^2+3/4/a^2*c/(-(4*a*c-b^2)^3)
^(1/2)/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))
*(4*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((
-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^7-10
/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*
c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b
*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*c^3+11/2/a/(
4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-
b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*a*b*c
+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*c^2*b^2-3/4/a^
2*c/(4*a*c-b^2)*2^(1/2)/((-4*a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4
*a*c-b^2)*c)^(1/2)*arctanh(1/2*(-8*a*c^2+2*b^2*c)*x*2^(1/2)/((-4*
a*b*c+b^3+(-(4*a*c-b^2)^3)^(1/2))*(4*a*c-b^2)*c)^(1/2))*b^4+64*a/
(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*
c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)
*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(
1/2))*b*c^4-44/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c
-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8
*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^
2)^3)^(1/2)))^(1/2))*b^3*c^3+10/a/(-(4*a*c-b^2)^3)^(1/2)/(4*a*c-b
^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^
(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)*c*(4*a*
b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^5*c^2-3/4/a^2*c/(-(4*a*
c-b^2)^3)^(1/2)/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(
-(4*a*c-b^2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1
/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b
^7-10/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^
2)^3)^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*
c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*c^3+11/2/a/
(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^
```

```
(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2)
*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*c^2*b^2-3/4/a^2*c
/(4*a*c-b^2)*2^(1/2)/((4*a*c-b^2)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)
^(1/2)))^(1/2)*arctan(1/2*(8*a*c^2-2*b^2*c)*x*2^(1/2)/((4*a*c-b^2
)*c*(4*a*b*c-b^3+(-(4*a*c-b^2)^3)^(1/2)))^(1/2))*b^4-1/a^2/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\,b^2c-10\,ac^2\right)x^4+2\,ab^2-8\,a^2c+\left(3\,b^3-11\,abc\right)x^2}{2\left((a^2b^2c-4\,a^3c^2)x^5+(a^2b^3-4\,a^3bc)x^3+(a^3b^2-4\,a^4c)x\right)}+\frac{-\int\frac{3\,b^3-13\,abc+\left(3\,b^2c-10\,ac^2\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(a^2b^2-4\,a^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="maxima")
```

```
[Out] -1/2*((3*b^2*c - 10*a*c^2)*x^4 + 2*a*b^2 - 8*a^2*c + (3*b^3 - 11*
a*b*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*
x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate(-(3*b^3 - 13*a*b*c +
 (3*b^2*c - 10*a*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a
^3*c)
```

Fricas [A] time = 0.391557, size = 3931, normalized size = 12.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(3*b^2*c - 10*a*c^2)*x^4 + 4*a*b^2 - 16*a^2*c + 2*(3*b^3
- 11*a*b*c)*x^2 - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b
^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4
*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^1
1 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^
4*b^3*c^4 - 5200*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7
*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*s
qrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^
13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*
```

b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sq
rt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)))
 + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)
*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2
*b^2*c^5 - 2500*a^3*c^6)*x - 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c
 + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 520
0*a^5*b*c^5 - (3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344
*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 9
18*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(
a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(
-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 + (a^5*b^
6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a
^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - sqrt(1/2)*((
a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2
 - 4*a^4*c)*x)*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420
*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^
3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c
^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6
4*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c
^3))*log(-(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500
*a^3*c^6)*x + 1/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7
*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (
3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 +
2176*a^9*b^2*c^4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a
*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*
c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 305
1*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4
*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a
^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*
sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3 - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((81*b^8
 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^
4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(189*
b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*x - 1
/2*sqrt(1/2)*(27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^
3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5 + (3*a^5*b^10 - 55
*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^
4 - 1280*a^10*c^5)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2
- 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*
a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(9*b^7 - 105*a*b^5*c + 385*a^
2*b^3*c^2 - 420*a^3*b*c^3 - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*

```
c^2 - 64*a^8*c^3)*sqrt((81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
 2550*a^3*b^2*c^3 + 625*a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a
^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2
*c^2 - 64*a^8*c^3))))/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4
*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

Sympy [A] time = 23.2677, size = 481, normalized size = 1.56

$$\text{RootSum}\Bigg(t^4\left(1048576a^{11}c^6-1572864a^{10}b^2c^5+983040a^9b^4c^4-327680a^8b^6c^3+61440a^7b^8c^2-6144a^6b^{10}c+256a^5b^{12}\right)+$$

$$-\frac{8a^2c-2ab^2+x^4\left(10ac^2-3b^2c\right)+x^2\left(11abc-3b^3\right)}{x^5\left(8a^3c^2-2a^2b^2c\right)+x^3\left(8a^3bc-2a^2b^3\right)+x\left(8a^4c-2a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] RootSum(_t**4*(1048576*a**11*c**6 - 1572864*a**10*b**2*c**5 + 983
040*a**9*b**4*c**4 - 327680*a**8*b**6*c**3 + 61440*a**7*b**8*c**2
 - 6144*a**6*b**10*c + 256*a**5*b**12) + _t**2*(430080*a**6*b*c**
6 - 716800*a**5*b**3*c**5 + 483840*a**4*b**5*c**4 - 170496*a**3*b
**7*c**3 + 33232*a**2*b**9*c**2 - 3408*a*b**11*c + 144*b**13) + 1
0000*a**2*c**7 - 4200*a*b**2*c**6 + 441*b**4*c**5, Lambda(_t, _t*
log(x + (-81920*_t**3*a**10*c**5 + 139264*_t**3*a**9*b**2*c**4 -
86016*_t**3*a**8*b**4*c**3 + 25088*_t**3*a**7*b**6*c**2 - 3520*_t
**3*a**6*b**8*c + 192*_t**3*a**5*b**10 - 27200*_t*a**5*b*c**5 + 6
0176*_t*a**4*b**3*c**4 - 42448*_t*a**3*b**5*c**3 + 13320*_t*a**2*
b**7*c**2 - 1944*_t*a*b**9*c + 108*_t*b**11)/(2500*a**3*c**6 - 56
25*a**2*b**2*c**5 + 1971*a*b**4*c**4 - 189*b**6*c**3)))) - (8*a**
2*c - 2*a*b**2 + x**4*(10*a*c**2 - 3*b**2*c) + x**2*(11*a*b*c - 3
*b**3))/(x**5*(8*a**3*c**2 - 2*a**2*b**2*c) + x**3*(8*a**3*b*c -
2*a**2*b**3) + x*(8*a**4*c - 2*a**3*b**2))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.873 $\int \frac{x^{11}}{(a+bx^2+cx^4)^3}\, dx$

Optimal. Leaf size=209

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^3}$$

```
[Out] -(b*(b^2 - 7*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^
2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a
*c) + b*(b^2 - 10*a*c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + Log[a + b*x^2 +
 c*x^4]/(4*c^3)
```

Rubi [A] time = 0.840838, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\left(b^2-4ac\right)^{5/2}}-\frac{bx^2\left(b^2-7ac\right)}{2c^2\left(b^2-4ac\right)^2}+\frac{x^4\left(bx^2\left(b^2-10ac\right)+a\left(b^2-16ac\right)\right)}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^8\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In]  Int[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b*(b^2 - 7*a*c)*x^2)/(2*c^2*(b^2 - 4*a*c)^2) + (x^8*(2*a + b*x^
2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^4*(a*(b^2 - 16*a
*c) + b*(b^2 - 10*a*c)*x^2))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (b*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/2)) + Log[a + b*x^2 +
 c*x^4]/(4*c^3)
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^3\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{x^8\left(2a+bx^2\right)}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^4\left(a\left(-16ac+b^2\right)+bx^2\left(-10ac+b^2\right)\right)}{4c\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}-\frac{\left(-7ac+b^2\right)\int^{x^2}b\,dx}{2c^2\left(-4ac+b^2\right)^2}+\frac{\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] b*(30*a**2*c**2 - 10*a*b**2*c + b**4)*atanh((b + 2*c*x**2)/sqrt(-
4*a*c + b**2))/(2*c**3*(-4*a*c + b**2)**(5/2)) + x**8*(2*a + b*x*
*2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)**2) + x**4*(a*(-16*a
*c + b**2) + b*x**2*(-10*a*c + b**2))/(4*c*(-4*a*c + b**2)**2*(a
+ b*x**2 + c*x**4)) - (-7*a*c + b**2)*Integral(b, (x, x**2))/(2*c
**2*(-4*a*c + b**2)**2) + log(a + b*x**2 + c*x**4)/(4*c**3)
```

Mathematica [A] time = 0.584226, size = 244, normalized size = 1.17

$$\frac{-\frac{2bc\left(30a^2c^2-10ab^2c+b^4\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}+\frac{2a^3c^2+a^2bc\left(5cx^2-4b\right)+ab^3\left(b-5cx^2\right)+b^5x^2}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{32a^3c^3-39a^2b^2c^2+50a^2bc^3x^2+11ab^4c-30ab^3c^2x^2-b^6+4b^5c}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}}{4c^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^11/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-b^6 + 11*a*b^4*c - 39*a^2*b^2*c^2 + 32*a^3*c^3 + 4*b^5*c*x^2 -
 30*a*b^3*c^2*x^2 + 50*a^2*b*c^3*x^2)/((b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)) + (2*a^3*c^2 + b^5*x^2 + a*b^3*(b - 5*c*x^2) + a^2*b*c
*(-4*b + 5*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*b*c
*(b^4 - 10*a*b^2*c + 30*a^2*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*Log[a + b*x^2 + c*x^4])/(4*c^4
)
```

Maple [B] time = 0.033, size = 819, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^11/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(1/c^2*b*(25*a^2*c^2-15*a*b^2*c+2*b^4)/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6+1/2*(32*a^3*c^3+11*a^2*b^2*c^2-19*a*b^4*c+3*b^6)/c^3/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*b*(31*a^2*c^2-22*a*b^2*c+3*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4)/c^3*x^2+3/2*a^2*(8*a^2*c^2-7*a*b^2*c+b^4)
/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/4/c^3*ln(c^2
*(16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))-15/(1024*a^5*c^9-128
0*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c
^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*x^2+c^2*(16*a^
2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^
4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))*a^2*b*c+5/(10
24*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*
b^8*c^5-b^10*c^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*b^2*c+b^4)*c^3*
x^2+c^2*(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9-1280*a^4*b^2*
c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2))
*a*b^3-1/2/(1024*a^5*c^9-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2
*b^6*c^6+20*a*b^8*c^5-b^10*c^4)^(1/2)*arctan((2*(16*a^2*c^2-8*a*b
^2*c+b^4)*c^3*x^2+c^2*(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^9
-1280*a^4*b^2*c^8+640*a^3*b^4*c^7-160*a^2*b^6*c^6+20*a*b^8*c^5-b^
10*c^4)^(1/2))*b^5/c
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298331, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10
*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*
c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4
+ 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*log((b^3 - 4*a*b
```

```
*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a
*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*(2*b^5*c - 15*a*
b^3*c^2 + 25*a^2*b*c^3)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c
^2 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 + 32*a^3*c^3)*x^4 + 2*(
3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x^2 + ((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^
6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^
3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^
4 + b*x^2 + a))*sqrt(b^2 - 4*a*c))/((a^2*b^4*c^3 - 8*a^3*b^2*c^4
+ 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^8 + 2*(b^5*
c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^6 + (b^6*c^3 - 6*a*b^4*c^4 +
32*a^3*c^6)*x^4 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*x^
2)*sqrt(b^2 - 4*a*c)), -1/4*(2*((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*
b*c^4)*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 1
0*a*b^4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3
*c^2 + 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c
^2)*x^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c))
- (2*(2*b^5*c - 15*a*b^3*c^2 + 25*a^2*b*c^3)*x^6 + 3*a^2*b^4 - 21
*a^3*b^2*c + 24*a^4*c^2 + (3*b^6 - 19*a*b^4*c + 11*a^2*b^2*c^2 +
32*a^3*c^3)*x^4 + 2*(3*a*b^5 - 22*a^2*b^3*c + 31*a^3*b*c^2)*x^2 +
 ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c
^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^
6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a))*sqrt(-b^2 + 4*a*c))/((a^2*b^
4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 16*
a^2*c^7)*x^8 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^6 + (b^
6*c^3 - 6*a*b^4*c^4 + 32*a^3*c^6)*x^4 + 2*(a*b^5*c^3 - 8*a^2*b^3*
c^4 + 16*a^3*b*c^5)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 54.5537, size = 1520, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**11/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/
(4*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**
3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(4*c**3))*log(
x**2 + (-128*a**3*c**5*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2
 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c
**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**
10)) + 1/(4*c**3)) + 32*a**3*c**2 + 96*a**2*b**2*c**4*(-b*sqrt(-(
4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(10
24*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**
2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(4*c**3)) - 9*a**2*b**2*c
 - 24*a*b**4*c**3*(-b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10
*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 +
```

```
 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10))
+ 1/(4*c**3)) + a*b**4 + 2*b**6*c**2*(-b*sqrt(-(4*a*c - b**2)**5)
*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 12
80*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*
a*b**8*c - b**10)) + 1/(4*c**3)))/(30*a**2*b*c**2 - 10*a*b**3*c +
 b**5)) + (b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c
 + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3
*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(4*c*
*3))*log(x**2 + (-128*a**3*c**5*(b*sqrt(-(4*a*c - b**2)**5)*(30*a
**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*a**
4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8
*c - b**10)) + 1/(4*c**3)) + 32*a**3*c**2 + 96*a**2*b**2*c**4*(b*
sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c
**3*(1024*a**5*c**5 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 -
160*a**2*b**6*c**2 + 20*a*b**8*c - b**10)) + 1/(4*c**3)) - 9*a**2
*b**2*c - 24*a*b**4*c**3*(b*sqrt(-(4*a*c - b**2)**5)*(30*a**2*c**
2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5 - 1280*a**4*b**2*
c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2 + 20*a*b**8*c - b*
*10)) + 1/(4*c**3)) + a*b**4 + 2*b**6*c**2*(b*sqrt(-(4*a*c - b**2
)**5)*(30*a**2*c**2 - 10*a*b**2*c + b**4)/(4*c**3*(1024*a**5*c**5
 - 1280*a**4*b**2*c**4 + 640*a**3*b**4*c**3 - 160*a**2*b**6*c**2
+ 20*a*b**8*c - b**10)) + 1/(4*c**3)))/(30*a**2*b*c**2 - 10*a*b**
3*c + b**5)) + (24*a**4*c**2 - 21*a**3*b**2*c + 3*a**2*b**4 + x**
6*(50*a**2*b*c**3 - 30*a*b**3*c**2 + 4*b**5*c) + x**4*(32*a**3*c*
*3 + 11*a**2*b**2*c**2 - 19*a*b**4*c + 3*b**6) + x**2*(62*a**3*b*
c**2 - 44*a**2*b**3*c + 6*a*b**5))/(64*a**4*c**5 - 32*a**3*b**2*c
**4 + 4*a**2*b**4*c**3 + x**8*(64*a**2*c**7 - 32*a*b**2*c**6 + 4*
b**4*c**5) + x**6*(128*a**2*b*c**6 - 64*a*b**3*c**5 + 8*b**5*c**4
) + x**4*(128*a**3*c**6 - 24*a*b**4*c**4 + 4*b**6*c**3) + x**2*(1
28*a**3*b*c**5 - 64*a**2*b**3*c**4 + 8*a*b**5*c**3))
```

GIAC/XCAS [A] time = 15.6605, size = 413, normalized size = 1.98

$$-\frac{\left(b^{5}-10\,ab^{3}c+30\,a^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}}-\frac{3\,b^{4}c^{2}x^{8}-24\,ab^{2}c^{3}x^{8}+48\,a^{2}c^{4}x^{8}-2\,b^{5}cx^{6}+12\,ab^{3}c^{2}x^{6}-4\,a^{2}bc^{3}x^{6}-3\,b^{6}x^{4}+20\,ab^{4}cx^{4}-22\,a^{2}b^{2}c^{2}x^{4}+32\,a^{3}c^{3}x^{4}-6}{8\,\left(b^{4}c^{3}-8\,ab^{2}c^{4}+16\,a^{2}c^{5}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}+\frac{\ln\left(cx^{4}+bx^{2}+a\right)}{4\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^11/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 +
4*a*c)) - 1/8*(3*b^4*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8
- 2*b^5*c*x^6 + 12*a*b^3*c^2*x^6 - 4*a^2*b*c^3*x^6 - 3*b^6*x^4 +
```

```
20*a*b^4*c*x^4 - 22*a^2*b^2*c^2*x^4 + 32*a^3*c^3*x^4 - 6*a*b^5*x^
2 + 40*a^2*b^3*c*x^2 - 28*a^3*b*c^2*x^2 - 3*a^2*b^4 + 18*a^3*b^2*
c)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2) +
 1/4*ln(c*x^4 + b*x^2 + a)/c^3
```

3.874 $\int \frac{x^9}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=121

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3ax^2\left(2a+bx^2\right)}{2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{x^6\left(2a+bx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*
a*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6
*a^2*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2
)
```

Rubi [A] time = 0.225365, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$-\frac{6a^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3ax^2\left(2a+bx^2\right)}{2(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{x^6\left(2a+bx^2\right)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^6*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*
a*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6
*a^2*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2
)
```

Rubi in Sympy [A] time = 26.4906, size = 112, normalized size = 0.93

$$-\frac{6a^2\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}-\frac{3ax^2\left(2a+bx^2\right)}{2(-4ac+b^2)^2(a+bx^2+cx^4)}+\frac{x^6\left(2a+bx^2\right)}{4(-4ac+b^2)(a+bx^2+cx^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -6*a**2*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)
**(5/2) - 3*a*x**2*(2*a + b*x**2)/(2*(-4*a*c + b**2)**2*(a + b*x*
*2 + c*x**4)) + x**6*(2*a + b*x**2)/(4*(-4*a*c + b**2)*(a + b*x**
```

```
2 + c*x**4)**2)
```

Mathematica [A] time = 0.303986, size = 194, normalized size = 1.6

$$\frac{1}{4}\left(\frac{24a^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{a^2c\left(2cx^2-3b\right)+ab^2\left(b-4cx^2\right)+b^4x^2}{c^3\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{22a^2bc^2-20a^2c^3x^2-8ab^3c+16ab^2c^2x^2+b^5-2b^4cx^2}{c^3\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5 - 8*a*b^3*c + 22*a^2*b*c^2 - 2*b^4*c*x^2 + 16*a*b^2*c^2*x^2
 - 20*a^2*c^3*x^2)/(c^3*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b
^4*x^2 + a*b^2*(b - 4*c*x^2) + a^2*c*(-3*b + 2*c*x^2))/(c^3*(-b^2
 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (24*a^2*ArcTan[(b + 2*c*x^2)/S
qrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] time = 0.021, size = 267, normalized size = 2.2

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-\frac{\left(10\,a^2c^2-8\,ab^2c+b^4\right)x^6}{c\left(16\,a^2c^2-8\,ab^2c+b^4\right)}+\frac{b\left(2\,a^2c^2+8\,ab^2c-b^4\right)x^4}{2\,c^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)}-\frac{a\left(6\,a^2c^2-10\,ab^2c+b^4\right)x^2}{c^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)}+\frac{a^2b\,(10}{2\,c^2\,(16\,a^2c^2}\right.$$
$$+6\,\frac{a^2}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-1/c*(10*a^2*c^2-8*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x
^6+1/2*b*(2*a^2*c^2+8*a*b^2*c-b^4)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)
*x^4-a*(6*a^2*c^2-10*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*
x^2+1/2*a^2*b*(10*a*c-b^2)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4
+b*x^2+a)^2+6*a^2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*ar
ctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.27215, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c
^2 + (a^2*b^2*c^2 + 2*a^3*c^3)*x^4)*log(-(b^3 - 4*a*b*c + 2*(b^2*
c - 4*a*c^2)*x^2 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (2*(b^4*c - 8*a*b^2*c^2 + 10*a^
2*c^3)*x^6 + a^2*b^3 - 10*a^3*b*c + (b^5 - 8*a*b^3*c - 2*a^2*b*c^
2)*x^4 + 2*(a*b^4 - 10*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(b^2 - 4*a
*c))/(((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8
*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c
^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2
 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(b^2 - 4*a*c)), 1/4*(24
*(a^2*c^4*x^8 + 2*a^2*b*c^3*x^6 + 2*a^3*b*c^2*x^2 + a^4*c^2 + (a^
2*b^2*c^2 + 2*a^3*c^3)*x^4)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a
*c)/(b^2 - 4*a*c)) - (2*(b^4*c - 8*a*b^2*c^2 + 10*a^2*c^3)*x^6 +
a^2*b^3 - 10*a^3*b*c + (b^5 - 8*a*b^3*c - 2*a^2*b*c^2)*x^4 + 2*(a
*b^4 - 10*a^2*b^2*c + 6*a^3*c^2)*x^2)*sqrt(-b^2 + 4*a*c))/(((b^4*
c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b
^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3
*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 39.2561, size = 554, normalized size = 4.58

$$-3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+\cdots}{6a^2c}\right)$$
$$+3a^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^5c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^4b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^3b^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3a^2b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}+3a\cdots}{6a^2c}\right)$$
$$-\frac{-10a^3bc+a^2b^3+x^6\left(20a^2c^3-16ab^2c^2+2b^4c\right)+x^4\left(-2a^2bc^2-8ab^3c+b^5\right)+x^2\left(12a^3c^2-\cdots\right.}{64a^4c^4-32a^3b^2c^3+4a^2b^4c^2+x^8\left(64a^2c^6-32ab^2c^5+4b^4c^4\right)+x^6\left(128a^2bc^5-64ab^3c^4+8b^5c^3\right)+x^4\left(128a^3c^5-24ab^4c^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*a**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**5*c**3*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**
2)**5) - 36*a**3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b**6*
sqrt(-1/(4*a*c - b**2)**5) + 3*a**2*b)/(6*a**2*c)) + 3*a**2*sqrt(
-1/(4*a*c - b**2)**5)*log(x**2 + (192*a**5*c**3*sqrt(-1/(4*a*c -
b**2)**5) - 144*a**4*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5) + 36*a*
*3*b**4*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a**2*b**6*sqrt(-1/(4*a*c
 - b**2)**5) + 3*a**2*b)/(6*a**2*c)) - (-10*a**3*b*c + a**2*b**3
+ x**6*(20*a**2*c**3 - 16*a*b**2*c**2 + 2*b**4*c) + x**4*(-2*a**2
*b*c**2 - 8*a*b**3*c + b**5) + x**2*(12*a**3*c**2 - 20*a**2*b**2*
c + 2*a*b**4))/(64*a**4*c**4 - 32*a**3*b**2*c**3 + 4*a**2*b**4*c*
*2 + x**8*(64*a**2*c**6 - 32*a*b**2*c**5 + 4*b**4*c**4) + x**6*(1
28*a**2*b*c**5 - 64*a*b**3*c**4 + 8*b**5*c**3) + x**4*(128*a**3*c
**5 - 24*a*b**4*c**3 + 4*b**6*c**2) + x**2*(128*a**3*b*c**4 - 64*
a**2*b**3*c**3 + 8*a*b**5*c**2))
```

GIAC/XCAS [A] time = 15.6508, size = 286, normalized size = 2.36

$$\frac{6\,a^2\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{2\,b^4cx^6-16\,ab^2c^2x^6+20\,a^2c^3x^6+b^5x^4-8\,ab^3cx^4-2\,a^2bc^2x^4+2\,ab^4x^2-20\,a^2b^2cx^2+12\,a^3c^2x^2+a^2b^3-10\,a^3bc}{4\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^4+bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*b^4*c*x^6 - 16*a*b^2*c
```

```
^2*x^6 + 20*a^2*c^3*x^6 + b^5*x^4 - 8*a*b^3*c*x^4 - 2*a^2*b*c^2*x
^4 + 2*a*b^4*x^2 - 20*a^2*b^2*c*x^2 + 12*a^3*c^2*x^2 + a^2*b^3 -
10*a^3*b*c)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2
+ a)^2)
```

3.875 $\int \frac{x^7}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=119

$$\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] -(x^6*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3
*b*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
3*a*b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/
2)
```

Rubi [A] time = 0.19605, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{3ab\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3bx^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^6(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^6*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3
*b*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
3*a*b*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/
2)
```

Rubi in Sympy [A] time = 26.0316, size = 112, normalized size = 0.94

$$\frac{3ab\,\mathrm{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}+\frac{3bx^2(2a+bx^2)}{4(-4ac+b^2)^2(a+bx^2+cx^4)}-\frac{x^6(b+2cx^2)}{4(-4ac+b^2)(a+bx^2+cx^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*b*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**
(5/2) + 3*b*x**2*(2*a + b*x**2)/(4*(-4*a*c + b**2)**2*(a + b*x**2
 + c*x**4)) - x**6*(b + 2*c*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2
```

```
+ c*x**4)**2)
```

Mathematica [A] time = 0.372837, size = 137, normalized size = 1.15

$$-\frac{8a^3c+a^2\left(b^2+10bcx^2+16c^2x^4\right)+abx^2\left(2b^2+bcx^2+6c^2x^4\right)+b^4x^4}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}-\frac{3ab\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(8*a^3*c + b^4*x^4 + a*b*x^2*(2*b^2 + b*c*x^2 + 6*c^2*x^4) + a^2
*(b^2 + 10*b*c*x^2 + 16*c^2*x^4))/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)^2) - (3*a*b*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(
-b^2 + 4*a*c)^(5/2)
```

Maple [B] time = 0.02, size = 230, normalized size = 1.9

$$\frac{1}{2\left(cx^4+bx^2+a\right)^2}\left(-3\frac{x^6abc}{16a^2c^2-8ab^2c+b^4}-\frac{\left(16a^2c^2+ab^2c+b^4\right)x^4}{2c\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{\left(5ac+b^2\right)abx^2}{c\left(16a^2c^2-8ab^2c+b^4\right)}-\frac{a^2\left(8ac+b^2\right)}{2c\left(16a^2c^2-8ab^2c\right.}\right.$$
$$-3\frac{ab}{\left(16a^2c^2-8ab^2c+b^4\right)\sqrt{4ac-b^2}}\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(-3*a*b*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-1/2*(16*a^2*c^2+a*b^
2*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*a*c+b^2)*a*b/c/(16*a
^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a^2*(8*a*c+b^2)/c/(16*a^2*c^2-8*a*b
^2*c+b^4))/(c*x^4+b*x^2+a)^2-3*a*b/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269551, size = 1, normalized size = 0.01

```
[6 (abc^3x^8 + 2 ab^2c^2x^6 + 2 a^2b^2cx^2 + a^3bc + (ab^3c + 2 a^2bc^2) x^4) log((b^3-4 abc+2 (b^2c-4 ac^2) x^2+(2 c^2x^4+2 bcx^2+b^2-2 ac) sqrt(b^2-4 ac))/(cx^4+bx^2+a)) -
 / 4 ((b^4c^3 - 8 ab^2c^4 + 16 a^2c^5)x^8 + a^2b^4c - 8 a^3b^2c^2 + 16 a^4c^3 + 2 (b^5c^2 - 8 ab^3c^3 + 16 a^2bc^4)x^6 + (b^6c - 6

- 12 (abc^3x^8 + 2 ab^2c^2x^6 + 2 a^2b^2cx^2 + a^3bc + (ab^3c + 2 a^2bc^2) x^4) arctan(-((2 cx^2+b) sqrt(-b^2+4 ac))/(b^2-4 ac)) + (6 abc^2x^6 + (b^4 + ab^2c + 1
 / 4 ((b^4c^3 - 8 ab^2c^4 + 16 a^2c^5)x^8 + a^2b^4c - 8 a^3b^2c^2 + 16 a^4c^3 + 2 (b^5c^2 - 8 ab^3c^3 + 16 a^2bc^4)x^6 + (b^6c - 6 ab^4c^2 + 32 a^3
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b*
c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*log((b^3 - 4*a*b*c + 2*(b^2*c -
4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - (6*a*b*c^2*x^6 + (b^4 + a*b^2*c + 1
6*a^2*c^2)*x^4 + a^2*b^2 + 8*a^3*c + 2*(a*b^3 + 5*a^2*b*c)*x^2)*s
qrt(b^2 - 4*a*c))/(((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^
2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 +
 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(
a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)),
-1/4*(12*(a*b*c^3*x^8 + 2*a*b^2*c^2*x^6 + 2*a^2*b^2*c*x^2 + a^3*b
*c + (a*b^3*c + 2*a^2*b*c^2)*x^4)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + (6*a*b*c^2*x^6 + (b^4 + a*b^2*c + 16*a
^2*c^2)*x^4 + a^2*b^2 + 8*a^3*c + 2*(a*b^3 + 5*a^2*b*c)*x^2)*sqrt
(-b^2 + 4*a*c))/(((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*
b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 1
6*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*
b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 35.8604, size = 520, normalized size = 4.37

$$\frac{3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^2}{6abc}\right)}{2} - \frac{3ab\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^4bc^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^3b^3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+36a^2b^5c\sqrt{-\frac{1}{(4ac-b^2)^5}}-3ab^7\sqrt{-\frac{1}{(4ac-b^2)^5}}+3ab^2}{6abc}\right)}{2} - \frac{8a^3c+a^2b^2+6abc^2x^6+x^4\left(16a^2c^2+ab^2c+b^4\right)+x^2\left(10a^2bc+2ab^3\right)}{64a^4c^3-32a^3b^2c^2+4a^2b^4c+x^8\left(64a^2c^5-32ab^2c^4+4b^4c^3\right)+x^6\left(128a^2bc^4-64ab^3c^3+8b^5c^2\right)+x^4\left(128a^3c^4-24ab^4c^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*a*b*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**4*b*c**3*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**
2)**5) - 36*a**2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) + 3*a*b**7*sqr
t(-1/(4*a*c - b**2)**5) + 3*a*b**2)/(6*a*b*c))/2 - 3*a*b*sqrt(-1/
(4*a*c - b**2)**5)*log(x**2 + (192*a**4*b*c**3*sqrt(-1/(4*a*c - b
**2)**5) - 144*a**3*b**3*c**2*sqrt(-1/(4*a*c - b**2)**5) + 36*a**
2*b**5*c*sqrt(-1/(4*a*c - b**2)**5) - 3*a*b**7*sqrt(-1/(4*a*c - b
**2)**5) + 3*a*b**2)/(6*a*b*c))/2 - (8*a**3*c + a**2*b**2 + 6*a*b
*c**2*x**6 + x**4*(16*a**2*c**2 + a*b**2*c + b**4) + x**2*(10*a**
2*b*c + 2*a*b**3))/(64*a**4*c**3 - 32*a**3*b**2*c**2 + 4*a**2*b**
4*c + x**8*(64*a**2*c**5 - 32*a*b**2*c**4 + 4*b**4*c**3) + x**6*(
128*a**2*b*c**4 - 64*a*b**3*c**3 + 8*b**5*c**2) + x**4*(128*a**3*
c**4 - 24*a*b**4*c**2 + 4*b**6*c) + x**2*(128*a**3*b*c**3 - 64*a*
*2*b**3*c**2 + 8*a*b**5*c))
```

GIAC/XCAS [A] time = 15.6391, size = 231, normalized size = 1.94

$$-\frac{3\,ab\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}} - \frac{6\,abc^2x^6+b^4x^4+ab^2cx^4+16\,a^2c^2x^4+2\,ab^3x^2+10\,a^2bcx^2+a^2b^2+8\,a^3c}{4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^4+bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -3*a*b*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*a*b*c^2*x^6 + b^4*x^4
```

```
 + a*b^2*c*x^4 + 16*a^2*c^2*x^4 + 2*a*b^3*x^2 + 10*a^2*b*c*x^2 +
a^2*b^2 + 8*a^3*c)/((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b
*x^2 + a)^2)
```

3.876 $\int \frac{x^5}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=130

$$-\frac{(2ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^2(2ac+b^2)+3ab}{2(b^2-4ac)^2(a+bx^2+cx^4)}$$

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*
a*b + (b^2 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
- ((b^2 + 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 -
 4*a*c)^(5/2)
```

Rubi [A] time = 0.250301, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{(2ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{x^2(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{x^2(2ac+b^2)+3ab}{2(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*
a*b + (b^2 + 2*a*c)*x^2)/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
- ((b^2 + 2*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 -
 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 29.8488, size = 119, normalized size = 0.92

$$\frac{x^2(2a+bx^2)}{4(-4ac+b^2)(a+bx^2+cx^4)^2}+\frac{6ab+x^2(4ac+2b^2)}{4(-4ac+b^2)^2(a+bx^2+cx^4)}-\frac{(2ac+b^2)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] x**2*(2*a + b*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)**2)
+ (6*a*b + x**2*(4*a*c + 2*b**2))/(4*(-4*a*c + b**2)**2*(a + b*x*
*2 + c*x**4)) - (2*a*c + b**2)*atanh((b + 2*c*x**2)/sqrt(-4*a*c +
```

```
b**2))/(-4*a*c + b**2)**(5/2)
```

Mathematica [A] time = 0.259719, size = 145, normalized size = 1.12

$$\frac{1}{4}\left(\frac{4\left(2ac+b^2\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{5/2}}+\frac{\left(2ac+b^2\right)\left(b+2cx^2\right)}{c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{a\left(b-2cx^2\right)+b^2x^2}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 + 2*a*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (b^2*x^2 + a*(b - 2*c*x^2))/(c*(-b^2 + 4*a*c)*(a + b*x^2
+ c*x^4)^2) + (4*(b^2 + 2*a*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4
*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] time = 0.019, size = 270, normalized size = 2.1

$$\frac{1}{2\,(cx^4+bx^2+a)^2}\left(\frac{c\left(2\,ac+b^2\right)x^6}{16\,a^2c^2-8\,ab^2c+b^4}+\frac{3\,b\left(2\,ac+b^2\right)x^4}{32\,a^2c^2-16\,ab^2c+2\,b^4}-\frac{a\left(2\,ac-5\,b^2\right)x^2}{16\,a^2c^2-8\,ab^2c+b^4}+3\,\frac{a^2b}{16\,a^2c^2-8\,ab^2c+b^4}\right)$$
$$+2\,\frac{ac}{(16\,a^2c^2-8\,ab^2c+b^4)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$
$$+\frac{b^2}{16\,a^2c^2-8\,ab^2c+b^4}\arctan\left((2\,cx^2+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/2*(c*(2*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+3/2*b*(2*a*c+b^
2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(2*a*c-5*b^2)/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^2+3*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+
a)^2+2/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*a*c+1/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.272466, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*c + 2*a*b*c^2)*x^6 + (b
^4 + 4*a*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2 + 2*a^3*c + 2*(a*b^3 +
2*a^2*b*c)*x^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 - (
2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 +
b*x^2 + a)) + (2*(b^2*c + 2*a*c^2)*x^6 + 3*(b^3 + 2*a*b*c)*x^4 +
6*a^2*b + 2*(5*a*b^2 - 2*a^2*c)*x^2)*sqrt(b^2 - 4*a*c))/(((b^4*c^
2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a
^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b
^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x
^2)*sqrt(b^2 - 4*a*c)), 1/4*(4*((b^2*c^2 + 2*a*c^3)*x^8 + 2*(b^3*
c + 2*a*b*c^2)*x^6 + (b^4 + 4*a*b^2*c + 4*a^2*c^2)*x^4 + a^2*b^2
+ 2*a^3*c + 2*(a*b^3 + 2*a^2*b*c)*x^2)*arctan(-(2*c*x^2 + b)*sqrt
(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*(b^2*c + 2*a*c^2)*x^6 + 3*(b^3
 + 2*a*b*c)*x^4 + 6*a^2*b + 2*(5*a*b^2 - 2*a^2*c)*x^2)*sqrt(-b^2
+ 4*a*c))/(((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*
c + 16*a^3*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 35.7218, size = 580, normalized size = 4.46

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(2ac+b^2\right)\log\left(x^2+\frac{-64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+2abc+b^6\sqrt{\cdots}}{4ac^2+2b^2c}\cdots\right.}{2}$$

$$+\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\left(2ac+b^2\right)\log\left(x^2+\frac{64a^3c^3\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)-48a^2b^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+12ab^4c\sqrt{-\frac{1}{\left(4ac-b^2\right)^5}}\left(2ac+b^2\right)+2abc-b^6\sqrt{\cdots}}{4ac^2+2b^2c}\cdots\right.}{2}$$

$$+\frac{6a^2b+x^6\left(4ac^2+2b^2c\right)+x^4\left(6abc+3b^3\right)+x^2\left(-4a^2c+10ab^2\right)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (-64*a**3*c
**3*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 48*a**2*b**2*c**2
*sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 12*a*b**4*c*sqrt(-1/
(4*a*c - b**2)**5)*(2*a*c + b**2) + 2*a*b*c + b**6*sqrt(-1/(4*a*c
 - b**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + sq
rt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2)*log(x**2 + (64*a**3*c**3*
sqrt(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) - 48*a**2*b**2*c**2*sqr
t(-1/(4*a*c - b**2)**5)*(2*a*c + b**2) + 12*a*b**4*c*sqrt(-1/(4*a
*c - b**2)**5)*(2*a*c + b**2) + 2*a*b*c - b**6*sqrt(-1/(4*a*c - b
**2)**5)*(2*a*c + b**2) + b**3)/(4*a*c**2 + 2*b**2*c))/2 + (6*a**
2*b + x**6*(4*a*c**2 + 2*b**2*c) + x**4*(6*a*b*c + 3*b**3) + x**2
*(-4*a**2*c + 10*a*b**2))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2
*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6
*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c
**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**
3*c + 8*a*b**5))
```

GIAC/XCAS [A] time = 15.6234, size = 217, normalized size = 1.67

$$\frac{\left(b^2+2\,ac\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\sqrt{-b^2+4\,ac}}+\frac{2\,b^2cx^6+4\,ac^2x^6+3\,b^3x^4+6\,abcx^4+10\,ab^2x^2-4\,a^2cx^2+6\,a^2b}{4\left(cx^4+bx^2+a\right)^2\left(b^4-8\,ab^2c+16\,a^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] (b^2 + 2*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*
a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^2*c*x^6 + 4*
a*c^2*x^6 + 3*b^3*x^4 + 6*a*b*c*x^4 + 10*a*b^2*x^2 - 4*a^2*c*x^2
+ 6*a^2*b)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.877 $\int \frac{x^3}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=113

$$\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}$$

```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b +
 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.181919, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{3bc\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{2a+bx^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a + b*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (3*b*(b +
 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*b*c*ArcTa
nh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 21.3898, size = 105, normalized size = 0.93

$$\frac{3bc\,\text{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{(-4ac+b^2)^{\frac{5}{2}}}-\frac{3b(b+2cx^2)}{4(-4ac+b^2)^2(a+bx^2+cx^4)}+\frac{2a+bx^2}{4(-4ac+b^2)(a+bx^2+cx^4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*b*c*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)**
(5/2) - 3*b*(b + 2*c*x**2)/(4*(-4*a*c + b**2)**2*(a + b*x**2 + c*
x**4)) + (2*a + b*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)*
*2)
```

Mathematica [A] time = 0.179406, size = 114, normalized size = 1.01

$$\frac{-\frac{12bc\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{(b^2-4ac)(2a+bx^2)}{(a+bx^2+cx^4)^2}-\frac{3b(b+2cx^2)}{a+bx^2+cx^4}}{4(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2 - 4*a*c)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 - (3*b*(b +
2*c*x^2))/(a + b*x^2 + c*x^4) - (12*b*c*ArcTan[(b + 2*c*x^2)/Sqrt
[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.011, size = 142, normalized size = 1.3

$$\frac{-bx^2-2a}{(16ac-4b^2)(cx^4+bx^2+a)^2}-\frac{3bcx^2}{2(4ac-b^2)^2(cx^4+bx^2+a)}-\frac{3b^2}{4(4ac-b^2)^2(cx^4+bx^2+a)}-3\frac{bc}{(4ac-b^2)^{5/2}}\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(-b*x^2-2*a)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2-3/2*b/(4*a*c-b^2)^
2/(c*x^4+b*x^2+a)*x^2*c-3/4*b^2/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)-3*b
/(4*a*c-b^2)^(5/2)*c*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271086, size = 1, normalized size = 0.01

$$\left[\frac{6\left(bc^3x^8+2\,b^2c^2x^6+2\,ab^2cx^2+\left(b^3c+2\,abc^2\right)x^4+a^2bc\right)\log\left(\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2+\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(6\,bc\right.}{4\left((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^8+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^6-6\,ab^4c+32\,a^3c\right.}\right.$$

$$-\frac{12\left(bc^3x^8+2\,b^2c^2x^6+2\,ab^2cx^2+\left(b^3c+2\,abc^2\right)x^4+a^2bc\right)\arctan\left(-\frac{\left(2\,cx^2+b\right)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+\left(6\,bc^2x^6+9\,b^2cx^4+ab^2+8\right.}{4\left((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^8+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^6-6\,ab^4c+32\,a^3c^3)x^4+2\,(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(6*(b*c^3*x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 + (b^3*c + 2*a
*b*c^2)*x^4 + a^2*b*c)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x
^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*
x^4 + b*x^2 + a)) - (6*b*c^2*x^6 + 9*b^2*c*x^4 + a*b^2 + 8*a^2*c
+ 2*(b^3 + 5*a*b*c)*x^2)*sqrt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*
c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^
6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^
3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(b^2
 - 4*a*c)), -1/4*(12*(b*c^3*x^8 + 2*b^2*c^2*x^6 + 2*a*b^2*c*x^2 +
 (b^3*c + 2*a*b*c^2)*x^4 + a^2*b*c)*arctan(-(2*c*x^2 + b)*sqrt(-b
^2 + 4*a*c)/(b^2 - 4*a*c)) + (6*b*c^2*x^6 + 9*b^2*c*x^4 + a*b^2 +
 8*a^2*c + 2*(b^3 + 5*a*b*c)*x^2)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
)*sqrt(-b^2 + 4*a*c))]
```

Sympy [A] time = 33.3551, size = 490, normalized size = 4.34

$$\frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)}{2}$$

$$-\frac{3bc\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^3bc^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^5c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^7c\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^2c}{6bc^2}\right)}{2}$$

$$-\frac{8a^2c+ab^2+9b^2cx^4+6bc^2x^6+x^2\left(10abc+2b^3\right)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 3*b*c*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*b*c**4*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**
2)**5) - 36*a*b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) + 3*b**7*c*sqr
t(-1/(4*a*c - b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 - 3*b*c*sqrt(-1
/(4*a*c - b**2)**5)*log(x**2 + (192*a**3*b*c**4*sqrt(-1/(4*a*c -
b**2)**5) - 144*a**2*b**3*c**3*sqrt(-1/(4*a*c - b**2)**5) + 36*a*
b**5*c**2*sqrt(-1/(4*a*c - b**2)**5) - 3*b**7*c*sqrt(-1/(4*a*c -
b**2)**5) + 3*b**2*c)/(6*b*c**2))/2 - (8*a**2*c + a*b**2 + 9*b**2
*c*x**4 + 6*b*c**2*x**6 + x**2*(10*a*b*c + 2*b**3))/(64*a**4*c**2
 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*
c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*
b**5*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128
*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

GIAC/XCAS [A] time = 15.6378, size = 193, normalized size = 1.71

$$-\frac{3\, bc \arctan\left(\frac{2\, cx^2+b}{\sqrt{-b^2+4\, ac}}\right)}{(b^4 - 8\, ab^2c + 16\, a^2c^2)\sqrt{-b^2 + 4\, ac}} - \frac{6\, bc^2x^6 + 9\, b^2cx^4 + 2\, b^3x^2 + 10\, abcx^2 + ab^2 + 8\, a^2c}{4\,(cx^4 + bx^2 + a)^2(b^4 - 8\, ab^2c + 16\, a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] -3*b*c*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c
 + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*b*c^2*x^6 + 9*b^2*c*x
^4 + 2*b^3*x^2 + 10*a*b*c*x^2 + a*b^2 + 8*a^2*c)/((c*x^4 + b*x^2
+ a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.878 $\int \frac{x}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=113

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] -(b + 2*c*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b
+ 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi [A] time = 0.172935, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{6c^2\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3c\left(b+2cx^2\right)}{2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(b + 2*c*x^2)/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*c*(b
+ 2*c*x^2))/(2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (6*c^2*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rubi in Sympy [A] time = 18.9878, size = 105, normalized size = 0.93

$$-\frac{6c^2\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3c\left(b+2cx^2\right)}{2\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}-\frac{b+2cx^2}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -6*c**2*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(-4*a*c + b**2)
**(5/2) + 3*c*(b + 2*c*x**2)/(2*(-4*a*c + b**2)**2*(a + b*x**2 +
c*x**4)) - (b + 2*c*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4
)**2)
```

Mathematica [A] time = 0.193789, size = 106, normalized size = 0.94

$$\frac{\frac{24c^2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{(b+2cx^2)\left(-2c\left(5a+3cx^4\right)+b^2-6bcx^2\right)}{(a+bx^2+cx^4)^2}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-(((b + 2*c*x^2)*(b^2 - 6*b*c*x^2 - 2*c*(5*a + 3*c*x^4)))/(a + b
*x^2 + c*x^4)^2) + (24*c^2*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c
]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4*a*c)^2)
```

Maple [A] time = 0.009, size = 141, normalized size = 1.3

$$\frac{2\,cx^2+b}{(16\,ac-4\,b^2)\,(cx^4+bx^2+a)^2}+3\,\frac{c^2x^2}{(4\,ac-b^2)^2\,(cx^4+bx^2+a)}+\frac{3\,bc}{2\,(4\,ac-b^2)^2\,(cx^4+bx^2+a)}+6\,\frac{c^2}{(4\,ac-b^2)^{5/2}}\arctan\left(\frac{2\,cx^2+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 1/4*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3*c^2/(4*a*c-b^2)^2
/(c*x^4+b*x^2+a)*x^2+3/2*c/(4*a*c-b^2)^2/(c*x^4+b*x^2+a)*b+6*c^2/
(4*a*c-b^2)^(5/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269147, size = 1, normalized size = 0.01

$$\left[\frac{12\left(c^4x^8+2\,bc^3x^6+2\,abc^2x^2+\left(b^2c^2+2\,ac^3\right)x^4+a^2c^2\right)\log\left(-\frac{b^3-4\,abc+2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4+2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+\left(12\,c\right.}{4\left((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^8+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^6-6\,ab^4c+32\,a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(12*(c^4*x^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*c^2 + 2*a*
c^3)*x^4 + a^2*c^2)*log(-(b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2
 - (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^
4 + b*x^2 + a)) + (12*c^3*x^6 + 18*b*c^2*x^4 - b^3 + 10*a*b*c + 4
*(b^2*c + 5*a*c^2)*x^2)*sqrt(b^2 - 4*a*c))/(((b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6
 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3
*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(b^2
- 4*a*c)), 1/4*(24*(c^4*x^8 + 2*b*c^3*x^6 + 2*a*b*c^2*x^2 + (b^2*
c^2 + 2*a*c^3)*x^4 + a^2*c^2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4
*a*c)/(b^2 - 4*a*c)) + (12*c^3*x^6 + 18*b*c^2*x^4 - b^3 + 10*a*b*
c + 4*(b^2*c + 5*a*c^2)*x^2)*sqrt(-b^2 + 4*a*c))/(((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c +
32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqr
t(-b^2 + 4*a*c))]
```

Sympy [A] time = 33.359, size = 481, normalized size = 4.26

$$-3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{-192a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}+144a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}-36ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3\right.}{6c^3}\right.$$
$$+3c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}\log\left(x^2+\frac{192a^3c^5\sqrt{-\frac{1}{(4ac-b^2)^5}}-144a^2b^2c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}+36ab^4c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}-3b^6c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}+3b\right.}{6c^3}\right.$$
$$+\frac{10abc-b^3+18bc^2x^4+12c^3x^6+x^2\left(20ac^2+4b^2c\right)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\left(64a^2c^4-32ab^2c^3+4b^4c^2\right)+x^6\left(128a^2bc^3-64ab^3c^2+8b^5c\right)+x^4\left(128a^3c^3-24ab^4c+4b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*c**2*sqrt(-1/(4*a*c - b**2)**5)*log(x**2 + (-192*a**3*c**5*sqr
t(-1/(4*a*c - b**2)**5) + 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**
2)**5) - 36*a*b**4*c**3*sqrt(-1/(4*a*c - b**2)**5) + 3*b**6*c**2*
sqrt(-1/(4*a*c - b**2)**5) + 3*b*c**2)/(6*c**3)) + 3*c**2*sqrt(-1
/(4*a*c - b**2)**5)*log(x**2 + (192*a**3*c**5*sqrt(-1/(4*a*c - b*
*2)**5) - 144*a**2*b**2*c**4*sqrt(-1/(4*a*c - b**2)**5) + 36*a*b*
*4*c**3*sqrt(-1/(4*a*c - b**2)**5) - 3*b**6*c**2*sqrt(-1/(4*a*c -
 b**2)**5) + 3*b*c**2)/(6*c**3)) + (10*a*b*c - b**3 + 18*b*c**2*x
**4 + 12*c**3*x**6 + x**2*(20*a*c**2 + 4*b**2*c))/(64*a**4*c**2 -
 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c*
*3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b*
*5*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a
**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

GIAC/XCAS [A] time = 15.728, size = 194, normalized size = 1.72

$$\frac{6\,c^2 \arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4 - 8\,ab^2c + 16\,a^2c^2)\sqrt{-b^2 + 4\,ac}} + \frac{12\,c^3x^6 + 18\,bc^2x^4 + 4\,b^2cx^2 + 20\,ac^2x^2 - b^3 + 10\,abc}{4\,(cx^4 + bx^2 + a)^2(b^4 - 8\,ab^2c + 16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 6*c^2*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^2*c
+ 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(12*c^3*x^6 + 18*b*c^2*x^
4 + 4*b^2*c*x^2 + 20*a*c^2*x^2 - b^3 + 10*a*b*c)/((c*x^4 + b*x^2
+ a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

3.879 $\int \frac{1}{x(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=200

$$-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4
*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c
+ 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b
^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x^2 + c*x^4]/(4*a^3)
```

Rubi [A] time = 0.674616, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444

$$-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{\log(x)}{a^3}+\frac{16a^2c^2+2bcx^2\left(b^2-7ac\right)-15ab^2c+2b^4}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
 + (2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b*c*(b^2 - 7*a*c)*x^2)/(4
*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b*(b^4 - 10*a*b^2*c
+ 30*a^2*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b
^2 - 4*a*c)^(5/2)) + Log[x]/a^3 - Log[a + b*x^2 + c*x^4]/(4*a^3)
```

Rubi in Sympy [A] time = 79.0637, size = 196, normalized size = 0.98

$$\frac{-2ac+b^2+bcx^2}{4a\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{16a^2c^2-15ab^2c+2b^4+2bcx^2\left(-7ac+b^2\right)}{4a^2\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}+\frac{b\left(30a^2c^2-10ab^2c+b^4\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^3\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{\log\left(x^2\right)}{2a^3}-\frac{\log\left(a+bx^2+cx^4\right)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(4*a*(-4*a*c + b**2)*(a + b*x**2 + c*x
**4)**2) + (16*a**2*c**2 - 15*a*b**2*c + 2*b**4 + 2*b*c*x**2*(-7*
a*c + b**2))/(4*a**2*(-4*a*c + b**2)**2*(a + b*x**2 + c*x**4)) +
b*(30*a**2*c**2 - 10*a*b**2*c + b**4)*atanh((b + 2*c*x**2)/sqrt(-
4*a*c + b**2))/(2*a**3*(-4*a*c + b**2)**(5/2)) + log(x**2)/(2*a**
3) - log(a + b*x**2 + c*x**4)/(4*a**3)
```

Mathematica [A] time = 0.927753, size = 342, normalized size = 1.71

$$\frac{\frac{a^2(-2ac+b^2+bcx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{a(16a^2c^2-15ab^2c-14abc^2x^2+2b^4+2b^3cx^2)}{(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{\left(16a^2c^2\sqrt{b^2-4ac}+30a^2bc^2-10ab^3c-8ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}+b^5\right)\log\left(-\sqrt{b^2-\right.}{(b^2-4ac)^{5/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
^2) + (a*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*
c^2*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*Log[x] - ((b^
5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c
*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + ((b^5 - 10*a*b^3*c
+ 30*a^2*b*c^2 - b^4*Sqrt[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a
*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [B] time = 0.044, size = 1200, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -7/2/a/(c*x^4+b*x^2+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2
/a^2/(c*x^4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+4/(
c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-29/4/a/(c*x^4
+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^2+1/a^2/(c*x^4+b
```

```
*x^2+a)^2*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4*b^4-1/2/(c*x^4+b*x^2+a
)^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^2-3/a/(c*x^4+b*x^2+a)^2*b^
3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c+1/2/a^2/(c*x^4+b*x^2+a)^2*b^5/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+6*a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-
8*a*b^2*c+b^4)*c^2-21/4/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b
^4)*b^2*c+3/4/a/(c*x^4+b*x^2+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*b^4-
4/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln((16*a^2*c^2-8*a*b^2*c+b^4)*
(c*x^4+b*x^2+a))+2/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln((16*a^2*c^
2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*b^2-1/4/a^3/(16*a^2*c^2-8*a*b^2
*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*b^4-15/a/(
1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*
a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(1
6*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^
3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*b*c^2+5/a^2/(10
24*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*
b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*
a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*
b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*b^3*c-1/2/a^3/(10
24*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*
b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4)+(16*
a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*
b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*b^5+ln(x)/a^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.712955, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x),x, algorithm="fricas")

[Out] [1/4*(((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)*x^8 + a^2*b^5 - 10
*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^4*c^2 + 30*a^2*b^2*
c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 + 60*a^3*b*c^3)*x^4
+ 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^2)*log((b^3 - 4*a*b
```

```
*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a
*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (2*(a*b^3*c^2 - 7*a
^2*b*c^3)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*c^2 + (4*a*b^4*
c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*x^4 + 2*(a*b^5 - 6*a^2*b^3*c - a
^3*b*c^2)*x^2 - ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2
*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2
*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4
*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2
)*log(x))*sqrt(b^2 - 4*a*c))/((a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2
 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32
*a^6*c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2)*sqr
t(b^2 - 4*a*c)), -1/4*(2*((b^5*c^2 - 10*a*b^3*c^3 + 30*a^2*b*c^4)
*x^8 + a^2*b^5 - 10*a^3*b^3*c + 30*a^4*b*c^2 + 2*(b^6*c - 10*a*b^
4*c^2 + 30*a^2*b^2*c^3)*x^6 + (b^7 - 8*a*b^5*c + 10*a^2*b^3*c^2 +
 60*a^3*b*c^3)*x^4 + 2*(a*b^6 - 10*a^2*b^4*c + 30*a^3*b^2*c^2)*x^
2)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (2*(
a*b^3*c^2 - 7*a^2*b*c^3)*x^6 + 3*a^2*b^4 - 21*a^3*b^2*c + 24*a^4*
c^2 + (4*a*b^4*c - 29*a^2*b^2*c^2 + 16*a^3*c^3)*x^4 + 2*(a*b^5 -
6*a^2*b^3*c - a^3*b*c^2)*x^2 - ((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2
*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a)
 + 4*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b
^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 +
 (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16
*a^3*b*c^2)*x^2)*log(x))*sqrt(-b^2 + 4*a*c))/((a^5*b^4 - 8*a^6*b^
2*c + 16*a^7*c^2 + (a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8
 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 -
6*a^4*b^4*c + 32*a^6*c^3)*x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6
*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.6035, size = 436, normalized size = 2.18

$$-\frac{\left(b^5 - 10\,ab^3c + 30\,a^2bc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2)\sqrt{-b^2+4\,ac}}$$
$$+\frac{3\,b^4c^2x^8 - 24\,ab^2c^3x^8 + 48\,a^2c^4x^8 + 6\,b^5cx^6 - 44\,ab^3c^2x^6 + 68\,a^2bc^3x^6 + 3\,b^6x^4 - 10\,ab^4cx^4 - 58\,a^2b^2c^2x^4 + 128\,a^3c^3x^4 +}{8\,(a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2)(cx^4 + bx^2 + a)^2}$$
$$-\frac{\ln\left(cx^4 + bx^2 + a\right)}{4\,a^3} + \frac{\ln\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x),x, algorithm="giac")
```

```
[Out] -1/2*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 +
4*a*c)) + 1/8*(3*b^4*c^2*x^8 - 24*a*b^2*c^3*x^8 + 48*a^2*c^4*x^8
+ 6*b^5*c*x^6 - 44*a*b^3*c^2*x^6 + 68*a^2*b*c^3*x^6 + 3*b^6*x^4 -
 10*a*b^4*c*x^4 - 58*a^2*b^2*c^2*x^4 + 128*a^3*c^3*x^4 + 10*a*b^5
*x^2 - 72*a^2*b^3*c*x^2 + 92*a^3*b*c^2*x^2 + 9*a^2*b^4 - 66*a^3*b
^2*c + 96*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 +
 b*x^2 + a)^2) - 1/4*ln(c*x^4 + b*x^2 + a)/a^3 + 1/2*ln(x^2)/a^3
```

3.880 $\int \frac{1}{x^3(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=255

$$\frac{3b\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx^2}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b
^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^
2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/
(4*a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) - (3*(b^6 - 10*a*
b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3
*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi [A] time = 0.894886, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{3b\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{3b\log(x)}{a^4}-\frac{3\left(b^2-5ac\right)\left(b^2-2ac\right)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{20a^2c^2+3bcx^2\left(b^2-6ac\right)-20ab^2c+3b^4}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}+\frac{-2ac+b^2+bcx^2}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(b^2 - 5*a*c)*(b^2 - 2*a*c))/(2*a^3*(b^2 - 4*a*c)^2*x^2) + (b
^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)^
2) + (3*b^4 - 20*a*b^2*c + 20*a^2*c^2 + 3*b*c*(b^2 - 6*a*c)*x^2)/
(4*a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) - (3*(b^6 - 10*a*
b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b
^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - (3*b*Log[x])/a^4 + (3
*b*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rubi in Sympy [A] time = 138.41, size = 258, normalized size = 1.01

$$\frac{-2ac+b^2+bcx^2}{4ax^2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{20a^2c^2-20ab^2c+3b^4+3bcx^2\left(-6ac+b^2\right)}{4a^2x^2\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)} - \frac{3\left(-5ac+b^2\right)\left(-2ac+b^2\right)}{2a^3x^2\left(-4ac+b^2\right)^2}-\frac{3b\log\left(x^2\right)}{2a^4}+\frac{3b\log\left(a+bx^2+cx^4\right)}{4a^4} - \frac{3\left(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6\right)\operatorname{atanh}\left(\frac{b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^4\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(4*a*x**2*(-4*a*c + b**2)*(a + b*x**2
+ c*x**4)**2) + (20*a**2*c**2 - 20*a*b**2*c + 3*b**4 + 3*b*c*x**2
*(-6*a*c + b**2))/(4*a**2*x**2*(-4*a*c + b**2)**2*(a + b*x**2 + c
*x**4)) - 3*(-5*a*c + b**2)*(-2*a*c + b**2)/(2*a**3*x**2*(-4*a*c
+ b**2)**2) - 3*b*log(x**2)/(2*a**4) + 3*b*log(a + b*x**2 + c*x**
4)/(4*a**4) - 3*(-20*a**3*c**3 + 30*a**2*b**2*c**2 - 10*a*b**4*c
+ b**6)*atanh((b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a**4*(-4*a*c
 + b**2)**(5/2))
```

Mathematica [A] time = 1.12735, size = 402, normalized size = 1.58

$$\frac{\frac{a^2\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{(4ac-b^2)(a+bx^2+cx^4)^2}-\frac{a\left(46a^2bc^2+28a^2c^3x^2-29ab^3c-26ab^2c^2x^2+4b^5+4b^4cx^2\right)}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\left(-20a^3c^3+30a^2b^2c^2+16a^2bc^2\sqrt{b^2-4ac}-10ab^4c+b^5\sqrt{b^2-4ac}\right.}{(b^2-4ac)^{5/2}}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a)/x^2 + (a^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) - (a*(4*b^5 - 29*a*b^3*c + 46
*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2))/((
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - 12*b*Log[x] + (3*(b^6 - 10*
a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8
*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])*Log[
b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) + (3*(-b^6
+ 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c
] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c])
*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^
4)
```

Maple [B] time = 0.038, size = 1486, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -7/a/(c*x^4+b*x^2+a)^2*c^4/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+13/2/a^
2/(c*x^4+b*x^2+a)^2*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^2-1/a^3/
(c*x^4+b*x^2+a)^2*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6*b^4-37/2/a/(
c*x^4+b*x^2+a)^2*b*c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+55/4/a^2/(c
*x^4+b*x^2+a)^2*b^3*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-2/a^3/(c*x
^4+b*x^2+a)^2*b^5*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-9/(c*x^4+b*x^2
+a)^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*c^3-7/2/a/(c*x^4+b*x^2+a)^2/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^2*c^2+6/a^2/(c*x^4+b*x^2+a)^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^2*b^4*c-1/a^3/(c*x^4+b*x^2+a)^2/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^2*b^6-29/2/(c*x^4+b*x^2+a)^2*b/(16*a^2*c^2
-8*a*b^2*c+b^4)*c^2+9/a/(c*x^4+b*x^2+a)^2*b^3/(16*a^2*c^2-8*a*b^2
*c+b^4)*c-5/4/a^2/(c*x^4+b*x^2+a)^2*b^5/(16*a^2*c^2-8*a*b^2*c+b^4
)+12/a^2/(16*a^2*c^2-8*a*b^2*c+b^4)*c^2*ln((16*a^2*c^2-8*a*b^2*c+
b^4)*(c*x^4+b*x^2+a))*b-6/a^3/(16*a^2*c^2-8*a*b^2*c+b^4)*c*ln((16
*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*b^3+3/4/a^4/(16*a^2*c^2-
8*a*b^2*c+b^4)*ln((16*a^2*c^2-8*a*b^2*c+b^4)*(c*x^4+b*x^2+a))*b^5
-30/a/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*
c^2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+
b^4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4
+640*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*c^3+45/a
^2/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^2
+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^4
)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+64
0*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*b^2*c^2-15/
a^3/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^
2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^
4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+6
40*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*b^4*c+3/2/
a^4/(1024*a^5*c^5-1280*a^4*b^2*c^4+640*a^3*b^4*c^3-160*a^2*b^6*c^
2+20*a*b^8*c-b^10)^(1/2)*arctan((2*c*x^2*(16*a^2*c^2-8*a*b^2*c+b^
4)+(16*a^2*c^2-8*a*b^2*c+b^4)*b)/(1024*a^5*c^5-1280*a^4*b^2*c^4+6
40*a^3*b^4*c^3-160*a^2*b^6*c^2+20*a*b^8*c-b^10)^(1/2))*b^6-1/2/a^
3/x^2-3*b*ln(x)/a^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.965477, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*((b^6*c^2 - 10*a*b^4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*
x^10 + 2*(b^7*c - 10*a*b^5*c^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x
^8 + (b^8 - 8*a*b^6*c + 10*a^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*
c^4)*x^6 + 2*(a*b^7 - 10*a^2*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^
3)*x^4 + (a^2*b^6 - 10*a^3*b^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x
^2)*log((b^3 - 4*a*b*c + 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 + 2
*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) +
 (6*(a*b^4*c^2 - 7*a^2*b^2*c^3 + 10*a^3*c^4)*x^8 + 2*a^3*b^4 - 16
*a^4*b^2*c + 32*a^5*c^2 + 3*(4*a*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*
b*c^3)*x^6 + 2*(3*a*b^6 - 18*a^2*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c
^3)*x^4 + (9*a^2*b^5 - 68*a^3*b^3*c + 122*a^4*b*c^2)*x^2 - 3*((b^
5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^10 + 2*(b^6*c - 8*a*b^4*c^2
 + 16*a^2*b^2*c^3)*x^8 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^6 + 2
*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^4 + (a^2*b^5 - 8*a^3*b^
3*c + 16*a^4*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) + 12*((b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*x^10 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^
2*b^2*c^3)*x^8 + (b^7 - 6*a*b^5*c + 32*a^3*b*c^3)*x^6 + 2*(a*b^6
- 8*a^2*b^4*c + 16*a^3*b^2*c^2)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16
*a^4*b*c^2)*x^2)*log(x))*sqrt(b^2 - 4*a*c))/(((a^4*b^4*c^2 - 8*a^
5*b^2*c^3 + 16*a^6*c^4)*x^10 + 2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*
a^6*b*c^3)*x^8 + (a^4*b^6 - 6*a^5*b^4*c + 32*a^7*c^3)*x^6 + 2*(a^
5*b^5 - 8*a^6*b^3*c + 16*a^7*b*c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c
+ 16*a^8*c^2)*x^2)*sqrt(b^2 - 4*a*c)), 1/4*(6*((b^6*c^2 - 10*a*b^
4*c^3 + 30*a^2*b^2*c^4 - 20*a^3*c^5)*x^10 + 2*(b^7*c - 10*a*b^5*c
^2 + 30*a^2*b^3*c^3 - 20*a^3*b*c^4)*x^8 + (b^8 - 8*a*b^6*c + 10*a
^2*b^4*c^2 + 40*a^3*b^2*c^3 - 40*a^4*c^4)*x^6 + 2*(a*b^7 - 10*a^2
*b^5*c + 30*a^3*b^3*c^2 - 20*a^4*b*c^3)*x^4 + (a^2*b^6 - 10*a^3*b
^4*c + 30*a^4*b^2*c^2 - 20*a^5*c^3)*x^2)*arctan(-(2*c*x^2 + b)*sq
rt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (6*(a*b^4*c^2 - 7*a^2*b^2*c^3 +
 10*a^3*c^4)*x^8 + 2*a^3*b^4 - 16*a^4*b^2*c + 32*a^5*c^2 + 3*(4*a
*b^5*c - 29*a^2*b^3*c^2 + 46*a^3*b*c^3)*x^6 + 2*(3*a*b^6 - 18*a^2
*b^4*c + 7*a^3*b^2*c^2 + 50*a^4*c^3)*x^4 + (9*a^2*b^5 - 68*a^3*b^
3*c + 122*a^4*b*c^2)*x^2 - 3*((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c
^4)*x^10 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x^8 + (b^7 -
6*a*b^5*c + 32*a^3*b*c^3)*x^6 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b
^2*c^2)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*log(c*x
^4 + b*x^2 + a) + 12*((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^10
```

```
 + 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*x^8 + (b^7 - 6*a*b^5*
c + 32*a^3*b*c^3)*x^6 + 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*log(x))*sqrt(-b
^2 + 4*a*c))/(((a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)*x^10 +
2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)*x^8 + (a^4*b^6 - 6*a
^5*b^4*c + 32*a^7*c^3)*x^6 + 2*(a^5*b^5 - 8*a^6*b^3*c + 16*a^7*b*
c^2)*x^4 + (a^6*b^4 - 8*a^7*b^2*c + 16*a^8*c^2)*x^2)*sqrt(-b^2 +
4*a*c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [A] time = 15.6525, size = 516, normalized size = 2.02

$$\frac{3\left(b^{6}-10\,ab^{4}c+30\,a^{2}b^{2}c^{2}-20\,a^{3}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}} - \frac{9\,b^{5}c^{2}x^{8}-72\,ab^{3}c^{3}x^{8}+144\,a^{2}bc^{4}x^{8}+18\,b^{6}cx^{6}-136\,ab^{4}c^{2}x^{6}+236\,a^{2}b^{2}c^{3}x^{6}+56\,a^{3}c^{4}x^{6}+9\,b^{7}x^{4}-38\,ab^{5}cx^{4}-110\,a^{2}b^{3}}{8\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)(c} + \frac{3\,b\ln\left(cx^{4}+bx^{2}+a\right)}{4\,a^{4}} - \frac{3\,b\ln\left(x^{2}\right)}{2\,a^{4}} + \frac{3\,bx^{2}-a}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x^3),x, algorithm="giac")
```

```
[Out] 3/2*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3)*arctan((2*c*
x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2
)*sqrt(-b^2 + 4*a*c)) - 1/8*(9*b^5*c^2*x^8 - 72*a*b^3*c^3*x^8 + 1
44*a^2*b*c^4*x^8 + 18*b^6*c*x^6 - 136*a*b^4*c^2*x^6 + 236*a^2*b^2
*c^3*x^6 + 56*a^3*c^4*x^6 + 9*b^7*x^4 - 38*a*b^5*c*x^4 - 110*a^2*
b^3*c^2*x^4 + 436*a^3*b*c^3*x^4 + 26*a*b^6*x^2 - 192*a^2*b^4*c*x^
2 + 316*a^3*b^2*c^2*x^2 + 72*a^4*c^3*x^2 + 19*a^2*b^5 - 144*a^3*b
^3*c + 260*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*(c*x^
4 + b*x^2 + a)^2) + 3/4*b*ln(c*x^4 + b*x^2 + a)/a^4 - 3/2*b*ln(x^
2)/a^4 + 1/2*(3*b*x^2 - a)/(a^4*x^2)
```

3.881 $\int \frac{x^{10}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=400

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3bx\left(b^2-8ac\right)}{8c^2\left(b^2-4ac\right)^2}+\frac{x^3\left(b^2-28ac\right)}{8c\left(b^2-4ac\right)^2}+\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^5\left(12ab-x^2\left(b^2-28ac\right)\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*
x^3)/(8*c*(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a
^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*
c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^4 -
9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rubi [A] time = 3.34115, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{3\left(-\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{44a^2bc^2-11ab^3c+b^5}{\sqrt{b^2-4ac}}+28a^2c^2-9ab^2c+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{5/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{3bx\left(b^2-8ac\right)}{8c^2\left(b^2-4ac\right)^2}+\frac{x^3\left(b^2-28ac\right)}{8c\left(b^2-4ac\right)^2}+\frac{x^7\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^5\left(12ab-x^2\left(b^2-28ac\right)\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*b*(b^2 - 8*a*c)*x)/(8*c^2*(b^2 - 4*a*c)^2) + ((b^2 - 28*a*c)*
x^3)/(8*c*(b^2 - 4*a*c)^2) + (x^7*(2*a + b*x^2))/(4*(b^2 - 4*a*c)
```

```
*(a + b*x^2 + c*x^4)^2) + (x^5*(12*a*b - (b^2 - 28*a*c)*x^2))/(8*
(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^4 - 9*a*b^2*c + 28*a
^2*c^2 - (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*
c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^4 -
9*a*b^2*c + 28*a^2*c^2 + (b^5 - 11*a*b^3*c + 44*a^2*b*c^2)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 2.22221, size = 455, normalized size = 1.14

$$\frac{-\frac{4\left(a^2cx\left(2cx^2-3b\right)+ab^2x\left(b-4cx^2\right)+b^4x^3\right)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3\sqrt{2}\sqrt{c}\left(28a^2c^2\sqrt{b^2-4ac}-44a^2bc^2+11ab^3c-9ab^2c\sqrt{b^2-4ac}+b^4\sqrt{b^2-4ac}-b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\cdots}{\cdots}}{16c\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^10/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5 - 17*a*b^3*c + 48*a^2*b*c^2 - 5*b^4*c*x^2 + 37*a*b^2
*c^2*x^2 - 44*a^2*c^3*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 - (4*(b^4*x^3 + a*b^2*x*(b - 4*c*x^2) + a^2*c*x*(-3*b + 2*c*x^2)
))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b
^5 + 11*a*b^3*c - 44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*
c*Sqrt[b^2 - 4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^5 - 11*a*b^3
*c + 44*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 9*a*b^2*c*Sqrt[b^2 -
4*a*c] + 28*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]))/(16*c^3)
```

Maple [B] time = 0.153, size = 5425, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^10/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(5\,b^4c - 37\,ab^2c^2 + 44\,a^2c^3)x^7 + (3\,b^5 - 20\,ab^3c - 4\,a^2bc^2)x^5 + (6\,ab^4 - 49\,a^2b^2c + 28\,a^3c^2)x^3 + 3\,(}{8\,((b^4c^4 - 8\,ab^2c^5 + 16\,a^2c^6)x^8 + a^2b^4c^2 - 8\,a^3b^2c^3 + 16\,a^4c^4 + 2\,(b^5c^3 - 8\,ab^3c^4 + 16\,a^2bc^5)x^6 + (b^6c^2 - 6\,ab^4c^3 + 32\,a^3c^5)x} + \frac{3\int\frac{ab^3-8\,a^2bc+(b^4-9\,ab^2c+28\,a^2c^2)x^2}{cx^4+bx^2+a}\,dx}{8\,(b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + (3*b^5 - 20*a*b
^3*c - 4*a^2*b*c^2)*x^5 + (6*a*b^4 - 49*a^2*b^2*c + 28*a^3*c^2)*x
^3 + 3*(a^2*b^3 - 8*a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*
c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3
- 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a
^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2) +
 3/8*integrate((a*b^3 - 8*a^2*b*c + (b^4 - 9*a*b^2*c + 28*a^2*c^2
)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)
```

Fricas [A] time = 0.552127, size = 5777, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*b^4*c - 37*a*b^2*c^2 + 44*a^2*c^3)*x^7 + 2*(3*b^5 - 2
0*a*b^3*c - 4*a^2*b*c^2)*x^5 + 2*(6*a*b^4 - 49*a^2*b^2*c + 28*a^3
```

```
*c^2)*x^3 + 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8
 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^
3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*
x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^
9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c
^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219
*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a
*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^
14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7
- 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21
*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 +
 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*
b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^
5 + 25088*a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^
7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 8
1920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*
a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*
b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^1
4 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 -
840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160
*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10
)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2
401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*
a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 2
0*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^
9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a
^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c
^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 3
2*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2
)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1
680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*
a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*
b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*
c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280
*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a
^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))
*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^
5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c
 + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584
*a^5*b^3*c^5 + 25088*a^6*b*c^6 - (b^14*c^5 - 30*a*b^12*c^6 + 416*
a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b
^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b
^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c
^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*
a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2
*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (b^10*c^5 - 20*a*b^
8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 10
24*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*
b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*
c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b
^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280
*a^4*b^2*c^9 - 1024*a^5*c^10))) + 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4
```

+ 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x + 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) - 3*sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(27*(21*a^2*b^8 - 447*a^3*b^6*c + 4189*a^4*b^4*c^2 - 19208*a^5*b^2*c^3 + 38416*a^6*c^4)*x - 27/2*sqrt(1/2)*(b^13 - 31*a*b^11*c + 413*a^2*b^9*c^2 - 3012*a^3*b^7*c^3 + 12496*a^4*b^5*c^4 - 27584*a^5*b^3*c^5 + 25088*a^6*b*c^6 + (b^14*c^5 - 30*a*b^12*c^6 + 416*a^2*b^10*c^7 - 3360*a^3*b^8*c^8 + 16640*a^4*b^6*c^9 - 49664*a^5*b^4*c^10 + 81920*a^6*b^2*c^11 - 57344*a^7*c^12)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + 6*(a^2*b^3 - 8*a^3*b*c)*x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 +

```
2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)
```

Sympy [A] time = 74.3966, size = 804, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**10/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(x**7*(44*a**2*c**3 - 37*a*b**2*c**2 + 5*b**4*c) + x**5*(-4*a**2
*b*c**2 - 20*a*b**3*c + 3*b**5) + x**3*(28*a**3*c**2 - 49*a**2*b*
*2*c + 6*a*b**4) + x*(-24*a**3*b*c + 3*a**2*b**3))/(128*a**4*c**4
 - 64*a**3*b**2*c**3 + 8*a**2*b**4*c**2 + x**8*(128*a**2*c**6 - 6
4*a*b**2*c**5 + 8*b**4*c**4) + x**6*(256*a**2*b*c**5 - 128*a*b**3
*c**4 + 16*b**5*c**3) + x**4*(256*a**3*c**5 - 48*a*b**4*c**3 + 8*
b**6*c**2) + x**2*(256*a**3*b*c**4 - 128*a**2*b**3*c**3 + 16*a*b*
*5*c**2)) + RootSum(_t**4*(68719476736*a**10*c**15 - 171798691840
*a**9*b**2*c**14 + 193273528320*a**8*b**4*c**13 - 128849018880*a*
*7*b**6*c**12 + 56371445760*a**6*b**8*c**11 - 16911433728*a**5*b*
*10*c**10 + 3523215360*a**4*b**12*c**9 - 503316480*a**3*b**14*c**
8 + 47185920*a**2*b**16*c**7 - 2621440*a*b**18*c**6 + 65536*b**20
*c**5) + _t**2*(-3963617280*a**9*b*c**9 + 6936330240*a**8*b**3*c*
*8 - 5400428544*a**7*b**5*c**7 + 2464874496*a**6*b**7*c**6 - 7300
54656*a**5*b**9*c**5 + 146165760*a**4*b**11*c**4 - 19860480*a**3*
b**13*c**3 + 1771776*a**2*b**15*c**2 - 94464*a*b**17*c + 2304*b**
19) + 49787136*a**9*c**4 - 27433728*a**8*b**2*c**3 + 6446304*a**7
*b**4*c**2 - 734832*a**6*b**6*c + 35721*a**5*b**8, Lambda(_t, _t*
log(x + (234881024*_t**3*a**7*c**12 - 335544320*_t**3*a**6*b**2*c
**11 + 203423744*_t**3*a**5*b**4*c**10 - 68157440*_t**3*a**4*b**6
*c**9 + 13762560*_t**3*a**3*b**8*c**8 - 1703936*_t**3*a**2*b**10*
c**7 + 122880*_t**3*a*b**12*c**6 - 4096*_t**3*b**14*c**5 - 858009
6*_t*a**6*b*c**6 + 6582528*_t*a**5*b**3*c**5 - 2387520*_t*a**4*b*
*5*c**4 + 498096*_t*a**3*b**7*c**3 - 62496*_t*a**2*b**9*c**2 + 44
64*_t*a*b**11*c - 144*_t*b**13)/(1037232*a**6*c**4 - 518616*a**5*
b**2*c**3 + 113103*a**4*b**4*c**2 - 12069*a**3*b**6*c + 567*a**2*
b**8))))
```

GIAC/XCAS [A] time = 35.0393, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^10/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Done
```

3.882 $\int \frac{x^8}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=348

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^3\left(x^2\left(20ac+b^2\right)+12ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20ac+b^2\right)}{8c\left(b^2-4ac\right)^2}$$

```
[Out] -((b^2 + 20*a*c)*x)/(8*c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(
4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20
*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*
a*b*c - (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(
3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 16*a*
b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/
2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.9039, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278

$$\frac{\left(-\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2c^2-18ab^2c+b^4}{\sqrt{b^2-4ac}}-16abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}c^{3/2}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^5\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^3\left(x^2\left(20ac+b^2\right)+12ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20ac+b^2\right)}{8c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((b^2 + 20*a*c)*x)/(8*c*(b^2 - 4*a*c)^2) + (x^5*(2*a + b*x^2))/(
4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x^3*(12*a*b + (b^2 + 20
*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3 - 16*
```

```
a*b*c - (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(
3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3 - 16*a*
b*c + (b^4 - 18*a*b^2*c - 40*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/
2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 140.816, size = 340, normalized size = 0.98

$$\frac{x^5\left(2a+bx^2\right)}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^3\left(12ab+x^2\left(20ac+b^2\right)\right)}{8\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}-\frac{x\left(20ac+b^2\right)}{8c\left(-4ac+b^2\right)^2}$$
$$+\frac{\sqrt{2}\left(-2ac\left(20ac+b^2\right)+b^2\left(-16ac+b^2\right)+b\left(-16ac+b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{16c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}$$
$$-\frac{\sqrt{2}\left(-2ac\left(20ac+b^2\right)+b^2\left(-16ac+b^2\right)-b\left(-16ac+b^2\right)\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{16c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] x**5*(2*a + b*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)**2)
+ x**3*(12*a*b + x**2*(20*a*c + b**2))/(8*(-4*a*c + b**2)**2*(a +
 b*x**2 + c*x**4)) - x*(20*a*c + b**2)/(8*c*(-4*a*c + b**2)**2) +
 sqrt(2)*(-2*a*c*(20*a*c + b**2) + b**2*(-16*a*c + b**2) + b*(-16
*a*c + b**2)*sqrt(-4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b +
 sqrt(-4*a*c + b**2)))/(16*c**(3/2)*sqrt(b + sqrt(-4*a*c + b**2))
*(-4*a*c + b**2)**(5/2)) - sqrt(2)*(-2*a*c*(20*a*c + b**2) + b**2
*(-16*a*c + b**2) - b*(-16*a*c + b**2)*sqrt(-4*a*c + b**2))*atan(
sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(16*c**(3/2)*sqr
t(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2))
```

Mathematica [A] time = 1.79321, size = 381, normalized size = 1.09

$$\frac{\frac{4\left(-2a^2cx+abx\left(b-3cx^2\right)+b^3x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{2x\left(-36a^2c^2+11ab^2c-16abc^2x^2-2b^4+b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(40a^2c^2+18ab^2c-16abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b}}}\right.}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}}{16c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^8/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4 + 11*a*b^2*c - 36*a^2*c^2 + b^3*c*x^2 - 16*a*b*c^2*
x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (4*(-2*a^2*c*x + b^
3*x^3 + a*b*x*(b - 3*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^
2) + (Sqrt[2]*Sqrt[c]*(-b^4 + 18*a*b^2*c + 40*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b^4 - 18*a*b^2*c - 40*a^2
*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 16*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)
^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [B] time = 0.143, size = 4840, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^8/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-1/8*b*(16*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7-1/8*(36*a^2*c
^2+5*a*b^2*c+b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/4*a/c*b*(14*
a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-1/8*a^2*(20*a*c+b^2)/c/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+17/8*c/(-c^2*(4*a*c
-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3
+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^
2*c+b^4))^(1/2)*arctanh(1/2*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*
x*2^(1/2)/c/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5
)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2))*a*b^10+9*c/(16*a^2*c^
2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*
(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arctanh(1
/2*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^2*b*c
^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*
b^2*c+b^4))^(1/2))*a^2*b^3-3/2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)
/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(1
6*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arctanh(1/2*(-32*a^2*c^4+16*a*b^2
*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c
^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2))*a*b^5
-84*c^3/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(
1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+
(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^4-16*a*b^
2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*
b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^3*b
^6-17/8*c/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2
^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*
c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^4-16*a*
b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^
2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2))*a*b
^10-352*c^5/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)
*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(
```

1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arctanh(1/2*(-32*a^2*c^4+
16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^2*b*c^3+8*a*b^3*c^2-b
^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2
))*a^5*b^2-64*c^4/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*
c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2
)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arctanh(1/2*(-32*a^
2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^2*b*c^3+8*a*b^3
*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)
)^(1/2))*a^4*b^4-640*c^6/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8
*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(3
2*a^2*c^4-16*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^
2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/
2)))^(1/2))*a^6+43/2*c^2/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8
*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(3
2*a^2*c^4-16*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^
2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/
2)))^(1/2))*a^2*b^8+1/16/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8
*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(3
2*a^2*c^4-16*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^
2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/
2)))^(1/2))*b^12-16*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a
^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*
c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^4-16*a*b^2*c^3+2*b^4
*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b
^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^3*b-3/2/(16*a^
2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*
b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan
(1/2*(32*a^2*c^4-16*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2
-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)
^5)^(1/2)))^(1/2))*a*b^5+640*c^6/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a
^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(
-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arct
anh(1/2*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^
2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2
-8*a*b^2*c+b^4))^(1/2))*a^6-43/2*c^2/(-c^2*(4*a*c-b^2)^5)^(1/2)/(
16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5
c+(-c^2(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*
arctanh(1/2*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-1
6*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2
*c^2-8*a*b^2*c+b^4))^(1/2))*a^2*b^8-1/16/(-c^2*(4*a*c-b^2)^5)^(1/
2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2
-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1
/2)*arctanh(1/2*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/
((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16
*a^2*c^2-8*a*b^2*c+b^4))^(1/2))*b^12-16*c^2/(16*a^2*c^2-8*a*b^2*c
+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)
^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arctanh(1/2*(-32*a^2
*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^2*b*c^3+8*a*b^3*
c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))
^(1/2))*a^3*b+352*c^5/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*
b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*

```
b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a
^2*c^4-16*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2*c
+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))
)^(1/2))*a^5*b^2+64*c^4/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*
a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32
*a^2*c^4-16*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2
*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2
)))^(1/2))*a^4*b^4+84*c^3/(-c^2*(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-
8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4
*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1/2)*arctanh(1/2
*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/((-16*a^2*b*c^3
+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^
2*c+b^4))^(1/2))*a^3*b^6+1/16/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2
)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c
^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^4-16*a*b^2*c
^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c
^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2))*b^7+1/16
/c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^3+8*a*b^3*c^2
-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4))^(1
/2)*arctanh(1/2*(-32*a^2*c^4+16*a*b^2*c^3-2*b^4*c^2)*x*2^(1/2)/c/
((-16*a^2*b*c^3+8*a*b^3*c^2-b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2))*(16
*a^2*c^2-8*a*b^2*c+b^4))^(1/2))*b^7+9*c/(16*a^2*c^2-8*a*b^2*c+b^4
)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*(16*a^2*b*c^3-8*a*b^3*c^2+b
^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^4-16
*a*b^2*c^3+2*b^4*c^2)*x*2^(1/2)/c/((16*a^2*c^2-8*a*b^2*c+b^4)*(16
*a^2*b*c^3-8*a*b^3*c^2+b^5*c+(-c^2*(4*a*c-b^2)^5)^(1/2)))^(1/2))*
a^2*b^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^3c-16\,abc^2\right)x^7-\left(b^4+5\,ab^2c+36\,a^2c^2\right)x^5-2\,\left(ab^3+14\,a^2bc\right)x^3-\left(a^2b^2+20\,a^3c\right)x}{8\left((b^4c^3-8\,ab^2c^4+16\,a^2c^5)x^8+a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3+2\,(b^5c^2-8\,ab^3c^3+16\,a^2bc^4)x^6+(b^6c-6\,ab^4c^2+32\,a^3c^4)x^4+\right.}$$
$$-\frac{-\int\frac{ab^2+20\,a^2c+\left(b^3-16\,abc\right)x^2}{cx^4+bx^2+a}\,dx}{8\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^3*c - 16*a*b*c^2)*x^7 - (b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^
5 - 2*(a*b^3 + 14*a^2*b*c)*x^3 - (a^2*b^2 + 20*a^3*c)*x)/((b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 1
6*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c
 - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 1
6*a^3*b*c^3)*x^2) - 1/8*integrate(-(a*b^2 + 20*a^2*c + (b^3 - 16*
a*b*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)
```

Fricas [A] time = 0.407899, size = 5029, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^3*c - 16*a*b*c^2)*x^7 - 2*(b^4 + 5*a*b^2*c + 36*a^2*c^
2)*x^5 - 4*(a*b^3 + 14*a^2*b*c)*x^3 + sqrt(1/2)*((b^4*c^3 - 8*a*b
^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3
 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^
4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c
^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6
 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*
a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4
*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*
c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024
*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 1
0000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*
c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 - (b^1
3*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 345
60*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 -
 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6
*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqr
t(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c
^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*
b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^
10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*
a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^
2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)))
- sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c
 - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2
*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c
 - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 2
80*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*
a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 +
160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*
c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*
c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*
b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x - 1/2*sqrt(1/2)*(b^1
0 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b
^2*c^4 + 32000*a^5*c^5 - (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9
*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 +
 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6
 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^
2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c
^2 + 1680*a^3*b*c^3 + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5
```

- 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x + 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log((35*a*b^6 - 1491*a^2*b^4*c + 15000*a^3*b^2*c^2 + 10000*a^4*c^3)*x - 1/2*sqrt(1/2)*(b^10 - 17*a*b^8*c - 392*a^2*b^6*c^2 + 5696*a^3*b^4*c^3 - 23680*a^4*b^2*c^4 + 32000*a^5*c^5 + (b^13*c^3 - 72*a*b^11*c^4 + 1200*a^2*b^9*c^5 - 8960*a^3*b^7*c^6 + 34560*a^4*b^5*c^7 - 67584*a^5*b^3*c^8 + 53248*a^6*b*c^9)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))) - 2*(a^2*b^2 + 20*a^3*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)

Sympy [A] time = 56.3217, size = 716, normalized size = 2.06

$$-\frac{x^7\left(16abc^2-b^3c\right)+x^5\left(36a^2c^2+5ab^2c+b^4\right)+x^3\left(28a^2bc+2ab^3\right)+x\left(20a^3c+\right.}{128a^4c^3-64a^3b^2c^2+8a^2b^4c+x^8\left(128a^2c^5-64ab^2c^4+8b^4c^3\right)+x^6\left(256a^2bc^4-128ab^3c^3+16b^5c^2\right)+x^4\left(256a^3c^4-48ab^4\right.}$$
$$+\mathrm{RootSum}\left(t^4\left(68719476736a^{10}c^{13}-171798691840a^9b^2c^{12}+193273528320a^8b^4c^{11}-128849018880a^7b^6c^{10}+56371445760\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**8/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(x**7*(16*a*b*c**2 - b**3*c) + x**5*(36*a**2*c**2 + 5*a*b**2*c +
 b**4) + x**3*(28*a**2*b*c + 2*a*b**3) + x*(20*a**3*c + a**2*b**2
))/(128*a**4*c**3 - 64*a**3*b**2*c**2 + 8*a**2*b**4*c + x**8*(128
*a**2*c**5 - 64*a*b**2*c**4 + 8*b**4*c**3) + x**6*(256*a**2*b*c**
4 - 128*a*b**3*c**3 + 16*b**5*c**2) + x**4*(256*a**3*c**4 - 48*a*
b**4*c**2 + 8*b**6*c) + x**2*(256*a**3*b*c**3 - 128*a**2*b**3*c**
2 + 16*a*b**5*c)) + RootSum(_t**4*(68719476736*a**10*c**13 - 1717
98691840*a**9*b**2*c**12 + 193273528320*a**8*b**4*c**11 - 1288490
18880*a**7*b**6*c**10 + 56371445760*a**6*b**8*c**9 - 16911433728*
a**5*b**10*c**8 + 3523215360*a**4*b**12*c**7 - 503316480*a**3*b**
14*c**6 + 47185920*a**2*b**16*c**5 - 2621440*a*b**18*c**4 + 65536
*b**20*c**3) + _t**2*(-440401920*a**8*b*c**8 + 477102080*a**7*b**
3*c**7 - 174325760*a**6*b**5*c**6 + 11206656*a**5*b**7*c**5 + 892
9280*a**4*b**9*c**4 - 2600960*a**3*b**11*c**3 + 291840*a**2*b**13
*c**2 - 14080*a*b**15*c + 256*b**17) + 160000*a**7*c**4 + 492800*
a**6*b**2*c**3 + 351456*a**5*b**4*c**2 - 43120*a**4*b**6*c + 1225
*a**3*b**8, Lambda(_t, _t*log(x + (218103808*_t**3*a**6*b*c**9 -
276824064*_t**3*a**5*b**3*c**8 + 141557760*_t**3*a**4*b**5*c**7 -
 36700160*_t**3*a**3*b**7*c**6 + 4915200*_t**3*a**2*b**9*c**5 - 2
94912*_t**3*a*b**11*c**4 + 4096*_t**3*b**13*c**3 + 256000*_t*a**5
*c**5 - 888320*_t*a**4*b**2*c**4 - 57472*_t*a**3*b**4*c**3 + 1366
4*_t*a**2*b**6*c**2 - 832*_t*a*b**8*c + 16*_t*b**10)/(10000*a**4*
c**3 + 15000*a**3*b**2*c**2 - 1491*a**2*b**4*c + 35*a*b**6))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^8/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.883 $\int \frac{x^6}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=298

$$\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*
x*(4*a*b + (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (3*(b^2 + 4*a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2
]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2
+ 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 -
 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.5028, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{3x\left(x^2\left(4ac+b^2\right)+4ab\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^3\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\left(-\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{b\left(12ac+b^2\right)}{\sqrt{b^2-4ac}}+4ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^3*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*
x*(4*a*b + (b^2 + 4*a*c)*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (3*(b^2 + 4*a*c - (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2
]*Sqrt[c]*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2
+ 4*a*c + (b*(b^2 + 12*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 -
 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 76.4623, size = 286, normalized size = 0.96

$$\frac{x^3\left(2a+bx^2\right)}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(12ab+x^2\left(12ac+3b^2\right)\right)}{8\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\left(b\left(12ac+b^2\right)+\sqrt{-4ac+b^2}\left(4ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{16\sqrt{c}\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}-\frac{3\sqrt{2}\left(b\left(12ac+b^2\right)-\sqrt{-4ac+b^2}\left(4ac+b^2\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{16\sqrt{c}\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] x**3*(2*a + b*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)**2)
+ x*(12*a*b + x**2*(12*a*c + 3*b**2))/(8*(-4*a*c + b**2)**2*(a +
b*x**2 + c*x**4)) + 3*sqrt(2)*(b*(12*a*c + b**2) + sqrt(-4*a*c +
b**2)*(4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c
 + b**2)))/(16*sqrt(c)*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b*
*2)**(5/2)) - 3*sqrt(2)*(b*(12*a*c + b**2) - sqrt(-4*a*c + b**2)*
(4*a*c + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**
2)))/(16*sqrt(c)*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(
5/2))
```

Mathematica [A] time = 1.60246, size = 343, normalized size = 1.15

$$\frac{-\frac{4\left(ax\left(b-2cx^2\right)+b^2x^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{8abcx+24ac^2x^3+4b^3x+6b^2cx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}-12abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+\cdots\right.}{\cdots}}{16c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*b^3*x + 8*a*b*c*x + 6*b^2*c*x^3 + 24*a*c^2*x^3)/((b^2 - 4*a*c
)^2*(a + b*x^2 + c*x^4)) - (4*(b^2*x^3 + a*x*(b - 2*c*x^2)))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*Sqrt[2]*Sqrt[c]*(-b^3 - 12
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3 + 12
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^
```

```
(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)
```

Maple [B] time = 0.128, size = 4017, normalized size = 13.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (3/8*c*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*b*(16*a*c+5
*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(4*a*c-19*b^2)*a/(16*a^2
*c^2-8*a*b^2*c+b^4)*x^3+3/2*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(
c*x^4+b*x^2+a)^2+576*c^5/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b
^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a
*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-
16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16
*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^5*b-52
8*c^4/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/(
(16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c
-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*
c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*
c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^4*b^3+168*c^3/(-(4*a*c-b^
2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b
^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^
(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16
*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^
2)^5)^(1/2)))^(1/2))*a^3*b^5-18*c^2/(-(4*a*c-b^2)^5)^(1/2)/(16*a^
2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^
2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(
32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c
+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2
))*a^2*b^7+18/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*c
^2*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*
(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a
*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*
c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a^2*b^7+3/1
6/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*
a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2
)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x
*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^
5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*b^11-3/(16*a^2*c^2-8*a*b^2*c+b^
4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+
b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^
2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*
c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^2*b^2*c^2+3/4
*c/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-1
6*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a
*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*
```

c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2)
)*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a*b^9-3/16/(-(4*a*c-b^2)^5
)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^
3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/
2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16
*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*
b^2*c+b^4)*c)^(1/2))*b^11-3/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((
-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8
*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^
4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/
2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a^2*b^2*c^2-576/(-(4*a*c
-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*c^5*2^(1/2)/((-16*a^2*b
*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+
b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^
(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a
^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a^5*b+528/(-(4*a*c-b^2)^5)^(1/2)/
(16*a^2*c^2-8*a*b^2*c+b^4)*c^4*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-
b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*a
rctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2
*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*
c+b^4)*c)^(1/2))*a^4*b^3-168/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*c^3*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-
b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-
32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^
3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/
2))*a^3*b^5+12/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*
a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)
))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/(
(16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c
-b^2)^5)^(1/2)))^(1/2))*a^3*c^3-3/4*c/(16*a^2*c^2-8*a*b^2*c+b^4)*
2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5
+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c
^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2
-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a*b^4+3/16/(16*a^2
*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2
*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(3
2*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+
b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)
)*b^6+12/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b
^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1
/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-1
6*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a
*b^2*c+b^4)*c)^(1/2))*a^3*c^3-3/4*c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^
(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a
^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*
c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2
)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a*b^4+3/16/(16*a
^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*
a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/
2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*
a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)
^(1/2))*b^6-3/4*c/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^
4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+
b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^

```
2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*
c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a*b^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^{2}c+4\,ac^{2}\right)x^{7}+\left(5\,b^{3}+16\,abc\right)x^{5}+12\,a^{2}bx+\left(19\,ab^{2}-4\,a^{2}c\right)x^{3}}{8\left((b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{8}+2\,(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{6}+a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+(b^{6}-6\,ab^{4}c+32\,a^{3}c^{3})x^{4}+2\,(ab\right.}$$
$$+\frac{3\int\frac{(b^{2}+4\,ac)\,x^{2}-4\,ab}{cx^{4}+bx^{2}+a}\,dx}{8\,(b^{4}-8\,ab^{2}c+16\,a^{2}c^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^2*c + 4*a*c^2)*x^7 + (5*b^3 + 16*a*b*c)*x^5 + 12*a^2*b*
x + (19*a*b^2 - 4*a^2*c)*x^3)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2) + 3/8*integrate(((b^2 +
 4*a*c)*x^2 - 4*a*b)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 1
6*a^2*c^2)
```

Fricas [A] time = 0.324563, size = 4223, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^2*c + 4*a*c^2)*x^7 + 2*(5*b^3 + 16*a*b*c)*x^5 + 24*a^2
*b*x + 2*(19*a*b^2 - 4*a^2*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b
^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)
*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32
*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(
-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*
a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/
sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5
+ 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*
a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))
*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 -
 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 + (3*b^12*c - 56*a*b
^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 +
 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 +
```

160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 + (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x + 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 + 2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*a^5*c^6))*log(3*(5*b^4 + 40*a*b^2*c + 16*a^2*c^2)*x - 3*sqrt(1/2)*(2*b^7 - 24*a*b^5*c + 96*a^2*b^3*c^2 - 128*a^3*b*c^3 - (3*b^12*c - 56*a*b^

```
10*c^2 + 400*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 1280*a^4*b^4*c^5 +
2048*a^5*b^2*c^6 - 4096*a^6*c^7)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 1
60*a^2*b^6*c^4 - 640*a^3*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^
7))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (b^10*c - 20*a*b^8*c
^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*
a^5*c^6)/sqrt(b^10*c^2 - 20*a*b^8*c^3 + 160*a^2*b^6*c^4 - 640*a^3
*b^4*c^5 + 1280*a^4*b^2*c^6 - 1024*a^5*c^7))/(b^10*c - 20*a*b^8*c
^2 + 160*a^2*b^6*c^3 - 640*a^3*b^4*c^4 + 1280*a^4*b^2*c^5 - 1024*
a^5*c^6))))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c
- 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^
4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3
*c + 16*a^3*b*c^2)*x^2)
```

Sympy [A] time = 47.6673, size = 627, normalized size = 2.1

$$\frac{12a^2bx + x^7\left(12ac^2 + 3b^2c\right) + x^5\left(16abc + 5b^3\right) + x^3\left(-4a^2c + 19ab^2\right)}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\left(128a^2c^4 - 64ab^2c^3 + 8b^4c^2\right) + x^6\left(256a^2bc^3 - 128ab^3c^2 + 16b^5c\right) + x^4\left(256a^3c^3 - 48ab^4c + 8\right.}$$
$$+\mathrm{RootSum}\left(t^4\left(68719476736a^{10}c^{11} - 171798691840a^9b^2c^{10} + 193273528320a^8b^4c^9 - 128849018880a^7b^6c^8 + 56371445760a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (12*a**2*b*x + x**7*(12*a*c**2 + 3*b**2*c) + x**5*(16*a*b*c + 5*b
**3) + x**3*(-4*a**2*c + 19*a*b**2))/(128*a**4*c**2 - 64*a**3*b**
2*c + 8*a**2*b**4 + x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**4
*c**2) + x**6*(256*a**2*b*c**3 - 128*a*b**3*c**2 + 16*b**5*c) + x
**4*(256*a**3*c**3 - 48*a*b**4*c + 8*b**6) + x**2*(256*a**3*b*c**
2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_t**4*(68719476736*a*
*10*c**11 - 171798691840*a**9*b**2*c**10 + 193273528320*a**8*b**4
*c**9 - 128849018880*a**7*b**6*c**8 + 56371445760*a**6*b**8*c**7
- 16911433728*a**5*b**10*c**6 + 3523215360*a**4*b**12*c**5 - 5033
16480*a**3*b**14*c**4 + 47185920*a**2*b**16*c**3 - 2621440*a*b**1
8*c**2 + 65536*b**20*c) + _t**2*(-188743680*a**7*b*c**7 + 1415577
60*a**6*b**3*c**6 - 2359296*a**5*b**5*c**5 - 26542080*a**4*b**7*c
**4 + 9584640*a**3*b**9*c**3 - 1290240*a**2*b**11*c**2 + 46080*a*
b**13*c + 2304*b**15) + 20736*a**5*c**4 + 103680*a**4*b**2*c**3 +
 142560*a**3*b**4*c**2 + 32400*a**2*b**6*c + 2025*a*b**8, Lambda(
_t, _t*log(x + (33554432*_t**3*a**6*c**7 - 16777216*_t**3*a**5*b*
*2*c**6 - 10485760*_t**3*a**4*b**4*c**5 + 10485760*_t**3*a**3*b**
6*c**4 - 3276800*_t**3*a**2*b**8*c**3 + 458752*_t**3*a*b**10*c**2
 - 24576*_t**3*b**12*c - 64512*_t*a**3*b*c**3 - 43776*_t*a**2*b**
3*c**2 - 21312*_t*a*b**5*c - 144*_t*b**7)/(432*a**2*c**2 + 1080*a
*b**2*c + 135*b**4))))
```

GIAC/XCAS [A] time = 31.299, size = 1, normalized size = 0.

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Done
```

3.884 $\int \frac{x^4}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=289

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7
*b^2 - 4*a*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 -
 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[c]*(3*b^2 +
4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.34589, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{x\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{4\sqrt{2}\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x*(7
*b^2 - 4*a*c + 12*b*c*x^2))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + (3*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 -
```

```
 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[c]*(3*b^2 +
4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(4*Sqrt[2]*(b^2 - 4*a*c)^(5/2)*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 87.4232, size = 274, normalized size = 0.95

$$-\frac{3\sqrt{2}\sqrt{c}\left(ac+\frac{3b^2}{4}+\frac{b\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{3\sqrt{2}\sqrt{c}\left(ac+\frac{3b^2}{4}-\frac{b\sqrt{-4ac+b^2}}{2}\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{5}{2}}}+\frac{x\left(2a+bx^2\right)}{4\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)^2}-\frac{x\left(-4ac+7b^2+12bcx^2\right)}{8\left(-4ac+b^2\right)^2\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -3*sqrt(2)*sqrt(c)*(a*c + 3*b**2/4 + b*sqrt(-4*a*c + b**2)/2)*ata
n(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*sqrt(b + sq
rt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + 3*sqrt(2)*sqrt(c)*(a
*c + 3*b**2/4 - b*sqrt(-4*a*c + b**2)/2)*atan(sqrt(2)*sqrt(c)*x/s
qrt(b - sqrt(-4*a*c + b**2)))/(2*sqrt(b - sqrt(-4*a*c + b**2))*(-
4*a*c + b**2)**(5/2)) + x*(2*a + b*x**2)/(4*(-4*a*c + b**2)*(a +
b*x**2 + c*x**4)**2) - x*(-4*a*c + 7*b**2 + 12*b*c*x**2)/(8*(-4*a
*c + b**2)**2*(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 1.31458, size = 285, normalized size = 0.99

$$\frac{1}{8}\left(\frac{2\left(2ax+bx^3\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{4acx-7b^2x-12bcx^3}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*(2*a*x + b*x^3))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (-7*
b^2*x + 4*a*c*x - 12*b*c*x^3)/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + (3*Sqrt[2]*Sqrt[c]*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(3*
b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b
^2 - 4*a*c]]))/8
```

Maple [B] time = 0.075, size = 3657, normalized size = 12.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] (-3/2*b*c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c*(4*a*c-19*b^2)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*b*(16*a*c+5*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3-3/8*a*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/
(c*x^4+b*x^2+a)^2-384/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*
c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^
3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*
a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^
2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^5*c^6-3/4
*c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)
*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arc
tan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-
8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/
2)))^(1/2))*b^5-3/4*c/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^
2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2
*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x
*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(1
6*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*b^5-9/8*c/(-(4*a*c-b^2)^5)^(1/
2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)
*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arc
tan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-
8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/
2)))^(1/2))*b^10+9/8*c/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2
*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1
/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3
+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-
(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*b^10-8
4/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*
```

a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2
)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x
*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^
5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^2*b^6*c^3+33/2/(-(4*a*c-b^2)^
5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*
c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/
2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^
2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^
5)^(1/2)))^(1/2))*a*b^8*c^2-96/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2
-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^
2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32
*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*
c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)
)*a^4*c^5*b^2-144/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^
4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*
(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a
*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*
c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a^3*c^4*b^4
+84/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-
16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*
a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4
*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2
))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*a^2*b^6*c^3-33/2/(-(4*a*c
-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2
+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)
*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2
)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c
^2-8*a*b^2*c+b^4)*c)^(1/2))*a*b^8*c^2+96/(-(4*a*c-b^2)^5)^(1/2)/(
16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(
16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(
1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*
b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))
^(1/2))*a^4*c^5*b^2+144/(-(4*a*c-b^2)^5)^(1/2)/(16*a^2*c^2-8*a*b^
2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*
b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-1
6*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*
a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*a^3*c^4*b
^4-12/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b
^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2)*
arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)*x*2^(1/2)/((16*a^2*c
^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b^2)^5)^
(1/2)))^(1/2))*b*a^2*c^3+6/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((1
6*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+b^5+(-(4*a*c-b
^2)^5)^(1/2)))^(1/2)*arctan(1/2*(32*a^2*c^3-16*a*b^2*c^2+2*b^4*c)
*x*2^(1/2)/((16*a^2*c^2-8*a*b^2*c+b^4)*c*(16*a^2*b*c^2-8*a*b^3*c+
b^5+(-(4*a*c-b^2)^5)^(1/2)))^(1/2))*b^3*a*c^2+384/(-(4*a*c-b^2)^5
)^(1/2)/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^
3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/
2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16
*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*
b^2*c+b^4)*c)^(1/2))*a^5*c^6-12/(16*a^2*c^2-8*a*b^2*c+b^4)*2^(1/2
)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c
^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(-32*a^2*c^3+16*a*b^2*c^2-
2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c-b^2)^5)

```
^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2))*b*a^2*c^3+6/(16*a^2*
c^2-8*a*b^2*c+b^4)*2^(1/2)/((-16*a^2*b*c^2+8*a*b^3*c-b^5+(-(4*a*c
-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1/2)*arctanh(1/2*(
-32*a^2*c^3+16*a*b^2*c^2-2*b^4*c)*x*2^(1/2)/((-16*a^2*b*c^2+8*a*b
^3*c-b^5+(-(4*a*c-b^2)^5)^(1/2))*(16*a^2*c^2-8*a*b^2*c+b^4)*c)^(1
/2))*b^3*a*c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{12\,bc^2x^7 + \left(19\,b^2c - 4\,ac^2\right)x^5 + \left(5\,b^3 + 16\,abc\right)x^3 + 3\,\left(ab^2 + 4\,a^2c\right)x}{8\left((b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^8 + 2\,(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^6 + a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^6 - 6\,ab^4c + 32\,a^3c^3)x^4 + 2\,(a\right.}$$
$$-\frac{3\int \frac{4\,bcx^2 - b^2 - 4\,ac}{cx^4 + bx^2 + a}\,dx}{8\,(b^4 - 8\,ab^2c + 16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(12*b*c^2*x^7 + (19*b^2*c - 4*a*c^2)*x^5 + (5*b^3 + 16*a*b*c
)*x^3 + 3*(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c
^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2
*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2) - 3/8*integrate((4*b*c
*x^2 - b^2 - 4*a*c)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16
*a^2*c^2)
```

Fricas [A] time = 0.34205, size = 4223, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(24*b*c^2*x^7 + 2*(19*b^2*c - 4*a*c^2)*x^5 + 2*(5*b^3 + 16*
a*b*c)*x^3 + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^
8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^
5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80
*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b
^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*
b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 10
24*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b
^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*
```

b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 - (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 + (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c + 40*a*b^2*c^2 + 16*a^2*c^3)*x + 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 128*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 - 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(b^5 + 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4

```
 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^6*c^2 - 640*
a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))*log(3*(5*b^4*c +
40*a*b^2*c^2 + 16*a^2*c^3)*x - 3/2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 1
28*a^3*b^2*c^3 - 256*a^4*c^4 + (a*b^13 - 8*a^2*b^11*c - 80*a^3*b^
9*c^2 + 1280*a^4*b^7*c^3 - 6400*a^5*b^5*c^4 + 14336*a^6*b^3*c^5 -
 12288*a^7*b*c^6)/sqrt(a^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2
- 640*a^5*b^4*c^3 + 1280*a^6*b^2*c^4 - 1024*a^7*c^5))*sqrt(-(b^5
+ 40*a*b^3*c + 80*a^2*b*c^2 - (a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5)/sqrt(a
^2*b^10 - 20*a^3*b^8*c + 160*a^4*b^6*c^2 - 640*a^5*b^4*c^3 + 1280
*a^6*b^2*c^4 - 1024*a^7*c^5))/(a*b^10 - 20*a^2*b^8*c + 160*a^3*b^
6*c^2 - 640*a^4*b^4*c^3 + 1280*a^5*b^2*c^4 - 1024*a^6*c^5))) + 6*
(a*b^2 + 4*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 -
8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)
```

Sympy [A] time = 48.1098, size = 644, normalized size = 2.23

$$-\frac{12bc^2x^7 + x^5\left(-4ac^2 + 19b^2c\right) + x^3\left(16abc + 5b^3\right) + x\left(12a^2c + 3ab^2\right)}{128a^4c^2 - 64a^3b^2c + 8a^2b^4 + x^8\left(128a^2c^4 - 64ab^2c^3 + 8b^4c^2\right) + x^6\left(256a^2bc^3 - 128ab^3c^2 + 16b^5c\right) + x^4\left(256a^3c^3 - 48ab^4c + \right.}$$

$$+\text{RootSum}\left(t^4\left(68719476736a^{11}c^{10} - 171798691840a^{10}b^2c^9 + 193273528320a^9b^4c^8 - 128849018880a^8b^6c^7 + 56371445760a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(12*b*c**2*x**7 + x**5*(-4*a*c**2 + 19*b**2*c) + x**3*(16*a*b*c
+ 5*b**3) + x*(12*a**2*c + 3*a*b**2))/(128*a**4*c**2 - 64*a**3*b*
*2*c + 8*a**2*b**4 + x**8*(128*a**2*c**4 - 64*a*b**2*c**3 + 8*b**
4*c**2) + x**6*(256*a**2*b*c**3 - 128*a*b**3*c**2 + 16*b**5*c) +
x**4*(256*a**3*c**3 - 48*a*b**4*c + 8*b**6) + x**2*(256*a**3*b*c*
*2 - 128*a**2*b**3*c + 16*a*b**5)) + RootSum(_t**4*(68719476736*a
**11*c**10 - 171798691840*a**10*b**2*c**9 + 193273528320*a**9*b**
4*c**8 - 128849018880*a**8*b**6*c**7 + 56371445760*a**7*b**8*c**6
 - 16911433728*a**6*b**10*c**5 + 3523215360*a**5*b**12*c**4 - 503
316480*a**4*b**14*c**3 + 47185920*a**3*b**16*c**2 - 2621440*a**2*
b**18*c + 65536*a*b**20) + _t**2*(-188743680*a**7*b*c**7 + 141557
760*a**6*b**3*c**6 - 2359296*a**5*b**5*c**5 - 26542080*a**4*b**7*
c**4 + 9584640*a**3*b**9*c**3 - 1290240*a**2*b**11*c**2 + 46080*a
*b**13*c + 2304*b**15) + 20736*a**4*c**5 + 103680*a**3*b**2*c**4
+ 142560*a**2*b**4*c**3 + 32400*a*b**6*c**2 + 2025*b**8*c, Lambda
(_t, _t*log(x + (50331648*_t**3*a**7*b*c**6 - 58720256*_t**3*a**6
*b**3*c**5 + 26214400*_t**3*a**5*b**5*c**4 - 5242880*_t**3*a**4*b
**7*c**3 + 327680*_t**3*a**3*b**9*c**2 + 32768*_t**3*a**2*b**11*c
 - 4096*_t**3*a*b**13 + 18432*_t*a**4*c**4 - 78336*_t*a**3*b**2*c
**3 - 40320*_t*a**2*b**4*c**2 - 3168*_t*a*b**6*c - 144*_t*b**8)/(
```

```
432*a**2*c**3 + 1080*a*b**2*c**2 + 135*b**4*c))))
```

GIAC/XCAS [A] time = 29.2936, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Done
```

3.885 $\int \frac{x^2}{(a+bx^2+cx^4)^3} \, dx$

Optimal. Leaf size=311

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] -(x*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(
b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a
+ b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.50605, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{x\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x\left(cx^2\left(20ac+b^2\right)+b\left(8ac+b^2\right)\right)}{8a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{b\left(b^2-52ac\right)}{\sqrt{b^2-4ac}}+20ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (x*(
b*(b^2 + 8*a*c) + c*(b^2 + 20*a*c)*x^2))/(8*a*(b^2 - 4*a*c)^2*(a
+ b*x^2 + c*x^4)) + (Sqrt[c]*(b^2 + 20*a*c + (b*(b^2 - 52*a*c))/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
```

```
4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]
]) + (Sqrt[c]*(b^2 + 20*a*c - (b*(b^2 - 52*a*c))/Sqrt[b^2 - 4*a*c
])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sq
rt[2]*a*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 81.3171, size = 291, normalized size = 0.94

$$-\frac{x\left(b+2cx^{2}\right)}{4\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{2}}-\frac{\sqrt{2}\sqrt{c}\left(b\left(-52ac+b^{2}\right)-\sqrt{-4ac+b^{2}}\left(20ac+b^{2}\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^{2}}}}\right)}{16a\sqrt{b+\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{5}{2}}}+\frac{\sqrt{2}\sqrt{c}\left(b\left(-52ac+b^{2}\right)+\sqrt{-4ac+b^{2}}\left(20ac+b^{2}\right)\right)\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^{2}}}}\right)}{16a\sqrt{b-\sqrt{-4ac+b^{2}}}\left(-4ac+b^{2}\right)^{\frac{5}{2}}}+\frac{x\left(b\left(8ac+b^{2}\right)+cx^{2}\left(20ac+b^{2}\right)\right)}{8a\left(-4ac+b^{2}\right)^{2}\left(a+bx^{2}+cx^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -x*(b + 2*c*x**2)/(4*(-4*a*c + b**2)*(a + b*x**2 + c*x**4)**2) -
sqrt(2)*sqrt(c)*(b*(-52*a*c + b**2) - sqrt(-4*a*c + b**2)*(20*a*c
 + b**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(
16*a*sqrt(b + sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + sqrt
(2)*sqrt(c)*(b*(-52*a*c + b**2) + sqrt(-4*a*c + b**2)*(20*a*c + b
**2))*atan(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(16*a
*sqrt(b - sqrt(-4*a*c + b**2))*(-4*a*c + b**2)**(5/2)) + x*(b*(8*
a*c + b**2) + c*x**2*(20*a*c + b**2))/(8*a*(-4*a*c + b**2)**2*(a
+ b*x**2 + c*x**4))
```

Mathematica [A] time = 1.54539, size = 334, normalized size = 1.07

$$\frac{1}{16}\left(-\frac{4x(b+2cx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(8abc+20ac^2x^2+b^3+b^2cx^2)}{a(b^2-4ac)^2(a+bx^2+cx^4)} +\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} +\frac{\sqrt{2}\sqrt{c}\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}+52abc-b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*x*(b + 2*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*
x*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2))/(a*(b^2 - 4*a*c)^2*
(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(b^3 - 52*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-b^3 + 52*a*b*c + b^2*S
qrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[
b + Sqrt[b^2 - 4*a*c]]))/16
```

Maple [B] time = 0.165, size = 2958, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] -9*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
)*b^2-20*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))-4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-
1/2/c*(-4*a*c+b^2)^(1/2))^2*a^2*x^3*b+3*c^2/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*b^3*x^3-42
*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+
```

b^2)^(1/2))^2*a^2*b^2*x+21/2*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*a*b^4-3/4*c^2/(-4*a*c+b
^2)^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+20*c^4/(-
4*a*c+b^2)^2/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-9*c^2
/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b
/c)^2*a*x^3*b^2+9*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))*b^2+3/4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2
b/c-1/2/c(-4*a*c+b^2)^(1/2))^2*x*b^5-7/8/(-4*a*c+b^2)^(5/2)/(4*
a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*b^6+7/8/(-4
*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b
/c)^2*x*b^6+12*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/
c*(-4*a*c+b^2)^(1/2))^2*x*a^2*b-6*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/
(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*a*b^3-9*c^2/(-4*a*c+b^
2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x^3
*b^2-3*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^
2)^(1/2)+1/2*b/c)^2*a*b^3*x^3+42*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^
2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a^2*b^2*x-21/2*c/(-
4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*
b/c)^2*x*a*b^4+4*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*
(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a^2*x^3*b+3/4/(-4*a*c+b^2)^2/(4*a*c
-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*b^5+15/4*c^2/(
-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5+
15/4*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2))*b^5+3/4*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*b^4+12*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*
(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a^2*b-6*c/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a*b^3+52*c^4/(-4
*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-
27*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^3-1/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2))*b^7+52*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b-27*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c
-b^2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-1/16*c/(-4*a*c+b^2)
^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^7+1/16*c
/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^6-
1/16*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2))*b^6+3/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^
2)^(1/2)+1/2*b/c)^2*x^3*b^4+1/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2
/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x^3*b^7+20*c^3/(-4*a*
c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a

```
^2*x^3+3/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+
b^2)^(1/2)+1/2*b/c)^2*x^3*b^5-56*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^
2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*a^3+3/4*c/(-4*a*c
+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^
3*b^4+20*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*
a*c+b^2)^(1/2))^2*a^2*x^3-3/4*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/
(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*b^5+56*c^3/(-4*a*c+b
^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*
x*a^3+1/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(
1/2)+1/2*b/c)^2/a*x^3*b^6+1/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+
1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x^3*b^6-1/16/(-4*a*c+b^2)^(
5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x^3
*b^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^2c^2+20\,ac^3\right)x^7+2\left(b^3c+14\,abc^2\right)x^5+\left(b^4+5\,ab^2c+36\,a^2c^2\right)x^3-\left(ab^3-16\,a^2bc\right)x}{8\left((ab^4c^2-8\,a^2b^2c^3+16\,a^3c^4)x^8+a^3b^4-8\,a^4b^2c+16\,a^5c^2+2\,(ab^5c-8\,a^2b^3c^2+16\,a^3bc^3)x^6+(ab^6-6\,a^2b^4c+32\,a^4c^3)x^4\right.}$$
$$+\frac{\int\frac{b^3-16\,abc+\left(b^2c+20\,ac^2\right)x^2}{cx^4+bx^2+a}\,dx}{8\left(ab^4-8\,a^2b^2c+16\,a^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^2*c^2 + 20*a*c^3)*x^7 + 2*(b^3*c + 14*a*b*c^2)*x^5 + (b^4
 + 5*a*b^2*c + 36*a^2*c^2)*x^3 - (a*b^3 - 16*a^2*b*c)*x)/((a*b^4*
c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b
^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 1
6*a^4*b*c^2)*x^2) + 1/8*integrate((b^3 - 16*a*b*c + (b^2*c + 20*a
*c^2)*x^2)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*
c^2)
```

Fricas [A] time = 0.42304, size = 5099, normalized size = 16.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(b^2*c^2 + 20*a*c^3)*x^7 + 4*(b^3*c + 14*a*b*c^2)*x^5 + 2
*(b^4 + 5*a*b^2*c + 36*a^2*c^2)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a
```

```
^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2
 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^
2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c
^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c
^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3
 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*
a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4
*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^
8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024
*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 +
 10000*a^3*c^5)*x + 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^
7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 -
(a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 +
 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*
a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^
7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 -
 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 168
0*a^3*b*c^3 + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^
6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2
*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 64
0*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 -
20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c
^4 - 1024*a^8*c^5))) - sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16
*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c -
 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^
4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-
(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (a^3*b^10
- 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2
*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b
^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^1
0*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b
^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log(
(35*b^6*c^2 - 1491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)
*x - 1/2*sqrt(1/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^
3*b^5*c^3 + 21824*a^4*b^3*c^4 - 25600*a^5*b*c^5 - (a^3*b^14 - 38*
a^4*b^12*c + 480*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c
^4 + 1536*a^8*b^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt(
(b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a
^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)
))*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 + (
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^
2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c +
160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c
^5))) + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 +
 a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2
+ 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*
(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5
*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c
+ 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8
*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^
8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 102
4*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^
```

```
6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1
491*a*b^4*c^3 + 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x + 1/2*sqrt(1
/2)*(b^11 - 53*a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 218
24*a^4*b^3*c^4 - 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 48
0*a^5*b^10*c^2 - 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b
^4*c^5 - 32768*a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2
*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 64
0*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 -
 35*a*b^5*c + 280*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a
^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
 1024*a^8*c^5)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 -
20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*
c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2
 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/
2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)
*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^
3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(b^7 - 35*a*b^5*c + 280*a^2*b^
3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c
^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((b^4
 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b
^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(
a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 128
0*a^7*b^2*c^4 - 1024*a^8*c^5))*log((35*b^6*c^2 - 1491*a*b^4*c^3 +
 15000*a^2*b^2*c^4 + 10000*a^3*c^5)*x - 1/2*sqrt(1/2)*(b^11 - 53*
a*b^9*c + 940*a^2*b^7*c^2 - 6832*a^3*b^5*c^3 + 21824*a^4*b^3*c^4
- 25600*a^5*b*c^5 + (a^3*b^14 - 38*a^4*b^12*c + 480*a^5*b^10*c^2
- 2720*a^6*b^8*c^3 + 6400*a^7*b^6*c^4 + 1536*a^8*b^4*c^5 - 32768*
a^9*b^2*c^6 + 40960*a^10*c^7)*sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^
2)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 +
 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))*sqrt(-(b^7 - 35*a*b^5*c + 2
80*a^2*b^3*c^2 + 1680*a^3*b*c^3 - (a^3*b^10 - 20*a^4*b^8*c + 160*
a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*
sqrt((b^4 - 50*a*b^2*c + 625*a^2*c^2)/(a^6*b^10 - 20*a^7*b^8*c +
160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11
*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*
c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - 2*(a*b^3 - 16*a^2*b*c)
*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3
)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a
^3*b^3*c + 16*a^4*b*c^2)*x^2)
```

Sympy [A] time = 53.3318, size = 733, normalized size = 2.36

$$\frac{x^7\left(20ac^3+b^2c^2\right)+x^5\left(28abc^2+2b^3c\right)+x^3\left(36a^2c^2+5ab^2c+b^4\right)+x\left(16a^2bc-\right.}{128a^5c^2-64a^4b^2c+8a^3b^4+x^8\left(128a^3c^4-64a^2b^2c^3+8ab^4c^2\right)+x^6\left(256a^3bc^3-128a^2b^3c^2+16ab^5c\right)+x^4\left(256a^4c^3-48a^2b\right.}$$

$$+\mathrm{RootSum}\left(t^4\left(68719476736a^{13}c^{10}-171798691840a^{12}b^2c^9+193273528320a^{11}b^4c^8-128849018880a^{10}b^6c^7+56371445760\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] (x**7*(20*a*c**3 + b**2*c**2) + x**5*(28*a*b*c**2 + 2*b**3*c) + x
**3*(36*a**2*c**2 + 5*a*b**2*c + b**4) + x*(16*a**2*b*c - a*b**3)
)/(128*a**5*c**2 - 64*a**4*b**2*c + 8*a**3*b**4 + x**8*(128*a**3*
c**4 - 64*a**2*b**2*c**3 + 8*a*b**4*c**2) + x**6*(256*a**3*b*c**3
 - 128*a**2*b**3*c**2 + 16*a*b**5*c) + x**4*(256*a**4*c**3 - 48*a
**2*b**4*c + 8*a*b**6) + x**2*(256*a**4*b*c**2 - 128*a**3*b**3*c
+ 16*a**2*b**5)) + RootSum(_t**4*(68719476736*a**13*c**10 - 17179
8691840*a**12*b**2*c**9 + 193273528320*a**11*b**4*c**8 - 12884901
8880*a**10*b**6*c**7 + 56371445760*a**9*b**8*c**6 - 16911433728*a
**8*b**10*c**5 + 3523215360*a**7*b**12*c**4 - 503316480*a**6*b**1
4*c**3 + 47185920*a**5*b**16*c**2 - 2621440*a**4*b**18*c + 65536*
a**3*b**20) + _t**2*(-440401920*a**8*b*c**8 + 477102080*a**7*b**3
*c**7 - 174325760*a**6*b**5*c**6 + 11206656*a**5*b**7*c**5 + 8929
280*a**4*b**9*c**4 - 2600960*a**3*b**11*c**3 + 291840*a**2*b**13*
c**2 - 14080*a*b**15*c + 256*b**17) + 160000*a**4*c**7 + 492800*a
**3*b**2*c**6 + 351456*a**2*b**4*c**5 - 43120*a*b**6*c**4 + 1225*
b**8*c**3, Lambda(_t, _t*log(x + (167772160*_t**3*a**10*c**7 - 13
4217728*_t**3*a**9*b**2*c**6 + 6291456*_t**3*a**8*b**4*c**5 + 262
14400*_t**3*a**7*b**6*c**4 - 11141120*_t**3*a**6*b**8*c**3 + 1966
080*_t**3*a**5*b**10*c**2 - 155648*_t**3*a**4*b**12*c + 4096*_t**
3*a**3*b**14 - 742400*_t*a**5*b*c**5 - 156928*_t*a**4*b**3*c**4 -
 70336*_t*a**3*b**5*c**3 + 14480*_t*a**2*b**7*c**2 - 848*_t*a*b**
9*c + 16*_t*b**11)/(10000*a**3*c**5 + 15000*a**2*b**2*c**4 - 1491
*a*b**4*c**3 + 35*b**6*c**2))))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.886 $\int \frac{1}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=355

$$\frac{x\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + (x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^
2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4
 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2
]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[
c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8
*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 3.62638, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$\frac{x\left(3bcx^2\left(b^2-8ac\right)+\left(b^2-7ac\right)\left(3b^2-4ac\right)\right)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{c}\left(56a^2c^2-10ab^2c+b\left(b^2-8ac\right)\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{c}\left(-\frac{56a^2c^2-10ab^2c+b^4}{\sqrt{b^2-4ac}}-8abc+b^3\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4
)^2) + (x*((b^2 - 7*a*c)*(3*b^2 - 4*a*c) + 3*b*c*(b^2 - 8*a*c)*x^
2))/(8*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[c]*(b^4
```

```
 - 10*a*b^2*c + 56*a^2*c^2 + b*(b^2 - 8*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2
]*a^2*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[
c]*(b^3 - 8*a*b*c - (b^4 - 10*a*b^2*c + 56*a^2*c^2)/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8
*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 1.95865, size = 372, normalized size = 1.05

$$\frac{\frac{2x\left(28a^2c^2-25ab^2c-24abc^2x^2+3b^4+3b^3cx^2\right)}{\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-10ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(56a^2c^2-10ab^2c+8 \cdots\right.}{\cdots}}{16a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(-3),x]
```

```
[Out] ((4*a*x*(b^2 - 2*a*c + b*c*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (2*x*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*
a*b*c^2*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*
Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] -
8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) - (3*Sqrt[2]*Sqrt[c]*(b^4 - 10*a*b^2*c + 56*a^2*c^2 - b^3*Sqr
t[b^2 - 4*a*c] + 8*a*b*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*a^2)
```

Maple [B] time = 0.136, size = 3360, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 114*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))*b^2-45/2*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(
-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x^3*b^4+15/4*c/(-4*a*c+b^2)^(5/2)/(4
*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*b^5+5/16/(
-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(
1/2))^2/a*x*b^7+45/2*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/
2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*b^4-15/4*c/(-4*a*c+b^2)^(5/
2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x*b^5+3
/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/
2*b/c)^2/a^2*x^3*b^7+3/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1
/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a^2*x^3*b^8+3/16/(-4*a*c+b^2)^
2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a^2*x^3*
b^7+5/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+
b^2)^(1/2))^2/a*x*b^6+5/16/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/
c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x*b^6-3/16/(-4*a*c+b^2)^(5/2)/(
4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a^2*x^3*b^8
-5/16/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1
/2)+1/2*b/c)^2/a*x*b^7-72*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x
^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x^3*a^2+15*c^2/(-4*a*c+b^2
)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*b^
3-21/4*c/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+
b^2)^(1/2))^2*x*b^4+15*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/
c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x^3*b^3-21/4*c/(-4*a*c+b^2)^2/(4*
a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x*b^4-44*c^3/
(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/
c)^2*a^2*x-44*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c
*(-4*a*c+b^2)^(1/2))^2*a^2*x-57/2*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b
^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4+66*c^3/(-4*a*c+b^2)^(5/2)/
(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*x^3*a*b^2+
20*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(
1/2)+1/2*b/c)^2*a^2*b*x+27/8*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(
x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2/a*x^3*b^6-15*c^2/(-4*a*c+
b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2
*a*x*b^3+15*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2
/c*(-4*a*c+b^2)^(1/2))^2*a*x*b^3-57/2*c^3/(-4*a*c+b^2)^(5/2)/(4*a
*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^4-66*c^3/(-4*a*c+b^2)
^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3
*a*b^2-20*c^3/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c
*(-4*a*c+b^2)^(1/2))^2*a^2*b*x-168*c^5/(-4*a*c+b^2)^(5/2)/(4*a*c-
b^2)^2*a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+27*c^2/(-4*a*c+b^2)^2/(4*
a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^2*a*x*b^2-168*c
^5/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2*a^2*2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))-27/8*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2/c
*(-4*a*c+b^2)^(1/2))^2/a*x^3*b^6-24*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2
```

```
)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x^3*b-15*c^3/(-4*a
*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^3-24*c^
3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*
b/c)^2*a*x^3*b+15*c^3/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b^3+27*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2*b
/c-1/2/c*(-4*a*c+b^2)^(1/2))^2*a*x*b^2-3*c/(-4*a*c+b^2)^2/(4*a*c-
b^2)^2/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2/a*x^3*b^5-3*c/(-4
*a*c+b^2)^2/(4*a*c-b^2)^2/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)^
2/a*x^3*b^5+72*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/(x^2+1/2*b/c-
1/2/c*(-4*a*c+b^2)^(1/2))^2*x^3*a^2+3*c^2/(-4*a*c+b^2)^2/(4*a*c-b
^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^5-3/16*c/(-4*a*c+b^2)^2
/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^7+27/8*c^2/(
-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*b^6+27/8*c^2/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))*b^6-3/16*c/(-4*a*c+b^2)^(5/2)/(4*a*c-b^2)^2/a^2*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*b^8+114*c^4/(-4*a*c+b^2)^(5/2)/(4*a*c-b^
2)^2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2-3/16*c/(-4*a*c+b^2)^(5
/2)/(4*a*c-b^2)^2/a^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^8+3/16*c/
(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^7
-24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*b-3*c^2/(-4*a*c+b^2)^2/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2))*b^5+24*c^4/(-4*a*c+b^2)^2/(4*a*c-b^2)^2*a*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^3c^2-8\,abc^3\right)x^7+\left(6\,b^4c-49\,ab^2c^2+28\,a^2c^3\right)x^5+\left(3\,b^5-20\,ab^3c-4\,a^2bc^2\right)x^3+\left(5\,ab^4-37\,a^2b\right.}{8\left(\left(a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4\right)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\left(a^2b^5c-8\,a^3b^3c^2+16\,a^4bc^3\right)x^6+\left(a^2b^6-6\,a^3b^4c+32\,a^5c^3\right)\right.}$$
$$-\frac{-3\int\frac{b^4-9\,ab^2c+28\,a^2c^2+\left(b^3c-8\,abc^2\right)x^2}{cx^4+bx^2+a}\,dx}{8\left(a^2b^4-8\,a^3b^2c+16\,a^4c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-3),x, algorithm="maxima")
```

```
[Out] 1/8*(3*(b^3*c^2 - 8*a*b*c^3)*x^7 + (6*b^4*c - 49*a*b^2*c^2 + 28*a
```

```
^2*c^3)*x^5 + (3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 + (5*a*b^4 -
 37*a^2*b^2*c + 44*a^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16
*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c
 - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 3
2*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2) -
3/8*integrate(-(b^4 - 9*a*b^2*c + 28*a^2*c^2 + (b^3*c - 8*a*b*c^2
)*x^2)/(c*x^4 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^
2)
```

Fricas [A] time = 0.602577, size = 5836, normalized size = 16.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-3),x, algorithm="fricas")
```

```
[Out] 1/16*(6*(b^3*c^2 - 8*a*b*c^3)*x^7 + 2*(6*b^4*c - 49*a*b^2*c^2 + 2
8*a^2*c^3)*x^5 + 2*(3*b^5 - 20*a*b^3*c - 4*a^2*b*c^2)*x^3 - 3*sqr
t(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4
- 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^
4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*
b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a*b^7*c +
189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 -
20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c
^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 10
78*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a
^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^
5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3
 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^8*c^3 - 447*a*
b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*x
 + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a
^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b
^2*c^6 - 87808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11
*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5
+ 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c +
 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 -
20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b
^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c
^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c
+ 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^1
0*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^
3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 -
 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^1
0 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b
^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*
b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2
)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^
```

3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7 - (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) - 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19208*a^3*b^2*c^6 + 38416*a^4*c^7)*x + 27/2*sqrt(1/2)*(b^14 - 32*a*b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4 - 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7 + (a^5*b^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 3*sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2

```
- 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 -
 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/
(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3
+ 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c +
 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10
*c^5))*log(27*(21*b^8*c^3 - 447*a*b^6*c^4 + 4189*a^2*b^4*c^5 - 19
208*a^3*b^2*c^6 + 38416*a^4*c^7)*x - 27/2*sqrt(1/2)*(b^14 - 32*a*
b^12*c + 464*a^2*b^10*c^2 - 3885*a^3*b^8*c^3 + 20088*a^4*b^6*c^4
- 63680*a^5*b^4*c^5 + 113792*a^6*b^2*c^6 - 87808*a^7*c^7 + (a^5*b
^15 - 31*a^6*b^13*c + 424*a^7*b^11*c^2 - 3280*a^8*b^9*c^3 + 15360
*a^9*b^7*c^4 - 43264*a^10*b^5*c^5 + 67584*a^11*b^3*c^6 - 45056*a^
12*b*c^7)*sqrt((b^8 - 22*a*b^6*c + 219*a^2*b^4*c^2 - 1078*a^3*b^2
*c^3 + 2401*a^4*c^4)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^
2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(
-(b^9 - 21*a*b^7*c + 189*a^2*b^5*c^2 - 840*a^3*b^3*c^3 + 1680*a^4
*b*c^4 - (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4
*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((b^8 - 22*a*b^6*c +
 219*a^2*b^4*c^2 - 1078*a^3*b^2*c^3 + 2401*a^4*c^4)/(a^10*b^10 -
20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b
^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*
c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))) + 2*(
5*a*b^4 - 37*a^2*b^2*c + 44*a^3*c^2)*x)/((a^2*b^4*c^2 - 8*a^3*b^2
*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(
a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 - 6*a^3*
b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2
)*x^2)
```

Sympy [A] time = 66.4071, size = 818, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -(x**7*(24*a*b*c**3 - 3*b**3*c**2) + x**5*(-28*a**2*c**3 + 49*a*b
**2*c**2 - 6*b**4*c) + x**3*(4*a**2*b*c**2 + 20*a*b**3*c - 3*b**5
) + x*(-44*a**3*c**2 + 37*a**2*b**2*c - 5*a*b**4))/(128*a**6*c**2
 - 64*a**5*b**2*c + 8*a**4*b**4 + x**8*(128*a**4*c**4 - 64*a**3*b
**2*c**3 + 8*a**2*b**4*c**2) + x**6*(256*a**4*b*c**3 - 128*a**3*b
**3*c**2 + 16*a**2*b**5*c) + x**4*(256*a**5*c**3 - 48*a**3*b**4*c
 + 8*a**2*b**6) + x**2*(256*a**5*b*c**2 - 128*a**4*b**3*c + 16*a*
*3*b**5)) + RootSum(_t**4*(68719476736*a**15*c**10 - 171798691840
*a**14*b**2*c**9 + 193273528320*a**13*b**4*c**8 - 128849018880*a*
*12*b**6*c**7 + 56371445760*a**11*b**8*c**6 - 16911433728*a**10*b
**10*c**5 + 3523215360*a**9*b**12*c**4 - 503316480*a**8*b**14*c**
3 + 47185920*a**7*b**16*c**2 - 2621440*a**6*b**18*c + 65536*a**5*
b**20) + _t**2*(-3963617280*a**9*b*c**9 + 6936330240*a**8*b**3*c*
*8 - 5400428544*a**7*b**5*c**7 + 2464874496*a**6*b**7*c**6 - 7300
```

```
54656*a**5*b**9*c**5 + 146165760*a**4*b**11*c**4 - 19860480*a**3*
b**13*c**3 + 1771776*a**2*b**15*c**2 - 94464*a*b**17*c + 2304*b**
19) + 49787136*a**4*c**9 - 27433728*a**3*b**2*c**8 + 6446304*a**2
*b**4*c**7 - 734832*a*b**6*c**6 + 35721*b**8*c**5, Lambda(_t, _t*
log(x + (184549376*_t**3*a**12*b*c**7 - 276824064*_t**3*a**11*b**
3*c**6 + 177209344*_t**3*a**10*b**5*c**5 - 62914560*_t**3*a**9*b*
*7*c**4 + 13434880*_t**3*a**8*b**9*c**3 - 1736704*_t**3*a**7*b**1
1*c**2 + 126976*_t**3*a**6*b**13*c - 4096*_t**3*a**5*b**15 + 6322
176*_t*a**7*c**7 - 13515264*_t*a**6*b**2*c**6 + 8576640*_t*a**5*b
**4*c**5 - 2831328*_t*a**4*b**6*c**4 + 556416*_t*a**3*b**8*c**3 -
 66816*_t*a**2*b**10*c**2 + 4608*_t*a*b**12*c - 144*_t*b**14)/(10
37232*a**4*c**7 - 518616*a**3*b**2*c**6 + 113103*a**2*b**4*c**5 -
 12069*a*b**6*c**4 + 567*b**8*c**3))))
```

GIAC/XCAS [A] time = 16.3985, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-3),x, algorithm="giac")
```

```
[Out] Done
```

3.887 $\int \frac{1}{x^2(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=425

$$-\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3x(b^2-4ac)^2}+\frac{36a^2c^2+bcx^2(5b^2-32ac)-35ab^2c+5b^4}{8a^2x(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{4ax(b^2-4ac)(a+bx^2+cx^4)^2}$$

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (
b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2
) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/
(8*a^2*(b^2 - 4*a*c)^2*x*(a + b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^
2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 4
7*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c
)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 1.90172, antiderivative size = 425, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{3(5b^2-12ac)(b^2-5ac)}{8a^3x(b^2-4ac)^2}+\frac{36a^2c^2+bcx^2(5b^2-32ac)-35ab^2c+5b^4}{8a^2x(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{3\sqrt{c}\left(\frac{b(124a^2c^2-47ab^2c+5b^4)}{\sqrt{b^2-4ac}}+(5b^2-12ac)(b^2-5ac)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{c}\left((5b^2-12ac)(b^2-5ac)-\frac{124a^2bc^2-47ab^3c+5b^5}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}a^3(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{-2ac+b^2+bcx^2}{4ax(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (-3*(5*b^2 - 12*a*c)*(b^2 - 5*a*c))/(8*a^3*(b^2 - 4*a*c)^2*x) + (
b^2 - 2*a*c + b*c*x^2)/(4*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)^2
) + (5*b^4 - 35*a*b^2*c + 36*a^2*c^2 + b*c*(5*b^2 - 32*a*c)*x^2)/
(8*a^2*(b^2 - 4*a*c)^2*x*(a + b*x^2 + c*x^4)) - (3*Sqrt[c]*((5*b^
2 - 12*a*c)*(b^2 - 5*a*c) + (b*(5*b^4 - 47*a*b^2*c + 124*a^2*c^2)
)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) - (3*Sqrt[c]*((5*b^2 - 12*a*c)*(b^2 - 5*a*c) - (5*b^5 - 4
7*a*b^3*c + 124*a^2*b*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^3*(b^2 - 4*a*c
)^2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [A] time = 3.2704, size = 454, normalized size = 1.07

$$-\frac{\frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}+124a^2bc^2-47ab^3c-37ab^2c\sqrt{b^2-4ac}+5b^4\sqrt{b^2-4ac}+5b^5\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(60a^2c^2\sqrt{b^2-4ac}-124a^2bc^2+47ab^3c-37a\cdots\right.}{(b^2-4ac)^{5\cdots}}}{1\cdots}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -(16/x + (4*a*x*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)^2) + (2*x*(7*b^5 - 52*a*b^3*c + 84*a
^2*b*c^2 + 7*b^4*c*x^2 - 47*a*b^2*c^2*x^2 + 52*a^2*c^3*x^2))/((b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^5 - 4
7*a*b^3*c + 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*
Sqrt[b^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^5 + 47*a*b^
3*c - 124*a^2*b*c^2 + 5*b^4*Sqrt[b^2 - 4*a*c] - 37*a*b^2*c*Sqrt[b
```

```
^2 - 4*a*c] + 60*a^2*c^2*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b +
 Sqrt[b^2 - 4*a*c]]))/(16*a^3)
```

Maple [B] time = 0.093, size = 5263, normalized size = 12.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3\,(5\,b^4c^2-37\,ab^2c^3+60\,a^2c^4)\,x^8+(30\,b^5c-227\,ab^3c^2+392\,a^2bc^3)\,x^6+8\,a^2b^4-64\,a^3b^2c+128\,a^4c^2+(15\,b^6-91\,ab^4c+}{8\,((a^3b^4c^2-8\,a^4b^2c^3+16\,a^5c^4)x^9+2\,(a^3b^5c-8\,a^4b^3c^2+16\,a^5bc^3)x^7+(a^3b^6-6\,a^4b^4c+32\,a^6c^3)x^5+2\,(a^4b^5-}$$
$$-\frac{3\int\frac{5\,b^5-42\,ab^3c+92\,a^2bc^2+(5\,b^4c-37\,ab^2c^2+60\,a^2c^3)\,x^2}{cx^4+bx^2+a}\,dx}{8\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x^2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + (30*b^5*c -
 227*a*b^3*c^2 + 392*a^2*b*c^3)*x^6 + 8*a^2*b^4 - 64*a^3*b^2*c +
128*a^4*c^2 + (15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*a^3*c^3
)*x^4 + (25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*x^2)/((a^3*b^4
*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3
*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x
^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*
a^6*b^2*c + 16*a^7*c^2)*x) - 3/8*integrate((5*b^5 - 42*a*b^3*c +
92*a^2*b*c^2 + (5*b^4*c - 37*a*b^2*c^2 + 60*a^2*c^3)*x^2)/(c*x^4
+ b*x^2 + a), x)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)
```

Fricas [A] time = 0.931407, size = 6647, normalized size = 15.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x^2),x, algorithm="fricas")
```

```
[Out] -1/16*(6*(5*b^4*c^2 - 37*a*b^2*c^3 + 60*a^2*c^4)*x^8 + 2*(30*b^5*
c - 227*a*b^3*c^2 + 392*a^2*b*c^3)*x^6 + 16*a^2*b^4 - 128*a^3*b^2
*c + 256*a^4*c^2 + 2*(15*b^6 - 91*a*b^4*c + 25*a^2*b^2*c^2 + 324*
a^3*c^3)*x^4 + 2*(25*a*b^5 - 194*a^2*b^3*c + 364*a^3*b*c^2)*x^2 +
 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4
*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 -
 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b
^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6
*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt
((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*
c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a
^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 +
1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 1
60*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12
*c^5))*log(-27*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*
c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)
*x + 27/2*sqrt(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^
2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c
^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^
8 - (5*a^7*b^16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10
*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^1
3*b^4*c^6 - 323584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12
 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 5918
86*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 -
 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*
b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^
2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c
^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^
3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b
^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c
^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8
*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 10
24*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a
^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) - 3*sqrt(1/2)*
((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c -
8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a
^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c
+ 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 1848
0*a^5*b*c^5 + (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^
10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 -
12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*
a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20
*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2
*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^
2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-2
7*(4125*b^10*c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349
*a^3*b^4*c^7 + 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x - 27/2*sqr
t(1/2)*(125*b^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^
```

3*b^11*c^3 + 1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*
a^6*b^5*c^6 - 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 - (5*a^7*b^
16 - 152*a^8*b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 6
8640*a^11*b^8*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 3
23584*a^14*b^2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^
10*c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^
4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*
c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 102
4*a^19*c^5)))*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 1
5015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 + (a^7*b^1
0 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11
*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725
*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a
^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16
*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5))
)/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
 1280*a^11*b^2*c^4 - 1024*a^12*c^5))) + 3*sqrt(1/2)*((a^3*b^4*c^2
 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2
 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 +
 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^
7*c^2 - 15015*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 -
 (a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 +
1280*a^11*b^2*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*
c + 94725*a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 -
 312300*a^5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c +
 160*a^16*b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a
^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*
b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*
c^4 - 77825*a*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7
+ 2835000*a^4*b^2*c^8 - 810000*a^5*c^9)*x + 27/2*sqrt(1/2)*(125*b
^17 - 3775*a*b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 +
1623534*a^4*b^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 -
 5684672*a^7*b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*
b^14*c + 2006*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8
*c^4 - 197120*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^
2*c^7 + 122880*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*
a^2*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^
5*b^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*
b^6*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))
*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*
c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^
8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 10
24*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2
- 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 +
50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 64
0*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 -
 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^
2*c^4 - 1024*a^12*c^5))) - 3*sqrt(1/2)*((a^3*b^4*c^2 - 8*a^4*b^2*
c^3 + 16*a^5*c^4)*x^9 + 2*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c
^3)*x^7 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 -
 8*a^5*b^3*c + 16*a^6*b*c^2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^
7*c^2)*x)*sqrt(-(25*b^11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015
*a^3*b^5*c^3 + 27720*a^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 -

```
20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2
*c^4 - 1024*a^12*c^5)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2
*b^8*c^2 - 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b
^2*c^5 + 50625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6
*c^2 - 640*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a
^7*b^10 - 20*a^8*b^8*c + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 128
0*a^11*b^2*c^4 - 1024*a^12*c^5))*log(-27*(4125*b^10*c^4 - 77825*a
*b^8*c^5 + 571030*a^2*b^6*c^6 - 1957349*a^3*b^4*c^7 + 2835000*a^4
*b^2*c^8 - 810000*a^5*c^9)*x - 27/2*sqrt(1/2)*(125*b^17 - 3775*a*
b^15*c + 49360*a^2*b^13*c^2 - 362733*a^3*b^11*c^3 + 1623534*a^4*b
^9*c^4 - 4463140*a^5*b^7*c^5 + 7146736*a^6*b^5*c^6 - 5684672*a^7*
b^3*c^7 + 1324800*a^8*b*c^8 + (5*a^7*b^16 - 152*a^8*b^14*c + 2006
*a^9*b^12*c^2 - 14960*a^10*b^10*c^3 + 68640*a^11*b^8*c^4 - 197120
*a^12*b^6*c^5 + 342528*a^13*b^4*c^6 - 323584*a^14*b^2*c^7 + 12288
0*a^15*c^8)*sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 -
 351310*a^3*b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 5
0625*a^6*c^6)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640
*a^17*b^4*c^3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))*sqrt(-(25*b^
11 - 495*a*b^9*c + 3894*a^2*b^7*c^2 - 15015*a^3*b^5*c^3 + 27720*a
^4*b^3*c^4 - 18480*a^5*b*c^5 - (a^7*b^10 - 20*a^8*b^8*c + 160*a^9
*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*a^12*c^5)*
sqrt((625*b^12 - 12250*a*b^10*c + 94725*a^2*b^8*c^2 - 351310*a^3*
b^6*c^3 + 591886*a^4*b^4*c^4 - 312300*a^5*b^2*c^5 + 50625*a^6*c^6
)/(a^14*b^10 - 20*a^15*b^8*c + 160*a^16*b^6*c^2 - 640*a^17*b^4*c^
3 + 1280*a^18*b^2*c^4 - 1024*a^19*c^5)))/(a^7*b^10 - 20*a^8*b^8*c
 + 160*a^9*b^6*c^2 - 640*a^10*b^4*c^3 + 1280*a^11*b^2*c^4 - 1024*
a^12*c^5))))/((a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^9 + 2*
(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*x^7 + (a^3*b^6 - 6*a^4
*b^4*c + 32*a^6*c^3)*x^5 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^
2)*x^3 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*x^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.888 $\int \frac{x^5}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=82

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2
)
```

Rubi [A] time = 0.201119, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] x^2/(2*c) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c
]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + (b*Log[a - b*x^2 + c*x^4])/(4*c^2
)
```

Rubi in Sympy [A] time = 28.6306, size = 73, normalized size = 0.89

$$\frac{b\log\left(a-bx^2+cx^4\right)}{4c^2}+\frac{x^2}{2c}-\frac{(-2ac+b^2)\,\mathrm{atanh}\left(\frac{-b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4-b*x**2+a),x)
```

```
[Out] b*log(a - b*x**2 + c*x**4)/(4*c**2) + x**2/(2*c) - (-2*a*c + b**2
)*atanh((-b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c**2*sqrt(-4*a*c
+ b**2))
```

Mathematica [A] time = 0.0916441, size = 80, normalized size = 0.98

$$\frac{\frac{2(b^2-2ac)\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+b\log\left(a-bx^2+cx^4\right)+2cx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a - b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x^2 + (2*(b^2 - 2*a*c)*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a
*c]])/Sqrt[-b^2 + 4*a*c] + b*Log[a - b*x^2 + c*x^4])/(4*c^2)
```

Maple [A] time = 0.008, size = 116, normalized size = 1.4

$$\frac{x^2}{2\,c}+\frac{b\ln\left(cx^4-bx^2+a\right)}{4\,c^2}-\frac{a}{c}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,c^2}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/2*x^2/c+1/4*b*ln(c*x^4-b*x^2+a)/c^2-1/c/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2-b)/(4*a*c-b^2)^(1/2))*a+1/2/c^2/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2-b)/(4*a*c-b^2)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 - b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.267537, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)\log\left(-\frac{b^3-4\,abc-2\,\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4-2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)-\left(2\,cx^2+b\log\left(cx^4-bx^2+a\right)\right)\sqrt{b^2-4\,ac}}{4\,\sqrt{b^2-4\,ac}c^2}, \ 2\,(b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 - b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*log(-(b^3 - 4*a*b*c - 2*(b^2*c - 4*a*c^2)*x^
2 - (2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x
^4 - b*x^2 + a)) - (2*c*x^2 + b*log(c*x^4 - b*x^2 + a))*sqrt(b^2
- 4*a*c))/(sqrt(b^2 - 4*a*c)*c^2), 1/4*(2*(b^2 - 2*a*c)*arctan(-(
2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (2*c*x^2 + b*log
(c*x^4 - b*x^2 + a))*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
]
```

Sympy [A] time = 5.33681, size = 311, normalized size = 3.79

$$\left(\frac{b}{4c^2} - \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{ab-8ac^2\left(\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(\frac{b}{4c^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right) + \left(\frac{b}{4c^2} + \frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4c^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{ab-8ac^2\left(\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)+2b^2c\left(\frac{b}{4c^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4c^2(4ac-b^2)}\right)}{2ac-b^2}\right) + \frac{x^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4-b*x**2+a),x)
```

```
[Out] (b/(4*c**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c -
 b**2)))*log(x**2 + (a*b - 8*a*c**2*(b/(4*c**2) - sqrt(-4*a*c + b
```

```
**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(b/(4*c**
2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))))
/(2*a*c - b**2)) + (b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**
2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (a*b - 8*a*c**2*(b/(4*c**2
) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**2*(4*a*c - b**2))) +
 2*b**2*c*(b/(4*c**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*c**
2*(4*a*c - b**2))))/(2*a*c - b**2)) + x**2/(2*c)
```

GIAC/XCAS [A] time = 0.313658, size = 105, normalized size = 1.28

$$\frac{x^2}{2\,c} + \frac{b\ln\left(cx^4 - bx^2 + a\right)}{4\,c^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2 - b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 - b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/c + 1/4*b*ln(c*x^4 - b*x^2 + a)/c^2 + 1/2*(b^2 - 2*a*c)*a
rctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^2)
```

3.889 $\int \frac{x^3}{a-bx^2+cx^4} \, dx$

Optimal. Leaf size=64

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*
c]) + Log[a - b*x^2 + c*x^4]/(4*c)
```

Rubi [A] time = 0.134761, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*
c]) + Log[a - b*x^2 + c*x^4]/(4*c)
```

Rubi in Sympy [A] time = 18.6457, size = 54, normalized size = 0.84

$$-\frac{b\,\mathrm{atanh}\left(\frac{-b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2c\sqrt{-4ac+b^2}}+\frac{\log\left(a-bx^2+cx^4\right)}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] -b*atanh((-b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*c*sqrt(-4*a*c +
b**2)) + log(a - b*x**2 + c*x**4)/(4*c)
```

Mathematica [A] time = 0.0419203, size = 65, normalized size = 1.02

$$\frac{\frac{2b\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\log\left(a-bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a - b*x^2 + c*x^4),x]
```

```
[Out] ((2*b*ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*
c] + Log[a - b*x^2 + c*x^4])/(4*c)
```

Maple [A] time = 0.005, size = 63, normalized size = 1.

$$\frac{\ln\left(cx^4-bx^2+a\right)}{4\,c}+\frac{b}{2\,c}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/4*ln(c*x^4-b*x^2+a)/c+1/2*b/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
-b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 - b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.265208, size = 1, normalized size = 0.02

$$\left[\frac{b\log\left(\frac{b^3-4\,abc-2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4-2\,bcx^2+b^2-2\,ac)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\sqrt{b^2-4\,ac}\log\left(cx^4-bx^2+a\right)}{4\sqrt{b^2-4\,ac}c},\ \frac{2\,b\arctan\left(-\frac{(2\,cx^2-b)\sqrt{-b^2+4\,a}}{b^2-4\,ac}\right.}{4\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 - b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((b^3 - 4*a*b*c - 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4
 - 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a
)) + sqrt(b^2 - 4*a*c)*log(c*x^4 - b*x^2 + a))/(sqrt(b^2 - 4*a*c)
*c), 1/4*(2*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) + sqrt(-b^2 + 4*a*c)*log(c*x^4 - b*x^2 + a))/(sqrt(-b^2 + 4*
a*c)*c)]
```

Sympy [A] time = 2.51163, size = 223, normalized size = 3.48

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)-2a-2b^2\left(-\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)+\left(\frac{b\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}+\frac{1}{4c}\right)\log\left(x^2+\frac{8ac\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)-2a-2b^2\left(\frac{b\sqrt{-4ac+b^2}}{4c(4ac-b^2)}+\frac{1}{4c}\right)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))*log(x**2
+ (8*a*c*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c))
- 2*a - 2*b**2*(-b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(
4*c)))/b) + (b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) + 1/(4*c)
)*log(x**2 + (8*a*c*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)) +
 1/(4*c)) - 2*a - 2*b**2*(b*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**
2)) + 1/(4*c)))/b)
```

GIAC/XCAS [A] time = 0.295392, size = 84, normalized size = 1.31

$$\frac{b\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,acc}}+\frac{\ln\left(cx^4-bx^2+a\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 - b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*c) + 1/4*ln(c*x^4 - b*x^2 + a)/c
```

3.890 $\int \frac{x}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0752139, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176

$$\frac{\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]]/Sqrt[b^2 - 4*a*c]
```

Rubi in Sympy [A] time = 9.64212, size = 34, normalized size = 0.97

$$-\frac{\operatorname{atanh}\left(\frac{-b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4-b*x**2+a),x)
```

```
[Out] -atanh((-b + 2*c*x**2)/sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2)
```

Mathematica [A] time = 0.014396, size = 41, normalized size = 1.17

$$\frac{\tan^{-1}\left(\frac{2cx^2-b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a - b*x^2 + c*x^4),x]
```

```
[Out] ArcTan[(-b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.002, size = 38, normalized size = 1.1

$$1 \arctan \left((2\,c x^2 - b) \frac{1}{\sqrt{4\,ac - b^2}} \right) \frac{1}{\sqrt{4\,ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2-b)/(4*a*c-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 - b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.264619, size = 1, normalized size = 0.03

$$\left[\frac{\log \left(\frac{b^3 - 4\,abc - 2\,(b^2 c - 4\,ac^2)\,x^2 + (2\,c^2 x^4 - 2\,bcx^2 + b^2 - 2\,ac)\,\sqrt{b^2 - 4\,ac}}{cx^4 - bx^2 + a} \right)}{2\,\sqrt{b^2 - 4\,ac}}, \frac{\arctan \left(-\frac{(2\,cx^2 - b)\,\sqrt{-b^2 + 4\,ac}}{b^2 - 4\,ac} \right)}{\sqrt{-b^2 + 4\,ac}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 - b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^3 - 4*a*b*c - 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4 -
2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a))/
sqrt(b^2 - 4*a*c), arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2
- 4*a*c))/sqrt(-b^2 + 4*a*c)]
```

Sympy [A] time = 1.33128, size = 131, normalized size = 3.74

$$-\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}+\frac{\sqrt{-\frac{1}{4ac-b^2}}\log\left(x^2+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}-b}{2c}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x**2 + (-4*a*c*sqrt(-1/(4*a*c - b**2
)) + b**2*sqrt(-1/(4*a*c - b**2)) - b)/(2*c))/2 + sqrt(-1/(4*a*c
- b**2))*log(x**2 + (4*a*c*sqrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1
/(4*a*c - b**2)) - b)/(2*c))/2
```

GIAC/XCAS [A] time = 0.29417, size = 50, normalized size = 1.43

$$\frac{\arctan\left(\frac{2\,cx^2-b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 - b*x^2 + a),x, algorithm="giac")
```

```
[Out] arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.891 $\int \frac{1}{x(a-bx^2+cx^4)}\,dx$

Optimal. Leaf size=70

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rubi [A] time = 0.160301, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368

$$\frac{b\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}+\frac{\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (b*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*
c]) + Log[x]/a - Log[a - b*x^2 + c*x^4]/(4*a)
```

Rubi in Sympy [A] time = 25.9092, size = 63, normalized size = 0.9

$$-\frac{b\,\mathrm{atanh}\left(\frac{-b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a\sqrt{-4ac+b^2}}+\frac{\log\left(x^2\right)}{2a}-\frac{\log\left(a-bx^2+cx^4\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4-b*x**2+a),x)
```

```
[Out] -b*atanh((-b + 2*c*x**2)/sqrt(-4*a*c + b**2))/(2*a*sqrt(-4*a*c +
b**2)) + log(x**2)/(2*a) - log(a - b*x**2 + c*x**4)/(4*a)
```

Mathematica [A] time = 0.138204, size = 117, normalized size = 1.67

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)-\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)+4\log(x)\sqrt{b^2-4ac}}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a - b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*Log[x] + (b - Sqrt[b^2 - 4*a*c])*Log[-b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2] - (b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.01, size = 69, normalized size = 1.

$$-\frac{\ln\left(cx^4-bx^2+a\right)}{4\,a}+\frac{b}{2\,a}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{\ln(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/4*ln(c*x^4-b*x^2+a)/a+1/2/a*b/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2-b)/(4*a*c-b^2)^(1/2))+ln(x)/a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.271966, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(\frac{b^3-4\,abc-2\,(b^2c-4\,ac^2)\,x^2+(2\,c^2x^4-2\,bcx^2+b^2-2\,ac)\,\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)-\sqrt{b^2-4\,ac}\left(\log\left(cx^4-bx^2+a\right)-4\,\log\left(x\right)\right)}{4\,\sqrt{b^2-4\,ac}a}, \frac{2\,b\arctan\left(-\frac{(2}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(b*log((b^3 - 4*a*b*c - 2*(b^2*c - 4*a*c^2)*x^2 + (2*c^2*x^4
 - 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/(c*x^4 - b*x^2 + a
)) - sqrt(b^2 - 4*a*c)*(log(c*x^4 - b*x^2 + a) - 4*log(x)))/(sqrt
(b^2 - 4*a*c)*a), 1/4*(2*b*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) - sqrt(-b^2 + 4*a*c)*(log(c*x^4 - b*x^2 + a) -
4*log(x)))/(sqrt(-b^2 + 4*a*c)*a)]
```

Sympy [A] time = 9.01307, size = 253, normalized size = 3.61

$$\left(-\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ab^2\left(-\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)$$
$$+\left(\frac{b\sqrt{-4ac+b^2}}{4a\left(4ac-b^2\right)}-\frac{1}{4a}\right)\log\left(x^2+\frac{8a^2c\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)-2ab^2\left(\frac{b\sqrt{-4ac+b^2}}{4a(4ac-b^2)}-\frac{1}{4a}\right)+2ac-b^2}{bc}\right)$$
$$+\frac{\log\left(x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a))*log(x**2
+ (8*a**2*c*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4*a
)) - 2*a*b**2*(-b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c - b**2)) - 1/(4
*a)) + 2*a*c - b**2)/(b*c)) + (b*sqrt(-4*a*c + b**2)/(4*a*(4*a*c
- b**2)) - 1/(4*a))*log(x**2 + (8*a**2*c*(b*sqrt(-4*a*c + b**2)/(
4*a*(4*a*c - b**2)) - 1/(4*a)) - 2*a*b**2*(b*sqrt(-4*a*c + b**2)/
(4*a*(4*a*c - b**2)) - 1/(4*a)) + 2*a*c - b**2)/(b*c)) + log(x)/a
```

GIAC/XCAS [A] time = 0.297119, size = 96, normalized size = 1.37

$$\frac{b \arctan\left(\frac{2 cx^2 - b}{\sqrt{-b^2 + 4 ac}}\right)}{2 \sqrt{-b^2 + 4 ac} a} - \frac{\ln\left(cx^4 - bx^2 + a\right)}{4 a} + \frac{\ln\left(x^2\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*b*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c
)*a) - 1/4*ln(c*x^4 - b*x^2 + a)/a + 1/2*ln(x^2)/a
```

3.892 $\int \frac{1}{x^3(a-bx^2+cx^4)} \, dx$

Optimal. Leaf size=89

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/(2*a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*
x^2 + c*x^4])/(4*a^2)
```

Rubi [A] time = 0.309356, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$

$$\frac{(b^2-2ac)\tanh^{-1}\left(\frac{b-2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}+\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -1/(2*a*x^2) + ((b^2 - 2*a*c)*ArcTanh[(b - 2*c*x^2)/Sqrt[b^2 - 4*
a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) + (b*Log[x])/a^2 - (b*Log[a - b*
x^2 + c*x^4])/(4*a^2)
```

Rubi in Sympy [A] time = 41.955, size = 87, normalized size = 0.98

$$-\frac{1}{2ax^2}+\frac{b\log\left(x^2\right)}{2a^2}-\frac{b\log\left(a-bx^2+cx^4\right)}{4a^2}-\frac{\left(-2ac+b^2\right)\operatorname{atanh}\left(\frac{-b+2cx^2}{\sqrt{-4ac+b^2}}\right)}{2a^2\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] -1/(2*a*x**2) + b*log(x**2)/(2*a**2) - b*log(a - b*x**2 + c*x**4)
/(4*a**2) - (-2*a*c + b**2)*atanh((-b + 2*c*x**2)/sqrt(-4*a*c + b
**2))/(2*a**2*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.33315, size = 139, normalized size = 1.56

$$\frac{\frac{\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(-\sqrt{b^2-4ac}-b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\log\left(\sqrt{b^2-4ac}-b+2cx^2\right)}{\sqrt{b^2-4ac}}-\frac{2a}{x^2}+4b\log(x)}{4a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a - b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a)/x^2 + 4*b*Log[x] + ((b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*L
og[-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((b^2 -
 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

Maple [A] time = 0.012, size = 123, normalized size = 1.4

$$-\frac{b\ln\left(cx^4-bx^2+a\right)}{4\,a^2}-\frac{c}{a}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}+\frac{b^2}{2\,a^2}\arctan\left((2\,cx^2-b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}}-\frac{1}{2\,ax^2}+\frac{b\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/4*b*ln(c*x^4-b*x^2+a)/a^2-1/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2-b)/(4*a*c-b^2)^(1/2))*c+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^2-b)/(4*a*c-b^2)^(1/2))*b^2-1/2/a/x^2+b*ln(x)/a^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283412, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2-2\,ac\right)x^2\log\left(-\frac{b^3-4\,abc-2\left(b^2c-4\,ac^2\right)x^2-\left(2\,c^2x^4-2\,bcx^2+b^2-2\,ac\right)\sqrt{b^2-4\,ac}}{cx^4-bx^2+a}\right)+\left(bx^2\log\left(cx^4-bx^2+a\right)-4\,bx^2\log\left(x\right)+2\,a\right.}{4\,\sqrt{b^2-4\,ac}a^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 2*a*c)*x^2*log(-(b^3 - 4*a*b*c - 2*(b^2*c - 4*a*c^2
)*x^2 - (2*c^2*x^4 - 2*b*c*x^2 + b^2 - 2*a*c)*sqrt(b^2 - 4*a*c))/
(c*x^4 - b*x^2 + a)) + (b*x^2*log(c*x^4 - b*x^2 + a) - 4*b*x^2*lo
g(x) + 2*a)*sqrt(b^2 - 4*a*c))/(sqrt(b^2 - 4*a*c)*a^2*x^2), 1/4*(
2*(b^2 - 2*a*c)*x^2*arctan(-(2*c*x^2 - b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)) - (b*x^2*log(c*x^4 - b*x^2 + a) - 4*b*x^2*log(x) + 2*a
)*sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a^2*x^2)]
```

Sympy [A] time = 21.2533, size = 350, normalized size = 3.93

$$\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(-\frac{b}{4a^2}-\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)-3abc+b^3}{2ac^2-b^2c}\right)$$
$$+\left(-\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}\left(2ac-b^2\right)}{4a^2\left(4ac-b^2\right)}\right)\log\left(x^2+\frac{-8a^3c\left(-\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)+2a^2b^2\left(-\frac{b}{4a^2}+\frac{\sqrt{-4ac+b^2}(2ac-b^2)}{4a^2(4ac-b^2)}\right)-3abc+b^3}{2ac^2-b^2c}\right)$$
$$-\frac{1}{2ax^2}+\frac{b\log\left(x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4-b*x**2+a),x)
```

```
[Out] (-b/(4*a**2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c
- b**2)))*log(x**2 + (-8*a**3*c*(-b/(4*a**2) - sqrt(-4*a*c + b**2
)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a*
```

```
*2) - sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))
 - 3*a*b*c + b**3)/(2*a*c**2 - b**2*c)) + (-b/(4*a**2) + sqrt(-4*
a*c + b**2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2)))*log(x**2 + (-
8*a**3*c*(-b/(4*a**2) + sqrt(-4*a*c + b**2)*(2*a*c - b**2)/(4*a**
2*(4*a*c - b**2))) + 2*a**2*b**2*(-b/(4*a**2) + sqrt(-4*a*c + b**
2)*(2*a*c - b**2)/(4*a**2*(4*a*c - b**2))) - 3*a*b*c + b**3)/(2*a
*c**2 - b**2*c)) - 1/(2*a*x**2) + b*log(x)/a**2
```

GIAC/XCAS [A] time = 0.296755, size = 128, normalized size = 1.44

$$-\frac{b\ln\left(cx^4 - bx^2 + a\right)}{4\,a^2} + \frac{b\ln\left(x^2\right)}{2\,a^2} + \frac{\left(b^2 - 2\,ac\right)\arctan\left(\frac{2\,cx^2 - b}{\sqrt{-b^2 + 4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}a^2} - \frac{bx^2 + a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/4*b*ln(c*x^4 - b*x^2 + a)/a^2 + 1/2*b*ln(x^2)/a^2 + 1/2*(b^2 -
 2*a*c)*arctan((2*c*x^2 - b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a
*c)*a^2) - 1/2*(b*x^2 + a)/(a^2*x^2)
```

3.893 $\int \frac{x^4}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.753893, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c - ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 53.8676, size = 189, normalized size = 1.06

$$\frac{x}{c}-\frac{\sqrt{2}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{2}}\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4-b*x**2+a),x)
```

```
[Out] x/c - sqrt(2)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atanh(sqrt(
2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b +
sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt(2)*(-2*a*c + b**
2 - b*sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(
-4*a*c + b**2)))/(2*c**(3/2)*sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-
4*a*c + b**2))
```

Mathematica [A] time = 0.224945, size = 208, normalized size = 1.16

$$\frac{\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a - b*x^2 + c*x^4),x]
```

```
[Out] x/c + ((b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4
*a*c]*Sqrt[-b - Sqrt[b^2 - 4*a*c]]) + ((-b^2 + 2*a*c + b*Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[b^2 - 4*a*c
]])
```

Maple [B] time = 0.035, size = 343, normalized size = 1.9

$$\frac{x}{c}+\frac{\sqrt{2}b}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\sqrt{2}a\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b^2}{2c}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b}{2c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$+\sqrt{2}aArtanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b^2}{2c}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4-b*x^2+a),x)
```

```
[Out] x/c+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2))*b^2-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+
1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*a-1/2/c/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{c} + \frac{\int \frac{bx^2-a}{cx^4-bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 - b*x^2 + a),x, algorithm="maxima")
```

```
[Out] x/c + integrate((b*x^2 - a)/(c*x^4 - b*x^2 + a), x)/c
```

Fricas [A] time = 0.279194, size = 1419, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 - b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*
c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a*b^2*c + 4*a
^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4
*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c^4)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3
- 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4 - 5*a*b^2*c
 + 4*a^2*c^2 - (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c + (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c
^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^3 - 4*
a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/(b^
2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x + sqrt(1/2)*(b^4 - 5*a
*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2*c^3 -
 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))/
(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt((b^3 - 3*a*b*c - (b^2*c^
3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)
))/(b^2*c^3 - 4*a*c^4))*log(-2*(a*b^2 - a^2*c)*x - sqrt(1/2)*(b^4
 - 5*a*b^2*c + 4*a^2*c^2 + (b^3*c^3 - 4*a*b*c^4)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c^7)))*sqrt((b^3 - 3*a*b*c - (b^2
*c^3 - 4*a*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))) - 2*x)/c
```

Sympy [A] time = 5.65699, size = 129, normalized size = 0.72

$$\operatorname{RootSum}\left(t^4\left(256a^2c^5-128ab^2c^4+16b^4c^3\right)+t^2\left(-48a^2bc^2+28ab^3c-4b^5\right)+a^3,\left(t\mapsto t\log\left(x+\frac{-32t^3abc^4+8t^3b^3c^3-\dots}{a^2c-\dots}\right.\right.\right.$$
$$+\frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**5 - 128*a*b**2*c**4 + 16*b**4*c**3) +
_t**2*(-48*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + a**3, Lambda(_t,
 _t*log(x + (-32*_t**3*a*b*c**4 + 8*_t**3*b**3*c**3 - 4*_t*a**2*c
**2 + 8*_t*a*b**2*c - 2*_t*b**4)/(a**2*c - a*b**2)))) + x/c
```

GIAC/XCAS [A] time = 0.786163, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 - b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.894 $\int \frac{x^2}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt
[b + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.265014, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{\sqrt{b-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{\sqrt{b^2-4ac}+b}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - (Sqrt
[b + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 24.0255, size = 141, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{b-\sqrt{-4ac+b^2}}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}-\frac{\sqrt{2}\sqrt{b+\sqrt{-4ac+b^2}}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] sqrt(2)*sqrt(b - sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c)*x/sqr
t(b - sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2)) - sqr
```

```
t(2)*sqrt(b + sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c)*x/sqrt(b
 + sqrt(-4*a*c + b**2)))/(2*sqrt(c)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.214989, size = 137, normalized size = 0.91

$$\frac{\sqrt{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)-\sqrt{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a - b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[-b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-
b - Sqrt[b^2 - 4*a*c]]]) + Sqrt[-b + Sqrt[b^2 - 4*a*c]]*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]
*Sqrt[b^2 - 4*a*c])
```

Maple [A] time = 0.016, size = 208, normalized size = 1.4

$$\frac{\sqrt{2}}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b}{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}}{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$
$$-\frac{\sqrt{2}b}{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] 1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2))*b-1/2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{cx^4 - bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 - b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 - b*x^2 + a), x)
```

Fricas [A] time = 0.267439, size = 744, normalized size = 4.96

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b+\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}\log\left(-\frac{\sqrt{\frac{1}{2}}\left(b^2c-4\,ac^2\right)\sqrt{\frac{b-\frac{b^2c-4\,ac^2}{\sqrt{b^2c^2-4\,ac^3}}}{b^2c-4\,ac^2}}}{\sqrt{b^2c^2-4\,ac^3}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 - b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3
))/(b^2*c - 4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b + (
b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt
(b^2*c^2 - 4*a*c^3) + x) + 1/2*sqrt(1/2)*sqrt((b + (b^2*c - 4*a*c
^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b
^2*c - 4*a*c^2)*sqrt((b + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^
3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x) + 1/2*sqrt(1/
2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
4*a*c^2))*log(sqrt(1/2)*(b^2*c - 4*a*c^2)*sqrt((b - (b^2*c - 4*a*
c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))/sqrt(b^2*c^2 - 4
*a*c^3) + x) - 1/2*sqrt(1/2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2
*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log(-sqrt(1/2)*(b^2*c - 4*a*c
^2)*sqrt((b - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c -
 4*a*c^2))/sqrt(b^2*c^2 - 4*a*c^3) + x)
```

Sympy [A] time = 2.40363, size = 75, normalized size = 0.5

$$\operatorname{RootSum}\left(t^4\left(256a^2c^3-128ab^2c^2+16b^4c\right)+t^2\left(16abc-4b^3\right)+a,\left(t\mapsto t\log\left(64t^3ac^2-16t^3b^2c+2tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**2*c**3 - 128*a*b**2*c**2 + 16*b**4*c) + _t*
*2*(16*a*b*c - 4*b**3) + a, Lambda(_t, _t*log(64*_t**3*a*c**2 - 1
6*_t**3*b**2*c + 2*_t*b + x)))
```

GIAC/XCAS [A] time = 0.734357, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 - b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.895 $\int \frac{1}{a-bx^2+cx^4}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.182899, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[2]*Sqrt[c]*ArcTanh[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 21.844, size = 138, normalized size = 0.92

$$-\frac{\sqrt{2}\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*atanh(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(-4*a*c + b
**2)))/(sqrt(b + sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) + sqrt
```

```
(2)*sqrt(c)*atanh(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2))
)/(sqrt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.139217, size = 137, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{c}\left(\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{-\sqrt{b^2-4ac}-b}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b*x^2 + c*x^4)^(-1),x]
```

```
[Out] (Sqrt[2]*Sqrt[c]*(ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 -
 4*a*c]]]/Sqrt[-b - Sqrt[b^2 - 4*a*c]] - ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]]/Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/S
qrt[b^2 - 4*a*c]
```

Maple [A] time = 0.015, size = 116, normalized size = 0.8

$$-c\sqrt{2}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-c\sqrt{2}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4-b*x^2+a),x)
```

```
[Out] -c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{cx^4 - bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 - b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/(c*x^4 - b*x^2 + a), x)
```

Fricas [A] time = 0.264581, size = 817, normalized size = 5.45

$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(b^2 - 4\,ac - \frac{ab^3 - 4\,a^2bc}{\sqrt{a^2b^2 - 4\,a^3c}}\right)\sqrt{\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\log\left(2\,cx - \sqrt{\frac{1}{2}}\left(b^2 - 4\,ac - \frac{ab^3 - 4\,a^2bc}{\sqrt{a^2b^2 - 4\,a^3c}}\right)\sqrt{\frac{b + \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\right)$$
$$-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b - \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\log\left(2\,cx + \sqrt{\frac{1}{2}}\left(b^2 - 4\,ac + \frac{ab^3 - 4\,a^2bc}{\sqrt{a^2b^2 - 4\,a^3c}}\right)\sqrt{\frac{b - \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\right)$$
$$+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\frac{b - \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\log\left(2\,cx - \sqrt{\frac{1}{2}}\left(b^2 - 4\,ac + \frac{ab^3 - 4\,a^2bc}{\sqrt{a^2b^2 - 4\,a^3c}}\right)\sqrt{\frac{b - \frac{ab^2 - 4\,a^2c}{\sqrt{a^2b^2 - 4\,a^3c}}}{ab^2 - 4\,a^2c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 - b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c
))/(a*b^2 - 4*a^2*c))*log(2*c*x + sqrt(1/2)*(b^2 - 4*a*c - (a*b^3
 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))) + 1/2*sqrt(1/2)*sq
rt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2
*c))*log(2*c*x - sqrt(1/2)*(b^2 - 4*a*c - (a*b^3 - 4*a^2*b*c)/sqr
t(a^2*b^2 - 4*a^3*c))*sqrt((b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
4*a^3*c))/(a*b^2 - 4*a^2*c))) - 1/2*sqrt(1/2)*sqrt((b - (a*b^2 -
4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x +
sqrt(1/2)*(b^2 - 4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3
*c))*sqrt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2
- 4*a^2*c))) + 1/2*sqrt(1/2)*sqrt((b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(2*c*x - sqrt(1/2)*(b^2 -
4*a*c + (a*b^3 - 4*a^2*b*c)/sqrt(a^2*b^2 - 4*a^3*c))*sqrt((b - (a
```

```
*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c)))
```

Sympy [A] time = 2.86741, size = 87, normalized size = 0.58

$$\operatorname{RootSum}\left(t^{4}\left(256a^{3}c^{2}-128a^{2}b^{2}c+16ab^{4}\right)+t^{2}\left(16abc-4b^{3}\right)+c,\left(t\mapsto t\log\left(x+\frac{-32t^{3}a^{2}bc+8t^{3}ab^{3}+4tac-2tb^{2}}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**3*c**2 - 128*a**2*b**2*c + 16*a*b**4) + _t*
*2*(16*a*b*c - 4*b**3) + c, Lambda(_t, _t*log(x + (-32*_t**3*a**2
*b*c + 8*_t**3*a*b**3 + 4*_t*a*c - 2*_t*b**2)/c)))
```

GIAC/XCAS [A] time = 0.369976, size = 1, normalized size = 0.01

$$\mathit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^4 - b*x^2 + a),x, algorithm="giac")
```

```
[Out] Done
```

3.896 $\int \frac{1}{x^2(a-bx^2+cx^4)} dx$

Optimal. Leaf size=172

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.383553, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$\frac{\sqrt{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*x)) + (Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt
[b^2 - 4*a*c]]) + (Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b +
 Sqrt[b^2 - 4*a*c]])
```

Rubi in Sympy [A] time = 43.5976, size = 177, normalized size = 1.03

$$-\frac{\sqrt{2}\sqrt{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt{2}\sqrt{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt{b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] -sqrt(2)*sqrt(c)*(b - sqrt(-4*a*c + b**2))*atanh(sqrt(2)*sqrt(c)*
x/sqrt(b + sqrt(-4*a*c + b**2)))/(2*a*sqrt(b + sqrt(-4*a*c + b**2
))*sqrt(-4*a*c + b**2)) + sqrt(2)*sqrt(c)*(b + sqrt(-4*a*c + b**2
))*atanh(sqrt(2)*sqrt(c)*x/sqrt(b - sqrt(-4*a*c + b**2)))/(2*a*sq
rt(b - sqrt(-4*a*c + b**2))*sqrt(-4*a*c + b**2)) - 1/(a*x)
```

Mathematica [A] time = 0.789264, size = 199, normalized size = 1.16

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}-b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}+b\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}-b}}+\frac{2}{x}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a - b*x^2 + c*x^4)),x]
```

```
[Out] -(2/x + (Sqrt[2]*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[-b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[-b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[-b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[b^2 - 4*a*c]*Sqrt[-b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

Maple [A] time = 0.022, size = 232, normalized size = 1.4

$$-\frac{c\sqrt{2}}{2\,a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{c\sqrt{2}b}{2\,a}\arctan\left(cx\sqrt{2}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(-b+\sqrt{-4\,ac+b^2}\right)c}}+\frac{c\sqrt{2}}{2\,a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{c\sqrt{2}b}{2\,a}Artanh\left(cx\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}\right)\frac{1}{\sqrt{-4\,ac+b^2}}\frac{1}{\sqrt{\left(b+\sqrt{-4\,ac+b^2}\right)c}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4-b*x^2+a),x)
```

```
[Out] -1/2*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))-1/2*c/a/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b+1/2*c/a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))-1/2*c/a/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*
b-1/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{cx^2-b}{cx^4-bx^2+a}\,dx}{a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^2 - b)/(c*x^4 - b*x^2 + a), x)/a - 1/(a*x)
```

Fricas [A] time = 0.28067, size = 1496, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt
((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a
^4*c))*log(-2*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c +
4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c + (a^
3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^
7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c
+ (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 -
 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqr
t(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 - (a^3*b^4 - 6*a^4*b^2*c +
8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))
*sqrt((b^3 - 3*a*b*c + (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2
)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b
```

```
^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-2
*(b^2*c^2 - a*c^3)*x + sqrt(1/2)*(b^5 - 5*a*b^3*c + 4*a^2*b*c^2 +
 (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 - 3*a*b*c - (a^3*b^2 - 4*a^4
*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b
^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt((b^3 - 3*a*b*c - (a^3*b^2 -
4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))/(
a^3*b^2 - 4*a^4*c))*log(-2*(b^2*c^2 - a*c^3)*x - sqrt(1/2)*(b^5 -
 5*a*b^3*c + 4*a^2*b*c^2 + (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^2 - 4*a^7*c)))*sqrt((b^3 -
3*a*b*c - (a^3*b^2 - 4*a^4*c)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2)/(a*x)
```

Sympy [A] time = 6.25798, size = 148, normalized size = 0.86

$$\operatorname{RootSum}\left(t^4\left(256a^5c^2 - 128a^4b^2c + 16a^3b^4\right) + t^2\left(-48a^2bc^2 + 28ab^3c - 4b^5\right) + c^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^5c^2 + 48t^3a^4b^2c -\right.\right.\right.$$

$$-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4-b*x**2+a),x)
```

```
[Out] RootSum(_t**4*(256*a**5*c**2 - 128*a**4*b**2*c + 16*a**3*b**4) +
_t**2*(-48*a**2*b*c**2 + 28*a*b**3*c - 4*b**5) + c**3, Lambda(_t,
 _t*log(x + (-64*_t**3*a**5*c**2 + 48*_t**3*a**4*b**2*c - 8*_t**3
*a**3*b**4 + 10*_t*a**2*b*c**2 - 10*_t*a*b**3*c + 2*_t*b**5)/(a*c
**3 - b**2*c**2)))) - 1/(a*x)
```

GIAC/XCAS [A] time = 0.792232, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 - b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] Done
```

3.897 $\int \frac{x^5}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=69

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(
3/2)*Sqrt[b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rubi [A] time = 0.18313, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) - ((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(
3/2)*Sqrt[b]) - Log[a - b + 2*a*x^2 + a*x^4]/(2*a)
```

Rubi in Sympy [A] time = 30.7995, size = 58, normalized size = 0.84

$$\frac{x^2}{2a}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{2a}-\frac{(a+b)\,\mathrm{atanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] x**2/(2*a) - log(a*x**4 + 2*a*x**2 + a - b)/(2*a) - (a + b)*atanh
(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*a**(3/2)*sqrt(b))
```

Mathematica [A] time = 0.0622546, size = 62, normalized size = 0.9

$$\frac{x^2 - \log\left(a\left(x^2+1\right)^2 - b\right)}{2a} - \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3/2)*Sqrt[b
]) + (x^2 - Log[-b + a*(1 + x^2)^2])/(2*a)
```

Maple [A] time = 0.005, size = 86, normalized size = 1.3

$$\frac{x^2}{2\,a} - \frac{\ln\left(ax^4 + 2\,ax^2 + a - b\right)}{2\,a} - \frac{1}{2}Artanh\left(\frac{2\,ax^2 + 2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}} - \frac{b}{2\,a}Artanh\left(\frac{2\,ax^2 + 2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a-b)/a-1/2/(a*b)^(1/2)*arctanh(1/2
*(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctanh(1/2*(2*a*x^
2+2*a)/(a*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278857, size = 1, normalized size = 0.01

$$\left[\frac{(a+b)\log\left(-\frac{2\,abx^2+2\,ab-\left(ax^4+2\,ax^2+a+b\right)\sqrt{ab}}{ax^4+2\,ax^2+a-b}\right)+2\,\sqrt{ab}\left(x^2-\log\left(ax^4+2\,ax^2+a-b\right)\right)}{4\,\sqrt{ab}a},\ \frac{(a+b)\arctan\left(\frac{b}{\sqrt{-ab}(x^2+1)}\right)+\sqrt{-ab}}{2\,\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*log(-(2*a*b*x^2 + 2*a*b - (a*x^4 + 2*a*x^2 + a + b)
*sqrt(a*b))/(a*x^4 + 2*a*x^2 + a - b)) + 2*sqrt(a*b)*(x^2 - log(a
*x^4 + 2*a*x^2 + a - b)))/(sqrt(a*b)*a), 1/2*((a + b)*arctan(b/(s
qrt(-a*b)*(x^2 + 1))) + sqrt(-a*b)*(x^2 - log(a*x^4 + 2*a*x^2 + a
 - b)))/(sqrt(-a*b)*a)]
```

Sympy [A] time = 3.68541, size = 138, normalized size = 2.

$$\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}-\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}\,(a+b)}{4a^3b}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{2a}+\frac{\sqrt{a^3b}(a+b)}{4a^3b}\right)+a-b}{a+b}\right)+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(
-1/(2*a) - sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + (
-1/(2*a) + sqrt(a**3*b)*(a + b)/(4*a**3*b))*log(x**2 + (-4*a*b*(-
1/(2*a) + sqrt(a**3*b)*(a + b)/(4*a**3*b)) + a - b)/(a + b)) + x*
*2/(2*a)
```

GIAC/XCAS [A] time = 0.54788, size = 81, normalized size = 1.17

$$\frac{x^2}{2\,a}+\frac{(a+b)\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}a}-\frac{\ln\left(ax^4+2\,ax^2+a-b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a + b)*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b
)*a) - 1/2*ln(a*x^4 + 2*a*x^2 + a - b)/a
```

3.898 $\int \frac{x^3}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}$$

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a
- b + 2*a*x^2 + a*x^4]/(4*a)
```

Rubi [A] time = 0.107204, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a
- b + 2*a*x^2 + a*x^4]/(4*a)
```

Rubi in Sympy [A] time = 19.7315, size = 48, normalized size = 0.86

$$\frac{\log\left(ax^4+2ax^2+a-b\right)}{4a}+\frac{\operatorname{atanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] log(a*x**4 + 2*a*x**2 + a - b)/(4*a) + atanh(sqrt(a)*(x**2 + 1)/s
qrt(b))/(2*sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.0300339, size = 51, normalized size = 0.91

$$\frac{\log \left(a \left(x^2+1\right)^2-b\right)+\frac{2 \sqrt{a} \tanh ^{-1}\left(\frac{\sqrt{a} \left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4 a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((2*Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[-
b + a*(1 + x^2)^2])/(4*a)
```

Maple [A] time = 0.004, size = 49, normalized size = 0.9

$$\frac{\ln \left(ax^4+2\,ax^2+a-b\right)}{4\,a}+\frac{1}{2}Artanh\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a-b)/a+1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+
2*a)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.281923, size = 1, normalized size = 0.02

$$\left[\frac{a\log\left(\frac{2\,abx^2+2\,ab+\left(ax^4+2\,ax^2+a+b\right)\sqrt{ab}}{ax^4+2\,ax^2+a-b}\right)+\sqrt{ab}\log\left(ax^4+2\,ax^2+a-b\right)}{4\,\sqrt{ab}a},\right.$$
$$\left.-\frac{2\,a\arctan\left(\frac{b}{\sqrt{-ab}(x^2+1)}\right)-\sqrt{-ab}\log\left(ax^4+2\,ax^2+a-b\right)}{4\,\sqrt{-ab}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="fricas")
```

```
[Out] [1/4*(a*log((2*a*b*x^2 + 2*a*b + (a*x^4 + 2*a*x^2 + a + b)*sqrt(a
*b))/(a*x^4 + 2*a*x^2 + a - b)) + sqrt(a*b)*log(a*x^4 + 2*a*x^2 +
 a - b))/(sqrt(a*b)*a), -1/4*(2*a*arctan(b/(sqrt(-a*b)*(x^2 + 1))
) - sqrt(-a*b)*log(a*x^4 + 2*a*x^2 + a - b))/(sqrt(-a*b)*a)]
```

Sympy [A] time = 1.47693, size = 110, normalized size = 1.96

$$\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}-\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)\log\left(x^2+\frac{4ab\left(\frac{1}{4a}+\frac{\sqrt{a^3b}}{4a^2b}\right)+a-b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (1/(4*a) - sqrt(a**3*b)/(4*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) -
sqrt(a**3*b)/(4*a**2*b)) + a - b)/a) + (1/(4*a) + sqrt(a**3*b)/(4
*a**2*b))*log(x**2 + (4*a*b*(1/(4*a) + sqrt(a**3*b)/(4*a**2*b)) +
 a - b)/a)
```

GIAC/XCAS [A] time = 0.548466, size = 62, normalized size = 1.11

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}}+\frac{\ln\left(ax^4+2\,ax^2+a-b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b) + 1/4*ln(a*x^4 + 2
*a*x^2 + a - b)/a
```

3.899 $\int \frac{x}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0632392, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 9.82348, size = 29, normalized size = 0.94

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -atanh(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.0148255, size = 31, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.002, size = 26, normalized size = 0.8

$$-\frac{1}{2}Artanh\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278685, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{2\,abx^2+2\,ab-\left(ax^4+2\,ax^2+a+b\right)\sqrt{ab}}{ax^4+2\,ax^2+a-b}\right)}{4\,\sqrt{ab}},\ \frac{\arctan\left(\frac{b}{\sqrt{-ab}\left(x^2+1\right)}\right)}{2\,\sqrt{-ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(2*a*b*x^2 + 2*a*b - (a*x^4 + 2*a*x^2 + a + b)*sqrt(a*b
))/(a*x^4 + 2*a*x^2 + a - b))/sqrt(a*b), 1/2*arctan(b/(sqrt(-a*b)
*(x^2 + 1)))/sqrt(-a*b)]
```

Sympy [A] time = 0.788382, size = 53, normalized size = 1.71

$$\frac{\sqrt{\frac{1}{ab}}\log\left(-b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}-\frac{\sqrt{\frac{1}{ab}}\log\left(b\sqrt{\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] sqrt(1/(a*b))*log(-b*sqrt(1/(a*b)) + x**2 + 1)/4 - sqrt(1/(a*b))*
log(b*sqrt(1/(a*b)) + x**2 + 1)/4
```

GIAC/XCAS [A] time = 0.54731, size = 31, normalized size = 1.

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(-a*b))/sqrt(-a*b)
```

3.900 $\int \frac{1}{x(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log(x)}{a-b}$$

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]
) + Log[x]/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rubi [A] time = 0.15787, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4(a-b)}+\frac{\log(x)}{a-b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (Sqrt[a]*ArcTanh[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*(a - b)*Sqrt[b]
) + Log[x]/(a - b) - Log[a - b + 2*a*x^2 + a*x^4]/(4*(a - b))
```

Rubi in Sympy [A] time = 31.189, size = 66, normalized size = 0.86

$$\frac{\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)}+\frac{\log\left(x^2\right)}{2\,(a-b)}-\frac{\log\left(ax^4+2ax^2+a-b\right)}{4\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] sqrt(a)*atanh(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*sqrt(b)*(a - b)) + l
og(x**2)/(2*(a - b)) - log(a*x**4 + 2*a*x**2 + a - b)/(4*(a - b))
```

Mathematica [A] time = 0.0844579, size = 90, normalized size = 1.17

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)+\left(\sqrt{b}-\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)-4\sqrt{b}\log(x)}{4\sqrt{b}(b-a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-4*Sqrt[b]*Log[x] + (Sqrt[a] + Sqrt[b])*Log[-Sqrt[b] + Sqrt[a]*(
1 + x^2)] + (-Sqrt[a] + Sqrt[b])*Log[Sqrt[b] + Sqrt[a]*(1 + x^2)]
)/(4*Sqrt[b]*(-a + b))
```

Maple [A] time = 0.009, size = 71, normalized size = 0.9

$$\frac{\ln(x)}{a-b}-\frac{\ln\left(ax^4+2\,ax^2+a-b\right)}{4\,a-4\,b}+\frac{a}{2\,a-2\,b}Artanh\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] ln(x)/(a-b)-1/4*ln(a*x^4+2*a*x^2+a-b)/(a-b)+1/2*a/(a-b)/(a*b)^(1/
2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284392, size = 1, normalized size = 0.01

$$\left[-\frac{\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+\log\left(ax^4+2\,ax^2+a-b\right)-4\,\log\left(x\right)}{4\,(a-b)},\ -\frac{2\,\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{-\frac{a}{b}}}{ax^2+a}\right)+\log\left(ax^4+2\,ax^2+a-b\right)-4\,\log\left(x\right)}{4\,(a-b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(a/b) +
 a + b)/(a*x^4 + 2*a*x^2 + a - b)) + log(a*x^4 + 2*a*x^2 + a - b)
 - 4*log(x))/(a - b), -1/4*(2*sqrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x
^2 + a)) + log(a*x^4 + 2*a*x^2 + a - b) - 4*log(x))/(a - b)]
```

Sympy [A] time = 7.44558, size = 184, normalized size = 2.39

$$\left(-\frac{1}{4\,(a-b)}-\frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}-\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\left(-\frac{1}{4\,(a-b)}+\frac{\sqrt{ab}}{4b\,(a-b)}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+a-4b^2\left(-\frac{1}{4(a-b)}+\frac{\sqrt{ab}}{4b(a-b)}\right)+b}{a}\right)+\frac{\log\left(x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] (-1/(4*(a - b)) - sqrt(a*b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/
(4*(a - b)) - sqrt(a*b)/(4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b
)) - sqrt(a*b)/(4*b*(a - b))) + b)/a) + (-1/(4*(a - b)) + sqrt(a*
b)/(4*b*(a - b)))*log(x**2 + (4*a*b*(-1/(4*(a - b)) + sqrt(a*b)/(
4*b*(a - b))) + a - 4*b**2*(-1/(4*(a - b)) + sqrt(a*b)/(4*b*(a -
b))) + b)/a) + log(x)/(a - b)
```

GIAC/XCAS [A] time = 0.535827, size = 96, normalized size = 1.25

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\sqrt{-ab}(a-b)}-\frac{\ln\left(ax^4+2\,ax^2+a-b\right)}{4\,(a-b)}+\frac{\ln\left(x^2\right)}{2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(-a*b))/(sqrt(-a*b)*(a - b)) - 1/4*
ln(a*x^4 + 2*a*x^2 + a - b)/(a - b) + 1/2*ln(x^2)/(a - b)
```

3.901 $\int \frac{1}{x^3(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2x^2(a-b)}-\frac{\sqrt{a}(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)^2}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

```
[Out] -1/(2*(a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))
/Sqrt[b]])/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Lo
g[a - b + 2*a*x^2 + a*x^4])/(2*(a - b)^2)
```

Rubi [A] time = 0.307805, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318

$$-\frac{1}{2x^2(a-b)}-\frac{\sqrt{a}(a+b)\tanh^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a-b)^2}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{2a\log(x)}{(a-b)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a - b)*x^2) - (Sqrt[a]*(a + b)*ArcTanh[(Sqrt[a]*(1 + x^2))
/Sqrt[b]])/(2*(a - b)^2*Sqrt[b]) - (2*a*Log[x])/(a - b)^2 + (a*Lo
g[a - b + 2*a*x^2 + a*x^4])/(2*(a - b)^2)
```

Rubi in Sympy [A] time = 47.232, size = 83, normalized size = 0.86

$$-\frac{\sqrt{a}\,(a+b)\,\operatorname{atanh}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a-b)^2}-\frac{a\log\left(x^2\right)}{(a-b)^2}+\frac{a\log\left(ax^4+2ax^2+a-b\right)}{2\,(a-b)^2}-\frac{1}{2x^2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -sqrt(a)*(a + b)*atanh(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*sqrt(b)*(a
- b)**2) - a*log(x**2)/(a - b)**2 + a*log(a*x**4 + 2*a*x**2 + a -
 b)/(2*(a - b)**2) - 1/(2*x**2*(a - b))
```

Mathematica [A] time = 0.21427, size = 146, normalized size = 1.51

$$\frac{-8a\sqrt{b}x^2\log(x)+\sqrt{a}x^2\left(\sqrt{a}+\sqrt{b}\right)^2\log\left(\sqrt{a}\left(x^2+1\right)-\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)\left(\left(ax^2-\sqrt{a}\sqrt{b}x^2\right)\log\left(\sqrt{a}\left(x^2+1\right)+\sqrt{b}\right)+2\left(\right.\right.}{4\sqrt{b}x^2(a-b)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (-8*a*Sqrt[b]*x^2*Log[x] + Sqrt[a]*(Sqrt[a] + Sqrt[b])^2*x^2*Log[
-Sqrt[b] + Sqrt[a]*(1 + x^2)] - (Sqrt[a] - Sqrt[b])*(2*(Sqrt[a]*S
qrt[b] + b) + (a*x^2 - Sqrt[a]*Sqrt[b]*x^2)*Log[Sqrt[b] + Sqrt[a]
*(1 + x^2)]))/(4*(a - b)^2*Sqrt[b]*x^2)
```

Maple [A] time = 0.014, size = 122, normalized size = 1.3

$$-\frac{1}{(2a-2b)x^2}-2\frac{a\ln(x)}{(a-b)^2}+\frac{a\ln\left(ax^4+2ax^2+a-b\right)}{2(a-b)^2}-\frac{a^2}{2(a-b)^2}\mathit{Artanh}\left(\frac{2ax^2+2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{ab}{2(a-b)^2}\mathit{Artanh}\left(\frac{2ax^2+2a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a-b)/x^2-2*a*ln(x)/(a-b)^2+1/2*a*ln(a*x^4+2*a*x^2+a-b)/(a-b
)^2-1/2/(a-b)^2*a^2/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(
1/2))-1/2/(a-b)^2*a/(a*b)^(1/2)*arctanh(1/2*(2*a*x^2+2*a)/(a*b)^(
1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290063, size = 1, normalized size = 0.01

$$\left[\frac{(a+b)x^2\sqrt{\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{\frac{a}{b}}+a+b}{ax^4+2\,ax^2+a-b}\right)+2\,ax^2\log\left(ax^4+2\,ax^2+a-b\right)-8\,ax^2\log\left(x\right)-2\,a+2\,b}{4\,(a^2-2\,ab+b^2)x^2}, \frac{(a+b)x^2\sqrt{-\frac{a}{b}}\,\text{ar}\ldots}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*((a + b)*x^2*sqrt(a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*
sqrt(a/b) + a + b)/(a*x^4 + 2*a*x^2 + a - b)) + 2*a*x^2*log(a*x^4
 + 2*a*x^2 + a - b) - 8*a*x^2*log(x) - 2*a + 2*b)/((a^2 - 2*a*b +
 b^2)*x^2), 1/2*((a + b)*x^2*sqrt(-a/b)*arctan(b*sqrt(-a/b)/(a*x^
2 + a)) + a*x^2*log(a*x^4 + 2*a*x^2 + a - b) - 4*a*x^2*log(x) - a
 + b)/((a^2 - 2*a*b + b^2)*x^2)]
```

Sympy [A] time = 17.3448, size = 372, normalized size = 3.84

$$-\frac{2a\log(x)}{(a-b)^2}+\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)\log\left(x^2+\frac{-4a^2b\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a-b)^2}-\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+3ab-4b^3\left(\frac{a}{2(a-b)^2}-\ldots\right.}{a^2+ab}\right.$$
$$+\left(\frac{a}{2(a-b)^2}+\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)\log\left(x^2+\frac{-4a^2b\left(\frac{a}{2(a-b)^2}+\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a-b)^2}+\frac{\sqrt{ab}(a+b)}{4b(a^2-2ab+b^2)}\right)+3ab-4b^3\left(\frac{a}{2(a-b)^2}+\ldots\right.}{a^2+ab}\right.$$
$$-\frac{1}{x^2(2a-2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -2*a*log(x)/(a - b)**2 + (a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4
*b*(a**2 - 2*a*b + b**2)))*log(x**2 + (-4*a**2*b*(a/(2*(a - b)**2
) - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))) + a**2 + 8*a*b
```

```
**2*(a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b*
*2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) - sqrt(a*b)*(a + b)/(4*b
*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) + (a/(2*(a - b)**2) + sqr
t(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)))*log(x**2 + (-4*a**2*b
*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2)
)) + a**2 + 8*a*b**2*(a/(2*(a - b)**2) + sqrt(a*b)*(a + b)/(4*b*(
a**2 - 2*a*b + b**2))) + 3*a*b - 4*b**3*(a/(2*(a - b)**2) + sqrt(
a*b)*(a + b)/(4*b*(a**2 - 2*a*b + b**2))))/(a**2 + a*b)) - 1/(x**
2*(2*a - 2*b))
```

GIAC/XCAS [A] time = 0.551049, size = 170, normalized size = 1.75

$$\frac{a\ln\left(ax^4 + 2\,ax^2 + a - b\right)}{2\,(a^2 - 2\,ab + b^2)} - \frac{a\ln\left(x^2\right)}{a^2 - 2\,ab + b^2} + \frac{\left(a^2 + ab\right)\,\arctan\left(\frac{ax^2+a}{\sqrt{-ab}}\right)}{2\,(a^2 - 2\,ab + b^2)\sqrt{-ab}} + \frac{2\,ax^2 - a + b}{2\,(a^2 - 2\,ab + b^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*a*ln(a*x^4 + 2*a*x^2 + a - b)/(a^2 - 2*a*b + b^2) - a*ln(x^2)
/(a^2 - 2*a*b + b^2) + 1/2*(a^2 + a*b)*arctan((a*x^2 + a)/sqrt(-a
*b))/((a^2 - 2*a*b + b^2)*sqrt(-a*b)) + 1/2*(2*a*x^2 - a + b)/((a
^2 - 2*a*b + b^2)*x^2)
```

3.902 $\int \frac{x^4}{a-b+2ax^2+ax^4} dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a]
- Sqrt[b]]])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*Arc
Tan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rubi [A] time = 0.337668, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{5/4}\sqrt{b}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a]
- Sqrt[b]]])/(2*a^(5/4)*Sqrt[b]) - ((Sqrt[a] + Sqrt[b])^(3/2)*Arc
Tan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(5/4)*Sqrt[b])
```

Rubi in Sympy [A] time = 43.3603, size = 128, normalized size = 1.12

$$\frac{x}{a}-\frac{\left(2\sqrt{a}\sqrt{b}+a+b\right)\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{\frac{5}{4}}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{\frac{5}{4}}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] x/a - (2*sqrt(a)*sqrt(b) + a + b)*atan(a**(1/4)*x/sqrt(sqrt(a) +
sqrt(b)))/(2*a**(5/4)*sqrt(b)*sqrt(sqrt(a) + sqrt(b))) + (-2*sqrt
```

```
(a)*sqrt(b) + a + b)*atan(a**(1/4)*x/sqrt(sqrt(a) - sqrt(b)))/(2*
a**(5/4)*sqrt(b)*sqrt(sqrt(a) - sqrt(b)))
```

Mathematica [A] time = 0.150328, size = 144, normalized size = 1.26

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2a\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a - b + 2*a*x^2 + a*x^4),x]

[Out] x/a + ((Sqrt[a] - Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*
Sqrt[b]]])/(2*a*Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((Sqrt[a] +
Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(2*a*Sq
rt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] time = 0.04, size = 210, normalized size = 1.8

$$\frac{x}{a}-1\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{a}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{b}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}+1Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}-\frac{a}{2}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}-\frac{b}{2}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] x/a-1/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(x*a/(((a*b)^(1/2)+a)*a)^(1
/2))-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(x*a/(((a*b)
^(1/2)+a)*a)^(1/2))*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*a
rctan(x*a/(((a*b)^(1/2)+a)*a)^(1/2))*b+1/(((a*b)^(1/2)-a)*a)^(1/2
)*arctanh(x*a/(((a*b)^(1/2)-a)*a)^(1/2))-1/2/(a*b)^(1/2)/(((a*b)^
(1/2)-a)*a)^(1/2)*arctanh(x*a/(((a*b)^(1/2)-a)*a)^(1/2))*a-1/2/(a
*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(x*a/(((a*b)^(1/2)-a)*
a)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{a} - \frac{\int \frac{2\,ax^2+a-b}{ax^4+2\,ax^2+a-b}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a - b)/(a*x^4 + 2*a*x^2 + a - b), x)/a
```

Fricas [A] time = 0.285181, size = 814, normalized size = 7.14

$$a\sqrt{-\frac{a^2 b\sqrt{\frac{9\,a^2+6\,ab+b^2}{a^5 b}}+a+3\,b}{a^2 b}}\log\left(-\left(3\,a^2-2\,ab-b^2\right)x+\left(a^4 b\sqrt{\frac{9\,a^2+6\,ab+b^2}{a^5 b}}-3\,a^2 b-ab^2\right)\sqrt{-\frac{a^2 b\sqrt{\frac{9\,a^2+6\,ab+b^2}{a^5 b}}+a+3\,b}{a^2 b}}\right)-a\sqrt{-\frac{a^2 b \cdots}{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b
)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*sqrt((9*a^2 + 6*
a*b + b^2)/(a^5*b)) - 3*a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 +
 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt
((9*a^2 + 6*a*b + b^2)/(a^5*b)) + a + 3*b)/(a^2*b))*log(-(3*a^2 -
 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) - 3*
a^2*b - a*b^2)*sqrt(-(a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) +
 a + 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a
^5*b)) - a - 3*b)/(a^2*b))*log(-(3*a^2 - 2*a*b - b^2)*x + (a^4*b*
sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*
b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))) + a*sq
rt((a^2*b*sqrt((9*a^2 + 6*a*b + b^2)/(a^5*b)) - a - 3*b)/(a^2*b))
*log(-(3*a^2 - 2*a*b - b^2)*x - (a^4*b*sqrt((9*a^2 + 6*a*b + b^2)
/(a^5*b)) + 3*a^2*b + a*b^2)*sqrt((a^2*b*sqrt((9*a^2 + 6*a*b + b^
2)/(a^5*b)) - a - 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A] time = 3.7333, size = 105, normalized size = 0.92

$$\operatorname{RootSum}\left(256t^4a^5b^2+t^2\left(32a^4b+96a^3b^2\right)+a^3-3a^2b+3ab^2-b^3,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b+4ta^3+24ta^2b+4tab^2}{3a^2-2ab-b^2}\right)\right)\right)+\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(32*a**4*b + 96*a**3*b**2) +
a**3 - 3*a**2*b + 3*a*b**2 - b**3, Lambda(_t, _t*log(x + (64*_t**
3*a**4*b + 4*_t*a**3 + 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 - 2*a*
b - b**2)))) + x/a
```

GIAC/XCAS [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.903 $\int \frac{x^2}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[
b]]])/(2*a^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1
/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rubi [A] time = 0.128659, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/4}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[
b]]])/(2*a^(3/4)*Sqrt[b]) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(a^(1
/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/4)*Sqrt[b])
```

Rubi in Sympy [A] time = 19.6964, size = 94, normalized size = 0.86

$$-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\mathrm{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{\frac{3}{4}}\sqrt{b}}+\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\mathrm{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{\frac{3}{4}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -sqrt(sqrt(a) - sqrt(b))*atan(a**(1/4)*x/sqrt(sqrt(a) - sqrt(b)))
/(2*a**(3/4)*sqrt(b)) + sqrt(sqrt(a) + sqrt(b))*atan(a**(1/4)*x/s
```

```
qrt(sqrt(a) + sqrt(b)))/(2*a**(3/4)*sqrt(b))
```

Mathematica [A] time = 0.189585, size = 128, normalized size = 1.17

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a - b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (-(((Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[
b]]])/Sqrt[a - Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])*ArcTan[(S
qrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]])/
(2*Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.017, size = 134, normalized size = 1.2

$$\frac{1}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}+\frac{a}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}$$
$$-\frac{1}{2}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}+\frac{a}{2}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] 1/2/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(x*a/(((a*b)^(1/2)+a)*a)^(1/2
))+1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(x*a/(((a*b)^(
1/2)+a)*a)^(1/2))*a-1/2/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(x*a/(((
a*b)^(1/2)-a)*a)^(1/2))+1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)
*arctanh(x*a/(((a*b)^(1/2)-a)*a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ax^4 + 2\,ax^2 + a - b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a - b), x)
```

Fricas [A] time = 0.277647, size = 360, normalized size = 3.3

$$\frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{\frac{1}{a^3b}}+1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right) + \frac{1}{4}\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{\frac{1}{a^3b}}-1}{ab}}\sqrt{\frac{1}{a^3b}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*sqrt(1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt(-(a*b*s
qrt(1/(a^3*b)) + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*
sqrt(1/(a^3*b)) + 1)/(a*b))*log(-a^2*b*sqrt(-(a*b*sqrt(1/(a^3*b))
 + 1)/(a*b))*sqrt(1/(a^3*b)) + x) - 1/4*sqrt((a*b*sqrt(1/(a^3*b))
 - 1)/(a*b))*log(a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt
(1/(a^3*b)) + x) + 1/4*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*log(
-a^2*b*sqrt((a*b*sqrt(1/(a^3*b)) - 1)/(a*b))*sqrt(1/(a^3*b)) + x)
```

Sympy [A] time = 1.07186, size = 44, normalized size = 0.4

$$\mathrm{RootSum}\left(256t^4a^3b^2 + 32t^2a^2b + a - b, \left(t \mapsto t \log\left(-64t^3a^2b - 4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 + 32*_t**2*a**2*b + a - b, Lambda(_t,
 _t*log(-64*_t**3*a**2*b - 4*_t*a + x)))
```

GIAC/XCAS [A] time = 2.25456, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="giac")
```

```
[Out] Done
```

3.904 $\int \frac{1}{a-b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=109

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[
a] - Sqrt[b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rubi [A] time = 0.118191, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*a^(1/4)*Sqrt[Sqrt[
a] - Sqrt[b]]*Sqrt[b]) - ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b])
```

Rubi in Sympy [A] time = 17.7047, size = 94, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt[4]{a}\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] -atan(a**(1/4)*x/sqrt(sqrt(a) + sqrt(b)))/(2*a**(1/4)*sqrt(b)*sqr
t(sqrt(a) + sqrt(b))) + atan(a**(1/4)*x/sqrt(sqrt(a) - sqrt(b)))/
```

```
(2*a**(1/4)*sqrt(b)*sqrt(sqrt(a) - sqrt(b)))
```

Mathematica [A] time = 0.111674, size = 105, normalized size = 0.96

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a - b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]*Sqrt[b]]]/(2*Sqrt[a - Sqrt[a]
*Sqrt[b]]*Sqrt[b]) - ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]
]/(2*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Maple [A] time = 0.014, size = 74, normalized size = 0.7

$$-\frac{a}{2}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}-\frac{a}{2}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(x*a/(((a*b)^(1/
2)+a)*a)^(1/2))*a-1/2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arcta
nh(x*a/(((a*b)^(1/2)-a)*a)^(1/2))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^4+2\,ax^2+a-b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="maxima")
```

```
[Out] integrate(1/(a*x^4 + 2*a*x^2 + a - b), x)
```

Fricas [A] time = 0.282818, size = 747, normalized size = 6.85

$$-\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}\log\left(-\left(b-\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{-\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}+1}{ab-b^2}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)$$
$$+\frac{1}{4}\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}\log\left(-\left(b+\frac{a^2b-ab^2}{\sqrt{a^3b-2a^2b^2+ab^3}}\right)\sqrt{\frac{\frac{ab-b^2}{\sqrt{a^3b-2a^2b^2+ab^3}}-1}{ab-b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b
 - b^2))*log((b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
)*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3) + 1)/(a*b -
b^2)) + x) + 1/4*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^
3) + 1)/(a*b - b^2))*log(-(b - (a^2*b - a*b^2)/sqrt(a^3*b - 2*a^2
*b^2 + a*b^3))*sqrt(-((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3)
 + 1)/(a*b - b^2)) + x) - 1/4*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^
2*b^2 + a*b^3) - 1)/(a*b - b^2))*log((b + (a^2*b - a*b^2)/sqrt(a^
3*b - 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b - 2*a^2*b^
2 + a*b^3) - 1)/(a*b - b^2)) + x) + 1/4*sqrt(((a*b - b^2)/sqrt(a^
3*b - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2))*log(-(b + (a^2*b - a*b
^2)/sqrt(a^3*b - 2*a^2*b^2 + a*b^3))*sqrt(((a*b - b^2)/sqrt(a^3*b
 - 2*a^2*b^2 + a*b^3) - 1)/(a*b - b^2)) + x)
```

Sympy [A] time = 1.96923, size = 63, normalized size = 0.58

$$\operatorname{RootSum}\left(t^4\left(256a^2b^2-256ab^3\right)+32t^2ab+1,\left(t\mapsto t\log\left(-64t^3a^2b+64t^3ab^2-4ta-4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 - 256*a*b**3) + 32*_t**2*a*b + 1, La
mbda(_t, _t*log(-64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a - 4*_
t*b + x)))
```

GIAC/XCAS [A] time = 0.687337, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^4 + 2*a*x^2 + a - b),x, algorithm="giac")
```

```
[Out] Done
```

3.905 $\int \frac{1}{x^2(a-b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=121

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqr
t[b]]])/(2*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(
a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)
*Sqrt[b])
```

Rubi [A] time = 0.265028, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{1}{x(a-b)}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a - b)*x)) - (a^(1/4)*ArcTan[(a^(1/4)*x)/Sqrt[Sqrt[a] - Sqr
t[b]]])/(2*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]) + (a^(1/4)*ArcTan[(
a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*(Sqrt[a] + Sqrt[b])^(3/2)
*Sqrt[b])
```

Rubi in Sympy [A] time = 37.6009, size = 128, normalized size = 1.06

$$\frac{\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{\sqrt{a}+\sqrt{b}}\,(a-b)}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)\operatorname{atan}\left(\frac{\sqrt[4]{a}x}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{\sqrt{a}-\sqrt{b}}\,(a-b)}-\frac{1}{x\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] a**(1/4)*(sqrt(a) - sqrt(b))*atan(a**(1/4)*x/sqrt(sqrt(a) + sqrt(
b)))/(2*sqrt(b)*sqrt(sqrt(a) + sqrt(b))*(a - b)) - a**(1/4)*(sqrt
(a) + sqrt(b))*atan(a**(1/4)*x/sqrt(sqrt(a) - sqrt(b)))/(2*sqrt(b
)*sqrt(sqrt(a) - sqrt(b))*(a - b)) - 1/(x*(a - b))
```

Mathematica [A] time = 0.280051, size = 143, normalized size = 1.18

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+a\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-\sqrt{a}\sqrt{b}}}\right)}{\sqrt{b}\sqrt{a-\sqrt{a}\sqrt{b}}}-\frac{\left(a-\sqrt{a}\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2}{x}}{2(b-a)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a - b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (2/x + ((a + Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - Sqrt[a]
*Sqrt[b]]])/(Sqrt[a - Sqrt[a]*Sqrt[b]]*Sqrt[b]) - ((a - Sqrt[a]*S
qrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a +
Sqrt[a]*Sqrt[b]]*Sqrt[b]))/(2*(-a + b))
```

Maple [B] time = 0.017, size = 180, normalized size = 1.5

$$-\frac{1}{(a-b)x}-\frac{a}{2a-2b}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}$$
$$+\frac{a^2}{2a-2b}\arctan\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}+a\right)a}}$$
$$+\frac{a}{2a-2b}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}$$
$$+\frac{a^2}{2a-2b}Artanh\left(ax\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}\right)\frac{1}{\sqrt{ab}}\frac{1}{\sqrt{\left(\sqrt{ab}-a\right)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a*x^4+2*a*x^2+a-b),x)
```

```
[Out] -1/(a-b)/x-1/2*a/(a-b)/(((a*b)^(1/2)+a)*a)^(1/2)*arctan(x*a/(((a*
b)^(1/2)+a)*a)^(1/2))+1/2*a^2/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+a)*
a)^(1/2)*arctan(x*a/(((a*b)^(1/2)+a)*a)^(1/2))+1/2*a/(a-b)/(((a*b
)^(1/2)-a)*a)^(1/2)*arctanh(x*a/(((a*b)^(1/2)-a)*a)^(1/2))+1/2*a^
2/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-a)*a)^(1/2)*arctanh(x*a/(((a*b)
^(1/2)-a)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\int \frac{x^2+2}{ax^4+2\,ax^2+a-b}\,dx}{a-b}-\frac{1}{(a-b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x^2),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a - b), x)/(a - b) - 1/
((a - b)*x)
```

Fricas [A] time = 0.289508, size = 2176, normalized size = 17.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*((a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3
- b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4
*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4))*log((3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 -
(a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2
)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a
*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*a^2*b^2 + 3*a*b^3 -
 b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*
b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b
^2 + 3*a*b^3 - b^4))) - (a - b)*x*sqrt(-(a^2 + 3*a*b + (a^3*b - 3
*a^2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b -
 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7
)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((3*a^2 + a*b)*x - (6
*a^2*b + 2*a*b^2 - (a^4*b - 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt((9*a^
3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4
```

```
 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 + 3*a*b + (a^3*b - 3*
a^2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b -
6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)
))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))) + (a - b)*x*sqrt(-(a^2 +
 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*
b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*
b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))*log((
3*a^2 + a*b)*x + (6*a^2*b + 2*a*b^2 + (a^4*b - 2*a^3*b^2 + 2*a*b^
4 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a
^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))*sqrt(-(a^2 +
3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sqrt((9*a^3 + 6*a^2*b
 + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b
^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4))) - (a
- b)*x*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*s
qrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 2
0*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*
a*b^3 - b^4))*log((3*a^2 + a*b)*x - (6*a^2*b + 2*a*b^2 + (a^4*b -
 2*a^3*b^2 + 2*a*b^4 - b^5)*sqrt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b
 - 6*a^5*b^2 + 15*a^4*b^3 - 20*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b
^7)))*sqrt(-(a^2 + 3*a*b - (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*sq
rt((9*a^3 + 6*a^2*b + a*b^2)/(a^6*b - 6*a^5*b^2 + 15*a^4*b^3 - 20
*a^3*b^4 + 15*a^2*b^5 - 6*a*b^6 + b^7)))/(a^3*b - 3*a^2*b^2 + 3*a
*b^3 - b^4))) - 4)/((a - b)*x)
```

Sympy [A] time = 7.41576, size = 134, normalized size = 1.11

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2-768a^2b^3+768ab^4-256b^5\right)+t^2\left(32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{64t^3a^4b-128t^3a^3b^2+128t\right.\right.\right.$$
$$-\frac{1}{x(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a*x**4+2*a*x**2+a-b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 - 768*a**2*b**3 + 768*a*b**4 - 256*b
**5) + _t**2*(32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x + (
64*_t**3*a**4*b - 128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 - 64*_t*
*3*b**5 + 4*_t*a**3 + 40*_t*a**2*b + 20*_t*a*b**2)/(3*a**2 + a*b)
))) - 1/(x*(a - b))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a - b)*x^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.906 $\int \frac{x^5}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=69

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3
/2)*Sqrt[b]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rubi [A] time = 0.176918, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{3/2}\sqrt{b}}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{x^2}{2a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x^2/(2*a) + ((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*a^(3
/2)*Sqrt[b]) - Log[a + b + 2*a*x^2 + a*x^4]/(2*a)
```

Rubi in Sympy [A] time = 28.6312, size = 58, normalized size = 0.84

$$\frac{x^2}{2a}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{2a}+\frac{(a-b)\,\mathrm{atan}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2a^{\frac{3}{2}}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] x**2/(2*a) - log(a*x**4 + 2*a*x**2 + a + b)/(2*a) + (a - b)*atan(
sqrt(a)*(x**2 + 1)/sqrt(b))/(2*a**(3/2)*sqrt(b))
```

Mathematica [A] time = 0.0622089, size = 62, normalized size = 0.9

$$\frac{\sqrt{a}\left(x^2-\log\left(a\left(x^2+1\right)^2+b\right)\right)+\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{a}\left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Sqrt[a]*
(x^2 - Log[b + a*(1 + x^2)^2]))/(2*a^(3/2))
```

Maple [A] time = 0.009, size = 84, normalized size = 1.2

$$\frac{x^2}{2\,a}-\frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{2\,a}+\frac{1}{2}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{b}{2\,a}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2*x^2/a-1/2*ln(a*x^4+2*a*x^2+a+b)/a+1/2/(a*b)^(1/2)*arctan(1/2*
(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/a/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+
2*a)/(a*b)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.283593, size = 1, normalized size = 0.01

$$\left[-\frac{(a-b)\log\left(-\frac{2\,abx^2+2\,ab-(ax^4+2\,ax^2+a-b)\sqrt{-ab}}{ax^4+2\,ax^2+a+b}\right)-2\,\sqrt{-ab}\left(x^2-\log\left(ax^4+2\,ax^2+a+b\right)\right)}{4\,\sqrt{-ab}a},\ -\frac{(a-b)\arctan\left(\frac{b}{\sqrt{ab}(x^2+1)}\right)-\sqrt{ab}\left(x^2-\log\left(ax^4+2\,ax^2+a+b\right)\right)}{2\,\sqrt{ab}a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)*log(-(2*a*b*x^2 + 2*a*b - (a*x^4 + 2*a*x^2 + a - b
)*sqrt(-a*b))/(a*x^4 + 2*a*x^2 + a + b)) - 2*sqrt(-a*b)*(x^2 - lo
g(a*x^4 + 2*a*x^2 + a + b)))/(sqrt(-a*b)*a), -1/2*((a - b)*arctan
(b/(sqrt(a*b)*(x^2 + 1))) - sqrt(a*b)*(x^2 - log(a*x^4 + 2*a*x^2
+ a + b)))/(sqrt(a*b)*a)]
```

Sympy [A] time = 3.66048, size = 144, normalized size = 2.09

$$\left(-\frac{1}{2a}-\frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{2a}-\frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right)+a+b}{a-b}\right)+\left(-\frac{1}{2a}+\frac{\sqrt{-a^3b}\,(a-b)}{4a^3b}\right)\log\left(x^2+\frac{4ab\left(-\frac{1}{2a}+\frac{\sqrt{-a^3b}(a-b)}{4a^3b}\right)+a+b}{a-b}\right)+\frac{x^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(
-1/(2*a) - sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) +
(-1/(2*a) + sqrt(-a**3*b)*(a - b)/(4*a**3*b))*log(x**2 + (4*a*b*(
-1/(2*a) + sqrt(-a**3*b)*(a - b)/(4*a**3*b)) + a + b)/(a - b)) +
x**2/(2*a)
```

GIAC/XCAS [A] time = 0.547772, size = 78, normalized size = 1.13

$$\frac{x^2}{2\,a} + \frac{(a-b)\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a} - \frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a + 1/2*(a - b)*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*
a) - 1/2*ln(a*x^4 + 2*a*x^2 + a + b)/a
```

3.907 $\int \frac{x^3}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=54

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] -ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a
+ b + 2*a*x^2 + a*x^4]/(4*a)
```

Rubi [A] time = 0.103545, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b]) + Log[a
+ b + 2*a*x^2 + a*x^4]/(4*a)
```

Rubi in Sympy [A] time = 18.2146, size = 48, normalized size = 0.89

$$\frac{\log\left(ax^4+2ax^2+a+b\right)}{4a}-\frac{\operatorname{atan}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] log(a*x**4 + 2*a*x**2 + a + b)/(4*a) - atan(sqrt(a)*(x**2 + 1)/sq
rt(b))/(2*sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.0297325, size = 49, normalized size = 0.91

$$\frac{\log \left(a \left(x^2+1\right)^2+b\right)-\frac{2 \sqrt{a} \tan ^{-1}\left(\frac{\sqrt{a} \left(x^2+1\right)}{\sqrt{b}}\right)}{\sqrt{b}}}{4 a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] ((-2*Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/Sqrt[b] + Log[b
 + a*(1 + x^2)^2])/(4*a)
```

Maple [A] time = 0.003, size = 47, normalized size = 0.9

$$\frac{\ln \left(a x^4+2\,a x^2+a+b\right)}{4\,a}-\frac{1}{2}\arctan \left(\frac{2\,a x^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/4*ln(a*x^4+2*a*x^2+a+b)/a-1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2
*a)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279579, size = 1, normalized size = 0.02

$$\left[\frac{a\log\left(-\frac{2\,abx^2+2\,ab-(ax^4+2\,ax^2+a-b)\sqrt{-ab}}{ax^4+2\,ax^2+a+b}\right)+\sqrt{-ab}\log\left(ax^4+2\,ax^2+a+b\right)}{4\,\sqrt{-ab}a}, \frac{2\,a\arctan\left(\frac{b}{\sqrt{ab}(x^2+1)}\right)+\sqrt{ab}\log\left(ax^4+2\,ax^2+\right.}{4\,\sqrt{ab}a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="fricas")
```

```
[Out] [1/4*(a*log(-(2*a*b*x^2 + 2*a*b - (a*x^4 + 2*a*x^2 + a - b)*sqrt(
-a*b))/(a*x^4 + 2*a*x^2 + a + b)) + sqrt(-a*b)*log(a*x^4 + 2*a*x^
2 + a + b))/(sqrt(-a*b)*a), 1/4*(2*a*arctan(b/(sqrt(a*b)*(x^2 + 1
))) + sqrt(a*b)*log(a*x^4 + 2*a*x^2 + a + b))/(sqrt(a*b)*a)]
```

Sympy [A] time = 1.40943, size = 117, normalized size = 2.17

$$\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}-\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)+\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)\log\left(x^2+\frac{-4ab\left(\frac{1}{4a}+\frac{\sqrt{-a^3b}}{4a^2b}\right)+a+b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (1/(4*a) - sqrt(-a**3*b)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a)
- sqrt(-a**3*b)/(4*a**2*b)) + a + b)/a) + (1/(4*a) + sqrt(-a**3*b
)/(4*a**2*b))*log(x**2 + (-4*a*b*(1/(4*a) + sqrt(-a**3*b)/(4*a**2
*b)) + a + b)/a)
```

GIAC/XCAS [A] time = 0.549735, size = 57, normalized size = 1.06

$$-\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}}+\frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="giac")
```

```
[Out] -1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b) + 1/4*ln(a*x^4 + 2*a
*x^2 + a + b)/a
```

3.908 $\int \frac{x}{a+b+2ax^2+ax^4} dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0605417, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b + 2*a*x^2 + a*x^4),x]

[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Rubi in Sympy [A] time = 9.06517, size = 27, normalized size = 0.87

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(a*x**4+2*a*x**2+a+b),x)

[Out] atan(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*sqrt(a)*sqrt(b))
```

Mathematica [A] time = 0.0122694, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b + 2*a*x^2 + a*x^4),x]

[Out] ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]]/(2*Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.002, size = 26, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\frac{2\,ax^{2}+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(a*x^4+2*a*x^2+a+b),x)

[Out] 1/2/(a*b)^(1/2)*arctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278096, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{2\,abx^{2}+2\,ab+\left(ax^{4}+2\,ax^{2}+a-b\right)\sqrt{-ab}}{ax^{4}+2\,ax^{2}+a+b}\right)}{4\,\sqrt{-ab}},-\frac{\arctan\left(\frac{b}{\sqrt{ab}(x^{2}+1)}\right)}{2\,\sqrt{ab}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="fricas")
```

```
[Out] [1/4*log((2*a*b*x^2 + 2*a*b + (a*x^4 + 2*a*x^2 + a - b)*sqrt(-a*b
))/(a*x^4 + 2*a*x^2 + a + b))/sqrt(-a*b), -1/2*arctan(b/(sqrt(a*b
)*(x^2 + 1)))/sqrt(a*b)]
```

Sympy [A] time = 0.734623, size = 60, normalized size = 1.94

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}+\frac{\sqrt{-\frac{1}{ab}}\log\left(b\sqrt{-\frac{1}{ab}}+x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-b*sqrt(-1/(a*b)) + x**2 + 1)/4 + sqrt(-1/(a*
b))*log(b*sqrt(-1/(a*b)) + x**2 + 1)/4
```

GIAC/XCAS [A] time = 0.542831, size = 28, normalized size = 0.9

$$\frac{\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="giac")
```

```
[Out] 1/2*arctan((a*x^2 + a)/sqrt(a*b))/sqrt(a*b)
```

3.909 $\int \frac{1}{x(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=69

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}+\frac{\log(x)}{a+b}$$

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*Sqrt[b]*(a + b)
) + Log[x]/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rubi [A] time = 0.148223, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4(a+b)}+\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(Sqrt[a]*(1 + x^2))/Sqrt[b]])/(2*Sqrt[b]*(a + b)
) + Log[x]/(a + b) - Log[a + b + 2*a*x^2 + a*x^4]/(4*(a + b))
```

Rubi in Sympy [A] time = 28.3211, size = 66, normalized size = 0.96

$$-\frac{\sqrt{a}\,\text{atan}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)}+\frac{\log\left(x^2\right)}{2\,(a+b)}-\frac{\log\left(ax^4+2ax^2+a+b\right)}{4\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(a)*atan(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*sqrt(b)*(a + b)) + l
og(x**2)/(2*(a + b)) - log(a*x**4 + 2*a*x**2 + a + b)/(4*(a + b))
```

Mathematica [C] time = 0.0908083, size = 105, normalized size = 1.52

$$\frac{i\left(\sqrt{a}+i\sqrt{b}\right)\log\left(\sqrt{a}\left(x^2+1\right)-i\sqrt{b}\right)+\left(-\sqrt{b}-i\sqrt{a}\right)\log\left(\sqrt{a}\left(x^2+1\right)+i\sqrt{b}\right)+4\sqrt{b}\log(x)}{4\sqrt{b}(a+b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] (4*Sqrt[b]*Log[x] + I*(Sqrt[a] + I*Sqrt[b])*Log[(-I)*Sqrt[b] + Sq
rt[a]*(1 + x^2)] + ((-I)*Sqrt[a] - Sqrt[b])*Log[I*Sqrt[b] + Sqrt[
a]*(1 + x^2)])/(4*Sqrt[b]*(a + b))
```

Maple [A] time = 0.009, size = 63, normalized size = 0.9

$$-\frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{4\,a+4\,b}-\frac{a}{2\,a+2\,b}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}+\frac{\ln(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/4*ln(a*x^4+2*a*x^2+a+b)/(a+b)-1/2*a/(a+b)/(a*b)^(1/2)*arctan(1
/2*(2*a*x^2+2*a)/(a*b)^(1/2))+ln(x)/(a+b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284861, size = 1, normalized size = 0.01

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-\log\left(ax^4+2\,ax^2+a+b\right)+4\,\log\left(x\right)}{4\,(a+b)}, \frac{2\,\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-\log\left(ax^4+2\,ax^2+a\right.}{4\,(a+b)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b)*sqrt(-a/b)
+ a - b)/(a*x^4 + 2*a*x^2 + a + b)) - log(a*x^4 + 2*a*x^2 + a + b
) + 4*log(x))/(a + b), 1/4*(2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*x^2
 + a)) - log(a*x^4 + 2*a*x^2 + a + b) + 4*log(x))/(a + b)]
```

Sympy [A] time = 7.15128, size = 194, normalized size = 2.81

$$\left(-\frac{1}{4\,(a+b)}-\frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}-\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right)$$
$$+\left(-\frac{1}{4\,(a+b)}+\frac{\sqrt{-ab}}{4b\,(a+b)}\right)\log\left(x^2+\frac{-4ab\left(-\frac{1}{4(a+b)}+\frac{\sqrt{-ab}}{4b(a+b)}\right)+a-4b^2\left(-\frac{1}{4(a+b)}+\frac{\sqrt{-ab}}{4b(a+b)}\right)-b}{a}\right)$$
$$+\frac{\log\left(x\right)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] (-1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-
1/(4*(a + b)) - sqrt(-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a
+ b)) - sqrt(-a*b)/(4*b*(a + b))) - b)/a) + (-1/(4*(a + b)) + sqr
t(-a*b)/(4*b*(a + b)))*log(x**2 + (-4*a*b*(-1/(4*(a + b)) + sqrt(
-a*b)/(4*b*(a + b))) + a - 4*b**2*(-1/(4*(a + b)) + sqrt(-a*b)/(4
*b*(a + b))) - b)/a) + log(x)/(a + b)
```

GIAC/XCAS [A] time = 0.545411, size = 82, normalized size = 1.19

$$-\frac{a\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,\sqrt{ab}(a+b)}-\frac{\ln\left(ax^4+2\,ax^2+a+b\right)}{4\,(a+b)}+\frac{\ln\left(x^2\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x),x, algorithm="giac")
```

```
[Out] -1/2*a*arctan((a*x^2 + a)/sqrt(a*b))/(sqrt(a*b)*(a + b)) - 1/4*ln
(a*x^4 + 2*a*x^2 + a + b)/(a + b) + 1/2*ln(x^2)/(a + b)
```

3.910 $\int \frac{1}{x^3(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2(a+b)}+\frac{\sqrt{a}(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

```
[Out] -1/(2*(a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/
Sqrt[b]])/(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log
[a + b + 2*a*x^2 + a*x^4])/(2*(a + b)^2)
```

Rubi [A] time = 0.294809, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{1}{2x^2(a+b)}+\frac{\sqrt{a}(a-b)\tan^{-1}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}(a+b)^2}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2(a+b)^2}-\frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a + b)*x^2) + (Sqrt[a]*(a - b)*ArcTan[(Sqrt[a]*(1 + x^2))/
Sqrt[b]])/(2*Sqrt[b]*(a + b)^2) - (2*a*Log[x])/(a + b)^2 + (a*Log
[a + b + 2*a*x^2 + a*x^4])/(2*(a + b)^2)
```

Rubi in Sympy [A] time = 43.8031, size = 83, normalized size = 0.93

$$\frac{\sqrt{a}\,(a-b)\,\mathrm{atan}\left(\frac{\sqrt{a}(x^2+1)}{\sqrt{b}}\right)}{2\sqrt{b}\,(a+b)^2}-\frac{a\log\left(x^2\right)}{(a+b)^2}+\frac{a\log\left(ax^4+2ax^2+a+b\right)}{2\,(a+b)^2}-\frac{1}{2x^2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] sqrt(a)*(a - b)*atan(sqrt(a)*(x**2 + 1)/sqrt(b))/(2*sqrt(b)*(a +
b)**2) - a*log(x**2)/(a + b)**2 + a*log(a*x**4 + 2*a*x**2 + a + b
)/(2*(a + b)**2) - 1/(2*x**2*(a + b))
```

Mathematica [C] time = 0.170537, size = 163, normalized size = 1.83

$$\frac{\left(2a^{3/2}\sqrt{b}-ia^2+iab\right)\log\left(\sqrt{a}x^2+\sqrt{a}-i\sqrt{b}\right)}{4\sqrt{a}\sqrt{b}(a+b)^2}+\frac{\left(2a^{3/2}\sqrt{b}+ia^2-iab\right)\log\left(\sqrt{a}x^2+\sqrt{a}+i\sqrt{b}\right)}{4\sqrt{a}\sqrt{b}(a+b)^2}-\frac{1}{2x^2(a+b)}-\frac{2a\log(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -1/(2*(a + b)*x^2) - (2*a*Log[x])/(a + b)^2 + (((-I)*a^2 + 2*a^(3
/2)*Sqrt[b] + I*a*b)*Log[Sqrt[a] - I*Sqrt[b] + Sqrt[a]*x^2])/(4*S
qrt[a]*Sqrt[b]*(a + b)^2) + ((I*a^2 + 2*a^(3/2)*Sqrt[b] - I*a*b)*
Log[Sqrt[a] + I*Sqrt[b] + Sqrt[a]*x^2])/(4*Sqrt[a]*Sqrt[b]*(a + b
)^2)
```

Maple [A] time = 0.012, size = 110, normalized size = 1.2

$$\frac{a\ln\left(ax^4+2\,ax^2+a+b\right)}{2\,(a+b)^2}+\frac{a^2}{2\,(a+b)^2}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{ab}{2\,(a+b)^2}\arctan\left(\frac{2\,ax^2+2\,a}{2}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}-\frac{1}{(2\,a+2\,b)\,x^2}-2\,\frac{a\ln(x)}{(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/2*a*ln(a*x^4+2*a*x^2+a+b)/(a+b)^2+1/2/(a+b)^2*a^2/(a*b)^(1/2)*a
rctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))-1/2/(a+b)^2*a/(a*b)^(1/2)*ar
ctan(1/2*(2*a*x^2+2*a)/(a*b)^(1/2))*b-1/2/(a+b)/x^2-2*a*ln(x)/(a+
b)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290071, size = 1, normalized size = 0.01

$$\left[-\frac{(a-b)x^2\sqrt{-\frac{a}{b}}\log\left(\frac{ax^4+2\,ax^2-2\,(bx^2+b)\sqrt{-\frac{a}{b}}+a-b}{ax^4+2\,ax^2+a+b}\right)-2\,ax^2\log\left(ax^4+2\,ax^2+a+b\right)+8\,ax^2\log\left(x\right)+2\,a+2\,b}{4\,(a^2+2\,ab+b^2)x^2}, -\frac{(a-b)x^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{\frac{a}{b}}}{ax^2+a}\right)-ax^2\log\left(ax^4+2\,ax^2+a+b\right)+4\,ax^2\log\left(x\right)+a+b}{2\,(a^2+2\,ab+b^2)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/4*((a - b)*x^2*sqrt(-a/b)*log((a*x^4 + 2*a*x^2 - 2*(b*x^2 + b
)*sqrt(-a/b) + a - b)/(a*x^4 + 2*a*x^2 + a + b)) - 2*a*x^2*log(a*
x^4 + 2*a*x^2 + a + b) + 8*a*x^2*log(x) + 2*a + 2*b)/((a^2 + 2*a*
b + b^2)*x^2), -1/2*((a - b)*x^2*sqrt(a/b)*arctan(b*sqrt(a/b)/(a*
x^2 + a)) - a*x^2*log(a*x^4 + 2*a*x^2 + a + b) + 4*a*x^2*log(x) +
 a + b)/((a^2 + 2*a*b + b^2)*x^2)]
```

Sympy [A] time = 17.2805, size = 386, normalized size = 4.34

$$-\frac{2a\log(x)}{(a+b)^2}+\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}\,(a-b)}{4b\,(a^2+2ab+b^2)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}-\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3ab+4b^3\left(\frac{a}{2(a+b)^2}-\ldots\right.}{a^2-ab}\right.$$

$$+\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}\,(a-b)}{4b\,(a^2+2ab+b^2)}\right)\log\left(x^2+\frac{4a^2b\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)+a^2+8ab^2\left(\frac{a}{2(a+b)^2}+\frac{\sqrt{-ab}(a-b)}{4b(a^2+2ab+b^2)}\right)-3ab+4b^3\left(\frac{a}{2(a+b)^2}+\ldots\right.}{a^2-ab}\right.$$

$$-\frac{1}{x^2\,(2a+2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -2*a*log(x)/(a + b)**2 + (a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(
4*b*(a**2 + 2*a*b + b**2)))*log(x**2 + (4*a**2*b*(a/(2*(a + b)**2
) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))) + a**2 + 8*a*
b**2*(a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b +
b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) - sqrt(-a*b)*(a - b)/(
4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) + (a/(2*(a + b)**2) +
sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2)))*log(x**2 + (4*a**
2*b*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b
**2))) + a**2 + 8*a*b**2*(a/(2*(a + b)**2) + sqrt(-a*b)*(a - b)/(
4*b*(a**2 + 2*a*b + b**2))) - 3*a*b + 4*b**3*(a/(2*(a + b)**2) +
sqrt(-a*b)*(a - b)/(4*b*(a**2 + 2*a*b + b**2))))/(a**2 - a*b)) -
1/(x**2*(2*a + 2*b))
```

GIAC/XCAS [A] time = 0.537401, size = 169, normalized size = 1.9

$$\frac{a\ln\left(ax^4 + 2\,ax^2 + a + b\right)}{2\,(a^2 + 2\,ab + b^2)} - \frac{a\ln\left(x^2\right)}{a^2 + 2\,ab + b^2} + \frac{\left(a^2 - ab\right)\,\arctan\left(\frac{ax^2+a}{\sqrt{ab}}\right)}{2\,(a^2 + 2\,ab + b^2)\sqrt{ab}} + \frac{2\,ax^2 - a - b}{2\,(a^2 + 2\,ab + b^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x^3),x, algorithm="giac")
```

```
[Out] 1/2*a*ln(a*x^4 + 2*a*x^2 + a + b)/(a^2 + 2*a*b + b^2) - a*ln(x^2)
/(a^2 + 2*a*b + b^2) + 1/2*(a^2 - a*b)*arctan((a*x^2 + a)/sqrt(a*
b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/2*(2*a*x^2 - a - b)/((a^2
 + 2*a*b + b^2)*x^2)
```

3.911 $\int \frac{x^4}{a+b+2ax^2+ax^4} \, dx$

Optimal. Leaf size=432

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}} - \frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}} + \frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}} - \frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}} + \frac{x}{a}$$

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sq
rt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*
Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a +
b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] +
 Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5
/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a + b - 2*Sqrt[a
]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] +
Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqr
t[-Sqrt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log
[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + S
qrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[
a + b]])
```

Rubi [A] time = 2.05947, antiderivative size = 432, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}} - \frac{\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}} + \frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}} - \frac{\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{5/4}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}} + \frac{x}{a}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a + ((a + b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sq
rt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*
Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - ((a +
b + 2*Sqrt[a]*Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] +
 Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*Sqrt[2]*a^(5
/4)*Sqrt[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) + ((a + b - 2*Sqrt[a
]*Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] +
Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqr
t[-Sqrt[a] + Sqrt[a + b]]) - ((a + b - 2*Sqrt[a]*Sqrt[a + b])*Log
[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + S
qrt[a]*x^2])/(4*Sqrt[2]*a^(5/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[
a + b]])
```

Rubi in Sympy [A] time = 158.426, size = 393, normalized size = 0.91

$$\frac{x}{a}-\frac{\sqrt{2}\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x-\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4a^{\frac{5}{4}}\sqrt{\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}-\frac{\sqrt{2}\left(2\sqrt{a}\sqrt{a+b}+a+b\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x+\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4a^{\frac{5}{4}}\sqrt{\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}+\frac{\sqrt{2}\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(x^2+\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8a^{\frac{5}{4}}\sqrt{-\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}-\frac{\sqrt{2}\left(-2\sqrt{a}\sqrt{a+b}+a+b\right)\log\left(x^2+\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8a^{\frac{5}{4}}\sqrt{-\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] x/a - sqrt(2)*(2*sqrt(a)*sqrt(a + b) + a + b)*atan(sqrt(2)*(a**(1
/4)*x - sqrt(-2*sqrt(a) + 2*sqrt(a + b))/2)/sqrt(sqrt(a) + sqrt(a
 + b)))/(4*a**(5/4)*sqrt(sqrt(a) + sqrt(a + b))*sqrt(a + b)) - sq
rt(2)*(2*sqrt(a)*sqrt(a + b) + a + b)*atan(sqrt(2)*(a**(1/4)*x +
sqrt(-2*sqrt(a) + 2*sqrt(a + b))/2)/sqrt(sqrt(a) + sqrt(a + b)))/
(4*a**(5/4)*sqrt(sqrt(a) + sqrt(a + b))*sqrt(a + b)) + sqrt(2)*(-
2*sqrt(a)*sqrt(a + b) + a + b)*log(x**2 + sqrt(a + b)/sqrt(a) - s
qrt(2)*x*sqrt(-sqrt(a) + sqrt(a + b))/a**(1/4))/(8*a**(5/4)*sqrt(
-sqrt(a) + sqrt(a + b))*sqrt(a + b)) - sqrt(2)*(-2*sqrt(a)*sqrt(a
 + b) + a + b)*log(x**2 + sqrt(a + b)/sqrt(a) + sqrt(2)*x*sqrt(-s
qrt(a) + sqrt(a + b))/a**(1/4))/(8*a**(5/4)*sqrt(-sqrt(a) + sqrt(
a + b))*sqrt(a + b))
```

Mathematica [C] time = 0.162432, size = 164, normalized size = 0.38

$$-\frac{i\left(\sqrt{a}-i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{i\left(\sqrt{a}+i\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2a\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}}+\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] x/a - ((I/2)*(Sqrt[a] - I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a -
I*Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((
I/2)*(Sqrt[a] + I*Sqrt[b])^2*ArcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a
]*Sqrt[b]]])/(a*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] time = 0.128, size = 1658, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] x/a+1/8/a/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)
^(1/2))*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a^2/b*ln(-a
^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2))*(a+b)^(1/
2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/a^(3/2)/b*ln
(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2))*(2*(a^
2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/a^(1/2)/b*ln(-a^(1/2)
*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2))*(2*(a^2+a*b)^(1
/2)-2*a)^(1/2)+1/a/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^
(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1
/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)-1/4/a/b
/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a
^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*
(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(
1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^2/b/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))
^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^
(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/
2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^
(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(
1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
*(a^2+a*b)^(1/2)+1/2/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))
^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1
/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(
a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a/b*ln(a
^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(a+b)^(1/
2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a^2/b*ln(a^(1/2)*x^2+(2*(a*(
a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(a+b)^(1/2)*(2*(a^2+a*b)^(1
/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/a^(3/2)/b*ln(a^(1/2)*x^2+(2*(a
*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(
1/2)*(a^2+a*b)^(1/2)+1/4/a^(1/2)/b*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1
```

```
/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/a/(
4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1
/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*
(a+b))^(1/2)+2*a)^(1/2))*(a+b)^(1/2)+1/4/a/b/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^
(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(
1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2
)-2*a)^(1/2)+1/4/a^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2
*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a
^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/
2)-2*a)^(1/2)*(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)
^(1/2)-1/2/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a
)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(
1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)
-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/2/a^(
1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan
((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/
2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2
*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{x}{a} - \frac{\int \frac{2\,ax^2+a+b}{ax^4+2\,ax^2+a+b}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="maxima")
```

```
[Out] x/a - integrate((2*a*x^2 + a + b)/(a*x^4 + 2*a*x^2 + a + b), x)/a
```

Fricas [A] time = 0.285478, size = 830, normalized size = 1.92

$$a\sqrt{\frac{a^2b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+a-3\,b}{a^2b}}\,\log\left(-\left(3\,a^2+2\,ab-b^2\right)x+\left(a^4b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+3\,a^2b-ab^2\right)\sqrt{\frac{a^2b\sqrt{-\frac{9\,a^2-6\,ab+b^2}{a^5b}}+a-3\,b}{a^2b}}\right)-a\sqrt{\frac{a^2b}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="fricas")
```

```
[Out] 1/4*(a*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b
)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^4*b*sqrt(-(9*a^2 - 6
```

```
*a*b + b^2)/(a^5*b)) + 3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2
- 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b))) - a*sqrt((a^2*b*sqrt
(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) + a - 3*b)/(a^2*b))*log(-(3*a^2
+ 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) +
3*a^2*b - a*b^2)*sqrt((a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b))
 + a - 3*b)/(a^2*b))) - a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2
)/(a^5*b)) - a + 3*b)/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x + (a^
4*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(
-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b)))
 + a*sqrt(-(a^2*b*sqrt(-(9*a^2 - 6*a*b + b^2)/(a^5*b)) - a + 3*b)
/(a^2*b))*log(-(3*a^2 + 2*a*b - b^2)*x - (a^4*b*sqrt(-(9*a^2 - 6*
a*b + b^2)/(a^5*b)) - 3*a^2*b + a*b^2)*sqrt(-(a^2*b*sqrt(-(9*a^2
- 6*a*b + b^2)/(a^5*b)) - a + 3*b)/(a^2*b))) + 4*x)/a
```

Sympy [A] time = 3.71654, size = 105, normalized size = 0.24

$$\operatorname{RootSum}\left(256t^4a^5b^2 + t^2\left(-32a^4b + 96a^3b^2\right) + a^3 + 3a^2b + 3ab^2 + b^3, \left(t \mapsto t\log\left(x + \frac{-64t^3a^4b + 4ta^3 - 24ta^2b + 4tab^2}{3a^2 + 2ab - b^2}\right)\right)\right) + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**5*b**2 + _t**2*(-32*a**4*b + 96*a**3*b**2) +
 a**3 + 3*a**2*b + 3*a*b**2 + b**3, Lambda(_t, _t*log(x + (-64*_t
**3*a**4*b + 4*_t*a**3 - 24*_t*a**2*b + 4*_t*a*b**2)/(3*a**2 + 2*
a*b - b**2)))) + x/a
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.912 $\int \frac{x^2}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=331

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[S
qrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a +
 b]]) + ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)
/Sqrt[Sqrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + S
qrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] +
Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] +
Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] +
 Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] +
 Sqrt[a + b]])
```

Rubi [A] time = 0.619689, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}a^{3/4}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[S
qrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[a +
 b]]) + ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2]*a^(1/4)*x)
/Sqrt[Sqrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(3/4)*Sqrt[Sqrt[a] + S
qrt[a + b]]) + Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] +
```

```
Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] +
Sqrt[a + b]]) - Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] +
 Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*a^(3/4)*Sqrt[-Sqrt[a] +
 Sqrt[a + b]])
```

Rubi in Sympy [A] time = 65.3461, size = 294, normalized size = 0.89

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x-\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x+\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4a^{\frac{3}{4}}\sqrt{\sqrt{a}+\sqrt{a+b}}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}\sqrt{-\sqrt{a}+\sqrt{a+b}}}-\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8a^{\frac{3}{4}}\sqrt{-\sqrt{a}+\sqrt{a+b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(a**(1/4)*x - sqrt(-2*sqrt(a) + 2*sqrt(a + b
))/2)/sqrt(sqrt(a) + sqrt(a + b)))/(4*a**(3/4)*sqrt(sqrt(a) + sqr
t(a + b))) + sqrt(2)*atan(sqrt(2)*(a**(1/4)*x + sqrt(-2*sqrt(a) +
 2*sqrt(a + b))/2)/sqrt(sqrt(a) + sqrt(a + b)))/(4*a**(3/4)*sqrt(
sqrt(a) + sqrt(a + b))) + sqrt(2)*log(x**2 + sqrt(a + b)/sqrt(a)
- sqrt(2)*x*sqrt(-sqrt(a) + sqrt(a + b))/a**(1/4))/(8*a**(3/4)*sq
rt(-sqrt(a) + sqrt(a + b))) - sqrt(2)*log(x**2 + sqrt(a + b)/sqrt
(a) + sqrt(2)*x*sqrt(-sqrt(a) + sqrt(a + b))/a**(1/4))/(8*a**(3/4
)*sqrt(-sqrt(a) + sqrt(a + b)))
```

Mathematica [C] time = 0.198852, size = 143, normalized size = 0.43

$$\frac{\frac{\left(\sqrt{b}+i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a-i\sqrt{a}\sqrt{b}}}+\frac{\left(\sqrt{b}-i\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a+i\sqrt{a}\sqrt{b}}}}{2\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b + 2*a*x^2 + a*x^4),x]
```

```
[Out] (((I*Sqrt[a] + Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqr
t[b]]])/Sqrt[a - I*Sqrt[a]*Sqrt[b]] + (((-I)*Sqrt[a] + Sqrt[b])*A
rcTan[(Sqrt[a]*x)/Sqrt[a + I*Sqrt[a]*Sqrt[b]]])/Sqrt[a + I*Sqrt[a
]*Sqrt[b]])/(2*Sqrt[a]*Sqrt[b])
```

Maple [B] time = 0.049, size = 724, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] 1/8/a^(3/2)/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+
b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/a^(3/
2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((
-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*
(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/8/a^(1/2)/b*ln(-a^(1
/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2))*(2*(a^2+a*b)
^(1/2)-2*a)^(1/2)-1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b)
)^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(
1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*
(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/a^(3/2)
/b*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2
*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/a^(3/2)/b/(4*a^(1
/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+
(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))
^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/
2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8/a^(1/2)/b*ln(a^(1/2)*x^2+(2*(a*
(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1
/2)+1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(
1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2
)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*
a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x^4 + 2*a*x^2 + a + b), x)
```

Fricas [A] time = 0.280333, size = 377, normalized size = 1.14

$$\frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\log\left(-a^2b\sqrt{\frac{ab\sqrt{-\frac{1}{a^3b}}+1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right) - \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right) + \frac{1}{4}\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\log\left(-a^2b\sqrt{-\frac{ab\sqrt{-\frac{1}{a^3b}}-1}{ab}}\sqrt{-\frac{1}{a^3b}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((a*b*sqrt(-1/(a^3*b)) + 1)/(a*b))*log(a^2*b*sqrt((a*b*sq
rt(-1/(a^3*b)) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt((a*b*
sqrt(-1/(a^3*b)) + 1)/(a*b))*log(-a^2*b*sqrt((a*b*sqrt(-1/(a^3*b)
) + 1)/(a*b))*sqrt(-1/(a^3*b)) + x) - 1/4*sqrt(-(a*b*sqrt(-1/(a^3
*b)) - 1)/(a*b))*log(a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b)
)*sqrt(-1/(a^3*b)) + x) + 1/4*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a
*b))*log(-a^2*b*sqrt(-(a*b*sqrt(-1/(a^3*b)) - 1)/(a*b))*sqrt(-1/(
a^3*b)) + x)
```

Sympy [A] time = 1.07097, size = 44, normalized size = 0.13

$$\operatorname{RootSum}\left(256t^4a^3b^2 - 32t^2a^2b + a + b, \left(t \mapsto t\log\left(64t^3a^2b - 4ta + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**2 - 32*_t**2*a**2*b + a + b, Lambda(_t,
 _t*log(64*_t**3*a**2*b - 4*_t*a + x)))
```

GIAC/XCAS [A] time = 2.24044, size = 1, normalized size = 0.

$$\textit{Done}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="giac")
```

```
[Out] Done
```

3.913 $\int \frac{1}{a+b+2ax^2+ax^4}\,dx$

Optimal. Leaf size=359

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[S
qrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a
] + Sqrt[a + b]]) + ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2
]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] - Sqrt[2]*
a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*
a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a +
b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2
]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rubi [A] time = 0.690382, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}+\frac{\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt{a+b}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[S
qrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[Sqrt[a
] + Sqrt[a + b]]) + ArcTan[(Sqrt[-Sqrt[a] + Sqrt[a + b]] + Sqrt[2
]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]]/(2*Sqrt[2]*a^(1/4)*Sqrt
[a + b]*Sqrt[Sqrt[a] + Sqrt[a + b]]) - Log[Sqrt[a + b] - Sqrt[2]*
a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2]/(4*Sqrt[2]*
```

```
a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]]) + Log[Sqrt[a +
b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2
]/(4*Sqrt[2]*a^(1/4)*Sqrt[a + b]*Sqrt[-Sqrt[a] + Sqrt[a + b]])
```

Rubi in Sympy [A] time = 88.3596, size = 321, normalized size = 0.89

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x-\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x+\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}-\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}+\frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\sqrt{-\sqrt{a}+\sqrt{a+b}}\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*(a**(1/4)*x - sqrt(-2*sqrt(a) + 2*sqrt(a + b
))/2)/sqrt(sqrt(a) + sqrt(a + b)))/(4*a**(1/4)*sqrt(sqrt(a) + sqr
t(a + b))*sqrt(a + b)) + sqrt(2)*atan(sqrt(2)*(a**(1/4)*x + sqrt(
-2*sqrt(a) + 2*sqrt(a + b))/2)/sqrt(sqrt(a) + sqrt(a + b)))/(4*a*
*(1/4)*sqrt(sqrt(a) + sqrt(a + b))*sqrt(a + b)) - sqrt(2)*log(x**
2 + sqrt(a + b)/sqrt(a) - sqrt(2)*x*sqrt(-sqrt(a) + sqrt(a + b))/
a**(1/4))/(8*a**(1/4)*sqrt(-sqrt(a) + sqrt(a + b))*sqrt(a + b)) +
 sqrt(2)*log(x**2 + sqrt(a + b)/sqrt(a) + sqrt(2)*x*sqrt(-sqrt(a)
 + sqrt(a + b))/a**(1/4))/(8*a**(1/4)*sqrt(-sqrt(a) + sqrt(a + b)
)*sqrt(a + b))
```

Mathematica [C] time = 0.119604, size = 119, normalized size = 0.33

$$\frac{i\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}}-\frac{i\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b + 2*a*x^2 + a*x^4)^(-1),x]
```

```
[Out] ((-I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a - I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a
- I*Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((I/2)*ArcTan[(Sqrt[a]*x)/Sqrt[a
+ I*Sqrt[a]*Sqrt[b]]])/(Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b])
```

Maple [B] time = 0.062, size = 913, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/8/(a+b)^(1/2)/a/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2
)*x-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/
8/(a+b)^(1/2)/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(
a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/(a+b)^(1/2)/(4*a^(1/2
)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(
2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^
(1/2)+2*a)^(1/2))+1/4/(a+b)^(1/2)/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a
*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-
2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*
(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+
a*b)^(1/2)+1/4/(a+b)^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(
1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2
))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+
b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/(a+b)^(1/2
)/a/b*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))
*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/8/(a+b)^(1/2)/b*
ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a
^2+a*b)^(1/2)-2*a)^(1/2)+1/(a+b)^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(
a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-
2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))-
1/4/(a+b)^(1/2)/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)
^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1
/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-
2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/(a+b
)^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arc
tan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^
(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)
*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{ax^4 + 2\,ax^2 + a + b}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="maxima")

[Out] integrate(1/(a*x^4 + 2*a*x^2 + a + b), x)
```

Fricas [A] time = 0.282999, size = 765, normalized size = 2.13

$$\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)$$
$$-\frac{1}{4}\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}\log\left(-\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+b\right)\sqrt{\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}+1}{ab+b^2}}+x\right)$$
$$-\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}+x\right)$$
$$+\frac{1}{4}\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}\log\left(-\left(\left(a^2b+ab^2\right)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-b\right)\sqrt{-\frac{(ab+b^2)\sqrt{-\frac{1}{a^3b+2a^2b^2+ab^3}}-1}{ab+b^2}}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="fricas")

[Out] 1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) + 1)/(
a*b + b^2))*log(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b
```

```
^3)) + b)*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3))
+ 1)/(a*b + b^2)) + x) - 1/4*sqrt(((a*b + b^2)*sqrt(-1/(a^3*b + 2
*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2))*log(-((a^2*b + a*b^2)*sqrt(-
1/(a^3*b + 2*a^2*b^2 + a*b^3)) + b)*sqrt(((a*b + b^2)*sqrt(-1/(a^
3*b + 2*a^2*b^2 + a*b^3)) + 1)/(a*b + b^2)) + x) - 1/4*sqrt(-((a*
b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b + b^2))*l
og(((a^2*b + a*b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - b)*sqr
t(-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 + a*b^3)) - 1)/(a*b +
b^2)) + x) + 1/4*sqrt(-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^2*b^2 +
a*b^3)) - 1)/(a*b + b^2))*log(-((a^2*b + a*b^2)*sqrt(-1/(a^3*b +
2*a^2*b^2 + a*b^3)) - b)*sqrt(-((a*b + b^2)*sqrt(-1/(a^3*b + 2*a^
2*b^2 + a*b^3)) - 1)/(a*b + b^2)) + x)
```

Sympy [A] time = 1.94328, size = 63, normalized size = 0.18

$$\operatorname{RootSum}\left(t^{4}\left(256a^{2}b^{2}+256ab^{3}\right)-32t^{2}ab+1,\left(t\mapsto t\log\left(64t^{3}a^{2}b+64t^{3}ab^{2}-4ta+4tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**2*b**2 + 256*a*b**3) - 32*_t**2*a*b + 1, La
mbda(_t, _t*log(64*_t**3*a**2*b + 64*_t**3*a*b**2 - 4*_t*a + 4*_t
*b + x)))
```

GIAC/XCAS [A] time = 0.677847, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*x^4 + 2*a*x^2 + a + b),x, algorithm="giac")
```

```
[Out] Done
```

3.914 $\int \frac{1}{x^2(a+b+2ax^2+ax^4)}\,dx$

Optimal. Leaf size=433

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{1}{x(a+b)}+\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqr
t[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqr
t[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
 - (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqr
t[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*S
qrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + (a^(1/4)*(2*S
qrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqr
t[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sq
rt[-Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*
Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x
+ Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a +
 b]])
```

Rubi [A] time = 1.27366, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(-\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(\sqrt{2}\sqrt[4]{a}x\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{a+b}+\sqrt{a}x^2\right)}{4\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}-\sqrt{a}}}-\frac{1}{x(a+b)}+\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+2\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a+b}-\sqrt{a}}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{\sqrt{a+b}+\sqrt{a}}}\right)}{2\sqrt{2}(a+b)^{3/2}\sqrt{\sqrt{a+b}+\sqrt{a}}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] -(1/((a + b)*x)) + (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqr
t[-Sqrt[a] + Sqrt[a + b]] - Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqr
t[a + b]]])/(2*Sqrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]])
 - (a^(1/4)*(2*Sqrt[a] + Sqrt[a + b])*ArcTan[(Sqrt[-Sqrt[a] + Sqr
t[a + b]] + Sqrt[2]*a^(1/4)*x)/Sqrt[Sqrt[a] + Sqrt[a + b]]])/(2*S
qrt[2]*(a + b)^(3/2)*Sqrt[Sqrt[a] + Sqrt[a + b]]) + (a^(1/4)*(2*S
qrt[a] - Sqrt[a + b])*Log[Sqrt[a + b] - Sqrt[2]*a^(1/4)*Sqrt[-Sqr
t[a] + Sqrt[a + b]]*x + Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sq
rt[-Sqrt[a] + Sqrt[a + b]]) - (a^(1/4)*(2*Sqrt[a] - Sqrt[a + b])*
Log[Sqrt[a + b] + Sqrt[2]*a^(1/4)*Sqrt[-Sqrt[a] + Sqrt[a + b]]*x
+ Sqrt[a]*x^2])/(4*Sqrt[2]*(a + b)^(3/2)*Sqrt[-Sqrt[a] + Sqrt[a +
 b]])
```

Rubi in Sympy [A] time = 128.938, size = 382, normalized size = 0.88

$$-\frac{\sqrt{2}\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x-\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4\sqrt{\sqrt{a}+\sqrt{a+b}}\,(a+b)^{\frac{3}{2}}}-\frac{\sqrt{2}\sqrt[4]{a}\left(2\sqrt{a}+\sqrt{a+b}\right)\operatorname{atan}\left(\frac{\sqrt{2}\left(\sqrt[4]{a}x+\frac{\sqrt{-2\sqrt{a}+2\sqrt{a+b}}}{2}\right)}{\sqrt{\sqrt{a}+\sqrt{a+b}}}\right)}{4\sqrt{\sqrt{a}+\sqrt{a+b}}\,(a+b)^{\frac{3}{2}}}+\frac{\sqrt{2}\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(x^{2}+\frac{\sqrt{a+b}}{\sqrt{a}}-\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8\sqrt{-\sqrt{a}+\sqrt{a+b}}\,(a+b)^{\frac{3}{2}}}-\frac{\sqrt{2}\sqrt[4]{a}\left(2\sqrt{a}-\sqrt{a+b}\right)\log\left(x^{2}+\frac{\sqrt{a+b}}{\sqrt{a}}+\frac{\sqrt{2}x\sqrt{-\sqrt{a}+\sqrt{a+b}}}{\sqrt[4]{a}}\right)}{8\sqrt{-\sqrt{a}+\sqrt{a+b}}\,(a+b)^{\frac{3}{2}}}-\frac{1}{x(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] -sqrt(2)*a**(1/4)*(2*sqrt(a) + sqrt(a + b))*atan(sqrt(2)*(a**(1/4
)*x - sqrt(-2*sqrt(a) + 2*sqrt(a + b))/2)/sqrt(sqrt(a) + sqrt(a +
```

```
 b)))/(4*sqrt(sqrt(a) + sqrt(a + b))*(a + b)**(3/2)) - sqrt(2)*a*
*(1/4)*(2*sqrt(a) + sqrt(a + b))*atan(sqrt(2)*(a**(1/4)*x + sqrt(
-2*sqrt(a) + 2*sqrt(a + b))/2)/sqrt(sqrt(a) + sqrt(a + b)))/(4*sq
rt(sqrt(a) + sqrt(a + b))*(a + b)**(3/2)) + sqrt(2)*a**(1/4)*(2*s
qrt(a) - sqrt(a + b))*log(x**2 + sqrt(a + b)/sqrt(a) - sqrt(2)*x*
sqrt(-sqrt(a) + sqrt(a + b))/a**(1/4))/(8*sqrt(-sqrt(a) + sqrt(a
+ b))*(a + b)**(3/2)) - sqrt(2)*a**(1/4)*(2*sqrt(a) - sqrt(a + b)
)*log(x**2 + sqrt(a + b)/sqrt(a) + sqrt(2)*x*sqrt(-sqrt(a) + sqrt
(a + b))/a**(1/4))/(8*sqrt(-sqrt(a) + sqrt(a + b))*(a + b)**(3/2)
) - 1/(x*(a + b))
```

Mathematica [C] time = 0.247111, size = 174, normalized size = 0.4

$$\frac{1}{x(-a-b)}+\frac{\left(-\sqrt{a}\sqrt{b}+ia\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a-i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a-i\sqrt{a}\sqrt{b}}(a+b)}+\frac{\left(-\sqrt{a}\sqrt{b}-ia\right)\tan^{-1}\left(\frac{\sqrt{a}x}{\sqrt{a+i\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{b}\sqrt{a+i\sqrt{a}\sqrt{b}}(a+b)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b + 2*a*x^2 + a*x^4)),x]
```

```
[Out] 1/((-a - b)*x) + ((I*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt
[a - I*Sqrt[a]*Sqrt[b]]])/(2*Sqrt[a - I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*
(a + b)) + (((-I)*a - Sqrt[a]*Sqrt[b])*ArcTan[(Sqrt[a]*x)/Sqrt[a
+ I*Sqrt[a]*Sqrt[b]]])/(2*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*Sqrt[b]*(a
+ b))
```

Maple [B] time = 0.058, size = 3318, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(a*x^4+2*a*x^2+a+b),x)
```

```
[Out] -1/4/(a+b)^(5/2)*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(
a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/(a+
b)^(5/2)*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1
/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8*a^(1/2)/(a
+b)^2*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2)
)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8*a^(1/2)/(a+b)^2*ln(a^(1/2)*x^
2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)
-2*a)^(1/2)+1/4*a/(a+b)^(5/2)*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-
```

2*a)^(1/2)*x-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+2*a^2/(a+
b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arct
an((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^
(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))-1/4*a/(a+b)^(5/2)*ln(a^(1/2)*
x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/
2)-2*a)^(1/2)-2*a^2/(a+b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b)
)^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1
/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))+1/4*a^(
1/2)/(a+b)^2/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2
)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*
(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)
^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8/a^(1/2)/
(a+b)^2*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/
2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8*a^(3/2)/(a+
b)^2/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2
))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2*a/(a+b)^(5/2)/b/(4*a^(1/2)*(
a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(
a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/
2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2
a)^(1/2)(a^2+a*b)^(1/2)-1/(a+b)/x+1/2*a/(a+b)^(5/2)/(4*a^(1/2)*
(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(
a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/
2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2
*a)^(1/2)+1/2/(a+b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2
)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(
4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^
(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/
2/(a+b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)
*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(
a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^
(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4*a/(a+b)^(
5/2)/b*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2)
)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4*a^(1/2)/(a+b)
^2/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2
*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-
2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a
^2+a*b)^(1/2)-2*a)^(1/2)+1/8*a^(1/2)/(a+b)^2/b*ln(a^(1/2)*x^2+(2*
(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)
^(1/2)*(a^2+a*b)^(1/2)-1/4*a^(1/2)/(a+b)^2/(4*a^(1/2)*(a+b)^(1/2)
-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1
/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/
2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1
/8*a^(1/2)/(a+b)^2/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2
)*x-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/
4*a/(a+b)^(5/2)/b*ln(-a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x
-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/2*a
/(a+b)^(5/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*
arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a
+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(
1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4*a^(1/2)/(a+b)^2/b/(4*a^(1/
2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(
2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^
(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2
)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/2*a/(a+b)^(5/2)/b/(4*a^(1/2)*(a+b)

```
^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+
b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*
a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(
1/2)*(a^2+a*b)^(1/2)+1/4*a^2/(a+b)^(5/2)/b*ln(-a^(1/2)*x^2+(2*(a*
(a+b))^(1/2)-2*a)^(1/2)*x-(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1
/2)+2*a/(a+b)^(5/2)*b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*
a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a
^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))-1/4*a^2/(a+b)^(5
/2)/b*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a)^(1/2)*x+(a+b)^(1/2))
*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-2*a/(a+b)^(5/2)*b/(4*a^(1/2)*(a+b)
^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+
b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*
a)^(1/2))+1/8/a^(1/2)/(a+b)^2*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2
*a)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^
(1/2)+1/8*a^(3/2)/(a+b)^2/b*ln(a^(1/2)*x^2+(2*(a*(a+b))^(1/2)-2*a
)^(1/2)*x+(a+b)^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/2*a^2/(a+b
)^(5/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arc
tan((-2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)
^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2
)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4*a^(3/2)/(a+b)^2/b/(4*a^(1/2)*
(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*
(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1
/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-
2*a)^(1/2)+1/2*a^2/(a+b)^(5/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b
))^(1/2)+2*a)^(1/2)*arctan((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(
1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*
(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/4/a^(1/2)
/(a+b)^2/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arct
an((2*a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(
1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*
(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4*a^(3/2)/(a+b)^2
/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*
a^(1/2)*x+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^
2+a*b)^(1/2)-2*a)^(1/2)+1/4/a^(1/2)/(a+b)^2/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*a^(1/2)*x+(2*(a*(a+b))^
(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(
1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
*(a^2+a*b)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{a\int\frac{x^2+2}{ax^4+2\,ax^2+a+b}\,dx}{a+b}-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x^2),x, algorithm="maxima")
```

```
[Out] -a*integrate((x^2 + 2)/(a*x^4 + 2*a*x^2 + a + b), x)/(a + b) - 1/
((a + b)*x)
```

Fricas [A] time = 0.290585, size = 2136, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x^2),x, algorithm="fricas")
```

```
[Out] 1/4*((a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3 +
 b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4
*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4))*log(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 +
 (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b
^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6
*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^
4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4))) - (a + b)*x*sqrt((a^2 - 3*a*b + (a^3*b +
3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b
 + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b
^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*log(-(3*a^2 - a*b)*x -
 (6*a^2*b - 2*a*b^2 + (a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqrt(-(
9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3
*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a^2 - 3*a*b + (a^3*b +
 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*
b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 +
b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))) + (a + b)*x*sqrt((a^
2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6*
a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*
a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))*l
og(-(3*a^2 - a*b)*x + (6*a^2*b - 2*a*b^2 - (a^4*b + 2*a^3*b^2 - 2
*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2
+ 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))*sqrt((a
^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(-(9*a^3 - 6
*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15
*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)))
 - (a + b)*x*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b
^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*b
^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b^
2 + 3*a*b^3 + b^4))*log(-(3*a^2 - a*b)*x - (6*a^2*b - 2*a*b^2 - (
a^4*b + 2*a^3*b^2 - 2*a*b^4 - b^5)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2
)/(a^6*b + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a
*b^6 + b^7)))*sqrt((a^2 - 3*a*b - (a^3*b + 3*a^2*b^2 + 3*a*b^3 +
b^4)*sqrt(-(9*a^3 - 6*a^2*b + a*b^2)/(a^6*b + 6*a^5*b^2 + 15*a^4*
b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)))/(a^3*b + 3*a^2*b
^2 + 3*a*b^3 + b^4))) - 4)/((a + b)*x)
```

Sympy [A] time = 7.30381, size = 134, normalized size = 0.31

$$\operatorname{RootSum}\left(t^4\left(256a^3b^2+768a^2b^3+768ab^4+256b^5\right)+t^2\left(-32a^2b+96ab^2\right)+a,\left(t\mapsto t\log\left(x+\frac{-64t^3a^4b-128t^3a^3b^2+12\right.\right.\right.$$

$$-\frac{1}{x(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(a*x**4+2*a*x**2+a+b),x)
```

```
[Out] RootSum(_t**4*(256*a**3*b**2 + 768*a**2*b**3 + 768*a*b**4 + 256*b
**5) + _t**2*(-32*a**2*b + 96*a*b**2) + a, Lambda(_t, _t*log(x +
(-64*_t**3*a**4*b - 128*_t**3*a**3*b**2 + 128*_t**3*a*b**4 + 64*_
t**3*b**5 + 4*_t*a**3 - 40*_t*a**2*b + 20*_t*a*b**2)/(3*a**2 - a*
b)))) - 1/(x*(a + b))
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((a*x^4 + 2*a*x^2 + a + b)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.915 $\int \frac{x}{1+x^2+x^4}\,dx$

Optimal. Leaf size=20

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0485539, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(1 + x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Rubi in Sympy [A] time = 4.46774, size = 22, normalized size = 1.1

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\sqrt{3}\left(\frac{2x^2}{3}+\frac{1}{3}\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+x**2+1),x)
```

```
[Out] sqrt(3)*atan(sqrt(3)*(2*x**2/3 + 1/3))/3
```

Mathematica [A] time = 0.00907408, size = 20, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(1 + x^2 + x^4),x]

[Out] ArcTan[(1 + 2*x^2)/Sqrt[3]]/Sqrt[3]
```

Maple [A] time = 0.002, size = 19, normalized size = 1.

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4+x^2+1),x)

[Out] 1/3*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 0.76745, size = 24, normalized size = 1.2

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + x^2 + 1),x, algorithm="maxima")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Fricas [A] time = 0.274031, size = 24, normalized size = 1.2

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^2+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + x^2 + 1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

Sympy [A] time = 0.19504, size = 26, normalized size = 1.3

$$\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+x**2+1),x)

[Out] sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/3
```

GIAC/XCAS [A] time = 0.285103, size = 24, normalized size = 1.2

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + x^2 + 1),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1))
```

3.916 $\int \frac{x}{10+2x^2+x^4} \, dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rubi [A] time = 0.0351037, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x/(10 + 2*x^2 + x^4),x]

[Out] ArcTan[(1 + x^2)/3]/6
```

Rubi in Sympy [A] time = 4.7103, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(x**4+2*x**2+10),x)

[Out] atan(x**2/3 + 1/3)/6
```

Mathematica [A] time = 0.00851059, size = 14, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(10 + 2*x^2 + x^4),x]

[Out] ArcTan[(1 + x^2)/3]/6
```

Maple [A] time = 0.004, size = 11, normalized size = 0.8

$$\frac{1}{6}\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(x^4+2*x^2+10),x)

[Out] 1/6*arctan(1/3*x^2+1/3)
```

Maxima [A] time = 0.761683, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 10),x, algorithm="maxima")

[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Fricas [A] time = 0.269194, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 10),x, algorithm="fricas")

[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Sympy [A] time = 0.187189, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3}+\frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

GIAC/XCAS [A] time = 0.268994, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(x^4 + 2*x^2 + 10),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

3.917 $\int \frac{x^2}{20+9x^2+x^4}\,dx$

Optimal. Leaf size=23

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rubi [A] time = 0.033035, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(20 + 9*x^2 + x^4),x]

[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Rubi in Sympy [A] time = 5.99943, size = 20, normalized size = 0.87

$$-2\,\mathrm{atan}\left(\frac{x}{2}\right)+\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4+9*x**2+20),x)

[Out] -2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)
```

Mathematica [A] time = 0.0213982, size = 23, normalized size = 1.

$$\sqrt{5}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)-2\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(20 + 9*x^2 + x^4),x]
```

```
[Out] -2*ArcTan[x/2] + Sqrt[5]*ArcTan[x/Sqrt[5]]
```

Maple [A] time = 0.012, size = 19, normalized size = 0.8

$$-2\ \arctan(x/2) + \arctan\left(\frac{x\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4+9*x^2+20),x)
```

```
[Out] -2*arctan(1/2*x)+arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 0.754431, size = 24, normalized size = 1.04

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\ \arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 9*x^2 + 20),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Fricas [A] time = 0.276274, size = 24, normalized size = 1.04

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right) - 2\ \arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 9*x^2 + 20),x, algorithm="fricas")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

Sympy [A] time = 0.361819, size = 20, normalized size = 0.87

$$-2\operatorname{atan}\left(\frac{x}{2}\right)+\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4+9*x**2+20),x)
```

```
[Out] -2*atan(x/2) + sqrt(5)*atan(sqrt(5)*x/5)
```

GIAC/XCAS [A] time = 0.26771, size = 24, normalized size = 1.04

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-2\arctan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 + 9*x^2 + 20),x, algorithm="giac")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*x) - 2*arctan(1/2*x)
```

3.918 $\int \frac{x^2}{1-x^2+x^4}\, dx$

Optimal. Leaf size=74

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt
[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi [A] time = 0.106697, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{\log\left(x^2-\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{\log\left(x^2+\sqrt{3}x+1\right)}{4\sqrt{3}}-\frac{1}{2}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{1}{2}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(1 - x^2 + x^4),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/2 + ArcTan[Sqrt[3] + 2*x]/2 + Log[1 - Sqrt
[3]*x + x^2]/(4*Sqrt[3]) - Log[1 + Sqrt[3]*x + x^2]/(4*Sqrt[3])
```

Rubi in Sympy [A] time = 22.0156, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4-x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)
*x + 1)/12 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

Mathematica [C] time = 0.243573, size = 94, normalized size = 1.27

$$\frac{\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)x\right)+\sqrt{-1+i\sqrt{3}}\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)x\right)}{2\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2/(1 - x^2 + x^4),x]
```

```
[Out] (Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x)/2]
 + Sqrt[-1 + I*Sqrt[3]]*(-I + Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x)
/2])/(2*Sqrt[6])
```

Maple [A] time = 0.029, size = 57, normalized size = 0.8

$$\frac{\arctan\left(2\,x-\sqrt{3}\right)}{2}+\frac{\arctan\left(2\,x+\sqrt{3}\right)}{2}+\frac{\ln\left(1+x^{2}-x\sqrt{3}\right)\sqrt{3}}{12}-\frac{\ln\left(1+x^{2}+x\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4-x^2+1),x)
```

```
[Out] 1/2*arctan(2*x-3^(1/2))+1/2*arctan(2*x+3^(1/2))+1/12*ln(1+x^2-x*3
^(1/2))*3^(1/2)-1/12*ln(1+x^2+x*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{2}}{x^{4}-x^{2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - x^2 + 1),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - x^2 + 1), x)
```

Fricas [A] time = 0.293435, size = 143, normalized size = 1.93

$$-\frac{1}{12}\sqrt{3}\left(4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^{2}+\sqrt{3}x+1}+3}\right)+4\sqrt{3}\arctan\left(\frac{\sqrt{3}}{2\sqrt{3}x+2\sqrt{3}\sqrt{x^{2}-\sqrt{3}x+1}-3}\right)+\log\left(x^{2}+\sqrt{3}x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - x^2 + 1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*(4*sqrt(3)*arctan(sqrt(3)/(2*sqrt(3)*x + 2*sqrt(3)*
sqrt(x^2 + sqrt(3)*x + 1) + 3)) + 4*sqrt(3)*arctan(sqrt(3)/(2*sqr
t(3)*x + 2*sqrt(3)*sqrt(x^2 - sqrt(3)*x + 1) - 3)) + log(x^2 + sq
rt(3)*x + 1) - log(x^2 - sqrt(3)*x + 1))
```

Sympy [A] time = 0.492911, size = 63, normalized size = 0.85

$$\frac{\sqrt{3}\log\left(x^2-\sqrt{3}x+1\right)}{12}-\frac{\sqrt{3}\log\left(x^2+\sqrt{3}x+1\right)}{12}+\frac{\operatorname{atan}\left(2x-\sqrt{3}\right)}{2}+\frac{\operatorname{atan}\left(2x+\sqrt{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4-x**2+1),x)
```

```
[Out] sqrt(3)*log(x**2 - sqrt(3)*x + 1)/12 - sqrt(3)*log(x**2 + sqrt(3)
*x + 1)/12 + atan(2*x - sqrt(3))/2 + atan(2*x + sqrt(3))/2
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^4-x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - x^2 + 1),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - x^2 + 1), x)
```

3.919 $\int \frac{x^2}{2-2x^2+x^4}\,dx$

Optimal. Leaf size=188

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt
[2*(-1 + Sqrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1
 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sq
rt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt
[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rubi [A] time = 0.387035, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$

$$\frac{\log\left(x^2-\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{\log\left(x^2+\sqrt{2\left(1+\sqrt{2}\right)}x+\sqrt{2}\right)}{4\sqrt{2\left(1+\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(1+\sqrt{2}\right)}-2x}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{2}\right)}\tan^{-1}\left(\frac{2x+\sqrt{2\left(1+\sqrt{2}\right)}}{\sqrt{2\left(\sqrt{2}-1\right)}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*x)/Sqrt
[2*(-1 + Sqrt[2])]])/2 + (Sqrt[(1 + Sqrt[2])/2]*ArcTan[(Sqrt[2*(1
 + Sqrt[2])] + 2*x)/Sqrt[2*(-1 + Sqrt[2])]])/2 + Log[Sqrt[2] - Sq
rt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])]) - Log[Sqrt
[2] + Sqrt[2*(1 + Sqrt[2])]*x + x^2]/(4*Sqrt[2*(1 + Sqrt[2])])
```

Rubi in Sympy [A] time = 30.0439, size = 185, normalized size = 0.98

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x\sqrt{1+\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{1+\sqrt{2}}}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x\sqrt{1+\sqrt{2}}+\sqrt{2}\right)}{8\sqrt{1+\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(x-\frac{\sqrt{2+2\sqrt{2}}}{2}\right)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(x+\frac{\sqrt{2+2\sqrt{2}}}{2}\right)}{\sqrt{-1+\sqrt{2}}}\right)}{4\sqrt{-1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(x**4-2*x**2+2),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x*sqrt(1 + sqrt(2)) + sqrt(2))/(8*sqrt
(1 + sqrt(2))) - sqrt(2)*log(x**2 + sqrt(2)*x*sqrt(1 + sqrt(2)) +
 sqrt(2))/(8*sqrt(1 + sqrt(2))) + sqrt(2)*atan(sqrt(2)*(x - sqrt(
2 + 2*sqrt(2))/2)/sqrt(-1 + sqrt(2)))/(4*sqrt(-1 + sqrt(2))) + sq
rt(2)*atan(sqrt(2)*(x + sqrt(2 + 2*sqrt(2))/2)/sqrt(-1 + sqrt(2))
)/(4*sqrt(-1 + sqrt(2)))
```

Mathematica [C] time = 0.0511342, size = 39, normalized size = 0.21

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1-i}}\right)}{(-1-i)^{3/2}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{-1+i}}\right)}{(-1+i)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(2 - 2*x^2 + x^4),x]
```

```
[Out] -(ArcTan[x/Sqrt[-1 - I]]/(-1 - I)^(3/2)) - ArcTan[x/Sqrt[-1 + I]]
/(-1 + I)^(3/2)
```

Maple [B] time = 0.055, size = 308, normalized size = 1.6

$$\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)-\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}-x\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{2+2\sqrt{2}}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x-\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)-\frac{\sqrt{2+2\sqrt{2}}\,\sqrt{2}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}+\frac{\sqrt{2}\left(2+2\sqrt{2}\right)}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)+\frac{\sqrt{2+2\sqrt{2}}\ln\left(x^2+\sqrt{2}+x\sqrt{2+2\sqrt{2}}\right)}{8}-\frac{2+2\sqrt{2}}{4\sqrt{-2+2\sqrt{2}}}\arctan\left(\frac{2x+\sqrt{2+2\sqrt{2}}}{\sqrt{-2+2\sqrt{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(x^4-2*x^2+2),x)
```

```
[Out] 1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1
/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*x-(
2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))-1/8*(2+2*2^(1/2))^(1/2)
*ln(x^2+2^(1/2)-x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2*2^
(1/2))^(1/2)*arctan((2*x-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2
))-1/8*(2+2*2^(1/2))^(1/2)*2^(1/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))
^(1/2))+1/4*2^(1/2)*(2+2*2^(1/2))/(-2+2*2^(1/2))^(1/2)*arctan((2*
x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+1/8*(2+2*2^(1/2))^(1
/2)*ln(x^2+2^(1/2)+x*(2+2*2^(1/2))^(1/2))-1/4*(2+2*2^(1/2))/(-2+2
*2^(1/2))^(1/2)*arctan((2*x+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{x^4-2\,x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - 2*x^2 + 2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

Fricas [A] time = 0.293952, size = 630, normalized size = 3.35

$$\sqrt{2}\left(2^{\frac{1}{4}}\left(\sqrt{2}-1\right)\log\left(24\sqrt{2}x^2+2^{\frac{3}{4}}\left(17\sqrt{2}x-24x\right)\sqrt{\frac{\sqrt{2}-2}{2\sqrt{2}-3}}-34x^2+2\sqrt{2}\left(12\sqrt{2}-17\right)\right)-2^{\frac{1}{4}}\left(\sqrt{2}-1\right)\log\left(24\sqrt{2}x^2-2^{\frac{3}{4}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - 2*x^2 + 2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(2^(1/4)*(sqrt(2) - 1)*log(24*sqrt(2)*x^2 + 2^(3/4)*(
17*sqrt(2)*x - 24*x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 34*x^2
 + 2*sqrt(2)*(12*sqrt(2) - 17)) - 2^(1/4)*(sqrt(2) - 1)*log(24*sq
rt(2)*x^2 - 2^(3/4)*(17*sqrt(2)*x - 24*x)*sqrt((sqrt(2) - 2)/(2*s
qrt(2) - 3)) - 34*x^2 + 2*sqrt(2)*(12*sqrt(2) - 17)) - 4*2^(1/4)*
arctan(2^(1/4)*(sqrt(2) - 1)/(sqrt(2)*sqrt(1/2)*(sqrt(2) - 1)*sqr
t((24*sqrt(2)*x^2 + 2^(3/4)*(17*sqrt(2)*x - 24*x)*sqrt((sqrt(2) -
 2)/(2*sqrt(2) - 3)) - 34*x^2 + 2*sqrt(2)*(12*sqrt(2) - 17))/(12*
sqrt(2) - 17))*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sqrt(2)*(sqr
t(2)*x - x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) - 2^(1/4))) - 4*2
^(1/4)*arctan(2^(1/4)*(sqrt(2) - 1)/(sqrt(2)*sqrt(1/2)*(sqrt(2) -
 1)*sqrt((24*sqrt(2)*x^2 - 2^(3/4)*(17*sqrt(2)*x - 24*x)*sqrt((sq
rt(2) - 2)/(2*sqrt(2) - 3)) - 34*x^2 + 2*sqrt(2)*(12*sqrt(2) - 17
))/(12*sqrt(2) - 17))*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + sqrt(
2)*(sqrt(2)*x - x)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)) + 2^(1/4))
))/((sqrt(2) - 1)*sqrt((sqrt(2) - 2)/(2*sqrt(2) - 3)))
```

Sympy [A] time = 1.6982, size = 24, normalized size = 0.13

$$\operatorname{RootSum}\left(128t^4+16t^2+1,\left(t\mapsto t\log\left(64t^3+4t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(x**4-2*x**2+2),x)
```

```
[Out] RootSum(128*_t**4 + 16*_t**2 + 1, Lambda(_t, _t*log(64*_t**3 + 4*
_t + x)))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{x^4-2x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(x^4 - 2*x^2 + 2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(x^4 - 2*x^2 + 2), x)
```

3.920 $\int x^7\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=171

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}$$

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*
c^4) + (x^4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c
 - 42*b*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 -
 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rubi [A] time = 0.387127, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{9/2}}-\frac{b\left(7b^2-12ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{\left(-32ac+35b^2-42bcx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{480c^3}+\frac{x^4\left(a+bx^2+cx^4\right)^{3/2}}{10c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(7*b^2 - 12*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*
c^4) + (x^4*(a + b*x^2 + c*x^4)^(3/2))/(10*c) + ((35*b^2 - 32*a*c
 - 42*b*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(480*c^3) + (b*(7*b^2 -
 12*a*c)*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4])])/(512*c^(9/2))
```

Rubi in Sympy [A] time = 27.2212, size = 163, normalized size = 0.95

$$-\frac{b\left(b+2cx^2\right)\left(-12ac+7b^2\right)\sqrt{a+bx^2+cx^4}}{256c^4}+\frac{b\left(-12ac+7b^2\right)\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{\frac{9}{2}}}+\frac{x^4\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{10c}+\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}\left(-8ac+\frac{35b^2}{4}-\frac{21bcx^2}{2}\right)}{120c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*(b + 2*c*x**2)*(-12*a*c + 7*b**2)*sqrt(a + b*x**2 + c*x**4)/(2
56*c**4) + b*(-12*a*c + 7*b**2)*(-4*a*c + b**2)*atanh((b + 2*c*x*
*2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(512*c**(9/2)) + x**4*
(a + b*x**2 + c*x**4)**(3/2)/(10*c) + (a + b*x**2 + c*x**4)**(3/2
)*(-8*a*c + 35*b**2/4 - 21*b*c*x**2/2)/(120*c**3)
```

Mathematica [A] time = 0.16346, size = 162, normalized size = 0.95

$$\frac{15b\left(7b^2-12ac\right)\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)-2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(-128c^2\left(-2a^2+acx^4+3c^2x^8\right)+b\right.}{7680c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(105*b^4 - 70*b^3*c*x^2 + 8*b
*c^2*x^2*(29*a - 6*c*x^4) + b^2*(-460*a*c + 56*c^2*x^4) - 128*c^2
*(-2*a^2 + a*c*x^4 + 3*c^2*x^8)) + 15*b*(7*b^2 - 12*a*c)*(b^2 - 4
*a*c)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(7680
*c^(9/2))
```

Maple [A] time = 0.046, size = 296, normalized size = 1.7

$$\frac{x^4}{10\,c}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{7\,bx^2}{80\,c^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{7\,b^2}{96\,c^3}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{7\,b^3x^2}{128\,c^3}\sqrt{cx^4+bx^2+a}$$
$$-\frac{7\,b^4}{256\,c^4}\sqrt{cx^4+bx^2+a}-\frac{5\,ab^3}{64}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}$$
$$+\frac{7\,b^5}{512}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{9}{2}}+\frac{3\,abx^2}{32\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,ab^2}{64\,c^3}\sqrt{cx^4+bx^2+a}$$
$$+\frac{3\,a^2b}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{a}{15\,c^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/10*x^4*(c*x^4+b*x^2+a)^(3/2)/c-7/80*b/c^2*x^2*(c*x^4+b*x^2+a)^(
3/2)+7/96*b^2/c^3*(c*x^4+b*x^2+a)^(3/2)-7/128*b^3/c^3*(c*x^4+b*x^
```

```
2+a)^(1/2)*x^2-7/256*b^4/c^4*(c*x^4+b*x^2+a)^(1/2)-5/64*b^3/c^(7/
2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a+7/512*b^5/c^
(9/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/32*b/c^2*
a*(c*x^4+b*x^2+a)^(1/2)*x^2+3/64*b^2/c^3*a*(c*x^4+b*x^2+a)^(1/2)+
3/32*b/c^(5/2)*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)
)-1/15/c^2*a*(c*x^4+b*x^2+a)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.307548, size = 1, normalized size = 0.01

$$\left[\frac{4\left(384\,c^4x^8 + 48\,bc^3x^6 - 8\left(7\,b^2c^2 - 16\,ac^3\right)x^4 - 105\,b^4 + 460\,ab^2c - 256\,a^2c^2 + 2\left(35\,b^3c - 116\,abc^2\right)x^2\right)\sqrt{cx^4 + bx^2 + a}\sqrt{c}}{15360\,c^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^7,x, algorithm="fricas")
```

```
[Out] [1/15360*(4*(384*c^4*x^8 + 48*b*c^3*x^6 - 8*(7*b^2*c^2 - 16*a*c^3
)*x^4 - 105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 2*(35*b^3*c - 116*a
*b*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + 15*(7*b^5 - 40*a*b
^3*c + 48*a^2*b*c^2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 +
b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^(9/2), 1
/7680*(2*(384*c^4*x^8 + 48*b*c^3*x^6 - 8*(7*b^2*c^2 - 16*a*c^3)*x
^4 - 105*b^4 + 460*a*b^2*c - 256*a^2*c^2 + 2*(35*b^3*c - 116*a*b*
c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) + 15*(7*b^5 - 40*a*b^3
*c + 48*a^2*b*c^2)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4
+ b*x^2 + a)*c)))/(sqrt(-c)*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^7\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.303072, size = 248, normalized size = 1.45

$$\frac{1}{3840}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\left(8x^2+\frac{b}{c}\right)x^2-\frac{7b^2c^5-16ac^6}{c^7}\right)x^2+\frac{35b^3c^4-116abc^5}{c^7}\right)x^2-\frac{105b^4c^3-460ab^2c^4+256a^2c^5}{c^7}\right)-\frac{\left(7b^5c^3-40ab^3c^4+48a^2bc^5\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{512c^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^7,x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*(8*x^2 + b/c)*x^2 - (7*b^
2*c^5 - 16*a*c^6)/c^7)*x^2 + (35*b^3*c^4 - 116*a*b*c^5)/c^7)*x^2
- (105*b^4*c^3 - 460*a*b^2*c^4 + 256*a^2*c^5)/c^7) - 1/512*(7*b^5
*c^3 - 40*a*b^3*c^4 + 48*a^2*b*c^5)*ln(abs(-2*(sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(15/2)
```

3.921 $\int x^5\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=153

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{(5b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\,(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2\,(a+bx^2+cx^4)^{3/2}}{8c}$$

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3)
 - (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c
*x^4)^(3/2))/(8*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(7/2))
```

Rubi [A] time = 0.305454, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{7/2}}+\frac{(5b^2-4ac)\,(b+2cx^2)\,\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{5b\,(a+bx^2+cx^4)^{3/2}}{48c^2}+\frac{x^2\,(a+bx^2+cx^4)^{3/2}}{8c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((5*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^3)
 - (5*b*(a + b*x^2 + c*x^4)^(3/2))/(48*c^2) + (x^2*(a + b*x^2 + c
*x^4)^(3/2))/(8*c) - ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(7/2))
```

Rubi in Sympy [A] time = 25.8435, size = 141, normalized size = 0.92

$$-\frac{5b\,(a+bx^2+cx^4)^{\frac{3}{2}}}{48c^2}+\frac{x^2\,(a+bx^2+cx^4)^{\frac{3}{2}}}{8c}+\frac{(b+2cx^2)\,(-4ac+5b^2)\,\sqrt{a+bx^2+cx^4}}{128c^3}-\frac{(-4ac+b^2)\,(-4ac+5b^2)\,\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -5*b*(a + b*x**2 + c*x**4)**(3/2)/(48*c**2) + x**2*(a + b*x**2 +
c*x**4)**(3/2)/(8*c) + (b + 2*c*x**2)*(-4*a*c + 5*b**2)*sqrt(a +
b*x**2 + c*x**4)/(128*c**3) - (-4*a*c + b**2)*(-4*a*c + 5*b**2)*a
tanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(256*c
**(7/2))
```

Mathematica [A] time = 0.142237, size = 134, normalized size = 0.88

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(b\left(8c^2x^4-52ac\right)+24c^2x^2\left(a+2cx^4\right)+15b^3-10b^2cx^2\right)-3\left(16a^2c^2-24ab^2c+5b^4\right)\log\left(2\sqrt{c}\sqrt{a+bx\right.}{768c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^3 - 10*b^2*c*x^2 + 24*c^
2*x^2*(a + 2*c*x^4) + b*(-52*a*c + 8*c^2*x^4)) - 3*(5*b^4 - 24*a*
b^2*c + 16*a^2*c^2)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 +
c*x^4]])/(768*c^(7/2))
```

Maple [A] time = 0.022, size = 247, normalized size = 1.6

$$\begin{aligned}&\frac{x^2}{8c}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{5b}{48c^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{5b^2x^2}{64c^2}\sqrt{cx^4+bx^2+a}\\&+\frac{5b^3}{128c^3}\sqrt{cx^4+bx^2+a}+\frac{3ab^2}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}\\&-\frac{5b^4}{256}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}-\frac{ax^2}{16c}\sqrt{cx^4+bx^2+a}\\&-\frac{ab}{32c^2}\sqrt{cx^4+bx^2+a}-\frac{a^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*x^2*(c*x^4+b*x^2+a)^(3/2)/c-5/48*b*(c*x^4+b*x^2+a)^(3/2)/c^2+
5/64*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)*x^2+5/128*b^3/c^3*(c*x^4+b*x^2
+a)^(1/2)+3/32*b^2/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+
a)^(1/2))*a-5/256*b^4/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))-1/16/c*a*(c*x^4+b*x^2+a)^(1/2)*x^2-1/32/c^2*a*(c*x^4
```

```
+b*x^2+a)^(1/2)*b-1/16/c^(3/2)*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300301, size = 1, normalized size = 0.01

$$\left[\frac{4\left(48\,c^3x^6+8\,bc^2x^4+15\,b^3-52\,abc-2\left(5\,b^2c-12\,ac^2\right)x^2\right)\sqrt{cx^4+bx^2+a}\sqrt{c}+3\left(5\,b^4-24\,ab^2c+16\,a^2c^2\right)\log\left(4\sqrt{cx^4+\right.}{1536\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^5,x, algorithm="fricas")
```

```
[Out] [1/1536*(4*(48*c^3*x^6 + 8*b*c^2*x^4 + 15*b^3 - 52*a*b*c - 2*(5*b
^2*c - 12*a*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + 3*(5*b^4
- 24*a*b^2*c + 16*a^2*c^2)*log(4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x
^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^(7/
2), 1/768*(2*(48*c^3*x^6 + 8*b*c^2*x^4 + 15*b^3 - 52*a*b*c - 2*(5
*b^2*c - 12*a*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) - 3*(5*b
^4 - 24*a*b^2*c + 16*a^2*c^2)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(
sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.304453, size = 197, normalized size = 1.29

$$\frac{1}{384}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(6\,x^2+\frac{b}{c}\right)x^2-\frac{5\,b^2c^3-12\,ac^4}{c^5}\right)x^2+\frac{15\,b^3c^2-52\,abc^3}{c^5}\right)+\frac{\left(5\,b^4c^2-24\,ab^2c^3+16\,a^2c^4\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^5,x, algorithm="giac")
```

```
[Out] 1/384*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(6*x^2 + b/c)*x^2 - (5*b^2*c^
3 - 12*a*c^4)/c^5)*x^2 + (15*b^3*c^2 - 52*a*b*c^3)/c^5) + 1/256*(
5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*ln(abs(-2*(sqrt(c)*x^2 - s
qrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(11/2)
```

3.922 $\int x^3\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=108

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2) + (a + b*x^2
+ c*x^4)^(3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2))
```

Rubi [A] time = 0.187908, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}}-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -(b*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c^2) + (a + b*x^2
+ c*x^4)^(3/2)/(6*c) + (b*(b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2))
```

Rubi in Sympy [A] time = 15.3244, size = 97, normalized size = 0.9

$$-\frac{b\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{16c^2}+\frac{b\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{5}{2}}}+\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*(b + 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(16*c**2) + b*(-4*a*c
 + b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4
)))/(32*c**(5/2)) + (a + b*x**2 + c*x**4)**(3/2)/(6*c)
```

Mathematica [A] time = 0.104838, size = 99, normalized size = 0.92

$$\frac{3\left(b^3-4abc\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(8c\left(a+cx^4\right)-3b^2+2bcx^2\right)}{96c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 2*b*c*x^2 + 8*c*(a +
 c*x^4)) + 3*(b^3 - 4*a*b*c)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4]])/(96*c^(5/2))
```

Maple [A] time = 0.017, size = 139, normalized size = 1.3

$$\frac{1}{6\,c}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{bx^2}{8\,c}\sqrt{cx^4+bx^2+a}-\frac{b^2}{16\,c^2}\sqrt{cx^4+bx^2+a} \\ -\frac{ab}{8}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{b^3}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*(c*x^4+b*x^2+a)^(1/2)*x^2-1/1
6*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))*a+1/32*b^3/c^(5/2)*ln((1/2*b+c*x^2)/
c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.296166, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8c^2x^4+2bcx^2-3b^2+8ac\right)\sqrt{cx^4+bx^2+a}\sqrt{c}-3\left(b^3-4abc\right)\log\left(4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\left(8c^2x^4+8bcx^2+\right.\right.}{192c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^3,x, algorithm="fricas")
```

```
[Out] [1/192*(4*(8*c^2*x^4 + 2*b*c*x^2 - 3*b^2 + 8*a*c)*sqrt(c*x^4 + b*
x^2 + a)*sqrt(c) - 3*(b^3 - 4*a*b*c)*log(4*sqrt(c*x^4 + b*x^2 + a
)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(
c)))/c^(5/2), 1/96*(2*(8*c^2*x^4 + 2*b*c*x^2 - 3*b^2 + 8*a*c)*sqr
t(c*x^4 + b*x^2 + a)*sqrt(-c) + 3*(b^3 - 4*a*b*c)*arctan(1/2*(2*c
*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.289286, size = 132, normalized size = 1.22

$$\frac{1}{48}\sqrt{cx^4+bx^2+a}\left(2\left(4x^2+\frac{b}{c}\right)x^2-\frac{3b^2-8ac}{c^2}\right)-\frac{\left(b^3-4abc\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^3,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*x^2 + b/c)*x^2 - (3*b^2 - 8*a*
c)/c^2) - 1/32*(b^3 - 4*a*b*c)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^
4 + b*x^2 + a))*sqrt(c) - b))/c^(5/2)
```

3.923 $\int x\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=83

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*Ar
cTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(
3/2))
```

Rubi [A] time = 0.121603, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*Ar
cTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(
3/2))
```

Rubi in Sympy [A] time = 9.08482, size = 73, normalized size = 0.88

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(-4ac+b^2)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] (b + 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(8*c) - (-4*a*c + b**2)*
atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(16*c
**(3/2))
```

Mathematica [A] time = 0.0726342, size = 81, normalized size = 0.98

$$\frac{(b+2cx^2)\sqrt{a+bx^2+cx^4}}{8c}-\frac{(b^2-4ac)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{16c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c) - ((b^2 - 4*a*c)*Lo
g[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(3/2))
```

Maple [A] time = 0.01, size = 101, normalized size = 1.2

$$\frac{2\,cx^2+b}{8\,c}\sqrt{cx^4+bx^2+a}+\frac{a}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{b^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((1/2*b+c*x
^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291955, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-\left(b^2-4\,ac\right)\log\left(-4\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)-\left(8\,c^2x^4+8\,bcx^2+b^2+4\,ac\right)\sqrt{c}\right)}{32\,c^{\frac{3}{2}}}, \frac{2\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x,x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - (b^2 - 4
*a*c)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x
^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^(3/2), 1/16*(2*sqrt(c*x
^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c) - (b^2 - 4*a*c)*arctan(1/2
*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.284176, size = 103, normalized size = 1.24

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(2\,x^2+\frac{b}{c}\right)+\frac{\left(b^2-4\,ac\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2 + b/c) + 1/16*(b^2 - 4*a*c)*ln
(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(
3/2)
```

3.924 $\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$

Optimal. Leaf size=109

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c])
```

Rubi [A] time = 0.278396, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}-\frac{1}{2}\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)+\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/2 - (Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4])])/2 + (b*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c])
```

Rubi in Sympy [A] time = 24.07, size = 95, normalized size = 0.87

$$-\frac{\sqrt{a}\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2}+\frac{b\,\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}}+\frac{\sqrt{a+bx^2+cx^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

```
[Out] -sqrt(a)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4
)))/2 + b*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**
4)))/(4*sqrt(c)) + sqrt(a + b*x**2 + c*x**4)/2
```

Mathematica [A] time = 0.306286, size = 111, normalized size = 1.02

$$\frac{1}{4}\left(2\sqrt{a+bx^2+cx^4}-2\sqrt{a}\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)+\frac{b\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{\sqrt{c}}+4\sqrt{a}\log(x)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x,x]
```

```
[Out] (2*Sqrt[a + b*x^2 + c*x^4] + 4*Sqrt[a]*Log[x] - 2*Sqrt[a]*Log[2*a
 + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]] + (b*Log[b + 2*c*x^
2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/Sqrt[c])/4
```

Maple [A] time = 0.015, size = 91, normalized size = 0.8

$$\frac{1}{2}\sqrt{cx^4+bx^2+a}+\frac{b}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{1}{2}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x,x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b
*x^2+a)^(1/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4
+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.319196, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(-4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\left(8c^2x^4+8bcx^2+b^2+4ac\right)\sqrt{c}\right)+2\sqrt{a}\sqrt{c}\log\left(-\frac{\left(b^2+4ac\right)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}(b\right.}{x^4}}{8\sqrt{c}}\right.$$

$$-\frac{4\sqrt{-a}\sqrt{c}\arctan\left(\frac{bx^2+2a}{2\sqrt{cx^4+bx^2+a}\sqrt{-a}}\right)-b\log\left(-4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\left(8c^2x^4+8bcx^2+b^2+4ac\right)\sqrt{c}\right)-4\sqrt{cx^4}}{8\sqrt{c}}$$

$$\left.-\frac{2\sqrt{-a}\sqrt{-c}\arctan\left(\frac{bx^2+2a}{2\sqrt{cx^4+bx^2+a}\sqrt{-a}}\right)-b\arctan\left(\frac{(2cx^2+b)\sqrt{-c}}{2\sqrt{cx^4+bx^2+a}c}\right)-2\sqrt{cx^4+bx^2+a}\sqrt{-c}}{4\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x,x, algorithm="fricas")
```

```
[Out] [1/8*(b*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2
*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)) + 2*sqrt(a)*sqrt(c)*log(
-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^
2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(c
))/sqrt(c), 1/4*(b*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4
+ b*x^2 + a)*c)) + sqrt(a)*sqrt(-c)*log(-((b^2 + 4*a*c)*x^4 + 8*a
*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)
/x^4) + 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))/sqrt(-c), -1/8*(4*sqr
t(-a)*sqrt(c)*arctan(1/2*(b*x^2 + 2*a)/(sqrt(c*x^4 + b*x^2 + a)*s
qrt(-a))) - b*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) -
(8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)) - 4*sqrt(c*x^4 + b
*x^2 + a)*sqrt(c))/sqrt(c), -1/4*(2*sqrt(-a)*sqrt(-c)*arctan(1/2*
(b*x^2 + 2*a)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))) - b*arctan(1/2*
(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)) - 2*sqrt(c*x^
4 + b*x^2 + a)*sqrt(-c))/sqrt(-c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x, x)
```

3.925 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3} dx$

Optimal. Leaf size=112

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sq
rt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(
b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rubi [A] time = 0.281461, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}+\frac{1}{2}\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sq
rt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]) + (Sqrt[c]*ArcTanh[(
b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/2
```

Rubi in Sympy [A] time = 23.3574, size = 99, normalized size = 0.88

$$\frac{\sqrt{c}\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2}-\frac{\sqrt{a+bx^2+cx^4}}{2x^2}-\frac{b\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] sqrt(c)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)
))/2 - sqrt(a + b*x**2 + c*x**4)/(2*x**2) - b*atanh((2*a + b*x**2
)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(4*sqrt(a))
```

Mathematica [A] time = 0.257574, size = 116, normalized size = 1.04

$$\frac{1}{2}\left(-\frac{\sqrt{a+bx^2+cx^4}}{x^2}-\frac{b\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)}{2\sqrt{a}}+\sqrt{c}\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+\frac{b\log(x)}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^3,x]
```

```
[Out] (-(Sqrt[a + b*x^2 + c*x^4]/x^2) + (b*Log[x])/Sqrt[a] - (b*Log[2*a
 + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[a]) + Sqrt
[c]*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/2
```

Maple [A] time = 0.015, size = 140, normalized size = 1.3

$$-\frac{1}{2\,ax^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b}{2\,a}\sqrt{cx^4+bx^2+a}-\frac{b}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}+\frac{cx^2}{2\,a}\sqrt{cx^4+bx^2+a}+\frac{1}{2}\sqrt{c}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^3,x)
```

```
[Out] -1/2/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2*b/a*(c*x^4+b*x^2+a)^(1/2)-1/
4*b/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1
/2*c/a*(c*x^4+b*x^2+a)^(1/2)*x^2+1/2*c^(1/2)*ln((1/2*b+c*x^2)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.315703, size = 1, normalized size = 0.01

$$\left[\frac{2\sqrt{a}\sqrt{c}x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+bx^2\log\left(\frac{4\sqrt{cx^4+bx^2+a}(abx^2+2\,a^2)-((b^2+4\,ac)x}{x^4}\right.}{8\sqrt{a}x^2}\right.$$

$$-\frac{bx^2\arctan\left(\frac{(bx^2+2\,a)\sqrt{-a}}{2\sqrt{cx^4+bx^2+a}a}\right)-\sqrt{-a}\sqrt{c}x^2\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\sqrt{cx^4+bx^2+a}(2\,cx^2+b)\sqrt{c}-4\,ac\right)+2\sqrt{cx^4+b}}{4\sqrt{-a}x^2}$$

$$\left.-\frac{bx^2\arctan\left(\frac{(bx^2+2\,a)\sqrt{-a}}{2\sqrt{cx^4+bx^2+a}a}\right)-2\sqrt{-a}\sqrt{-c}x^2\arctan\left(\frac{2\,cx^2+b}{2\sqrt{cx^4+bx^2+a}\sqrt{-c}}\right)+2\sqrt{cx^4+bx^2+a}\sqrt{-a}}{4\sqrt{-a}x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(a)*sqrt(c)*x^2*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + b*x^2*lo
g((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x
^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)
*sqrt(a))/(sqrt(a)*x^2), 1/8*(4*sqrt(a)*sqrt(-c)*x^2*arctan(1/2*(
2*c*x^2 + b)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) + b*x^2*log((4*s
qrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8
*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*sqrt(c*x^4 + b*x^2 + a)*sqrt(
a))/(sqrt(a)*x^2), -1/4*(b*x^2*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/
(sqrt(c*x^4 + b*x^2 + a)*a)) - sqrt(-a)*sqrt(c)*x^2*log(-8*c^2*x^
4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
t(c) - 4*a*c) + 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*x^2
), -1/4*(b*x^2*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*
x^2 + a)*a)) - 2*sqrt(-a)*sqrt(-c)*x^2*arctan(1/2*(2*c*x^2 + b)/(
sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) + 2*sqrt(c*x^4 + b*x^2 + a)*sq
rt(-a))/(sqrt(-a)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{a+bx^2+cx^4}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^3, x)
```

3.926 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$

Optimal. Leaf size=88

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*
c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(1
6*a^(3/2))
```

Rubi [A] time = 0.172143, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) + ((b^2 - 4*a*
c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(1
6*a^(3/2))
```

Rubi in Sympy [A] time = 16.0541, size = 76, normalized size = 0.86

$$-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}+\frac{(-4ac+b^2)\,\text{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] -(2*a + b*x**2)*sqrt(a + b*x**2 + c*x**4)/(8*a*x**4) + (-4*a*c +
b**2)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))
/(16*a**(3/2))
```

Mathematica [A] time = 0.234385, size = 93, normalized size = 1.06

$$-\frac{(b^2-4ac)\left(\log(x^2)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{16a^{3/2}}-\frac{(2a+bx^2)\sqrt{a+bx^2+cx^4}}{8ax^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^5,x]
```

```
[Out] -((2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a*x^4) - ((b^2 - 4*a*
c)*(Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4
]]))/(16*a^(3/2))
```

Maple [B] time = 0.015, size = 193, normalized size = 2.2

$$-\frac{1}{4\,ax^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b}{8\,a^2x^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{b^2}{8\,a^2}\sqrt{cx^4+bx^2+a}+\frac{b^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{bcx^2}{8\,a^2}\sqrt{cx^4+bx^2+a}+\frac{c}{4\,a}\sqrt{cx^4+bx^2+a}-\frac{c}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^5,x)
```

```
[Out] -1/4/a/x^4*(c*x^4+b*x^2+a)^(3/2)+1/8*b/a^2/x^2*(c*x^4+b*x^2+a)^(3
/2)-1/8*b^2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*b^2/a^(3/2)*ln((2*a+b*
x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/8*b/a^2*c*(c*x^4+b*x^
2+a)^(1/2)*x^2+1/4*c/a*(c*x^4+b*x^2+a)^(1/2)-1/4*c/a^(1/2)*ln((2*
a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295778, size = 1, normalized size = 0.01

$$\left[-\frac{(b^2-4ac)x^4\log\left(\frac{4\sqrt{cx^4+bx^2+a}(abx^2+2a^2)-((b^2+4ac)x^4+8abx^2+8a^2)\sqrt{a}}{x^4}\right)+4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a}}{32a^{\frac{3}{2}}x^4}, \frac{(b^2-4ac)x^4\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^5,x, algorithm="fricas")
```

```
[Out] [-1/32*((b^2 - 4*a*c)*x^4*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2
 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4)
 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a))/(a^(3/2)*x^4)
, 1/16*((b^2 - 4*a*c)*x^4*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt
(c*x^4 + b*x^2 + a)*a)) - 2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)
*sqrt(-a))/(sqrt(-a)*a*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**5, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^5, x)
```

3.927 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^7}\,dx$

Optimal. Leaf size=116

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x
^2 + c*x^4)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x
^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rubi [A] time = 0.236174, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{5/2}}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] (b*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a^2*x^4) - (a + b*x
^2 + c*x^4)^(3/2)/(6*a*x^6) - (b*(b^2 - 4*a*c)*ArcTanh[(2*a + b*x
^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(5/2))
```

Rubi in Sympy [A] time = 20.8608, size = 104, normalized size = 0.9

$$-\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6ax^6}+\frac{b\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{16a^2x^4}-\frac{b\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] -(a + b*x**2 + c*x**4)**(3/2)/(6*a*x**6) + b*(2*a + b*x**2)*sqrt(
a + b*x**2 + c*x**4)/(16*a**2*x**4) - b*(-4*a*c + b**2)*atanh((2*
a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(32*a**(5/2))
```

Mathematica [A] time = 0.281819, size = 113, normalized size = 0.97

$$\frac{b\left(b^2-4ac\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{32a^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(8a^2+2ax^2\left(b+4cx^2\right)-3b^2x^4\right)}{48a^2x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^7,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]*(8*a^2 - 3*b^2*x^4 + 2*a*x^2*(b + 4*c*x
^2)))/(48*a^2*x^6) + (b*(b^2 - 4*a*c)*(Log[x^2] - Log[2*a + b*x^2
 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(32*a^(5/2))
```

Maple [B] time = 0.019, size = 222, normalized size = 1.9

$$-\frac{1}{6\,a x^6}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b}{8\,a^2x^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{b^2}{16\,a^3x^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{b^3}{16\,a^3}\sqrt{cx^4+bx^2+a}-\frac{b^3}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{b^2cx^2}{16\,a^3}\sqrt{cx^4+bx^2+a}-\frac{bc}{8\,a^2}\sqrt{cx^4+bx^2+a}+\frac{bc}{8}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^7,x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(3/2)/a/x^6+1/8*b/a^2/x^4*(c*x^4+b*x^2+a)^(3
/2)-1/16*b^2/a^3/x^2*(c*x^4+b*x^2+a)^(3/2)+1/16*b^3/a^3*(c*x^4+b*
x^2+a)^(1/2)-1/32*b^3/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2)+1/16*b^2/a^3*c*(c*x^4+b*x^2+a)^(1/2)*x^2-1/8*b/a
^2*c*(c*x^4+b*x^2+a)^(1/2)+1/8*b/a^(3/2)*c*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.303721, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(b^{3}-4\,abc\right)x^{6}\log\left(-\frac{4\sqrt{cx^{4}+bx^{2}+a}\left(abx^{2}+2\,a^{2}\right)+\left(\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+8\,a^{2}\right)\sqrt{a}}{x^{4}}\right)-4\left(\left(3\,b^{2}-8\,ac\right)x^{4}-2\,abx^{2}-8\,a^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\sqrt{a}}{192\,a^{\frac{5}{2}}x^{6}},\right.$$
$$\left.-\frac{3\left(b^{3}-4\,abc\right)x^{6}\arctan\left(\frac{\left(bx^{2}+2\,a\right)\sqrt{-a}}{2\sqrt{cx^{4}+bx^{2}+a}a}\right)-2\left(\left(3\,b^{2}-8\,ac\right)x^{4}-2\,abx^{2}-8\,a^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\sqrt{-a}}{96\sqrt{-a}a^{2}x^{6}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^7,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3 - 4*a*b*c)*x^6*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a
*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a)
)/x^4) - 4*((3*b^2 - 8*a*c)*x^4 - 2*a*b*x^2 - 8*a^2)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(a))/(a^(5/2)*x^6), -1/96*(3*(b^3 - 4*a*b*c)*x^6*
arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) -
2*((3*b^2 - 8*a*c)*x^4 - 2*a*b*x^2 - 8*a^2)*sqrt(c*x^4 + b*x^2 +
a)*sqrt(-a))/(sqrt(-a)*a^2*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^{2}+cx^{4}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**7,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**7, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^7,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^7, x)
```

3.928 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$

Optimal. Leaf size=161

$$\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}-\frac{(5b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{5b\,(a+bx^2+cx^4)^{3/2}}{48a^2x^6}-\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8}$$

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^3
*x^4) - (a + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c
*x^4)^(3/2))/(48*a^2*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTan
h[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(7/2
))
```

Rubi [A] time = 0.386265, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{(b^2-4ac)\,(5b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{7/2}}-\frac{(5b^2-4ac)\,(2a+bx^2)\,\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{5b\,(a+bx^2+cx^4)^{3/2}}{48a^2x^6}-\frac{(a+bx^2+cx^4)^{3/2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] -((5*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^3
*x^4) - (a + b*x^2 + c*x^4)^(3/2)/(8*a*x^8) + (5*b*(a + b*x^2 + c
*x^4)^(3/2))/(48*a^2*x^6) + ((b^2 - 4*a*c)*(5*b^2 - 4*a*c)*ArcTan
h[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(256*a^(7/2
))
```

Rubi in Sympy [A] time = 30.3439, size = 148, normalized size = 0.92

$$-\frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{8ax^8}+\frac{5b\,(a+bx^2+cx^4)^{\frac{3}{2}}}{48a^2x^6}-\frac{(2a+bx^2)\,(-4ac+5b^2)\,\sqrt{a+bx^2+cx^4}}{128a^3x^4}+\frac{(-4ac+b^2)\,(-4ac+5b^2)\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] -(a + b*x**2 + c*x**4)**(3/2)/(8*a*x**8) + 5*b*(a + b*x**2 + c*x*
*4)**(3/2)/(48*a**2*x**6) - (2*a + b*x**2)*(-4*a*c + 5*b**2)*sqrt
(a + b*x**2 + c*x**4)/(128*a**3*x**4) + (-4*a*c + b**2)*(-4*a*c +
 5*b**2)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4
)))/(256*a**(7/2))
```

Mathematica [A] time = 0.228904, size = 143, normalized size = 0.89

$$\frac{-\frac{2\sqrt{a}\sqrt{a+bx^2+cx^4}\left(48a^3+8a^2x^2\left(b+3cx^2\right)-2abx^4\left(5b+26cx^2\right)+15b^3x^6\right)}{x^8}-3\left(b^2-4ac\right)\left(5b^2-4ac\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+\right.\right.}{768a^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^9,x]
```

```
[Out] ((-2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(48*a^3 + 15*b^3*x^6 + 8*a^2
*x^2*(b + 3*c*x^2) - 2*a*b*x^4*(5*b + 26*c*x^2)))/x^8 - 3*(b^2 -
4*a*c)*(5*b^2 - 4*a*c)*(Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4]]))/(768*a^(7/2))
```

Maple [B] time = 0.023, size = 387, normalized size = 2.4

$$-\frac{1}{8\,ax^8}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{5\,b}{48\,a^2x^6}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{5\,b^2}{64\,a^3x^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{5\,b^3}{128\,a^4x^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{5\,b^4}{128\,a^4}\sqrt{cx^4+bx^2+a}+\frac{5\,b^4}{256}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{5\,x^2b^3c}{128\,a^4}\sqrt{cx^4+bx^2+a}+\frac{7\,b^2c}{64\,a^3}\sqrt{cx^4+bx^2+a}-\frac{3\,b^2c}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{c}{16\,a^2x^4}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}-\frac{bc}{32\,a^3x^2}\left(cx^4+bx^2+a\right)^{\frac{3}{2}}+\frac{bc^2x^2}{32\,a^3}\sqrt{cx^4+bx^2+a}-\frac{c^2}{16\,a^2}\sqrt{cx^4+bx^2+a}+\frac{c^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^9,x)
```

```
[Out] -1/8*(c*x^4+b*x^2+a)^(3/2)/a/x^8+5/48*b*(c*x^4+b*x^2+a)^(3/2)/a^2
/x^6-5/64*b^2/a^3/x^4*(c*x^4+b*x^2+a)^(3/2)+5/128*b^3/a^4/x^2*(c*
x^4+b*x^2+a)^(3/2)-5/128*b^4/a^4*(c*x^4+b*x^2+a)^(1/2)+5/256*b^4/
a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-5/128
*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/2)*x^2+7/64*b^2/a^3*c*(c*x^4+b*x^2+
a)^(1/2)-3/32*b^2/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+
a)^(1/2))/x^2)+1/16*c/a^2/x^4*(c*x^4+b*x^2+a)^(3/2)-1/32*c/a^3*b/
x^2*(c*x^4+b*x^2+a)^(3/2)+1/32*c^2/a^3*b*(c*x^4+b*x^2+a)^(1/2)*x^
2-1/16*c^2/a^2*(c*x^4+b*x^2+a)^(1/2)+1/16*c^2/a^(3/2)*ln((2*a+b*x
^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.310012, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,b^{4}-24\,ab^{2}c+16\,a^{2}c^{2}\right)x^{8}\log\left(-\frac{4\,\sqrt{cx^{4}+bx^{2}+a}\left(abx^{2}+2\,a^{2}\right)+\left(\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+8\,a^{2}\right)\sqrt{a}}{x^{4}}\right)-4\left(\left(15\,b^{3}-52\,abc\right)x^{6}+8\,a^{2}bx^{2}-\right.}{1536\,a^{\frac{7}{2}}x^{8}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1536*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*x^8*log(-(4*sqrt(c*x
^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^
2 + 8*a^2)*sqrt(a))/x^4) - 4*((15*b^3 - 52*a*b*c)*x^6 + 8*a^2*b*x
^2 - 2*(5*a*b^2 - 12*a^2*c)*x^4 + 48*a^3)*sqrt(c*x^4 + b*x^2 + a)
*sqrt(a))/(a^(7/2)*x^8), 1/768*(3*(5*b^4 - 24*a*b^2*c + 16*a^2*c^
2)*x^8*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)
*a)) - 2*((15*b^3 - 52*a*b*c)*x^6 + 8*a^2*b*x^2 - 2*(5*a*b^2 - 12
*a^2*c)*x^4 + 48*a^3)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)
*a^3*x^8)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**9,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**9, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^9,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^9, x)
```

3.929 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\,dx$

Optimal. Leaf size=199

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6}+\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}}$$

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a
^4*x^4) - (a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2
 + c*x^4)^(3/2))/(80*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c
*x^4)^(3/2))/(480*a^3*x^6) - (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*Ar
cTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^
(9/2))
```

Rubi [A] time = 0.583054, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{9/2}}+\frac{b\left(7b^2-12ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{\left(35b^2-32ac\right)\left(a+bx^2+cx^4\right)^{3/2}}{480a^3x^6}+\frac{7b\left(a+bx^2+cx^4\right)^{3/2}}{80a^2x^8}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (b*(7*b^2 - 12*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a
^4*x^4) - (a + b*x^2 + c*x^4)^(3/2)/(10*a*x^10) + (7*b*(a + b*x^2
 + c*x^4)^(3/2))/(80*a^2*x^8) - ((35*b^2 - 32*a*c)*(a + b*x^2 + c
*x^4)^(3/2))/(480*a^3*x^6) - (b*(7*b^2 - 12*a*c)*(b^2 - 4*a*c)*Ar
cTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(512*a^
(9/2))
```

Rubi in Sympy [A] time = 44.3488, size = 185, normalized size = 0.93

$$-\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{10ax^{10}}+\frac{7b\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{80a^2x^8}-\frac{\left(-32ac+35b^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{480a^3x^6}+\frac{b\left(2a+bx^2\right)\left(-12ac+7b^2\right)\sqrt{a+bx^2+cx^4}}{256a^4x^4}-\frac{b\left(-12ac+7b^2\right)\left(-4ac+b^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] -(a + b*x**2 + c*x**4)**(3/2)/(10*a*x**10) + 7*b*(a + b*x**2 + c*
x**4)**(3/2)/(80*a**2*x**8) - (-32*a*c + 35*b**2)*(a + b*x**2 + c
*x**4)**(3/2)/(480*a**3*x**6) + b*(2*a + b*x**2)*(-12*a*c + 7*b**
2)*sqrt(a + b*x**2 + c*x**4)/(256*a**4*x**4) - b*(-12*a*c + 7*b**
2)*(-4*a*c + b**2)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**
2 + c*x**4)))/(512*a**(9/2))
```

Mathematica [A] time = 0.240012, size = 175, normalized size = 0.88

$$\frac{b\left(7b^2-12ac\right)\left(b^2-4ac\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{512a^{9/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(-384a^4-16a^3\left(3bx^2+8cx^4\right)+8a^2\left(7b^2x^4+29bcx^6+32c^2x^8\right)-10ab^2x^6\left(7b+46cx^2\right)+105b^4x^8\right)}{3840a^4x^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^11,x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-384*a^4 + 105*b^4*x^8 - 10*a*b^2*x^6*(
7*b + 46*c*x^2) - 16*a^3*(3*b*x^2 + 8*c*x^4) + 8*a^2*(7*b^2*x^4 +
 29*b*c*x^6 + 32*c^2*x^8)))/(3840*a^4*x^10) + (b*(7*b^2 - 12*a*c)
*(b^2 - 4*a*c)*(Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b
*x^2 + c*x^4]]))/(512*a^(9/2))
```

Maple [B] time = 0.025, size = 442, normalized size = 2.2

$$
\begin{aligned}
&-\frac{1}{10\,a x^{10}}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}+\frac{7\,b}{80\,a^2 x^8}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}-\frac{7\,b^2}{96\,a^3 x^6}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}\\
&+\frac{7\,b^3}{128\,a^4 x^4}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}-\frac{7\,b^4}{256\,a^5 x^2}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}\\
&+\frac{7\,b^5}{256\,a^5}\sqrt{c x^4+b x^2+a}-\frac{7\,b^5}{512}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{9}{2}}\\
&+\frac{7\,x^2 b^4 c}{256\,a^5}\sqrt{c x^4+b x^2+a}-\frac{13\,b^3 c}{128\,a^4}\sqrt{c x^4+b x^2+a}\\
&+\frac{5\,b^3 c}{64}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{3\,bc}{32\,a^3 x^4}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}\\
&+\frac{3\,b^2 c}{64\,a^4 x^2}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}-\frac{3\,b^2 c^2 x^2}{64\,a^4}\sqrt{c x^4+b x^2+a}+\frac{3\,c^2 b}{32\,a^3}\sqrt{c x^4+b x^2+a}\\
&-\frac{3\,c^2 b}{32}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{c}{15\,a^2 x^6}\left(c x^4+b x^2+a\right)^{\frac{3}{2}}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^11,x)
```

```
[Out] -1/10*(c*x^4+b*x^2+a)^(3/2)/a/x^10+7/80*b*(c*x^4+b*x^2+a)^(3/2)/a
^2/x^8-7/96*b^2/a^3/x^6*(c*x^4+b*x^2+a)^(3/2)+7/128*b^3/a^4/x^4*(
c*x^4+b*x^2+a)^(3/2)-7/256*b^4/a^5/x^2*(c*x^4+b*x^2+a)^(3/2)+7/25
6*b^5/a^5*(c*x^4+b*x^2+a)^(1/2)-7/512*b^5/a^(9/2)*ln((2*a+b*x^2+2
*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+7/256*b^4/a^5*c*(c*x^4+b*x^2
+a)^(1/2)*x^2-13/128*b^3/a^4*c*(c*x^4+b*x^2+a)^(1/2)+5/64*b^3/a^(
7/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/32*b
/a^3*c/x^4*(c*x^4+b*x^2+a)^(3/2)+3/64*b^2/a^4*c/x^2*(c*x^4+b*x^2+
a)^(3/2)-3/64*b^2/a^4*c^2*(c*x^4+b*x^2+a)^(1/2)*x^2+3/32*b/a^3*c^
2*(c*x^4+b*x^2+a)^(1/2)-3/32*b/a^(5/2)*c^2*ln((2*a+b*x^2+2*a^(1/2
)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/15*c/a^2/x^6*(c*x^4+b*x^2+a)^(3/2
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.325769, size = 1, normalized size = 0.01

$$\left[\frac{15\left(7 b^5-40 a b^3 c+48 a^2 b c^2\right) x^{10} \log \left(\frac{4 \sqrt{c x^4+b x^2+a}\left(a b x^2+2 a^2\right)-\left(\left(b^2+4 a c\right) x^4+8 a b x^2+8 a^2\right) \sqrt{a}}{x^4}\right)+4\left(\left(105 b^4-460 a b^2 c+256 a^2 c^2\right.\right.}{15360 a^{\frac{9}{2}} x^{10}}\right.$$

$$-\frac{15\left(7 b^5-40 a b^3 c+48 a^2 b c^2\right) x^{10} \arctan \left(\frac{\left(b x^2+2 a\right) \sqrt{-a}}{2 \sqrt{c x^4+b x^2+a} a}\right)-2\left(\left(105 b^4-460 a b^2 c+256 a^2 c^2\right) x^8-2\left(35 a b^3-116 a^2 b c\right) x^6\right.}{7680 \sqrt{-a} a^4 x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^11,x, algorithm="fricas")
```

```
[Out] [1/15360*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^2)*x^10*log((4*sqrt
(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*
b*x^2 + 8*a^2)*sqrt(a))/x^4) + 4*((105*b^4 - 460*a*b^2*c + 256*a^
2*c^2)*x^8 - 2*(35*a*b^3 - 116*a^2*b*c)*x^6 - 48*a^3*b*x^2 + 8*(7
*a^2*b^2 - 16*a^3*c)*x^4 - 384*a^4)*sqrt(c*x^4 + b*x^2 + a)*sqrt(
a))/(a^(9/2)*x^10), -1/7680*(15*(7*b^5 - 40*a*b^3*c + 48*a^2*b*c^
2)*x^10*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a
)*a)) - 2*((105*b^4 - 460*a*b^2*c + 256*a^2*c^2)*x^8 - 2*(35*a*b^
3 - 116*a^2*b*c)*x^6 - 48*a^3*b*x^2 + 8*(7*a^2*b^2 - 16*a^3*c)*x^
4 - 384*a^4)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^4*x^10
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**11,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**11, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^{11}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^11,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^11, x)
```

3.930 $\int x^4\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=395

$$\frac{bx\left(8b^2-29ac\right)\sqrt{a+bx^2+cx^4}}{105c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt[4]{a}b\left(8b^2-29ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{105c^{11/4}\sqrt{a+bx^2+cx^4}} - \frac{2x\left(2b^2-5ac\right)\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}\left(2b^2-5ac\right)-29abc+8b^3\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{210c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{x^3\left(b+5cx^2\right)\sqrt{a+bx^2+cx^4}}{35c}$$

```
[Out] (-2*(2*b^2 - 5*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^2) + (b*(8*
b^2 - 29*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(5/2)*(Sqrt[a] +
Sqrt[c]*x^2)) + (x^3*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c
) - (a^(1/4)*b*(8*b^2 - 29*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(105*c^(11/4)*Sq
rt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^3 - 29*a*b*c + 2*Sqrt[a]*S
qrt[c]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(11/4)*Sqrt[a + b
*x^2 + c*x^4])
```

Rubi [A] time = 0.661045, antiderivative size = 395, normalized size of antiderivative = 1., number

of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{bx(8b^2-29ac)\sqrt{a+bx^2+cx^4}}{105c^{5/2}(\sqrt{a}+\sqrt{c}x^2)} - \frac{\sqrt[4]{a}b(8b^2-29ac)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{105c^{11/4}\sqrt{a+bx^2+cx^4}} - \frac{2x(2b^2-5ac)\sqrt{a+bx^2+cx^4}}{105c^2} + \frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}(2b^2-5ac)-29abc+8b^3\right)(\sqrt{a}+\sqrt{c}x^2)\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{210c^{11/4}\sqrt{a+bx^2+cx^4}} + \frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(2*b^2 - 5*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^2) + (b*(8*
b^2 - 29*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(105*c^(5/2)*(Sqrt[a] +
Sqrt[c]*x^2)) + (x^3*(b + 5*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*c
) - (a^(1/4)*b*(8*b^2 - 29*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(105*c^(11/4)*Sq
rt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^3 - 29*a*b*c + 2*Sqrt[a]*S
qrt[c]*(2*b^2 - 5*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(210*c^(11/4)*Sqrt[a + b
*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 67.2463, size = 364, normalized size = 0.92

$$-\frac{\sqrt[4]{a}b\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}+\sqrt{c}x^2)(-29ac+8b^2)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{105c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}(\sqrt{a}+\sqrt{c}x^2)\left(\sqrt{a}\sqrt{c}(-10ac+4b^2)+b(-29ac+8b^2)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{210c^{\frac{11}{4}}\sqrt{a+bx^2+cx^4}} + \frac{bx(-29ac+8b^2)\sqrt{a+bx^2+cx^4}}{105c^{\frac{5}{2}}(\sqrt{a}+\sqrt{c}x^2)} + \frac{x^3(b+5cx^2)\sqrt{a+bx^2+cx^4}}{35c} - \frac{2x(-5ac+2b^2)\sqrt{a+bx^2+cx^4}}{105c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -a**(1/4)*b*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*(-29*a*c + 8*b**2)*elliptic_e(2*atan(
c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(105*c**(11/4)
*sqrt(a + b*x**2 + c*x**4)) + a**(1/4)*sqrt((a + b*x**2 + c*x**4)
/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*s
qrt(c)*(-10*a*c + 4*b**2) + b*(-29*a*c + 8*b**2))*elliptic_f(2*at
an(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(210*c**(11
/4)*sqrt(a + b*x**2 + c*x**4)) + b*x*(-29*a*c + 8*b**2)*sqrt(a +
b*x**2 + c*x**4)/(105*c**(5/2)*(sqrt(a) + sqrt(c)*x**2)) + x**3*(
b + 5*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(35*c) - 2*x*(-5*a*c + 2*
b**2)*sqrt(a + b*x**2 + c*x**4)/(105*c**2)
```

Mathematica [C] time = 2.87946, size = 538, normalized size = 1.36

$$4cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(10a^2c+a\left(-4b^2+13bcx^2+25c^2x^4\right)-4b^3x^2-b^2cx^4+18bc^2x^6+15c^3x^8\right)-i\left(-20a^2c^2+37ab^2c-29abc\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(10*a^2*c - 4*b^3*x^2 - b^
2*c*x^4 + 18*b*c^2*x^6 + 15*c^3*x^8 + a*(-4*b^2 + 13*b*c*x^2 + 25
*c^2*x^4)) + I*b*(8*b^2 - 29*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elli
pticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^4 + 37*a*b^
2*c - 20*a^2*c^2 + 8*b^3*Sqrt[b^2 - 4*a*c] - 29*a*b*c*Sqrt[b^2 -
4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*
c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(420*c
^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.053, size = 476, normalized size = 1.2

$$\frac{x^5}{7}\sqrt{cx^4+bx^2+a}+\frac{bx^3}{35c}\sqrt{cx^4+bx^2+a}+\frac{x}{3c}\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)\sqrt{cx^4+bx^2+a}$$
$$-\frac{a\sqrt{2}}{12c}\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\right.$$
$$-\frac{a\sqrt{2}}{2}\left(-\frac{3ab}{35c}-\frac{2b}{3c}\left(\frac{2a}{7}-\frac{4b^2}{35c}\right)\right)\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/7*x^5*(c*x^4+b*x^2+a)^(1/2)+1/35*b/c*x^3*(c*x^4+b*x^2+a)^(1/2)+
1/3*(2/7*a-4/35*b^2/c)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(2/7*a-4/35
*b^2/c)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)
^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2))-1/2*(-3/35*b/c*a-2/3*(2/7*a-4/35*b^2/c)/c*b)*a*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
)-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{cx^4+bx^2+a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^4, x)
```

3.931 $\int x^2\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=342

$$-\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c}$$

```
[Out] (-2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt[a]
 + Sqrt[c]*x^2)) + (x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*
c) + (2*a^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a
 + b*x^2 + c*x^4]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.391291, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$-\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*c^(3/2)*(Sqrt[a]
 + Sqrt[c]*x^2)) + (x*(b + 3*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(15*
c) + (2*a^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b
*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(15*c^(7/4)*Sqrt[a
 + b*x^2 + c*x^4]) - (a^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)
*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(30*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 43.9088, size = 314, normalized size = 0.92

$$\frac{2\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{15c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{30c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}} + \frac{x\left(b+3cx^2\right)\sqrt{a+bx^2+cx^4}}{15c} - \frac{2x\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{15c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2
)*(sqrt(a) + sqrt(c)*x**2)*(-3*a*c + b**2)*elliptic_e(2*atan(c**(
1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(15*c**(7/4)*sqrt(
a + b*x**2 + c*x**4)) - a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt
(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*b*sqrt(
c) - 6*a*c + 2*b**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2
- b/(4*sqrt(a)*sqrt(c)))/(30*c**(7/4)*sqrt(a + b*x**2 + c*x**4))
+ x*(b + 3*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(15*c) - 2*x*(-3*a*c
 + b**2)*sqrt(a + b*x**2 + c*x**4)/(15*c**(3/2)*(sqrt(a) + sqrt(c
)*x**2))
```

Mathematica [C] time = 2.29491, size = 479, normalized size = 1.4

$$-i\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+2cx\sqrt{\sqrt{b^2 \ldots$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 3*c*x^2)*(a + b*x^2 +
 c*x^4) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*S
qrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 -
 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[S
qrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c]
)/(b - Sqrt[b^2 - 4*a*c])])/(30*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.015, size = 417, normalized size = 1.2

$$\frac{x^3}{5}\sqrt{cx^4+bx^2+a}+\frac{bx}{15c}\sqrt{cx^4+bx^2+a}$$

$$-\frac{ab\sqrt{2}}{60c}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b(}{}}\right.$$

$$-\frac{a\sqrt{2}}{2}\left(\frac{2a}{5}-\frac{2b^2}{15c}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/5*x^3*(c*x^4+b*x^2+a)^(1/2)+1/15*b/c*x*(c*x^4+b*x^2+a)^(1/2)-1/
60*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
-1/2*(2/5*a-2/15*b^2/c)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)
*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1
/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*x^2,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*x^2, x)
```

3.932 $\int \sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/3 + (b*x*Sqrt[a + b*x^2 + c*x^4])/(3*
Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[
c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.298228, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt[4]{a}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{1}{3}x\sqrt{a+bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/3 + (b*x*Sqrt[a + b*x^2 + c*x^4])/(3*
Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
E[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3
*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(b + 2*Sqrt[a]*Sqrt[
```

```
c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(6*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 39.8969, size = 279, normalized size = 0.9

$$-\frac{\sqrt[4]{a}b\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{bx\sqrt{a+bx^2+cx^4}}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\sqrt{a+bx^2+cx^4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -a**(1/4)*b*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)
), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*c**(3/4)*sqrt(a + b*x**2 + c*x
**4)) + a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x*
*2)**2)*(sqrt(a) + sqrt(c)*x**2)*(2*sqrt(a)*sqrt(c) + b)*elliptic
_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(6*c
**(3/4)*sqrt(a + b*x**2 + c*x**4)) + b*x*sqrt(a + b*x**2 + c*x**4
)/(3*sqrt(c)*(sqrt(a) + sqrt(c)*x**2)) + x*sqrt(a + b*x**2 + c*x*
*4)/3
```

Mathematica [C] time = 1.53022, size = 445, normalized size = 1.44

$$\frac{-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+ib\left(\sqrt{b^2-4\right.}{12c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) + I*b*
(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
```

```
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*S
qrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*
ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*c*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.011, size = 379, normalized size = 1.2

$$\frac{x}{3}\sqrt{cx^4+bx^2+a}$$
$$+\frac{a\sqrt{2}}{6}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$
$$-\frac{ab\sqrt{2}}{6}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)+1/6*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2))-1/6*b*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2
)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a), x)
```

3.933 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^2} dx$

Optimal. Leaf size=303

$$\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^
4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.271116, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/x) + (2*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^
4])/(Sqrt[a] + Sqrt[c]*x^2) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[
c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellip
ticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])
```

```
/Sqrt[a + b*x^2 + c*x^4] + ((b + 2*Sqrt[a]*Sqrt[c])*(Sqrt[a] + Sq
rt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*El
lipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/
4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 35.5095, size = 274, normalized size = 0.9

$$-\frac{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\sqrt{a}+\sqrt{c}x^2}-\frac{\sqrt{a+bx^2+cx^4}}{x}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -2*a**(1/4)*c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c
)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x
/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/sqrt(a + b*x**2 + c*x**4
) + 2*sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2
) - sqrt(a + b*x**2 + c*x**4)/x + sqrt((a + b*x**2 + c*x**4)/(sqr
t(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(2*sqrt(a)*sqrt
(c) + b)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(
a)*sqrt(c)))/(2*a**(1/4)*c**(1/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 1.4592, size = 435, normalized size = 1.44

$$\frac{-i\sqrt{2}x\sqrt{b^2-4ac}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+ix\left(\sqrt{b^2-4ac}-b\right)\sqrt{\cdots}}{2x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^2,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*(-b +
 Sqrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(
b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + S
qrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*
```

```
a*c])] - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*x*Sqrt[(b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*
Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
Sqrt[b^2 - 4*a*c])])/(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a
+ b*x^2 + c*x^4])
```

Maple [A] time = 0.018, size = 381, normalized size = 1.3

$$-\frac{1}{x}\sqrt{cx^4+bx^2+a}$$
$$+\frac{b\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$
$$-ac\sqrt{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2-\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^2,x)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/x+1/4*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))-c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2
)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*
(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(
1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^2, x)
```

3.934 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^4} dx$

Optimal. Leaf size=341

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} + \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*x^3) - (b*Sqrt[a + b*x^2 + c*x^4])/(3
*a*x) + (b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*(Sqrt[a] + Sqr
t[c]*x^2)) - (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(3/4)*Sqrt[a + b*x^
2 + c*x^4]) + ((b + 2*Sqrt[a]*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
6*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.46503, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{c}\left(2\sqrt{a}\sqrt{c}+b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{a+bx^2+cx^4}}{3ax} + \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{a+bx^2+cx^4}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*x^3) - (b*Sqrt[a + b*x^2 + c*x^4])/(3
*a*x) + (b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a*(Sqrt[a] + Sqr
t[c]*x^2)) - (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
```

```
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(3/4)*Sqrt[a + b*x^
2 + c*x^4]) + ((b + 2*Sqrt[a]*Sqrt[c])*c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
6*a^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 56.9775, size = 304, normalized size = 0.89

$$-\frac{\sqrt{a+bx^2+cx^4}}{3x^3}+\frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{b\sqrt{a+bx^2+cx^4}}{3ax}-\frac{b\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3a^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6a^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**4,x)
```

```
[Out] -sqrt(a + b*x**2 + c*x**4)/(3*x**3) + b*sqrt(c)*x*sqrt(a + b*x**2
 + c*x**4)/(3*a*(sqrt(a) + sqrt(c)*x**2)) - b*sqrt(a + b*x**2 + c
*x**4)/(3*a*x) - b*c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) +
 sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**
(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*a**(3/4)*sqrt(
a + b*x**2 + c*x**4)) + c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt
(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(2*sqrt(a)*sqrt(
c) + b)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a
)*sqrt(c)))/(6*a**(3/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 1.62425, size = 459, normalized size = 1.35

$$\frac{-4\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a+bx^2\right)\left(a+bx^2+cx^4\right)-ix^3\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\cdots\right.\right.}{12ax^3\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^4,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2)*(a + b*x^2 + c*x^
4) + I*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a
*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*x
^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*a*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.02, size = 404, normalized size = 1.2

$$-\frac{1}{3x^3}\sqrt{cx^4+bx^2+a}-\frac{b}{3ax}\sqrt{cx^4+bx^2+a}$$
$$+\frac{c\sqrt{2}}{6}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b\left(b\right.}{}}\right.$$
$$-\frac{bc\sqrt{2}}{6}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^4,x)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/x^3-1/3*b*(c*x^4+b*x^2+a)^(1/2)/a/x+1/
6*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/6*
b*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)
^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4+bx^2+a}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^4,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^4,x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**4,x)

[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^4,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^4, x)
```

3.935 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^6}\,dx$

Optimal. Leaf size=397

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x}-\frac{2\sqrt{c}x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}-\frac{b\sqrt{a+bx^2+cx^4}}{15ax^3}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(5*x^5) - (b*Sqrt[a + b*x^2 + c*x^4])/(1
5*a*x^3) + (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*x) -
 (2*Sqrt[c]*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*(Sqr
t[a] + Sqrt[c]*x^2)) + (2*c^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(15*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2 + Sqrt[a]*
b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*a^(7/4)*Sqrt[a + b*x^2 +
c*x^4])
```

Rubi [A] time = 0.726037, antiderivative size = 397, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2x}-\frac{2\sqrt{c}x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}-\frac{b\sqrt{a+bx^2+cx^4}}{15ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(5*x^5) - (b*Sqrt[a + b*x^2 + c*x^4])/(1
5*a*x^3) + (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*x) -
 (2*Sqrt[c]*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(15*a^2*(Sqr
t[a] + Sqrt[c]*x^2)) + (2*c^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipt
icE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/
(15*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(2*b^2 + Sqrt[a]*
b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^
4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1
/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*a^(7/4)*Sqrt[a + b*x^2 +
c*x^4])
```

Rubi in Sympy [A] time = 79.3718, size = 366, normalized size = 0.92

$$-\frac{\sqrt{a+bx^2+cx^4}}{5x^5}-\frac{b\sqrt{a+bx^2+cx^4}}{15ax^3}-\frac{2\sqrt{c}x\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{15a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{2\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{15a^2x}+\frac{2\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{15a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{30a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] -sqrt(a + b*x**2 + c*x**4)/(5*x**5) - b*sqrt(a + b*x**2 + c*x**4)
/(15*a*x**3) - 2*sqrt(c)*x*(-3*a*c + b**2)*sqrt(a + b*x**2 + c*x*
*4)/(15*a**2*(sqrt(a) + sqrt(c)*x**2)) + 2*(-3*a*c + b**2)*sqrt(a
 + b*x**2 + c*x**4)/(15*a**2*x) + 2*c**(1/4)*sqrt((a + b*x**2 + c
*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(-3*
a*c + b**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sq
rt(a)*sqrt(c)))/(15*a**(7/4)*sqrt(a + b*x**2 + c*x**4)) - c**(1/4
)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a
) + sqrt(c)*x**2)*(sqrt(a)*b*sqrt(c) - 6*a*c + 2*b**2)*elliptic_f
(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(30*a*
*(7/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 2.54995, size = 530, normalized size = 1.34

$$-2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(3a^3+a^2\left(4bx^2+9cx^4\right)+a\left(-b^2x^4+7bcx^6+6c^2x^8\right)-2b^2x^6\left(b+cx^2\right)\right)-ix^5\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/x^6,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(3*a^3 - 2*b^2*x^6*(b + c*x^2
) + a^2*(4*b*x^2 + 9*c*x^4) + a*(-(b^2*x^4) + 7*b*c*x^6 + 6*c^2*x
^8)) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*
Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I
*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^
2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sq
rt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[
b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(30*a^2*Sqrt[c/(b + Sqrt[b^2 - 4
*a*c])]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.023, size = 452, normalized size = 1.1

$$-\frac{1}{5x^5}\sqrt{cx^4+bx^2+a}-\frac{b}{15ax^3}\sqrt{cx^4+bx^2+a}-\frac{6ac-2b^2}{15a^2x}\sqrt{cx^4+bx^2+a}$$
$$-\frac{bc\sqrt{2}}{60a}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b\left(\right.}{}}\right.$$
$$-\frac{c\left(3ac-b^2\right)\sqrt{2}}{15a}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/x^6,x)
```

```
[Out] -1/5*(c*x^4+b*x^2+a)^(1/2)/x^5-1/15*b*(c*x^4+b*x^2+a)^(1/2)/a/x^3
-2/15*(3*a*c-b^2)/a^2*(c*x^4+b*x^2+a)^(1/2)/x-1/60*b*c/a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
```

```
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/15*c*(3*a*c-b^
2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(
1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2
)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^6,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^6,x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{x^6} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/x**6, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/x^6, x)
```

3.936 $\int x^7 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=223

$$-\frac{3b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}+\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4}+\frac{\left(-16ac+21b^2-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3}+\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}$$

```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2
 + c*x^4)^(3/2))/(256*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*
c) + ((21*b^2 - 16*a*c - 30*b*c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(
560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c^(11/2))
```

Rubi [A] time = 0.506986, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{3b\left(b^2-4ac\right)^2\left(3b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4096c^{11/2}}+\frac{3b\left(b^2-4ac\right)\left(3b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{2048c^5}-\frac{b\left(3b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{256c^4}+\frac{\left(-16ac+21b^2-30bcx^2\right)\left(a+bx^2+cx^4\right)^{5/2}}{560c^3}+\frac{x^4\left(a+bx^2+cx^4\right)^{5/2}}{14c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (3*b*(b^2 - 4*a*c)*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(2048*c^5) - (b*(3*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2
 + c*x^4)^(3/2))/(256*c^4) + (x^4*(a + b*x^2 + c*x^4)^(5/2))/(14*
c) + ((21*b^2 - 16*a*c - 30*b*c*x^2)*(a + b*x^2 + c*x^4)^(5/2))/(
560*c^3) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4096*c^(11/2))
```

Rubi in Sympy [A] time = 35.8066, size = 218, normalized size = 0.98

$$-\frac{b\left(b+2cx^{2}\right)\left(-4ac+3b^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{256c^{4}}+\frac{3b\left(b+2cx^{2}\right)\left(-4ac+b^{2}\right)\left(-4ac+3b^{2}\right)\sqrt{a+bx^{2}+cx^{4}}}{2048c^{5}}-\frac{3b\left(-4ac+b^{2}\right)^{2}\left(-4ac+3b^{2}\right)\operatorname{atanh}\left(\frac{b+2cx^{2}}{2\sqrt{c}\sqrt{a+bx^{2}+cx^{4}}}\right)}{4096c^{\frac{11}{2}}}+\frac{x^{4}\left(a+bx^{2}+cx^{4}\right)^{\frac{5}{2}}}{14c}+\frac{\left(a+bx^{2}+cx^{4}\right)^{\frac{5}{2}}\left(-12ac+\frac{63b^{2}}{4}-\frac{45bcx^{2}}{2}\right)}{420c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -b*(b + 2*c*x**2)*(-4*a*c + 3*b**2)*(a + b*x**2 + c*x**4)**(3/2)/
(256*c**4) + 3*b*(b + 2*c*x**2)*(-4*a*c + b**2)*(-4*a*c + 3*b**2)
*sqrt(a + b*x**2 + c*x**4)/(2048*c**5) - 3*b*(-4*a*c + b**2)**2*(
-4*a*c + 3*b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2
+ c*x**4)))/(4096*c**(11/2)) + x**4*(a + b*x**2 + c*x**4)**(5/2)/
(14*c) + (a + b*x**2 + c*x**4)**(5/2)*(-12*a*c + 63*b**2/4 - 45*b
*c*x**2/2)/(420*c**3)
```

Mathematica [A] time = 0.212549, size = 220, normalized size = 0.99

$$\frac{\sqrt{a+bx^{2}+cx^{4}}\left(16b^{2}c^{2}\left(343a^{2}-62acx^{4}+8c^{2}x^{8}\right)+32bc^{3}x^{2}\left(-73a^{2}+22acx^{4}+200c^{2}x^{8}\right)+168b^{4}c\left(cx^{4}-15a\right)+16b^{3}c^{2}x^{2}\right.}{71680c^{5}}$$

$$-\frac{3b\left(b^{2}-4ac\right)^{2}\left(3b^{2}-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^{2}+cx^{4}}+b+2cx^{2}\right)}{4096c^{11/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(315*b^6 - 210*b^5*c*x^2 + 16*b^3*c^2*x^
2*(91*a - 9*c*x^4) + 168*b^4*c*(-15*a + c*x^4) + 1024*c^3*(a + c*
x^4)^2*(-2*a + 5*c*x^4) + 16*b^2*c^2*(343*a^2 - 62*a*c*x^4 + 8*c^
2*x^8) + 32*b*c^3*x^2*(-73*a^2 + 22*a*c*x^4 + 200*c^2*x^8)))/(716
80*c^5) - (3*b*(b^2 - 4*a*c)^2*(3*b^2 - 4*a*c)*Log[b + 2*c*x^2 +
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4096*c^(11/2))
```

Maple [B] time = 0.041, size = 534, normalized size = 2.4

$$
\begin{aligned}
&-\frac{a^{3}}{35 c^{2}} \sqrt{c x^{4}+b x^{2}+a}+\frac{b^{2} x^{8}}{560 c} \sqrt{c x^{4}+b x^{2}+a}-\frac{9 b^{3} x^{6}}{4480 c^{2}} \sqrt{c x^{4}+b x^{2}+a} \\
&+\frac{3 b^{4} x^{4}}{1280 c^{3}} \sqrt{c x^{4}+b x^{2}+a}-\frac{3 b^{5} x^{2}}{1024 c^{4}} \sqrt{c x^{4}+b x^{2}+a} \\
&-\frac{9 b^{7}}{4096} \ln \left(1\left(\frac{b}{2}+c x^{2}\right) \frac{1}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right) c^{-\frac{11}{2}} \\
&-\frac{9 b^{4} a}{256 c^{4}} \sqrt{c x^{4}+b x^{2}+a}+\frac{a^{2} x^{4}}{70 c} \sqrt{c x^{4}+b x^{2}+a}+\frac{49 a^{2} b^{2}}{640 c^{3}} \sqrt{c x^{4}+b x^{2}+a} \\
&-\frac{15 a^{2} b^{3}}{256} \ln \left(1\left(\frac{b}{2}+c x^{2}\right) \frac{1}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right) c^{-\frac{7}{2}} \\
&+\frac{3 a^{3} b}{64} \ln \left(1\left(\frac{b}{2}+c x^{2}\right) \frac{1}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right) c^{-\frac{5}{2}}+\frac{4 a x^{8}}{35} \sqrt{c x^{4}+b x^{2}+a} \\
&+\frac{5 b x^{10}}{56} \sqrt{c x^{4}+b x^{2}+a}+\frac{c x^{12}}{14} \sqrt{c x^{4}+b x^{2}+a}+\frac{9 b^{6}}{2048 c^{5}} \sqrt{c x^{4}+b x^{2}+a} \\
&-\frac{73 a^{2} b x^{2}}{2240 c^{2}} \sqrt{c x^{4}+b x^{2}+a}+\frac{21 b^{5} a}{1024} \ln \left(1\left(\frac{b}{2}+c x^{2}\right) \frac{1}{\sqrt{c}}+\sqrt{c x^{4}+b x^{2}+a}\right) c^{-\frac{9}{2}} \\
&+\frac{11 a b x^{6}}{1120 c} \sqrt{c x^{4}+b x^{2}+a}+\frac{13 a b^{3} x^{2}}{640 c^{3}} \sqrt{c x^{4}+b x^{2}+a}-\frac{31 a x^{4} b^{2}}{2240 c^{2}} \sqrt{c x^{4}+b x^{2}+a}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^7*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/35*a^3/c^2*(c*x^4+b*x^2+a)^(1/2)+1/560/c*b^2*x^8*(c*x^4+b*x^2+
a)^(1/2)-9/4480/c^2*b^3*x^6*(c*x^4+b*x^2+a)^(1/2)+3/1280/c^3*b^4*
x^4*(c*x^4+b*x^2+a)^(1/2)-3/1024/c^4*b^5*x^2*(c*x^4+b*x^2+a)^(1/2
)-9/4096/c^(11/2)*b^7*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))-9/256/c^4*b^4*a*(c*x^4+b*x^2+a)^(1/2)+1/70*a^2*x^4/c*(c*x^4+
b*x^2+a)^(1/2)+49/640*a^2*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-15/256*a^
2*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/6
4*a^3*b/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+4
/35*a*x^8*(c*x^4+b*x^2+a)^(1/2)+5/56*b*x^10*(c*x^4+b*x^2+a)^(1/2)
+1/14*c*x^12*(c*x^4+b*x^2+a)^(1/2)+9/2048/c^5*b^6*(c*x^4+b*x^2+a)
^(1/2)-73/2240*a^2*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+21/1024/c^(9/2
)*b^5*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+11/1120/c
*b*a*x^6*(c*x^4+b*x^2+a)^(1/2)+13/640/c^3*b^3*a*x^2*(c*x^4+b*x^2+
a)^(1/2)-31/2240/c^2*b^2*a*x^4*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^7,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.32462, size = 1, normalized size = 0.

$$\left[\frac{4\left(5120\,c^{6}x^{12}+6400\,bc^{5}x^{10}+128\left(b^{2}c^{4}+64\,ac^{5}\right)x^{8}-16\left(9\,b^{3}c^{3}-44\,abc^{4}\right)x^{6}+315\,b^{6}-2520\,ab^{4}c+5488\,a^{2}b^{2}c^{2}-2048\,a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^7,x, algorithm="fricas")
```

```
[Out] [1/286720*(4*(5120*c^6*x^12 + 6400*b*c^5*x^10 + 128*(b^2*c^4 + 64
*a*c^5)*x^8 - 16*(9*b^3*c^3 - 44*a*b*c^4)*x^6 + 315*b^6 - 2520*a*
b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3 + 8*(21*b^4*c^2 - 124*a*b
^2*c^3 + 128*a^2*c^4)*x^4 - 2*(105*b^5*c - 728*a*b^3*c^2 + 1168*a
^2*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) - 105*(3*b^7 - 28*
a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*log(-4*sqrt(c*x^4 + b*x^
2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*
sqrt(c)))/c^(11/2), 1/143360*(2*(5120*c^6*x^12 + 6400*b*c^5*x^10
+ 128*(b^2*c^4 + 64*a*c^5)*x^8 - 16*(9*b^3*c^3 - 44*a*b*c^4)*x^6
+ 315*b^6 - 2520*a*b^4*c + 5488*a^2*b^2*c^2 - 2048*a^3*c^3 + 8*(2
1*b^4*c^2 - 124*a*b^2*c^3 + 128*a^2*c^4)*x^4 - 2*(105*b^5*c - 728
*a*b^3*c^2 + 1168*a^2*b*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c
) - 105*(3*b^7 - 28*a*b^5*c + 80*a^2*b^3*c^2 - 64*a^3*b*c^3)*arct
an(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt
(-c)*c^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{7}\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.308764, size = 374, normalized size = 1.68

$$\frac{1}{71680}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\left(10\left(4\,cx^2+5\,b\right)x^2+\frac{b^2c^{10}+64\,ac^{11}}{c^{11}}\right)x^2-\frac{9\,b^3c^9-44\,abc^{10}}{c^{11}}\right)x^2+\frac{21\,b^4c^8-124\,ab^2c^9+\cdots}{c^{11}}\cdots\right.\right.\right.$$
$$+\frac{3\left(3\,b^7c^6-28\,ab^5c^7+80\,a^2b^3c^8-64\,a^3bc^9\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4096\,c^{\frac{23}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^7,x, algorithm="giac")
```

```
[Out] 1/71680*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*(4*c*x^2 + 5*b)*x
^2 + (b^2*c^10 + 64*a*c^11)/c^11)*x^2 - (9*b^3*c^9 - 44*a*b*c^10)
/c^11)*x^2 + (21*b^4*c^8 - 124*a*b^2*c^9 + 128*a^2*c^10)/c^11)*x^
2 - (105*b^5*c^7 - 728*a*b^3*c^8 + 1168*a^2*b*c^9)/c^11)*x^2 + (3
15*b^6*c^6 - 2520*a*b^4*c^7 + 5488*a^2*b^2*c^8 - 2048*a^3*c^9)/c^
11) + 3/4096*(3*b^7*c^6 - 28*a*b^5*c^7 + 80*a^2*b^3*c^8 - 64*a^3*
b*c^9)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c)
- b))/c^(23/2)
```

3.937 $\int x^5 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=204

$$\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}} - \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3} - \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{12c}$$

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*
x^4])/(1024*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*
x^4)^(3/2))/(384*c^3) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2)
 + (x^2*(a + b*x^2 + c*x^4)^(5/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4])])/(2048*c^(9/2))
```

Rubi [A] time = 0.405896, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{9/2}} - \frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4} + \frac{\left(7b^2-4ac\right)\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384c^3} - \frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120c^2} + \frac{x^2\left(a+bx^2+cx^4\right)^{5/2}}{12c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*
x^4])/(1024*c^4) + ((7*b^2 - 4*a*c)*(b + 2*c*x^2)*(a + b*x^2 + c*
x^4)^(3/2))/(384*c^3) - (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*c^2)
 + (x^2*(a + b*x^2 + c*x^4)^(5/2))/(12*c) + ((b^2 - 4*a*c)^2*(7*b
^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x
^4])])/(2048*c^(9/2))
```

Rubi in Sympy [A] time = 35.5485, size = 190, normalized size = 0.93

$$-\frac{7b\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{120c^2}+\frac{x^2\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{12c}+\frac{\left(b+2cx^2\right)\left(-4ac+7b^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{384c^3}-\frac{\left(b+2cx^2\right)\left(-4ac+b^2\right)\left(-4ac+7b^2\right)\sqrt{a+bx^2+cx^4}}{1024c^4}+\frac{\left(-4ac+b^2\right)^2\left(-4ac+7b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2048c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -7*b*(a + b*x**2 + c*x**4)**(5/2)/(120*c**2) + x**2*(a + b*x**2 +
 c*x**4)**(5/2)/(12*c) + (b + 2*c*x**2)*(-4*a*c + 7*b**2)*(a + b*
x**2 + c*x**4)**(3/2)/(384*c**3) - (b + 2*c*x**2)*(-4*a*c + b**2)
*(-4*a*c + 7*b**2)*sqrt(a + b*x**2 + c*x**4)/(1024*c**4) + (-4*a*
c + b**2)**2*(-4*a*c + 7*b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sq
rt(a + b*x**2 + c*x**4)))/(2048*c**(9/2))
```

Mathematica [A] time = 0.196005, size = 194, normalized size = 0.95

$$\frac{15\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)-2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(-16bc^2\left(-81a^2+18acx^4+104c^2x^8\right)\right.}{30720c^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(105*b^5 - 70*b^4*c*x^2 - 48*
b^2*c^2*x^2*(-9*a + c*x^4) + 8*b^3*c*(-95*a + 7*c*x^4) - 160*c^3*
x^2*(3*a^2 + 14*a*c*x^4 + 8*c^2*x^8) - 16*b*c^2*(-81*a^2 + 18*a*c
*x^4 + 104*c^2*x^8)) + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c)*Log[b +
 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(30720*c^(9/2))
```

Maple [B] time = 0.031, size = 432, normalized size = 2.1

$$\frac{b^2x^6}{320\,c}\sqrt{cx^4+bx^2+a}-\frac{7\,b^3x^4}{1920\,c^2}\sqrt{cx^4+bx^2+a}+\frac{7\,b^4x^2}{1536\,c^3}\sqrt{cx^4+bx^2+a}+\frac{7\,b^6}{2048}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{9}{2}}+\frac{19\,ab^3}{384\,c^3}\sqrt{cx^4+bx^2+a}+\frac{cx^{10}}{12}\sqrt{cx^4+bx^2+a}+\frac{13\,bx^8}{120}\sqrt{cx^4+bx^2+a}-\frac{7\,b^5}{1024\,c^4}\sqrt{cx^4+bx^2+a}+\frac{7\,ax^6}{48}\sqrt{cx^4+bx^2+a}+\frac{a^2x^2}{32\,c}\sqrt{cx^4+bx^2+a}-\frac{27\,a^2b}{320\,c^2}\sqrt{cx^4+bx^2+a}+\frac{9\,a^2b^2}{128}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{15\,b^4a}{512}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}-\frac{a^3}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{3\,abx^4}{160\,c}\sqrt{cx^4+bx^2+a}-\frac{9\,ab^2x^2}{320\,c^2}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/320/c*b^2*x^6*(c*x^4+b*x^2+a)^(1/2)-7/1920/c^2*b^3*x^4*(c*x^4+b
*x^2+a)^(1/2)+7/1536/c^3*b^4*x^2*(c*x^4+b*x^2+a)^(1/2)+7/2048/c^(
9/2)*b^6*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+19/384/c
^3*b^3*a*(c*x^4+b*x^2+a)^(1/2)+1/12*c*x^10*(c*x^4+b*x^2+a)^(1/2)+
13/120*b*x^8*(c*x^4+b*x^2+a)^(1/2)-7/1024/c^4*b^5*(c*x^4+b*x^2+a)
^(1/2)+7/48*a*x^6*(c*x^4+b*x^2+a)^(1/2)+1/32*a^2*x^2/c*(c*x^4+b*x
^2+a)^(1/2)-27/320*a^2*b/c^2*(c*x^4+b*x^2+a)^(1/2)+9/128*a^2*b^2/
c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-15/512/c^
(7/2)*b^4*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/32*
a^3/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/160
/c*b*a*x^4*(c*x^4+b*x^2+a)^(1/2)-9/320/c^2*b^2*a*x^2*(c*x^4+b*x^2
+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.309279, size = 1, normalized size = 0.

$$\left[\frac{4\left(1280\,c^5x^{10} + 1664\,bc^4x^8 + 16\left(3\,b^2c^3 + 140\,ac^4\right)x^6 - 105\,b^5 + 760\,ab^3c - 1296\,a^2bc^2 - 8\left(7\,b^3c^2 - 36\,abc^3\right)x^4 + 2\left(35\,b^4c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5,x, algorithm="fricas")
```

```
[Out] [1/61440*(4*(1280*c^5*x^10 + 1664*b*c^4*x^8 + 16*(3*b^2*c^3 + 140
*a*c^4)*x^6 - 105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2 - 8*(7*b^3*c
^2 - 36*a*b*c^3)*x^4 + 2*(35*b^4*c - 216*a*b^2*c^2 + 240*a^2*c^3)
*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) - 15*(7*b^6 - 60*a*b^4*c +
144*a^2*b^2*c^2 - 64*a^3*c^3)*log(4*sqrt(c*x^4 + b*x^2 + a)*(2*c^
2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^
(9/2), 1/30720*(2*(1280*c^5*x^10 + 1664*b*c^4*x^8 + 16*(3*b^2*c^3
 + 140*a*c^4)*x^6 - 105*b^5 + 760*a*b^3*c - 1296*a^2*b*c^2 - 8*(7
*b^3*c^2 - 36*a*b*c^3)*x^4 + 2*(35*b^4*c - 216*a*b^2*c^2 + 240*a^
2*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) + 15*(7*b^6 - 60*a*b
^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*arctan(1/2*(2*c*x^2 + b)*sqr
t(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^5\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.305323, size = 311, normalized size = 1.52

$$\frac{1}{15360}\sqrt{cx^4 + bx^2 + a}\left(2\left(4\left(2\left(8\left(10\,cx^2 + 13\,b\right)x^2 + \frac{3\,b^2c^8 + 140\,ac^9}{c^9}\right)x^2 - \frac{7\,b^3c^7 - 36\,abc^8}{c^9}\right)x^2 + \frac{35\,b^4c^6 - 216\,ab^2c^7 + 24}{c^9}\right.\right.$$
$$-\frac{\left(7\,b^6c^5 - 60\,ab^4c^6 + 144\,a^2b^2c^7 - 64\,a^3c^8\right)\ln\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{2048\,c^{\frac{19}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^5,x, algorithm="giac")
```

```
[Out] 1/15360*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*(10*c*x^2 + 13*b)*x^2
 + (3*b^2*c^8 + 140*a*c^9)/c^9)*x^2 - (7*b^3*c^7 - 36*a*b*c^8)/c^
9)*x^2 + (35*b^4*c^6 - 216*a*b^2*c^7 + 240*a^2*c^8)/c^9)*x^2 - (1
05*b^5*c^5 - 760*a*b^3*c^6 + 1296*a^2*b*c^7)/c^9) - 1/2048*(7*b^6
*c^5 - 60*a*b^4*c^6 + 144*a^2*b^2*c^7 - 64*a^3*c^8)*ln(abs(-2*(sq
rt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(19/2)
```

3.938 $\int x^3 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=150

$$-\frac{3b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}+\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}$$

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^
3) - (b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a +
b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2
*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2))
```

Rubi [A] time = 0.259659, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{3b\left(b^2-4ac\right)^2 \tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}}+\frac{3b\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{32c^2}+\frac{\left(a+bx^2+cx^4\right)^{5/2}}{10c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (3*b*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*c^
3) - (b*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*c^2) + (a +
b*x^2 + c*x^4)^(5/2)/(10*c) - (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2
*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(512*c^(7/2))
```

Rubi in Sympy [A] time = 21.5849, size = 141, normalized size = 0.94

$$-\frac{b\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{32c^2}+\frac{3b\left(b+2cx^2\right)\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}{256c^3}-\frac{3b\left(-4ac+b^2\right)^2 \operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{\frac{7}{2}}}+\frac{\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{10c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -b*(b + 2*c*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(32*c**2) + 3*b*(b
 + 2*c*x**2)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/(256*c**3)
 - 3*b*(-4*a*c + b**2)**2*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a
+ b*x**2 + c*x**4)))/(512*c**(7/2)) + (a + b*x**2 + c*x**4)**(5/2
)/(10*c)
```

Mathematica [A] time = 0.14079, size = 142, normalized size = 0.95

$$\frac{2\sqrt{c}\sqrt{a+bx^2+cx^4}\left(4b^2c\left(2cx^4-25a\right)+8bc^2x^2\left(7a+22cx^4\right)+128c^2\left(a+cx^4\right)^2+15b^4-10b^3cx^2\right)-15b\left(b^2-4ac\right)^2\log}{2560c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^4 - 10*b^3*c*x^2 + 128*c
^2*(a + c*x^4)^2 + 4*b^2*c*(-25*a + 2*c*x^4) + 8*b*c^2*x^2*(7*a +
 22*c*x^4)) - 15*b*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sq
rt[a + b*x^2 + c*x^4]])/(2560*c^(7/2))
```

Maple [B] time = 0.025, size = 316, normalized size = 2.1

$$\frac{a^2}{10\,c}\sqrt{cx^4+bx^2+a}-\frac{3\,a^2b}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{cx^8}{10}\sqrt{cx^4+bx^2+a}+\frac{11\,bx^6}{80}\sqrt{cx^4+bx^2+a}+\frac{b^2x^4}{160\,c}\sqrt{cx^4+bx^2+a}-\frac{b^3x^2}{128\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^4}{256\,c^3}\sqrt{cx^4+bx^2+a}-\frac{3\,b^5}{512}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}+\frac{3\,ab^3}{64}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{5\,ab^2}{64\,c^2}\sqrt{cx^4+bx^2+a}+\frac{7\,abx^2}{160\,c}\sqrt{cx^4+bx^2+a}+\frac{ax^4}{5}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)-3/32*a^2*b/c^(3/2)*ln((1/2*b+c*x
^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10*c*x^8*(c*x^4+b*x^2+a)^(1/
```

```
2)+11/80*b*x^6*(c*x^4+b*x^2+a)^(1/2)+1/160/c*b^2*x^4*(c*x^4+b*x^2
+a)^(1/2)-1/128/c^2*b^3*x^2*(c*x^4+b*x^2+a)^(1/2)+3/256/c^3*b^4*(
c*x^4+b*x^2+a)^(1/2)-3/512/c^(7/2)*b^5*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))+3/64/c^(5/2)*b^3*a*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))-5/64/c^2*b^2*a*(c*x^4+b*x^2+a)^(1/2)+7/16
0/c*b*a*x^2*(c*x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.304455, size = 1, normalized size = 0.01

$$\left[\frac{4\left(128\,c^4x^8 + 176\,bc^3x^6 + 8\left(b^2c^2 + 32\,ac^3\right)x^4 + 15\,b^4 - 100\,ab^2c + 128\,a^2c^2 - 2\left(5\,b^3c - 28\,abc^2\right)x^2\right)\sqrt{cx^4 + bx^2 + a}\sqrt{c} + 1}{5120\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^3,x, algorithm="fricas")
```

```
[Out] [1/5120*(4*(128*c^4*x^8 + 176*b*c^3*x^6 + 8*(b^2*c^2 + 32*a*c^3)*
x^4 + 15*b^4 - 100*a*b^2*c + 128*a^2*c^2 - 2*(5*b^3*c - 28*a*b*c^
2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + 15*(b^5 - 8*a*b^3*c + 1
6*a^2*b*c^2)*log(4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8
*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^(7/2), 1/2560*(2*
(128*c^4*x^8 + 176*b*c^3*x^6 + 8*(b^2*c^2 + 32*a*c^3)*x^4 + 15*b^
4 - 100*a*b^2*c + 128*a^2*c^2 - 2*(5*b^3*c - 28*a*b*c^2)*x^2)*sqr
t(c*x^4 + b*x^2 + a)*sqrt(-c) - 15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^
2)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c))
)/(sqrt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.288637, size = 232, normalized size = 1.55

$$\frac{1}{1280}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\left(8\,cx^2+11\,b\right)x^2+\frac{b^2c^3+32\,ac^4}{c^4}\right)x^2-\frac{5\,b^3c^2-28\,abc^3}{c^4}\right)x^2+\frac{15\,b^4c-100\,ab^2c^2+128\,a^2c^3}{c^4}\right)+\frac{3\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{512\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^3,x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*(8*c*x^2 + 11*b)*x^2 + (b
^2*c^3 + 32*a*c^4)/c^4)*x^2 - (5*b^3*c^2 - 28*a*b*c^3)/c^4)*x^2 +
 (15*b^4*c - 100*a*b^2*c^2 + 128*a^2*c^3)/c^4) + 3/512*(b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))*sqrt(c) - b))/c^(7/2)
```

3.939 $\int x \left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2
) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 -
4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]
)])/(256*c^(5/2))
```

Rubi [A] time = 0.181564, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}}-\frac{3\left(b^2-4ac\right)\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16c}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-3*(b^2 - 4*a*c)*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*c^2
) + ((b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*c) + (3*(b^2 -
4*a*c)^2*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]
)])/(256*c^(5/2))
```

Rubi in Sympy [A] time = 14.045, size = 116, normalized size = 0.94

$$\frac{\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{16c}-\frac{3\left(b+2cx^2\right)\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}{128c^2}+\frac{3\left(-4ac+b^2\right)^2\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (b + 2*c*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(16*c) - 3*(b + 2*c*x
**2)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/(128*c**2) + 3*(-4
*a*c + b**2)**2*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 +
 c*x**4)))/(256*c**(5/2))
```

Mathematica [A] time = 0.102101, size = 111, normalized size = 0.9

$$\frac{2\sqrt{c}\left(b+2cx^2\right)\sqrt{a+bx^2+cx^4}\left(4c\left(5a+2cx^4\right)-3b^2+8bcx^2\right)+3\left(b^2-4ac\right)^2\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{256c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2 + 8*b*c*
x^2 + 4*c*(5*a + 2*c*x^4)) + 3*(b^2 - 4*a*c)^2*Log[b + 2*c*x^2 +
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(256*c^(5/2))
```

Maple [B] time = 0.019, size = 242, normalized size = 2.

$$\frac{3\,a^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}+\frac{cx^6}{8}\sqrt{cx^4+bx^2+a}+\frac{3\,bx^4}{16}\sqrt{cx^4+bx^2+a}+\frac{b^2x^2}{64\,c}\sqrt{cx^4+bx^2+a}-\frac{3\,b^3}{128\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^4}{256}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{3\,ab^2}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}+\frac{5\,ab}{32\,c}\sqrt{cx^4+bx^2+a}+\frac{5\,ax^2}{16}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 3/16*a^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+
1/8*c*x^6*(c*x^4+b*x^2+a)^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+
1/64/c*b^2*x^2*(c*x^4+b*x^2+a)^(1/2)-3/128/c^2*b^3*(c*x^4+b*x^2+a
)^(1/2)+3/256/c^(5/2)*b^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a
)^(1/2))-3/32/c^(3/2)*b^2*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))+5/32/c*b*a*(c*x^4+b*x^2+a)^(1/2)+5/16*a*x^2*(c*x^4+b*x
^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.299633, size = 1, normalized size = 0.01

$$\left[\frac{4\left(16\,c^{3}x^{6} + 24\,bc^{2}x^{4} - 3\,b^{3} + 20\,abc + 2\left(b^{2}c + 20\,ac^{2}\right)x^{2}\right)\sqrt{cx^{4} + bx^{2} + a}\sqrt{c} + 3\left(b^{4} - 8\,ab^{2}c + 16\,a^{2}c^{2}\right)\log\left(-4\sqrt{cx^{4} + bx\right.}{512\,c^{\frac{5}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] [1/512*(4*(16*c^3*x^6 + 24*b*c^2*x^4 - 3*b^3 + 20*a*b*c + 2*(b^2*
c + 20*a*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + 3*(b^4 - 8*a
*b^2*c + 16*a^2*c^2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 +
b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^(5/2), 1
/256*(2*(16*c^3*x^6 + 24*b*c^2*x^4 - 3*b^3 + 20*a*b*c + 2*(b^2*c
+ 20*a*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) + 3*(b^4 - 8*a*
b^2*c + 16*a^2*c^2)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4
 + b*x^2 + a)*c)))/(sqrt(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.290358, size = 182, normalized size = 1.47

$$\frac{1}{128}\sqrt{cx^4+bx^2+a}\left(2\left(4\left(2\,cx^2+3\,b\right)x^2+\frac{b^2c^2+20\,ac^3}{c^3}\right)x^2-\frac{3\,b^3c-20\,abc^2}{c^3}\right)-\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{256\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(c*x^4 + b*x^2 + a)*(2*(4*(2*c*x^2 + 3*b)*x^2 + (b^2*c^
2 + 20*a*c^3)/c^3)*x^2 - (3*b^3*c - 20*a*b*c^2)/c^3) - 3/256*(b^4
 - 8*a*b^2*c + 16*a^2*c^2)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))*sqrt(c) - b))/c^(5/2)
```

3.940 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=155

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2}$$

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a +
 b*x^2 + c*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2))
```

Rubi [A] time = 0.462733, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{1}{2}a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)-\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}}+\frac{\left(8ac+b^2+2bcx^2\right)\sqrt{a+bx^2+cx^4}}{16c}+\frac{1}{6}\left(a+bx^2+cx^4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c) + (a +
 b*x^2 + c*x^4)^(3/2)/6 - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[
a]*Sqrt[a + b*x^2 + c*x^4])])/2 - (b*(b^2 - 12*a*c)*ArcTanh[(b +
2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2))
```

Rubi in Sympy [A] time = 36.3782, size = 139, normalized size = 0.9

$$-\frac{a^{\frac{3}{2}}\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2}-\frac{b\left(-12ac+b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{3}{2}}}+\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6}+\frac{\sqrt{a+bx^2+cx^4}\left(4ac+\frac{b^2}{2}+bcx^2\right)}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

```
[Out] -a**(3/2)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**
4)))/2 - b*(-12*a*c + b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(
a + b*x**2 + c*x**4)))/(32*c**(3/2)) + (a + b*x**2 + c*x**4)**(3/
2)/6 + sqrt(a + b*x**2 + c*x**4)*(4*a*c + b**2/2 + b*c*x**2)/(8*c
)
```

Mathematica [A] time = 0.622195, size = 150, normalized size = 0.97

$$\frac{1}{96}\left(-48a^{3/2}\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)+48a^{3/2}\log\left(x^2\right)-\frac{3\left(b^3-12abc\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{c^{3/2}}+\frac{2\sqrt{a+bx^2+cx^4}\left(8c\left(4a+cx^4\right)+3b^2+14bcx^2\right)}{c}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x,x]
```

```
[Out] ((2*Sqrt[a + b*x^2 + c*x^4]*(3*b^2 + 14*b*c*x^2 + 8*c*(4*a + c*x^
4)))/c + 48*a^(3/2)*Log[x^2] - 48*a^(3/2)*Log[2*a + b*x^2 + 2*Sqr
t[a]*Sqrt[a + b*x^2 + c*x^4]] - (3*(b^3 - 12*a*b*c)*Log[b + 2*c*x
^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/c^(3/2))/96
```

Maple [A] time = 0.023, size = 192, normalized size = 1.2

$$\frac{b^2}{16\,c}\sqrt{cx^4+bx^2+a}-\frac{b^3}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}-\frac{1}{2}a^{\frac{3}{2}}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)+\frac{cx^4}{6}\sqrt{cx^4+bx^2+a}+\frac{7\,bx^2}{24}\sqrt{cx^4+bx^2+a}+\frac{3\,ab}{8}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}+\frac{2\,a}{3}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x,x)
```

```
[Out] 1/16*b^2/c*(c*x^4+b*x^2+a)^(1/2)-1/32*b^3/c^(3/2)*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2*a^(3/2)*ln((2*a+b*x^2+2*a^(1
```

```
/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7
/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+3/8/c^(1/2)*b*a*ln((1/2*b+c*x^2)/
c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+2/3*a*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.417914, size = 1, normalized size = 0.01

$$\left[\frac{48\,a^{\frac{3}{2}}c^{\frac{3}{2}}\log\left(-\frac{(b^2+4\,ac)\,x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\,\sqrt{a}+8\,a^2}{x^4}\right)+4\left(8\,c^2x^4+14\,bcx^2+3\,b^2+32\,ac\right)\sqrt{cx^4+bx^2+a}\sqrt{c}-3\,(b^3}{192\,c^{\frac{3}{2}}}\right.$$

$$-\frac{96\,\sqrt{-a}ac^{\frac{3}{2}}\arctan\left(\frac{bx^2+2\,a}{2\,\sqrt{cx^4+bx^2+a}\sqrt{-a}}\right)-4\left(8\,c^2x^4+14\,bcx^2+3\,b^2+32\,ac\right)\sqrt{cx^4+bx^2+a}\sqrt{c}+3\left(b^3-12\,abc\right)\log\left(-4\,\sqrt{cx}\right.}{192\,c^{\frac{3}{2}}}$$

$$-\frac{48\,\sqrt{-a}a\sqrt{-c}c\arctan\left(\frac{bx^2+2\,a}{2\,\sqrt{cx^4+bx^2+a}\sqrt{-a}}\right)-2\left(8\,c^2x^4+14\,bcx^2+3\,b^2+32\,ac\right)\sqrt{cx^4+bx^2+a}\sqrt{-c}+3\left(b^3-12\,abc\right)\arctan}{96\,\sqrt{-c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/192*(48*a^(3/2)*c^(3/2)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 -
4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4
*(8*c^2*x^4 + 14*b*c*x^2 + 3*b^2 + 32*a*c)*sqrt(c*x^4 + b*x^2 + a
)*sqrt(c) - 3*(b^3 - 12*a*b*c)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*
c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/
c^(3/2), 1/96*(24*a^(3/2)*sqrt(-c)*c*log(-((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2
)/x^4) + 2*(8*c^2*x^4 + 14*b*c*x^2 + 3*b^2 + 32*a*c)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(-c) - 3*(b^3 - 12*a*b*c)*arctan(1/2*(2*c*x^2 + b
)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c), -1/192*(96
*sqrt(-a)*a*c^(3/2)*arctan(1/2*(b*x^2 + 2*a)/(sqrt(c*x^4 + b*x^2
```

```
+ a)*sqrt(-a))) - 4*(8*c^2*x^4 + 14*b*c*x^2 + 3*b^2 + 32*a*c)*sqr
t(c*x^4 + b*x^2 + a)*sqrt(c) + 3*(b^3 - 12*a*b*c)*log(-4*sqrt(c*x
^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2
+ 4*a*c)*sqrt(c)))/c^(3/2), -1/96*(48*sqrt(-a)*a*sqrt(-c)*c*arcta
n(1/2*(b*x^2 + 2*a)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))) - 2*(8*c^
2*x^4 + 14*b*c*x^2 + 3*b^2 + 32*a*c)*sqrt(c*x^4 + b*x^2 + a)*sqrt
(-c) + 3*(b^3 - 12*a*b*c)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt
(c*x^4 + b*x^2 + a)*c)))/(sqrt(-c)*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x, x)
```

3.941 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=150

$$\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{(a+bx^2+cx^4)^{3/2}}{2x^2}+\frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)$$

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^
4)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c])
```

Rubi [A] time = 0.449783, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}-\frac{(a+bx^2+cx^4)^{3/2}}{2x^2}+\frac{3}{8}(3b+2cx^2)\sqrt{a+bx^2+cx^4}-\frac{3}{4}\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/8 - (a + b*x^2 + c*x^
4)^(3/2)/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/4 + (3*(b^2 + 4*a*c)*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c])
```

Rubi in Sympy [A] time = 35.5785, size = 139, normalized size = 0.93

$$-\frac{3\sqrt{a}b\,\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4}+\frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8}-\frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{2x^2}+\frac{3(4ac+b^2)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*sqrt(a)*b*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*
x**4)))/4 + 3*(3*b + 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/8 - (a +
 b*x**2 + c*x**4)**(3/2)/(2*x**2) + 3*(4*a*c + b**2)*atanh((b + 2
*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(16*sqrt(c))
```

Mathematica [A] time = 0.524087, size = 146, normalized size = 0.97

$$\frac{3}{16}\left(\frac{(4ac+b^2)\log\left(2\sqrt{c}\sqrt{a+x^2(b+cx^2)}+b+2cx^2\right)}{\sqrt{c}}-4\sqrt{a}b\log\left(2\sqrt{a}\sqrt{a+x^2(b+cx^2)}+2a+bx^2\right)+4\sqrt{a}b\log\left(x^2\right)\right)+\sqrt{a+bx^2+cx^4}\left(-\frac{a}{2x^2}+\frac{5b}{8}+\frac{cx^2}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^3,x]
```

```
[Out] ((5*b)/8 - a/(2*x^2) + (c*x^2)/4)*Sqrt[a + b*x^2 + c*x^4] + (3*(4
*Sqrt[a]*b*Log[x^2] - 4*Sqrt[a]*b*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqr
t[a + x^2*(b + c*x^2)]] + ((b^2 + 4*a*c)*Log[b + 2*c*x^2 + 2*Sqrt
[c]*Sqrt[a + x^2*(b + c*x^2)]])/Sqrt[c]))/16
```

Maple [A] time = 0.025, size = 170, normalized size = 1.1

$$\frac{3\,b^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}-\frac{a}{2\,x^2}\sqrt{cx^4+bx^2+a}-\frac{3\,b}{4}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)+\frac{cx^2}{4}\sqrt{cx^4+bx^2+a}+\frac{5\,b}{8}\sqrt{cx^4+bx^2+a}+\frac{3\,a}{4}\sqrt{c}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^3,x)
```

```
[Out] 3/16*b^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-
1/2*a/x^2*(c*x^4+b*x^2+a)^(1/2)-3/4*a^(1/2)*b*ln((2*a+b*x^2+2*a^(
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+
5/8*b*(c*x^4+b*x^2+a)^(1/2)+3/4*c^(1/2)*a*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.36825, size = 1, normalized size = 0.01

$$\left[\frac{12\sqrt{a}b\sqrt{c}x^2\log\left(-\frac{(b^2+4ac)x^4+8abx^2-4\sqrt{cx^4+bx^2+a}(bx^2+2a)\sqrt{a}+8a^2}{x^4}\right)+3\left(b^2+4ac\right)x^2\log\left(-4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\right.}{32\sqrt{c}x^2}\right.$$

$$-\frac{24\sqrt{-a}b\sqrt{c}x^2\arctan\left(\frac{bx^2+2a}{2\sqrt{cx^4+bx^2+a}\sqrt{-a}}\right)-3\left(b^2+4ac\right)x^2\log\left(-4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\left(8c^2x^4+8bcx^2+b^2+4\right.\right.}{32\sqrt{c}x^2}$$

$$-\frac{12\sqrt{-a}b\sqrt{-c}x^2\arctan\left(\frac{bx^2+2a}{2\sqrt{cx^4+bx^2+a}\sqrt{-a}}\right)-3\left(b^2+4ac\right)x^2\arctan\left(\frac{(2cx^2+b)\sqrt{-c}}{2\sqrt{cx^4+bx^2+ac}}\right)-2\left(2cx^4+5bx^2-4a\right)\sqrt{cx^4+bx^2+}}{16\sqrt{-c}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/32*(12*sqrt(a)*b*sqrt(c)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4
) + 3*(b^2 + 4*a*c)*x^2*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2
 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)) + 4*(2*c
*x^4 + 5*b*x^2 - 4*a)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c))/(sqrt(c)*x
^2), 1/16*(6*sqrt(a)*b*sqrt(-c)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a
*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)
/x^4) + 3*(b^2 + 4*a*c)*x^2*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sq
rt(c*x^4 + b*x^2 + a)*c)) + 2*(2*c*x^4 + 5*b*x^2 - 4*a)*sqrt(c*x^
```

```
4 + b*x^2 + a)*sqrt(-c))/(sqrt(-c)*x^2), -1/32*(24*sqrt(-a)*b*sqr
t(c)*x^2*arctan(1/2*(b*x^2 + 2*a)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-
a))) - 3*(b^2 + 4*a*c)*x^2*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*
x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)) - 4*(
2*c*x^4 + 5*b*x^2 - 4*a)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c))/(sqrt(c
)*x^2), -1/16*(12*sqrt(-a)*b*sqrt(-c)*x^2*arctan(1/2*(b*x^2 + 2*a
)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))) - 3*(b^2 + 4*a*c)*x^2*arcta
n(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)) - 2*(2*
c*x^4 + 5*b*x^2 - 4*a)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))/(sqrt(-c
)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**3, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^3, x)
```

3.942 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=151

$$-\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{(a+bx^2+cx^4)^{3/2}}{4x^4}-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 +
 c*x^4)^(3/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2
*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/4
```

Rubi [A] time = 0.428448, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$-\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}-\frac{(a+bx^2+cx^4)^{3/2}}{4x^4}-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}+\frac{3}{4}b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (-3*(b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*x^2) - (a + b*x^2 +
 c*x^4)^(3/2)/(4*x^4) - (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x^2)/(2
*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[a]) + (3*b*Sqrt[c]*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/4
```

Rubi in Sympy [A] time = 35.9646, size = 141, normalized size = 0.93

$$\frac{3b\sqrt{c}\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4}-\frac{3(b-2cx^2)\sqrt{a+bx^2+cx^4}}{8x^2}-\frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{4x^4}-\frac{3(4ac+b^2)\,\mathrm{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)
```

```
[Out] 3*b*sqrt(c)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x
**4)))/4 - 3*(b - 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(8*x**2) -
(a + b*x**2 + c*x**4)**(3/2)/(4*x**4) - 3*(4*a*c + b**2)*atanh((2
*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(16*sqrt(a))
```

Mathematica [A] time = 0.631744, size = 153, normalized size = 1.01

$$\frac{3\left(\log\left(x^2\right)\left(4ac+b^2\right)-\left(4ac+b^2\right)\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)+4\sqrt{a}b\sqrt{c}\log\left(2\sqrt{c}\sqrt{a+x^2\left(b+cx^2\right)}+b+2cx^2\right.}{16\sqrt{a}}$$
$$+\sqrt{a+bx^2+cx^4}\left(-\frac{a}{4x^4}-\frac{5b}{8x^2}+\frac{c}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^5,x]
```

```
[Out] (c/2 - a/(4*x^4) - (5*b)/(8*x^2))*Sqrt[a + b*x^2 + c*x^4] + (3*((
b^2 + 4*a*c)*Log[x^2] - (b^2 + 4*a*c)*Log[2*a + b*x^2 + 2*Sqrt[a]
*Sqrt[a + x^2*(b + c*x^2)]] + 4*Sqrt[a]*b*Sqrt[c]*Log[b + 2*c*x^2
 + 2*Sqrt[c]*Sqrt[a + x^2*(b + c*x^2)]]))/(16*Sqrt[a])
```

Maple [A] time = 0.025, size = 174, normalized size = 1.2

$$\frac{c}{2}\sqrt{cx^4+bx^2+a}+\frac{3b}{4}\sqrt{c}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)-\frac{a}{4x^4}\sqrt{cx^4+bx^2+a}$$
$$-\frac{5b}{8x^2}\sqrt{cx^4+bx^2+a}-\frac{3b^2}{16}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$
$$-\frac{3c}{4}\sqrt{a}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^5,x)
```

```
[Out] 1/2*c*(c*x^4+b*x^2+a)^(1/2)+3/4*c^(1/2)*b*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/x^4*(c*x^4+b*x^2+a)^(1/2)-5/8*b/x^
2*(c*x^4+b*x^2+a)^(1/2)-3/16/a^(1/2)*b^2*ln((2*a+b*x^2+2*a^(1/2)*
```

```
(c*x^4+b*x^2+a)^(1/2))/x^2)-3/4*a^(1/2)*c*ln((2*a+b*x^2+2*a^(1/2)
*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.354103, size = 1, normalized size = 0.01

$$\left[\frac{12\sqrt{a}b\sqrt{c}x^4\log\left(-8\,c^2x^4-8\,bcx^2-b^2-4\,\sqrt{cx^4+bx^2+a}\left(2\,cx^2+b\right)\sqrt{c}-4\,ac\right)+3\left(b^2+4\,ac\right)x^4\log\left(\frac{4\,\sqrt{cx^4+bx^2+a}\left(abx^2+\cdots\right)}{\cdots}\right)}{32\,\sqrt{a}x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^5,x, algorithm="fricas")

[Out] [1/32*(12*sqrt(a)*b*sqrt(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2
- 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 3*(b
^2 + 4*a*c)*x^4*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2)
- ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) + 4*(4*c*
x^4 - 5*b*x^2 - 2*a)*sqrt(c*x^4 + b*x^2 + a)*sqrt(a))/(sqrt(a)*x^
4), 1/32*(24*sqrt(a)*b*sqrt(-c)*x^4*arctan(1/2*(2*c*x^2 + b)/(sqr
t(c*x^4 + b*x^2 + a)*sqrt(-c))) + 3*(b^2 + 4*a*c)*x^4*log((4*sqrt
(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*
b*x^2 + 8*a^2)*sqrt(a))/x^4) + 4*(4*c*x^4 - 5*b*x^2 - 2*a)*sqrt(c
*x^4 + b*x^2 + a)*sqrt(a))/(sqrt(a)*x^4), 1/16*(6*sqrt(-a)*b*sqrt
(c)*x^4*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^2 + 4*a*c)*x^4*arctan(1
/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) + 2*(4*c*x
^4 - 5*b*x^2 - 2*a)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*x
^4), 1/16*(12*sqrt(-a)*b*sqrt(-c)*x^4*arctan(1/2*(2*c*x^2 + b)/(s
qrt(c*x^4 + b*x^2 + a)*sqrt(-c))) - 3*(b^2 + 4*a*c)*x^4*arctan(1/
2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) + 2*(4*c*x^
4 - 5*b*x^2 - 2*a)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*x^
4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**5,x)

[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**5, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^5,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^5, x)
```

3.943 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=163

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{\left(x^2\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6}$$

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a*x^4)
 - (a + b*x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[
(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2))
+ (c^(3/2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/2
```

Rubi [A] time = 0.463887, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{b\left(b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{3/2}}-\frac{\left(x^2\left(8ac+b^2\right)+2ab\right)\sqrt{a+bx^2+cx^4}}{16ax^4}+\frac{1}{2}c^{3/2}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] -((2*a*b + (b^2 + 8*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*a*x^4)
 - (a + b*x^2 + c*x^4)^(3/2)/(6*x^6) + (b*(b^2 - 12*a*c)*ArcTanh[
(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(3/2))
+ (c^(3/2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/2
```

Rubi in Sympy [A] time = 36.1987, size = 146, normalized size = 0.9

$$\frac{c^{\frac{3}{2}}\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2}-\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{6x^6}-\frac{\left(ab+x^2\left(4ac+\frac{b^2}{2}\right)\right)\sqrt{a+bx^2+cx^4}}{8ax^4}+\frac{b\left(-12ac+b^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] c**(3/2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4
)))/2 - (a + b*x**2 + c*x**4)**(3/2)/(6*x**6) - (a*b + x**2*(4*a*
c + b**2/2))*sqrt(a + b*x**2 + c*x**4)/(8*a*x**4) + b*(-12*a*c +
b**2)*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))
/(32*a**(3/2))
```

Mathematica [A] time = 0.597306, size = 169, normalized size = 1.04

$$\frac{16a^{3/2}c^{3/2}\log\left(2\sqrt{c}\sqrt{a+x^2\left(b+cx^2\right)}+b+2cx^2\right)-\log\left(x^2\right)\left(b^3-12abc\right)+\left(b^3-12abc\right)\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+\right.}{32a^{3/2}}$$
$$+\sqrt{a+bx^2+cx^4}\left(\frac{-32ac-3b^2}{48ax^2}-\frac{a}{6x^6}-\frac{7b}{24x^4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^7,x]
```

```
[Out] (-a/(6*x^6) - (7*b)/(24*x^4) + (-3*b^2 - 32*a*c)/(48*a*x^2))*Sqrt
[a + b*x^2 + c*x^4] + (-((b^3 - 12*a*b*c)*Log[x^2]) + (b^3 - 12*a
*b*c)*Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]] + 16
*a^(3/2)*c^(3/2)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + x^2*(b + c*
x^2)]])/(32*a^(3/2))
```

Maple [A] time = 0.025, size = 202, normalized size = 1.2

$$\frac{1}{2}c^{\frac{3}{2}}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)-\frac{a}{6x^6}\sqrt{cx^4+bx^2+a}-\frac{7b}{24x^4}\sqrt{cx^4+bx^2+a}$$
$$-\frac{b^2}{16ax^2}\sqrt{cx^4+bx^2+a}+\frac{b^3}{32}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$
$$-\frac{3bc}{8}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}-\frac{2c}{3x^2}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^7,x)
```

```
[Out] 1/2*c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/6*a
/x^6*(c*x^4+b*x^2+a)^(1/2)-7/24*b/x^4*(c*x^4+b*x^2+a)^(1/2)-1/16/
```

```
a*b^2/x^2*(c*x^4+b*x^2+a)^(1/2)+1/32/a^(3/2)*b^3*ln((2*a+b*x^2+2*
a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/8/a^(1/2)*b*c*ln((2*a+b*x^2
+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-2/3*c/x^2*(c*x^4+b*x^2+a)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^7,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.377694, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^7,x, algorithm="fricas")

[Out] [1/192*(48*a^(3/2)*c^(3/2)*x^6*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 -
 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 3*(b^
3 - 12*a*b*c)*x^6*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2
) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*((3
*b^2 + 32*a*c)*x^4 + 14*a*b*x^2 + 8*a^2)*sqrt(c*x^4 + b*x^2 + a)*
sqrt(a))/(a^(3/2)*x^6), 1/192*(96*a^(3/2)*sqrt(-c)*c*x^6*arctan(1
/2*(2*c*x^2 + b)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) - 3*(b^3 - 1
2*a*b*c)*x^6*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - (
(b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*((3*b^2
+ 32*a*c)*x^4 + 14*a*b*x^2 + 8*a^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(
a))/(a^(3/2)*x^6), 1/96*(24*sqrt(-a)*a*c^(3/2)*x^6*log(-8*c^2*x^4
 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt
(c) - 4*a*c) + 3*(b^3 - 12*a*b*c)*x^6*arctan(1/2*(b*x^2 + 2*a)*sq
rt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*((3*b^2 + 32*a*c)*x^4 + 1
4*a*b*x^2 + 8*a^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a*
x^6), 1/96*(48*sqrt(-a)*a*sqrt(-c)*c*x^6*arctan(1/2*(2*c*x^2 + b)
/(sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))) + 3*(b^3 - 12*a*b*c)*x^6*arc
tan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*(
(3*b^2 + 32*a*c)*x^4 + 14*a*b*x^2 + 8*a^2)*sqrt(c*x^4 + b*x^2 + a
)*sqrt(-a))/(sqrt(-a)*a*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**7,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**7, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^7, x)
```

3.944 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=133

$$-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}+\frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8}$$

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*
x^4) - ((2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*
(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
 c*x^4])])/(256*a^(5/2))
```

Rubi [A] time = 0.259164, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3\left(b^2-4ac\right)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{5/2}}+\frac{3\left(b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{16ax^8}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] (3*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(128*a^2*
x^4) - ((2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(16*a*x^8) - (3*
(b^2 - 4*a*c)^2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 +
 c*x^4])])/(256*a^(5/2))
```

Rubi in Sympy [A] time = 23.7682, size = 122, normalized size = 0.92

$$-\frac{\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{16ax^8}+\frac{3\left(2a+bx^2\right)\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}{128a^2x^4}-\frac{3\left(-4ac+b^2\right)^2\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{256a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)
```

```
[Out] -(2*a + b*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(16*a*x**8) + 3*(2*a
 + b*x**2)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/(128*a**2*x*
*4) - 3*(-4*a*c + b**2)**2*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a
 + b*x**2 + c*x**4)))/(256*a**(5/2))
```

Mathematica [A] time = 0.234631, size = 125, normalized size = 0.94

$$\frac{3\left(b^2-4ac\right)^2\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)-\frac{2\sqrt{a}\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}\left(8a^2+8abx^2+20acx^4-3b^2x^4\right)}{x^8}}{256a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^9,x]
```

```
[Out] ((-2*Sqrt[a]*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 8*a*b
*x^2 - 3*b^2*x^4 + 20*a*c*x^4))/x^8 + 3*(b^2 - 4*a*c)^2*(Log[x^2]
 - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(256*a^
(5/2))
```

Maple [B] time = 0.026, size = 260, normalized size = 2.

$$-\frac{a}{8x^8}\sqrt{cx^4+bx^2+a}-\frac{3b}{16x^6}\sqrt{cx^4+bx^2+a}-\frac{b^2}{64ax^4}\sqrt{cx^4+bx^2+a} +\frac{3b^3}{128a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{3b^4}{256}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}} +\frac{3b^2c}{32}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{5bc}{32ax^2}\sqrt{cx^4+bx^2+a} -\frac{5c}{16x^4}\sqrt{cx^4+bx^2+a}-\frac{3c^2}{16}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^9,x)
```

```
[Out] -1/8*a/x^8*(c*x^4+b*x^2+a)^(1/2)-3/16*b/x^6*(c*x^4+b*x^2+a)^(1/2)
-1/64/a*b^2/x^4*(c*x^4+b*x^2+a)^(1/2)+3/128/a^2*b^3/x^2*(c*x^4+b*
x^2+a)^(1/2)-3/256/a^(5/2)*b^4*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x
^2+a)^(1/2))/x^2)+3/32/a^(3/2)*b^2*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x
^4+b*x^2+a)^(1/2))/x^2)-5/32/a*b*c/x^2*(c*x^4+b*x^2+a)^(1/2)-5/16
*c/x^4*(c*x^4+b*x^2+a)^(1/2)-3/16/a^(1/2)*c^2*ln((2*a+b*x^2+2*a^(
```

```
1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.309997, size = 1, normalized size = 0.01

$$\left[\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)x^8\log\left(\frac{4\,\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)+4\left(\left(3\,b^3-20\,abc\right)x^6-24\,a^2bx^2-2\left(a\right.\right.}{512\,a^{\frac{5}{2}}x^8}\right.$$

$$-\frac{3\left(b^4-8\,ab^2c+16\,a^2c^2\right)x^8\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^2+a}a}\right)-2\left(\left(3\,b^3-20\,abc\right)x^6-24\,a^2bx^2-2\left(ab^2+20\,a^2c\right)x^4-16\,a^3\right)\sqrt{cx^4+}}{256\,\sqrt{-a}a^2x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/512*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*x^8*log((4*sqrt(c*x^4 +
b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8
*a^2)*sqrt(a))/x^4) + 4*((3*b^3 - 20*a*b*c)*x^6 - 24*a^2*b*x^2 -
2*(a*b^2 + 20*a^2*c)*x^4 - 16*a^3)*sqrt(c*x^4 + b*x^2 + a)*sqrt(a
))/(a^(5/2)*x^8), -1/256*(3*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*x^8*ar
ctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*
((3*b^3 - 20*a*b*c)*x^6 - 24*a^2*b*x^2 - 2*(a*b^2 + 20*a^2*c)*x^4
 - 16*a^3)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^2*x^8)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**9,x)

[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**9, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{x^9} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^9,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^9, x)
```

3.945 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^{11}}\,dx$

Optimal. Leaf size=162

$$\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}-\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8}-\frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}}$$

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a
^3*x^4) + (b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8
) - (a + b*x^2 + c*x^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(512*
a^(7/2))
```

Rubi [A] time = 0.327918, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{7/2}}-\frac{3b(b^2-4ac)(2a+bx^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{3/2}}{32a^2x^8}-\frac{(a+bx^2+cx^4)^{5/2}}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] (-3*b*(b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(256*a
^3*x^4) + (b*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(32*a^2*x^8
) - (a + b*x^2 + c*x^4)^(5/2)/(10*a*x^10) + (3*b*(b^2 - 4*a*c)^2*
ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(512*
a^(7/2))
```

Rubi in Sympy [A] time = 30.1936, size = 151, normalized size = 0.93

$$-\frac{(a+bx^2+cx^4)^{\frac{5}{2}}}{10ax^{10}}+\frac{b(2a+bx^2)(a+bx^2+cx^4)^{\frac{3}{2}}}{32a^2x^8}-\frac{3b(2a+bx^2)(-4ac+b^2)\sqrt{a+bx^2+cx^4}}{256a^3x^4}+\frac{3b(-4ac+b^2)^2\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{512a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

```
[Out] -(a + b*x**2 + c*x**4)**(5/2)/(10*a*x**10) + b*(2*a + b*x**2)*(a
+ b*x**2 + c*x**4)**(3/2)/(32*a**2*x**8) - 3*b*(2*a + b*x**2)*(-4
*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/(256*a**3*x**4) + 3*b*(-4*
a*c + b**2)**2*atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 +
c*x**4)))/(512*a**(7/2))
```

Mathematica [A] time = 0.218082, size = 163, normalized size = 1.01

$$-\frac{3b\left(b^2-4ac\right)^2\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{512a^{7/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(128a^4+16a^3\left(11bx^2+16cx^4\right)+8a^2x^4\left(b^2+7bcx^2+16c^2x^4\right)-10ab^2x^6\left(b+10cx^2\right)+15b^4x^8\right)}{1280a^3x^{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^11,x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]*(128*a^4 + 15*b^4*x^8 - 10*a*b^2*x^6*(b
 + 10*c*x^2) + 16*a^3*(11*b*x^2 + 16*c*x^4) + 8*a^2*x^4*(b^2 + 7*
b*c*x^2 + 16*c^2*x^4)))/(1280*a^3*x^10) - (3*b*(b^2 - 4*a*c)^2*(L
og[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/
(512*a^(7/2))
```

Maple [B] time = 0.028, size = 337, normalized size = 2.1

$$-\frac{a}{10\,x^{10}}\sqrt{cx^4+bx^2+a}-\frac{11\,b}{80\,x^8}\sqrt{cx^4+bx^2+a}-\frac{b^2}{160\,ax^6}\sqrt{cx^4+bx^2+a}+\frac{b^3}{128\,a^2x^4}\sqrt{cx^4+bx^2+a}-\frac{3\,b^4}{256\,a^3x^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^5}{512}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{3\,b^3c}{64}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{5\,b^2c}{64\,a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{7\,bc}{160\,ax^4}\sqrt{cx^4+bx^2+a}+\frac{3\,c^2b}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}-\frac{c}{5\,x^6}\sqrt{cx^4+bx^2+a}-\frac{c^2}{10\,ax^2}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^11,x)
```

```
[Out] -1/10*a/x^10*(c*x^4+b*x^2+a)^(1/2)-11/80*b/x^8*(c*x^4+b*x^2+a)^(1
/2)-1/160/a*b^2/x^6*(c*x^4+b*x^2+a)^(1/2)+1/128/a^2*b^3/x^4*(c*x^
4+b*x^2+a)^(1/2)-3/256/a^3*b^4/x^2*(c*x^4+b*x^2+a)^(1/2)+3/512/a^
(7/2)*b^5*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/6
4/a^(5/2)*b^3*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^
2)+5/64/a^2*b^2*c/x^2*(c*x^4+b*x^2+a)^(1/2)-7/160/a*b*c/x^4*(c*x^
4+b*x^2+a)^(1/2)+3/32/a^(3/2)*b*c^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x^
4+b*x^2+a)^(1/2))/x^2)-1/5*c/x^6*(c*x^4+b*x^2+a)^(1/2)-1/10/a*c^2
/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^11,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.324237, size = 1, normalized size = 0.01

$$\left[\frac{15\left(b^5-8\,ab^3c+16\,a^2bc^2\right)x^{10}\log\left(-\frac{4\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)+\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)-4\left(\left(15\,b^4-100\,ab^2c+128\,a^2c^2\right)x\right.}{5120\,a^{\frac{7}{2}}x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^11,x, algorithm="fricas")
```

```
[Out] [1/5120*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^10*log(-(4*sqrt(c*
x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x
^2 + 8*a^2)*sqrt(a))/x^4) - 4*((15*b^4 - 100*a*b^2*c + 128*a^2*c^
2)*x^8 - 2*(5*a*b^3 - 28*a^2*b*c)*x^6 + 176*a^3*b*x^2 + 8*(a^2*b^
2 + 32*a^3*c)*x^4 + 128*a^4)*sqrt(c*x^4 + b*x^2 + a)*sqrt(a))/(a^
(7/2)*x^10), 1/2560*(15*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^10*arc
tan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*(
(15*b^4 - 100*a*b^2*c + 128*a^2*c^2)*x^8 - 2*(5*a*b^3 - 28*a^2*b*
```

```
c)*x^6 + 176*a^3*b*x^2 + 8*(a^2*b^2 + 32*a^3*c)*x^4 + 128*a^4)*sq
rt(c*x^4 + b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^10)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**11,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**11, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^11,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^11, x)
```

3.946 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{x^{13}}\,dx$

Optimal. Leaf size=216

$$-\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}+\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384a^3x^8}+\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120a^2x^{10}}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}}$$

```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x
^4])/(1024*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 +
 c*x^4)^(3/2))/(384*a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^
12) + (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*a^2*x^10) - ((b^2 - 4*
a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rubi [A] time = 0.505013, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{9/2}}+\frac{\left(b^2-4ac\right)\left(7b^2-4ac\right)\left(2a+bx^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{\left(7b^2-4ac\right)\left(2a+bx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{384a^3x^8}+\frac{7b\left(a+bx^2+cx^4\right)^{5/2}}{120a^2x^{10}}-\frac{\left(a+bx^2+cx^4\right)^{5/2}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```


```
[Out] ((b^2 - 4*a*c)*(7*b^2 - 4*a*c)*(2*a + b*x^2)*Sqrt[a + b*x^2 + c*x
^4])/(1024*a^4*x^4) - ((7*b^2 - 4*a*c)*(2*a + b*x^2)*(a + b*x^2 +
 c*x^4)^(3/2))/(384*a^3*x^8) - (a + b*x^2 + c*x^4)^(5/2)/(12*a*x^
12) + (7*b*(a + b*x^2 + c*x^4)^(5/2))/(120*a^2*x^10) - ((b^2 - 4*
a*c)^2*(7*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a +
b*x^2 + c*x^4])])/(2048*a^(9/2))
```

Rubi in Sympy [A] time = 42.0037, size = 201, normalized size = 0.93

$$-\frac{\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{12ax^{12}}+\frac{7b\left(a+bx^2+cx^4\right)^{\frac{5}{2}}}{120a^2x^{10}}-\frac{\left(2a+bx^2\right)\left(-4ac+7b^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{384a^3x^8}+\frac{\left(2a+bx^2\right)\left(-4ac+b^2\right)\left(-4ac+7b^2\right)\sqrt{a+bx^2+cx^4}}{1024a^4x^4}-\frac{\left(-4ac+b^2\right)^2\left(-4ac+7b^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2048a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
```

```
[Out] -(a + b*x**2 + c*x**4)**(5/2)/(12*a*x**12) + 7*b*(a + b*x**2 + c*
x**4)**(5/2)/(120*a**2*x**10) - (2*a + b*x**2)*(-4*a*c + 7*b**2)*
(a + b*x**2 + c*x**4)**(3/2)/(384*a**3*x**8) + (2*a + b*x**2)*(-4
*a*c + b**2)*(-4*a*c + 7*b**2)*sqrt(a + b*x**2 + c*x**4)/(1024*a*
*4*x**4) - (-4*a*c + b**2)**2*(-4*a*c + 7*b**2)*atanh((2*a + b*x*
*2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2048*a**(9/2))
```

Mathematica [A] time = 0.329111, size = 206, normalized size = 0.95

$$\frac{15\left(b^2-4ac\right)^2\left(7b^2-4ac\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)-\frac{2\sqrt{a}\sqrt{a+bx^2+cx^4}\left(1280a^5+64a^4\left(26bx^2+35cx^4\right)+48a^3x}{}}{30720a^{9/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^13,x]
```

```
[Out] ((-2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(1280*a^5 - 105*b^5*x^10 + 1
0*a*b^3*x^8*(7*b + 76*c*x^2) + 64*a^4*(26*b*x^2 + 35*c*x^4) + 48*
a^3*x^4*(b^2 + 6*b*c*x^2 + 10*c^2*x^4) - 8*a^2*b*x^6*(7*b^2 + 54*
b*c*x^2 + 162*c^2*x^4)))/x^12 + 15*(b^2 - 4*a*c)^2*(7*b^2 - 4*a*c
)*(Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]
]))/(30720*a^(9/2))
```

Maple [B] time = 0.036, size = 457, normalized size = 2.1

$$
\begin{aligned}
&-\frac{b^2}{320\,a x^8}\sqrt{c x^4+b x^2+a}+\frac{7\,b^3}{1920\,a^2 x^6}\sqrt{c x^4+b x^2+a}-\frac{7\,b^4}{1536\,a^3 x^4}\sqrt{c x^4+b x^2+a}\\
&+\frac{7\,b^5}{1024\,a^4 x^2}\sqrt{c x^4+b x^2+a}-\frac{7\,b^6}{2048}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{9}{2}}\\
&-\frac{c^2}{32\,a x^4}\sqrt{c x^4+b x^2+a}+\frac{c^3}{32}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{3}{2}}\\
&-\frac{a}{12\,x^{12}}\sqrt{c x^4+b x^2+a}-\frac{13\,b}{120\,x^{10}}\sqrt{c x^4+b x^2+a}-\frac{7\,c}{48\,x^8}\sqrt{c x^4+b x^2+a}\\
&+\frac{15\,b^4 c}{512}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{19\,b^3 c}{384\,a^3 x^2}\sqrt{c x^4+b x^2+a}\\
&+\frac{9\,b^2 c}{320\,a^2 x^4}\sqrt{c x^4+b x^2+a}-\frac{9\,b^2 c^2}{128}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{5}{2}}\\
&-\frac{3\,b c}{160\,a x^6}\sqrt{c x^4+b x^2+a}+\frac{27\,c^2 b}{320\,a^2 x^2}\sqrt{c x^4+b x^2+a}
\end{aligned}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^13,x)
```

```
[Out] -1/320/a*b^2/x^8*(c*x^4+b*x^2+a)^(1/2)+7/1920/a^2*b^3/x^6*(c*x^4+
b*x^2+a)^(1/2)-7/1536/a^3*b^4/x^4*(c*x^4+b*x^2+a)^(1/2)+7/1024/a^
4*b^5/x^2*(c*x^4+b*x^2+a)^(1/2)-7/2048/a^(9/2)*b^6*ln((2*a+b*x^2+
2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-1/32/a*c^2/x^4*(c*x^4+b*x^2
+a)^(1/2)+1/32/a^(3/2)*c^3*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)-1/12*a/x^12*(c*x^4+b*x^2+a)^(1/2)-13/120*b/x^10*(c*
x^4+b*x^2+a)^(1/2)-7/48*c/x^8*(c*x^4+b*x^2+a)^(1/2)+15/512/a^(7/2
)*b^4*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-19/38
4/a^3*b^3*c/x^2*(c*x^4+b*x^2+a)^(1/2)+9/320/a^2*b^2*c/x^4*(c*x^4+
b*x^2+a)^(1/2)-9/128/a^(5/2)*b^2*c^2*ln((2*a+b*x^2+2*a^(1/2)*(c*x
^4+b*x^2+a)^(1/2))/x^2)-3/160/a*b*c/x^6*(c*x^4+b*x^2+a)^(1/2)+27/
320/a^2*b*c^2/x^2*(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.355406, size = 1, normalized size = 0.

$$\left[-\frac{15\left(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)x^{12}\log\left(-\frac{4\,\sqrt{cx^{4}+bx^{2}+a}\left(abx^{2}+2\,a^{2}\right)+\left(\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+8\,a^{2}\right)\sqrt{a}}{x^{4}}\right)-4\left(\left(105\,b^{5}-760\right.\right.}{}\right.$$

$$-\frac{15\left(7\,b^{6}-60\,ab^{4}c+144\,a^{2}b^{2}c^{2}-64\,a^{3}c^{3}\right)x^{12}\arctan\left(\frac{\left(bx^{2}+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^{4}+bx^{2}+a}a}\right)-2\left(\left(105\,b^{5}-760\,ab^{3}c+1296\,a^{2}bc^{2}\right)x^{10}-2\left(35\,ab^{4}\right.\right.}{30720}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^13,x, algorithm="fricas")
```

```
[Out] [-1/61440*(15*(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)
*x^12*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 +
 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*((105*b^5 - 76
0*a*b^3*c + 1296*a^2*b*c^2)*x^10 - 2*(35*a*b^4 - 216*a^2*b^2*c +
240*a^3*c^2)*x^8 - 1664*a^4*b*x^2 + 8*(7*a^2*b^3 - 36*a^3*b*c)*x^
6 - 1280*a^5 - 16*(3*a^3*b^2 + 140*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2
 + a)*sqrt(a))/(a^(9/2)*x^12), -1/30720*(15*(7*b^6 - 60*a*b^4*c +
 144*a^2*b^2*c^2 - 64*a^3*c^3)*x^12*arctan(1/2*(b*x^2 + 2*a)*sqrt
(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*((105*b^5 - 760*a*b^3*c + 1
296*a^2*b*c^2)*x^10 - 2*(35*a*b^4 - 216*a^2*b^2*c + 240*a^3*c^2)*
x^8 - 1664*a^4*b*x^2 + 8*(7*a^2*b^3 - 36*a^3*b*c)*x^6 - 1280*a^5
- 16*(3*a^3*b^2 + 140*a^4*c)*x^4)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a
))/(sqrt(-a)*a^4*x^12)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{x^{13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**13,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**13, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^{13}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^13,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^13, x)
```

3.947 $\int x^4 \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=495

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(2b^2-9ac\right)\left(b^2-3ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2310c^{15/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{8bx\left(2b^2-9ac\right)\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{8\sqrt[4]{a}b\left(2b^2-9ac\right)\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{1155c^{15/4}\sqrt{a+bx^2+cx^4}}-\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{385c^2}+\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}$$

```
[Out] ((8*b^4 - 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(11
55*c^3) - (8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(1155*c^(7/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (x^3*(b*(2*b^2 +
a*c) + 10*c*(b^2 - 3*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(385*c^2)
 + (x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(33*c) + (8*a^(1
/4)*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(1155*c^(15/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(8*b*(2*b^2 - 9*a*c)*(b^2
- 3*a*c) + Sqrt[a]*Sqrt[c]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*
Sqrt[c]))/4])/(2310*c^(15/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 1.08315, antiderivative size = 495, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(2b^2-9ac\right)\left(b^2-3ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2310c^{15/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(60a^2c^2-51ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{1155c^3}-\frac{8bx\left(2b^2-9ac\right)\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{1155c^{7/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{8\sqrt[4]{a}b\left(2b^2-9ac\right)\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{1155c^{15/4}\sqrt{a+bx^2+cx^4}}-\frac{x^3\left(10cx^2\left(b^2-3ac\right)+b\left(ac+2b^2\right)\right)\sqrt{a+bx^2+cx^4}}{385c^2}+\frac{x^3\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{33c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 51*a*b^2*c + 60*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(11
55*c^3) - (8*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(1155*c^(7/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (x^3*(b*(2*b^2 +
a*c) + 10*c*(b^2 - 3*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(385*c^2)
 + (x^3*(b + 3*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(33*c) + (8*a^(1
/4)*b*(2*b^2 - 9*a*c)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(1155*c^(15/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(8*b*(2*b^2 - 9*a*c)*(b^2
- 3*a*c) + Sqrt[a]*Sqrt[c]*(8*b^4 - 51*a*b^2*c + 60*a^2*c^2))*(Sq
rt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*
x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*
Sqrt[c]))/4])/(2310*c^(15/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 105.571, size = 469, normalized size = 0.95

$$\frac{8\sqrt[4]{a}b\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-9ac+2b^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{1155c^{\frac{15}{4}}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}\left(60a^2c^2-51ab^2c+8b^4\right)+8b\left(-9ac+2b^2\right)\left(-3ac+b^2\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2310c^{\frac{15}{4}}\sqrt{a+bx^2+cx^4}} - \frac{8bx\left(-9ac+2b^2\right)\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{1155c^{\frac{7}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x^3\left(3b+9cx^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{99c} - \frac{x^3\left(3b\left(ac+2b^2\right)+30cx^2\left(-3ac+b^2\right)\right)\sqrt{a+bx^2+cx^4}}{1155c^2} + \frac{x\sqrt{a+bx^2+cx^4}\left(60a^2c^2-51ab^2c+8b^4\right)}{1155c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 8*a**(1/4)*b*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)*
*2)*(sqrt(a) + sqrt(c)*x**2)*(-9*a*c + 2*b**2)*(-3*a*c + b**2)*el
liptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c))
)/(1155*c**(15/4)*sqrt(a + b*x**2 + c*x**4)) - a**(1/4)*sqrt((a +
 b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)
*x**2)*(sqrt(a)*sqrt(c)*(60*a**2*c**2 - 51*a*b**2*c + 8*b**4) + 8
*b*(-9*a*c + 2*b**2)*(-3*a*c + b**2))*elliptic_f(2*atan(c**(1/4)*
x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2310*c**(15/4)*sqrt(a
+ b*x**2 + c*x**4)) - 8*b*x*(-9*a*c + 2*b**2)*(-3*a*c + b**2)*sqr
t(a + b*x**2 + c*x**4)/(1155*c**(7/2)*(sqrt(a) + sqrt(c)*x**2)) +
 x**3*(3*b + 9*c*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(99*c) - x**3
```

```
*(3*b*(a*c + 2*b**2) + 30*c*x**2*(-3*a*c + b**2))*sqrt(a + b*x**2
 + c*x**4)/(1155*c**2) + x*sqrt(a + b*x**2 + c*x**4)*(60*a**2*c**
2 - 51*a*b**2*c + 8*b**4)/(1155*c**3)
```

Mathematica [C] time = 4.12576, size = 657, normalized size = 1.33

$$-4ib\left(27a^2c^2-15ab^2c+2b^4\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-}}{b-\sqrt{b^2-}}\right.$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(60*a^3*c^2 + a^2*c*(-51*b
^2 + 92*b*c*x^2 + 255*c^2*x^4) + a*(8*b^4 - 57*b^3*c*x^2 - 14*b^2
*c^2*x^4 + 367*b*c^3*x^6 + 300*c^4*x^8) + x^2*(8*b^5 + 2*b^4*c*x^
2 - b^3*c^2*x^4 + 145*b^2*c^3*x^6 + 245*b*c^4*x^8 + 105*c^5*x^10)
) - (4*I)*b*(2*b^4 - 15*a*b^2*c + 27*a^2*c^2)*(-b + Sqrt[b^2 - 4*
a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-8*b
^6 + 68*a*b^4*c - 159*a^2*b^2*c^2 + 60*a^3*c^3 + 8*b^5*Sqrt[b^2 -
 4*a*c] - 60*a*b^3*c*Sqrt[b^2 - 4*a*c] + 108*a^2*b*c^2*Sqrt[b^2 -
 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2
- 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(2310
*c^4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.017, size = 674, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/11*c*x^9*(c*x^4+b*x^2+a)^(1/2)+4/33*b*x^7*(c*x^4+b*x^2+a)^(1/2)
+1/7*(13/11*a*c+1/33*b^2)/c*x^5*(c*x^4+b*x^2+a)^(1/2)+1/5*(38/33*
a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3
*(a^2-5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+
```

```
1/33*b^2)/c*b)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(a^2-5/7*(13/1
1*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/
c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
-1/2*(-3/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b^2)/c*b)/c*a-2/3*(a^2-
5/7*(13/11*a*c+1/33*b^2)/c*a-4/5*(38/33*a*b-6/7*(13/11*a*c+1/33*b
^2)/c*b)/c*b)/c*b)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^8 + bx^6 + ax^4\right)\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^8 + b*x^6 + a*x^4)*sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^4, x)
```

3.948 $\int x^2 \left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=443

$$\frac{x\left(84a^2c^2-57ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}$$

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(31
5*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (x*(b*(4*b^2 - 9*a*c) + 6*c*
(2*b^2 - 7*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + (x*(3*b
 + 7*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(63*c) - (a^(1/4)*(8*b^4 -
 57*a*b^2*c + 84*a^2*c^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*
Sqrt[a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*S
qrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.581442, antiderivative size = 443, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{x\left(84a^2c^2-57ab^2c+8b^4\right)\sqrt{a+bx^2+cx^4}}{315c^{5/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+4\sqrt{a}b\sqrt{c}\left(b^2-6ac\right)+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{630c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{315c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{x\left(6cx^2\left(2b^2-7ac\right)+b\left(4b^2-9ac\right)\right)\sqrt{a+bx^2+cx^4}}{315c^2}+\frac{x\left(3b+7cx^2\right)\left(a+bx^2+cx^4\right)^{3/2}}{63c}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*x*Sqrt[a + b*x^2 + c*x^4])/(31
5*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (x*(b*(4*b^2 - 9*a*c) + 6*c*
(2*b^2 - 7*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(315*c^2) + (x*(3*b
 + 7*c*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(63*c) - (a^(1/4)*(8*b^4 -
 57*a*b^2*c + 84*a^2*c^2)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2
 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*
x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(315*c^(11/4)*Sqrt[a +
 b*x^2 + c*x^4]) + (a^(1/4)*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2 + 4*
Sqrt[a]*b*Sqrt[c]*(b^2 - 6*a*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c
^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(630*c^(11/4)*S
qrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 66.3376, size = 413, normalized size = 0.93

$$- \frac{\sqrt[4]{a} \sqrt{\frac{a + bx^2 + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}} \left(\sqrt{a} + \sqrt{c}x^2\right) \left(84a^2c^2 - 57ab^2c + 8b^4\right) E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle| \frac{1}{2} - \frac{b}{4\sqrt{a}\sqrt{c}}\right)}{315c^{\frac{11}{4}}\sqrt{a + bx^2 + cx^4}} + \frac{\sqrt[4]{a} \sqrt{\frac{a + bx^2 + cx^4}{\left(\sqrt{a} + \sqrt{c}x^2\right)^2}} \left(\sqrt{a} + \sqrt{c}x^2\right) \left(4\sqrt{a}b\sqrt{c}\left(-6ac + b^2\right) + 84a^2c^2 - 57ab^2c + 8b^4\right) F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle| \frac{1}{2} - \frac{b}{4\sqrt{a}\sqrt{c}}\right)}{630c^{\frac{11}{4}}\sqrt{a + bx^2 + cx^4}} + \frac{x\left(3b + 7cx^2\right)\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{63c} - \frac{x\left(b\left(-9ac + 4b^2\right) + 6cx^2\left(-7ac + 2b^2\right)\right)\sqrt{a + bx^2 + cx^4}}{315c^2} + \frac{x\sqrt{a + bx^2 + cx^4}\left(84a^2c^2 - 57ab^2c + 8b^4\right)}{315c^{\frac{5}{2}}\left(\sqrt{a} + \sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)
*(sqrt(a) + sqrt(c)*x**2)*(84*a**2*c**2 - 57*a*b**2*c + 8*b**4)*e
lliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)
))/(315*c**(11/4)*sqrt(a + b*x**2 + c*x**4)) + a**(1/4)*sqrt((a +
 b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)
*x**2)*(4*sqrt(a)*b*sqrt(c)*(-6*a*c + b**2) + 84*a**2*c**2 - 57*a
*b**2*c + 8*b**4)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b
/(4*sqrt(a)*sqrt(c)))/(630*c**(11/4)*sqrt(a + b*x**2 + c*x**4)) +
 x*(3*b + 7*c*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(63*c) - x*(b*(-
9*a*c + 4*b**2) + 6*c*x**2*(-7*a*c + 2*b**2))*sqrt(a + b*x**2 + c
*x**4)/(315*c**2) + x*sqrt(a + b*x**2 + c*x**4)*(84*a**2*c**2 - 5
7*a*b**2*c + 8*b**4)/(315*c**(5/2)*(sqrt(a) + sqrt(c)*x**2))
```

Mathematica [C] time = 3.64503, size = 602, normalized size = 1.36

$$\frac{i\left(84a^2c^2-57ab^2c+8b^4\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b^4*x^2 - b^3*c*x^4 +
53*b^2*c^2*x^6 + 85*b*c^3*x^8 + 35*c^4*x^10 + a^2*c*(24*b + 77*c*
x^2) + a*(-4*b^3 + 27*b^2*c*x^2 + 151*b*c^2*x^4 + 112*c^3*x^6)) +
 I*(8*b^4 - 57*a*b^2*c + 84*a^2*c^2)*(-b + Sqrt[b^2 - 4*a*c])*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-8*b^5 + 65*a
*b^3*c - 132*a^2*b*c^2 + 8*b^4*Sqrt[b^2 - 4*a*c] - 57*a*b^2*c*Sqr
t[b^2 - 4*a*c] + 84*a^2*c^2*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[
b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(1260*c^3*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.016, size = 545, normalized size = 1.2

$$\frac{cx^7}{9}\sqrt{cx^4+bx^2+a}+\frac{10\,bx^5}{63}\sqrt{cx^4+bx^2+a}+\frac{x^3}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)\sqrt{cx^4+bx^2+a}$$
$$+\frac{x}{3\,c}\left(\frac{76\,ab}{63}-\frac{4\,b}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)\right)\sqrt{cx^4+bx^2+a}$$
$$-\frac{a\sqrt{2}}{12\,c}\left(\frac{76\,ab}{63}-\frac{4\,b}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\right.}\right.$$
$$-\frac{a\sqrt{2}}{2}\left(a^2-\frac{3\,a}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)-\frac{2\,b}{3\,c}\left(\frac{76\,ab}{63}-\frac{4\,b}{5\,c}\left(\frac{11\,ac}{9}+\frac{b^2}{21}\right)\right)\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b\right.}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/9*c*x^7*(c*x^4+b*x^2+a)^(1/2)+10/63*b*x^5*(c*x^4+b*x^2+a)^(1/2)
+1/5*(11/9*a*c+1/21*b^2)/c*x^3*(c*x^4+b*x^2+a)^(1/2)+1/3*(76/63*a
*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*x*(c*x^4+b*x^2+a)^(1/2)-1/12*(7
6/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*a*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2
*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*Ellipt
icF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(
b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(a^2-3/5*(11/9*a*c+1/21*b^2
)/c*a-2/3*(76/63*a*b-4/5*(11/9*a*c+1/21*b^2)/c*b)/c*b)*a*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+
a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^4 + ax^2\right)\sqrt{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^4 + a*x^2)*sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*x^2, x)
```

3.949 $\int \left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=381

$$-\frac{2bx\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}b\left(b^2-8ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

```
[Out] (-2*b*(b^2 - 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*c^(3/2)*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a + b*x^2
 + c*x^4])/(35*c) + (x*(a + b*x^2 + c*x^4)^(3/2))/7 + (2*a^(1/4)*
b*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a + b*x^2 + c*x
^4]) - (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*
c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(70*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.499336, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$

$$-\frac{2bx\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}b\left(b^2-8ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(10ac+b^2+3bcx^2\right)\sqrt{a+bx^2+cx^4}}{35c}+\frac{1}{7}x\left(a+bx^2+cx^4\right)^{3/2}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*b*(b^2 - 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*c^(3/2)*(Sqrt[
a] + Sqrt[c]*x^2)) + (x*(b^2 + 10*a*c + 3*b*c*x^2)*Sqrt[a + b*x^2
 + c*x^4])/(35*c) + (x*(a + b*x^2 + c*x^4)^(3/2))/7 + (2*a^(1/4)*
b*(b^2 - 8*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)
], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(35*c^(7/4)*Sqrt[a + b*x^2 + c*x
^4]) - (a^(1/4)*(Sqrt[a]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*
c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + S
qrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(S
qrt[a]*Sqrt[c]))/4])/(70*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 62.0745, size = 354, normalized size = 0.93

$$\frac{2\sqrt[4]{a}b\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-8ac+b^2\right)E\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{35c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}\left(-20ac+b^2\right)+2b\left(-8ac+b^2\right)\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{70c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}} - \frac{2bx\left(-8ac+b^2\right)\sqrt{a+bx^2+cx^4}}{35c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{7} + \frac{x\sqrt{a+bx^2+cx^4}\left(10ac+b^2+3bcx^2\right)}{35c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*a**(1/4)*b*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)*
*2)*(sqrt(a) + sqrt(c)*x**2)*(-8*a*c + b**2)*elliptic_e(2*atan(c*
*(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(35*c**(7/4)*sqr
t(a + b*x**2 + c*x**4)) - a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sq
rt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*sqrt(
c)*(-20*a*c + b**2) + 2*b*(-8*a*c + b**2))*elliptic_f(2*atan(c**(
1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(70*c**(7/4)*sqrt(
a + b*x**2 + c*x**4)) - 2*b*x*(-8*a*c + b**2)*sqrt(a + b*x**2 + c
*x**4)/(35*c**(3/2)*(sqrt(a) + sqrt(c)*x**2)) + x*(a + b*x**2 + c
*x**4)**(3/2)/7 + x*sqrt(a + b*x**2 + c*x**4)*(10*a*c + b**2 + 3*
b*c*x**2)/(35*c)
```

Mathematica [C] time = 2.78979, size = 533, normalized size = 1.4

$$\frac{2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(15a^2c+a\left(b^2+23bcx^2+20c^2x^4\right)+x^2\left(b^3+9b^2cx^2+13bc^2x^4+5c^3x^6\right)\right)+i\left(-20a^2c^2+9ab^2c-8abc\sqrt{b^2}\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(15*a^2*c + a*(b^2 + 23*b*
c*x^2 + 20*c^2*x^4) + x^2*(b^3 + 9*b^2*c*x^2 + 13*b*c^2*x^4 + 5*c
^3*x^6)) - I*b*(b^2 - 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE
[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c - 20
*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(70*c^2*Sqrt[c/(b
 + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.012, size = 471, normalized size = 1.2

$$\frac{cx^5}{7}\sqrt{cx^4+bx^2+a}+\frac{8\,bx^3}{35}\sqrt{cx^4+bx^2+a}+\frac{x}{3\,c}\left(\frac{9\,ac}{7}+\frac{3\,b^2}{35}\right)\sqrt{cx^4+bx^2+a}$$

$$+\frac{\sqrt{2}}{4}\left(a^2-\frac{a}{3\,c}\left(\frac{9\,ac}{7}+\frac{3\,b^2}{35}\right)\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\right.}}\right.$$

$$-\frac{a\sqrt{2}}{2}\left(\frac{46\,ab}{35}-\frac{2\,b}{3\,c}\left(\frac{9\,ac}{7}+\frac{3\,b^2}{35}\right)\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/7*c*x^5*(c*x^4+b*x^2+a)^(1/2)+8/35*b*x^3*(c*x^4+b*x^2+a)^(1/2)+
1/3*(9/7*a*c+3/35*b^2)/c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(a^2-1/3*(9/
7*a*c+3/35*b^2)/c*a)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
```

```
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))
/a/c)^(1/2))-1/2*(46/35*a*b-2/3*(9/7*a*c+3/35*b^2)/c*b)*a*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a
*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c
)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2), x)
```

3.950 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=361

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}+\frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}$$

```
[Out] ((b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*Sqrt[c]*(Sqrt[a] +
Sqrt[c]*x^2)) + (x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/5 - (
a + b*x^2 + c*x^4)^(3/2)/x - (a^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(5*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(b^2 + 8*Sqrt
[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*c^(3/4)*Sqrt[a + b*x
^2 + c*x^4])
```

Rubi [A] time = 0.423403, antiderivative size = 361, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{a}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{x\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{x}+\frac{1}{5}x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```


```
[Out] ((b^2 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*Sqrt[c]*(Sqrt[a] +
Sqrt[c]*x^2)) + (x*(7*b + 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/5 - (
```

```
a + b*x^2 + c*x^4)^(3/2)/x - (a^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*E
llipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))
/4])/(5*c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(b^2 + 8*Sqrt
[a]*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 +
 c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)
/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*c^(3/4)*Sqrt[a + b*x
^2 + c*x^4])
```

Rubi in Sympy [A] time = 56.8031, size = 328, normalized size = 0.91

$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(12ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{5c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{10c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}} + \frac{x\left(7b+6cx^2\right)\sqrt{a+bx^2+cx^4}}{5} - \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x} + \frac{x\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)
*(sqrt(a) + sqrt(c)*x**2)*(12*a*c + b**2)*elliptic_e(2*atan(c**(1
/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(5*c**(3/4)*sqrt(a
+ b*x**2 + c*x**4)) + a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a
) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(8*sqrt(a)*b*sqrt(
c) + 12*a*c + b**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 -
 b/(4*sqrt(a)*sqrt(c)))/(10*c**(3/4)*sqrt(a + b*x**2 + c*x**4)) +
 x*(7*b + 6*c*x**2)*sqrt(a + b*x**2 + c*x**4)/5 - (a + b*x**2 + c
*x**4)**(3/2)/x + x*(12*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/(5*
sqrt(c)*(sqrt(a) + sqrt(c)*x**2))
```

Mathematica [C] time = 2.37414, size = 505, normalized size = 1.4

$$4c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-5a^2-3abx^2-4acx^4+2b^2x^4+3bcx^6+c^2x^8\right)+ix\left(12ac+b^2\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2\cdots}}{b-\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^2,x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-5*a^2 - 3*a*b*x^2 + 2*b^2*
x^4 - 4*a*c*x^4 + 3*b*c*x^6 + c^2*x^8) + I*(b^2 + 12*a*c)*(-b + S
qrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])] - I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 12*a*c*Sqrt[b^
2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt
[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/
(20*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.019, size = 430, normalized size = 1.2

$$-\frac{a}{x}\sqrt{cx^4+bx^2+a}+\frac{cx^3}{5}\sqrt{cx^4+bx^2+a}+\frac{2\,bx}{5}\sqrt{cx^4+bx^2+a}$$
$$+\frac{2\,ab\sqrt{2}}{5}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2-\ldots b}\right.$$
$$-\frac{a\sqrt{2}}{2}\left(\frac{12\,ac}{5}+\frac{b^2}{5}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^2,x)
```

```
[Out] -a*(c*x^4+b*x^2+a)^(1/2)/x+1/5*c*x^3*(c*x^4+b*x^2+a)^(1/2)+2/5*b*
x*(c*x^4+b*x^2+a)^(1/2)+2/5*a*b*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-1/2*(12/5*a*c+1/5*b^2)*a*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-
EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4
+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**2, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^2, x)
```

3.951 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^4} dx$

Optimal. Leaf size=353

$$\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x} + \frac{8b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{8\sqrt[4]{a}b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3\sqrt{a+bx^2+cx^4}} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

```
[Out] (8*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*(Sqrt[a] + Sqrt[c]*x^2
)) - ((3*b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(3*x) - (a + b*x^2
 + c*x^4)^(3/2)/(3*x^3) - (8*a^(1/4)*b*c^(1/4)*(Sqrt[a] + Sqrt[c]
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
3*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 + 8*Sqrt[a]*b*Sqrt[c] + 4*a*
c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(6*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.397476, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}} - \frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x} + \frac{8b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{8\sqrt[4]{a}b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3\sqrt{a+bx^2+cx^4}} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (8*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*(Sqrt[a] + Sqrt[c]*x^2
)) - ((3*b - 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(3*x) - (a + b*x^2
 + c*x^4)^(3/2)/(3*x^3) - (8*a^(1/4)*b*c^(1/4)*(Sqrt[a] + Sqrt[c]
```

```
*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Ellipti
cE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
3*Sqrt[a + b*x^2 + c*x^4]) + ((3*b^2 + 8*Sqrt[a]*b*Sqrt[c] + 4*a*
c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sq
rt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sq
rt[a]*Sqrt[c]))/4])/(6*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 55.4525, size = 325, normalized size = 0.92

$$-\frac{8\sqrt[4]{a}b\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3\sqrt{a+bx^2+cx^4}}+\frac{8b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(3b-2cx^2\right)\sqrt{a+bx^2+cx^4}}{3x}-\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{3x^3}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(8\sqrt{a}b\sqrt{c}+4ac+3b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] -8*a**(1/4)*b*c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt
(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)
*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*sqrt(a + b*x**2 + c
*x**4)) + 8*b*sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/(3*(sqrt(a) + s
qrt(c)*x**2)) - (3*b - 2*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(3*x)
- (a + b*x**2 + c*x**4)**(3/2)/(3*x**3) + sqrt((a + b*x**2 + c*x*
*4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(8*sqrt
(a)*b*sqrt(c) + 4*a*c + 3*b**2)*elliptic_f(2*atan(c**(1/4)*x/a**(
1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(6*a**(1/4)*c**(1/4)*sqrt(a +
 b*x**2 + c*x**4))
```

Mathematica [C] time = 1.73466, size = 473, normalized size = 1.34

$$\frac{2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-a^2-5abx^2-4b^2x^4-3bcx^6+c^2x^8\right)-ix^3\left(4b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}}{6x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^4,x]
```

```
[Out] (2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-a^2 - 5*a*b*x^2 - 4*b^2*x^4
- 3*b*c*x^6 + c^2*x^8) + (4*I)*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*E
llipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b
 + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^2 + 4*a*c
+ 4*b*Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])])/(6*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*
x^2 + c*x^4])
```

Maple [A] time = 0.022, size = 428, normalized size = 1.2

$$-\frac{a}{3x^3}\sqrt{cx^4+bx^2+a}-\frac{4b}{3x}\sqrt{cx^4+bx^2+a}+\frac{cx}{3}\sqrt{cx^4+bx^2+a}$$
$$+\frac{\sqrt{2}}{4}\left(\frac{4ac}{3}+b^2\right)\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{\right.$$
$$-\frac{4abc\sqrt{2}}{3}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^4,x)
```

```
[Out] -1/3*a*(c*x^4+b*x^2+a)^(1/2)/x^3-4/3*b*(c*x^4+b*x^2+a)^(1/2)/x+1/
3*c*x*(c*x^4+b*x^2+a)^(1/2)+1/4*(4/3*a*c+b^2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*
(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*El
lipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2
*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-4/3*b*c*a*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/
(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)
^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-E
llipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+
2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**4,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**4, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^4, x)
```

3.952 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=400

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{c}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5ax} + \frac{\sqrt{c}x\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5} - \frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}$$

```
[Out] -((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(5*a*x) + (Sqrt[c]*(b^2
 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2
)) - ((b - 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(5*x^3) - (a + b*x^2
 + c*x^4)^(3/2)/(5*x^5) - (c^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(5*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(b^2 + 8*Sqrt[a]
*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*a^(3/4)*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi [A] time = 0.613534, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\sqrt[4]{c}\left(8\sqrt{a}b\sqrt{c}+12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{10a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{c}\left(12ac+b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{5a^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5ax} + \frac{\sqrt{c}x\left(12ac+b^2\right)\sqrt{a+bx^2+cx^4}}{5a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\left(a+bx^2+cx^4\right)^{3/2}}{5x^5} - \frac{\left(b-6cx^2\right)\sqrt{a+bx^2+cx^4}}{5x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] -((b^2 + 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(5*a*x) + (Sqrt[c]*(b^2
 + 12*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(5*a*(Sqrt[a] + Sqrt[c]*x^2
)) - ((b - 6*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(5*x^3) - (a + b*x^2
 + c*x^4)^(3/2)/(5*x^5) - (c^(1/4)*(b^2 + 12*a*c)*(Sqrt[a] + Sqrt
[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]
)/(5*a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(b^2 + 8*Sqrt[a]
*b*Sqrt[c] + 12*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(10*a^(3/4)*Sqrt[a + b*x^2
+ c*x^4])
```

Rubi in Sympy [A] time = 77.0867, size = 362, normalized size = 0.9

$$- \frac{\left(b - 6cx^{2}\right)\sqrt{a + bx^{2} + cx^{4}}}{5x^{3}} - \frac{\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{5x^{5}} + \frac{\sqrt{c}x\left(12ac + b^{2}\right)\sqrt{a + bx^{2} + cx^{4}}}{5a\left(\sqrt{a} + \sqrt{c}x^{2}\right)} - \frac{\left(12ac + b^{2}\right)\sqrt{a + bx^{2} + cx^{4}}}{5ax} - \frac{\sqrt[4]{c}\sqrt{\frac{a + bx^{2} + cx^{4}}{\left(\sqrt{a} + \sqrt{c}x^{2}\right)^{2}}}\left(\sqrt{a} + \sqrt{c}x^{2}\right)\left(12ac + b^{2}\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2} - \frac{b}{4\sqrt{a}\sqrt{c}}\right)}{5a^{\frac{3}{4}}\sqrt{a + bx^{2} + cx^{4}}} + \frac{\sqrt[4]{c}\sqrt{\frac{a + bx^{2} + cx^{4}}{\left(\sqrt{a} + \sqrt{c}x^{2}\right)^{2}}}\left(\sqrt{a} + \sqrt{c}x^{2}\right)\left(8\sqrt{a}b\sqrt{c} + 12ac + b^{2}\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2} - \frac{b}{4\sqrt{a}\sqrt{c}}\right)}{10a^{\frac{3}{4}}\sqrt{a + bx^{2} + cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] -(b - 6*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(5*x**3) - (a + b*x**2
+ c*x**4)**(3/2)/(5*x**5) + sqrt(c)*x*(12*a*c + b**2)*sqrt(a + b*
x**2 + c*x**4)/(5*a*(sqrt(a) + sqrt(c)*x**2)) - (12*a*c + b**2)*s
qrt(a + b*x**2 + c*x**4)/(5*a*x) - c**(1/4)*sqrt((a + b*x**2 + c*
x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(12*a
*c + b**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqr
t(a)*sqrt(c)))/(5*a**(3/4)*sqrt(a + b*x**2 + c*x**4)) + c**(1/4)*
sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a)
+ sqrt(c)*x**2)*(8*sqrt(a)*b*sqrt(c) + 12*a*c + b**2)*elliptic_f(
2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(10*a**
(3/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 2.55586, size = 527, normalized size = 1.32

$$-4\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a^3+a^2\left(3bx^2+8cx^4\right)+a\left(3b^2x^4+9bcx^6+7c^2x^8\right)+b^2x^6\left(b+cx^2\right)\right)+ix^5\left(12ac+b^2\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^6,x]
```

```
[Out] (-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a^3 + b^2*x^6*(b + c*x^2) +
a^2*(3*b*x^2 + 8*c*x^4) + a*(3*b^2*x^4 + 9*b*c*x^6 + 7*c^2*x^8))
+ I*(b^2 + 12*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt
[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*Arc
Sinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(-b^3 + 4*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] + 12*a*c*Sqrt[b^2 - 4*a*c])*x^5*Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2
 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh
[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*
c])/(b - Sqrt[b^2 - 4*a*c])])/(20*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]
)]*x^5*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.023, size = 450, normalized size = 1.1

$$-\frac{a}{5x^5}\sqrt{cx^4+bx^2+a}-\frac{2b}{5x^3}\sqrt{cx^4+bx^2+a}-\frac{7ac+b^2}{5ax}\sqrt{cx^4+bx^2+a}$$
$$+\frac{2bc\sqrt{2}}{5}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b}{}}\right.$$
$$-\frac{a\sqrt{2}}{2}\left(c^2+\frac{c\left(7ac+b^2\right)}{5a}\right)\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^6,x)
```

```
[Out] -1/5*a*(c*x^4+b*x^2+a)^(1/2)/x^5-2/5*b*(c*x^4+b*x^2+a)^(1/2)/x^3-
1/5*(7*a*c+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x+2/5*b*c*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
```

```
/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2
)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c^2+1/5*c*(7*a*c+b
^2)/a)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1
/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c
+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**6,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**6, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^6, x)
```

3.953 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=447

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\left(b^2-8ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}x\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}$$

```
[Out] -((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a*x^3) + (2*b*(b^2
- 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a^2*x) - (2*b*Sqrt[c]*(b^2
- 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*a^2*(Sqrt[a] + Sqrt[c]*x^
2)) - (3*(b + 10*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*x^5) - (a +
b*x^2 + c*x^4)^(3/2)/(7*x^7) + (2*b*c^(1/4)*(b^2 - 8*a*c)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(35*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a
]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(7
0*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.931427, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}\left(b^2-20ac\right)+2b\left(b^2-8ac\right)\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{70a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\left(b^2-8ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{35a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2x}-\frac{2b\sqrt{c}x\left(b^2-8ac\right)\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\left(b^2-20ac\right)\sqrt{a+bx^2+cx^4}}{35ax^3}-\frac{\left(a+bx^2+cx^4\right)^{3/2}}{7x^7}-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] -((b^2 - 20*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a*x^3) + (2*b*(b^2
- 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(35*a^2*x) - (2*b*Sqrt[c]*(b^2
- 8*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(35*a^2*(Sqrt[a] + Sqrt[c]*x^
2)) - (3*(b + 10*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(35*x^5) - (a +
b*x^2 + c*x^4)^(3/2)/(7*x^7) + (2*b*c^(1/4)*(b^2 - 8*a*c)*(Sqrt[a
] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt
[c]))/4])/(35*a^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a
]*Sqrt[c]*(b^2 - 20*a*c) + 2*b*(b^2 - 8*a*c))*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(7
0*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 103.207, size = 416, normalized size = 0.93

$$-\frac{3\left(b+10cx^2\right)\sqrt{a+bx^2+cx^4}}{35x^5}-\frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{7x^7}-\frac{\left(-20ac+b^2\right)\sqrt{a+bx^2+cx^4}}{35ax^3}-\frac{2b\sqrt{c}x\left(-8ac+b^2\right)\sqrt{a+bx^2+cx^4}}{35a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{2b\left(-8ac+b^2\right)\sqrt{a+bx^2+cx^4}}{35a^2x}+\frac{2b\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-8ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{35a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}\left(-20ac+b^2\right)+2b\left(-8ac+b^2\right)\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{70a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] -3*(b + 10*c*x**2)*sqrt(a + b*x**2 + c*x**4)/(35*x**5) - (a + b*x
**2 + c*x**4)**(3/2)/(7*x**7) - (-20*a*c + b**2)*sqrt(a + b*x**2
+ c*x**4)/(35*a*x**3) - 2*b*sqrt(c)*x*(-8*a*c + b**2)*sqrt(a + b*
x**2 + c*x**4)/(35*a**2*(sqrt(a) + sqrt(c)*x**2)) + 2*b*(-8*a*c +
 b**2)*sqrt(a + b*x**2 + c*x**4)/(35*a**2*x) + 2*b*c**(1/4)*sqrt(
(a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqr
t(c)*x**2)*(-8*a*c + b**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4))
, 1/2 - b/(4*sqrt(a)*sqrt(c)))/(35*a**(7/4)*sqrt(a + b*x**2 + c*x
**4)) - c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x*
*2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*sqrt(c)*(-20*a*c + b**2
) + 2*b*(-8*a*c + b**2))*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)),
1/2 - b/(4*sqrt(a)*sqrt(c)))/(70*a**(7/4)*sqrt(a + b*x**2 + c*x**
```

```
4))
```

Mathematica [C] time = 3.0043, size = 572, normalized size = 1.28

$$\frac{ix^7\left(-20a^2c^2+9ab^2c-8abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}-b^4\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-\cdots}}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/x^8,x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(5*a^4 - 2*b^3*x^8*(b + c*x^2
) + a^3*(13*b*x^2 + 20*c*x^4) + a*b*x^6*(-b^2 + 17*b*c*x^2 + 16*c
^2*x^4) + 3*a^2*(3*b^2*x^4 + 13*b*c*x^6 + 5*c^2*x^8)) - I*b*(b^2
- 8*a*c)*(-b + Sqrt[b^2 - 4*a*c])*x^7*Sqrt[(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a
*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
 - Sqrt[b^2 - 4*a*c])] + I*(-b^4 + 9*a*b^2*c - 20*a^2*c^2 + b^3*S
qrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c])*x^7*Sqrt[(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sq
rt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*A
rcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
- 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]])/(70*a^2*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*x^7*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.028, size = 495, normalized size = 1.1

$$-\frac{a}{7x^7}\sqrt{cx^4+bx^2+a}-\frac{8b}{35x^5}\sqrt{cx^4+bx^2+a}-\frac{15ac+b^2}{35ax^3}\sqrt{cx^4+bx^2+a}-\frac{2b\left(8ac-b^2\right)}{35a^2x}\sqrt{cx^4+bx^2+a}$$
$$+\frac{\sqrt{2}}{4}\left(c^2-\frac{c\left(15ac+b^2\right)}{35a}\right)\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+\cdots}\right.}\right.$$
$$-\frac{bc\left(8ac-b^2\right)\sqrt{2}}{35a}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/x^8,x)
```

```
[Out] -1/7*a*(c*x^4+b*x^2+a)^(1/2)/x^7-8/35*b*(c*x^4+b*x^2+a)^(1/2)/x^5
-1/35*(15*a*c+b^2)/a*(c*x^4+b*x^2+a)^(1/2)/x^3-2/35*b*(8*a*c-b^2)
/a^2*(c*x^4+b*x^2+a)^(1/2)/x+1/4*(c^2-1/35*c*(15*a*c+b^2)/a)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*
x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/35*b*c*(8*
a*c-b^2)/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2
*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a
*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}{x^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/x**8,x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/x**8, x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/x^8, x)
```

3.954 $\int \sqrt{3-2x^2-x^4}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt
[3] + (4*EllipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rubi [A] time = 0.139859, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312

$$\frac{1}{3}\sqrt{-x^4-2x^2+3}x+\frac{4F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}-\frac{2E\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4])/3 - (2*EllipticE[ArcSin[x], -1/3])/Sqrt
[3] + (4*EllipticF[ArcSin[x], -1/3])/Sqrt[3]
```

Rubi in Sympy [A] time = 22.9189, size = 49, normalized size = 1.02

$$\frac{x\sqrt{-x^4-2x^2+3}}{3}-\frac{2\sqrt{3}E\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)}{3}+\frac{4\sqrt{3}F\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(-x**4 - 2*x**2 + 3)/3 - 2*sqrt(3)*elliptic_e(asin(x), -1/3
)/3 + 4*sqrt(3)*elliptic_f(asin(x), -1/3)/3
```

Mathematica [C] time = 0.10745, size = 59, normalized size = 1.23

$$\frac{1}{3}\left(\sqrt{-x^4-2x^2+3}x-4iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)-2iE\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3 - 2*x^2 - x^4],x]
```

```
[Out] (x*Sqrt[3 - 2*x^2 - x^4] - (2*I)*EllipticE[I*ArcSinh[x/Sqrt[3]],
-3] - (4*I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3])/3
```

Maple [B] time = 0.019, size = 114, normalized size = 2.4

$$\frac{x}{3}\sqrt{-x^4-2x^2+3}+\frac{2\,EllipticF\left(x,i/3\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}+\frac{2\,EllipticF\left(x,i/3\sqrt{3}\right)-2\,EllipticE\left(x,i/3\sqrt{3}\right)}{3}\sqrt{-x^2+1}\sqrt{3x^2+9}\frac{1}{\sqrt{-x^4-2x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^4-2*x^2+3)^(1/2),x)
```

```
[Out] 1/3*x*(-x^4-2*x^2+3)^(1/2)+2/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x
^4-2*x^2+3)^(1/2)*EllipticF(x,1/3*I*3^(1/2))+2/3*(-x^2+1)^(1/2)*(
3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*(EllipticF(x,1/3*I*3^(1/2))-E
llipticE(x,1/3*I*3^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^4-2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 - 2*x^2 + 3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-x^4-2x^2+3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 - 2*x^2 + 3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 - 2x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**4-2*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**4 - 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^4 - 2\,x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^4 - 2*x^2 + 3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^4 - 2*x^2 + 3), x)
```

3.955 $\int \frac{x^7}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=121

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*
x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/
2))
```

Rubi [A] time = 0.252468, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{\left(-16ac+15b^2-10bcx^2\right)\sqrt{a+bx^2+cx^4}}{48c^3}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2 - 16*a*c - 10*b*c*
x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - (b*(5*b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(7/
2))
```

Rubi in Sympy [A] time = 22.3458, size = 114, normalized size = 0.94

$$-\frac{b\left(-12ac+5b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{\frac{7}{2}}}+\frac{x^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{\sqrt{a+bx^2+cx^4}\left(-4ac+\frac{15b^2}{4}-\frac{5bcx^2}{2}\right)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*(-12*a*c + 5*b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*
x**2 + c*x**4)))/(32*c**(7/2)) + x**4*sqrt(a + b*x**2 + c*x**4)/(
6*c) + sqrt(a + b*x**2 + c*x**4)*(-4*a*c + 15*b**2/4 - 5*b*c*x**2
```

```
/2)/(12*c**3)
```

Mathematica [A] time = 0.128596, size = 102, normalized size = 0.84

$$\frac{(36abc-15b^3)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+2\sqrt{c}\sqrt{a+bx^2+cx^4}\,(8c\,(cx^4-2a)+15b^2-10bcx^2)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]*(15*b^2 - 10*b*c*x^2 + 8*c*(-2
*a + c*x^4)) + (-15*b^3 + 36*a*b*c)*Log[b + 2*c*x^2 + 2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4]])/(96*c^(7/2))
```

Maple [A] time = 0.022, size = 162, normalized size = 1.3

$$\frac{x^4}{6\,c}\sqrt{cx^4+bx^2+a}-\frac{5\,bx^2}{24\,c^2}\sqrt{cx^4+bx^2+a}+\frac{5\,b^2}{16\,c^3}\sqrt{cx^4+bx^2+a}-\frac{5\,b^3}{32}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}+\frac{3\,ab}{8}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{a}{3\,c^2}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(c*x^4+b*x^2+a)^(1
/2)+5/16*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*b^3/c^(7/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*ln((1/2*b+
c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/3/c^2*a*(c*x^4+b*x^2+a)^(
1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.303668, size = 1, normalized size = 0.01

$$\left[\frac{4\left(8\,c^2x^4 - 10\,bcx^2 + 15\,b^2 - 16\,ac\right)\sqrt{cx^4 + bx^2 + a}\sqrt{c} - 3\left(5\,b^3 - 12\,abc\right)\log\left(-4\,\sqrt{cx^4 + bx^2 + a}\left(2\,c^2x^2 + bc\right) - \left(8\,c^2x^4 + \right.\right.}{192\,c^{\frac{7}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/192*(4*(8*c^2*x^4 - 10*b*c*x^2 + 15*b^2 - 16*a*c)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(c) - 3*(5*b^3 - 12*a*b*c)*log(-4*sqrt(c*x^4 + b*
x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c
)*sqrt(c)))/c^(7/2), 1/96*(2*(8*c^2*x^4 - 10*b*c*x^2 + 15*b^2 - 1
6*a*c)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) - 3*(5*b^3 - 12*a*b*c)*ar
ctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sq
rt(-c)*c^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.302297, size = 147, normalized size = 1.21

$$\frac{1}{48}\,\sqrt{cx^4 + bx^2 + a}\left(2\,x^2\left(\frac{4\,x^2}{c} - \frac{5\,b}{c^2}\right) + \frac{15\,b^2c - 16\,ac^2}{c^4}\right) + \frac{\left(5\,b^3c - 12\,abc^2\right)\ln\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{32\,c^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c - 5*b/c^2) + (15*b^2
*c - 16*a*c^2)/c^4) + 1/32*(5*b^3*c - 12*a*b*c^2)*ln(abs(-2*(sqrt
(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(9/2)
```

3.956 $\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=104

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*
x^4])/(4*c) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))
```

Rubi [A] time = 0.189989, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + (x^2*Sqrt[a + b*x^2 + c*
x^4])/(4*c) + ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*S
qrt[a + b*x^2 + c*x^4])])/(16*c^(5/2))
```

Rubi in Sympy [A] time = 21.4166, size = 94, normalized size = 0.9

$$-\frac{3b\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{x^2\sqrt{a+bx^2+cx^4}}{4c}+\frac{(-4ac+3b^2)\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -3*b*sqrt(a + b*x**2 + c*x**4)/(8*c**2) + x**2*sqrt(a + b*x**2 +
c*x**4)/(4*c) + (-4*a*c + 3*b**2)*atanh((b + 2*c*x**2)/(2*sqrt(c)
*sqrt(a + b*x**2 + c*x**4)))/(16*c**(5/2))
```

Mathematica [A] time = 0.0747087, size = 86, normalized size = 0.83

$$\frac{(3b^2-4ac)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)+2\sqrt{c}\,(2cx^2-3b)\,\sqrt{a+bx^2+cx^4}}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[c]*(-3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (3*b^2 - 4*
a*c)*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(16*c^
(5/2))
```

Maple [A] time = 0.021, size = 116, normalized size = 1.1

$$\frac{x^2}{4\,c}\sqrt{cx^4+bx^2+a}-\frac{3\,b}{8\,c^2}\sqrt{cx^4+bx^2+a}+\frac{3\,b^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}-\frac{a}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3
/16*b^2/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1
/4*a/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.295822, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^4+bx^2+a}\left(2\,cx^2-3\,b\right)\sqrt{c}-\left(3\,b^2-4\,ac\right)\log\left(4\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)-\left(8\,c^2x^4+8\,bcx^2+b^2+4\,ac\right)\sqrt{c}\right)}{32\,c^{\frac{5}{2}}},\ 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/32*(4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 - 3*b)*sqrt(c) - (3*b^2
 - 4*a*c)*log(4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^
2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/c^(5/2), 1/16*(2*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 - 3*b)*sqrt(-c) + (3*b^2 - 4*a*c)*arc
tan(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(sqr
t(-c)*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.302504, size = 111, normalized size = 1.07

$$\frac{1}{8}\sqrt{cx^4+bx^2+a}\left(\frac{2\,x^2}{c}-\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2-4\,ac\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*x^2/c - 3*b/c^2) - 1/16*(3*b^2 - 4
*a*c)*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) -
 b))/c^(5/2)
```

3.957 $\int \frac{x^3}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[
c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi [A] time = 0.106927, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[
c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rubi in Sympy [A] time = 12.2634, size = 58, normalized size = 0.85

$$-\frac{b\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{\frac{3}{2}}}+\frac{\sqrt{a+bx^2+cx^4}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(4
*c**(3/2)) + sqrt(a + b*x**2 + c*x**4)/(2*c)
```

Mathematica [A] time = 0.036847, size = 66, normalized size = 0.97

$$\frac{\sqrt{a+bx^2+cx^4}}{2c}-\frac{b\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c) - (b*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sq
rt[a + b*x^2 + c*x^4]])/(4*c^(3/2))
```

Maple [A] time = 0.015, size = 56, normalized size = 0.8

$$\frac{1}{2c}\sqrt{cx^4+bx^2+a}-\frac{b}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29296, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(4\sqrt{cx^4+bx^2+a}\left(2c^2x^2+bc\right)-\left(8c^2x^4+8bcx^2+b^2+4ac\right)\sqrt{c}\right)+4\sqrt{cx^4+bx^2+a}\sqrt{c}}{8c^{\frac{3}{2}}},\right.$$
$$\left.-\frac{b\arctan\left(\frac{(2cx^2+b)\sqrt{-c}}{2\sqrt{cx^4+bx^2+a}c}\right)-2\sqrt{cx^4+bx^2+a}\sqrt{-c}}{4\sqrt{-c}c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/8*(b*log(4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*
x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)) + 4*sqrt(c*x^4 + b*x^2 +
a)*sqrt(c))/c^(3/2), -1/4*(b*arctan(1/2*(2*c*x^2 + b)*sqrt(-c)/(s
qrt(c*x^4 + b*x^2 + a)*c)) - 2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c))/
(sqrt(-c)*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.294992, size = 82, normalized size = 1.21

$$\frac{b\ln\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{4\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^4 + bx^2 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/4*b*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) -
 b))/c^(3/2) + 1/2*sqrt(c*x^4 + b*x^2 + a)/c
```

3.958 $\int \frac{x}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=43

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqr
t[c])
```

Rubi [A] time = 0.0609932, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqr
t[c])
```

Rubi in Sympy [A] time = 6.96255, size = 37, normalized size = 0.86

$$\frac{\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(2*sq
rt(c))
```

Mathematica [A] time = 0.0188294, size = 41, normalized size = 0.95

$$\frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + b*x^2 + c*x^4],x]

[Out] Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/(2*Sqrt[c])
```

Maple [A] time = 0.01, size = 35, normalized size = 0.8

$$\frac{1}{2}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2+a)^(1/2),x)

[Out] 1/2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.314407, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(-4\sqrt{cx^4+bx^2+a}\left(2\,c^2x^2+bc\right)-\left(8\,c^2x^4+8\,bcx^2+b^2+4\,ac\right)\sqrt{c}\right)}{4\sqrt{c}}, \frac{\arctan\left(\frac{(2\,cx^2+b)\sqrt{-c}}{2\sqrt{cx^4+bx^2+a}c}\right)}{2\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^
4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c))/sqrt(c), 1/2*arctan(1/2*(2*
c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c))/sqrt(-c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.290886, size = 54, normalized size = 1.26

$$-\frac{\ln\left(\left|-2\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) -
b))/sqrt(c)
```

3.959 $\int \frac{1}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sq
rt[a])
```

Rubi [A] time = 0.0861922, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sq
rt[a])
```

Rubi in Sympy [A] time = 10.8014, size = 39, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(2*s
qrt(a))
```

Mathematica [A] time = 0.11974, size = 50, normalized size = 1.14

$$\frac{\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]
])/(2*Sqrt[a])
```

Maple [A] time = 0.014, size = 39, normalized size = 0.9

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287385, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)}{4\,\sqrt{a}},\ -\frac{\arctan\left(\frac{\left(bx^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^4+bx^2+a}a}\right)}{2\,\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4
*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4)/sqrt(a), -1/2*arctan
(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a))/sqrt(-a)
]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x), x)
```

3.960 $\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*
Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi [A] time = 0.127957, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*
Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi in Sympy [A] time = 14.6344, size = 61, normalized size = 0.85

$$-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{b\,\mathrm{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2 + c*x**4)/(2*a*x**2) + b*atanh((2*a + b*x**2)/(2
*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(4*a**(3/2))
```

Mathematica [A] time = 0.123607, size = 78, normalized size = 1.08

$$-\frac{b\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)\right)}{4a^{3/2}}-\frac{\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(2*a*x^2) - (b*(Log[x^2] - Log[2*a + b*x
^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*x^2)]]))/(4*a^(3/2))
```

Maple [A] time = 0.016, size = 63, normalized size = 0.9

$$-\frac{1}{2\,a x^2}\sqrt{c x^4+b x^2+a}+\frac{b}{4}\ln\left(\frac{1}{x^2}\left(2\,a+b x^2+2\,\sqrt{a}\sqrt{c x^4+b x^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^
(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298947, size = 1, normalized size = 0.01

$$\left[\frac{b x^2\log\left(-\frac{4\,\sqrt{c x^4+b x^2+a}\left(a b x^2+2\,a^2\right)+\left(\left(b^2+4\,a c\right)x^4+8\,a b x^2+8\,a^2\right)\sqrt{a}}{x^4}\right)-4\,\sqrt{c x^4+b x^2+a}\sqrt{a}}{8\,a^{\frac{3}{2}}x^2},\ \frac{b x^2\arctan\left(\frac{\left(b x^2+2\,a\right)\sqrt{-a}}{2\,\sqrt{c x^4+b x^2+a}a}\right)-2\,\sqrt{c x^4+}}{4\,\sqrt{-a}a x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(b*x^2*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) + (
(b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4) - 4*sqrt(c*x
^4 + b*x^2 + a)*sqrt(a))/(a^(3/2)*x^2), 1/4*(b*x^2*arctan(1/2*(b*
x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^3), x)
```

3.961 $\int \frac{1}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=108

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4]
)/(8*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]
*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rubi [A] time = 0.239125, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[a + b*x^2 + c*x^4]
)/(8*a^2*x^2) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]
*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rubi in Sympy [A] time = 22.2489, size = 97, normalized size = 0.9

$$-\frac{\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{(-4ac+3b^2)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2 + c*x**4)/(4*a*x**4) + 3*b*sqrt(a + b*x**2 + c*x
**4)/(8*a**2*x**2) - (-4*a*c + 3*b**2)*atanh((2*a + b*x**2)/(2*sq
rt(a)*sqrt(a + b*x**2 + c*x**4)))/(16*a**(5/2))
```

Mathematica [A] time = 0.252742, size = 96, normalized size = 0.89

$$\frac{(3b^2-4ac)\left(\log(x^2)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{16a^{5/2}}+\frac{(3bx^2-2a)\sqrt{a+bx^2+cx^4}}{8a^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-2*a + 3*b*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*a^2*x^4) + ((3*b^2
- 4*a*c)*(Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 +
 c*x^4]]))/(16*a^(5/2))
```

Maple [A] time = 0.018, size = 127, normalized size = 1.2

$$-\frac{1}{4ax^4}\sqrt{cx^4+bx^2+a}+\frac{3b}{8a^2x^2}\sqrt{cx^4+bx^2+a}-\frac{3b^2}{16}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{c}{4}\ln\left(\frac{1}{x^2}\left(2a+bx^2+2\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/
x^2-3/16*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2
))/x^2)+1/4*c/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/
2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302769, size = 1, normalized size = 0.01

$$\left[-\frac{\left(3\,b^{2}-4\,ac\right)x^{4}\log\left(-\frac{4\,\sqrt{cx^{4}+bx^{2}+a}\left(abx^{2}+2\,a^{2}\right)+\left(\left(b^{2}+4\,ac\right)x^{4}+8\,abx^{2}+8\,a^{2}\right)\sqrt{a}}{x^{4}}\right)-4\,\sqrt{cx^{4}+bx^{2}+a}\left(3\,bx^{2}-2\,a\right)\sqrt{a}}{32\,a^{\frac{5}{2}}x^{4}},\ -\frac{\left(3\,b^{2}-4\,ac\right)x^{4}\arctan\left(\frac{\left(bx^{2}+2\,a\right)\sqrt{-a}}{2\,\sqrt{cx^{4}+bx^{2}+a}a}\right)-2\,\sqrt{cx^{4}+bx^{2}+a}\left(3\,bx^{2}-2\,a\right)\sqrt{-a}}{16\,\sqrt{-a}a^{2}x^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^5),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2 - 4*a*c)*x^4*log(-(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*
x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x
^4) - 4*sqrt(c*x^4 + b*x^2 + a)*(3*b*x^2 - 2*a)*sqrt(a))/(a^(5/2)
*x^4), -1/16*((3*b^2 - 4*a*c)*x^4*arctan(1/2*(b*x^2 + 2*a)*sqrt(-
a)/(sqrt(c*x^4 + b*x^2 + a)*a)) - 2*sqrt(c*x^4 + b*x^2 + a)*(3*b*
x^2 - 2*a)*sqrt(-a))/(sqrt(-a)*a^2*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{5}\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^{4}+bx^{2}+a}x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^5),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^5), x)
```

3.962 $\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4]
)/(24*a^2*x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*
a^3*x^2) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*S
qrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi [A] time = 0.388971, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{b\left(5b^2-12ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{\left(15b^2-16ac\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[a + b*x^2 + c*x^4]
)/(24*a^2*x^4) - ((15*b^2 - 16*a*c)*Sqrt[a + b*x^2 + c*x^4])/(48*
a^3*x^2) + (b*(5*b^2 - 12*a*c)*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*S
qrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi in Sympy [A] time = 35.4144, size = 133, normalized size = 0.92

$$-\frac{\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{5b\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{\left(-16ac+15b^2\right)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{b\left(-12ac+5b^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2 + c*x**4)/(6*a*x**6) + 5*b*sqrt(a + b*x**2 + c*x
**4)/(24*a**2*x**4) - (-16*a*c + 15*b**2)*sqrt(a + b*x**2 + c*x**
4)/(48*a**3*x**2) + b*(-12*a*c + 5*b**2)*atanh((2*a + b*x**2)/(2*
sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(32*a**(7/2))
```

Mathematica [A] time = 0.439272, size = 117, normalized size = 0.81

$$\frac{b\left(12ac-5b^2\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{32a^{7/2}}-\frac{\sqrt{a+bx^2+cx^4}\left(8a^2-2a\left(5bx^2+8cx^4\right)+15b^2x^4\right)}{48a^3x^6}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]*(8*a^2 + 15*b^2*x^4 - 2*a*(5*b*x^2 + 8*
c*x^4)))/(48*a^3*x^6) + (b*(-5*b^2 + 12*a*c)*(Log[x^2] - Log[2*a
+ b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(32*a^(7/2))
```

Maple [A] time = 0.02, size = 176, normalized size = 1.2

$$-\frac{1}{6\,ax^6}\sqrt{cx^4+bx^2+a}+\frac{5\,b}{24\,a^2x^4}\sqrt{cx^4+bx^2+a}-\frac{5\,b^2}{16\,a^3x^2}\sqrt{cx^4+bx^2+a}+\frac{5\,b^3}{32}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{7}{2}}-\frac{3\,bc}{8}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}+\frac{c}{3\,a^2x^2}\sqrt{cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*(c*x^4+b*x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2
/x^4-5/16*b^2/a^3/x^2*(c*x^4+b*x^2+a)^(1/2)+5/32*b^3/a^(7/2)*ln((
2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-3/8*b/a^(5/2)*c*l
n((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/3*c/a^2/x^2*
(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.309992, size = 1, normalized size = 0.01

$$\left[-\frac{3\left(5\,b^3-12\,abc\right)x^6\log\left(\frac{4\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left(\left(b^2+4\,ac\right)x^4+8\,abx^2+8\,a^2\right)\sqrt{a}}{x^4}\right)+4\left(\left(15\,b^2-16\,ac\right)x^4-10\,abx^2+8\,a^2\right)\sqrt{cx^4}}{192\,a^{\frac{7}{2}}x^6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^7),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(5*b^3 - 12*a*b*c)*x^6*log((4*sqrt(c*x^4 + b*x^2 + a)*
(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(
a))/x^4) + 4*((15*b^2 - 16*a*c)*x^4 - 10*a*b*x^2 + 8*a^2)*sqrt(c*
x^4 + b*x^2 + a)*sqrt(a))/(a^(7/2)*x^6), 1/96*(3*(5*b^3 - 12*a*b*
c)*x^6*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)
*a)) - 2*((15*b^2 - 16*a*c)*x^4 - 10*a*b*x^2 + 8*a^2)*sqrt(c*x^4
+ b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a^3*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^7\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^7),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^7), x)
```

3.963 $\int \frac{x^4}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=313

$$-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx^2+cx^4}}{3c}$$

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (2*b*x*Sqrt[a + b*x^2 + c*x^4
])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a^(1/4)*b*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b + Sqrt[
a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.28488, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx^2+cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - (2*b*x*Sqrt[a + b*x^2 + c*x^4
])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (2*a^(1/4)*b*(Sqrt[a] +
Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*
EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
```

```
)/4])/(3*c^(7/4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b + Sqrt[
a]*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqr
t[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (
2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 40.3614, size = 284, normalized size = 0.91

$$\frac{2\sqrt[4]{a}b\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}+2b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6c^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}} - \frac{2bx\sqrt{a+bx^2+cx^4}}{3c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x\sqrt{a+bx^2+cx^4}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*a**(1/4)*b*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)*
*2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4
)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*c**(7/4)*sqrt(a + b*x**2 + c*
x**4)) - a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x
**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*sqrt(c) + 2*b)*ellipti
c_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(6*
c**(7/4)*sqrt(a + b*x**2 + c*x**4)) - 2*b*x*sqrt(a + b*x**2 + c*x
**4)/(3*c**(3/2)*(sqrt(a) + sqrt(c)*x**2)) + x*sqrt(a + b*x**2 +
c*x**4)/(3*c)
```

Mathematica [C] time = 1.63343, size = 444, normalized size = 1.42

$$\frac{i\left(b\sqrt{b^2-4ac}+ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-ib\left(\sqrt{b^2-4ac}\right.}{6c^2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*b*
(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
```

```
b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b
+ Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 -
 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqr
t[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 -
 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*c^2*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.016, size = 388, normalized size = 1.2

$$\frac{x}{3c}\sqrt{cx^4+bx^2+a}$$
$$-\frac{a\sqrt{2}}{12c}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b\left(b\right.}{}}\right.$$
$$+\frac{ab\sqrt{2}}{3c}\sqrt{4-2\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12/c*a*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(
-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*a*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*
(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elliptic
E(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+
(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

3.964 $\int \frac{x^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (
a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(
1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.194392, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}} + \frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```


```
[Out] (x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (
a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a]
 + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 -
b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(
1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] +
Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(
Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 32.1305, size = 240, normalized size = 0.9

$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{(\sqrt{a}+\sqrt{c}x^2)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2c^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)
*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)),
 1/2 - b/(4*sqrt(a)*sqrt(c)))/(c**(3/4)*sqrt(a + b*x**2 + c*x**4)
) + a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)*
*2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4
)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*c**(3/4)*sqrt(a + b*x**2 + c*
x**4)) + x*sqrt(a + b*x**2 + c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*
x**2))
```

Mathematica [C] time = 0.262022, size = 278, normalized size = 1.04

$$\frac{i\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right.\right.\right.}{2\sqrt{2}c\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a
*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - Elli
pticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b +
Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.014, size = 216, normalized size = 0.8

$$-\frac{a\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2
)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(x**2/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] integrate(x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

3.965 $\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=114

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0489158, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt
[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt
[a]*Sqrt[c]))/4])/(2*a^(1/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 7.38118, size = 102, normalized size = 0.89

$$\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a)
+ sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(
4*sqrt(a)*sqrt(c)))/(2*a**(1/4)*c**(1/4)*sqrt(a + b*x**2 + c*x**4
```

```
))
```

Mathematica [C] time = 0.146586, size = 186, normalized size = 1.63

$$-\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.01, size = 144, normalized size = 1.3

$$\frac{\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b+\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + b*x^2 + a), x)
```

3.966 $\int \frac{1}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=294

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + b*x^2 + c*
x^4])/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^
(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(
3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.283344, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```


```
[Out] -(Sqrt[a + b*x^2 + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + b*x^2 + c*
x^4])/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x
^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE
[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^
```

```
(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)
*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*
ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(
3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 44.6218, size = 260, normalized size = 0.88

$$\frac{\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)} - \frac{\sqrt{a+bx^2+cx^4}}{ax} - \frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}} + \frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{3}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/(a*(sqrt(a) + sqrt(c)*x**2))
- sqrt(a + b*x**2 + c*x**4)/(a*x) - c**(1/4)*sqrt((a + b*x**2 + c
*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elli
ptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/
(a**(3/4)*sqrt(a + b*x**2 + c*x**4)) + c**(1/4)*sqrt((a + b*x**2
+ c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*e
lliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)
))/(2*a**(3/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.909468, size = 298, normalized size = 1.01

$$\frac{-\frac{4\left(a+bx^2+cx^4\right)}{x} + \frac{i\sqrt{2}\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-}}{b-\sqrt{b^2-}}\right.\right.}{\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}}}{4a\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4))/x + (I*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c])
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c
])/(b - Sqrt[b^2 - 4*a*c])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
```

```
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^
2 - 4*a*c])]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [A] time = 0.017, size = 239, normalized size = 0.8

$$-\frac{1}{ax}\sqrt{cx^4+bx^2+a}$$
$$-\frac{c\sqrt{2}}{2}\sqrt{4-2\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)
^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2+a}x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

3.967 $\int \frac{1}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=345

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 + c*x^4]
)/(3*a^2*x) - (2*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(Sqr
t[a] + Sqrt[c]*x^2)) + (2*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*S
qrt[a + b*x^2 + c*x^4]) - ((2*b + Sqrt[a]*Sqrt[c])*c^(1/4)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(6*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.415169, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}\sqrt{c}+2b\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{2b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```



```
[Out] -Sqrt[a + b*x^2 + c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 + c*x^4]
)/(3*a^2*x) - (2*b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(3*a^2*(Sqr
t[a] + Sqrt[c]*x^2)) + (2*b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
```

```
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*S
qrt[a + b*x^2 + c*x^4]) - ((2*b + Sqrt[a]*Sqrt[c])*c^(1/4)*(Sqrt[
a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2
)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqr
t[c]))/4])/(6*a^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 55.6473, size = 314, normalized size = 0.91

$$-\frac{\sqrt{a+bx^2+cx^4}}{3ax^3}-\frac{2b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{2b\sqrt{a+bx^2+cx^4}}{3a^2x}+\frac{2b\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{3a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}\sqrt{c}+2b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{6a^{\frac{7}{4}}\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2 + c*x**4)/(3*a*x**3) - 2*b*sqrt(c)*x*sqrt(a + b*
x**2 + c*x**4)/(3*a**2*(sqrt(a) + sqrt(c)*x**2)) + 2*b*sqrt(a + b
*x**2 + c*x**4)/(3*a**2*x) + 2*b*c**(1/4)*sqrt((a + b*x**2 + c*x*
*4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*ellipti
c_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(3*
a**(7/4)*sqrt(a + b*x**2 + c*x**4)) - c**(1/4)*sqrt((a + b*x**2 +
 c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(s
qrt(a)*sqrt(c) + 2*b)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2
 - b/(4*sqrt(a)*sqrt(c)))/(6*a**(7/4)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 1.69985, size = 459, normalized size = 1.33

$$\frac{-2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a-2bx^2\right)\left(a+bx^2+cx^4\right)+ix^3\left(b\sqrt{b^2-4ac}+ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\cdots\right.\right.}{6a^2x^3\sqrt{\sqrt{b\cdots}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a - 2*b*x^2)*(a + b*x^2 + c*
x^4) - I*b*(-b + Sqrt[b^2 - 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4
*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqr
t[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/
(b - Sqrt[b^2 - 4*a*c])] + I*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c])*x
^3*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*
c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*
x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(6*a^2*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c])]*x^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.022, size = 413, normalized size = 1.2

$$-\frac{1}{3\,ax^3}\sqrt{cx^4+bx^2+a}+\frac{2\,b}{3\,a^2x}\sqrt{cx^4+bx^2+a}$$
$$-\frac{c\sqrt{2}}{12\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b\left(b\right.}{}}\right.$$
$$+\frac{bc\sqrt{2}}{3\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+2\,\frac{b}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/
x-1/12*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2
))+1/3*b*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2 + a}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

3.968 $\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

```
[Out] -(x^4*Sqrt[a + b*x^2 - c*x^4])/(6*c) - ((15*b^2 + 16*a*c + 10*b*c
*x^2)*Sqrt[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*Arc
Tan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/
2))
```

Rubi [A] time = 0.264882, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{7/2}}-\frac{\left(16ac+15b^2+10bcx^2\right)\sqrt{a+bx^2-cx^4}}{48c^3}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(x^4*Sqrt[a + b*x^2 - c*x^4])/(6*c) - ((15*b^2 + 16*a*c + 10*b*c
*x^2)*Sqrt[a + b*x^2 - c*x^4])/(48*c^3) - (b*(5*b^2 + 12*a*c)*Arc
Tan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])])/(32*c^(7/
2))
```

Rubi in Sympy [A] time = 23.4444, size = 116, normalized size = 0.94

$$-\frac{b\left(12ac+5b^2\right)\operatorname{atan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{32c^{\frac{7}{2}}}-\frac{x^4\sqrt{a+bx^2-cx^4}}{6c}-\frac{\sqrt{a+bx^2-cx^4}\left(4ac+\frac{15b^2}{4}+\frac{5bcx^2}{2}\right)}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*(12*a*c + 5*b**2)*atan((b - 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x*
*2 - c*x**4)))/(32*c**(7/2)) - x**4*sqrt(a + b*x**2 - c*x**4)/(6*
c) - sqrt(a + b*x**2 - c*x**4)*(4*a*c + 15*b**2/4 + 5*b*c*x**2/2)
```

```
/(12*c**3)
```

Mathematica [C] time = 0.240083, size = 112, normalized size = 0.9

$$\frac{-2\sqrt{c}\sqrt{a+bx^2-cx^4}\left(8c\left(2a+cx^4\right)+15b^2+10bcx^2\right)+3i\left(12abc+5b^3\right)\log\left(2\sqrt{a+bx^2-cx^4}+\frac{i(b-2cx^2)}{\sqrt{c}}\right)}{96c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]*(15*b^2 + 10*b*c*x^2 + 8*c*(2
*a + c*x^4)) + (3*I)*(5*b^3 + 12*a*b*c)*Log[(I*(b - 2*c*x^2))/Sqr
t[c] + 2*Sqrt[a + b*x^2 - c*x^4]])/(96*c^(7/2))
```

Maple [A] time = 0.028, size = 168, normalized size = 1.4

$$-\frac{x^4}{6\,c}\sqrt{-cx^4+bx^2+a}-\frac{5\,bx^2}{24\,c^2}\sqrt{-cx^4+bx^2+a}-\frac{5\,b^2}{16\,c^3}\sqrt{-cx^4+bx^2+a} +\frac{5\,b^3}{32}\arctan\left(1\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{7}{2}} +\frac{3\,ab}{8}\arctan\left(1\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{5}{2}}-\frac{a}{3\,c^2}\sqrt{-cx^4+bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/6*x^4*(-c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(-c*x^4+b*x^2+a)
^(1/2)-5/16*b^2/c^3*(-c*x^4+b*x^2+a)^(1/2)+5/32*b^3/c^(7/2)*arcta
n(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*a
rctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))-1/3/c^2*a*(-c
*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.300477, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(8c^2x^4+10bcx^2+15b^2+16ac\right)\sqrt{-cx^4+bx^2+a}\sqrt{-c}-3\left(5b^3+12abc\right)\log\left(4\sqrt{-cx^4+bx^2+a}\left(2c^2x^2-bc\right)+\left(8c^2\right.\right.}{192\sqrt{-c}c^3}\right.$$
$$\left.-\frac{2\left(8c^2x^4+10bcx^2+15b^2+16ac\right)\sqrt{-cx^4+bx^2+a}\sqrt{c}-3\left(5b^3+12abc\right)\arctan\left(\frac{2cx^2-b}{2\sqrt{-cx^4+bx^2+a}\sqrt{c}}\right)}{96c^{\frac{7}{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/192*(4*(8*c^2*x^4 + 10*b*c*x^2 + 15*b^2 + 16*a*c)*sqrt(-c*x^4
 + b*x^2 + a)*sqrt(-c) - 3*(5*b^3 + 12*a*b*c)*log(4*sqrt(-c*x^4 +
 b*x^2 + a)*(2*c^2*x^2 - b*c) + (8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*
a*c)*sqrt(-c)))/(sqrt(-c)*c^3), -1/96*(2*(8*c^2*x^4 + 10*b*c*x^2
+ 15*b^2 + 16*a*c)*sqrt(-c*x^4 + b*x^2 + a)*sqrt(c) - 3*(5*b^3 +
12*a*b*c)*arctan(1/2*(2*c*x^2 - b)/(sqrt(-c*x^4 + b*x^2 + a)*sqrt
(c))))/c^(7/2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**7/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [A] time = 0.304953, size = 159, normalized size = 1.28

$$-\frac{1}{48}\sqrt{-cx^4+bx^2+a}\left(2\,x^2\left(\frac{4\,x^2}{c}+\frac{5\,b}{c^2}\right)+\frac{15\,b^2c+16\,ac^2}{c^4}\right)-\frac{\left(5\,b^3c+12\,abc^2\right)\ln\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{32\sqrt{-c}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/48*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2*(4*x^2/c + 5*b/c^2) + (15*b
^2*c + 16*a*c^2)/c^4) - 1/32*(5*b^3*c + 12*a*b*c^2)*ln(abs(2*(sqr
t(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c) + b))/(sqrt(-c)*c^
4)
```

3.969 $\int \frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=107

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*
x^4])/(4*c) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sq
rt[a + b*x^2 - c*x^4])])/(16*c^(5/2))
```

Rubi [A] time = 0.202522, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{5/2}}-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (-3*b*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) - (x^2*Sqrt[a + b*x^2 - c*
x^4])/(4*c) - ((3*b^2 + 4*a*c)*ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sq
rt[a + b*x^2 - c*x^4])])/(16*c^(5/2))
```

Rubi in Sympy [A] time = 22.7034, size = 95, normalized size = 0.89

$$-\frac{3b\sqrt{a+bx^2-cx^4}}{8c^2}-\frac{x^2\sqrt{a+bx^2-cx^4}}{4c}-\frac{(4ac+3b^2)\operatorname{atan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{16c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -3*b*sqrt(a + b*x**2 - c*x**4)/(8*c**2) - x**2*sqrt(a + b*x**2 -
c*x**4)/(4*c) - (4*a*c + 3*b**2)*atan((b - 2*c*x**2)/(2*sqrt(c)*s
qrt(a + b*x**2 - c*x**4)))/(16*c**(5/2))
```

Mathematica [C] time = 0.129357, size = 94, normalized size = 0.88

$$-\frac{(3b+2cx^2)\sqrt{a+bx^2-cx^4}}{8c^2}+\frac{i(4ac+3b^2)\log\left(2\sqrt{a+bx^2-cx^4}+\frac{i(b-2cx^2)}{\sqrt{c}}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((3*b + 2*c*x^2)*Sqrt[a + b*x^2 - c*x^4])/(8*c^2) + ((I/16)*(3*b
^2 + 4*a*c)*Log[(I*(b - 2*c*x^2))/Sqrt[c] + 2*Sqrt[a + b*x^2 - c*
x^4]])/c^(5/2)
```

Maple [A] time = 0.021, size = 120, normalized size = 1.1

$$-\frac{x^2}{4\,c}\sqrt{-cx^4+bx^2+a}-\frac{3\,b}{8\,c^2}\sqrt{-cx^4+bx^2+a}+\frac{3\,b^2}{16}\arctan\left(1\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{5}{2}}+\frac{a}{4}\arctan\left(1\sqrt{c}\left(x^2-\frac{b}{2\,c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/4*x^2*(-c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(-c*x^4+b*x^2+a)^(1/2)/c^
2+3/16*b^2/c^(5/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^
(1/2))+1/4*a/c^(3/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.294223, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{-cx^4+bx^2+a}\left(2\,cx^2+3\,b\right)\sqrt{-c}-\left(3\,b^2+4\,ac\right)\log\left(4\sqrt{-cx^4+bx^2+a}\left(2\,c^2x^2-bc\right)+\left(8\,c^2x^4-8\,bcx^2+b^2-4\,ac\right)\right.}{32\sqrt{-c}c^2}\right.$$

$$\left.-\frac{2\sqrt{-cx^4+bx^2+a}\left(2\,cx^2+3\,b\right)\sqrt{c}-\left(3\,b^2+4\,ac\right)\arctan\left(\frac{2\,cx^2-b}{2\sqrt{-cx^4+bx^2+a}\sqrt{c}}\right)}{16\,c^{\frac{5}{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [-1/32*(4*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 + 3*b)*sqrt(-c) - (3*
b^2 + 4*a*c)*log(4*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 - b*c) + (
8*c^2*x^4 - 8*b*c*x^2 + b^2 - 4*a*c)*sqrt(-c)))/(sqrt(-c)*c^2), -
1/16*(2*sqrt(-c*x^4 + b*x^2 + a)*(2*c*x^2 + 3*b)*sqrt(c) - (3*b^2
 + 4*a*c)*arctan(1/2*(2*c*x^2 - b)/(sqrt(-c*x^4 + b*x^2 + a)*sqrt
(c))))/c^(5/2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^5}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [A] time = 0.306066, size = 123, normalized size = 1.15

$$-\frac{1}{8}\sqrt{-cx^4+bx^2+a}\left(\frac{2\,x^2}{c}+\frac{3\,b}{c^2}\right)-\frac{\left(3\,b^2+4\,ac\right)\ln\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{16\sqrt{-c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-c*x^4 + b*x^2 + a)*(2*x^2/c + 3*b/c^2) - 1/16*(3*b^2 +
 4*a*c)*ln(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-
c) + b))/(sqrt(-c)*c^2)
```

3.970 $\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[
c]*Sqrt[a + b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rubi [A] time = 0.114199, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19

$$-\frac{b\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{3/2}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) - (b*ArcTan[(b - 2*c*x^2)/(2*Sqrt[
c]*Sqrt[a + b*x^2 - c*x^4])])/(4*c^(3/2))
```

Rubi in Sympy [A] time = 12.5072, size = 60, normalized size = 0.86

$$-\frac{b\,\mathrm{atan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{4c^{\frac{3}{2}}}-\frac{\sqrt{a+bx^2-cx^4}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -b*atan((b - 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 - c*x**4)))/(4*
c**(3/2)) - sqrt(a + b*x**2 - c*x**4)/(2*c)
```

Mathematica [C] time = 0.0709748, size = 77, normalized size = 1.1

$$-\frac{\sqrt{a+bx^2-cx^4}}{2c}+\frac{ib\log\left(2\sqrt{a+bx^2-cx^4}-\frac{i(2cx^2-b)}{\sqrt{c}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(2*c) + ((I/4)*b*Log[((-I)*(-b + 2*c*x^2
))/Sqrt[c] + 2*Sqrt[a + b*x^2 - c*x^4]])/c^(3/2)
```

Maple [A] time = 0.017, size = 58, normalized size = 0.8

$$-\frac{1}{2c}\sqrt{-cx^4+bx^2+a}+\frac{b}{4}\arctan\left(1\sqrt{c}\left(x^2-\frac{b}{2c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(-c*x^4+b*x^2+a)^(1/2)/c+1/4*b/c^(3/2)*arctan(c^(1/2)*(x^2-1
/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.292197, size = 1, normalized size = 0.01

$$\left[\frac{b\log\left(4\sqrt{-cx^4+bx^2+a}\left(2c^2x^2-bc\right)+\left(8c^2x^4-8bcx^2+b^2-4ac\right)\sqrt{-c}\right)-4\sqrt{-cx^4+bx^2+a}\sqrt{-c}}{8\sqrt{-c}c}, \frac{b\arctan\left(\frac{2cx^2-}{2\sqrt{-cx^4+b}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/8*(b*log(4*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 - b*c) + (8*c^2
*x^4 - 8*b*c*x^2 + b^2 - 4*a*c)*sqrt(-c)) - 4*sqrt(-c*x^4 + b*x^2
 + a)*sqrt(-c))/(sqrt(-c)*c), 1/4*(b*arctan(1/2*(2*c*x^2 - b)/(sq
rt(-c*x^4 + b*x^2 + a)*sqrt(c))) - 2*sqrt(-c*x^4 + b*x^2 + a)*sqr
t(c))/c^(3/2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [A] time = 0.293335, size = 95, normalized size = 1.36

$$-\frac{b\ln\left(\left|2\left(\sqrt{-c}x^2-\sqrt{-cx^4+bx^2+a}\right)\sqrt{-c}+b\right|\right)}{4\sqrt{-c}c}-\frac{\sqrt{-cx^4+bx^2+a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/4*b*ln(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c
) + b))/(sqrt(-c)*c) - 1/2*sqrt(-c*x^4 + b*x^2 + a)/c
```

3.971 $\int \frac{x}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqr
t[c])
```

Rubi [A] time = 0.063748, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158

$$-\frac{\tan^{-1}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -ArcTan[(b - 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])]/(2*Sqr
t[c])
```

Rubi in Sympy [A] time = 7.31274, size = 39, normalized size = 0.89

$$-\frac{\operatorname{atan}\left(\frac{b-2cx^2}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -atan((b - 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 - c*x**4)))/(2*sq
rt(c))
```

Mathematica [C] time = 0.0280529, size = 51, normalized size = 1.16

$$\frac{i\log\left(2\sqrt{a+bx^2-cx^4}-\frac{i(2cx^2-b)}{\sqrt{c}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((I/2)*Log[((-I)*(-b + 2*c*x^2))/Sqrt[c] + 2*Sqrt[a + b*x^2 - c*x
^4]])/Sqrt[c]
```

Maple [A] time = 0.011, size = 36, normalized size = 0.8

$$\frac{1}{2}\arctan\left(1\sqrt{c}\left(x^2-\frac{b}{2c}\right)\frac{1}{\sqrt{-cx^4+bx^2+a}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/2/c^(1/2)*arctan(c^(1/2)*(x^2-1/2*b/c)/(-c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288194, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(4\sqrt{-cx^4+bx^2+a}\left(2c^2x^2-bc\right)+\left(8c^2x^4-8bcx^2+b^2-4ac\right)\sqrt{-c}\right)}{4\sqrt{-c}},\ \frac{\arctan\left(\frac{2cx^2-b}{2\sqrt{-cx^4+bx^2+a}\sqrt{c}}\right)}{2\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*log(4*sqrt(-c*x^4 + b*x^2 + a)*(2*c^2*x^2 - b*c) + (8*c^2*x^
4 - 8*b*c*x^2 + b^2 - 4*a*c)*sqrt(-c))/sqrt(-c), 1/2*arctan(1/2*(
2*c*x^2 - b)/(sqrt(-c*x^4 + b*x^2 + a)*sqrt(c)))/sqrt(c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + bx^2 - cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [A] time = 0.292851, size = 61, normalized size = 1.39

$$-\frac{\ln\left(\left|2\left(\sqrt{-c}x^2 - \sqrt{-cx^4 + bx^2 + a}\right)\sqrt{-c} + b\right|\right)}{2\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(2*(sqrt(-c)*x^2 - sqrt(-c*x^4 + b*x^2 + a))*sqrt(-c)
+ b))/sqrt(-c)
```

3.972 $\int \frac{1}{x\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sq
rt[a])
```

Rubi [A] time = 0.0940548, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$-\frac{\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*Sqrt[-a + b*x^2 + c*x^4])]/(2*Sq
rt[a])
```

Rubi in Sympy [A] time = 11.6656, size = 37, normalized size = 0.79

$$\frac{\operatorname{atan}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] atan((-2*a + b*x**2)/(2*sqrt(a)*sqrt(-a + b*x**2 + c*x**4)))/(2*s
qrt(a))
```

Mathematica [A] time = 0.161289, size = 55, normalized size = 1.17

$$\frac{2\log(x) - \log\left(2\sqrt{-a}\sqrt{-a + bx^2 + cx^4} - 2a + bx^2\right)}{2\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Log[x] - Log[-2*a + b*x^2 + 2*Sqrt[-a]*Sqrt[-a + b*x^2 + c*x^4
]])/(2*Sqrt[-a])
```

Maple [A] time = 0.016, size = 45, normalized size = 1.

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(-2\,a + bx^2 + 2\,\sqrt{-a}\sqrt{cx^4 + bx^2 - a}\right)\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] -1/2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2)
)/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.287647, size = 1, normalized size = 0.02

$$\left[\frac{\log\left(\frac{4\,\sqrt{cx^4+bx^2-a}\left(abx^2-2\,a^2\right)+\left(\left(b^2-4\,ac\right)x^4-8\,abx^2+8\,a^2\right)\sqrt{-a}}{x^4}\right)}{4\,\sqrt{-a}}, \frac{\arctan\left(\frac{bx^2-2\,a}{2\,\sqrt{cx^4+bx^2-a}\sqrt{a}}\right)}{2\,\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*log((4*sqrt(c*x^4 + b*x^2 - a)*(a*b*x^2 - 2*a^2) + ((b^2 - 4
*a*c)*x^4 - 8*a*b*x^2 + 8*a^2)*sqrt(-a))/x^4)/sqrt(-a), 1/2*arcta
n(1/2*(b*x^2 - 2*a)/(sqrt(c*x^4 + b*x^2 - a)*sqrt(a)))/sqrt(a)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.325491, size = 59, normalized size = 1.26

$$\frac{\ln\left(\left|-2\sqrt{-a}\left(\sqrt{c + \frac{b}{x^2} - \frac{a}{x^4}} - \frac{\sqrt{-a}}{x^2}\right) + b\right|\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x),x, algorithm="giac")
```

```
[Out] 1/2*ln(abs(-2*sqrt(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^2) +
 b))/sqrt(-a)
```

3.973 $\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*S
qrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi [A] time = 0.141881, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}-\frac{b\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(2*a*x^2) - (b*ArcTan[(2*a - b*x^2)/(2*S
qrt[a]*Sqrt[-a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rubi in Sympy [A] time = 15.1498, size = 61, normalized size = 0.79

$$\frac{\sqrt{-a+bx^2+cx^4}}{2ax^2}+\frac{b\,\mathrm{atan}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(-a + b*x**2 + c*x**4)/(2*a*x**2) + b*atan((-2*a + b*x**2)/(2
*sqrt(a)*sqrt(-a + b*x**2 + c*x**4)))/(4*a**(3/2))
```

Mathematica [A] time = 0.118517, size = 93, normalized size = 1.21

$$-\frac{2\sqrt{-a}\sqrt{-a+bx^2+cx^4}-bx^2\log\left(2\sqrt{-a}\sqrt{-a+bx^2+cx^4}-2a+bx^2\right)+2bx^2\log(x)}{4(-a)^{3/2}x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] -(2*Sqrt[-a]*Sqrt[-a + b*x^2 + c*x^4] + 2*b*x^2*Log[x] - b*x^2*Lo
g[-2*a + b*x^2 + 2*Sqrt[-a]*Sqrt[-a + b*x^2 + c*x^4]])/(4*(-a)^(3
/2)*x^2)
```

Maple [A] time = 0.019, size = 74, normalized size = 1.

$$\frac{1}{2\,ax^2}\sqrt{cx^4+bx^2-a}-\frac{b}{4\,a}\ln\left(\frac{1}{x^2}\left(-2\,a+bx^2+2\,\sqrt{-a}\sqrt{cx^4+bx^2-a}\right)\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1/2)/a/x^2-1/4*b/a/(-a)^(1/2)*ln((-2*a+b*x^2
+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.293889, size = 1, normalized size = 0.01

$$\left[\frac{bx^2\log\left(\frac{4\,\sqrt{cx^4+bx^2-a}\left(abx^2-2\,a^2\right)+\left(\left(b^2-4\,ac\right)x^4-8\,abx^2+8\,a^2\right)\sqrt{-a}}{x^4}\right)+4\,\sqrt{cx^4+bx^2-a}\sqrt{-a}}{8\,\sqrt{-a}ax^2},\ \frac{bx^2\arctan\left(\frac{bx^2-2\,a}{2\,\sqrt{cx^4+bx^2-a}\sqrt{a}}\right)+2\,\sqrt{cx}}{4\,a^{\frac{3}{2}}x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(b*x^2*log((4*sqrt(c*x^4 + b*x^2 - a)*(a*b*x^2 - 2*a^2) + ((
b^2 - 4*a*c)*x^4 - 8*a*b*x^2 + 8*a^2)*sqrt(-a))/x^4) + 4*sqrt(c*x
^4 + b*x^2 - a)*sqrt(-a))/(sqrt(-a)*a*x^2), 1/4*(b*x^2*arctan(1/2
*(b*x^2 - 2*a)/(sqrt(c*x^4 + b*x^2 - a)*sqrt(a))) + 2*sqrt(c*x^4
+ b*x^2 - a)*sqrt(a))/(a^(3/2)*x^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(-a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.324625, size = 93, normalized size = 1.21

$$\frac{b\ln\left(\left|-2\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{4\sqrt{-a}a}+\frac{\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^3),x, algorithm="giac")
```

```
[Out] 1/4*b*ln(abs(-2*sqrt(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^2)
 + b))/(sqrt(-a)*a) + 1/2*sqrt(c + b/x^2 - a/x^4)/a
```

3.974 $\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=115

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4
])/(8*a^2*x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]
*Sqrt[-a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rubi [A] time = 0.269914, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227

$$-\frac{(4ac+3b^2)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(4*a*x^4) + (3*b*Sqrt[-a + b*x^2 + c*x^4
])/(8*a^2*x^2) - ((3*b^2 + 4*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]
*Sqrt[-a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rubi in Sympy [A] time = 23.6516, size = 97, normalized size = 0.84

$$\frac{\sqrt{-a+bx^2+cx^4}}{4ax^4}+\frac{3b\sqrt{-a+bx^2+cx^4}}{8a^2x^2}+\frac{(4ac+3b^2)\,\operatorname{atan}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{16a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(-a + b*x**2 + c*x**4)/(4*a*x**4) + 3*b*sqrt(-a + b*x**2 + c*
x**4)/(8*a**2*x**2) + (4*a*c + 3*b**2)*atan((-2*a + b*x**2)/(2*sq
rt(a)*sqrt(-a + b*x**2 + c*x**4)))/(16*a**(5/2))
```

Mathematica [A] time = 0.189887, size = 118, normalized size = 1.03

$$\frac{(4ac+3b^2)\left(\frac{\log(x)}{\sqrt{-a}}-\frac{\log\left(2\sqrt{-a}\sqrt{-a+bx^2+cx^4}-2a+bx^2\right)}{2\sqrt{-a}}\right)}{8a^2}+\left(\frac{3b}{8a^2x^2}+\frac{1}{4ax^4}\right)\sqrt{-a+bx^2+cx^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (1/(4*a*x^4) + (3*b)/(8*a^2*x^2))*Sqrt[-a + b*x^2 + c*x^4] + ((3*
b^2 + 4*a*c)*(Log[x]/Sqrt[-a] - Log[-2*a + b*x^2 + 2*Sqrt[-a]*Sqr
t[-a + b*x^2 + c*x^4]]/(2*Sqrt[-a])))/(8*a^2)
```

Maple [A] time = 0.02, size = 149, normalized size = 1.3

$$\frac{1}{4\,ax^4}\sqrt{cx^4+bx^2-a}+\frac{3\,b}{8\,a^2x^2}\sqrt{cx^4+bx^2-a}-\frac{3\,b^2}{16\,a^2}\ln\left(\frac{1}{x^2}\left(-2\,a+bx^2+2\,\sqrt{-a}\sqrt{cx^4+bx^2-a}\right)\right)\frac{1}{\sqrt{-a}}-\frac{c}{4\,a}\ln\left(\frac{1}{x^2}\left(-2\,a+bx^2+2\,\sqrt{-a}\sqrt{cx^4+bx^2-a}\right)\right)\frac{1}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/4*(c*x^4+b*x^2-a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2-a)^(1/2)/a^2/x
^2-3/16*b^2/a^2/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x
^2-a)^(1/2))/x^2)-1/4*c/a/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*
(c*x^4+b*x^2-a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.30244, size = 1, normalized size = 0.01

$$\left[\frac{\left(3\,b^2+4\,ac\right)x^4\log\left(\frac{4\,\sqrt{cx^4+bx^2-a}\left(abx^2-2\,a^2\right)+\left(\left(b^2-4\,ac\right)x^4-8\,abx^2+8\,a^2\right)\sqrt{-a}}{x^4}\right)+4\,\sqrt{cx^4+bx^2-a}\left(3\,bx^2+2\,a\right)\sqrt{-a}}{32\,\sqrt{-a}a^2x^4},\,\frac{\left(3\,b^2+4\,ac\right)}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^5),x, algorithm="fricas")
```

```
[Out] [1/32*((3*b^2 + 4*a*c)*x^4*log((4*sqrt(c*x^4 + b*x^2 - a)*(a*b*x^
2 - 2*a^2) + ((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 + 8*a^2)*sqrt(-a))/x^
4) + 4*sqrt(c*x^4 + b*x^2 - a)*(3*b*x^2 + 2*a)*sqrt(-a))/(sqrt(-a
)*a^2*x^4), 1/16*((3*b^2 + 4*a*c)*x^4*arctan(1/2*(b*x^2 - 2*a)/(s
qrt(c*x^4 + b*x^2 - a)*sqrt(a))) + 2*sqrt(c*x^4 + b*x^2 - a)*(3*b
*x^2 + 2*a)*sqrt(a))/(a^(5/2)*x^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{-a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(-a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.522576, size = 122, normalized size = 1.06

$$\frac{1}{8}\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}\left(\frac{3\,b}{a^2}+\frac{2}{ax^2}\right)+\frac{\left(3\,b^2+4\,ac\right)\ln\left(\left|-2\,\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{16\,\sqrt{-a}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^5),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c + b/x^2 - a/x^4)*(3*b/a^2 + 2/(a*x^2)) + 1/16*(3*b^2 +
 4*a*c)*ln(abs(-2*sqrt(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^
2) + b))/(sqrt(-a)*a^2)
```

3.975 $\int \frac{1}{x^7\sqrt{-a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=154

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4
])/(24*a^2*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(4
8*a^3*x^2) - (b*(5*b^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*
Sqrt[-a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi [A] time = 0.434747, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{b\left(12ac+5b^2\right)\tan^{-1}\left(\frac{2a-bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{7/2}}+\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[-a + b*x^2 + c*x^4]/(6*a*x^6) + (5*b*Sqrt[-a + b*x^2 + c*x^4
])/(24*a^2*x^4) + ((15*b^2 + 16*a*c)*Sqrt[-a + b*x^2 + c*x^4])/(4
8*a^3*x^2) - (b*(5*b^2 + 12*a*c)*ArcTan[(2*a - b*x^2)/(2*Sqrt[a]*
Sqrt[-a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rubi in Sympy [A] time = 37.0042, size = 133, normalized size = 0.86

$$\frac{\sqrt{-a+bx^2+cx^4}}{6ax^6}+\frac{5b\sqrt{-a+bx^2+cx^4}}{24a^2x^4}+\frac{\left(16ac+15b^2\right)\sqrt{-a+bx^2+cx^4}}{48a^3x^2}+\frac{b\left(12ac+5b^2\right)\operatorname{atan}\left(\frac{-2a+bx^2}{2\sqrt{a}\sqrt{-a+bx^2+cx^4}}\right)}{32a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] sqrt(-a + b*x**2 + c*x**4)/(6*a*x**6) + 5*b*sqrt(-a + b*x**2 + c*
x**4)/(24*a**2*x**4) + (16*a*c + 15*b**2)*sqrt(-a + b*x**2 + c*x*
*4)/(48*a**3*x**2) + b*(12*a*c + 5*b**2)*atan((-2*a + b*x**2)/(2*
sqrt(a)*sqrt(-a + b*x**2 + c*x**4)))/(32*a**(7/2))
```

Mathematica [A] time = 0.277932, size = 139, normalized size = 0.9

$$\frac{b\left(12ac+5b^2\right)\left(\frac{\log(x)}{\sqrt{-a}}-\frac{\log\left(2\sqrt{-a}\sqrt{-a+bx^2+cx^4}-2a+bx^2\right)}{2\sqrt{-a}}\right)}{16a^3}+\sqrt{-a+bx^2+cx^4}\left(\frac{16ac+15b^2}{48a^3x^2}+\frac{5b}{24a^2x^4}+\frac{1}{6ax^6}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^7*Sqrt[-a + b*x^2 + c*x^4]),x]
```

```
[Out] (1/(6*a*x^6) + (5*b)/(24*a^2*x^4) + (15*b^2 + 16*a*c)/(48*a^3*x^2
))*Sqrt[-a + b*x^2 + c*x^4] + (b*(5*b^2 + 12*a*c)*(Log[x]/Sqrt[-a
] - Log[-2*a + b*x^2 + 2*Sqrt[-a]*Sqrt[-a + b*x^2 + c*x^4]]/(2*Sq
rt[-a])))/(16*a^3)
```

Maple [A] time = 0.023, size = 202, normalized size = 1.3

$$\frac{1}{6\,a{x}^{6}}\sqrt{c{x}^{4}+b{x}^{2}-a}+\frac{5\,b}{24\,{a}^{2}{x}^{4}}\sqrt{c{x}^{4}+b{x}^{2}-a}+\frac{5\,{b}^{2}}{16\,{a}^{3}{x}^{2}}\sqrt{c{x}^{4}+b{x}^{2}-a}-\frac{5\,{b}^{3}}{32\,{a}^{3}}\ln\left(\frac{1}{{x}^{2}}\left(-2\,a+b{x}^{2}+2\,\sqrt{-a}\sqrt{c{x}^{4}+b{x}^{2}-a}\right)\right)\frac{1}{\sqrt{-a}}-\frac{3\,bc}{8\,{a}^{2}}\ln\left(\frac{1}{{x}^{2}}\left(-2\,a+b{x}^{2}+2\,\sqrt{-a}\sqrt{c{x}^{4}+b{x}^{2}-a}\right)\right)\frac{1}{\sqrt{-a}}+\frac{c}{3\,{a}^{2}{x}^{2}}\sqrt{c{x}^{4}+b{x}^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^7/(c*x^4+b*x^2-a)^(1/2),x)
```

```
[Out] 1/6*(c*x^4+b*x^2-a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2-a)^(1/2)/a^2/
x^4+5/16*b^2/a^3/x^2*(c*x^4+b*x^2-a)^(1/2)-5/32*b^3/a^3/(-a)^(1/2
)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2))/x^2)-3/8*b/a
^2*c/(-a)^(1/2)*ln((-2*a+b*x^2+2*(-a)^(1/2)*(c*x^4+b*x^2-a)^(1/2)
)/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2-a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^7),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.30754, size = 1, normalized size = 0.01

$$\left[\frac{3\left(5\,b^{3}+12\,abc\right)x^{6}\log\left(\frac{4\,\sqrt{cx^{4}+bx^{2}-a}\left(abx^{2}-2\,a^{2}\right)+\left(\left(b^{2}-4\,ac\right)x^{4}-8\,abx^{2}+8\,a^{2}\right)\sqrt{-a}}{x^{4}}\right)+4\left(\left(15\,b^{2}+16\,ac\right)x^{4}+10\,abx^{2}+8\,a^{2}\right)\sqrt{cx^{4}+}}{192\,\sqrt{-a}a^{3}x^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^7),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*b^3 + 12*a*b*c)*x^6*log((4*sqrt(c*x^4 + b*x^2 - a)*(
a*b*x^2 - 2*a^2) + ((b^2 - 4*a*c)*x^4 - 8*a*b*x^2 + 8*a^2)*sqrt(-
a))/x^4) + 4*((15*b^2 + 16*a*c)*x^4 + 10*a*b*x^2 + 8*a^2)*sqrt(c*
x^4 + b*x^2 - a)*sqrt(-a))/(sqrt(-a)*a^3*x^6), 1/96*(3*(5*b^3 + 1
2*a*b*c)*x^6*arctan(1/2*(b*x^2 - 2*a)/(sqrt(c*x^4 + b*x^2 - a)*sq
rt(a))) + 2*((15*b^2 + 16*a*c)*x^4 + 10*a*b*x^2 + 8*a^2)*sqrt(c*x
^4 + b*x^2 - a)*sqrt(a))/(a^(7/2)*x^6)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{7}\sqrt{-a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**7/(c*x**4+b*x**2-a)**(1/2),x)
```

```
[Out] Integral(1/(x**7*sqrt(-a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [A] time = 0.521242, size = 158, normalized size = 1.03

$$\frac{1}{48}\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}\left(\frac{2\left(\frac{5b}{a^2}+\frac{4}{ax^2}\right)}{x^2}+\frac{15ab^2+16a^2c}{a^4}\right)+\frac{\left(5ab^3+12a^2bc\right)\ln\left(\left|-2\sqrt{-a}\left(\sqrt{c+\frac{b}{x^2}-\frac{a}{x^4}}-\frac{\sqrt{-a}}{x^2}\right)+b\right|\right)}{32\sqrt{-a}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 - a)*x^7),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c + b/x^2 - a/x^4)*(2*(5*b/a^2 + 4/(a*x^2))/x^2 + (15*a
*b^2 + 16*a^2*c)/a^4) + 1/32*(5*a*b^3 + 12*a^2*b*c)*ln(abs(-2*sqr
t(-a)*(sqrt(c + b/x^2 - a/x^4) - sqrt(-a)/x^2) + b))/(sqrt(-a)*a^
4)
```

3.976 $\int \frac{x^4}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=409

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} - \frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} - \frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

```
[Out] -(x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - (b*(b - Sqrt[b^2 + 4*a*c])*S
qrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a
*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSi
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a +
b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqr
t[b^2 + 4*a*c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[
1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/
(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x
^4])
```

Rubi [A] time = 1.40518, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} - \frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}c^{5/2}\sqrt{a+bx^2-cx^4}} - \frac{x\sqrt{a+bx^2-cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -(x*Sqrt[a + b*x^2 - c*x^4])/(3*c) - (b*(b - Sqrt[b^2 + 4*a*c])*S
qrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a
```

```
*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSi
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2
 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a +
b*x^2 - c*x^4]) + (Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqr
t[b^2 + 4*a*c])*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[
1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/
(b - Sqrt[b^2 + 4*a*c])])/(3*Sqrt[2]*c^(5/2)*Sqrt[a + b*x^2 - c*x
^4])
```

Rubi in Sympy [A] time = 117.025, size = 359, normalized size = 0.88

$$-\frac{\sqrt{2}b\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{6c^{\frac{5}{2}}\sqrt{a+bx^2-cx^4}} - \frac{x\sqrt{a+bx^2-cx^4}}{3c} + \frac{\sqrt{2}\sqrt{b+\sqrt{4ac+b^2}}\left(ac+b\left(b-\sqrt{4ac+b^2}\right)\right)\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{6c^{\frac{5}{2}}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(2)*b*(b - sqrt(4*a*c + b**2))*sqrt(b + sqrt(4*a*c + b**2))*
sqrt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(-2*c*x**2/(b +
sqrt(4*a*c + b**2)) + 1)*elliptic_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b
 + sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c
 + b**2)))/(6*c**(5/2)*sqrt(a + b*x**2 - c*x**4)) - x*sqrt(a + b*
x**2 - c*x**4)/(3*c) + sqrt(2)*sqrt(b + sqrt(4*a*c + b**2))*(a*c
+ b*(b - sqrt(4*a*c + b**2)))*sqrt(-2*c*x**2/(b - sqrt(4*a*c + b*
*2)) + 1)*sqrt(-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*elliptic_f
(asin(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), (b + sqrt(
4*a*c + b**2))/(b - sqrt(4*a*c + b**2)))/(6*c**(5/2)*sqrt(a + b*x
**2 - c*x**4))
```

Mathematica [C] time = 1.38583, size = 459, normalized size = 1.12

$$\frac{i\sqrt{2}\left(b\sqrt{4ac+b^2}-ac-b^2\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-i\sqrt{2}b\left(\sqrt{4ac\cdots}\right.}{6c^2\sqrt{-\frac{c}{\sqrt{4ac+b^2\cdots}}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (2*c*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x*(-a - b*x^2 + c*x^4) -
I*Sqrt[2]*b*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c]
- 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c]
+ 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*
Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b
 - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - a*c + b*Sqrt[b^2 + 4*a
*c])*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c
])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*
c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c])
)]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(6*c^2*S
qrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.057, size = 391, normalized size = 1.

$$-\frac{x}{3\,c}\sqrt{-cx^4+bx^2+a}$$
$$+\frac{a\sqrt{2}}{12\,c}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{\right.}}{a}}\right.$$
$$-\frac{ab\sqrt{2}}{3\,c}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*x*(-c*x^4+b*x^2+a)^(1/2)/c+1/12/c*a*2^(1/2)/((-b+(4*a*c+b^2)
^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF
(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4
*a*c+b^2)^(1/2))/a/c)^(1/2))-1/3*b/c*a*2^(1/2)/((-b+(4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b
^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1
/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b
^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{-cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")

[Out] integrate(x^4/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.977 $\int \frac{x^2}{\sqrt{a+bx^2-cx^4}} dx$

Optimal. Leaf size=377

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}} - \frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2
*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4]) + ((b - Sqrt[b^2 + 4*a
*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2
 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF
[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sq
rt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqr
t[a + b*x^2 - c*x^4])
```

Rubi [A] time = 0.852446, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}} - \frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}c^{3/2}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] -((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2
*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2
 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^
2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(
2*Sqrt[2]*c^(3/2)*Sqrt[a + b*x^2 - c*x^4]) + ((b - Sqrt[b^2 + 4*a
*c])*Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2
 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF
[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sq
```

```
rt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*c^(3/2)*Sqr
t[a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 96.4373, size = 332, normalized size = 0.88

$$-\frac{\sqrt{2}\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4c^{\frac{3}{2}}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{2}\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4c^{\frac{3}{2}}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(2)*(b - sqrt(4*a*c + b**2))*sqrt(b + sqrt(4*a*c + b**2))*sq
rt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(-2*c*x**2/(b + sq
rt(4*a*c + b**2)) + 1)*elliptic_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b +
 sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c +
 b**2)))/(4*c**(3/2)*sqrt(a + b*x**2 - c*x**4)) + sqrt(2)*(b - sq
rt(4*a*c + b**2))*sqrt(b + sqrt(4*a*c + b**2))*sqrt(-2*c*x**2/(b
- sqrt(4*a*c + b**2)) + 1)*sqrt(-2*c*x**2/(b + sqrt(4*a*c + b**2)
) + 1)*elliptic_f(asin(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b*
*2))), (b + sqrt(4*a*c + b**2))/(b - sqrt(4*a*c + b**2)))/(4*c**(
3/2)*sqrt(a + b*x**2 - c*x**4))
```

Mathematica [C] time = 0.215214, size = 271, normalized size = 0.72

$$-\frac{i\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b}}\right.\right.\right.}{2\sqrt{2}c\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I/2)*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^
2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*(Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b
+ Sqrt[b^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))] - EllipticF[I*Arc
Sinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b
```

```
^2 + 4*a*c])/(-b + Sqrt[b^2 + 4*a*c]))]))/(Sqrt[2]*c*Sqrt[-(c/(b
+ Sqrt[b^2 + 4*a*c]))]*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.014, size = 217, normalized size = 0.6

$$-\frac{a\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b
^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(
-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(
1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))
/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^2}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^2}{\sqrt{-cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-cx^4 + bx^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.978 $\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=169

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[Arc
Sin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a +
b*x^2 - c*x^4])
```

Rubi [A] time = 0.196483, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] (Sqrt[b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4
*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[Arc
Sin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b
^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[a +
b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 43.7662, size = 151, normalized size = 0.89

$$\frac{\sqrt{2}\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{2\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(b + sqrt(4*a*c + b**2))*sqrt(-2*c*x**2/(b - sqrt(4*a
*c + b**2)) + 1)*sqrt(-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*ell
iptic_f(asin(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), (b
+ sqrt(4*a*c + b**2))/(b - sqrt(4*a*c + b**2)))/(2*sqrt(c)*sqrt(a
 + b*x**2 - c*x**4))
```

Mathematica [C] time = 0.141634, size = 177, normalized size = 1.05

$$-\frac{i\sqrt{\frac{2cx^2}{\sqrt{4ac+b^2}-b}+1}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)|-\frac{b+\sqrt{b^2+4ac}}{\sqrt{b^2+4ac}-b}\right)}{\sqrt{2}\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b*x^2 - c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*
x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(
c/(b + Sqrt[b^2 + 4*a*c]))]*x], -((b + Sqrt[b^2 + 4*a*c])/(-b + S
qrt[b^2 + 4*a*c]))])/(Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*
Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.011, size = 145, normalized size = 0.9

$$\frac{\sqrt{2}}{4}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{4\,a\right.}}{ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)
^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(-c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^2-cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*x**2 - c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-c*x^4 + b*x^2 + a), x)
```

3.979 $\int \frac{1}{x^2\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=408

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]/(a*x)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[
b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])
]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4
*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*
x^2 - c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*
c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 1.01604, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286

$$-\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{2\sqrt{2}a\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{\sqrt{a+bx^2-cx^4}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -(Sqrt[a + b*x^2 - c*x^4]/(a*x)) + ((b - Sqrt[b^2 + 4*a*c])*Sqrt[
b + Sqrt[b^2 + 4*a*c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])
]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(S
```

```
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4
*a*c])/(b - Sqrt[b^2 + 4*a*c])])/(2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*
x^2 - c*x^4]) - ((b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*
c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(2*Sqrt[2]*a*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 117.214, size = 354, normalized size = 0.87

$$-\frac{\sqrt{a+bx^2-cx^4}}{ax}+\frac{\sqrt{2}\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4a\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{\sqrt{2}\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{4a\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2 - c*x**4)/(a*x) + sqrt(2)*(b - sqrt(4*a*c + b**2
))*sqrt(b + sqrt(4*a*c + b**2))*sqrt(-2*c*x**2/(b - sqrt(4*a*c +
b**2)) + 1)*sqrt(-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*elliptic
_e(asin(sqrt(2)*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), (b + sqr
t(4*a*c + b**2))/(b - sqrt(4*a*c + b**2)))/(4*a*sqrt(c)*sqrt(a +
b*x**2 - c*x**4)) - sqrt(2)*(b - sqrt(4*a*c + b**2))*sqrt(b + sqr
t(4*a*c + b**2))*sqrt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqr
t(-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*elliptic_f(asin(sqrt(2)
*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2)
)/(b - sqrt(4*a*c + b**2)))/(4*a*sqrt(c)*sqrt(a + b*x**2 - c*x**4
))
```

Mathematica [C] time = 0.824807, size = 283, normalized size = 0.69

$$\frac{\frac{i\left(\sqrt{4ac+b^2}-b\right)\sqrt{\frac{4cx^2}{\sqrt{4ac+b^2}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}\left(E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)-F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{c}{b+\sqrt{b^2+4ac}}}x\right)\middle|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)\right)}{\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}}-\frac{4a}{x}-4bx+}{4a\sqrt{a+bx^2-cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] ((-4*a)/x - 4*b*x + 4*c*x^3 + (I*(-b + Sqrt[b^2 + 4*a*c])*Sqrt[2
+ (4*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqr
t[b^2 + 4*a*c])]*(EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[
b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c
])] - EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]
))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])]))/Sqrt[-
(c/(b + Sqrt[b^2 + 4*a*c]))])/(4*a*Sqrt[a + b*x^2 - c*x^4])
```

Maple [A] time = 0.019, size = 241, normalized size = 0.6

$$-\frac{1}{ax}\sqrt{-cx^4+bx^2+a}$$
$$+\frac{c\sqrt{2}}{2}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -(-c*x^4+b*x^2+a)^(1/2)/a/x+1/2*c*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))
/a)^(1/2)*(4-2*(-b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(
1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(
1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4+bx^2+a}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-cx^4 + bx^2 + a}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2),x, algorithm="fricas")

[Out] integral(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(-c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a + b*x**2 - c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2),x, algorithm="giac")

[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^2), x)
```

3.980 $\int \frac{1}{x^4\sqrt{a+bx^2-cx^4}}\,dx$

Optimal. Leaf size=445

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 - c*x^4]
)/(3*a^2*x) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*
c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]) + (
Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*c])*Sqr
t[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a
*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi [A] time = 1.33006, antiderivative size = 445, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{\sqrt{\sqrt{4ac+b^2}+b}\left(-b\sqrt{4ac+b^2}+ac+b^2\right)\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}F\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}-\frac{b\left(b-\sqrt{4ac+b^2}\right)\sqrt{\sqrt{4ac+b^2}+b}\sqrt{1-\frac{2cx^2}{b-\sqrt{4ac+b^2}}}\sqrt{1-\frac{2cx^2}{\sqrt{4ac+b^2}+b}}E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2+4ac}}}\right)|\frac{b+\sqrt{b^2+4ac}}{b-\sqrt{b^2+4ac}}\right)}{3\sqrt{2}a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2 - c*x^4]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2 - c*x^4]
)/(3*a^2*x) - (b*(b - Sqrt[b^2 + 4*a*c])*Sqrt[b + Sqrt[b^2 + 4*a*
```

```
c]]*Sqrt[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2
)/(b + Sqrt[b^2 + 4*a*c])]*EllipticE[ArcSin[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^
2 + 4*a*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4]) + (
Sqrt[b + Sqrt[b^2 + 4*a*c]]*(b^2 + a*c - b*Sqrt[b^2 + 4*a*c])*Sqr
t[1 - (2*c*x^2)/(b - Sqrt[b^2 + 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b +
Sqrt[b^2 + 4*a*c])]*EllipticF[ArcSin[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 + 4*a*c]]], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 + 4*a
*c])])/(3*Sqrt[2]*a^2*Sqrt[c]*Sqrt[a + b*x^2 - c*x^4])
```

Rubi in Sympy [A] time = 158.343, size = 393, normalized size = 0.88

$$-\frac{\sqrt{a+bx^2-cx^4}}{3ax^3}+\frac{2b\sqrt{a+bx^2-cx^4}}{3a^2x}-\frac{\sqrt{2}b\left(b-\sqrt{4ac+b^2}\right)\sqrt{b+\sqrt{4ac+b^2}}\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{6a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}+\frac{\sqrt{2}\sqrt{b+\sqrt{4ac+b^2}}\left(ac+b\left(b-\sqrt{4ac+b^2}\right)\right)\sqrt{-\frac{2cx^2}{b-\sqrt{4ac+b^2}}+1}\sqrt{-\frac{2cx^2}{b+\sqrt{4ac+b^2}}+1}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{4ac+b^2}}}\right)\middle|\frac{b+\sqrt{4ac+b^2}}{b-\sqrt{4ac+b^2}}\right)}{6a^2\sqrt{c}\sqrt{a+bx^2-cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2 - c*x**4)/(3*a*x**3) + 2*b*sqrt(a + b*x**2 - c*x
**4)/(3*a**2*x) - sqrt(2)*b*(b - sqrt(4*a*c + b**2))*sqrt(b + sqr
t(4*a*c + b**2))*sqrt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqr
t(-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*elliptic_e(asin(sqrt(2)
*sqrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2)
)/(b - sqrt(4*a*c + b**2)))/(6*a**2*sqrt(c)*sqrt(a + b*x**2 - c*x
**4)) + sqrt(2)*sqrt(b + sqrt(4*a*c + b**2))*(a*c + b*(b - sqrt(4
*a*c + b**2)))*sqrt(-2*c*x**2/(b - sqrt(4*a*c + b**2)) + 1)*sqrt(
-2*c*x**2/(b + sqrt(4*a*c + b**2)) + 1)*elliptic_f(asin(sqrt(2)*s
qrt(c)*x/sqrt(b + sqrt(4*a*c + b**2))), (b + sqrt(4*a*c + b**2))/
(b - sqrt(4*a*c + b**2)))/(6*a**2*sqrt(c)*sqrt(a + b*x**2 - c*x**
4))
```

Mathematica [C] time = 1.40933, size = 472, normalized size = 1.06

$$\frac{-2\sqrt{-\frac{c}{\sqrt{4ac+b^2}+b}}\left(a-2bx^2\right)\left(a+bx^2-cx^4\right)+i\sqrt{2}x^3\left(b\sqrt{4ac+b^2}-ac-b^2\right)\sqrt{\frac{\sqrt{4ac+b^2}+b-2cx^2}{\sqrt{4ac+b^2}+b}}\sqrt{\frac{\sqrt{4ac+b^2}-b+2cx^2}{\sqrt{4ac+b^2}-b}}F\left(i\sinh^{-1}\right.}{6a^2x^3\sqrt{-}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + b*x^2 - c*x^4]),x]
```

```
[Out] (-2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*(a - 2*b*x^2)*(a + b*x^2 -
 c*x^4) - I*Sqrt[2]*b*(-b + Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt
[b^2 + 4*a*c] - 2*c*x^2)/(b + Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt
[b^2 + 4*a*c] + 2*c*x^2)/(-b + Sqrt[b^2 + 4*a*c])]*EllipticE[I*Ar
cSinh[Sqrt[2]*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^
2 + 4*a*c])/(b - Sqrt[b^2 + 4*a*c])] + I*Sqrt[2]*(-b^2 - a*c + b*
Sqrt[b^2 + 4*a*c])*x^3*Sqrt[(b + Sqrt[b^2 + 4*a*c] - 2*c*x^2)/(b
+ Sqrt[b^2 + 4*a*c])]*Sqrt[(-b + Sqrt[b^2 + 4*a*c] + 2*c*x^2)/(-b
 + Sqrt[b^2 + 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[-(c/(b +
Sqrt[b^2 + 4*a*c]))]*x], (b + Sqrt[b^2 + 4*a*c])/(b - Sqrt[b^2 +
4*a*c])])/(6*a^2*Sqrt[-(c/(b + Sqrt[b^2 + 4*a*c]))]*x^3*Sqrt[a +
b*x^2 - c*x^4])
```

Maple [A] time = 0.021, size = 417, normalized size = 0.9

$$-\frac{1}{3\,ax^3}\sqrt{-cx^4+bx^2+a}+\frac{2\,b}{3\,a^2x}\sqrt{-cx^4+bx^2+a}$$
$$+\frac{c\sqrt{2}}{12\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\sqrt{\right.}}{a}}\right.$$
$$-\frac{bc\sqrt{2}}{3\,a}\sqrt{4-2\,\frac{\left(-b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4-2\,\frac{b\left(b+\right.}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(-c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] -1/3*(-c*x^4+b*x^2+a)^(1/2)/a/x^3+2/3*b*(-c*x^4+b*x^2+a)^(1/2)/a^
2/x+1/12*c/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4
*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(
1/2)/(-c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2))
-1/3*b*c/a*2^(1/2)/((-b+(4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(4*a
*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)/(-c*x^4+b*x^2+a)^(1/2)/(b+(4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*
2^(1/2)*((-b+(4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^
2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4-2*b*(b+(4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{-cx^4 + bx^2 + a}x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{a + bx^2 - cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(-c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a + b*x**2 - c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-cx^4 + bx^2 + a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c*x^4 + b*x^2 + a)*x^4), x)
```

3.981 $\int \frac{x^9}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=190

$$\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5b^2-12ac\right)\right)\sqrt{a+bx^2+cx^4}}{8c^3\left(b^2-4ac\right)}-\frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
x^4*Sqrt[a + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52
*a*c) - 2*c*(5*b^2 - 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3
*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7/2))
```

Rubi [A] time = 0.537616, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{7/2}}-\frac{\left(b\left(15b^2-52ac\right)-2cx^2\left(5b^2-12ac\right)\right)\sqrt{a+bx^2+cx^4}}{8c^3\left(b^2-4ac\right)}-\frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{x^6\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^6*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
x^4*Sqrt[a + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) - ((b*(15*b^2 - 52
*a*c) - 2*c*(5*b^2 - 12*a*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^3
*(b^2 - 4*a*c)) + (3*(5*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*c^(7/2))
```

Rubi in Sympy [A] time = 40.9247, size = 177, normalized size = 0.93

$$-\frac{bx^4\sqrt{a+bx^2+cx^4}}{c\left(-4ac+b^2\right)}+\frac{x^6\left(2a+bx^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(b\left(-39ac+\frac{45b^2}{4}\right)-\frac{3cx^2\left(-12ac+5b^2\right)}{2}\right)\sqrt{a+bx^2+cx^4}}{6c^3\left(-4ac+b^2\right)}+\frac{3\left(-4ac+5b^2\right)\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -b*x**4*sqrt(a + b*x**2 + c*x**4)/(c*(-4*a*c + b**2)) + x**6*(2*a
 + b*x**2)/((-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) - (b*(-39*
a*c + 45*b**2/4) - 3*c*x**2*(-12*a*c + 5*b**2)/2)*sqrt(a + b*x**2
 + c*x**4)/(6*c**3*(-4*a*c + b**2)) + 3*(-4*a*c + 5*b**2)*atanh((
b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(16*c**(7/2)
)
```

Mathematica [A] time = 0.235533, size = 147, normalized size = 0.77

$$\frac{\sqrt{a+bx^2+cx^4}\left(-\frac{8\left(a^2c\left(2cx^2-3b\right)+ab^2\left(b-4cx^2\right)+b^4x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-7b+2cx^2\right)}{8c^3}+\frac{3\left(5b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{16c^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^9/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-7*b + 2*c*x^2 - (8*(b^4*x^2 + a*b^2*(b
 - 4*c*x^2) + a^2*c*(-3*b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2
+ c*x^4))))/(8*c^3) + (3*(5*b^2 - 4*a*c)*Log[b + 2*c*x^2 + 2*Sqrt
[c]*Sqrt[a + b*x^2 + c*x^4]])/(16*c^(7/2))
```

Maple [B] time = 0.023, size = 354, normalized size = 1.9

$$\frac{x^6}{4\,c}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{5\,bx^4}{8\,c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{15\,b^2x^2}{16\,c^3}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{15\,b^3}{32\,c^4}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{15\,b^4x^2}{16\,c^3\,(4\,ac-b^2)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{15\,b^5}{32\,c^4\,(4\,ac-b^2)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{15\,b^2}{16}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{7}{2}}-\frac{13\,ab}{8\,c^3}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{13\,ab^2x^2}{4\,(4\,ac-b^2)\,c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{13\,ab^3}{8\,c^3\,(4\,ac-b^2)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{3\,ax^2}{4\,c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3\,a}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^9/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x^6/c/(c*x^4+b*x^2+a)^(1/2)-5/8*b/c^2*x^4/(c*x^4+b*x^2+a)^(1/
2)-15/16*b^2/c^3*x^2/(c*x^4+b*x^2+a)^(1/2)+15/32*b^3/c^4/(c*x^4+b
*x^2+a)^(1/2)+15/16*b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2
+15/32*b^5/c^4/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+15/16*b^2/c^(7/2
)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-13/8*b/c^3*a/(c
*x^4+b*x^2+a)^(1/2)-13/4*b^2/c^2*a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1
/2)*x^2-13/8*b^3/c^3*a/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4/c^2*
a*x^2/(c*x^4+b*x^2+a)^(1/2)-3/4/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2
)+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.357921, size = 1, normalized size = 0.01

$$\left[\frac{4\left(2\left(b^2c^2-4\,ac^3\right)x^6-5\left(b^3c-4\,abc^2\right)x^4-15\,ab^3+52\,a^2bc-\left(15\,b^4-62\,ab^2c+24\,a^2c^2\right)x^2\right)\sqrt{cx^4+bx^2+a}\sqrt{c}-3\left(5\,ab\right.}{32\left(ab^2c^3-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(4*(2*(b^2*c^2 - 4*a*c^3)*x^6 - 5*(b^3*c - 4*a*b*c^2)*x^4 -
 15*a*b^3 + 52*a^2*b*c - (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)*x^2)*
sqrt(c*x^4 + b*x^2 + a)*sqrt(c) - 3*(5*a*b^4 - 24*a^2*b^2*c + 16*
a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (5*b^5 - 24
*a*b^3*c + 16*a^2*b*c^2)*x^2)*log(4*sqrt(c*x^4 + b*x^2 + a)*(2*c^
2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/((
a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*
b*c^4)*x^2)*sqrt(c)), 1/16*(2*(2*(b^2*c^2 - 4*a*c^3)*x^6 - 5*(b^3
*c - 4*a*b*c^2)*x^4 - 15*a*b^3 + 52*a^2*b*c - (15*b^4 - 62*a*b^2*
c + 24*a^2*c^2)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-c) + 3*(5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2 + (5*b^4*c - 24*a*b^2*c^2 + 16*a^2*
```

```
c^3)*x^4 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^2)*arctan(1/2*(2
*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/((a*b^2*c^3 -
4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2)*
sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^9}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**9/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**9/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.334651, size = 490, normalized size = 2.58

$$\frac{\left(\left(\frac{2\left(b^4c^3-8\,ab^2c^4+16\,a^2c^5\right)x^2}{b^4c^4-8\,ab^2c^5+16\,a^2c^6}-\frac{5\left(b^5c^2-8\,ab^3c^3+16\,a^2bc^4\right)}{b^4c^4-8\,ab^2c^5+16\,a^2c^6}\right)x^2-\frac{15\,b^6c-122\,ab^4c^2+272\,a^2b^2c^3-96\,a^3c^4}{b^4c^4-8\,ab^2c^5+16\,a^2c^6}\right)x^2-\frac{15\,ab^5c-112\,a^2b^3c^2+208\,a^3bc^3}{b^4c^4-8\,ab^2c^5+16\,a^2c^6}}{8\sqrt{cx^4+bx^2+a}}-\frac{3\left(5\,b^6c-44\,ab^4c^2+112\,a^2b^2c^3-64\,a^3c^4\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{16\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^9/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(((2*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^2/(b^4*c^4 - 8*a*
b^2*c^5 + 16*a^2*c^6) - 5*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)/
(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x^2 - (15*b^6*c - 122*a*b^4
*c^2 + 272*a^2*b^2*c^3 - 96*a^3*c^4)/(b^4*c^4 - 8*a*b^2*c^5 + 16*
a^2*c^6))*x^2 - (15*a*b^5*c - 112*a^2*b^3*c^2 + 208*a^3*b*c^3)/(b
^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/sqrt(c*x^4 + b*x^2 + a) - 3/1
6*(5*b^6*c - 44*a*b^4*c^2 + 112*a^2*b^2*c^3 - 64*a^3*c^4)*ln(abs(
-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/((b^4*c^
4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(c))
```

3.982 $\int \frac{x^7}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=134

$$\frac{(-8ac+3b^2-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)}+\frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3
*b^2 - 8*a*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 -
4*a*c)) - (3*b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
c*x^4])])/(4*c^(5/2))
```

Rubi [A] time = 0.254923, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{(-8ac+3b^2-2bcx^2)\sqrt{a+bx^2+cx^4}}{2c^2(b^2-4ac)}+\frac{x^4(2a+bx^2)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{3b\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + ((3
*b^2 - 8*a*c - 2*b*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(2*c^2*(b^2 -
4*a*c)) - (3*b*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 +
c*x^4])])/(4*c^(5/2))
```

Rubi in Sympy [A] time = 25.3105, size = 124, normalized size = 0.93

$$-\frac{3b\,\mathrm{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{\frac{5}{2}}}+\frac{x^4(2a+bx^2)}{(-4ac+b^2)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{a+bx^2+cx^4}(-8ac+3b^2-2bcx^2)}{2c^2(-4ac+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -3*b*atanh((b + 2*c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/
(4*c**(5/2)) + x**4*(2*a + b*x**2)/((-4*a*c + b**2)*sqrt(a + b*x*
```

```
*2 + c*x**4)) + sqrt(a + b*x**2 + c*x**4)*(-8*a*c + 3*b**2 - 2*b*
c*x**2)/(2*c**2*(-4*a*c + b**2))
```

Mathematica [A] time = 0.181717, size = 127, normalized size = 0.95

$$\frac{1}{2}\sqrt{a+bx^2+cx^4}\left(\frac{2\left(2a^2c-ab^2+3abcx^2+b^3\left(-x^2\right)\right)}{c^2\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}+\frac{1}{c^2}\right)-\frac{3b\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^7/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(c^(-2) + (2*(-(a*b^2) + 2*a^2*c - b^3*x
^2 + 3*a*b*c*x^2))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4))))/2 -
 (3*b*Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(4*c^
(5/2))
```

Maple [B] time = 0.019, size = 264, normalized size = 2.

$$\frac{x^4}{2c}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{3bx^2}{4c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3b^2}{8c^3}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3b^3x^2}{4\left(4ac-b^2\right)c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3b^4}{8c^3\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3b}{4}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{5}{2}}+\frac{a}{c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+2\frac{abx^2}{\left(4ac-b^2\right)c\sqrt{cx^4+bx^2+a}}+\frac{ab^2}{\left(4ac-b^2\right)c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2*x^4/c/(c*x^4+b*x^2+a)^(1/2)+3/4*b/c^2*x^2/(c*x^4+b*x^2+a)^(1/
2)-3/8*b^2/c^3/(c*x^4+b*x^2+a)^(1/2)-3/4*b^3/c^2/(4*a*c-b^2)/(c*x
^4+b*x^2+a)^(1/2)*x^2-3/8*b^4/c^3/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/
2)-3/4*b/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+
1/c^2*a/(c*x^4+b*x^2+a)^(1/2)+2/c*a*b/(4*a*c-b^2)/(c*x^4+b*x^2+a)
^(1/2)*x^2+1/c^2*a*b^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.325895, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(b^2c-4\,ac^2\right)x^4+3\,ab^2-8\,a^2c+\left(3\,b^3-10\,abc\right)x^2\right)\sqrt{cx^4+bx^2+a}\sqrt{c}+3\left(\left(b^3c-4\,abc^2\right)x^4+ab^3-4\,a^2bc+\left(b^4-4\,ab\right.\right.}{8\left(ab^2c^2-4\,a^2c^3+(b^2c^3-4\,ac^4)x^4+(b^3c^2-4\,abc^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*((b^2*c - 4*a*c^2)*x^4 + 3*a*b^2 - 8*a^2*c + (3*b^3 - 10*
a*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(c) + 3*((b^3*c - 4*a*b*c
^2)*x^4 + a*b^3 - 4*a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*log(4*sqrt(c
*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4 + 8*b*c*x^2 + b^
2 + 4*a*c)*sqrt(c)))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4
)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(c)), 1/4*(2*((b^2*c - 4*a
*c^2)*x^4 + 3*a*b^2 - 8*a^2*c + (3*b^3 - 10*a*b*c)*x^2)*sqrt(c*x^
4 + b*x^2 + a)*sqrt(-c) - 3*((b^3*c - 4*a*b*c^2)*x^4 + a*b^3 - 4*
a^2*b*c + (b^4 - 4*a*b^2*c)*x^2)*arctan(1/2*(2*c*x^2 + b)*sqrt(-c
)/(sqrt(c*x^4 + b*x^2 + a)*c)))/((a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^
3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.331666, size = 359, normalized size = 2.68

$$\frac{\left(\frac{\left(b^4c-8\,ab^2c^2+16\,a^2c^3\right)x^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}+\frac{3\,b^5-22\,ab^3c+40\,a^2bc^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}\right)x^2+\frac{3\,ab^4-20\,a^2b^2c+32\,a^3c^2}{b^4c^2-8\,ab^2c^3+16\,a^2c^4}}{2\,\sqrt{cx^4+bx^2+a}}+\frac{3\,\left(b^5-8\,ab^3c+16\,a^2bc^2\right)\ln\left(\left|-2\,\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{4\,\left(b^4c^2-8\,ab^2c^3+16\,a^2c^4\right)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^7/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^2/(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4) + (3*b^5 - 22*a*b^3*c + 40*a^2*b*c^2)/(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4))*x^2 + (3*a*b^4 - 20*a^2*b^2*c + 32*a^
3*c^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/sqrt(c*x^4 + b*x^2 +
 a) + 3/4*(b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*ln(abs(-2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/((b^4*c^2 - 8*a*b^2*c^3
 + 16*a^2*c^4)*sqrt(c))
```

3.983 $\int \frac{x^5}{(a+bx^2+cx^4)^{3/2}}\, dx$

Optimal. Leaf size=115

$$\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
Sqrt[a + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2
)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rubi [A] time = 0.189373, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{x^2\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^2*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
Sqrt[a + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + ArcTanh[(b + 2*c*x^2
)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2))
```

Rubi in Sympy [A] time = 24., size = 100, normalized size = 0.87

$$-\frac{b\sqrt{a+bx^2+cx^4}}{c\left(-4ac+b^2\right)}+\frac{x^2\left(2a+bx^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{\operatorname{atanh}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -b*sqrt(a + b*x**2 + c*x**4)/(c*(-4*a*c + b**2)) + x**2*(2*a + b*
x**2)/((-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) + atanh((b + 2*
c*x**2)/(2*sqrt(c)*sqrt(a + b*x**2 + c*x**4)))/(2*c**(3/2))
```

Mathematica [A] time = 0.140785, size = 92, normalized size = 0.8

$$\frac{ab-2acx^2+b^2x^2}{c\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{\log\left(2\sqrt{c}\sqrt{a+bx^2+cx^4}+b+2cx^2\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^5/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*b + b^2*x^2 - 2*a*c*x^2)/(c*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*
x^4]) + Log[b + 2*c*x^2 + 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]/(2*c
^(3/2))
```

Maple [A] time = 0.019, size = 149, normalized size = 1.3

$$-\frac{x^2}{2\,c}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{b}{4\,c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{b^2x^2}{2\,(4\,ac-b^2)\,c}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{b^3}{4\,(4\,ac-b^2)\,c^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{1}{2}\ln\left(1\left(\frac{b}{2}+cx^2\right)\frac{1}{\sqrt{c}}+\sqrt{cx^4+bx^2+a}\right)c^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4*b/c^2/(c*x^4+b*x^2+a)^(1/2)+
1/2*b^2/c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*b^3/c^2/(4*a*
c-b^2)/(c*x^4+b*x^2+a)^(1/2)+1/2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.312969, size = 1, normalized size = 0.01

$$\left[-\frac{4\sqrt{cx^4+bx^2+a}\left(\left(b^2-2\,ac\right)x^2+ab\right)\sqrt{c}-\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\log\left(-4\sqrt{cx^4+bx^2+a}\left(2\,c^2\right.\right.}{4\,((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2)\sqrt{c}}\right.$$

$$\left.-\frac{2\sqrt{cx^4+bx^2+a}\left(\left(b^2-2\,ac\right)x^2+ab\right)\sqrt{-c}-\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\arctan\left(\frac{(2\,cx^2+b)\sqrt{-c}}{2\sqrt{cx^4+bx^2+a}c}\right)}{2\,((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2)\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(c*x^4 + b*x^2 + a)*((b^2 - 2*a*c)*x^2 + a*b)*sqrt(c
) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*log(-4*sqrt(c*x^4 + b*x^2 + a)*(2*c^2*x^2 + b*c) - (8*c^2*x^4
+ 8*b*c*x^2 + b^2 + 4*a*c)*sqrt(c)))/(((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(c)), -1/2*(2*
sqrt(c*x^4 + b*x^2 + a)*((b^2 - 2*a*c)*x^2 + a*b)*sqrt(-c) - ((b^
2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*arcta
n(1/2*(2*c*x^2 + b)*sqrt(-c)/(sqrt(c*x^4 + b*x^2 + a)*c)))/(((b^2
*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x
^2)*sqrt(-c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.328639, size = 255, normalized size = 2.22

$$-\frac{\frac{\left(b^4-6\,ab^2c+8\,a^2c^2\right)x^2}{b^4c-8\,ab^2c^2+16\,a^2c^3}+\frac{ab^3-4\,a^2bc}{b^4c-8\,ab^2c^2+16\,a^2c^3}}{\sqrt{cx^4+bx^2+a}}-\frac{\left(b^4-8\,ab^2c+16\,a^2c^2\right)\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{2\,(b^4c-8\,ab^2c^2+16\,a^2c^3)\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^5/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -((b^4 - 6*a*b^2*c + 8*a^2*c^2)*x^2/(b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3) + (a*b^3 - 4*a^2*b*c)/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3))/s
qrt(c*x^4 + b*x^2 + a) - 1/2*(b^4 - 8*a*b^2*c + 16*a^2*c^2)*ln(ab
s(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/((b^4*
c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(c))
```

3.984 $\int \frac{x^3}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.0684588, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/(a + b*x^2 + c*x^4)^(3/2),x]

[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 9.65654, size = 34, normalized size = 0.94

$$\frac{4a+2bx^2}{2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)

[Out] (4*a + 2*b*x**2)/(2*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0346583, size = 36, normalized size = 1.

$$\frac{2a+bx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a + b*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.008, size = 38, normalized size = 1.1

$$-\frac{bx^2+2a}{4ac-b^2}\frac{1}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -(b*x^2+2*a)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.291994, size = 90, normalized size = 2.5

$$\frac{\sqrt{cx^4+bx^2+a}\left(bx^2+2a\right)}{(b^2c-4ac^2)x^4+ab^2-4a^2c+(b^3-4abc)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)/((b^2*c - 4*a*c^2)*x^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(x**3/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.312015, size = 59, normalized size = 1.64

$$\frac{\frac{bx^2}{b^2-4\,ac}+\frac{2\,a}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")

[Out] (b*x^2/(b^2 - 4*a*c) + 2*a/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a)
```

3.985 $\int \frac{x}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=36

$$-\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rubi [A] time = 0.0492076, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(a + b*x^2 + c*x^4)^(3/2),x]

[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Rubi in Sympy [A] time = 6.18673, size = 36, normalized size = 1.

$$-\frac{2b+4cx^2}{2\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2+a)**(3/2),x)

[Out] -(2*b + 4*c*x**2)/(2*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [A] time = 0.0260914, size = 36, normalized size = 1.

$$-\frac{b+2cx^2}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((b + 2*c*x^2)/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]))
```

Maple [A] time = 0.006, size = 36, normalized size = 1.

$$\frac{2\,c x^{2}+b}{4\,a c -b^{2}}\frac{1}{\sqrt{c x^{4}+b x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] (2*c*x^2+b)/(c*x^4+b*x^2+a)^(1/2)/(4*a*c-b^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.29328, size = 90, normalized size = 2.5

$$-\frac{\sqrt{c x^{4}+b x^{2}+a}\left(2\,c x^{2}+b\right)}{(b^{2} c -4\,a c^{2}) x^{4}+a b^{2}-4\,a^{2} c +(b^{3}-4\,a b c) x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)/((b^2*c - 4*a*c^2)*x^4 + a
*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.314461, size = 61, normalized size = 1.69

$$-\frac{\frac{2\,cx^2}{b^2-4\,ac}+\frac{b}{b^2-4\,ac}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] -(2*c*x^2/(b^2 - 4*a*c) + b/(b^2 - 4*a*c))/sqrt(c*x^4 + b*x^2 + a
)
```

3.986 $\int \frac{1}{x(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
 - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*
a^(3/2))
```

Rubi [A] time = 0.170805, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{-2ac+b^2+bcx^2}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
 - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(2*
a^(3/2))
```

Rubi in Sympy [A] time = 19.3991, size = 78, normalized size = 0.88

$$\frac{-2ac+b^2+bcx^2}{a\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(a*(-4*a*c + b**2)*sqrt(a + b*x**2 + c
*x**4)) - atanh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**
4)))/(2*a**(3/2))
```

Mathematica [A] time = 0.274109, size = 100, normalized size = 1.12

$$\frac{\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+x^2\left(b+cx^2\right)}+2a+bx^2\right)}{2a^{3/2}}+\frac{2ac-b^2-bcx^2}{a\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-b^2 + 2*a*c - b*c*x^2)/(a*(-b^2 + 4*a*c)*Sqrt[a + b*x^2 + c*x^4
]) + (Log[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + x^2*(b + c*
x^2)]])/(2*a^(3/2))
```

Maple [A] time = 0.018, size = 99, normalized size = 1.1

$$\frac{1}{2\,a}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{b\left(2\,cx^2+b\right)}{2\,a\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] 1/2/a/(c*x^4+b*x^2+a)^(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^
4+b*x^2+a)^(1/2)-1/2/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2
+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.317143, size = 1, normalized size = 0.01

$$\left[\frac{4\sqrt{cx^4+bx^2+a}\left(bcx^2+b^2-2\,ac\right)\sqrt{a}+\left(\left(b^2c-4\,ac^2\right)x^4+ab^2-4\,a^2c+\left(b^3-4\,abc\right)x^2\right)\log\left(\frac{4\sqrt{cx^4+bx^2+a}\left(abx^2+2\,a^2\right)-\left((b^2+\right.}{x^4}\right.}{4\left(\left(ab^2c-4\,a^2c^2\right)x^4+a^2b^2-4\,a^3c+\left(ab^3-4\,a^2bc\right)x^2\right)\sqrt{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(c*x^4 + b*x^2 + a)*(b*c*x^2 + b^2 - 2*a*c)*sqrt(a) +
 ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
log((4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) - ((b^2 + 4*a*c)
*x^4 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4))/(((a*b^2*c - 4*a^2*c^2)*
x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(a)), 1/2*
(2*sqrt(c*x^4 + b*x^2 + a)*(b*c*x^2 + b^2 - 2*a*c)*sqrt(-a) - ((b
^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*arct
an(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)))/(((a*
b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*
x^2)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x), x)
```

3.987 $\int \frac{1}{x^3(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=139

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2x^2(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x
^4]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*
a*c)*x^2) + (3*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
+ c*x^4])])/(4*a^(5/2))
```

Rubi [A] time = 0.299634, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2x^2(b^2-4ac)}+\frac{-2ac+b^2+bcx^2}{ax^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x
^4]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*
a*c)*x^2) + (3*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2
+ c*x^4])])/(4*a^(5/2))
```

Rubi in Sympy [A] time = 31.865, size = 128, normalized size = 0.92

$$\frac{-2ac+b^2+bcx^2}{ax^2(-4ac+b^2)\sqrt{a+bx^2+cx^4}}-\frac{(-8ac+3b^2)\sqrt{a+bx^2+cx^4}}{2a^2x^2(-4ac+b^2)}+\frac{3b\,\mathrm{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(a*x**2*(-4*a*c + b**2)*sqrt(a + b*x**
2 + c*x**4)) - (-8*a*c + 3*b**2)*sqrt(a + b*x**2 + c*x**4)/(2*a**
2*x**2*(-4*a*c + b**2)) + 3*b*atanh((2*a + b*x**2)/(2*sqrt(a)*sqr
```

```
t(a + b*x**2 + c*x**4)))/(4*a**(5/2))
```

Mathematica [A] time = 0.199472, size = 132, normalized size = 0.95

$$\frac{-4a^2c+a\left(b^2-10bcx^2-8c^2x^4\right)+3b^2x^2\left(b+cx^2\right)}{2a^2x^2\left(4ac-b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{3b\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{4a^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-4*a^2*c + 3*b^2*x^2*(b + c*x^2) + a*(b^2 - 10*b*c*x^2 - 8*c^2*x
^4))/(2*a^2*(-b^2 + 4*a*c)*x^2*Sqrt[a + b*x^2 + c*x^4]) - (3*b*(L
og[x^2] - Log[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/
(4*a^(5/2))
```

Maple [A] time = 0.019, size = 195, normalized size = 1.4

$$-\frac{1}{2\,ax^2}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3\,b}{4\,a^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{3\,b^2cx^2}{2\,a^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{3\,b^3}{4\,a^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{3\,b}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}-2\,\frac{c\left(2\,cx^2+b\right)}{a\left(4\,ac-b^2\right)\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/2/a/x^2/(c*x^4+b*x^2+a)^(1/2)-3/4*b/a^2/(c*x^4+b*x^2+a)^(1/2)+
3/2*b^2/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*c*x^2+3/4*b^3/a^2/(
4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)+3/4*b/a^(5/2)*ln((2*a+b*x^2+2*a^
(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-2*c/a*(2*c*x^2+b)/(4*a*c-b^2)/(
c*x^4+b*x^2+a)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.3242, size = 1, normalized size = 0.01

$$\left[-\frac{4\left(\left(3\,b^2c-8\,ac^2\right)x^4+ab^2-4\,a^2c+\left(3\,b^3-10\,abc\right)x^2\right)\sqrt{cx^4+bx^2+a}\sqrt{a}-3\left(\left(b^3c-4\,abc^2\right)x^6+\left(b^4-4\,ab^2c\right)x^4+\left(ab^3\right.\right.}{8\left((a^2b^2c-4\,a^3c^2)x^6+(a^2b^3-4\,a^3bc)x^4+(a^3b^2-4\,a^4c)\right.}\right.$$

$$-\frac{2\left(\left(3\,b^2c-8\,ac^2\right)x^4+ab^2-4\,a^2c+\left(3\,b^3-10\,abc\right)x^2\right)\sqrt{cx^4+bx^2+a}\sqrt{-a}-3\left(\left(b^3c-4\,abc^2\right)x^6+\left(b^4-4\,ab^2c\right)x^4+\left(ab\right.\right.}{4\left((a^2b^2c-4\,a^3c^2)x^6+(a^2b^3-4\,a^3bc)x^4+(a^3b^2-4\,a^4c)x^2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^3),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*((3*b^2*c - 8*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (3*b^3 - 10
*a*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(a) - 3*((b^3*c - 4*a*b*
c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*x^2)*log(-
(4*sqrt(c*x^4 + b*x^2 + a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 + 8*a^2)*sqrt(a))/x^4))/(((a^2*b^2*c - 4*a^3*c^2)*x^
6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2)*sqrt(a))
, -1/4*(2*((3*b^2*c - 8*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (3*b^3 - 1
0*a*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a)*sqrt(-a) - 3*((b^3*c - 4*a*
b*c^2)*x^6 + (b^4 - 4*a*b^2*c)*x^4 + (a*b^3 - 4*a^2*b*c)*x^2)*arc
tan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x^2 + a)*a)))/(((a
^2*b^2*c - 4*a^3*c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2
- 4*a^4*c)*x^2)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^3), x)
```

3.988 $\int \frac{1}{x^5(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=195

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x
^4]) - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4
*a*c)*x^4) + (b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3
*(b^2 - 4*a*c)*x^2) - (3*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2
*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(7/2))
```

Rubi [A] time = 0.483718, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{3\left(5b^2-4ac\right)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{7/2}}+\frac{b\left(15b^2-52ac\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(b^2-4ac\right)}-\frac{\left(5b^2-12ac\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(b^2-4ac\right)}+\frac{-2ac+b^2+bcx^2}{ax^4\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x^4*Sqrt[a + b*x^2 + c*x
^4]) - ((5*b^2 - 12*a*c)*Sqrt[a + b*x^2 + c*x^4])/(4*a^2*(b^2 - 4
*a*c)*x^4) + (b*(15*b^2 - 52*a*c)*Sqrt[a + b*x^2 + c*x^4])/(8*a^3
*(b^2 - 4*a*c)*x^2) - (3*(5*b^2 - 4*a*c)*ArcTanh[(2*a + b*x^2)/(2
*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(7/2))
```

Rubi in Sympy [A] time = 46.7064, size = 180, normalized size = 0.92

$$\frac{-2ac+b^2+bcx^2}{ax^4\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\left(-12ac+5b^2\right)\sqrt{a+bx^2+cx^4}}{4a^2x^4\left(-4ac+b^2\right)}+\frac{b\left(-52ac+15b^2\right)\sqrt{a+bx^2+cx^4}}{8a^3x^2\left(-4ac+b^2\right)}-\frac{3\left(-4ac+5b^2\right)\operatorname{atanh}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(a*x**4*(-4*a*c + b**2)*sqrt(a + b*x**
2 + c*x**4)) - (-12*a*c + 5*b**2)*sqrt(a + b*x**2 + c*x**4)/(4*a*
*2*x**4*(-4*a*c + b**2)) + b*(-52*a*c + 15*b**2)*sqrt(a + b*x**2
+ c*x**4)/(8*a**3*x**2*(-4*a*c + b**2)) - 3*(-4*a*c + 5*b**2)*ata
nh((2*a + b*x**2)/(2*sqrt(a)*sqrt(a + b*x**2 + c*x**4)))/(16*a**(
7/2))
```

Mathematica [A] time = 0.326525, size = 159, normalized size = 0.82

$$\frac{3\left(5b^2-4ac\right)\left(\log\left(x^2\right)-\log\left(2\sqrt{a}\sqrt{a+bx^2+cx^4}+2a+bx^2\right)\right)}{16a^{7/2}}+\frac{\sqrt{a+bx^2+cx^4}\left(\frac{8\left(2a^2c^2-4ab^2c-3abc^2x^2+b^4+b^3cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{2a}{x^4}+\frac{7b}{x^2}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^5*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*((-2*a)/x^4 + (7*b)/x^2 + (8*(b^4 - 4*a*
b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^2))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4))))/(8*a^3) + (3*(5*b^2 - 4*a*c)*(Log[x^2] - Log
[2*a + b*x^2 + 2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]]))/(16*a^(7/2))
```

Maple [A] time = 0.023, size = 314, normalized size = 1.6

$$-\frac{1}{4\,ax^4}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{5\,b}{8\,a^2x^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{15\,b^2}{16\,a^3}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{15\,x^2b^3c}{8\,a^3\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{15\,b^4}{16\,a^3\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{15\,b^2}{16}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{7}{2}}+\frac{13\,bc^2x^2}{2\,a^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{13\,b^2c}{4\,a^2\left(4\,ac-b^2\right)}\frac{1}{\sqrt{cx^4+bx^2+a}}-\frac{3\,c}{4\,a^2}\frac{1}{\sqrt{cx^4+bx^2+a}}+\frac{3\,c}{4}\ln\left(\frac{1}{x^2}\left(2\,a+bx^2+2\,\sqrt{a}\sqrt{cx^4+bx^2+a}\right)\right)a^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^5/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -1/4/a/x^4/(c*x^4+b*x^2+a)^(1/2)+5/8*b/a^2/x^2/(c*x^4+b*x^2+a)^(1
/2)+15/16*b^2/a^3/(c*x^4+b*x^2+a)^(1/2)-15/8*b^3/a^3/(4*a*c-b^2)/
(c*x^4+b*x^2+a)^(1/2)*c*x^2-15/16*b^4/a^3/(4*a*c-b^2)/(c*x^4+b*x^
2+a)^(1/2)-15/16*b^2/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2
+a)^(1/2))/x^2)+13/2*b/a^2*c^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*
x^2+13/4*b^2/a^2*c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-3/4*c/a^2/(c
*x^4+b*x^2+a)^(1/2)+3/4*c/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+
b*x^2+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^5),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.354233, size = 1, normalized size = 0.01

$$\left[\frac{4\left(\left(15\,b^{3}c-52\,abc^{2}\right)x^{6}+\left(15\,b^{4}-62\,ab^{2}c+24\,a^{2}c^{2}\right)x^{4}-2\,a^{2}b^{2}+8\,a^{3}c+5\left(ab^{3}-4\,a^{2}bc\right)x^{2}\right)\sqrt{cx^{4}+bx^{2}+a}\sqrt{a}-3\left(\left(5\,b\right.\right.}{32\left(\left(a^{3}b^{2}c-4\,a^{4}c^{2}\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^5),x, algorithm="fricas")
```

```
[Out] [1/32*(4*((15*b^3*c - 52*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 24
*a^2*c^2)*x^4 - 2*a^2*b^2 + 8*a^3*c + 5*(a*b^3 - 4*a^2*b*c)*x^2)*
sqrt(c*x^4 + b*x^2 + a)*sqrt(a) - 3*((5*b^4*c - 24*a*b^2*c^2 + 16
*a^2*c^3)*x^8 + (5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a*b^
4 - 24*a^2*b^2*c + 16*a^3*c^2)*x^4)*log(-(4*sqrt(c*x^4 + b*x^2 +
a)*(a*b*x^2 + 2*a^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*sq
rt(a))/x^4))/(((a^3*b^2*c - 4*a^4*c^2)*x^8 + (a^3*b^3 - 4*a^4*b*c
)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4)*sqrt(a)), 1/16*(2*((15*b^3*c - 5
2*a*b*c^2)*x^6 + (15*b^4 - 62*a*b^2*c + 24*a^2*c^2)*x^4 - 2*a^2*b
^2 + 8*a^3*c + 5*(a*b^3 - 4*a^2*b*c)*x^2)*sqrt(c*x^4 + b*x^2 + a)
*sqrt(-a) - 3*((5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*x^8 + (5*b^5
 - 24*a*b^3*c + 16*a^2*b*c^2)*x^6 + (5*a*b^4 - 24*a^2*b^2*c + 16*
```

```
a^3*c^2)*x^4)*arctan(1/2*(b*x^2 + 2*a)*sqrt(-a)/(sqrt(c*x^4 + b*x
^2 + a)*a)))/(((a^3*b^2*c - 4*a^4*c^2)*x^8 + (a^3*b^3 - 4*a^4*b*c
)*x^6 + (a^4*b^2 - 4*a^5*c)*x^4)*sqrt(-a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5 (a + bx^2 + cx^4)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**5/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**5*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^5),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^5), x)
```

3.989 $\int \frac{x^6}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=408

$$\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
x*Sqrt[a + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + (2*(b^2 - 3*a*c)*x
*Sqrt[a + b*x^2 + c*x^4])/(c^(3/2)*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[
c]*x^2)) - (2*a^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(7/4)*(b^
2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*b^2 + Sqrt[a]*b
*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(7/4)*(b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.558177, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{c^{3/2}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{\sqrt[4]{a}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{c\left(b^2-4ac\right)}+\frac{x^3\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x^3*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*
x*Sqrt[a + b*x^2 + c*x^4])/(c*(b^2 - 4*a*c)) + (2*(b^2 - 3*a*c)*x
*Sqrt[a + b*x^2 + c*x^4])/(c^(3/2)*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[
c]*x^2)) - (2*a^(1/4)*(b^2 - 3*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[
(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan
[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(7/4)*(b^
2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(2*b^2 + Sqrt[a]*b
*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4
)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/
4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(7/4)*(b^2 - 4*a*c)*Sqrt[
a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 66.3315, size = 372, normalized size = 0.91

$$-\frac{2\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{c^{\frac{7}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2c^{\frac{7}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{c\left(-4ac+b^2\right)}+\frac{x^3\left(2a+bx^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{2x\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{c^{\frac{3}{2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**6/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -2*a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**
2)*(sqrt(a) + sqrt(c)*x**2)*(-3*a*c + b**2)*elliptic_e(2*atan(c**
(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(c**(7/4)*(-4*a*c
 + b**2)*sqrt(a + b*x**2 + c*x**4)) + a**(1/4)*sqrt((a + b*x**2 +
 c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(s
qrt(a)*b*sqrt(c) - 6*a*c + 2*b**2)*elliptic_f(2*atan(c**(1/4)*x/a
**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*c**(7/4)*(-4*a*c + b**2
)*sqrt(a + b*x**2 + c*x**4)) - b*x*sqrt(a + b*x**2 + c*x**4)/(c*(
-4*a*c + b**2)) + x**3*(2*a + b*x**2)/((-4*a*c + b**2)*sqrt(a + b
*x**2 + c*x**4)) + 2*x*(-3*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)/
(c**(3/2)*(sqrt(a) + sqrt(c)*x**2)*(-4*a*c + b**2))
```

Mathematica [C] time = 2.38976, size = 489, normalized size = 1.2

$$\frac{2cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(a\left(b-2cx^2\right)+b^2x^2\right)-i\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}E\left(i\sinh^{-1}\left(\sqrt{2}\right.\right.}{2c^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^6/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2*x^2 + a*(b - 2*c*x^2)
) - I*(b^2 - 3*a*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^
2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2
- 4*a*c] - 3*a*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c
] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]
*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b -
 Sqrt[b^2 - 4*a*c])])/(2*c^2*(-b^2 + 4*a*c)*Sqrt[c/(b + Sqrt[b^2
- 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.031, size = 482, normalized size = 1.2

$$-2\,c\left(1/2\,\frac{\left(2\,ac-b^2\right)x^3}{\left(4\,ac-b^2\right)c^2}-1/2\,\frac{abx}{\left(4\,ac-b^2\right)c^2}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}$$

$$-\frac{ab\sqrt{2}}{4\,\left(4\,ac-b^2\right)c}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-\right.$$

$$-\frac{a\sqrt{2}}{2}\left(c^{-1}+\frac{2\,ac-b^2}{\left(4\,ac-b^2\right)c}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,a\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^6/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2/c^2*(2*a*c-b^2)/(4*a*c-b^2)*x^3-1/2*a*b/c^2/(4*a*c-b^2)
*x)/((x^4+b/c*x^2+1/c*a)*c)^(1/2)-1/4*a*b/c/(4*a*c-b^2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x
```

```
^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a
)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)
,1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(1/c+(2*a*c-b
^2)/c/(4*a*c-b^2))*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4
-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2
))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(Ell
ipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*
b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b
+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^6}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^6/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**6/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**6/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^6/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.990 $\int \frac{x^4}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=342

$$\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*
Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c
]*x^2)) + (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*(b^2 - 4*a*c)*Sqrt
[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*(b - 2*Sqrt[a
]*Sqrt[c])*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.350067, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt[4]{a}b\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)}+\frac{x\left(2a+bx^2\right)}{\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(2*a + b*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (b*x*
Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(b^2 - 4*a*c)*(Sqrt[a] + Sqrt[c
]*x^2)) + (a^(1/4)*b*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(c^(3/4)*(b^2 - 4*a*c)*Sqrt
[a + b*x^2 + c*x^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*(b - 2*Sqrt[a
```

```
]*Sqrt[c])*c^(3/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 46.5995, size = 316, normalized size = 0.92

$$\frac{\sqrt[4]{a}b\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{c^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{a}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2c^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}} - \frac{bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)} + \frac{x\left(2a+bx^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] a**(1/4)*b*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2
)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4))
, 1/2 - b/(4*sqrt(a)*sqrt(c)))/(c**(3/4)*(-4*a*c + b**2)*sqrt(a +
 b*x**2 + c*x**4)) - a**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a)
 + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(2*sqrt(a)*sqrt(c)
+ b)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*s
qrt(c)))/(2*c**(3/4)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) -
 b*x*sqrt(a + b*x**2 + c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x**2)*
(-4*a*c + b**2)) + x*(2*a + b*x**2)/((-4*a*c + b**2)*sqrt(a + b*x
**2 + c*x**4))
```

Mathematica [C] time = 1.52742, size = 452, normalized size = 1.32

$$\frac{4cx\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(2a+bx^2\right)+i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right.\right.}{4c\left(b^2-4ac\right)\sqrt{\frac{c}{\sqrt{b^2-4ac+}}}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(2*a + b*x^2) - I*b*(-b +
Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b -
```

```
 Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt
[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
])] + I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^
2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSin
h[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*c*(b^2 - 4*a*c)*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.021, size = 450, normalized size = 1.3

$$-2\,c\left(1/2\,{\frac{b{x}^{3}}{\left(4\,ac-{b}^{2}\right)c}}+{\frac{ax}{\left(4\,ac-{b}^{2}\right)c}}\right){\frac{1}{\sqrt{\left({x}^{4}+{\frac{b{x}^{2}}{c}}+{\frac{a}{c}}\right)c}}}+{\frac{a\sqrt{2}}{8\,ac-2\,{b}^{2}}}\sqrt{4-2\,{\frac{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right){x}^{2}}{a}}}\sqrt{4+2\,{\frac{\left(b+\sqrt{-4\,ac+{b}^{2}}\right){x}^{2}}{a}}}EllipticF\left({\frac{\sqrt{2}x}{2}\sqrt{{\frac{1}{a}}\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)}},{\frac{1}{2}}\sqrt{-4+ \ldots$$

$$-{\frac{ab\sqrt{2}}{8\,ac-2\,{b}^{2}}}\sqrt{4-2\,{\frac{\left(-b+\sqrt{-4\,ac+{b}^{2}}\right){x}^{2}}{a}}}\sqrt{4+2\,{\frac{\left(b+\sqrt{-4\,ac+{b}^{2}}\right){x}^{2}}{a}}}\left(EllipticF\left({\frac{\sqrt{2}x}{2}\sqrt{{\frac{1}{a}}\left(-b+\sqrt{-4\,ac+{b}^{2}}\right)}},{\frac{1}{2}}\sqrt{-4 \ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]   int(x^4/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*b/(4*a*c-b^2)/c*x^3+a/c/(4*a*c-b^2)*x)/((x^4+b/c*x^2+1/
c*a)*c)^(1/2)+1/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/
a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*
c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*
2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+
b^2)^(1/2))/a/c)^(1/2))-1/2*b/(4*a*c-b^2)*a*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4
+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(
-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Ellip
ticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")

[Out] integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")

[Out] integral(x^4/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(x**4/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.991 $\int \frac{x^2}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=341

$$\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{x\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] -((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (2
*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/((b^2 - 4*a*c)*(Sqrt[a] + Sqr
t[c]*x^2)) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/((b^2 - 4*a*c)*S
qrt[a + b*x^2 + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(b - 2*Sqr
t[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.330507, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{(b^2-4ac)\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{x\left(b+2cx^2\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}-\frac{2\sqrt[4]{a}\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -((x*(b + 2*c*x^2))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])) + (2
*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/((b^2 - 4*a*c)*(Sqrt[a] + Sqr
```

```
t[c]*x^2)) - (2*a^(1/4)*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a +
 b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^
(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/((b^2 - 4*a*c)*S
qrt[a + b*x^2 + c*x^4]) + ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^
2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)
*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*a^(1/4)*(b - 2*Sqr
t[a]*Sqrt[c])*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 44.8875, size = 316, normalized size = 0.93

$$-\frac{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\sqrt{a+bx^2+cx^4}}{\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)}-\frac{x\left(b+2cx^2\right)}{\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -2*a**(1/4)*c**(1/4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c
)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x
/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/((-4*a*c + b**2)*sqrt(a
+ b*x**2 + c*x**4)) + 2*sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/((sqr
t(a) + sqrt(c)*x**2)*(-4*a*c + b**2)) - x*(b + 2*c*x**2)/((-4*a*c
 + b**2)*sqrt(a + b*x**2 + c*x**4)) + sqrt((a + b*x**2 + c*x**4)/
(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(2*sqrt(a)*
sqrt(c) + b)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*s
qrt(a)*sqrt(c)))/(2*a**(1/4)*c**(1/4)*(-4*a*c + b**2)*sqrt(a + b*
x**2 + c*x**4))
```

Mathematica [C] time = 1.4652, size = 437, normalized size = 1.28

$$\frac{-2x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(b+2cx^2\right)-i\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{-\sqrt{b^2-4ac}+b+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{2\left(b^2-4ac\right)\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (-2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b + 2*c*x^2) + I*(-b + Sqr
t[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^
2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]
 - I*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(
b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])])/(2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqr
t[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.021, size = 446, normalized size = 1.3

$$-2\,c\left(-\frac{x^3}{4\,ac-b^2}-1/2\,\frac{bx}{(4\,ac-b^2)\,c}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}$$

$$-\frac{b\sqrt{2}}{16\,ac-4\,b^2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\,EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4\right.$$

$$+\frac{ac\sqrt{2}}{4\,ac-b^2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(EllipticF\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac+b^2}\right)},\frac{1}{2}\sqrt{-4+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(-1/(4*a*c-b^2)*x^3-1/2*b/(4*a*c-b^2)/c*x)/((x^4+b/c*x^2+1/c
*a)*c)^(1/2)-1/4*b/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c
+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2
^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b
^2)^(1/2))/a/c)^(1/2))+c/(4*a*c-b^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b
+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*
c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(
1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-
4*a*c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**2/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.992 $\int \frac{1}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=353

$$\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x
^4]) - (b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sq
rt[a] + Sqrt[c]*x^2)) + (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2
 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.308149, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{b\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a+bx^2+cx^4}} - \frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] (x*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x
^4]) - (b*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(a*(b^2 - 4*a*c)*(Sq
rt[a] + Sqrt[c]*x^2)) + (b*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
```

```
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[
(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*(b^2
 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(3/4)*(b - 2*Sqrt[a]*Sqrt[c])*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 49.3478, size = 326, normalized size = 0.92

$$-\frac{b\sqrt{c}x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)}+\frac{x\left(-2ac+b^2+bcx^2\right)}{a\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{b\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(2\sqrt{a}\sqrt{c}+b\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{3}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -b*sqrt(c)*x*sqrt(a + b*x**2 + c*x**4)/(a*(sqrt(a) + sqrt(c)*x**2
)*(-4*a*c + b**2)) + x*(-2*a*c + b**2 + b*c*x**2)/(a*(-4*a*c + b*
*2)*sqrt(a + b*x**2 + c*x**4)) + b*c**(1/4)*sqrt((a + b*x**2 + c*
x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*ellip
tic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(
a**(3/4)*(-4*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)) - c**(1/4)*sq
rt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) +
sqrt(c)*x**2)*(2*sqrt(a)*sqrt(c) + b)*elliptic_f(2*atan(c**(1/4)*
x/a**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(3/4)*(-4*a*c + b
**2)*sqrt(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 1.57781, size = 456, normalized size = 1.29

$$-\frac{-4x\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-2ac+b^2+bcx^2\right)-i\left(b\sqrt{b^2-4ac}+4ac-b^2\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{-2\sqrt{b^2-4ac}+2b+4cx^2}{b-\sqrt{b^2-4ac}}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\cdots}\right.\right.}{4a\left(b^2-4ac\right)\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(-3/2),x]
```

```
[Out] -(-4*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(b^2 - 2*a*c + b*c*x^2) +
I*b*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c
*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c
/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b
^2 - 4*a*c])] - I*(-b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b -
 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Elliptic
F[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt
[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(4*a*(b^2 - 4*a*c)*Sqrt[
c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A] time = 0.018, size = 481, normalized size = 1.4

$$-2c\left(1/2\,\frac{bx^3}{a(4ac-b^2)}-1/2\,\frac{(2ac-b^2)x}{c(4ac-b^2)a}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}$$

$$+\frac{\sqrt{2}}{4}\left(a^{-1}-\frac{2ac-b^2}{a(4ac-b^2)}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+\right.}}\right.$$

$$-\frac{bc\sqrt{2}}{8ac-2b^2}\sqrt{4-2\,\frac{\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4ac+b^2}\right)},\frac{1}{2}\sqrt{-4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*b/a/(4*a*c-b^2)*x^3-1/2*(2*a*c-b^2)/a/(4*a*c-b^2)/c*x)/
((x^4+b/c*x^2+1/c*a)*c)^(1/2)+1/4*(1/a-(2*a*c-b^2)/a/(4*a*c-b^2))
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2)
)/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*b/(
4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(
-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1
/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4
*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/
2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(-3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(-3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(-3/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.993 $\int \frac{1}{x^2(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=428

$$\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2x\left(b^2-4ac\right)} + \frac{2\sqrt{c}x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4
]) - (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)
*x) + (2*Sqrt[c]*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b
^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*c^(1/4)*(b^2 - 3*a*c)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) +
 (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2
*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.553542, antiderivative size = 428, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{2\sqrt[4]{c}\left(b^2-3ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{7/4}\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}} - \frac{2\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2x\left(b^2-4ac\right)} + \frac{2\sqrt{c}x\left(b^2-3ac\right)\sqrt{a+bx^2+cx^4}}{a^2\left(b^2-4ac\right)\left(\sqrt{a}+\sqrt{c}x^2\right)} + \frac{-2ac+b^2+bcx^2}{ax\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*x*Sqrt[a + b*x^2 + c*x^4
]) - (2*(b^2 - 3*a*c)*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b^2 - 4*a*c)
*x) + (2*Sqrt[c]*(b^2 - 3*a*c)*x*Sqrt[a + b*x^2 + c*x^4])/(a^2*(b
^2 - 4*a*c)*(Sqrt[a] + Sqrt[c]*x^2)) - (2*c^(1/4)*(b^2 - 3*a*c)*(
Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a
]*Sqrt[c]))/4]])/(a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4]) +
 (c^(1/4)*(2*b^2 + Sqrt[a]*b*Sqrt[c] - 6*a*c)*(Sqrt[a] + Sqrt[c]*
x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elliptic
F[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4]])/(2
*a^(7/4)*(b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 74.6487, size = 393, normalized size = 0.92

$$\frac{-2ac+b^2+bcx^2}{ax\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{2\sqrt{c}x\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{a^2\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-4ac+b^2\right)}-\frac{2\left(-3ac+b^2\right)\sqrt{a+bx^2+cx^4}}{a^2x\left(-4ac+b^2\right)}-\frac{2\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(-3ac+b^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{a^{\frac{7}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)\left(\sqrt{a}b\sqrt{c}-6ac+2b^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}-\frac{b}{4\sqrt{a}\sqrt{c}}\right)}{2a^{\frac{7}{4}}\left(-4ac+b^2\right)\sqrt{a+bx^2+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (-2*a*c + b**2 + b*c*x**2)/(a*x*(-4*a*c + b**2)*sqrt(a + b*x**2 +
 c*x**4)) + 2*sqrt(c)*x*(-3*a*c + b**2)*sqrt(a + b*x**2 + c*x**4)
/(a**2*(sqrt(a) + sqrt(c)*x**2)*(-4*a*c + b**2)) - 2*(-3*a*c + b*
*2)*sqrt(a + b*x**2 + c*x**4)/(a**2*x*(-4*a*c + b**2)) - 2*c**(1/
4)*sqrt((a + b*x**2 + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(
a) + sqrt(c)*x**2)*(-3*a*c + b**2)*elliptic_e(2*atan(c**(1/4)*x/a
**(1/4)), 1/2 - b/(4*sqrt(a)*sqrt(c)))/(a**(7/4)*(-4*a*c + b**2)*
sqrt(a + b*x**2 + c*x**4)) + c**(1/4)*sqrt((a + b*x**2 + c*x**4)/
(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*(sqrt(a)*b*
sqrt(c) - 6*a*c + 2*b**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)),
 1/2 - b/(4*sqrt(a)*sqrt(c)))/(2*a**(7/4)*(-4*a*c + b**2)*sqrt(a
+ b*x**2 + c*x**4))
```

Mathematica [C] time = 2.43801, size = 515, normalized size = 1.2

$$-\frac{2\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\left(-4a^2c+a\left(b^2-7bcx^2-6c^2x^4\right)+2b^2x^2\left(b+cx^2\right)\right)-ix\left(b^2-3ac\right)\left(\sqrt{b^2-4ac}-b\right)\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{-2\cdots}}{}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -(2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(-4*a^2*c + 2*b^2*x^2*(b + c*
x^2) + a*(b^2 - 7*b*c*x^2 - 6*c^2*x^4)) - I*(b^2 - 3*a*c)*(-b + S
qrt[b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqr
t[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*
c])] + I*(-b^3 + 4*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 3*a*c*Sqrt[b^2
 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 -
 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(
2*a^2*(b^2 - 4*a*c)*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*
x^2 + c*x^4])
```

Maple [A] time = 0.028, size = 536, normalized size = 1.3

$$-2\,c\left(1/2\,\frac{\left(2\,ac-b^2\right)x^3}{\left(4\,ac-b^2\right)a^2}+1/2\,\frac{b\left(3\,ac-b^2\right)x}{a^2\left(4\,ac-b^2\right)c}\right)\frac{1}{\sqrt{\left(x^4+\frac{bx^2}{c}+\frac{a}{c}\right)c}}-\frac{1}{a^2x}\sqrt{cx^4+bx^2+a}$$
$$+\frac{\sqrt{2}}{4}\left(-\frac{b}{a^2}+\frac{b\left(3\,ac-b^2\right)}{\left(4\,ac-b^2\right)a^2}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\,ac\cdots}\right.}\right.$$
$$-\frac{a\sqrt{2}}{2}\left(\frac{c\left(2\,ac-b^2\right)}{\left(4\,ac-b^2\right)a^2}+\frac{c}{a^2}\right)\sqrt{4-2\,\frac{\left(-b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\sqrt{4+2\,\frac{\left(b+\sqrt{-4\,ac+b^2}\right)x^2}{a}}\left(\mathit{EllipticF}\left(\frac{\sqrt{2}x}{2}\sqrt{\frac{1}{a}\left(-b+\sqrt{-4\cdots}\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] -2*c*(1/2*(2*a*c-b^2)/(4*a*c-b^2)/a^2*x^3+1/2*b*(3*a*c-b^2)/a^2/(
4*a*c-b^2)/c*x)/((x^4+b/c*x^2+1/c*a)*c)^(1/2)-1/a^2*(c*x^4+b*x^2+
```

```
a)^(1/2)/x+1/4*(-b/a^2+b*(3*a*c-b^2)/a^2/(4*a*c-b^2))*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2
)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1
/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(c*(2*a*c-b^2)/
(4*a*c-b^2)/a^2+c/a^2)*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^
(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*
(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*
((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^6 + bx^4 + ax^2)\sqrt{cx^4 + bx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^6 + b*x^4 + a*x^2)*sqrt(c*x^4 + b*x^2 + a)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \left(a + bx^2 + cx^4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2+a)**(3/2),x)

[Out] Integral(1/(x**2*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*x^2),x, algorithm="giac")

[Out] Exception raised: RuntimeError
```

3.994 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=50

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3
*c)
```

Rubi [A] time = 0.0893297, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{x\sqrt{bx^2+cx^4}}{3c}-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (-2*b*Sqrt[b*x^2 + c*x^4])/(3*c^2*x) + (x*Sqrt[b*x^2 + c*x^4])/(3
*c)
```

Rubi in Sympy [A] time = 13.477, size = 41, normalized size = 0.82

$$-\frac{2b\sqrt{bx^2+cx^4}}{3c^2x}+\frac{x\sqrt{bx^2+cx^4}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -2*b*sqrt(b*x**2 + c*x**4)/(3*c**2*x) + x*sqrt(b*x**2 + c*x**4)/(
3*c)
```

Mathematica [A] time = 0.0310374, size = 34, normalized size = 0.68

$$\frac{\left(cx^2-2b\right)\sqrt{x^2\left(b+cx^2\right)}}{3c^2x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] ((-2*b + c*x^2)*Sqrt[x^2*(b + c*x^2)])/(3*c^2*x)
```

Maple [A] time = 0.007, size = 37, normalized size = 0.7

$$-\frac{\left(cx^{2}+b\right)\left(-cx^{2}+2\,b\right)x}{3\,c^{2}}\frac{1}{\sqrt{cx^{4}+bx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-c*x^2+2*b)*x/c^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.710697, size = 46, normalized size = 0.92

$$\frac{c^{2}x^{4}-bcx^{2}-2\,b^{2}}{3\,\sqrt{cx^{2}+b}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] 1/3*(c^2*x^4 - b*c*x^2 - 2*b^2)/(sqrt(c*x^2 + b)*c^2)
```

Fricas [A] time = 0.272115, size = 41, normalized size = 0.82

$$\frac{\sqrt{cx^{4}+bx^{2}}\left(cx^{2}-2\,b\right)}{3\,c^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(c*x^2 - 2*b)/(c^2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+b*x**2)**(1/2),x)

[Out] Integral(x**4/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4+bx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + b*x^2),x, algorithm="giac")

[Out] integrate(x^4/sqrt(c*x^4 + b*x^2), x)
```

3.995 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(3/2))
```

Rubi [A] time = 0.13859, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179

$$\frac{\sqrt{bx^2+cx^4}}{2c}-\frac{b\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] Sqrt[b*x^2 + c*x^4]/(2*c) - (b*ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 +
 c*x^4]])/(2*c^(3/2))
```

Rubi in Sympy [A] time = 12.3511, size = 48, normalized size = 0.83

$$-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{2c^{\frac{3}{2}}}+\frac{\sqrt{bx^2+cx^4}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -b*atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/(2*c**(3/2)) + sqrt(
b*x**2 + c*x**4)/(2*c)
```

Mathematica [A] time = 0.0582484, size = 76, normalized size = 1.31

$$\frac{x\left(\sqrt{c}x\left(b+cx^2\right)-b\sqrt{b+cx^2}\log\left(\sqrt{c}\sqrt{b+cx^2}+cx\right)\right)}{2c^{3/2}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*(Sqrt[c]*x*(b + c*x^2) - b*Sqrt[b + c*x^2]*Log[c*x + Sqrt[c]*S
qrt[b + c*x^2]]))/(2*c^(3/2)*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.007, size = 64, normalized size = 1.1

$$-\frac{x}{2}\sqrt{cx^2+b}\left(-x\sqrt{cx^2+b}c^{\frac{3}{2}}+b\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)c\right)\frac{1}{\sqrt{cx^4+bx^2}}c^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/2*x*(c*x^2+b)^(1/2)*(-x*(c*x^2+b)^(1/2)*c^(3/2)+b*ln(x*c^(1/2)
+(c*x^2+b)^(1/2))*c)/(c*x^4+b*x^2)^(1/2)/c^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284583, size = 1, normalized size = 0.02

$$\left[\frac{b\sqrt{c}\log\left(-\left(2\,cx^2+b\right)\sqrt{c}+2\,\sqrt{cx^4+bx^2}c\right)+2\,\sqrt{cx^4+bx^2}c}{4\,c^2},\ \frac{b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}c}{2\,c^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(b*sqrt(c)*log(-(2*c*x^2 + b)*sqrt(c) + 2*sqrt(c*x^4 + b*x^2
)*c) + 2*sqrt(c*x^4 + b*x^2)*c)/c^2, 1/2*(b*sqrt(-c)*arctan(sqrt(
-c)*x^2/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 + b*x^2)*c)/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.298499, size = 80, normalized size = 1.38

$$\frac{b\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{4\,c^{\frac{3}{2}}}+\frac{\sqrt{cx^4+bx^2}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] 1/4*b*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))
/c^(3/2) + 1/2*sqrt(c*x^4 + b*x^2)/c
```

3.996 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=22

$$\frac{\sqrt{bx^2 + cx^4}}{cx}$$

```
[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rubi [A] time = 0.0131807, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{\sqrt{bx^2 + cx^4}}{cx}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]

[Out] Sqrt[b*x^2 + c*x^4]/(c*x)
```

Rubi in Sympy [A] time = 7.59124, size = 15, normalized size = 0.68

$$\frac{\sqrt{bx^2 + cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+b*x**2)**(1/2),x)

[Out] sqrt(b*x**2 + c*x**4)/(c*x)
```

Mathematica [A] time = 0.00949294, size = 22, normalized size = 1.

$$\frac{\sqrt{x^2 (b + cx^2)}}{cx}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]

[Out] Sqrt[x^2*(b + c*x^2)]/(c*x)
```

Maple [A] time = 0.003, size = 26, normalized size = 1.2

$$\frac{x\left(cx^2+b\right)}{c}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+b*x^2)^(1/2),x)

[Out] (c*x^2+b)/c*x/(c*x^4+b*x^2)^(1/2)
```

Maxima [A] time = 0.714677, size = 18, normalized size = 0.82

$$\frac{\sqrt{cx^2+b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")

[Out] sqrt(c*x^2 + b)/c
```

Fricas [A] time = 0.272006, size = 27, normalized size = 1.23

$$\frac{\sqrt{cx^4+bx^2}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")

[Out] sqrt(c*x^4 + b*x^2)/(c*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.278081, size = 42, normalized size = 1.91

$$-\frac{2\sqrt{b}}{\left(\sqrt{c+\frac{b}{x^2}}-\frac{\sqrt{b}}{x}\right)^2-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b)/((sqrt(c + b/x^2) - sqrt(b)/x)^2 - c)
```

3.997 $\int \frac{x}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi [A] time = 0.0768295, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[b*x^2 + c*x^4]]/Sqrt[c]
```

Rubi in Sympy [A] time = 7.55694, size = 27, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+b*x**2)**(1/2),x)

[Out] atanh(sqrt(c)*x**2/sqrt(b*x**2 + c*x**4))/sqrt(c)
```

Mathematica [A] time = 0.0238048, size = 52, normalized size = 1.68

$$\frac{x\sqrt{b+cx^2}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{b+cx^2}}\right)}{\sqrt{c}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*ArcTanh[(Sqrt[c]*x)/Sqrt[b + c*x^2]])/(Sqrt[c]
*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0., size = 44, normalized size = 1.4

$$x\sqrt{cx^2+b}\ln\left(x\sqrt{c}+\sqrt{cx^2+b}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)*ln(x*c^(1/2)+(c*x^2+b)^(1
/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280522, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\left(2\,cx^2+b\right)\sqrt{c}-2\,\sqrt{cx^4+bx^2}c\right)}{2\,\sqrt{c}}, -\frac{\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x^2}{\sqrt{cx^4+bx^2}}\right)}{c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*c*x^2 + b)*sqrt(c) - 2*sqrt(c*x^4 + b*x^2)*c)/sqrt(c
), -sqrt(-c)*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + b*x^2))/c]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2\,(b+cx^2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(b + c*x**2)), x)
```

GIAC/XCAS [A] time = 0.292217, size = 53, normalized size = 1.71

$$-\frac{\ln\left(\left|-2\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2}\right)\sqrt{c}-b\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2))*sqrt(c) - b))/
sqrt(c)
```

3.998 $\int \frac{1}{\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

```
[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rubi [A] time = 0.0227959, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]

[Out] -(ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]]/Sqrt[b])
```

Rubi in Sympy [A] time = 5.44413, size = 27, normalized size = 0.9

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+b*x**2)**(1/2),x)

[Out] -atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/sqrt(b)
```

Mathematica [A] time = 0.0425539, size = 58, normalized size = 1.93

$$\frac{x\sqrt{b+cx^2}\left(\log(x)-\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)\right)}{\sqrt{b}\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[b + c*x^2]*(Log[x] - Log[b + Sqrt[b]*Sqrt[b + c*x^2]]))/(
Sqrt[b]*Sqrt[x^2*(b + c*x^2)])
```

Maple [B] time = 0., size = 50, normalized size = 1.7

$$-x\sqrt{cx^2+b}\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)\frac{1}{\sqrt{cx^4+bx^2}}\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/(c*x^4+b*x^2)^(1/2)*x*(c*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(c
*x^2+b)^(1/2)+b)/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.278009, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}-2\,\sqrt{cx^4+bx^2}b}{x^3}\right)}{2\,\sqrt{b}},\ \frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c*x^4 + b*x^2)*b)/x^3
)/sqrt(b), sqrt(-b)*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2))/b]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b*x**2 + c*x**4), x)
```

GIAC/XCAS [A] time = 0.273501, size = 62, normalized size = 2.07

$$-\frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)\operatorname{sign}(x)}{\sqrt{-b}} + \frac{\arctan\left(\frac{\sqrt{cx^2+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\operatorname{sign}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + b*x^2),x, algorithm="giac")
```

```
[Out] -arctan(sqrt(b)/sqrt(-b))*sign(x)/sqrt(-b) + arctan(sqrt(c*x^2 +
b)/sqrt(-b))/(sqrt(-b)*sign(x))
```

3.999 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

```
[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rubi [A] time = 0.0659235, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]

[Out] -(Sqrt[b*x^2 + c*x^4]/(b*x^2))
```

Rubi in Sympy [A] time = 7.23402, size = 19, normalized size = 0.83

$$-\frac{\sqrt{bx^2+cx^4}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+b*x**2)**(1/2),x)

[Out] -sqrt(b*x**2 + c*x**4)/(b*x**2)
```

Mathematica [A] time = 0.0224359, size = 23, normalized size = 1.

$$-\frac{\sqrt{x^2\left(b+cx^2\right)}}{bx^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]

[Out] -(Sqrt[x^2*(b + c*x^2)]/(b*x^2))
```

Maple [A] time = 0.006, size = 26, normalized size = 1.1

$$-\frac{cx^2+b}{b}\frac{1}{\sqrt{cx^4+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+b*x^2)^(1/2),x)

[Out] -(c*x^2+b)/b/(c*x^4+b*x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.268479, size = 28, normalized size = 1.22

$$-\frac{\sqrt{cx^4+bx^2}}{bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x),x, algorithm="fricas")

[Out] -sqrt(c*x^4 + b*x^2)/(b*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [A] time = 0.276213, size = 19, normalized size = 0.83

$$-\frac{\sqrt{c+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x),x, algorithm="giac")
```

```
[Out] -sqrt(c + b/x^2)/b
```

3.1000 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=59

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^
2 + c*x^4]])/(2*b^(3/2))
```

Rubi [A] time = 0.0891354, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{c\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{3/2}}-\frac{\sqrt{bx^2+cx^4}}{2bx^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(2*b*x^3) + (c*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^
2 + c*x^4]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 12.3236, size = 49, normalized size = 0.83

$$-\frac{\sqrt{bx^2+cx^4}}{2bx^3}+\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(2*b*x**3) + c*atanh(sqrt(b)*x/sqrt(b*x**2
 + c*x**4))/(2*b**(3/2))
```

Mathematica [A] time = 0.0700673, size = 97, normalized size = 1.64

$$\frac{-\sqrt{b}\left(b+cx^2\right)-cx^2\log(x)\sqrt{b+cx^2}+cx^2\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)}{2b^{3/2}x\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (-(Sqrt[b]*(b + c*x^2)) - c*x^2*Sqrt[b + c*x^2]*Log[x] + c*x^2*Sq
rt[b + c*x^2]*Log[b + Sqrt[b]*Sqrt[b + c*x^2]])/(2*b^(3/2)*x*Sqrt
[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 73, normalized size = 1.2

$$\frac{1}{2x}\sqrt{cx^2+b}\left(c\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^2b-\sqrt{cx^2+b}\,b^{\frac{3}{2}}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(c*x^2+b)^(1/2)*(c*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x^2*
b-(c*x^2+b)^(1/2)*b^(3/2))/(c*x^4+b*x^2)^(1/2)/b^(5/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.285814, size = 1, normalized size = 0.02

$$\left[\frac{\sqrt{b}cx^3\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}+2\,\sqrt{cx^4+bx^2}b}{x^3}\right)-2\,\sqrt{cx^4+bx^2}b}{4\,b^2x^3},\,-\frac{\sqrt{-b}cx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}b}{2\,b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*c*x^3*log(-((c*x^3 + 2*b*x)*sqrt(b) + 2*sqrt(c*x^4
+ b*x^2)*b)/x^3) - 2*sqrt(c*x^4 + b*x^2)*b)/(b^2*x^3), -1/2*(sqrt
(-b)*c*x^3*arctan(sqrt(-b)*x/sqrt(c*x^4 + b*x^2)) + sqrt(c*x^4 +
b*x^2)*b)/(b^2*x^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1001 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2
*x^2)
```

Rubi [A] time = 0.132855, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107

$$\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}-\frac{\sqrt{bx^2+cx^4}}{3bx^4}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(3*b*x^4) + (2*c*Sqrt[b*x^2 + c*x^4])/(3*b^2
*x^2)
```

Rubi in Sympy [A] time = 12.5982, size = 44, normalized size = 0.85

$$-\frac{\sqrt{bx^2+cx^4}}{3bx^4}+\frac{2c\sqrt{bx^2+cx^4}}{3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(3*b*x**4) + 2*c*sqrt(b*x**2 + c*x**4)/(3*
b**2*x**2)
```

Mathematica [A] time = 0.0367212, size = 35, normalized size = 0.67

$$\frac{\sqrt{x^2\left(b+cx^2\right)}\left(2cx^2-b\right)}{3b^2x^4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[x^2*(b + c*x^2)]*(-b + 2*c*x^2))/(3*b^2*x^4)
```

Maple [A] time = 0.006, size = 37, normalized size = 0.7

$$-\frac{\left(cx^{2}+b\right)\left(-2\,cx^{2}+b\right)}{3\,b^{2}x^{2}}\frac{1}{\sqrt{cx^{4}+bx^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/3*(c*x^2+b)*(-2*c*x^2+b)/x^2/b^2/(c*x^4+b*x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.276295, size = 42, normalized size = 0.81

$$\frac{\sqrt{cx^{4}+bx^{2}}\left(2\,cx^{2}-b\right)}{3\,b^{2}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4 + b*x^2)*(2*c*x^2 - b)/(b^2*x^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [A] time = 0.288364, size = 36, normalized size = 0.69

$$-\frac{\left(c+\frac{b}{x^2}\right)^{\frac{3}{2}}-3\sqrt{c+\frac{b}{x^2}}\,c}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^3),x, algorithm="giac")
```

```
[Out] -1/3*((c + b/x^2)^(3/2) - 3*sqrt(c + b/x^2)*c)/b^2
```

3.1002 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+bx^2+cx^4}}\,dx$

Optimal. Leaf size=87

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2
*x^3) - (3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/
2))
```

Rubi [A] time = 0.168333, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{3c^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{5/2}}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{\sqrt{bx^2+cx^4}}{4bx^5}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[b*x^2 + c*x^4]/(4*b*x^5) + (3*c*Sqrt[b*x^2 + c*x^4])/(8*b^2
*x^3) - (3*c^2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + c*x^4]])/(8*b^(5/
2))
```

Rubi in Sympy [A] time = 18.7878, size = 78, normalized size = 0.9

$$-\frac{\sqrt{bx^2+cx^4}}{4bx^5}+\frac{3c\sqrt{bx^2+cx^4}}{8b^2x^3}-\frac{3c^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{bx^2+cx^4}}\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] -sqrt(b*x**2 + c*x**4)/(4*b*x**5) + 3*c*sqrt(b*x**2 + c*x**4)/(8*
b**2*x**3) - 3*c**2*atanh(sqrt(b)*x/sqrt(b*x**2 + c*x**4))/(8*b**
(5/2))
```

Mathematica [A] time = 0.0836093, size = 114, normalized size = 1.31

$$\frac{\sqrt{b}\left(-2b^2+bcx^2+3c^2x^4\right)+3c^2x^4\log(x)\sqrt{b+cx^2}-3c^2x^4\sqrt{b+cx^2}\log\left(\sqrt{b}\sqrt{b+cx^2}+b\right)}{8b^{5/2}x^3\sqrt{x^2\left(b+cx^2\right)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[b]*(-2*b^2 + b*c*x^2 + 3*c^2*x^4) + 3*c^2*x^4*Sqrt[b + c*x^
2]*Log[x] - 3*c^2*x^4*Sqrt[b + c*x^2]*Log[b + Sqrt[b]*Sqrt[b + c*
x^2]])/(8*b^(5/2)*x^3*Sqrt[x^2*(b + c*x^2)])
```

Maple [A] time = 0.006, size = 94, normalized size = 1.1

$$-\frac{1}{8\,x^3}\sqrt{cx^2+b}\left(3\,\ln\left(2\,\frac{\sqrt{b}\sqrt{cx^2+b}+b}{x}\right)x^4bc^2-3\,\sqrt{cx^2+b}b^{3/2}x^2c+2\,\sqrt{cx^2+b}b^{5/2}\right)\frac{1}{\sqrt{cx^4+bx^2}}b^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+b*x^2)^(1/2),x)
```

```
[Out] -1/8/x^3*(c*x^2+b)^(1/2)*(3*ln(2*(b^(1/2)*(c*x^2+b)^(1/2)+b)/x)*x
^4*b*c^2-3*(c*x^2+b)^(1/2)*b^(3/2)*x^2*c+2*(c*x^2+b)^(1/2)*b^(5/2
))/(c*x^4+b*x^2)^(1/2)/b^(7/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.290948, size = 1, normalized size = 0.01

$$\left[\frac{3\sqrt{b}c^2x^5\log\left(-\frac{(cx^3+2\,bx)\sqrt{b}-2\,\sqrt{cx^4+bx^2}b}{x^3}\right)+2\,\sqrt{cx^4+bx^2}\left(3\,bcx^2-2\,b^2\right)}{16\,b^3x^5},\ \frac{3\,\sqrt{-b}c^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{cx^4+bx^2}}\right)+\sqrt{cx^4+bx^2}\left(3\,bc\right.}{8\,b^3x^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^4),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*c^2*x^5*log(-((c*x^3 + 2*b*x)*sqrt(b) - 2*sqrt(c
*x^4 + b*x^2)*b)/x^3) + 2*sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2)
)/(b^3*x^5), 1/8*(3*sqrt(-b)*c^2*x^5*arctan(sqrt(-b)*x/sqrt(c*x^4
 + b*x^2)) + sqrt(c*x^4 + b*x^2)*(3*b*c*x^2 - 2*b^2))/(b^3*x^5)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4\sqrt{x^2\left(b+cx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+b*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**2*(b + c*x**2))), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2)*x^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.1003 $\int \frac{x^4}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}} \, dx$

Optimal. Leaf size=108

$$\frac{x\sqrt{a+cx^4}}{3c} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

```
[Out] (x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.07188, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{x\sqrt{a+cx^4}}{3c} - \frac{a^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6c^{5/4}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*Sqrt[a + c*x^4])/(3*c) - (a^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt
[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/
4)*x)/a^(1/4)], 1/2])/(6*c^(5/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 7.27848, size = 94, normalized size = 0.87

$$-\frac{a^{\frac{3}{4}}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6c^{\frac{5}{4}}\sqrt{a+cx^4}} + \frac{x\sqrt{a+cx^4}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] -a**(3/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a)
 + sqrt(c)*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(6*
```

```
c**(5/4)*sqrt(a + c*x**4)) + x*sqrt(a + c*x**4)/(3*c)
```

Mathematica [C] time = 0.215036, size = 92, normalized size = 0.85

$$\frac{x\left(a+cx^4\right)+\frac{ia\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3c\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (x*(a + c*x^4) + (I*a*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqr
t[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(3*c*S
qrt[a + c*x^4])
```

Maple [C] time = 0.052, size = 91, normalized size = 0.8

$$\frac{x}{3\,c}\sqrt{cx^4+a}-\frac{a}{3\,c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/3*x*(c*x^4+a)^(1/2)/c-1/3/c*a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^
(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a
)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4}{\sqrt{cx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral(x^4/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 2.15231, size = 37, normalized size = 0.34

$$\frac{x^5\,\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2}, \frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c*x**4*exp_polar(I*pi)/
a)/(4*sqrt(a)*gamma(9/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(c*x^4 + a), x)
```

3.1004 $\int \frac{x^3}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a+cx^4}}{2c}$$

```
[Out] Sqrt[a + c*x^4]/(2*c)
```

Rubi [A] time = 0.0116512, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]

[Out] Sqrt[a + c*x^4]/(2*c)
```

Rubi in Sympy [A] time = 2.12272, size = 12, normalized size = 0.67

$$\frac{\sqrt{a+cx^4}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4+a)**(1/2),x)

[Out] sqrt(a + c*x**4)/(2*c)
```

Mathematica [A] time = 0.00792534, size = 18, normalized size = 1.

$$\frac{\sqrt{a+cx^4}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]

[Out] Sqrt[a + c*x^4]/(2*c)
```

Maple [A] time = 0.007, size = 15, normalized size = 0.8

$$\frac{1}{2c}\sqrt{cx^4+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4+a)^(1/2),x)

[Out] 1/2*(c*x^4+a)^(1/2)/c
```

Maxima [A] time = 0.713995, size = 19, normalized size = 1.06

$$\frac{\sqrt{cx^4+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + a),x, algorithm="maxima")

[Out] 1/2*sqrt(c*x^4 + a)/c
```

Fricas [A] time = 0.270971, size = 19, normalized size = 1.06

$$\frac{\sqrt{cx^4+a}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + a),x, algorithm="fricas")

[Out] 1/2*sqrt(c*x^4 + a)/c
```

Sympy [A] time = 1.58558, size = 22, normalized size = 1.22

$$\begin{cases} \frac{\sqrt{a+cx^4}}{2c} & \text{for } c \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + c*x**4)/(2*c), Ne(c, 0)), (x**4/(4*sqrt(a)),
True))
```

GIAC/XCAS [A] time = 0.275757, size = 19, normalized size = 1.06

$$\frac{\sqrt{cx^4 + a}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + a)/c
```

3.1005 $\int \frac{x^2}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=210

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] (x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a
 + c*x^4]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.141058, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2c^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{c^{3/4}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```


```
[Out] (x*Sqrt[a + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(c^(3/4)*Sqrt[a
 + c*x^4]) + (a^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(S
qrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)],
 1/2])/(2*c^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 16.6567, size = 187, normalized size = 0.89

$$-\frac{\sqrt[4]{a}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{\sqrt[4]{a}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2c^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{x\sqrt{a+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] -a**(1/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a)
 + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(c*
*(3/4)*sqrt(a + c*x**4)) + a**(1/4)*sqrt((a + c*x**4)/(sqrt(a) +
sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(c**(
1/4)*x/a**(1/4)), 1/2)/(2*c**(3/4)*sqrt(a + c*x**4)) + x*sqrt(a +
 c*x**4)/(sqrt(c)*(sqrt(a) + sqrt(c)*x**2))
```

Mathematica [C] time = 0.084113, size = 104, normalized size = 0.5

$$\frac{i\sqrt{\frac{cx^4}{a}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\left(\frac{i\sqrt{c}}{\sqrt{a}}\right)^{3/2}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] (I*Sqrt[1 + (c*x^4)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt
[a]]*x], -1] - EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x],
-1]))/(((I*Sqrt[c])/Sqrt[a])^(3/2)*Sqrt[a + c*x^4])
```

Maple [C] time = 0.01, size = 97, normalized size = 0.5

$$i\sqrt{a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4+a)^(1/2),x)
```

```
[Out] I*a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/
2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)/c^(1/2)*(Ellip
ticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-EllipticE(x*(I/a^(1/2)*c^(1/2
))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2}{\sqrt{cx^4 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 1.9976, size = 37, normalized size = 0.18

$$\frac{x^3 \left(\frac{3}{4}\right) {}_2F_1\left(\left.\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\right| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a} \left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), c*x**4*exp_polar(I*pi)/
a)/(4*sqrt(a)*gamma(7/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(c*x^4 + a), x)
```

3.1006 $\int \frac{x}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=30

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rubi [A] time = 0.0374159, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Rubi in Sympy [A] time = 4.10321, size = 26, normalized size = 0.87

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4+a)**(1/2),x)

[Out] atanh(sqrt(c)*x**2/sqrt(a + c*x**4))/(2*sqrt(c))
```

Mathematica [A] time = 0.0137487, size = 30, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x^2}{\sqrt{a+cx^4}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]

[Out] ArcTanh[(Sqrt[c]*x^2)/Sqrt[a + c*x^4]]/(2*Sqrt[c])
```

Maple [A] time = 0.009, size = 24, normalized size = 0.8

$$\frac{1}{2}\ln\left(x^{2}\sqrt{c}+\sqrt{cx^{4}+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4+a)^(1/2),x)

[Out] 1/2*ln(x^2*c^(1/2)+(c*x^4+a)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.282995, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-2\sqrt{cx^{4}+a}cx^{2}-\left(2\,cx^{4}+a\right)\sqrt{c}\right)}{4\sqrt{c}},\ \frac{\arctan\left(\frac{\sqrt{-c}x^{2}}{\sqrt{cx^{4}+a}}\right)}{2\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + a),x, algorithm="fricas")

[Out] [1/4*log(-2*sqrt(c*x^4 + a)*c*x^2 - (2*c*x^4 + a)*sqrt(c))/sqrt(c
), 1/2*arctan(sqrt(-c)*x^2/sqrt(c*x^4 + a))/sqrt(-c)]
```

Sympy [A] time = 3.37999, size = 20, normalized size = 0.67

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x^2}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4+a)**(1/2),x)

[Out] asinh(sqrt(c)*x**2/sqrt(a))/(2*sqrt(c))
```

GIAC/XCAS [A] time = 0.286277, size = 34, normalized size = 1.13

$$-\frac{\ln\left(\left|-\sqrt{c}x^2+\sqrt{cx^4+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4 + a),x, algorithm="giac")

[Out] -1/2*ln(abs(-sqrt(c)*x^2 + sqrt(c*x^4 + a)))/sqrt(c)
```

3.1007 $\int \frac{1}{\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=88

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(
1/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0358771, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ((Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)
^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*c^(
1/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 3.55763, size = 78, normalized size = 0.89

$$\frac{\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)
*x**2)*elliptic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(1/4)*c
```

```
**(1/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.0520766, size = 74, normalized size = 0.84

$$-\frac{i\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4],x]
```

```
[Out] ((-I)*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sq
rt[a]]*x], -1])/(Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[a + c*x^4])
```

Maple [C] time = 0.007, size = 70, normalized size = 0.8

$$1\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4+a)^(1/2),x)
```

```
[Out] 1/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/
a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)
*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + a),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + a),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(c*x^4 + a), x)
```

Sympy [A] time = 1.89813, size = 36, normalized size = 0.41

$$\frac{x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4+a)**(1/2),x)
```

```
[Out] x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), c*x**4*exp_polar(I*pi)/a)/
(4*sqrt(a)*gamma(5/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4 + a),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*x^4 + a), x)
```

3.1008 $\int \frac{1}{x\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=27

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rubi [A] time = 0.048112, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]

[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Rubi in Sympy [A] time = 4.97162, size = 24, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4+a)**(1/2),x)

[Out] -atanh(sqrt(a + c*x**4)/sqrt(a))/(2*sqrt(a))
```

Mathematica [A] time = 0.0639467, size = 27, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^4}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]

[Out] -ArcTanh[Sqrt[a + c*x^4]/Sqrt[a]]/(2*Sqrt[a])
```

Maple [A] time = 0.013, size = 29, normalized size = 1.1

$$-\frac{1}{2}\ln\left(\frac{1}{x^2}\left(2\,a+2\,\sqrt{a}\sqrt{cx^4+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4+a)^(1/2),x)

[Out] -1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^4+a)^(1/2))/x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.28077, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{(cx^4+2\,a)\sqrt{a}-2\,\sqrt{cx^4+a}a}{x^4}\right)}{4\,\sqrt{a}}, \frac{\arctan\left(\frac{a}{\sqrt{cx^4+a}\sqrt{-a}}\right)}{2\,\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/4*log(((c*x^4 + 2*a)*sqrt(a) - 2*sqrt(c*x^4 + a)*a)/x^4)/sqrt(
a), 1/2*arctan(a/(sqrt(c*x^4 + a)*sqrt(-a)))/sqrt(-a)]
```

Sympy [A] time = 3.61092, size = 22, normalized size = 0.81

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{c}x^2}\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(c)*x**2))/(2*sqrt(a))
```

GIAC/XCAS [A] time = 0.274781, size = 31, normalized size = 1.15

$$\frac{\arctan\left(\frac{\sqrt{cx^4+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(c*x^4 + a)/sqrt(-a))/sqrt(-a)
```

3.1009 $\int \frac{1}{x^2\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=232

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

```
[Out] -(Sqrt[a + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + c*x^4])/(a*(Sqrt[a
] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(a^(3/4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.187823, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{ax}+\frac{\sqrt{c}x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -(Sqrt[a + c*x^4]/(a*x)) + (Sqrt[c]*x*Sqrt[a + c*x^4])/(a*(Sqrt[a
] + Sqrt[c]*x^2)) - (c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + c*
x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^
(1/4)], 1/2])/(a^(3/4)*Sqrt[a + c*x^4]) + (c^(1/4)*(Sqrt[a] + Sqr
t[c]*x^2)*Sqrt[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(3/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 22.1873, size = 202, normalized size = 0.87

$$\frac{\sqrt{c}x\sqrt{a+cx^4}}{a\left(\sqrt{a}+\sqrt{c}x^2\right)}-\frac{\sqrt{a+cx^4}}{ax}-\frac{\sqrt[4]{c}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)E\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{a^{\frac{3}{4}}\sqrt{a+cx^4}}+\frac{\sqrt[4]{c}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\operatorname{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{2a^{\frac{3}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] sqrt(c)*x*sqrt(a + c*x**4)/(a*(sqrt(a) + sqrt(c)*x**2)) - sqrt(a
+ c*x**4)/(a*x) - c**(1/4)*sqrt((a + c*x**4)/(sqrt(a) + sqrt(c)*x
**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_e(2*atan(c**(1/4)*x/a*
*(1/4)), 1/2)/(a**(3/4)*sqrt(a + c*x**4)) + c**(1/4)*sqrt((a + c*
x**4)/(sqrt(a) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*ellip
tic_f(2*atan(c**(1/4)*x/a**(1/4)), 1/2)/(2*a**(3/4)*sqrt(a + c*x*
*4))
```

Mathematica [C] time = 0.51068, size = 121, normalized size = 0.52

$$\frac{-\frac{a+cx^4}{ax}-i\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}\sqrt{\frac{cx^4}{a}+1}\left(E\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)-F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)\right)}{\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] (-((a + c*x^4)/(a*x)) - I*Sqrt[(I*Sqrt[c])/Sqrt[a]]*Sqrt[1 + (c*x
^4)/a]*(EllipticE[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1] - E
llipticF[I*ArcSinh[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1]))/Sqrt[a + c
*x^4]
```

Maple [C] time = 0.015, size = 115, normalized size = 0.5

$$-\frac{1}{ax}\sqrt{cx^4+a}$$
$$+i\sqrt{c}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\left(EllipticF\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)-EllipticE\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\right)\frac{1}{\sqrt{a}}\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4+a)^(1/2),x)
```

```
[Out] -(c*x^4+a)^(1/2)/a/x+I*c^(1/2)/a^(1/2)/(I/a^(1/2)*c^(1/2))^(1/2)*
(1-I/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(
c*x^4+a)^(1/2)*(EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)-Elliptic
E(x*(I/a^(1/2)*c^(1/2))^(1/2),I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + a}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^2),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + a)*x^2), x)
```

Sympy [A] time = 2.23794, size = 39, normalized size = 0.17

$$\frac{\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{1}{2} \\ \frac{3}{4} \end{matrix}\middle| \frac{cx^4 e^{i\pi}}{a}\right)}{4\sqrt{a}x\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), c*x**4*exp_polar(I*pi)/a)/
(4*sqrt(a)*x*gamma(3/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + a}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^2), x)
```

3.1010 $\int \frac{1}{x^3\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=21

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

```
[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Rubi [A] time = 0.0194559, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]

[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Rubi in Sympy [A] time = 2.65435, size = 17, normalized size = 0.81

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4+a)**(1/2),x)

[Out] -sqrt(a + c*x**4)/(2*a*x**2)
```

Mathematica [A] time = 0.0158417, size = 21, normalized size = 1.

$$-\frac{\sqrt{a+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]

[Out] -Sqrt[a + c*x^4]/(2*a*x^2)
```

Maple [A] time = 0.007, size = 18, normalized size = 0.9

$$-\frac{1}{2\,a{x}^{2}}\sqrt{c{x}^{4}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4+a)^(1/2),x)

[Out] -1/2*(c*x^4+a)^(1/2)/a/x^2
```

Maxima [A] time = 0.713417, size = 23, normalized size = 1.1

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^3),x, algorithm="maxima")

[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Fricas [A] time = 0.274018, size = 23, normalized size = 1.1

$$-\frac{\sqrt{cx^4+a}}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^3),x, algorithm="fricas")

[Out] -1/2*sqrt(c*x^4 + a)/(a*x^2)
```

Sympy [A] time = 1.94637, size = 20, normalized size = 0.95

$$-\frac{\sqrt{c}\sqrt{\frac{a}{cx^4}+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(c)*sqrt(a/(c*x**4) + 1)/(2*a)
```

GIAC/XCAS [A] time = 0.279358, size = 19, normalized size = 0.9

$$-\frac{\sqrt{c+\frac{a}{x^4}}}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(c + a/x^4)/a
```

3.1011 $\int \frac{1}{x^4\sqrt{a+(2+2b-2(1+b))x^2+cx^4}}\,dx$

Optimal. Leaf size=110

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

```
[Out] -Sqrt[a + c*x^4]/(3*a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Rubi [A] time = 0.0691406, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$-\frac{c^{3/4}\left(\sqrt{a}+\sqrt{c}x^2\right)\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{5/4}\sqrt{a+cx^4}}-\frac{\sqrt{a+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] -Sqrt[a + c*x^4]/(3*a*x^3) - (c^(3/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqr
t[(a + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1
/4)*x)/a^(1/4)], 1/2])/(6*a^(5/4)*Sqrt[a + c*x^4])
```

Rubi in Sympy [A] time = 6.99536, size = 97, normalized size = 0.88

$$-\frac{\sqrt{a+cx^4}}{3ax^3}-\frac{c^{\frac{3}{4}}\sqrt{\frac{a+cx^4}{\left(\sqrt{a}+\sqrt{c}x^2\right)^2}}\left(\sqrt{a}+\sqrt{c}x^2\right)F\left(2\,\mathrm{atan}\left(\frac{\sqrt[4]{c}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{6a^{\frac{5}{4}}\sqrt{a+cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] -sqrt(a + c*x**4)/(3*a*x**3) - c**(3/4)*sqrt((a + c*x**4)/(sqrt(a
) + sqrt(c)*x**2)**2)*(sqrt(a) + sqrt(c)*x**2)*elliptic_f(2*atan(
c**(1/4)*x/a**(1/4)), 1/2)/(6*a**(5/4)*sqrt(a + c*x**4))
```

Mathematica [C] time = 0.194277, size = 95, normalized size = 0.86

$$\frac{-\frac{a+cx^4}{x^3}+\frac{ic\sqrt{\frac{cx^4}{a}+1}F\left(i\sinh^{-1}\left(\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}x\right)\middle|-1\right)}{\sqrt{\frac{i\sqrt{c}}{\sqrt{a}}}}}{3a\sqrt{a+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + (2 + 2*b - 2*(1 + b))*x^2 + c*x^4]),x]
```

```
[Out] (-((a + c*x^4)/x^3) + (I*c*Sqrt[1 + (c*x^4)/a]*EllipticF[I*ArcSin
h[Sqrt[(I*Sqrt[c])/Sqrt[a]]*x], -1])/Sqrt[(I*Sqrt[c])/Sqrt[a]])/(
3*a*Sqrt[a + c*x^4])
```

Maple [C] time = 0.016, size = 93, normalized size = 0.9

$$-\frac{1}{3\,ax^3}\sqrt{cx^4+a}-\frac{c}{3\,a}\sqrt{1-ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{c}\frac{1}{\sqrt{a}}}\mathit{EllipticF}\left(x\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{c}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{cx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4+a)^(1/2),x)
```

```
[Out] -1/3*(c*x^4+a)^(1/2)/a/x^3-1/3*c/a/(I/a^(1/2)*c^(1/2))^(1/2)*(1-I
/a^(1/2)*c^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*c^(1/2)*x^2)^(1/2)/(c*x^
4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*c^(1/2))^(1/2),I)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{cx^4+a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{cx^4+a}x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^4),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + a)*x^4), x)
```

Sympy [A] time = 2.58375, size = 41, normalized size = 0.37

$$\frac{\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{cx^4e^{i\pi}}{a}\right)}{4\sqrt{a}x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4+a)**(1/2),x)
```

```
[Out] gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), c*x**4*exp_polar(I*pi)/a)/
(4*sqrt(a)*x**3*gamma(1/4))
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+a}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + a)*x^4),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + a)*x^4), x)
```

3.1012 $\int \frac{x^4}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=73

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) +
(3*a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi [A] time = 0.0685916, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (-3*a*x*Sqrt[a + b*x^2])/(8*b^2) + (x^3*Sqrt[a + b*x^2])/(4*b) +
(3*a^2*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rubi in Sympy [A] time = 8.48802, size = 66, normalized size = 0.9

$$\frac{3a^2\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{\frac{5}{2}}}-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] 3*a**2*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(8*b**(5/2)) - 3*a*x*sqr
t(a + b*x**2)/(8*b**2) + x**3*sqrt(a + b*x**2)/(4*b)
```

Mathematica [A] time = 0.0551104, size = 67, normalized size = 0.92

$$\frac{3a^2\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{8b^{5/2}}+\sqrt{a+bx^2}\left(\frac{x^3}{4b}-\frac{3ax}{8b^2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] Sqrt[a + b*x^2]*((-3*a*x)/(8*b^2) + x^3/(4*b)) + (3*a^2*Log[b*x +
 Sqrt[b]*Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A] time = 0.01, size = 59, normalized size = 0.8

$$\frac{x^3}{4\,b}\sqrt{bx^2+a}-\frac{3\,ax}{8\,b^2}\sqrt{bx^2+a}+\frac{3\,a^2}{8}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/4*x^3*(b*x^2+a)^(1/2)/b-3/8*a*x*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(
5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.288658, size = 1, normalized size = 0.01

$$\left[\frac{3\,a^2\log\left(-2\,\sqrt{bx^2+a}bx-\left(2\,bx^2+a\right)\sqrt{b}\right)+2\left(2\,bx^3-3\,ax\right)\sqrt{bx^2+a}\sqrt{b}}{16\,b^{\frac{5}{2}}},\frac{3\,a^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(2\,bx^3-3\,ax\right)\sqrt{bx^2+a}}{8\,\sqrt{-b}b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))
+ 2*(2*b*x^3 - 3*a*x)*sqrt(b*x^2 + a)*sqrt(b))/b^(5/2), 1/8*(3*a^
2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*b*x^3 - 3*a*x)*sqrt(b*x
^2 + a)*sqrt(-b))/(sqrt(-b)*b^2)]
```

Sympy [A] time = 12.429, size = 95, normalized size = 1.3

$$-\frac{3a^{\frac{3}{2}}x}{8b^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{\sqrt{a}x^{3}}{8b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^{5}}{4\sqrt{a}\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqr
t(1 + b*x**2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) +
 x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

GIAC/XCAS [A] time = 0.283285, size = 73, normalized size = 1.

$$\frac{1}{8}\sqrt{bx^{2}+a}x\left(\frac{2x^{2}}{b}-\frac{3a}{b^{2}}\right)-\frac{3a^{2}\ln\left(\left|-\sqrt{b}x+\sqrt{bx^{2}+a}\right|\right)}{8b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*ln(abs(-sqrt(
b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.1013 $\int \frac{x^3}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=36

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

```
[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rubi [A] time = 0.0595031, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{a\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] -((a*Sqrt[a + b*x^2])/b^2) + (a + b*x^2)^(3/2)/(3*b^2)
```

Rubi in Sympy [A] time = 7.47066, size = 29, normalized size = 0.81

$$-\frac{a\sqrt{a+bx^2}}{b^2}+\frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(b*x**2+a)**(1/2),x)

[Out] -a*sqrt(a + b*x**2)/b**2 + (a + b*x**2)**(3/2)/(3*b**2)
```

Mathematica [A] time = 0.019558, size = 27, normalized size = 0.75

$$\frac{\left(bx^2-2a\right)\sqrt{a+bx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] ((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Maple [A] time = 0.007, size = 25, normalized size = 0.7

$$-\frac{-bx^2+2a}{3b^2}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(b*x^2+a)^(1/2),x)

[Out] -1/3*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(b*x^2 + a),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273984, size = 31, normalized size = 0.86

$$\frac{\sqrt{bx^2+a}\left(bx^2-2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(b*x^2 + a),x, algorithm="fricas")

[Out] 1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A] time = 1.67197, size = 44, normalized size = 1.22

$$\begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2} + \frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)
/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

GIAC/XCAS [A] time = 0.278336, size = 36, normalized size = 1.

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}} - 3\sqrt{bx^2+a}a}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/3*((b*x^2 + a)^(3/2) - 3*sqrt(b*x^2 + a)*a)/b^2
```

3.1014 $\int \frac{x^2}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*b^(3/2))
```

Rubi [A] time = 0.0395918, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2
]])/(2*b^(3/2))
```

Rubi in Sympy [A] time = 5.40438, size = 41, normalized size = 0.84

$$-\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{\frac{3}{2}}}+\frac{x\sqrt{a+bx^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -a*atanh(sqrt(b)*x/sqrt(a + b*x**2))/(2*b**(3/2)) + x*sqrt(a + b*
x**2)/(2*b)
```

Mathematica [A] time = 0.0280654, size = 52, normalized size = 1.06

$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\log\left(\sqrt{b}\sqrt{a+bx^2}+bx\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] (x*Sqrt[a + b*x^2])/(2*b) - (a*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]
)/(2*b^(3/2))
```

Maple [A] time = 0.007, size = 39, normalized size = 0.8

$$\frac{x}{2b}\sqrt{bx^2+a}-\frac{a}{2}\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)b^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(b*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.284935, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{bx^2+a}\sqrt{b}x+a\log\left(2\sqrt{bx^2+a}abx-\left(2bx^2+a\right)\sqrt{b}\right)}{4b^{\frac{3}{2}}},\frac{\sqrt{bx^2+a}\sqrt{-b}x-a\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\sqrt{-b}b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*x^2 + a)*sqrt(b)*x + a*log(2*sqrt(b*x^2 + a)*b*x -
 (2*b*x^2 + a)*sqrt(b)))/b^(3/2), 1/2*(sqrt(b*x^2 + a)*sqrt(-b)*x
 - a*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(sqrt(-b)*b)]
```

Sympy [A] time = 6.95677, size = 42, normalized size = 0.86

$$\frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(
2*b**(3/2))
```

GIAC/XCAS [A] time = 0.283588, size = 54, normalized size = 1.1

$$\frac{\sqrt{bx^2+a}x}{2\,b}+\frac{a\ln\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*ln(abs(-sqrt(b)*x + sqrt(b*x^2 +
a)))/b^(3/2)
```

3.1015 $\int \frac{x}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{a+bx^2}}{b}$$

```
[Out] Sqrt[a + b*x^2]/b
```

Rubi [A] time = 0.00917807, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] Sqrt[a + b*x^2]/b
```

Rubi in Sympy [A] time = 2.04987, size = 10, normalized size = 0.67

$$\frac{\sqrt{a+bx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(b*x**2+a)**(1/2),x)

[Out] sqrt(a + b*x**2)/b
```

Mathematica [A] time = 0.00409738, size = 15, normalized size = 1.

$$\frac{\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] Sqrt[a + b*x^2]/b
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$\frac{1}{b}\sqrt{bx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(b*x^2+a)^(1/2),x)

[Out] (b*x^2+a)^(1/2)/b
```

Maxima [A] time = 0.708105, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b*x^2 + a),x, algorithm="maxima")

[Out] sqrt(b*x^2 + a)/b
```

Fricas [A] time = 0.271583, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b*x^2 + a),x, algorithm="fricas")

[Out] sqrt(b*x^2 + a)/b
```

Sympy [A] time = 1.3656, size = 20, normalized size = 1.33

$$\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True
))
```

GIAC/XCAS [A] time = 0.275567, size = 18, normalized size = 1.2

$$\frac{\sqrt{bx^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b
```

3.1016 $\int \frac{1}{\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] time = 0.0168151, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi in Sympy [A] time = 2.32599, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+a)**(1/2),x)

[Out] atanh(sqrt(b)*x/sqrt(a + b*x**2))/sqrt(b)
```

Mathematica [A] time = 0.0112528, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$1\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+a)^(1/2),x)
```

```
[Out] ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.280842, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(-2\sqrt{bx^2+a}bx-\left(2bx^2+a\right)\sqrt{b}\right)}{2\sqrt{b}},\ \frac{\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{\sqrt{-b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*sqrt(b*x^2 + a)*b*x - (2*b*x^2 + a)*sqrt(b))/sqrt(b),
 arctan(sqrt(-b)*x/sqrt(b*x^2 + a))/sqrt(-b)]
```

Sympy [A] time = 3.3579, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+a)**(1/2),x)
```

```
[Out] asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

GIAC/XCAS [A] time = 0.281835, size = 31, normalized size = 1.24

$$-\frac{\ln\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b*x^2 + a),x, algorithm="giac")
```

```
[Out] -ln(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

3.1017 $\int \frac{1}{x\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=25

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rubi [A] time = 0.0475632, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rubi in Sympy [A] time = 5.42323, size = 22, normalized size = 0.88

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(b*x**2+a)**(1/2),x)

[Out] -atanh(sqrt(a + b*x**2)/sqrt(a))/sqrt(a)
```

Mathematica [A] time = 0.0273563, size = 31, normalized size = 1.24

$$\frac{\log(x) - \log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] (Log[x] - Log[a + Sqrt[a]*Sqrt[a + b*x^2]])/Sqrt[a]
```

Maple [A] time = 0.006, size = 29, normalized size = 1.2

$$-1\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{bx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(b*x^2+a)^(1/2),x)

[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.279876, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(-\frac{(bx^2+2\,a)\sqrt{a}-2\,\sqrt{bx^2+a}a}{x^2}\right)}{2\,\sqrt{a}}, -\frac{\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{\sqrt{-a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((b*x^2 + 2*a)*sqrt(a) - 2*sqrt(b*x^2 + a)*a)/x^2)/sqrt
(a), -arctan(sqrt(-a)/sqrt(b*x^2 + a))/sqrt(-a)]
```

Sympy [A] time = 3.47171, size = 19, normalized size = 0.76

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

GIAC/XCAS [A] time = 0.265499, size = 30, normalized size = 1.2

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

3.1018 $\int \frac{1}{x^2\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a+bx^2}}{ax}$$

```
[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rubi [A] time = 0.0200969, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Rubi in Sympy [A] time = 3.02716, size = 14, normalized size = 0.74

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(b*x**2+a)**(1/2),x)

[Out] -sqrt(a + b*x**2)/(a*x)
```

Mathematica [A] time = 0.015299, size = 19, normalized size = 1.

$$-\frac{\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(Sqrt[a + b*x^2]/(a*x))
```

Maple [A] time = 0.005, size = 18, normalized size = 1.

$$-\frac{1}{ax}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(b*x^2+a)^(1/2),x)

[Out] -(b*x^2+a)^(1/2)/a/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.269518, size = 23, normalized size = 1.21

$$-\frac{\sqrt{bx^2+a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^2),x, algorithm="fricas")

[Out] -sqrt(b*x^2 + a)/(a*x)
```

Sympy [A] time = 1.79354, size = 19, normalized size = 1.

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

GIAC/XCAS [A] time = 0.267644, size = 41, normalized size = 2.16

$$\frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.1019 $\int \frac{1}{x^3\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=50

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/(2*a^(3/2))
```

Rubi [A] time = 0.0733049, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/(2*a^(3/2))
```

Rubi in Sympy [A] time = 7.23566, size = 41, normalized size = 0.82

$$-\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\,\text{atanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2)/(2*a*x**2) + b*atanh(sqrt(a + b*x**2)/sqrt(a))/
(2*a**(3/2))
```

Mathematica [A] time = 0.0364934, size = 64, normalized size = 1.28

$$\frac{b\log\left(\sqrt{a}\sqrt{a+bx^2}+a\right)}{2a^{3/2}}-\frac{b\log(x)}{2a^{3/2}}-\frac{\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(2*a*x^2) - (b*Log[x])/(2*a^(3/2)) + (b*Log[a +
Sqrt[a]*Sqrt[a + b*x^2]])/(2*a^(3/2))
```

Maple [A] time = 0.008, size = 48, normalized size = 1.

$$-\frac{1}{2\,a x^2}\sqrt{b x^2+a}+\frac{b}{2}\ln\left(\frac{1}{x}\left(2\,a+2\,\sqrt{a}\sqrt{b x^2+a}\right)\right)a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(b*x^2+a)^(1/2),x)
```

```
[Out] -1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2
+a)^(1/2))/x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.286097, size = 1, normalized size = 0.02

$$\left[\frac{b x^2\log\left(-\frac{(b x^2+2\,a)\sqrt{a}+2\,\sqrt{b x^2+a}a}{x^2}\right)-2\,\sqrt{b x^2+a}\sqrt{a}}{4\,a^{\frac{3}{2}}x^2},\ \frac{b x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^2+a}}\right)-\sqrt{b x^2+a}\sqrt{-a}}{2\,\sqrt{-a}a x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(b*x^2*log(-((b*x^2 + 2*a)*sqrt(a) + 2*sqrt(b*x^2 + a)*a)/x^
2) - 2*sqrt(b*x^2 + a)*sqrt(a))/(a^(3/2)*x^2), 1/2*(b*x^2*arctan(
sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*sqrt(-a))/(sqrt(-a)*a
*x^2)]
```

Sympy [A] time = 7.30155, size = 42, normalized size = 0.84

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*
x))/(2*a**(3/2))
```

GIAC/XCAS [A] time = 0.26404, size = 65, normalized size = 1.3

$$-\frac{1}{2}b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx^2+a}}{abx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^
2 + a)/(a*b*x^2))
```

3.1020 $\int \frac{1}{x^4\sqrt{a+bx^2+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=44

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rubi [A] time = 0.0413927, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103

$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]
```

```
[Out] -Sqrt[a + b*x^2]/(3*a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Rubi in Sympy [A] time = 4.97138, size = 36, normalized size = 0.82

$$-\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(a + b*x**2)/(3*a*x**3) + 2*b*sqrt(a + b*x**2)/(3*a**2*x)
```

Mathematica [A] time = 0.0208888, size = 29, normalized size = 0.66

$$-\frac{\left(a-2bx^2\right)\sqrt{a+bx^2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + b*x^2 + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -((a - 2*b*x^2)*Sqrt[a + b*x^2])/(3*a^2*x^3)
```

Maple [A] time = 0.006, size = 26, normalized size = 0.6

$$-\frac{-2\,bx^2+a}{3\,a^2x^3}\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(b*x^2+a)^(1/2),x)

[Out] -1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^4),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.273675, size = 36, normalized size = 0.82

$$\frac{\left(2\,bx^2-a\right)\sqrt{bx^2+a}}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^4),x, algorithm="fricas")

[Out] 1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] time = 2.42826, size = 46, normalized size = 1.05

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(3*a**2)
```

GIAC/XCAS [A] time = 0.26848, size = 74, normalized size = 1.68

$$\frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(b*x^2 + a)*x^4),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x -
 sqrt(b*x^2 + a))^2 - a)^3
```

3.1021 $\int \frac{x^4}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^5}{3\sqrt{cx^4}}$$

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Rubi [A] time = 0.00663293, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] x^5/(3*Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 1.85377, size = 12, normalized size = 0.75

$$\frac{x\sqrt{cx^4}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/(c*x**4)**(1/2),x)

[Out] x*sqrt(c*x**4)/(3*c)
```

Mathematica [A] time = 0.00433641, size = 16, normalized size = 1.

$$\frac{x^5}{3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^5/(3*Sqrt[c*x^4])
```

Maple [A] time = 0.004, size = 13, normalized size = 0.8

$$\frac{x^5}{3}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/(c*x^4)^(1/2),x)
```

```
[Out] 1/3*x^5/(c*x^4)^(1/2)
```

Maxima [A] time = 0.708321, size = 16, normalized size = 1.

$$\frac{x^5}{3\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4),x, algorithm="maxima")
```

```
[Out] 1/3*x^5/sqrt(c*x^4)
```

Fricas [A] time = 0.263066, size = 18, normalized size = 1.12

$$\frac{\sqrt{cx^4}x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(c*x^4)*x/c
```

Sympy [A] time = 1.88705, size = 15, normalized size = 0.94

$$\frac{x^5}{3\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/(c*x**4)**(1/2),x)

[Out] x**5/(3*sqrt(c)*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.261711, size = 18, normalized size = 1.12

$$\frac{\sqrt{cx^4}x}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(c*x^4),x, algorithm="giac")

[Out] 1/3*sqrt(c*x^4)*x/c
```

3.1022 $\int \frac{x^3}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$\frac{x^4}{2\sqrt{cx^4}}$$

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Rubi [A] time = 0.00688027, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 1.91306, size = 10, normalized size = 0.62

$$\frac{\sqrt{cx^4}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/(c*x**4)**(1/2),x)
```

```
[Out] sqrt(c*x**4)/(2*c)
```

Mathematica [A] time = 0.00335886, size = 16, normalized size = 1.

$$\frac{x^4}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^4/(2*Sqrt[c*x^4])
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$\frac{x^4}{2}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/(c*x^4)^(1/2),x)
```

```
[Out] 1/2*x^4/(c*x^4)^(1/2)
```

Maxima [A] time = 0.706185, size = 16, normalized size = 1.

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

Fricas [A] time = 0.263201, size = 16, normalized size = 1.

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

Sympy [A] time = 1.64345, size = 15, normalized size = 0.94

$$\frac{x^4}{2\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/(c*x**4)**(1/2),x)
```

```
[Out] x**4/(2*sqrt(c)*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.260677, size = 16, normalized size = 1.

$$\frac{\sqrt{cx^4}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(c*x^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4)/c
```

3.1023 $\int \frac{x^2}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$\frac{x^3}{\sqrt{cx^4}}$$

```
[Out] x^3/Sqrt[c*x^4]
```

Rubi [A] time = 0.00588769, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] x^3/Sqrt[c*x^4]
```

Rubi in Sympy [A] time = 2.12478, size = 10, normalized size = 0.77

$$\frac{\sqrt{cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/(c*x**4)**(1/2),x)

[Out] sqrt(c*x**4)/(c*x)
```

Mathematica [A] time = 0.00237811, size = 13, normalized size = 1.

$$\frac{x^3}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] x^3/Sqrt[c*x^4]
```

Maple [A] time = 0.006, size = 12, normalized size = 0.9

$$x^3\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/(c*x^4)^(1/2),x)
```

```
[Out] x^3/(c*x^4)^(1/2)
```

Maxima [A] time = 0.708495, size = 15, normalized size = 1.15

$$\frac{x^3}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4),x, algorithm="maxima")
```

```
[Out] x^3/sqrt(c*x^4)
```

Fricas [A] time = 0.2659, size = 19, normalized size = 1.46

$$\frac{\sqrt{cx^4}}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4)/(c*x)
```

Sympy [A] time = 1.52269, size = 14, normalized size = 1.08

$$\frac{x^3}{\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/(c*x**4)**(1/2),x)
```

```
[Out] x**3/(sqrt(c)*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.264578, size = 7, normalized size = 0.54

$$\frac{x}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(c*x^4),x, algorithm="giac")
```

```
[Out] x/sqrt(c)
```

3.1024 $\int \frac{x}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=15

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Rubi [A] time = 0.00614463, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]

[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Rubi in Sympy [A] time = 1.93994, size = 15, normalized size = 1.

$$\frac{\sqrt{cx^4}\log(x)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(c*x**4)**(1/2),x)

[Out] sqrt(c*x**4)*log(x)/(c*x**2)
```

Mathematica [A] time = 0.00298992, size = 15, normalized size = 1.

$$\frac{x^2\log(x)}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] (x^2*Log[x])/Sqrt[c*x^4]
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$x^2 \ln(x) \frac{1}{\sqrt{c x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(c*x^4)^(1/2),x)
```

```
[Out] x^2*ln(x)/(c*x^4)^(1/2)
```

Maxima [A] time = 0.713645, size = 18, normalized size = 1.2

$$\frac{x^2 \log(x)}{\sqrt{c x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4),x, algorithm="maxima")
```

```
[Out] x^2*log(x)/sqrt(c*x^4)
```

Fricas [A] time = 0.270119, size = 22, normalized size = 1.47

$$\frac{\sqrt{c x^4} \log(x)}{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^4)*log(x)/(c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(c*x**4)**(1/2),x)
```

```
[Out] Integral(x/sqrt(c*x**4), x)
```

GIAC/XCAS [A] time = 0.271099, size = 16, normalized size = 1.07

$$\frac{\ln\left(x^4|c|\right)}{4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(c*x^4),x, algorithm="giac")
```

```
[Out] 1/4*ln(x^4*abs(c))/sqrt(c)
```

3.1025 $\int \frac{1}{\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{x}{\sqrt{cx^4}}$$

```
[Out] -(x/Sqrt[c*x^4])
```

Rubi [A] time = 0.00608896, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] -(x/Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 1.37427, size = 14, normalized size = 1.17

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**4)**(1/2),x)
```

```
[Out] -sqrt(c*x**4)/(c*x**3)
```

Mathematica [A] time = 0.00211701, size = 12, normalized size = 1.

$$-\frac{x}{\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 2*a - 2*(1 + a) + c*x^4],x]
```

```
[Out] -(x/Sqrt[c*x^4])
```

Maple [A] time = 0.003, size = 11, normalized size = 0.9

$$-x\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^4)^(1/2),x)
```

```
[Out] -x/(c*x^4)^(1/2)
```

Maxima [A] time = 0.700816, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4),x, algorithm="maxima")
```

```
[Out] -x/sqrt(c*x^4)
```

Fricas [A] time = 0.260966, size = 20, normalized size = 1.67

$$-\frac{\sqrt{cx^4}}{cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^4)/(c*x^3)
```

Sympy [A] time = 1.40688, size = 14, normalized size = 1.17

$$-\frac{x}{\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**4)**(1/2),x)
```

```
[Out] -x/(sqrt(c)*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.26533, size = 11, normalized size = 0.92

$$-\frac{1}{\sqrt{c}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^4),x, algorithm="giac")
```

```
[Out] -1/(sqrt(c)*x)
```

3.1026 $\int \frac{1}{x\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{2\sqrt{cx^4}}$$

```
[Out] -1/(2*Sqrt[c*x^4])
```

Rubi [A] time = 0.00570434, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -1/(2*Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 2.12009, size = 12, normalized size = 0.92

$$-\frac{1}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/(c*x**4)**(1/2),x)

[Out] -1/(2*sqrt(c*x**4))
```

Mathematica [A] time = 0.00229332, size = 13, normalized size = 1.

$$-\frac{1}{2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]
```

```
[Out] -1/(2*Sqrt[c*x^4])
```

Maple [A] time = 0.004, size = 10, normalized size = 0.8

$$-\frac{1}{2}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/(c*x^4)^(1/2),x)
```

```
[Out] -1/2/(c*x^4)^(1/2)
```

Maxima [A] time = 0.708888, size = 12, normalized size = 0.92

$$-\frac{1}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x),x, algorithm="maxima")
```

```
[Out] -1/2/sqrt(c*x^4)
```

Fricas [A] time = 0.267883, size = 20, normalized size = 1.54

$$-\frac{\sqrt{cx^4}}{2\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^4)/(c*x^4)
```

Sympy [A] time = 1.52042, size = 15, normalized size = 1.15

$$-\frac{1}{2\sqrt{c}\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/(c*x**4)**(1/2),x)
```

```
[Out] -1/(2*sqrt(c)*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.26103, size = 12, normalized size = 0.92

$$-\frac{1}{2\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x),x, algorithm="giac")
```

```
[Out] -1/2/sqrt(c*x^4)
```

3.1027 $\int \frac{1}{x^2\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3x\sqrt{cx^4}}$$

```
[Out] -1/(3*x*Sqrt[c*x^4])
```

Rubi [A] time = 0.00762296, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -1/(3*x*Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 2.13506, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{3cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/(c*x**4)**(1/2),x)

[Out] -sqrt(c*x**4)/(3*c*x**5)
```

Mathematica [A] time = 0.00365389, size = 16, normalized size = 1.

$$-\frac{1}{3x\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -1/(3*x*Sqrt[c*x^4])
```

Maple [A] time = 0.004, size = 13, normalized size = 0.8

$$-\frac{1}{3x}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/(c*x^4)^(1/2),x)

[Out] -1/3/x/(c*x^4)^(1/2)
```

Maxima [A] time = 0.708985, size = 16, normalized size = 1.

$$-\frac{1}{3\sqrt{cx^4}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^2),x, algorithm="maxima")

[Out] -1/3/(sqrt(c*x^4)*x)
```

Fricas [A] time = 0.263154, size = 20, normalized size = 1.25

$$-\frac{\sqrt{cx^4}}{3\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^2),x, algorithm="fricas")

[Out] -1/3*sqrt(c*x^4)/(c*x^5)
```

Sympy [A] time = 1.67977, size = 17, normalized size = 1.06

$$-\frac{1}{3\sqrt{c}x\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/(c*x**4)**(1/2),x)

[Out] -1/(3*sqrt(c)*x*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.261782, size = 11, normalized size = 0.69

$$-\frac{1}{3\sqrt{c}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^2),x, algorithm="giac")

[Out] -1/3/(sqrt(c)*x^3)
```

3.1028 $\int \frac{1}{x^3\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

```
[Out] -1/(4*x^2*Sqrt[c*x^4])
```

Rubi [A] time = 0.00655485, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{1}{4x^2\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -1/(4*x^2*Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 2.12053, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{4cx^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/(c*x**4)**(1/2),x)

[Out] -sqrt(c*x**4)/(4*c*x**6)
```

Mathematica [A] time = 0.00572354, size = 17, normalized size = 1.06

$$-\frac{cx^2}{4\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -(c*x^2)/(4*(c*x^4)^(3/2))
```

Maple [A] time = 0.005, size = 13, normalized size = 0.8

$$-\frac{1}{4x^{2}}\frac{1}{\sqrt{cx^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/(c*x^4)^(1/2),x)

[Out] -1/4/x^2/(c*x^4)^(1/2)
```

Maxima [A] time = 0.722932, size = 16, normalized size = 1.

$$-\frac{1}{4\sqrt{cx^{4}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^3),x, algorithm="maxima")

[Out] -1/4/(sqrt(c*x^4)*x^2)
```

Fricas [A] time = 0.264467, size = 20, normalized size = 1.25

$$-\frac{\sqrt{cx^{4}}}{4\,cx^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^3),x, algorithm="fricas")

[Out] -1/4*sqrt(c*x^4)/(c*x^6)
```

Sympy [A] time = 1.92904, size = 19, normalized size = 1.19

$$-\frac{1}{4\sqrt{c}x^2\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/(c*x**4)**(1/2),x)
```

```
[Out] -1/(4*sqrt(c)*x**2*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.264185, size = 11, normalized size = 0.69

$$-\frac{1}{4\sqrt{c}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^3),x, algorithm="giac")
```

```
[Out] -1/4/(sqrt(c)*x^4)
```

3.1029 $\int \frac{1}{x^4\sqrt{2+2a-2(1+a)+cx^4}}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

```
[Out] -1/(5*x^3*Sqrt[c*x^4])
```

Rubi [A] time = 0.00611136, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13

$$-\frac{1}{5x^3\sqrt{cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -1/(5*x^3*Sqrt[c*x^4])
```

Rubi in Sympy [A] time = 2.12622, size = 15, normalized size = 0.94

$$-\frac{\sqrt{cx^4}}{5cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/(c*x**4)**(1/2),x)

[Out] -sqrt(c*x**4)/(5*c*x**7)
```

Mathematica [A] time = 0.00342446, size = 15, normalized size = 0.94

$$-\frac{cx}{5\left(cx^4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[2 + 2*a - 2*(1 + a) + c*x^4]),x]

[Out] -(c*x)/(5*(c*x^4)^(3/2))
```

Maple [A] time = 0.004, size = 13, normalized size = 0.8

$$-\frac{1}{5\,x^3}\frac{1}{\sqrt{cx^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/(c*x^4)^(1/2),x)

[Out] -1/5/x^3/(c*x^4)^(1/2)
```

Maxima [A] time = 0.71728, size = 16, normalized size = 1.

$$-\frac{1}{5\,\sqrt{cx^4}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^4),x, algorithm="maxima")

[Out] -1/5/(sqrt(c*x^4)*x^3)
```

Fricas [A] time = 0.260651, size = 20, normalized size = 1.25

$$-\frac{\sqrt{cx^4}}{5\,cx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^4),x, algorithm="fricas")

[Out] -1/5*sqrt(c*x^4)/(c*x^7)
```

Sympy [A] time = 2.24826, size = 19, normalized size = 1.19

$$-\frac{1}{5\sqrt{c}x^3\sqrt{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/(c*x**4)**(1/2),x)
```

```
[Out] -1/(5*sqrt(c)*x**3*sqrt(x**4))
```

GIAC/XCAS [A] time = 0.609977, size = 4, normalized size = 0.25

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4)*x^4),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.1030 $\int \frac{x^4}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^5}{5\sqrt{a}}$$

```
[Out] x^5/(5*Sqrt[a])
```

Rubi [A] time = 0.00592897, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^5/(5*Sqrt[a])
```

Rubi in Sympy [A] time = 1.53179, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4/a**(1/2),x)

[Out] x**5/(5*sqrt(a))
```

Mathematica [A] time = 0.00127929, size = 12, normalized size = 1.

$$\frac{x^5}{5\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^4/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^5/(5*Sqrt[a])
```

Maple [A] time = 0., size = 9, normalized size = 0.8

$$\frac{x^5}{5}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4/a^(1/2),x)
```

```
[Out] 1/5*x^5/a^(1/2)
```

Maxima [A] time = 0.711232, size = 11, normalized size = 0.92

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(a),x, algorithm="maxima")
```

```
[Out] 1/5*x^5/sqrt(a)
```

Fricas [A] time = 0.264471, size = 11, normalized size = 0.92

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5/sqrt(a)
```

Sympy [A] time = 0.057093, size = 8, normalized size = 0.67

$$\frac{x^5}{5\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4/a**(1/2),x)
```

```
[Out] x**5/(5*sqrt(a))
```

GIAC/XCAS [A] time = 0.263128, size = 11, normalized size = 0.92

$$\frac{x^5}{5\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^4/sqrt(a),x, algorithm="giac")
```

```
[Out] 1/5*x^5/sqrt(a)
```

3.1031 $\int \frac{x^3}{\sqrt{a+(2+2c-2(1+c))x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{x^4}{4\sqrt{a}}$$

```
[Out] x^4/(4*Sqrt[a])
```

Rubi [A] time = 0.00682588, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^4/(4*Sqrt[a])
```

Rubi in Sympy [A] time = 1.53352, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3/a**(1/2),x)

[Out] x**4/(4*sqrt(a))
```

Mathematica [A] time = 0.00102683, size = 12, normalized size = 1.

$$\frac{x^4}{4\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^3/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^4/(4*Sqrt[a])
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$\frac{x^4}{4}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3/a^(1/2),x)

[Out] 1/4*x^4/a^(1/2)
```

Maxima [A] time = 0.739844, size = 11, normalized size = 0.92

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(a),x, algorithm="maxima")

[Out] 1/4*x^4/sqrt(a)
```

Fricas [A] time = 0.266392, size = 11, normalized size = 0.92

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(a),x, algorithm="fricas")

[Out] 1/4*x^4/sqrt(a)
```

Sympy [A] time = 0.060962, size = 8, normalized size = 0.67

$$\frac{x^4}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3/a**(1/2),x)
```

```
[Out] x**4/(4*sqrt(a))
```

GIAC/XCAS [A] time = 0.26345, size = 11, normalized size = 0.92

$$\frac{x^4}{4\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^3/sqrt(a),x, algorithm="giac")
```

```
[Out] 1/4*x^4/sqrt(a)
```

3.1032 $\int \frac{x^2}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{x^3}{3\sqrt{a}}$$

```
[Out] x^3/(3*Sqrt[a])
```

Rubi [A] time = 0.00605888, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^3/(3*Sqrt[a])
```

Rubi in Sympy [A] time = 1.52938, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2/a**(1/2),x)

[Out] x**3/(3*sqrt(a))
```

Mathematica [A] time = 0.00108122, size = 12, normalized size = 1.

$$\frac{x^3}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^2/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^3/(3*Sqrt[a])
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$\frac{x^3}{3}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2/a^(1/2),x)
```

```
[Out] 1/3*x^3/a^(1/2)
```

Maxima [A] time = 0.746887, size = 11, normalized size = 0.92

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(a),x, algorithm="maxima")
```

```
[Out] 1/3*x^3/sqrt(a)
```

Fricas [A] time = 0.269022, size = 11, normalized size = 0.92

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(a),x, algorithm="fricas")
```

```
[Out] 1/3*x^3/sqrt(a)
```

Sympy [A] time = 0.055546, size = 8, normalized size = 0.67

$$\frac{x^3}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2/a**(1/2),x)
```

```
[Out] x**3/(3*sqrt(a))
```

GIAC/XCAS [A] time = 0.261061, size = 11, normalized size = 0.92

$$\frac{x^3}{3\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^2/sqrt(a),x, algorithm="giac")
```

```
[Out] 1/3*x^3/sqrt(a)
```

3.1033 $\int \frac{x}{\sqrt{a+(2+2c-2(1+c))x^4}} \, dx$

Optimal. Leaf size=12

$$\frac{x^2}{2\sqrt{a}}$$

```
[Out] x^2/(2*Sqrt[a])
```

Rubi [A] time = 0.00579073, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x^2/(2*Sqrt[a])
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int x \, dx}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/a**(1/2),x)

[Out] Integral(x, x)/sqrt(a)
```

Mathematica [A] time = 0.000953549, size = 12, normalized size = 1.

$$\frac{x^2}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]
```

```
[Out] x^2/(2*Sqrt[a])
```

Maple [A] time = 0., size = 9, normalized size = 0.8

$$\frac{x^2}{2}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/a^(1/2),x)
```

```
[Out] 1/2*x^2/a^(1/2)
```

Maxima [A] time = 0.737804, size = 11, normalized size = 0.92

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/sqrt(a)
```

Fricas [A] time = 0.266192, size = 11, normalized size = 0.92

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(a),x, algorithm="fricas")
```

```
[Out] 1/2*x^2/sqrt(a)
```

Sympy [A] time = 0.051983, size = 8, normalized size = 0.67

$$\frac{x^2}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/a**(1/2),x)
```

```
[Out] x**2/(2*sqrt(a))
```

GIAC/XCAS [A] time = 0.260769, size = 11, normalized size = 0.92

$$\frac{x^2}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/sqrt(a),x, algorithm="giac")
```

```
[Out] 1/2*x^2/sqrt(a)
```

3.1034 $\int \frac{1}{\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=7

$$\frac{x}{\sqrt{a}}$$

```
[Out] x/Sqrt[a]
```

Rubi [A] time = 0.00440841, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x/Sqrt[a]
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/a**(1/2),x)

[Out] Integral(1/sqrt(a), x)
```

Mathematica [A] time = 0.000984268, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4],x]

[Out] x/Sqrt[a]
```

Maple [A] time = 0., size = 6, normalized size = 0.9

$$x\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/a^(1/2),x)

[Out] x/a^(1/2)
```

Maxima [A] time = 0.737945, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a),x, algorithm="maxima")

[Out] x/sqrt(a)
```

Fricas [A] time = 0.262916, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a),x, algorithm="fricas")

[Out] x/sqrt(a)
```

Sympy [A] time = 0.040289, size = 5, normalized size = 0.71

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/a**(1/2),x)
```

```
[Out] x/sqrt(a)
```

GIAC/XCAS [A] time = 0.260934, size = 7, normalized size = 1.

$$\frac{x}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(a),x, algorithm="giac")
```

```
[Out] x/sqrt(a)
```

3.1035 $\int \frac{1}{x\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{\sqrt{a}}$$

```
[Out] Log[x]/Sqrt[a]
```

Rubi [A] time = 0.00515621, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] Log[x]/Sqrt[a]
```

Rubi in Sympy [A] time = 1.48567, size = 7, normalized size = 0.88

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x/a**(1/2),x)

[Out] log(x)/sqrt(a)
```

Mathematica [A] time = 0.000910992, size = 8, normalized size = 1.

$$\frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] Log[x]/Sqrt[a]
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$\ln(x)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x/a^(1/2),x)

[Out] ln(x)/a^(1/2)
```

Maxima [A] time = 0.752277, size = 8, normalized size = 1.

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x),x, algorithm="maxima")

[Out] log(x)/sqrt(a)
```

Fricas [A] time = 0.268515, size = 8, normalized size = 1.

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x),x, algorithm="fricas")

[Out] log(x)/sqrt(a)
```

Sympy [A] time = 0.068428, size = 7, normalized size = 0.88

$$\frac{\log(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x/a**(1/2),x)
```

```
[Out] log(x)/sqrt(a)
```

GIAC/XCAS [A] time = 0.262521, size = 9, normalized size = 1.12

$$\frac{\ln(|x|)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x),x, algorithm="giac")
```

```
[Out] ln(abs(x))/sqrt(a)
```

3.1036 $\int \frac{1}{x^2\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{\sqrt{a}x}$$

```
[Out] -(1/(Sqrt[a]*x))
```

Rubi [A] time = 0.00599616, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{1}{\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(1/(Sqrt[a]*x))
```

Rubi in Sympy [A] time = 1.50746, size = 8, normalized size = 0.8

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**2/a**(1/2),x)

[Out] -1/(sqrt(a)*x)
```

Mathematica [A] time = 0.000891473, size = 10, normalized size = 1.

$$-\frac{1}{\sqrt{a}x}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^2*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -(1/(Sqrt[a]*x))
```

Maple [A] time = 0.001, size = 9, normalized size = 0.9

$$-\frac{1}{x}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^2/a^(1/2),x)

[Out] -1/x/a^(1/2)
```

Maxima [A] time = 0.775061, size = 11, normalized size = 1.1

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^2),x, algorithm="maxima")

[Out] -1/(sqrt(a)*x)
```

Fricas [A] time = 0.268042, size = 11, normalized size = 1.1

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^2),x, algorithm="fricas")

[Out] -1/(sqrt(a)*x)
```

Sympy [A] time = 0.07421, size = 8, normalized size = 0.8

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**2/a**(1/2),x)
```

```
[Out] -1/(sqrt(a)*x)
```

GIAC/XCAS [A] time = 0.258761, size = 11, normalized size = 1.1

$$-\frac{1}{\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^2),x, algorithm="giac")
```

```
[Out] -1/(sqrt(a)*x)
```

3.1037 $\int \frac{1}{x^3\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2\sqrt{a}x^2}$$

```
[Out] -1/(2*Sqrt[a]*x^2)
```

Rubi [A] time = 0.00596704, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{1}{2\sqrt{a}x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -1/(2*Sqrt[a]*x^2)
```

Rubi in Sympy [A] time = 1.50852, size = 12, normalized size = 1.

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**3/a**(1/2),x)

[Out] -1/(2*sqrt(a)*x**2)
```

Mathematica [A] time = 0.00104314, size = 12, normalized size = 1.

$$-\frac{1}{2\sqrt{a}x^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^3*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -1/(2*Sqrt[a]*x^2)
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$-\frac{1}{2\,x^{2}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^3/a^(1/2),x)

[Out] -1/2/x^2/a^(1/2)
```

Maxima [A] time = 0.784501, size = 11, normalized size = 0.92

$$-\frac{1}{2\,\sqrt{a}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^3),x, algorithm="maxima")

[Out] -1/2/(sqrt(a)*x^2)
```

Fricas [A] time = 0.26636, size = 11, normalized size = 0.92

$$-\frac{1}{2\,\sqrt{a}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^3),x, algorithm="fricas")

[Out] -1/2/(sqrt(a)*x^2)
```

Sympy [A] time = 0.080459, size = 12, normalized size = 1.

$$-\frac{1}{2\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**3/a**(1/2),x)
```

```
[Out] -1/(2*sqrt(a)*x**2)
```

GIAC/XCAS [A] time = 0.261513, size = 11, normalized size = 0.92

$$-\frac{1}{2\,\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^3),x, algorithm="giac")
```

```
[Out] -1/2/(sqrt(a)*x^2)
```

3.1038 $\int \frac{1}{x^4\sqrt{a+(2+2c-2(1+c))x^4}}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{3\sqrt{a}x^3}$$

```
[Out] -1/(3*Sqrt[a]*x^3)
```

Rubi [A] time = 0.00583617, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{1}{3\sqrt{a}x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -1/(3*Sqrt[a]*x^3)
```

Rubi in Sympy [A] time = 1.50599, size = 12, normalized size = 1.

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**4/a**(1/2),x)

[Out] -1/(3*sqrt(a)*x**3)
```

Mathematica [A] time = 0.000914511, size = 12, normalized size = 1.

$$-\frac{1}{3\sqrt{a}x^3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^4*Sqrt[a + (2 + 2*c - 2*(1 + c))*x^4]),x]

[Out] -1/(3*Sqrt[a]*x^3)
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$-\frac{1}{3\,x^3}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^4/a^(1/2),x)

[Out] -1/3/x^3/a^(1/2)
```

Maxima [A] time = 0.743285, size = 11, normalized size = 0.92

$$-\frac{1}{3\,\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^4),x, algorithm="maxima")

[Out] -1/3/(sqrt(a)*x^3)
```

Fricas [A] time = 0.267776, size = 11, normalized size = 0.92

$$-\frac{1}{3\,\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^4),x, algorithm="fricas")

[Out] -1/3/(sqrt(a)*x^3)
```

Sympy [A] time = 0.076964, size = 12, normalized size = 1.

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**4/a**(1/2),x)
```

```
[Out] -1/(3*sqrt(a)*x**3)
```

GIAC/XCAS [A] time = 0.262034, size = 11, normalized size = 0.92

$$-\frac{1}{3\sqrt{a}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(a)*x^4),x, algorithm="giac")
```

```
[Out] -1/3/(sqrt(a)*x^3)
```

3.1039 $\int \frac{1}{\sqrt{3-2x^2-x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

```
[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rubi [A] time = 0.0316799, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\frac{F\left(\sin^{-1}(x)\middle|-\frac{1}{3}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 2*x^2 - x^4],x]

[Out] EllipticF[ArcSin[x], -1/3]/Sqrt[3]
```

Rubi in Sympy [A] time = 6.8272, size = 14, normalized size = 1.17

$$\frac{\sqrt{3}F\left(\operatorname{asin}(x)\middle|-\frac{1}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4-2*x**2+3)**(1/2),x)

[Out] sqrt(3)*elliptic_f(asin(x), -1/3)/3
```

Mathematica [C] time = 0.0310304, size = 18, normalized size = 1.5

$$-iF\left(i\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)\middle|-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 2*x^2 - x^4],x]

[Out] (-I)*EllipticF[I*ArcSinh[x/Sqrt[3]], -3]
```

Maple [B] time = 0.008, size = 43, normalized size = 3.6

$$\frac{\mathit{EllipticF}\left(x,\frac{i}{3}\sqrt{3}\right)}{3}\sqrt{-x^{2}+1}\sqrt{3\,x^{2}+9}\frac{1}{\sqrt{-x^{4}-2\,x^{2}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4-2*x^2+3)^(1/2),x)

[Out] 1/3*(-x^2+1)^(1/2)*(3*x^2+9)^(1/2)/(-x^4-2*x^2+3)^(1/2)*EllipticF
(x,1/3*I*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^{4}-2\,x^{2}+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 - 2*x^2 + 3),x, algorithm="maxima")

[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-x^{4}-2\,x^{2}+3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 - 2*x^2 + 3),x, algorithm="fricas")

[Out] integral(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 - 2x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4-2*x**2+3)**(1/2),x)

[Out] Integral(1/sqrt(-x**4 - 2*x**2 + 3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4 - 2\,x^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 - 2*x^2 + 3),x, algorithm="giac")

[Out] integrate(1/sqrt(-x^4 - 2*x^2 + 3), x)
```

3.1040 $\int \frac{1}{\sqrt{-1+5x^2-x^4}}\,dx$

Optimal. Leaf size=39

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

```
[Out] -(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/4
2]/21^(1/4))
```

Rubi [A] time = 0.15241, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{F\left(\cos^{-1}\left(\sqrt{\frac{2}{5+\sqrt{21}}}x\right)|\frac{1}{42}\left(21+5\sqrt{21}\right)\right)}{\sqrt[4]{21}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] -(EllipticF[ArcCos[Sqrt[2/(5 + Sqrt[21])]*x], (21 + 5*Sqrt[21])/4
2]/21^(1/4))
```

Rubi in Sympy [A] time = 12.5067, size = 61, normalized size = 1.56

$$-\frac{2\cdot 21^{\frac{3}{4}}F\left(\operatorname{acos}\left(\frac{\sqrt{2}x\sqrt{-\sqrt{21}+5}}{2}\right)\middle|\frac{1}{2}+\frac{5\sqrt{21}}{42}\right)}{21\sqrt{-\sqrt{21}+5}\sqrt{\sqrt{21}+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**4+5*x**2-1)**(1/2),x)
```

```
[Out] -2*21**(3/4)*elliptic_f(acos(sqrt(2)*x*sqrt(-sqrt(21) + 5)/2), 1/
2 + 5*sqrt(21)/42)/(21*sqrt(-sqrt(21) + 5)*sqrt(sqrt(21) + 5))
```

Mathematica [B] time = 0.194192, size = 87, normalized size = 2.23

$$\frac{\sqrt{-2x^2-\sqrt{21}+5}\sqrt{\left(\sqrt{21}-5\right)x^2+2}F\left(\sin^{-1}\left(\sqrt{\frac{1}{2}\left(5+\sqrt{21}\right)}x\right)|\frac{23}{2}-\frac{5\sqrt{21}}{2}\right)}{2\sqrt{-x^4+5x^2-1}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-1 + 5*x^2 - x^4],x]
```

```
[Out] (Sqrt[5 - Sqrt[21] - 2*x^2]*Sqrt[2 + (-5 + Sqrt[21])*x^2]*Ellipti
cF[ArcSin[Sqrt[(5 + Sqrt[21])/2]*x], 23/2 - (5*Sqrt[21])/2])/(2*S
qrt[-1 + 5*x^2 - x^4])
```

Maple [A] time = 0.435, size = 82, normalized size = 2.1

$$\frac{EllipticF\left(x\left(\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}\right),\frac{5}{2}+\frac{\sqrt{21}}{2}\right)}{\frac{\sqrt{7}}{2}-\frac{\sqrt{3}}{2}}\sqrt{1-\left(\frac{5}{2}-\frac{\sqrt{21}}{2}\right)x^2}\sqrt{1-\left(\frac{5}{2}+\frac{\sqrt{21}}{2}\right)x^2}\frac{1}{\sqrt{-x^4+5x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^4+5*x^2-1)^(1/2),x)
```

```
[Out] 1/(1/2*7^(1/2)-1/2*3^(1/2))*(1-(5/2-1/2*21^(1/2))*x^2)^(1/2)*(1-(
5/2+1/2*21^(1/2))*x^2)^(1/2)/(-x^4+5*x^2-1)^(1/2)*EllipticF(x*(1/
2*7^(1/2)-1/2*3^(1/2)),5/2+1/2*21^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^4+5x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 5*x^2 - 1),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{-x^4+5\,x^2-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 5*x^2 - 1),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**4+5*x**2-1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**4 + 5*x**2 - 1), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^4+5\,x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^4 + 5*x^2 - 1),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-x^4 + 5*x^2 - 1), x)
```

3.1041 $\int x^{5/2} \left(a + bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=31

$$\frac{2}{7}ax^{7/2} + \frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rubi [A] time = 0.0162375, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{2}{7}ax^{7/2} + \frac{2}{11}bx^{11/2} + \frac{2}{15}cx^{15/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2 + c*x^4),x]

[Out] (2*a*x^(7/2))/7 + (2*b*x^(11/2))/11 + (2*c*x^(15/2))/15
```

Rubi in Sympy [A] time = 4.40332, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11} + \frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(c*x**4+b*x**2+a),x)

[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

Mathematica [A] time = 0.0108929, size = 25, normalized size = 0.81

$$\frac{2x^{7/2}\left(165a + 105bx^2 + 77cx^4\right)}{1155}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(7/2)*(165*a + 105*b*x^2 + 77*c*x^4))/1155
```

Maple [A] time = 0.005, size = 22, normalized size = 0.7

$$\frac{154\,c x^{4}+210\,b x^{2}+330\,a}{1155}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 2/1155*x^(7/2)*(77*c*x^4+105*b*x^2+165*a)
```

Maxima [A] time = 0.749177, size = 26, normalized size = 0.84

$$\frac{2}{15}\,c x^{\frac{15}{2}}+\frac{2}{11}\,b x^{\frac{11}{2}}+\frac{2}{7}\,a x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Fricas [A] time = 0.266948, size = 32, normalized size = 1.03

$$\frac{2}{1155}\left(77\,c x^{7}+105\,b x^{5}+165\,a x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c*x^7 + 105*b*x^5 + 165*a*x^3)*sqrt(x)
```

Sympy [A] time = 16.1733, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{7}{2}}}{7}+\frac{2bx^{\frac{11}{2}}}{11}+\frac{2cx^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2+a),x)

[Out] 2*a*x**(7/2)/7 + 2*b*x**(11/2)/11 + 2*c*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.260385, size = 26, normalized size = 0.84

$$\frac{2}{15}\,cx^{\frac{15}{2}} + \frac{2}{11}\,bx^{\frac{11}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^(5/2),x, algorithm="giac")

[Out] 2/15*c*x^(15/2) + 2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

3.1042 $\int x^{3/2} \left(a + bx^2 + cx^4\right) \, dx$

Optimal. Leaf size=31

$$\frac{2}{5}ax^{5/2} + \frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rubi [A] time = 0.0151442, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{2}{5}ax^{5/2} + \frac{2}{9}bx^{9/2} + \frac{2}{13}cx^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2 + c*x^4),x]

[Out] (2*a*x^(5/2))/5 + (2*b*x^(9/2))/9 + (2*c*x^(13/2))/13
```

Rubi in Sympy [A] time = 4.74376, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{9}{2}}}{9} + \frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**4+b*x**2+a),x)

[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

Mathematica [A] time = 0.00931023, size = 25, normalized size = 0.81

$$\frac{2}{585}x^{5/2}\left(117a + 65bx^2 + 45cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(5/2)*(117*a + 65*b*x^2 + 45*c*x^4))/585
```

Maple [A] time = 0.004, size = 22, normalized size = 0.7

$$\frac{90\,cx^4+130\,bx^2+234\,a}{585}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 2/585*x^(5/2)*(45*c*x^4+65*b*x^2+117*a)
```

Maxima [A] time = 0.773322, size = 26, normalized size = 0.84

$$\frac{2}{13}\,cx^{\frac{13}{2}}+\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Fricas [A] time = 0.270299, size = 32, normalized size = 1.03

$$\frac{2}{585}\left(45\,cx^6+65\,bx^4+117\,ax^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c*x^6 + 65*b*x^4 + 117*a*x^2)*sqrt(x)
```

Sympy [A] time = 7.80846, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}+\frac{2cx^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(5/2)/5 + 2*b*x**(9/2)/9 + 2*c*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.259831, size = 26, normalized size = 0.84

$$\frac{2}{13}\,cx^{\frac{13}{2}} + \frac{2}{9}\,bx^{\frac{9}{2}} + \frac{2}{5}\,ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/13*c*x^(13/2) + 2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

3.1043 $\int \sqrt{x} \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=31

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rubi [A] time = 0.0160379, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}+\frac{2}{11}cx^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(7/2))/7 + (2*c*x^(11/2))/11
```

Rubi in Sympy [A] time = 4.40565, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

Mathematica [A] time = 0.00910448, size = 25, normalized size = 0.81

$$\frac{2}{231}x^{3/2}\left(77a+33bx^2+21cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2)*(77*a + 33*b*x^2 + 21*c*x^4))/231
```

Maple [A] time = 0.004, size = 22, normalized size = 0.7

$$\frac{42\,cx^4+66\,bx^2+154\,a}{231}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(c*x^4+b*x^2+a),x)
```

```
[Out] 2/231*x^(3/2)*(21*c*x^4+33*b*x^2+77*a)
```

Maxima [A] time = 0.772612, size = 26, normalized size = 0.84

$$\frac{2}{11}\,cx^{\frac{11}{2}}+\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Fricas [A] time = 0.269389, size = 30, normalized size = 0.97

$$\frac{2}{231}\left(21\,cx^5+33\,bx^3+77\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/231*(21*c*x^5 + 33*b*x^3 + 77*a*x)*sqrt(x)
```

Sympy [A] time = 2.50494, size = 29, normalized size = 0.94

$$\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}+\frac{2cx^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2+a),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(7/2)/7 + 2*c*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.261633, size = 26, normalized size = 0.84

$$\frac{2}{11}\,cx^{\frac{11}{2}} + \frac{2}{7}\,bx^{\frac{7}{2}} + \frac{2}{3}\,ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/11*c*x^(11/2) + 2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

3.1044 $\int \frac{a+bx^2+cx^4}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

```
[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rubi [A] time = 0.0148693, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}+\frac{2}{9}cx^{9/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/Sqrt[x],x]

[Out] 2*a*Sqrt[x] + (2*b*x^(5/2))/5 + (2*c*x^(9/2))/9
```

Rubi in Sympy [A] time = 4.43351, size = 27, normalized size = 0.93

$$2a\sqrt{x}+\frac{2bx^{\frac{5}{2}}}{5}+\frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**(1/2),x)

[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

Mathematica [A] time = 0.00889873, size = 25, normalized size = 0.86

$$\frac{2}{45}\sqrt{x}\left(45a+9bx^2+5cx^4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(45*a + 9*b*x^2 + 5*c*x^4))/45
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$\frac{10\,c x^4 + 18\,b x^2 + 90\,a}{45}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^(1/2),x)
```

```
[Out] 2/45*x^(1/2)*(5*c*x^4+9*b*x^2+45*a)
```

Maxima [A] time = 0.748569, size = 26, normalized size = 0.9

$$\frac{2}{9}\,c x^{\frac{9}{2}} + \frac{2}{5}\,b x^{\frac{5}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Fricas [A] time = 0.270266, size = 28, normalized size = 0.97

$$\frac{2}{45}\left(5\,c x^4 + 9\,b x^2 + 45\,a\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/45*(5*c*x^4 + 9*b*x^2 + 45*a)*sqrt(x)
```

Sympy [A] time = 1.96356, size = 27, normalized size = 0.93

$$2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**(1/2),x)
```

```
[Out] 2*a*sqrt(x) + 2*b*x**(5/2)/5 + 2*c*x**(9/2)/9
```

GIAC/XCAS [A] time = 0.25977, size = 26, normalized size = 0.9

$$\frac{2}{9}\,cx^{\frac{9}{2}} + \frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/9*c*x^(9/2) + 2/5*b*x^(5/2) + 2*a*sqrt(x)
```

3.1045 $\int \frac{a+bx^2+cx^4}{x^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rubi [A] time = 0.0155096, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}+\frac{2}{7}cx^{7/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + (2*b*x^(3/2))/3 + (2*c*x^(7/2))/7
```

Rubi in Sympy [A] time = 4.4136, size = 27, normalized size = 0.93

$$-\frac{2a}{\sqrt{x}}+\frac{2bx^{\frac{3}{2}}}{3}+\frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

Mathematica [A] time = 0.01184, size = 25, normalized size = 0.86

$$\frac{2\left(-21a+7bx^2+3cx^4\right)}{21\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^(3/2),x]

[Out] (2*(-21*a + 7*b*x^2 + 3*c*x^4))/(21*Sqrt[x])
```

Maple [A] time = 0.005, size = 22, normalized size = 0.8

$$-\frac{-6\,cx^{4}-14\,bx^{2}+42\,a}{21}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^(3/2),x)

[Out] -2/21*(-3*c*x^4-7*b*x^2+21*a)/x^(1/2)
```

Maxima [A] time = 0.733382, size = 26, normalized size = 0.9

$$\frac{2}{7}\,cx^{\frac{7}{2}}+\frac{2}{3}\,bx^{\frac{3}{2}}-\frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(3/2),x, algorithm="maxima")

[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Fricas [A] time = 0.273366, size = 28, normalized size = 0.97

$$\frac{2\left(3\,cx^{4}+7\,bx^{2}-21\,a\right)}{21\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(3/2),x, algorithm="fricas")

[Out] 2/21*(3*c*x^4 + 7*b*x^2 - 21*a)/sqrt(x)
```

Sympy [A] time = 3.10866, size = 27, normalized size = 0.93

$$-\frac{2a}{\sqrt{x}} + \frac{2bx^{\frac{3}{2}}}{3} + \frac{2cx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**(3/2),x)

[Out] -2*a/sqrt(x) + 2*b*x**(3/2)/3 + 2*c*x**(7/2)/7
```

GIAC/XCAS [A] time = 0.261352, size = 26, normalized size = 0.9

$$\frac{2}{7}\,cx^{\frac{7}{2}} + \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(3/2),x, algorithm="giac")

[Out] 2/7*c*x^(7/2) + 2/3*b*x^(3/2) - 2*a/sqrt(x)
```

3.1046 $\int \frac{a+bx^2+cx^4}{x^{5/2}} \, dx$

Optimal. Leaf size=29

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

```
[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rubi [A] time = 0.0152414, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{2a}{3x^{3/2}}+2b\sqrt{x}+\frac{2}{5}cx^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^(5/2),x]

[Out] (-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x] + (2*c*x^(5/2))/5
```

Rubi in Sympy [A] time = 4.47087, size = 27, normalized size = 0.93

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**(5/2),x)

[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

Mathematica [A] time = 0.0117338, size = 25, normalized size = 0.86

$$\frac{2\left(-5a+15bx^2+3cx^4\right)}{15x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^(5/2),x]
```

```
[Out] (2*(-5*a + 15*b*x^2 + 3*c*x^4))/(15*x^(3/2))
```

Maple [A] time = 0.005, size = 22, normalized size = 0.8

$$-\frac{-6\,cx^{4}-30\,bx^{2}+10\,a}{15}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^(5/2),x)
```

```
[Out] -2/15*(-3*c*x^4-15*b*x^2+5*a)/x^(3/2)
```

Maxima [A] time = 0.733164, size = 26, normalized size = 0.9

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Fricas [A] time = 0.271447, size = 28, normalized size = 0.97

$$\frac{2\,\left(3\,cx^{4}+15\,bx^{2}-5\,a\right)}{15\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*c*x^4 + 15*b*x^2 - 5*a)/x^(3/2)
```

Sympy [A] time = 3.89902, size = 27, normalized size = 0.93

$$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}+\frac{2cx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**(5/2),x)
```

```
[Out] -2*a/(3*x**(3/2)) + 2*b*sqrt(x) + 2*c*x**(5/2)/5
```

GIAC/XCAS [A] time = 0.261927, size = 26, normalized size = 0.9

$$\frac{2}{5}\,cx^{\frac{5}{2}}+2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/5*c*x^(5/2) + 2*b*sqrt(x) - 2/3*a/x^(3/2)
```

3.1047 $\int \frac{a+bx^2+cx^4}{x^{7/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

```
[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rubi [A] time = 0.0154632, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}+\frac{2}{3}cx^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)/x^(7/2),x]

[Out] (-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x] + (2*c*x^(3/2))/3
```

Rubi in Sympy [A] time = 4.44683, size = 27, normalized size = 0.93

$$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}+\frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)/x**(7/2),x)

[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

Mathematica [A] time = 0.0118586, size = 25, normalized size = 0.86

$$\frac{2\left(-3a-15bx^2+5cx^4\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)/x^(7/2),x]

[Out] (2*(-3*a - 15*b*x^2 + 5*c*x^4))/(15*x^(5/2))
```

Maple [A] time = 0.005, size = 22, normalized size = 0.8

$$-\frac{-10\,cx^4+30\,bx^2+6\,a}{15}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)/x^(7/2),x)

[Out] -2/15*(-5*c*x^4+15*b*x^2+3*a)/x^(5/2)
```

Maxima [A] time = 0.74058, size = 27, normalized size = 0.93

$$\frac{2}{3}\,cx^{\frac{3}{2}}-\frac{2\,\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(7/2),x, algorithm="maxima")

[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

Fricas [A] time = 0.270259, size = 28, normalized size = 0.97

$$\frac{2\,\left(5\,cx^4-15\,bx^2-3\,a\right)}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(7/2),x, algorithm="fricas")

[Out] 2/15*(5*c*x^4 - 15*b*x^2 - 3*a)/x^(5/2)
```

Sympy [A] time = 5.8852, size = 27, normalized size = 0.93

$$-\frac{2a}{5x^{\frac{5}{2}}} - \frac{2b}{\sqrt{x}} + \frac{2cx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)/x**(7/2),x)
```

```
[Out] -2*a/(5*x**(5/2)) - 2*b/sqrt(x) + 2*c*x**(3/2)/3
```

GIAC/XCAS [A] time = 0.262108, size = 27, normalized size = 0.93

$$\frac{2}{3}\,cx^{\frac{3}{2}} - \frac{2\left(5\,bx^2 + a\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/3*c*x^(3/2) - 2/5*(5*b*x^2 + a)/x^(5/2)
```

3.1048 $\int x^{5/2}\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=64

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/
2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rubi [A] time = 0.0581908, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/
2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Rubi in Sympy [A] time = 9.14314, size = 65, normalized size = 1.02

$$\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{4bcx^{\frac{19}{2}}}{19}+\frac{2c^2x^{\frac{23}{2}}}{23}+x^{\frac{15}{2}}\left(\frac{4ac}{15}+\frac{2b^2}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*b*c*x**(19/2)/19 + 2*c
**2*x**(23/2)/23 + x**(15/2)*(4*a*c/15 + 2*b**2/15)
```

Mathematica [A] time = 0.029601, size = 64, normalized size = 1.

$$\frac{2}{7}a^2x^{7/2}+\frac{2}{15}x^{15/2}\left(2ac+b^2\right)+\frac{4}{11}abx^{11/2}+\frac{4}{19}bcx^{19/2}+\frac{2}{23}c^2x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*(b^2 + 2*a*c)*x^(15/
2))/15 + (4*b*c*x^(19/2))/19 + (2*c^2*x^(23/2))/23
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$\frac{43890\,c^2x^8+106260\,bcx^6+134596\,x^4ac+67298\,b^2x^4+183540\,abx^2+144210\,a^2}{504735}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/504735*x^(7/2)*(21945*c^2*x^8+53130*b*c*x^6+67298*a*c*x^4+33649
*b^2*x^4+91770*a*b*x^2+72105*a^2)
```

Maxima [A] time = 0.75981, size = 59, normalized size = 0.92

$$\frac{2}{23}\,c^2x^{\frac{23}{2}}+\frac{4}{19}\,bcx^{\frac{19}{2}}+\frac{2}{15}\,\left(b^2+2\,ac\right)x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*(b^2 + 2*a*c)*x^(15/
2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Fricas [A] time = 0.271702, size = 66, normalized size = 1.03

$$\frac{2}{504735}\,\left(21945\,c^2x^{11}+53130\,bcx^9+33649\,\left(b^2+2\,ac\right)x^7+91770\,abx^5+72105\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/504735*(21945*c^2*x^11 + 53130*b*c*x^9 + 33649*(b^2 + 2*a*c)*x^
7 + 91770*a*b*x^5 + 72105*a^2*x^3)*sqrt(x)
```

Sympy [A] time = 62.8919, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{11}{2}}}{11} + \frac{4acx^{\frac{15}{2}}}{15} + \frac{2b^2x^{\frac{15}{2}}}{15} + \frac{4bcx^{\frac{19}{2}}}{19} + \frac{2c^2x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 4*a*c*x**(15/2)/15 + 2*b
**2*x**(15/2)/15 + 4*b*c*x**(19/2)/19 + 2*c**2*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.261276, size = 62, normalized size = 0.97

$$\frac{2}{23}\,c^2x^{\frac{23}{2}} + \frac{4}{19}\,bcx^{\frac{19}{2}} + \frac{2}{15}\,b^2x^{\frac{15}{2}} + \frac{4}{15}\,acx^{\frac{15}{2}} + \frac{4}{11}\,abx^{\frac{11}{2}} + \frac{2}{7}\,a^2x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/23*c^2*x^(23/2) + 4/19*b*c*x^(19/2) + 2/15*b^2*x^(15/2) + 4/15*
a*c*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

3.1049 $\int x^{3/2} \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=64

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2)
)/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rubi [A] time = 0.0523883, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2)
)/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Rubi in Sympy [A] time = 9.25919, size = 65, normalized size = 1.02

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}+x^{\frac{13}{2}}\left(\frac{4ac}{13}+\frac{2b^2}{13}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*b*c*x**(17/2)/17 + 2*c**
2*x**(21/2)/21 + x**(13/2)*(4*a*c/13 + 2*b**2/13)
```

Mathematica [A] time = 0.0290135, size = 64, normalized size = 1.

$$\frac{2}{5}a^2x^{5/2}+\frac{2}{13}x^{13/2}\left(2ac+b^2\right)+\frac{4}{9}abx^{9/2}+\frac{4}{17}bcx^{17/2}+\frac{2}{21}c^2x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*(b^2 + 2*a*c)*x^(13/2)
)/13 + (4*b*c*x^(17/2))/17 + (2*c^2*x^(21/2))/21
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$\frac{6630\,c^2x^8 + 16380\,bcx^6 + 21420\,x^4ac + 10710\,b^2x^4 + 30940\,abx^2 + 27846\,a^2}{69615}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/69615*x^(5/2)*(3315*c^2*x^8+8190*b*c*x^6+10710*a*c*x^4+5355*b^2
*x^4+15470*a*b*x^2+13923*a^2)
```

Maxima [A] time = 0.74861, size = 59, normalized size = 0.92

$$\frac{2}{21}\,c^2x^{\frac{21}{2}} + \frac{4}{17}\,bcx^{\frac{17}{2}} + \frac{2}{13}\left(b^2 + 2\,ac\right)x^{\frac{13}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*(b^2 + 2*a*c)*x^(13/
2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Fricas [A] time = 0.269163, size = 66, normalized size = 1.03

$$\frac{2}{69615}\left(3315\,c^2x^{10} + 8190\,bcx^8 + 5355\left(b^2 + 2\,ac\right)x^6 + 15470\,abx^4 + 13923\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^2*x^10 + 8190*b*c*x^8 + 5355*(b^2 + 2*a*c)*x^6 +
15470*a*b*x^4 + 13923*a^2*x^2)*sqrt(x)
```

Sympy [A] time = 34.0803, size = 70, normalized size = 1.09

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{9}{2}}}{9}+\frac{4acx^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{13}{2}}}{13}+\frac{4bcx^{\frac{17}{2}}}{17}+\frac{2c^2x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 4*a*c*x**(13/2)/13 + 2*b**
2*x**(13/2)/13 + 4*b*c*x**(17/2)/17 + 2*c**2*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.26227, size = 62, normalized size = 0.97

$$\frac{2}{21}\,c^2x^{\frac{21}{2}}+\frac{4}{17}\,bcx^{\frac{17}{2}}+\frac{2}{13}\,b^2x^{\frac{13}{2}}+\frac{4}{13}\,acx^{\frac{13}{2}}+\frac{4}{9}\,abx^{\frac{9}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/21*c^2*x^(21/2) + 4/17*b*c*x^(17/2) + 2/13*b^2*x^(13/2) + 4/13*
a*c*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

3.1050 $\int \sqrt{x}\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=64

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2)
)/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rubi [A] time = 0.0522043, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{3}a^2x^{3/2}+\frac{2}{11}x^{11/2}\left(2ac+b^2\right)+\frac{4}{7}abx^{7/2}+\frac{4}{15}bcx^{15/2}+\frac{2}{19}c^2x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*(b^2 + 2*a*c)*x^(11/2)
)/11 + (4*b*c*x^(15/2))/15 + (2*c^2*x^(19/2))/19
```

Rubi in Sympy [A] time = 9.11114, size = 65, normalized size = 1.02

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+x^{\frac{11}{2}}\left(\frac{4ac}{11}+\frac{2b^2}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**
2*x**(19/2)/19 + x**(11/2)*(4*a*c/11 + 2*b**2/11)
```

Mathematica [A] time = 0.0306717, size = 50, normalized size = 0.78

$$\frac{2x^{3/2}\left(7315a^2+1995x^4\left(2ac+b^2\right)+6270abx^2+2926bcx^6+1155c^2x^8\right)}{21945}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*x^(3/2)*(7315*a^2 + 6270*a*b*x^2 + 1995*(b^2 + 2*a*c)*x^4 + 29
26*b*c*x^6 + 1155*c^2*x^8))/21945
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$\frac{2310\,c^2x^8+5852\,bcx^6+7980\,x^4ac+3990\,b^2x^4+12540\,abx^2+14630\,a^2}{21945}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2/21945*x^(3/2)*(1155*c^2*x^8+2926*b*c*x^6+3990*a*c*x^4+1995*b^2*
x^4+6270*a*b*x^2+7315*a^2)
```

Maxima [A] time = 0.764077, size = 59, normalized size = 0.92

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\left(b^2+2\,ac\right)x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*(b^2 + 2*a*c)*x^(11/
2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Fricas [A] time = 0.267642, size = 63, normalized size = 0.98

$$\frac{2}{21945}\left(1155\,c^2x^9+2926\,bcx^7+1995\left(b^2+2\,ac\right)x^5+6270\,abx^3+7315\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/21945*(1155*c^2*x^9 + 2926*b*c*x^7 + 1995*(b^2 + 2*a*c)*x^5 + 6
270*a*b*x^3 + 7315*a^2*x)*sqrt(x)
```

Sympy [A] time = 7.70705, size = 63, normalized size = 0.98

$$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{4bcx^{\frac{15}{2}}}{15}+\frac{2c^2x^{\frac{19}{2}}}{19}+\frac{2x^{\frac{11}{2}}\left(2ac+b^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 4*b*c*x**(15/2)/15 + 2*c**
2*x**(19/2)/19 + 2*x**(11/2)*(2*a*c + b**2)/11
```

GIAC/XCAS [A] time = 0.261315, size = 62, normalized size = 0.97

$$\frac{2}{19}\,c^2x^{\frac{19}{2}}+\frac{4}{15}\,bcx^{\frac{15}{2}}+\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{11}\,acx^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/19*c^2*x^(19/2) + 4/15*b*c*x^(15/2) + 2/11*b^2*x^(11/2) + 4/11*
a*c*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

3.1051 $\int \frac{(a+bx^2+cx^4)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 +
 (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rubi [A] time = 0.0523175, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 +
 (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Rubi in Sympy [A] time = 9.13504, size = 63, normalized size = 1.02

$$2a^2\sqrt{x}+\frac{4abx^{\frac{5}{2}}}{5}+\frac{4bcx^{\frac{13}{2}}}{13}+\frac{2c^2x^{\frac{17}{2}}}{17}+x^{\frac{9}{2}}\left(\frac{4ac}{9}+\frac{2b^2}{9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*b*c*x**(13/2)/13 + 2*c**2*x
**(17/2)/17 + x**(9/2)*(4*a*c/9 + 2*b**2/9)
```

Mathematica [A] time = 0.0264786, size = 62, normalized size = 1.

$$2a^2\sqrt{x}+\frac{2}{9}x^{9/2}\left(2ac+b^2\right)+\frac{4}{5}abx^{5/2}+\frac{4}{13}bcx^{13/2}+\frac{2}{17}c^2x^{17/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*(b^2 + 2*a*c)*x^(9/2))/9 +
 (4*b*c*x^(13/2))/13 + (2*c^2*x^(17/2))/17
```

Maple [A] time = 0.008, size = 49, normalized size = 0.8

$$\frac{1170\,c^2x^8+3060\,bcx^6+4420\,x^4ac+2210\,b^2x^4+7956\,abx^2+19890\,a^2}{9945}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^(1/2),x)
```

```
[Out] 2/9945*x^(1/2)*(585*c^2*x^8+1530*b*c*x^6+2210*a*c*x^4+1105*b^2*x^
4+3978*a*b*x^2+9945*a^2)
```

Maxima [A] time = 0.75766, size = 65, normalized size = 1.05

$$\frac{2}{17}\,c^2x^{\frac{17}{2}}+\frac{4}{13}\,bcx^{\frac{13}{2}}+\frac{2}{9}\,b^2x^{\frac{9}{2}}+2\,a^2\sqrt{x}+\frac{4}{45}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 2*a^2*s
qrt(x) + 4/45*(5*c*x^(9/2) + 9*b*x^(5/2))*a
```

Fricas [A] time = 0.280816, size = 62, normalized size = 1.

$$\frac{2}{9945}\left(585\,c^2x^8+1530\,bcx^6+1105\left(b^2+2\,ac\right)x^4+3978\,abx^2+9945\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/9945*(585*c^2*x^8 + 1530*b*c*x^6 + 1105*(b^2 + 2*a*c)*x^4 + 397
8*a*b*x^2 + 9945*a^2)*sqrt(x)
```

Sympy [A] time = 13.933, size = 68, normalized size = 1.1

$$2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{4acx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{9}{2}}}{9} + \frac{4bcx^{\frac{13}{2}}}{13} + \frac{2c^2x^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**(1/2),x)
```

```
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 4*a*c*x**(9/2)/9 + 2*b**2*x**
(9/2)/9 + 4*b*c*x**(13/2)/13 + 2*c**2*x**(17/2)/17
```

GIAC/XCAS [A] time = 0.260313, size = 62, normalized size = 1.

$$\frac{2}{17}\,c^2x^{\frac{17}{2}} + \frac{4}{13}\,bcx^{\frac{13}{2}} + \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{9}\,acx^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/17*c^2*x^(17/2) + 4/13*b*c*x^(13/2) + 2/9*b^2*x^(9/2) + 4/9*a*c
*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

3.1052 $\int \frac{(a+bx^2+cx^4)^2}{x^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/
7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rubi [A] time = 0.054175, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2a^2}{\sqrt{x}}+\frac{2}{7}x^{7/2}\left(2ac+b^2\right)+\frac{4}{3}abx^{3/2}+\frac{4}{11}bcx^{11/2}+\frac{2}{15}c^2x^{15/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*(b^2 + 2*a*c)*x^(7/2))/
7 + (4*b*c*x^(11/2))/11 + (2*c^2*x^(15/2))/15
```

Rubi in Sympy [A] time = 9.17958, size = 63, normalized size = 1.02

$$-\frac{2a^2}{\sqrt{x}}+\frac{4abx^{\frac{3}{2}}}{3}+\frac{4bcx^{\frac{11}{2}}}{11}+\frac{2c^2x^{\frac{15}{2}}}{15}+x^{\frac{7}{2}}\left(\frac{4ac}{7}+\frac{2b^2}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*b*c*x**(11/2)/11 + 2*c**2*
x**(15/2)/15 + x**(7/2)*(4*a*c/7 + 2*b**2/7)
```

Mathematica [A] time = 0.0298845, size = 50, normalized size = 0.81

$$\frac{2\left(-1155a^2+165x^4\left(2ac+b^2\right)+770abx^2+210bcx^6+77c^2x^8\right)}{1155\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^(3/2),x]
```

```
[Out] (2*(-1155*a^2 + 770*a*b*x^2 + 165*(b^2 + 2*a*c)*x^4 + 210*b*c*x^6
 + 77*c^2*x^8))/(1155*Sqrt[x])
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$-\frac{-154\,c^2x^8-420\,bcx^6-660\,x^4ac-330\,b^2x^4-1540\,abx^2+2310\,a^2}{1155}\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^(3/2),x)
```

```
[Out] -2/1155*(-77*c^2*x^8-210*b*c*x^6-330*a*c*x^4-165*b^2*x^4-770*a*b*
x^2+1155*a^2)/x^(1/2)
```

Maxima [A] time = 0.758327, size = 59, normalized size = 0.95

$$\frac{2}{15}\,c^2x^{\frac{15}{2}}+\frac{4}{11}\,bcx^{\frac{11}{2}}+\frac{2}{7}\left(b^2+2\,ac\right)x^{\frac{7}{2}}+\frac{4}{3}\,abx^{\frac{3}{2}}-\frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*(b^2 + 2*a*c)*x^(7/2)
 + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Fricas [A] time = 0.27402, size = 62, normalized size = 1.

$$\frac{2\left(77\,c^2x^8+210\,bcx^6+165\left(b^2+2\,ac\right)x^4+770\,abx^2-1155\,a^2\right)}{1155\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(77*c^2*x^8 + 210*b*c*x^6 + 165*(b^2 + 2*a*c)*x^4 + 770*a*
b*x^2 - 1155*a^2)/sqrt(x)
```

Sympy [A] time = 15.9796, size = 68, normalized size = 1.1

$$-\frac{2a^2}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{4acx^{\frac{7}{2}}}{7} + \frac{2b^2x^{\frac{7}{2}}}{7} + \frac{4bcx^{\frac{11}{2}}}{11} + \frac{2c^2x^{\frac{15}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**(3/2),x)
```

```
[Out] -2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 4*a*c*x**(7/2)/7 + 2*b**2*x*
*(7/2)/7 + 4*b*c*x**(11/2)/11 + 2*c**2*x**(15/2)/15
```

GIAC/XCAS [A] time = 0.261894, size = 62, normalized size = 1.

$$\frac{2}{15}\,c^2x^{\frac{15}{2}} + \frac{4}{11}\,bcx^{\frac{11}{2}} + \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{7}\,acx^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*c^2*x^(15/2) + 4/11*b*c*x^(11/2) + 2/7*b^2*x^(7/2) + 4/7*a*c
*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

3.1053 $\int \frac{(a+bx^2+cx^4)^2}{x^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/
5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rubi [A] time = 0.0541078, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2a^2}{3x^{3/2}}+\frac{2}{5}x^{5/2}\left(2ac+b^2\right)+4ab\sqrt{x}+\frac{4}{9}bcx^{9/2}+\frac{2}{13}c^2x^{13/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*(b^2 + 2*a*c)*x^(5/2))/
5 + (4*b*c*x^(9/2))/9 + (2*c^2*x^(13/2))/13
```

Rubi in Sympy [A] time = 9.2278, size = 63, normalized size = 1.02

$$-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{4bcx^{\frac{9}{2}}}{9}+\frac{2c^2x^{\frac{13}{2}}}{13}+x^{\frac{5}{2}}\left(\frac{4ac}{5}+\frac{2b^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*b*c*x**(9/2)/9 + 2*c**2*
x**(13/2)/13 + x**(5/2)*(4*a*c/5 + 2*b**2/5)
```

Mathematica [A] time = 0.0294567, size = 50, normalized size = 0.81

$$\frac{2\left(-195a^2+117x^4\left(2ac+b^2\right)+1170abx^2+130bcx^6+45c^2x^8\right)}{585x^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^(5/2),x]
```

```
[Out] (2*(-195*a^2 + 1170*a*b*x^2 + 117*(b^2 + 2*a*c)*x^4 + 130*b*c*x^6
 + 45*c^2*x^8))/(585*x^(3/2))
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$-\frac{-90\,c^2x^8-260\,bcx^6-468\,x^4ac-234\,b^2x^4-2340\,abx^2+390\,a^2}{585}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^(5/2),x)
```

```
[Out] -2/585*(-45*c^2*x^8-130*b*c*x^6-234*a*c*x^4-117*b^2*x^4-1170*a*b*
x^2+195*a^2)/x^(3/2)
```

Maxima [A] time = 0.743577, size = 59, normalized size = 0.95

$$\frac{2}{13}\,c^2x^{\frac{13}{2}}+\frac{4}{9}\,bcx^{\frac{9}{2}}+\frac{2}{5}\,\left(b^2+2\,ac\right)x^{\frac{5}{2}}+4\,ab\sqrt{x}-\frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*(b^2 + 2*a*c)*x^(5/2) +
 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Fricas [A] time = 0.278471, size = 62, normalized size = 1.

$$\frac{2\,\left(45\,c^2x^8+130\,bcx^6+117\,\left(b^2+2\,ac\right)x^4+1170\,abx^2-195\,a^2\right)}{585\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/585*(45*c^2*x^8 + 130*b*c*x^6 + 117*(b^2 + 2*a*c)*x^4 + 1170*a*
b*x^2 - 195*a^2)/x^(3/2)
```

Sympy [A] time = 18.8174, size = 68, normalized size = 1.1

$$-\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{4acx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{5}{2}}}{5} + \frac{4bcx^{\frac{9}{2}}}{9} + \frac{2c^2x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**(5/2),x)
```

```
[Out] -2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 4*a*c*x**(5/2)/5 + 2*b**2*
x**(5/2)/5 + 4*b*c*x**(9/2)/9 + 2*c**2*x**(13/2)/13
```

GIAC/XCAS [A] time = 0.260597, size = 62, normalized size = 1.

$$\frac{2}{13}\,c^2x^{\frac{13}{2}} + \frac{4}{9}\,bcx^{\frac{9}{2}} + \frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{5}\,acx^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/13*c^2*x^(13/2) + 4/9*b*c*x^(9/2) + 2/5*b^2*x^(5/2) + 4/5*a*c*x
^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

3.1054 $\int \frac{(a+bx^2+cx^4)^2}{x^{7/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2)
)/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rubi [A] time = 0.0530036, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2a^2}{5x^{5/2}}+\frac{2}{3}x^{3/2}\left(2ac+b^2\right)-\frac{4ab}{\sqrt{x}}+\frac{4}{7}bcx^{7/2}+\frac{2}{11}c^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*(b^2 + 2*a*c)*x^(3/2)
)/3 + (4*b*c*x^(7/2))/7 + (2*c^2*x^(11/2))/11
```

Rubi in Sympy [A] time = 9.2531, size = 63, normalized size = 1.02

$$-\frac{2a^2}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{4bcx^{\frac{7}{2}}}{7}+\frac{2c^2x^{\frac{11}{2}}}{11}+x^{\frac{3}{2}}\left(\frac{4ac}{3}+\frac{2b^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*b*c*x**(7/2)/7 + 2*c**2*
x**(11/2)/11 + x**(3/2)*(4*a*c/3 + 2*b**2/3)
```

Mathematica [A] time = 0.0365881, size = 50, normalized size = 0.81

$$\frac{2\left(-231a^2+385x^4\left(2ac+b^2\right)-2310abx^2+330bcx^6+105c^2x^8\right)}{1155x^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^2/x^(7/2),x]
```

```
[Out] (2*(-231*a^2 - 2310*a*b*x^2 + 385*(b^2 + 2*a*c)*x^4 + 330*b*c*x^6
 + 105*c^2*x^8))/(1155*x^(5/2))
```

Maple [A] time = 0.009, size = 49, normalized size = 0.8

$$-\frac{-210\,c^2x^8-660\,bcx^6-1540\,x^4ac-770\,b^2x^4+4620\,abx^2+462\,a^2}{1155}x^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^2/x^(7/2),x)
```

```
[Out] -2/1155*(-105*c^2*x^8-330*b*c*x^6-770*a*c*x^4-385*b^2*x^4+2310*a*
b*x^2+231*a^2)/x^(5/2)
```

Maxima [A] time = 0.732638, size = 61, normalized size = 0.98

$$\frac{2}{11}\,c^2x^{\frac{11}{2}}+\frac{4}{7}\,bcx^{\frac{7}{2}}+\frac{2}{3}\left(b^2+2\,ac\right)x^{\frac{3}{2}}-\frac{2\left(10\,abx^2+a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*(b^2 + 2*a*c)*x^(3/2) -
 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Fricas [A] time = 0.27545, size = 62, normalized size = 1.

$$\frac{2\left(105\,c^2x^8+330\,bcx^6+385\left(b^2+2\,ac\right)x^4-2310\,abx^2-231\,a^2\right)}{1155\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*c^2*x^8 + 330*b*c*x^6 + 385*(b^2 + 2*a*c)*x^4 - 2310*
a*b*x^2 - 231*a^2)/x^(5/2)
```

Sympy [A] time = 26.66, size = 68, normalized size = 1.1

$$-\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{3}{2}}}{3} + \frac{4bcx^{\frac{7}{2}}}{7} + \frac{2c^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**2/x**(7/2),x)
```

```
[Out] -2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 4*a*c*x**(3/2)/3 + 2*b**2*
x**(3/2)/3 + 4*b*c*x**(7/2)/7 + 2*c**2*x**(11/2)/11
```

GIAC/XCAS [A] time = 0.263071, size = 63, normalized size = 1.02

$$\frac{2}{11}\,c^2x^{\frac{11}{2}} + \frac{4}{7}\,bcx^{\frac{7}{2}} + \frac{2}{3}\,b^2x^{\frac{3}{2}} + \frac{4}{3}\,acx^{\frac{3}{2}} - \frac{2\left(10\,abx^2 + a^2\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/11*c^2*x^(11/2) + 4/7*b*c*x^(7/2) + 2/3*b^2*x^(3/2) + 4/3*a*c*x
^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

3.1055 $\int x^{5/2} \left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(1
5/2))/5 + (2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(2
3/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rubi [A] time = 0.108892, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(1
5/2))/5 + (2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(2
3/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Rubi in Sympy [A] time = 14.0453, size = 102, normalized size = 0.99

$$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{2ax^{\frac{15}{2}}\left(ac+b^2\right)}{5}+\frac{2bc^2x^{\frac{27}{2}}}{9}+\frac{2bx^{\frac{19}{2}}\left(6ac+b^2\right)}{19}+\frac{2c^3x^{\frac{31}{2}}}{31}+\frac{6cx^{\frac{23}{2}}\left(ac+b^2\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*x**(15/2)*(a*c +
b**2)/5 + 2*b*c**2*x**(27/2)/9 + 2*b*x**(19/2)*(6*a*c + b**2)/19
+ 2*c**3*x**(31/2)/31 + 6*c*x**(23/2)*(a*c + b**2)/23
```

Mathematica [A] time = 0.0508146, size = 103, normalized size = 1.

$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{6}{23}cx^{23/2}\left(ac+b^2\right)+\frac{2}{19}bx^{19/2}\left(6ac+b^2\right)+\frac{2}{5}ax^{15/2}\left(ac+b^2\right)+\frac{2}{9}bc^2x^{27/2}+\frac{2}{31}c^3x^{31/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*(b^2 + a*c)*x^(1
5/2))/5 + (2*b*(b^2 + 6*a*c)*x^(19/2))/19 + (6*c*(b^2 + a*c)*x^(2
3/2))/23 + (2*b*c^2*x^(27/2))/9 + (2*c^3*x^(31/2))/31
```

Maple [A] time = 0.009, size = 90, normalized size = 0.9

$$\frac{3028410\,c^3x^{12}+10431190\,bc^2x^{10}+12245310\,x^8ac^2+12245310\,b^2cx^8+29646540\,x^6abc+4941090\,b^3x^6+18776142\,x^4a^2c+}{46940355}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/46940355*x^(7/2)*(1514205*c^3*x^12+5215595*b*c^2*x^10+6122655*a
*c^2*x^8+6122655*b^2*c*x^8+14823270*a*b*c*x^6+2470545*b^3*x^6+938
8071*a^2*c*x^4+9388071*a*b^2*x^4+12801915*a^2*b*x^2+6705765*a^3)
```

Maxima [A] time = 0.748226, size = 109, normalized size = 1.06

$$\frac{2}{31}\,c^3x^{\frac{31}{2}}+\frac{2}{9}\,bc^2x^{\frac{27}{2}}+\frac{6}{23}\left(b^2c+ac^2\right)x^{\frac{23}{2}}+\frac{2}{19}\left(b^3+6\,abc\right)x^{\frac{19}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{5}\left(ab^2+a^2c\right)x^{\frac{15}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*(b^2*c + a*c^2)*x^(
23/2) + 2/19*(b^3 + 6*a*b*c)*x^(19/2) + 6/11*a^2*b*x^(11/2) + 2/5
*(a*b^2 + a^2*c)*x^(15/2) + 2/7*a^3*x^(7/2)
```

Fricas [A] time = 0.282197, size = 116, normalized size = 1.13

$$\frac{2}{46940355}\left(1514205\,c^3x^{15} + 5215595\,bc^2x^{13} + 6122655\left(b^2c + ac^2\right)x^{11} + 2470545\left(b^3 + 6\,abc\right)x^9 + 12801915\,a^2bx^5 + 938807\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/46940355*(1514205*c^3*x^15 + 5215595*b*c^2*x^13 + 6122655*(b^2*
c + a*c^2)*x^11 + 2470545*(b^3 + 6*a*b*c)*x^9 + 12801915*a^2*b*x^
5 + 9388071*(a*b^2 + a^2*c)*x^7 + 6705765*a^3*x^3)*sqrt(x)
```

Sympy [A] time = 168.084, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{7}{2}}}{7} + \frac{6a^2bx^{\frac{11}{2}}}{11} + \frac{2a^2cx^{\frac{15}{2}}}{5} + \frac{2ab^2x^{\frac{15}{2}}}{5} + \frac{12abcx^{\frac{19}{2}}}{19} + \frac{6ac^2x^{\frac{23}{2}}}{23} + \frac{2b^3x^{\frac{19}{2}}}{19} + \frac{6b^2cx^{\frac{23}{2}}}{23} + \frac{2bc^2x^{\frac{27}{2}}}{9} + \frac{2c^3x^{\frac{31}{2}}}{31}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a**2*c*x**(15/2)/5
+ 2*a*b**2*x**(15/2)/5 + 12*a*b*c*x**(19/2)/19 + 6*a*c**2*x**(23/
2)/23 + 2*b**3*x**(19/2)/19 + 6*b**2*c*x**(23/2)/23 + 2*b*c**2*x*
*(27/2)/9 + 2*c**3*x**(31/2)/31
```

GIAC/XCAS [A] time = 0.263106, size = 117, normalized size = 1.14

$$\frac{2}{31}\,c^3x^{\frac{31}{2}} + \frac{2}{9}\,bc^2x^{\frac{27}{2}} + \frac{6}{23}\,b^2cx^{\frac{23}{2}} + \frac{6}{23}\,ac^2x^{\frac{23}{2}} + \frac{2}{19}\,b^3x^{\frac{19}{2}} + \frac{12}{19}\,abcx^{\frac{19}{2}} + \frac{2}{5}\,ab^2x^{\frac{15}{2}} + \frac{2}{5}\,a^2cx^{\frac{15}{2}} + \frac{6}{11}\,a^2bx^{\frac{11}{2}} + \frac{2}{7}\,a^3x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^(5/2),x, algorithm="giac")
```

```
[Out] 2/31*c^3*x^(31/2) + 2/9*b*c^2*x^(27/2) + 6/23*b^2*c*x^(23/2) + 6/
23*a*c^2*x^(23/2) + 2/19*b^3*x^(19/2) + 12/19*a*b*c*x^(19/2) + 2/
5*a*b^2*x^(15/2) + 2/5*a^2*c*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7
*a^3*x^(7/2)
```

3.1056 $\int x^{3/2}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/
2))/13 + (2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21
/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rubi [A] time = 0.101411, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/
2))/13 + (2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21
/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3*x^(29/2))/29
```

Rubi in Sympy [A] time = 14.0418, size = 102, normalized size = 0.99

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6ax^{\frac{13}{2}}\left(ac+b^2\right)}{13}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2bx^{\frac{17}{2}}\left(6ac+b^2\right)}{17}+\frac{2c^3x^{\frac{29}{2}}}{29}+\frac{2cx^{\frac{21}{2}}\left(ac+b^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*x**(13/2)*(a*c + b*
*2)/13 + 6*b*c**2*x**(25/2)/25 + 2*b*x**(17/2)*(6*a*c + b**2)/17
+ 2*c**3*x**(29/2)/29 + 2*c*x**(21/2)*(a*c + b**2)/7
```

Mathematica [A] time = 0.0469197, size = 103, normalized size = 1.

$$\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}cx^{21/2}\left(ac+b^2\right)+\frac{2}{17}bx^{17/2}\left(6ac+b^2\right)+\frac{6}{13}ax^{13/2}\left(ac+b^2\right)+\frac{6}{25}bc^2x^{25/2}+\frac{2}{29}c^3x^{29/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*(b^2 + a*c)*x^(13/
2))/13 + (2*b*(b^2 + 6*a*c)*x^(17/2))/17 + (2*c*(b^2 + a*c)*x^(21
/2))/7 + (6*b*c^2*x^(25/2))/25 + (2*c^3*x^(29/2))/29
```

Maple [A] time = 0.01, size = 90, normalized size = 0.9

$$\frac{232050\,c^3x^{12} + 807534\,bc^2x^{10} + 961350\,x^8ac^2 + 961350\,b^2cx^8 + 2375100\,x^6abc + 395850\,b^3x^6 + 1552950\,x^4a^2c + 1552950\,ax}{3364725}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/3364725*x^(5/2)*(116025*c^3*x^12+403767*b*c^2*x^10+480675*a*c^2
*x^8+480675*b^2*c*x^8+1187550*a*b*c*x^6+197925*b^3*x^6+776475*a^2
*c*x^4+776475*a*b^2*x^4+1121575*a^2*b*x^2+672945*a^3)
```

Maxima [A] time = 0.774649, size = 109, normalized size = 1.06

$$\frac{2}{29}\,c^3x^{\frac{29}{2}} + \frac{6}{25}\,bc^2x^{\frac{25}{2}} + \frac{2}{7}\left(b^2c + ac^2\right)x^{\frac{21}{2}} + \frac{2}{17}\left(b^3 + 6\,abc\right)x^{\frac{17}{2}} + \frac{2}{3}\,a^2bx^{\frac{9}{2}} + \frac{6}{13}\left(ab^2 + a^2c\right)x^{\frac{13}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*(b^2*c + a*c^2)*x^(
21/2) + 2/17*(b^3 + 6*a*b*c)*x^(17/2) + 2/3*a^2*b*x^(9/2) + 6/13*
(a*b^2 + a^2*c)*x^(13/2) + 2/5*a^3*x^(5/2)
```

Fricas [A] time = 0.273439, size = 116, normalized size = 1.13

$$\frac{2}{3364725}\left(116025\,c^3x^{14} + 403767\,bc^2x^{12} + 480675\left(b^2c + ac^2\right)x^{10} + 197925\left(b^3 + 6\,abc\right)x^8 + 1121575\,a^2bx^4 + 776475\left(ab^2 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3364725*(116025*c^3*x^14 + 403767*b*c^2*x^12 + 480675*(b^2*c +
a*c^2)*x^10 + 197925*(b^3 + 6*a*b*c)*x^8 + 1121575*a^2*b*x^4 + 77
6475*(a*b^2 + a^2*c)*x^6 + 672945*a^3*x^2)*sqrt(x)
```

Sympy [A] time = 109.808, size = 129, normalized size = 1.25

$$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6a^2cx^{\frac{13}{2}}}{13}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{12abcx^{\frac{17}{2}}}{17}+\frac{2ac^2x^{\frac{21}{2}}}{7}+\frac{2b^3x^{\frac{17}{2}}}{17}+\frac{2b^2cx^{\frac{21}{2}}}{7}+\frac{6bc^2x^{\frac{25}{2}}}{25}+\frac{2c^3x^{\frac{29}{2}}}{29}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a**2*c*x**(13/2)/13 +
 6*a*b**2*x**(13/2)/13 + 12*a*b*c*x**(17/2)/17 + 2*a*c**2*x**(21/
2)/7 + 2*b**3*x**(17/2)/17 + 2*b**2*c*x**(21/2)/7 + 6*b*c**2*x**(
25/2)/25 + 2*c**3*x**(29/2)/29
```

GIAC/XCAS [A] time = 0.261898, size = 117, normalized size = 1.14

$$\frac{2}{29}\,c^3x^{\frac{29}{2}}+\frac{6}{25}\,bc^2x^{\frac{25}{2}}+\frac{2}{7}\,b^2cx^{\frac{21}{2}}+\frac{2}{7}\,ac^2x^{\frac{21}{2}}+\frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{12}{17}\,abcx^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{6}{13}\,a^2cx^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*x^(3/2),x, algorithm="giac")
```

```
[Out] 2/29*c^3*x^(29/2) + 6/25*b*c^2*x^(25/2) + 2/7*b^2*c*x^(21/2) + 2/
7*a*c^2*x^(21/2) + 2/17*b^3*x^(17/2) + 12/17*a*b*c*x^(17/2) + 6/1
3*a*b^2*x^(13/2) + 6/13*a^2*c*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*
a^3*x^(5/2)
```

3.1057 $\int \sqrt{x}\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/
2))/11 + (2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19
/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rubi [A] time = 0.0993547, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/
2))/11 + (2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19
/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Rubi in Sympy [A] time = 14.0545, size = 102, normalized size = 0.99

$$\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6ax^{\frac{11}{2}}\left(ac+b^2\right)}{11}+\frac{6bc^2x^{\frac{23}{2}}}{23}+\frac{2bx^{\frac{15}{2}}\left(6ac+b^2\right)}{15}+\frac{2c^3x^{\frac{27}{2}}}{27}+\frac{6cx^{\frac{19}{2}}\left(ac+b^2\right)}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*x**(11/2)*(a*c + b*
*2)/11 + 6*b*c**2*x**(23/2)/23 + 2*b*x**(15/2)*(6*a*c + b**2)/15
+ 2*c**3*x**(27/2)/27 + 6*c*x**(19/2)*(a*c + b**2)/19
```

Mathematica [A] time = 0.0427862, size = 103, normalized size = 1.

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{19}cx^{19/2}\left(ac+b^2\right)+\frac{2}{15}bx^{15/2}\left(6ac+b^2\right)+\frac{6}{11}ax^{11/2}\left(ac+b^2\right)+\frac{6}{23}bc^2x^{23/2}+\frac{2}{27}c^3x^{27/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*(b^2 + a*c)*x^(11/
2))/11 + (2*b*(b^2 + 6*a*c)*x^(15/2))/15 + (6*c*(b^2 + a*c)*x^(19
/2))/19 + (6*b*c^2*x^(23/2))/23 + (2*c^3*x^(27/2))/27
```

Maple [A] time = 0.01, size = 90, normalized size = 0.9

$$\frac{336490\,c^3x^{12}+1185030\,bc^2x^{10}+1434510\,x^8ac^2+1434510\,b^2cx^8+3634092\,x^6abc+605682\,b^3x^6+2477790\,x^4a^2c+247779}{4542615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2/4542615*x^(3/2)*(168245*c^3*x^12+592515*b*c^2*x^10+717255*a*c^2
*x^8+717255*b^2*c*x^8+1817046*a*b*c*x^6+302841*b^3*x^6+1238895*a^
2*c*x^4+1238895*a*b^2*x^4+1946835*a^2*b*x^2+1514205*a^3)
```

Maxima [A] time = 0.76794, size = 109, normalized size = 1.06

$$\frac{2}{27}\,c^3x^{\frac{27}{2}}+\frac{6}{23}\,bc^2x^{\frac{23}{2}}+\frac{6}{19}\left(b^2c+ac^2\right)x^{\frac{19}{2}}+\frac{2}{15}\left(b^3+6\,abc\right)x^{\frac{15}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{6}{11}\left(ab^2+a^2c\right)x^{\frac{11}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*(b^2*c + a*c^2)*x^
(19/2) + 2/15*(b^3 + 6*a*b*c)*x^(15/2) + 6/7*a^2*b*x^(7/2) + 6/11
*(a*b^2 + a^2*c)*x^(11/2) + 2/3*a^3*x^(3/2)
```

Fricas [A] time = 0.270959, size = 113, normalized size = 1.1

$$\frac{2}{4542615}\left(168245\,c^3x^{13} + 592515\,bc^2x^{11} + 717255\left(b^2c + ac^2\right)x^9 + 302841\left(b^3 + 6\,abc\right)x^7 + 1946835\,a^2bx^3 + 1238895\left(ab^2 + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/4542615*(168245*c^3*x^13 + 592515*b*c^2*x^11 + 717255*(b^2*c +
a*c^2)*x^9 + 302841*(b^3 + 6*a*b*c)*x^7 + 1946835*a^2*b*x^3 + 123
8895*(a*b^2 + a^2*c)*x^5 + 1514205*a^3*x)*sqrt(x)
```

Sympy [A] time = 23.3839, size = 112, normalized size = 1.09

$$\frac{2a^3x^{\frac{3}{2}}}{3} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{6bc^2x^{\frac{23}{2}}}{23} + \frac{2c^3x^{\frac{27}{2}}}{27} + \frac{2x^{\frac{19}{2}}\left(3ac^2 + 3b^2c\right)}{19} + \frac{2x^{\frac{15}{2}}\left(6abc + b^3\right)}{15} + \frac{2x^{\frac{11}{2}}\left(3a^2c + 3ab^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] 2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*b*c**2*x**(23/2)/23 +
 2*c**3*x**(27/2)/27 + 2*x**(19/2)*(3*a*c**2 + 3*b**2*c)/19 + 2*x
**(15/2)*(6*a*b*c + b**3)/15 + 2*x**(11/2)*(3*a**2*c + 3*a*b**2)/
11
```

GIAC/XCAS [A] time = 0.260582, size = 117, normalized size = 1.14

$$\frac{2}{27}\,c^3x^{\frac{27}{2}} + \frac{6}{23}\,bc^2x^{\frac{23}{2}} + \frac{6}{19}\,b^2cx^{\frac{19}{2}} + \frac{6}{19}\,ac^2x^{\frac{19}{2}} + \frac{2}{15}\,b^3x^{\frac{15}{2}} + \frac{4}{5}\,abcx^{\frac{15}{2}} + \frac{6}{11}\,ab^2x^{\frac{11}{2}} + \frac{6}{11}\,a^2cx^{\frac{11}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*sqrt(x),x, algorithm="giac")
```

```
[Out] 2/27*c^3*x^(27/2) + 6/23*b*c^2*x^(23/2) + 6/19*b^2*c*x^(19/2) + 6
/19*a*c^2*x^(19/2) + 2/15*b^3*x^(15/2) + 4/5*a*b*c*x^(15/2) + 6/1
1*a*b^2*x^(11/2) + 6/11*a^2*c*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*
a^3*x^(3/2)
```

3.1058 $\int \frac{(a+bx^2+cx^4)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=101

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3
 + (2*b*(b^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/1
7 + (2*b*c^2*x^(21/2))/7 + (2*c^3*x^(25/2))/25
```

Rubi [A] time = 0.101397, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3
 + (2*b*(b^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/1
7 + (2*b*c^2*x^(21/2))/7 + (2*c^3*x^(25/2))/25
```

Rubi in Sympy [A] time = 14.0201, size = 100, normalized size = 0.99

$$2a^3\sqrt{x}+\frac{6a^2bx^{\frac{5}{2}}}{5}+\frac{2ax^{\frac{9}{2}}\left(ac+b^2\right)}{3}+\frac{2bc^2x^{\frac{21}{2}}}{7}+\frac{2bx^{\frac{13}{2}}\left(6ac+b^2\right)}{13}+\frac{2c^3x^{\frac{25}{2}}}{25}+\frac{6cx^{\frac{17}{2}}\left(ac+b^2\right)}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a*x**(9/2)*(a*c + b**2)/
3 + 2*b*c**2*x**(21/2)/7 + 2*b*x**(13/2)*(6*a*c + b**2)/13 + 2*c*
*3*x**(25/2)/25 + 6*c*x**(17/2)*(a*c + b**2)/17
```

Mathematica [A] time = 0.0425763, size = 101, normalized size = 1.

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{6}{17}cx^{17/2}\left(ac+b^2\right)+\frac{2}{13}bx^{13/2}\left(6ac+b^2\right)+\frac{2}{3}ax^{9/2}\left(ac+b^2\right)+\frac{2}{7}bc^2x^{21/2}+\frac{2}{25}c^3x^{25/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/Sqrt[x],x]
```

```
[Out] 2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*(b^2 + a*c)*x^(9/2))/3
 + (2*b*(b^2 + 6*a*c)*x^(13/2))/13 + (6*c*(b^2 + a*c)*x^(17/2))/1
7 + (2*b*c^2*x^(21/2))/7 + (2*c^3*x^(25/2))/25
```

Maple [A] time = 0.009, size = 90, normalized size = 0.9

$$\frac{9282\,c^3x^{12}+33150\,bc^2x^{10}+40950\,x^8ac^2+40950\,b^2cx^8+107100\,x^6abc+17850\,b^3x^6+77350\,x^4a^2c+77350\,ax^4b^2+139230}{116025}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^(1/2),x)
```

```
[Out] 2/116025*x^(1/2)*(4641*c^3*x^12+16575*b*c^2*x^10+20475*a*c^2*x^8+
20475*b^2*c*x^8+53550*a*b*c*x^6+8925*b^3*x^6+38675*a^2*c*x^4+3867
5*a*b^2*x^4+69615*a^2*b*x^2+116025*a^3)
```

Maxima [A] time = 0.739843, size = 119, normalized size = 1.18

$$\frac{2}{25}\,c^3x^{\frac{25}{2}}+\frac{2}{7}\,bc^2x^{\frac{21}{2}}+\frac{6}{17}\,b^2cx^{\frac{17}{2}}+\frac{2}{13}\,b^3x^{\frac{13}{2}}+2\,a^3\sqrt{x}+\frac{2}{15}\left(5\,cx^{\frac{9}{2}}+9\,bx^{\frac{5}{2}}\right)a^2+\frac{2}{663}\left(117\,c^2x^{\frac{17}{2}}+306\,bcx^{\frac{13}{2}}+221\,b^2x^{\frac{9}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/sqrt(x),x, algorithm="maxima")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 2/
13*b^3*x^(13/2) + 2*a^3*sqrt(x) + 2/15*(5*c*x^(9/2) + 9*b*x^(5/2)
)*a^2 + 2/663*(117*c^2*x^(17/2) + 306*b*c*x^(13/2) + 221*b^2*x^(9
/2))*a
```

Fricas [A] time = 0.276211, size = 112, normalized size = 1.11

$$\frac{2}{116025}\left(4641\,c^3x^{12}+16575\,bc^2x^{10}+20475\,\left(b^2c+ac^2\right)x^8+8925\,\left(b^3+6\,abc\right)x^6+69615\,a^2bx^2+38675\,\left(ab^2+a^2c\right)x^4+116\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/sqrt(x),x, algorithm="fricas")
```

```
[Out] 2/116025*(4641*c^3*x^12 + 16575*b*c^2*x^10 + 20475*(b^2*c + a*c^2
)*x^8 + 8925*(b^3 + 6*a*b*c)*x^6 + 69615*a^2*b*x^2 + 38675*(a*b^2
 + a^2*c)*x^4 + 116025*a^3)*sqrt(x)
```

Sympy [A] time = 55.9083, size = 128, normalized size = 1.27

$$2a^3\sqrt{x} + \frac{6a^2bx^{\frac{5}{2}}}{5} + \frac{2a^2cx^{\frac{9}{2}}}{3} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{12abcx^{\frac{13}{2}}}{13} + \frac{6ac^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{13}{2}}}{13} + \frac{6b^2cx^{\frac{17}{2}}}{17} + \frac{2bc^2x^{\frac{21}{2}}}{7} + \frac{2c^3x^{\frac{25}{2}}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**(1/2),x)
```

```
[Out] 2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a**2*c*x**(9/2)/3 + 2*a*
b**2*x**(9/2)/3 + 12*a*b*c*x**(13/2)/13 + 6*a*c**2*x**(17/2)/17 +
 2*b**3*x**(13/2)/13 + 6*b**2*c*x**(17/2)/17 + 2*b*c**2*x**(21/2)
/7 + 2*c**3*x**(25/2)/25
```

GIAC/XCAS [A] time = 0.260224, size = 117, normalized size = 1.16

$$\frac{2}{25}\,c^3x^{\frac{25}{2}} + \frac{2}{7}\,bc^2x^{\frac{21}{2}} + \frac{6}{17}\,b^2cx^{\frac{17}{2}} + \frac{6}{17}\,ac^2x^{\frac{17}{2}} + \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{12}{13}\,abcx^{\frac{13}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{2}{3}\,a^2cx^{\frac{9}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + 2\,a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/sqrt(x),x, algorithm="giac")
```

```
[Out] 2/25*c^3*x^(25/2) + 2/7*b*c^2*x^(21/2) + 6/17*b^2*c*x^(17/2) + 6/
17*a*c^2*x^(17/2) + 2/13*b^3*x^(13/2) + 12/13*a*b*c*x^(13/2) + 2/
3*a*b^2*x^(9/2) + 2/3*a^2*c*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*s
qrt(x)
```

3.1059 $\int \frac{(a+bx^2+cx^4)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7
+ (2*b*(b^2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5
+ (6*b*c^2*x^(19/2))/19 + (2*c^3*x^(23/2))/23
```

Rubi [A] time = 0.0998379, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7
+ (2*b*(b^2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5
+ (6*b*c^2*x^(19/2))/19 + (2*c^3*x^(23/2))/23
```

Rubi in Sympy [A] time = 14.037, size = 99, normalized size = 1.

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{\frac{3}{2}}+\frac{6ax^{\frac{7}{2}}\left(ac+b^2\right)}{7}+\frac{6bc^2x^{\frac{19}{2}}}{19}+\frac{2bx^{\frac{11}{2}}\left(6ac+b^2\right)}{11}+\frac{2c^3x^{\frac{23}{2}}}{23}+\frac{2cx^{\frac{15}{2}}\left(ac+b^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*x**(7/2)*(a*c + b**2)/7
 + 6*b*c**2*x**(19/2)/19 + 2*b*x**(11/2)*(6*a*c + b**2)/11 + 2*c*
*3*x**(23/2)/23 + 2*c*x**(15/2)*(a*c + b**2)/5
```

Mathematica [A] time = 0.0687992, size = 99, normalized size = 1.

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{2}{5}cx^{15/2}\left(ac+b^2\right)+\frac{2}{11}bx^{11/2}\left(6ac+b^2\right)+\frac{6}{7}ax^{7/2}\left(ac+b^2\right)+\frac{6}{19}bc^2x^{19/2}+\frac{2}{23}c^3x^{23/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*(b^2 + a*c)*x^(7/2))/7
+ (2*b*(b^2 + 6*a*c)*x^(11/2))/11 + (2*c*(b^2 + a*c)*x^(15/2))/5
+ (6*b*c^2*x^(19/2))/19 + (2*c^3*x^(23/2))/23
```

Maple [A] time = 0.009, size = 90, normalized size = 0.9

$$-\frac{-14630\,c^3x^{12}-53130\,bc^2x^{10}-67298\,x^8ac^2-67298\,b^2cx^8-183540\,x^6abc-30590\,b^3x^6-144210\,x^4a^2c-144210\,ax^4b^2}{168245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^(3/2),x)
```

```
[Out] -2/168245*(-7315*c^3*x^12-26565*b*c^2*x^10-33649*a*c^2*x^8-33649*
b^2*c*x^8-91770*a*b*c*x^6-15295*b^3*x^6-72105*a^2*c*x^4-72105*a*b
^2*x^4-168245*a^2*b*x^2+168245*a^3)/x^(1/2)
```

Maxima [A] time = 0.770219, size = 109, normalized size = 1.1

$$\frac{2}{23}\,c^3x^{\frac{23}{2}}+\frac{6}{19}\,bc^2x^{\frac{19}{2}}+\frac{2}{5}\left(b^2c+ac^2\right)x^{\frac{15}{2}}+\frac{2}{11}\left(b^3+6\,abc\right)x^{\frac{11}{2}}+2\,a^2bx^{\frac{3}{2}}+\frac{6}{7}\left(ab^2+a^2c\right)x^{\frac{7}{2}}-\frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*(b^2*c + a*c^2)*x^(
15/2) + 2/11*(b^3 + 6*a*b*c)*x^(11/2) + 2*a^2*b*x^(3/2) + 6/7*(a*
b^2 + a^2*c)*x^(7/2) - 2*a^3/sqrt(x)
```

Fricas [A] time = 0.302463, size = 112, normalized size = 1.13

$$\frac{2\left(7315\,c^3x^{12} + 26565\,bc^2x^{10} + 33649\left(b^2c + ac^2\right)x^8 + 15295\left(b^3 + 6\,abc\right)x^6 + 168245\,a^2bx^2 + 72105\left(ab^2 + a^2c\right)x^4 - 168245\,a^3\right)}{168245\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/168245*(7315*c^3*x^12 + 26565*b*c^2*x^10 + 33649*(b^2*c + a*c^2
)*x^8 + 15295*(b^3 + 6*a*b*c)*x^6 + 168245*a^2*b*x^2 + 72105*(a*b
^2 + a^2*c)*x^4 - 168245*a^3)/sqrt(x)
```

Sympy [A] time = 61.6181, size = 126, normalized size = 1.27

$$-\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6a^2cx^{\frac{7}{2}}}{7} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{12abcx^{\frac{11}{2}}}{11} + \frac{2ac^2x^{\frac{15}{2}}}{5} + \frac{2b^3x^{\frac{11}{2}}}{11} + \frac{2b^2cx^{\frac{15}{2}}}{5} + \frac{6bc^2x^{\frac{19}{2}}}{19} + \frac{2c^3x^{\frac{23}{2}}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**(3/2),x)
```

```
[Out] -2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a**2*c*x**(7/2)/7 + 6*a*b
**2*x**(7/2)/7 + 12*a*b*c*x**(11/2)/11 + 2*a*c**2*x**(15/2)/5 + 2
*b**3*x**(11/2)/11 + 2*b**2*c*x**(15/2)/5 + 6*b*c**2*x**(19/2)/19
 + 2*c**3*x**(23/2)/23
```

GIAC/XCAS [A] time = 0.264124, size = 117, normalized size = 1.18

$$\frac{2}{23}\,c^3x^{\frac{23}{2}} + \frac{6}{19}\,bc^2x^{\frac{19}{2}} + \frac{2}{5}\,b^2cx^{\frac{15}{2}} + \frac{2}{5}\,ac^2x^{\frac{15}{2}} + \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{12}{11}\,abcx^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + \frac{6}{7}\,a^2cx^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(3/2),x, algorithm="giac")
```

```
[Out] 2/23*c^3*x^(23/2) + 6/19*b*c^2*x^(19/2) + 2/5*b^2*c*x^(15/2) + 2/
5*a*c^2*x^(15/2) + 2/11*b^3*x^(11/2) + 12/11*a*b*c*x^(11/2) + 6/7
*a*b^2*x^(7/2) + 6/7*a^2*c*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt
(x)
```

3.1060 $\int \frac{(a+bx^2+cx^4)^3}{x^{5/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2)
)/5 + (2*b*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/
13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*x^(21/2))/21
```

Rubi [A] time = 0.101527, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2)
)/5 + (2*b*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/
13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*x^(21/2))/21
```

Rubi in Sympy [A] time = 14.2875, size = 100, normalized size = 0.99

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6ax^{\frac{5}{2}}\left(ac+b^2\right)}{5}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2bx^{\frac{9}{2}}\left(6ac+b^2\right)}{9}+\frac{2c^3x^{\frac{21}{2}}}{21}+\frac{6cx^{\frac{13}{2}}\left(ac+b^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a*x**(5/2)*(a*c + b**
2)/5 + 6*b*c**2*x**(17/2)/17 + 2*b*x**(9/2)*(6*a*c + b**2)/9 + 2*
c**3*x**(21/2)/21 + 6*c*x**(13/2)*(a*c + b**2)/13
```

Mathematica [A] time = 0.0640046, size = 101, normalized size = 1.

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{13}cx^{13/2}\left(ac+b^2\right)+\frac{2}{9}bx^{9/2}\left(6ac+b^2\right)+\frac{6}{5}ax^{5/2}\left(ac+b^2\right)+\frac{6}{17}bc^2x^{17/2}+\frac{2}{21}c^3x^{21/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x^(5/2),x]
```

```
[Out] (-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*(b^2 + a*c)*x^(5/2)
)/5 + (2*b*(b^2 + 6*a*c)*x^(9/2))/9 + (6*c*(b^2 + a*c)*x^(13/2))/
13 + (6*b*c^2*x^(17/2))/17 + (2*c^3*x^(21/2))/21
```

Maple [A] time = 0.009, size = 90, normalized size = 0.9

$$-\frac{-6630\,c^3x^{12}-24570\,bc^2x^{10}-32130\,x^8ac^2-32130\,b^2cx^8-92820\,x^6abc-15470\,b^3x^6-83538\,x^4a^2c-83538\,ax^4b^2-41}{69615}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^(5/2),x)
```

```
[Out] -2/69615*(-3315*c^3*x^12-12285*b*c^2*x^10-16065*a*c^2*x^8-16065*b
^2*c*x^8-46410*a*b*c*x^6-7735*b^3*x^6-41769*a^2*c*x^4-41769*a*b^2
*x^4-208845*a^2*b*x^2+23205*a^3)/x^(3/2)
```

Maxima [A] time = 0.754687, size = 109, normalized size = 1.08

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,\left(b^2c+ac^2\right)x^{\frac{13}{2}}+\frac{2}{9}\,\left(b^3+6\,abc\right)x^{\frac{9}{2}}+6\,a^2b\sqrt{x}+\frac{6}{5}\,\left(ab^2+a^2c\right)x^{\frac{5}{2}}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*(b^2*c + a*c^2)*x^
(13/2) + 2/9*(b^3 + 6*a*b*c)*x^(9/2) + 6*a^2*b*sqrt(x) + 6/5*(a*b
^2 + a^2*c)*x^(5/2) - 2/3*a^3/x^(3/2)
```

Fricas [A] time = 0.271782, size = 112, normalized size = 1.11

$$\frac{2\left(3315\,c^3x^{12}+12285\,bc^2x^{10}+16065\left(b^2c+ac^2\right)x^8+7735\left(b^3+6\,abc\right)x^6+208845\,a^2bx^2+41769\left(ab^2+a^2c\right)x^4-23205\,a^3\right)}{69615\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(5/2),x, algorithm="fricas")
```

```
[Out] 2/69615*(3315*c^3*x^12 + 12285*b*c^2*x^10 + 16065*(b^2*c + a*c^2)
*x^8 + 7735*(b^3 + 6*a*b*c)*x^6 + 208845*a^2*b*x^2 + 41769*(a*b^2
 + a^2*c)*x^4 - 23205*a^3)/x^(3/2)
```

Sympy [A] time = 70.2332, size = 128, normalized size = 1.27

$$-\frac{2a^3}{3x^{\frac{3}{2}}}+6a^2b\sqrt{x}+\frac{6a^2cx^{\frac{5}{2}}}{5}+\frac{6ab^2x^{\frac{5}{2}}}{5}+\frac{4abcx^{\frac{9}{2}}}{3}+\frac{6ac^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{9}{2}}}{9}+\frac{6b^2cx^{\frac{13}{2}}}{13}+\frac{6bc^2x^{\frac{17}{2}}}{17}+\frac{2c^3x^{\frac{21}{2}}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**(5/2),x)
```

```
[Out] -2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a**2*c*x**(5/2)/5 + 6
*a*b**2*x**(5/2)/5 + 4*a*b*c*x**(9/2)/3 + 6*a*c**2*x**(13/2)/13 +
 2*b**3*x**(9/2)/9 + 6*b**2*c*x**(13/2)/13 + 6*b*c**2*x**(17/2)/1
7 + 2*c**3*x**(21/2)/21
```

GIAC/XCAS [A] time = 0.262553, size = 117, normalized size = 1.16

$$\frac{2}{21}\,c^3x^{\frac{21}{2}}+\frac{6}{17}\,bc^2x^{\frac{17}{2}}+\frac{6}{13}\,b^2cx^{\frac{13}{2}}+\frac{6}{13}\,ac^2x^{\frac{13}{2}}+\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{4}{3}\,abcx^{\frac{9}{2}}+\frac{6}{5}\,ab^2x^{\frac{5}{2}}+\frac{6}{5}\,a^2cx^{\frac{5}{2}}+6\,a^2b\sqrt{x}-\frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(5/2),x, algorithm="giac")
```

```
[Out] 2/21*c^3*x^(21/2) + 6/17*b*c^2*x^(17/2) + 6/13*b^2*c*x^(13/2) + 6
/13*a*c^2*x^(13/2) + 2/9*b^3*x^(9/2) + 4/3*a*b*c*x^(9/2) + 6/5*a*
b^2*x^(5/2) + 6/5*a^2*c*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/
2)
```

3.1061 $\int \frac{(a+bx^2+cx^4)^3}{x^{7/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2
) + (2*b*(b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11
 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x^(19/2))/19
```

Rubi [A] time = 0.0998184, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2
) + (2*b*(b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11
 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x^(19/2))/19
```

Rubi in Sympy [A] time = 13.9115, size = 99, normalized size = 1.

$$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2ax^{\frac{3}{2}}\left(ac+b^2\right)+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2bx^{\frac{7}{2}}\left(6ac+b^2\right)}{7}+\frac{2c^3x^{\frac{19}{2}}}{19}+\frac{6cx^{\frac{11}{2}}\left(ac+b^2\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*x**(3/2)*(a*c + b**
2) + 2*b*c**2*x**(15/2)/5 + 2*b*x**(7/2)*(6*a*c + b**2)/7 + 2*c**
3*x**(19/2)/19 + 6*c*x**(11/2)*(a*c + b**2)/11
```

Mathematica [A] time = 0.0637128, size = 99, normalized size = 1.

$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+\frac{6}{11}cx^{11/2}\left(ac+b^2\right)+\frac{2}{7}bx^{7/2}\left(6ac+b^2\right)+2ax^{3/2}\left(ac+b^2\right)+\frac{2}{5}bc^2x^{15/2}+\frac{2}{19}c^3x^{19/2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^3/x^(7/2),x]
```

```
[Out] (-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*(b^2 + a*c)*x^(3/2
) + (2*b*(b^2 + 6*a*c)*x^(7/2))/7 + (6*c*(b^2 + a*c)*x^(11/2))/11
 + (2*b*c^2*x^(15/2))/5 + (2*c^3*x^(19/2))/19
```

Maple [A] time = 0.009, size = 90, normalized size = 0.9

$$-\frac{-770\,c^3x^{12}-2926\,bc^2x^{10}-3990\,x^8ac^2-3990\,b^2cx^8-12540\,x^6abc-2090\,b^3x^6-14630\,x^4a^2c-14630\,ax^4b^2+43890\,a^2b}{7315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^3/x^(7/2),x)
```

```
[Out] -2/7315*(-385*c^3*x^12-1463*b*c^2*x^10-1995*a*c^2*x^8-1995*b^2*c*
x^8-6270*a*b*c*x^6-1045*b^3*x^6-7315*a^2*c*x^4-7315*a*b^2*x^4+219
45*a^2*b*x^2+1463*a^3)/x^(5/2)
```

Maxima [A] time = 0.791726, size = 111, normalized size = 1.12

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,\left(b^2c+ac^2\right)x^{\frac{11}{2}}+\frac{2}{7}\,\left(b^3+6\,abc\right)x^{\frac{7}{2}}+2\,\left(ab^2+a^2c\right)x^{\frac{3}{2}}-\frac{2\,\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*(b^2*c + a*c^2)*x^(
11/2) + 2/7*(b^3 + 6*a*b*c)*x^(7/2) + 2*(a*b^2 + a^2*c)*x^(3/2) -
 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Fricas [A] time = 0.272333, size = 112, normalized size = 1.13

$$\frac{2\left(385\,c^3x^{12}+1463\,bc^2x^{10}+1995\left(b^2c+ac^2\right)x^8+1045\left(b^3+6\,abc\right)x^6-21945\,a^2bx^2+7315\left(ab^2+a^2c\right)x^4-1463\,a^3\right)}{7315\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/7315*(385*c^3*x^12 + 1463*b*c^2*x^10 + 1995*(b^2*c + a*c^2)*x^8
 + 1045*(b^3 + 6*a*b*c)*x^6 - 21945*a^2*b*x^2 + 7315*(a*b^2 + a^2
*c)*x^4 - 1463*a^3)/x^(5/2)
```

Sympy [A] time = 88.8243, size = 124, normalized size = 1.25

$$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a^2cx^{\frac{3}{2}}+2ab^2x^{\frac{3}{2}}+\frac{12abcx^{\frac{7}{2}}}{7}+\frac{6ac^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{7}{2}}}{7}+\frac{6b^2cx^{\frac{11}{2}}}{11}+\frac{2bc^2x^{\frac{15}{2}}}{5}+\frac{2c^3x^{\frac{19}{2}}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**3/x**(7/2),x)
```

```
[Out] -2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a**2*c*x**(3/2) + 2*a
*b**2*x**(3/2) + 12*a*b*c*x**(7/2)/7 + 6*a*c**2*x**(11/2)/11 + 2*
b**3*x**(7/2)/7 + 6*b**2*c*x**(11/2)/11 + 2*b*c**2*x**(15/2)/5 +
2*c**3*x**(19/2)/19
```

GIAC/XCAS [A] time = 0.263131, size = 119, normalized size = 1.2

$$\frac{2}{19}\,c^3x^{\frac{19}{2}}+\frac{2}{5}\,bc^2x^{\frac{15}{2}}+\frac{6}{11}\,b^2cx^{\frac{11}{2}}+\frac{6}{11}\,ac^2x^{\frac{11}{2}}+\frac{2}{7}\,b^3x^{\frac{7}{2}}+\frac{12}{7}\,abcx^{\frac{7}{2}}+2\,ab^2x^{\frac{3}{2}}+2\,a^2cx^{\frac{3}{2}}-\frac{2\left(15\,a^2bx^2+a^3\right)}{5\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/19*c^3*x^(19/2) + 2/5*b*c^2*x^(15/2) + 6/11*b^2*c*x^(11/2) + 6/
11*a*c^2*x^(11/2) + 2/7*b^3*x^(7/2) + 12/7*a*b*c*x^(7/2) + 2*a*b^
2*x^(3/2) + 2*a^2*c*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

3.1062 $\int \frac{x^{9/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=389

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2x^{3/2}}{3c}$$

```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b
^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1
/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^
(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c]
)^(1/4)])/(2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.7161, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{7/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2x^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*x^(3/2))/(3*c) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3
/4)*c^(7/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((b - (b^2 - 2*a*c)
/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b
^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1
/4)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^
(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(7/4)*
(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c]
)^(1/4)])/(2^(3/4)*c^(7/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 167.124, size = 403, normalized size = 1.04

$$\frac{2x^{\frac{3}{2}}}{3c} + \frac{\sqrt[4]{2}\left(-2ac + b^2 - b\sqrt{-4ac + b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b + \sqrt{-4ac + b^2}}}\right)}{2c^{\frac{7}{4}}\sqrt[4]{-b + \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} - \frac{\sqrt[4]{2}\left(-2ac + b^2 - b\sqrt{-4ac + b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b + \sqrt{-4ac + b^2}}}\right)}{2c^{\frac{7}{4}}\sqrt[4]{-b + \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} - \frac{\sqrt[4]{2}\left(-2ac + b^2 + b\sqrt{-4ac + b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b - \sqrt{-4ac + b^2}}}\right)}{2c^{\frac{7}{4}}\sqrt[4]{-b - \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}} + \frac{\sqrt[4]{2}\left(-2ac + b^2 + b\sqrt{-4ac + b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b - \sqrt{-4ac + b^2}}}\right)}{2c^{\frac{7}{4}}\sqrt[4]{-b - \sqrt{-4ac + b^2}}\sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] 2*x**(3/2)/(3*c) + 2**(1/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2
))*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/
4))/(2*c**(7/4)*(-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b
**2)) - 2**(1/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atanh(2*
*(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(2*c**
(7/4)*(-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2*
*(1/4)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(
```

```
1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**(7/4)*(-b -
 sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*(-2*
a*c + b**2 + b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*sqrt(
x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**(7/4)*(-b - sqrt(-4*a
*c + b**2))**(1/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0644452, size = 80, normalized size = 0.21

$$\frac{4x^{3/2} - 3\text{RootSum}\left[\#1^8c + \#1^4b + a\&, \frac{\#1^4 b \log(\sqrt{x}-\#1) + a \log(\sqrt{x}-\#1)}{2\#1^5 c + \#1 b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*x^(3/2) - 3*RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #
1] + b*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(6*c)
```

Maple [C] time = 0.092, size = 65, normalized size = 0.2

$$\frac{2}{3\,c}x^{\frac{3}{2}} - \frac{1}{2\,c}\sum_{\_R=RootOf\left(c\_Z^8+b\_Z^4+a\right)}\frac{\_R^6 b + \_R^2 a}{2\,\_R^7 c + \_R^3 b}\ln\left(\sqrt{x} - \_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2/3*x^(3/2)/c-1/2/c*sum((_R^6*b+_R^2*a)/(2*_R^7*c+_R^3*b)*ln(x^(1
/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\,x^{\frac{3}{2}}}{3\,c} - \int \frac{bx^{\frac{5}{2}} + a\sqrt{x}}{c^2x^4 + bcx^2 + ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/c - integrate((b*x^(5/2) + a*sqrt(x))/(c^2*x^4 + b*c*
x^2 + a*c), x)
```

Fricas [A] time = 1.30129, size = 8598, normalized size = 22.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -1/6*(12*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2
 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
- 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^
2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*ar
ctan(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*
a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6
 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*
a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10
*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12
*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^
16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^
2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sq
rt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4
*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 +
48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*
c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b
^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*
a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^1
7)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))/((a^5*b^6 - 5*a^6*b^4*
c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x) - sqrt((a^10*b^12 - 10*a^11*
b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12
*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 17*a^8*b^
15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 -
1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15
*b*c^8 - (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491
*a^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^1
3*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*
c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)
/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqr
t(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8
*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^
2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(
b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*
c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))))) - 12*c*sqrt(sqrt(1/2)*sqrt(-(
b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b
^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*
```

c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*arctan(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))/((a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x) - sqrt((a^10*b^12 - 10*a^11*b^10*c + 37*a^12*b^8*c^2 - 62*a^13*b^6*c^3 + 46*a^14*b^4*c^4 - 12*a^15*b^2*c^5 + a^16*c^6)*x - 1/2*sqrt(1/2)*(a^7*b^17 - 17*a^8*b^15*c + 119*a^9*b^13*c^2 - 441*a^10*b^11*c^3 + 924*a^11*b^9*c^4 - 1078*a^12*b^7*c^5 + 637*a^13*b^5*c^6 - 151*a^14*b^3*c^7 + 12*a^15*b*c^8 + (a^7*b^14*c^7 - 18*a^8*b^12*c^8 + 131*a^9*b^10*c^9 - 491*a^10*b^8*c^10 + 997*a^11*b^6*c^11 - 1052*a^12*b^4*c^12 + 496*a^13*b^2*c^13 - 64*a^14*c^14)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9))))) + 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 -

```
 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^
8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14
- 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a
*b^2*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c +
 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b
^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 - (b^11*c^7 - 17*a*b^9*c^8
+ 113*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5
*b*c^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c
^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*
b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-
(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*
b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6
*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7
 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^
3*c^2 - 7*a^3*b*c^3 + (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((
b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4
*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a
^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)
) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) +
3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*
b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^
5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16
- 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*log(1/2*s
qrt(1/2)*(b^14 - 16*a*b^12*c + 102*a^2*b^10*c^2 - 328*a^3*b^8*c^3
 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c^5 + 152*a^6*b^2*c^6 - 16*a^7*c
^7 + (b^11*c^7 - 17*a*b^9*c^8 + 113*a^2*b^7*c^9 - 364*a^3*b^5*c^1
0 + 560*a^4*b^3*c^11 - 320*a^5*b*c^12)*sqrt((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3
*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 -
 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*
c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)))*sqrt
(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*
a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b
^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c
^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) - (a^5*b^6 - 5*a^6*b^4*c + 6*a^7*
b^2*c^2 - a^8*c^3)*sqrt(x)) - 3*c*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a
*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 +
16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12
*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2
*c^8 + 16*a^2*c^9)))*log(-1/2*sqrt(1/2)*(b^14 - 16*a*b^12*c + 102
*a^2*b^10*c^2 - 328*a^3*b^8*c^3 + 553*a^4*b^6*c^4 - 457*a^5*b^4*c
^5 + 152*a^6*b^2*c^6 - 16*a^7*c^7 + (b^11*c^7 - 17*a*b^9*c^8 + 11
3*a^2*b^7*c^9 - 364*a^3*b^5*c^10 + 560*a^4*b^3*c^11 - 320*a^5*b*c
^12)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*
c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))*sqrt(sqrt(1/2)*sqrt(-(b^7
```

```
 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*
c^8 + 16*a^2*c^9)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*
a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^1
4 - 12*a*b^4*c^15 + 48*a^2*b^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8
*a*b^2*c^8 + 16*a^2*c^9)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^
2 - 7*a^3*b*c^3 - (b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*sqrt((b^12
 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4
 - 12*a^5*b^2*c^5 + a^6*c^6)/(b^6*c^14 - 12*a*b^4*c^15 + 48*a^2*b
^2*c^16 - 64*a^3*c^17)))/(b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)) -
(a^5*b^6 - 5*a^6*b^4*c + 6*a^7*b^2*c^2 - a^8*c^3)*sqrt(x)) - 4*x^
(3/2))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a), x)
```

3.1063 $\int \frac{x^{7/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=385

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\sqrt{x}}{c}$$

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 -
 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.58393, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2\sqrt{x}}{c}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[x])/c + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*
c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 -
 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
 + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(5/4)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 161.23, size = 401, normalized size = 1.04

$$\frac{2\sqrt{x}}{c}-\frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\left(-2ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{4}}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\left(-2ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{5}{4}}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] 2*sqrt(x)/c - 2**(3/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*at
an(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(
2*c**(5/4)*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2))
 - 2**(3/4)*(-2*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atanh(2**(1/4
)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(2*c**(5/4)
*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4
```

```
)*(-2*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*
sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**(5/4)*(-b - sqrt
(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*(-2*a*c +
 b**2 + b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-
b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**(5/4)*(-b - sqrt(-4*a*c +
b**2))**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0605926, size = 80, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log(\sqrt{x}-\#1)+a\log(\sqrt{x}-\#1)}{2\#1^7c+\#1^3b}\&\right]-4\sqrt{x}}{2c}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(-4*Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (a*Log[Sqrt[x] - #
1] + b*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(2*c)
```

Maple [C] time = 0.013, size = 64, normalized size = 0.2

$$2\,\frac{\sqrt{x}}{c}+\frac{1}{2\,c}\sum_{\_R=RootOf(\_Z^8c+\_Z^4b+a)}\frac{-\_R^4b-a}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 2*x^(1/2)/c+1/2/c*sum((-_R^4*b-a)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R
),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{7}{2}}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.490443, size = 5210, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] 1/2*(4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 4
8*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)))*arctan(-(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3 - (b^
5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^
2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 +
48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*
b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((
b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*
c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5
- 8*a*b^2*c^6 + 16*a^2*c^7)))/(2*(a*b^4 - 3*a^2*b^2*c + a^3*c^2)*
sqrt(x) + sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*
b^2*c^3 + a^6*c^4)*x + 2*sqrt(1/2)*(b^12 - 12*a*b^10*c + 55*a^2*b
^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a
^6*c^6 - (b^11*c^5 - 15*a*b^9*c^6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*
c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*
b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^
6 + 16*a^2*c^7))))) - 4*c*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c +
 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b
^2*c^6 + 16*a^2*c^7)))*arctan((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 -
 4*a^3*c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6
*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 -
12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*s
qrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16
*a^2*c^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))/(2*(a*b^4 - 3*a^2*b
^2*c + a^3*c^2)*sqrt(x) + sqrt(4*(a^2*b^8 - 6*a^3*b^6*c + 11*a^4*
b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x + 2*sqrt(1/2)*(b^12 - 12*a*b
^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a
^5*b^2*c^5 + 8*a^6*c^6 + (b^11*c^5 - 15*a*b^9*c^6 + 85*a^2*b^7*c^
7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*sqrt((b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10
```

```
 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b^5 -
5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sq
rt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(
b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*
c^5 - 8*a*b^2*c^6 + 16*a^2*c^7))))) + c*sqrt(sqrt(1/2)*sqrt(-(b^5
 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c +
 a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3
 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*c
^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^
4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(b^
5 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c
+ a^3*c^2)*sqrt(x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^
3 - (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4*
c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^5
 - 5*a*b^3*c + 5*a^2*b*c^2 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)
*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b
^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + c*sqrt(sqrt(1/2)*sqrt(-(b
^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*c
 + a^3*c^2)*sqrt(x) + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c
^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*c
 + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^4
*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b^
5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(
b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) - c*sqrt(sqrt(1/2)*sqrt(-(
b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c
^7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*
c^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))
/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*log(2*(a*b^4 - 3*a^2*b^2*
c + a^3*c^2)*sqrt(x) - (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*
c^3 + (b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*sqrt((b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(b^6*c^10 - 12*a*b^
4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(sqrt(1/2)*sqrt(-(b
^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^
7)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c
^4)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))) + 4*sqrt(x))/c
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a), x)
```

3.1064 $\int \frac{x^{5/2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=331

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*
a*c])) + ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b
^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*
c^(3/4)*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.874032, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}+\frac{\left(-\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}-\frac{\left(\sqrt{b^2-4ac}-b\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}c^{3/4}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x]
)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b^2 - 4*
a*c])) + ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*
Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)*Sqrt[b
^2 - 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*c^(3/4)
*Sqrt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(3/4)*ArcTanh[(2^
(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*
c^(3/4)*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 104.831, size = 304, normalized size = 0.92

$$-\frac{\sqrt[4]{2}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2c^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] -2**(1/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*atan(2**(1/4)*c**(1/4
)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**(3/4)*sqrt(-4*
a*c + b**2)) + 2**(1/4)*(-b - sqrt(-4*a*c + b**2))**(3/4)*atanh(2
**(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c*
*(3/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*(-b + sqrt(-4*a*c + b**2))
**(3/4)*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))
**(1/4))/(2*c**(3/4)*sqrt(-4*a*c + b**2)) - 2**(1/4)*(-b + sqrt(-
```

```
4*a*c + b**2))**(3/4)*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(
-4*a*c + b**2))**(1/4))/(2*c**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0381455, size = 48, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^3\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1^3)/(b + 2*c
*#1^4) & ]/2
```

Maple [C] time = 0.011, size = 45, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^6}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^6/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{5}{2}}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.400401, size = 5339, normalized size = 16.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 +
 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4
*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)))*arctan(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^
2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160
*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6
*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/
2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sq
rt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b
^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqr
t(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))/((a^2*b^2
- a^3*c)*sqrt(x) - sqrt((a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x - 1/2
*sqrt(1/2)*(a^3*b^7 - 6*a^4*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 -
 (a^3*b^8*c^3 - 13*a^4*b^6*c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6
 + 64*a^7*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b
^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c + (b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^
2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*
c^3 - 8*a*b^2*c^4 + 16*a^2*c^5))))) + 2*sqrt(sqrt(1/2)*sqrt(-(b^3
 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a
*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a
^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*arctan(-1/2*sqrt
(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 + (b^8*c^3
 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (
b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c
^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4
*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^
3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b
^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5))/((a^2*b^2 - a^3*c)*sqrt(x) - sqrt((a^
4*b^4 - 2*a^5*b^2*c + a^6*c^2)*x - 1/2*sqrt(1/2)*(a^3*b^7 - 6*a^4
*b^5*c + 9*a^5*b^3*c^2 - 4*a^6*b*c^3 + (a^3*b^8*c^3 - 13*a^4*b^6*
c^4 + 60*a^5*b^4*c^5 - 112*a^6*b^2*c^6 + 64*a^7*c^7)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 6
4*a^3*c^9)))*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a
^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*
c^5))))) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8
```

```
*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^
6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*
b^2*c^4 + 16*a^2*c^5)))*log(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a
^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*
c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 +
 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c
^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) -
 (a^2*b^2 - a^3*c)*sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a
*b*c + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c
 + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9
)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^
7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3 - (b^8*c^3 - 14*a*b
^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^
4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8
 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*
a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c + (b^4*c^3 - 8*a*b
^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 -
12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*
c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x)) + 1/2*sqrt(sqrt(1
/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*s
qrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*
b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*lo
g(1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3*c^2 - 16*a^3*b*c^3
+ (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 - 160*a^3*b^2*c^6 + 12
8*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c
^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3
*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c
^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*sqrt(-(b^3 - 3*a*b*c
 - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*b^2 - a^3*c)*sqrt(x)
) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*
c^4 + 16*a^2*c^5)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*
a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4
 + 16*a^2*c^5)))*log(-1/2*sqrt(1/2)*(b^7 - 9*a*b^5*c + 24*a^2*b^3
*c^2 - 16*a^3*b*c^3 + (b^8*c^3 - 14*a*b^6*c^4 + 72*a^2*b^4*c^5 -
160*a^3*b^2*c^6 + 128*a^4*c^7)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(
b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(sqrt
(1/2)*sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)
*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))*
sqrt(-(b^3 - 3*a*b*c - (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*
c^8 - 64*a^3*c^9)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)) - (a^2*
b^2 - a^3*c)*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a), x)
```

3.1065 $\int \frac{x^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*
c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2
- 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sq
rt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/
4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(
1/4)*Sqrt[b^2 - 4*a*c])
```

Rubi [A] time = 0.72769, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}+\frac{\sqrt[4]{-\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}-\frac{\sqrt[4]{\sqrt{b^2-4ac}-b}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}\sqrt[4]{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/
(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2 - 4*a*
c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqr
```

```
t[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sqrt[b^2
- 4*a*c]) + ((-b - Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/4)*c^(1
/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*Sq
rt[b^2 - 4*a*c]) - ((-b + Sqrt[b^2 - 4*a*c])^(1/4)*ArcTanh[(2^(1/
4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(
1/4)*Sqrt[b^2 - 4*a*c])
```

Rubi in Sympy [A] time = 104.714, size = 304, normalized size = 0.92

$$\frac{2^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{c}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{c}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{c}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2\sqrt[4]{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] 2**(3/4)*(-b - sqrt(-4*a*c + b**2))**(1/4)*atan(2**(1/4)*c**(1/4)
*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**(1/4)*sqrt(-4*a
*c + b**2)) + 2**(3/4)*(-b - sqrt(-4*a*c + b**2))**(1/4)*atanh(2*
*(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*c**
(1/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*(-b + sqrt(-4*a*c + b**2))*
*(1/4)*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))*
*(1/4))/(2*c**(1/4)*sqrt(-4*a*c + b**2)) - 2**(3/4)*(-b + sqrt(-4
*a*c + b**2))**(1/4)*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-
4*a*c + b**2))**(1/4))/(2*c**(1/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0344839, size = 46, normalized size = 0.14

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1\log\left(\sqrt{x}-\#1\right)}{2\#1^4c+b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (Log[Sqrt[x] - #1]*#1)/(b + 2*c*#
1^4) & ]/2
```

Maple [C] time = 0.012, size = 45, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^4}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^4/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^{\frac{3}{2}}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.328761, size = 2476, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/s
qrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c
 - 8*a*b^2*c^2 + 16*a^2*c^3)))*arctan((b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(sqrt(b^6*c^2 - 12*a*b^4*c^3
+ 48*a^2*b^2*c^4 - 64*a^3*c^5)*(sqrt(sqrt(1/2)*(b^2 - 4*a*c)*sqrt
(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^
2*c^3)) + x) + sqrt(x)))) + 2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c -
8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^
2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*arctan(
(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(s
qrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*(sqrt(s
qrt(1/2)*(b^2 - 4*a*c)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)) + x) + sqrt(x)))) + 1/2*sqrt(s
qrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c
^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^
2*c^2 + 16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt
(sqrt(1/2)*sqrt(-(b + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6
*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*
b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*
c^4 - 64*a^3*c^5) + sqrt(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*lo
g(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b + (
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 4
8*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqr
t(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a
^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5
))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))*log((b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)*sqrt(sqrt(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16
*a^2*c^3)/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c
^5))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4
*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5) + sqrt(x)) + 1/2*sqrt(sqrt(1/
2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2 - 1
2*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c^2
+ 16*a^2*c^3)))*log(-(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*sqrt(sqrt
(1/2)*sqrt(-(b - (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)/sqrt(b^6*c^2
- 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5))/(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)))/sqrt(b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 -
 64*a^3*c^5) + sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a), x)
```

3.1066 $\int \frac{\sqrt{x}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=331

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(
1/4))) + (2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*
a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b
^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b +
 Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.594358, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(
1/4))) + (2^(1/4)*c^(1/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*
a*c])^(1/4)) + (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])
/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b
^2 - 4*a*c])^(1/4)) - (2^(1/4)*c^(1/4)*ArcTanh[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b +
 Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 103.769, size = 298, normalized size = 0.9

$$\frac{\sqrt[4]{2}\sqrt[4]{c}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\,\mathrm{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\,\mathrm{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] 2**(1/4)*c**(1/4)*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*
c + b**2))**(1/4))/((-b + sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c
 + b**2)) - 2**(1/4)*c**(1/4)*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b
 + sqrt(-4*a*c + b**2))**(1/4))/((-b + sqrt(-4*a*c + b**2))**(1/4
)*sqrt(-4*a*c + b**2)) - 2**(1/4)*c**(1/4)*atan(2**(1/4)*c**(1/4)
*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/((-b - sqrt(-4*a*c +
b**2))**(1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*c**(1/4)*atanh(2**(
1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/((-b - s
qrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0360486, size = 47, normalized size = 0.14

$$\frac{1}{2}\mathrm{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(a + b*x^2 + c*x^4),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1 + 2*c*#1^
5) & ]/2
```

Maple [C] time = 0.011, size = 45, normalized size = 0.1

$$\frac{1}{2}\sum_{\_R=RootOf(\_Z^8c+\_Z^4b+a)}\frac{\_R^2}{2\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(_R^2/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z
^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x}}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

Fricas [A] time = 0.349508, size = 3776, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] -2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/s
qrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4
```

```
 - 8*a^2*b^2*c + 16*a^3*c^2)))*arctan(1/2*sqrt(1/2)*(b^4 - 8*a*b^
2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^
4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c +
 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^
5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))/(c*sqrt(x) + sqrt(c^2
*x - 1/2*sqrt(1/2)*(b^3*c - 4*a*b*c^2 - (a*b^6*c - 12*a^2*b^4*c^2
 + 48*a^3*b^2*c^3 - 64*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*
a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))))) + 2*sqrt(sqrt(1/2)*sqrt
(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b
^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^
3*c^2)))*arctan(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 + (a
*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sq
rt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3
*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2
*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^
2*b^2*c + 16*a^3*c^2))/(c*sqrt(x) + sqrt(c^2*x - 1/2*sqrt(1/2)*(b
^3*c - 4*a*b*c^2 + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 6
4*a^4*c^4)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*
c^3))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6
- 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2))))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a
^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sq
rt(1/2)*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 4
8*a^3*b^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^
4*b^2*c^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2
*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2
 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2))
+ c*sqrt(x)) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)
*(b^4 - 8*a*b^2*c + 16*a^2*c^2 - (a*b^7 - 12*a^2*b^5*c + 48*a^3*b
^3*c^2 - 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c
^2 - 64*a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a
^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b + (a*b^4 -
 8*a^2*b^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2 - 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqr
t(x)) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a
^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(1/2*sqrt(1/2)*(b^4 -
8*a*b^2*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 -
 64*a^4*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*
a^5*c^3))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3
*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))
```

```
/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b
^2*c + 16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 -
 64*a^5*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x)) +
1/2*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/
sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^
4 - 8*a^2*b^2*c + 16*a^3*c^2)))*log(-1/2*sqrt(1/2)*(b^4 - 8*a*b^2
*c + 16*a^2*c^2 + (a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4
*b*c^3)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3
))*sqrt(sqrt(1/2)*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)/s
qrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3))/(a*b^4
 - 8*a^2*b^2*c + 16*a^3*c^2)))*sqrt(-(b - (a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)/sqrt(a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5
*c^3))/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)) + c*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a), x)
```

3.1067 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=331

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/
4)) - (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c
])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sq
rt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.669277, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2^{3/4}c^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2 - 4*a*c])^(3/
4)) - (2^(3/4)*c^(3/4)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqr
t[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sqrt[b^2 - 4*a*c
])^(3/4)) + (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-
b - Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) - (2^(3/4)*c^(3/4)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(Sqrt[b^2 - 4*a*c]*(-b + Sq
rt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 103.835, size = 298, normalized size = 0.9

$$-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] -2**(3/4)*c**(3/4)*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a
*c + b**2))**(1/4))/((-b + sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*
c + b**2)) - 2**(3/4)*c**(3/4)*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-
b + sqrt(-4*a*c + b**2))**(1/4))/((-b + sqrt(-4*a*c + b**2))**(3/
4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*atan(2**(1/4)*c**(1/4
)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/((-b - sqrt(-4*a*c +
 b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*atanh(2**
(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/((-b -
sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0404772, size = 49, normalized size = 0.15

$$\frac{1}{2}\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , Log[Sqrt[x] - #1]/(b*#1^3 + 2*c*#
1^7) & ]/2
```

Maple [C] time = 0.01, size = 42, normalized size = 0.1

$$\frac{1}{2} \sum_{\_R = RootOf(\_Z^8 c + \_Z^4 b + a)} \frac{1}{2\_R^7 c + \_R^3 b} \ln\left(\sqrt{x} - \_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*sum(1/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*
b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{x}}{a} - \int \frac{cx^{\frac{7}{2}} + bx^{\frac{3}{2}}}{acx^4 + abx^2 + a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/a - integrate((c*x^(7/2) + b*x^(3/2))/(a*c*x^4 + a*b*x^
2 + a^2), x)
```

Fricas [A] time = 0.439182, size = 4016, normalized size = 12.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c +
16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^
```

4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*
a^5*c^2)))*arctan((b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4
*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sq
rt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(2*(b^2*
c - a*c^2)*sqrt(x) - sqrt(4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x +
2*sqrt(1/2)*(b^8 - 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 +
8*a^4*c^4 - (a^3*b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b
^3*c^3 + 64*a^7*b*c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*
b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c
+ a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)
))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))) - 2*sqrt(sqrt(1/2)*sq
rt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b
^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^
2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*arctan(-
(b^4 - 5*a*b^2*c + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*
c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c
- (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a
^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/
(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))/(2*(b^2*c - a*c^2)*sqrt(x)
- sqrt(4*(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*x + 2*sqrt(1/2)*(b^8
- 8*a*b^6*c + 21*a^2*b^4*c^2 - 22*a^3*b^2*c^3 + 8*a^4*c^4 + (a^3*
b^9 - 13*a^4*b^7*c + 60*a^5*b^5*c^2 - 112*a^6*b^3*c^3 + 64*a^7*b*
c^4)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8
*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^
6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^
4*b^2*c + 16*a^5*c^2))))) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b
*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c +
a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*s
qrt(x) + (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a^3*b^5 - 8*a^4*b^3*c +
16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*
b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3
- 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*
b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^
9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(
1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*
sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8
*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*l
og(-2*(b^2*c - a*c^2)*sqrt(x) - (b^4 - 5*a*b^2*c + 4*a^2*c^2 - (a
^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*
c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqr
t(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c + (a^3*b^4 - 8*a^4*b^2*c + 16*a^
5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c +
48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2)))) + 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^4 - 8*a
^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6
- 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 - 8*a^4*
b^2*c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^2)*sqrt(x) + (b^4 - 5*a

```
*b^2*c + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*sqrt(
(b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*
c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3*a*b*c - (a^3*b^
4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a
^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^4 -
 8*a^4*b^2*c + 16*a^5*c^2)))) - 1/2*sqrt(sqrt(1/2)*sqrt(-(b^3 - 3
*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 - 2*a*b^2
*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c
^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)))*log(-2*(b^2*c - a*c^
2)*sqrt(x) - (b^4 - 5*a*b^2*c + 4*a^2*c^2 + (a^3*b^5 - 8*a^4*b^3*
c + 16*a^5*b*c^2)*sqrt((b^4 - 2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(sqrt(1/2)*sqrt(-(
b^3 - 3*a*b*c - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt((b^4 -
2*a*b^2*c + a^2*c^2)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 6
4*a^9*c^3)))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(c*x**4+b*x**2+a),x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)),x, algorithm="giac")

[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x)), x)
```

3.1068 $\int \frac{1}{x^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=371

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{2}{a\sqrt{x}}$$

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/
4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(
-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c
])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 - b/
Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) +
 (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 -
 4*a*c])^(1/4))
```

Rubi [A] time = 1.02573, antiderivative size = 371, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (c^(1/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/
4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(
-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c
])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)])/(2^(3/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(1 - b/
Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) +
 (c^(1/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a*(-b + Sqrt[b^2 -
 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 147.074, size = 374, normalized size = 1.01

$$-\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2a\sqrt[4]{-b+\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}+\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2a\sqrt[4]{-b-\sqrt{-4ac+b^2}}\sqrt{-4ac+b^2}}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] -2**(1/4)*c**(1/4)*(b + sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/
4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(2*a*(-b + sqrt(-4*
a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) + 2**(1/4)*c**(1/4)*(b +
 sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(
-4*a*c + b**2))**(1/4))/(2*a*(-b + sqrt(-4*a*c + b**2))**(1/4)*sq
rt(-4*a*c + b**2)) + 2**(1/4)*c**(1/4)*(b - sqrt(-4*a*c + b**2))*
atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))
/(2*a*(-b - sqrt(-4*a*c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2*
*(1/4)*c**(1/4)*(b - sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)
*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*a*(-b - sqrt(-4*a*
c + b**2))**(1/4)*sqrt(-4*a*c + b**2)) - 2/(a*sqrt(x))
```

Mathematica [C] time = 0.0646622, size = 78, normalized size = 0.21

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4}{\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(4/Sqrt[x] + RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] - #1
] + c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(2*a)
```

Maple [C] time = 0.015, size = 65, normalized size = 0.2

$$-\frac{1}{2\,a}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\_R^6c+\_R^2b}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)-2\,\frac{1}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/a*sum((_R^6*c+_R^2*b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))-2/a/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{a\sqrt{x}}-\int\frac{cx^{\frac{5}{2}}+b\sqrt{x}}{acx^4+abx^2+a^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)),x, algorithm="maxima")
```

```
[Out] -2/(a*sqrt(x)) - integrate((c*x^(5/2) + b*sqrt(x))/(a*c*x^4 + a*b
*x^2 + a^2), x)
```

Fricas [A] time = 1.10767, size = 7055, normalized size = 19.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)),x, algorithm="fricas")
```

[Out] 1/2*(4*a*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))/((b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x) + sqrt((b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 - (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))))) - 4*a*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*arctan(-1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))/((b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x) + sqrt((b^8*c^8 - 6*a*b^6*c^9 + 11*a^2*b^4*c^10 - 6*a^3*b^2*c^11 + a^4*c^12)*x - 1/2*sqrt(1/2)*(b^13*c^5 - 13*a*b^11*c^6 + 65*a^2*b^9*c^7 - 155*a^3*b^7*c^8 + 175*a^4*b^5*c^9 - 79*a^5*b^3*c^10 + 12*a^6*b*c^11 + (a^5*b^12*c^5 - 16*a^6*b^10*c^6 + 100*a^7*b^8*c^7 - 305*a^8*b^6*c^8 + 460*a^9*b^4*c^9 - 304*a^10*b^2*c^10 + 64*a^11*c^11)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^

2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*
b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 -
64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2))))) + a*sqrt(
x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*
log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b
^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*
c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10
*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4
 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12
*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*
sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 1
6*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13
*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^
2*c^5 + a^2*c^6)*sqrt(x)) - a*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 -
 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*s
qrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11 - 13*
a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32
*a^5*b*c^5 - (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*
b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^
4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5
- 5*a*b^3*c + 5*a^2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*
sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)
/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^
5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^
2*b*c^2 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^
11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*
c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) + a
*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a
^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)))*log(1/2*sqrt(1/2)*(b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138
*a^3*b^5*c^3 + 128*a^4*b^3*c^4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^
6*b^8*c + 98*a^7*b^6*c^2 - 280*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 12
8*a^10*c^5)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^1
3*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a
^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)))*sqrt(-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2
*c + 16*a^7*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 6

```
4*a^13*c^3)))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 -
3*a*b^2*c^5 + a^2*c^6)*sqrt(x)) - a*sqrt(x)*sqrt(sqrt(1/2)*sqrt(-
(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*
c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3))
)/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*log(-1/2*sqrt(1/2)*(b^11
 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^
4 - 32*a^5*b*c^5 + (a^5*b^10 - 16*a^6*b^8*c + 98*a^7*b^6*c^2 - 28
0*a^8*b^4*c^3 + 352*a^9*b^2*c^4 - 128*a^10*c^5)*sqrt((b^8 - 6*a*b
^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 - 12*a
^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(sqrt(1/2)*sqrt(
-(b^5 - 5*a*b^3*c + 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7
*c^2)*sqrt((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^
4*c^4)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)
))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)))*sqrt(-(b^5 - 5*a*b^3*c
+ 5*a^2*b*c^2 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*sqrt((b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)/(a^10*b^6 -
 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^4 - 8*a^
6*b^2*c + 16*a^7*c^2)) + (b^4*c^4 - 3*a*b^2*c^5 + a^2*c^6)*sqrt(x
)) - 4)/(a*sqrt(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(3/2)), x)
```

3.1069 $\int \frac{1}{x^{5/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=371

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2}{3ax^{3/2}}$$

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a
*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 -
b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2
 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.903972, antiderivative size = 371, normalized size of antiderivative = 1., number

of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (c^(3/4)*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a
*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a
*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(2^(1/4)*a*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(1 -
b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt
[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 + (c^(3/4)*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*a*(-b + Sqrt[b^2
 - 4*a*c])^(3/4))
```

Rubi in Sympy [A] time = 142.524, size = 376, normalized size = 1.01

$$\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2a\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b+\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{2a\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2a\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{2a\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}\sqrt{-4ac+b^2}}-\frac{2}{3ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] 2**(3/4)*c**(3/4)*(b + sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4
)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(2*a*(-b + sqrt(-4*a
*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) + 2**(3/4)*c**(3/4)*(b +
sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqrt(-
4*a*c + b**2))**(1/4))/(2*a*(-b + sqrt(-4*a*c + b**2))**(3/4)*sqr
t(-4*a*c + b**2)) - 2**(3/4)*c**(3/4)*(b - sqrt(-4*a*c + b**2))*a
tan(2**(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/
(2*a*(-b - sqrt(-4*a*c + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2**
(3/4)*c**(3/4)*(b - sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*
sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(2*a*(-b - sqrt(-4*a*c
 + b**2))**(3/4)*sqrt(-4*a*c + b**2)) - 2/(3*a*x**(3/2))
```

Mathematica [C] time = 0.0703329, size = 82, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4c\log\left(\sqrt{x}-\#1\right)+b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4}{x^{3/2}}}{6a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(5/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(4/x^(3/2) + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*Log[Sqrt[x] -
#1] + c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(6*a)
```

Maple [C] time = 0.015, size = 64, normalized size = 0.2

$$-\frac{2}{3\,a}x^{-\frac{3}{2}}+\frac{1}{2\,a}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{-\_R^4c-b}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(5/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] -2/3/a/x^(3/2)+1/2/a*sum((-_R^4*c-b)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)
-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(3\,b\sqrt{x}+\frac{a}{x^{\frac{3}{2}}}\right)}{3\,a^2}+\int\frac{bcx^{\frac{7}{2}}+\left(b^2-ac\right)x^{\frac{3}{2}}}{a^2cx^4+a^2bx^2+a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)),x, algorithm="maxima")
```

```
[Out] -2/3*(3*b*sqrt(x) + a/x^(3/2))/a^2 + integrate((b*c*x^(7/2) + (b^
2 - a*c)*x^(3/2))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Fricas [A] time = 0.971757, size = 6602, normalized size = 17.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)),x, algorithm="fricas")
```

[Out] 1/6*(12*a*x^(3/2)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*arctan(-(b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x) - sqrt(4*(b^12*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 - (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))))) - 12*a*x^(3/2)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*arctan((b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))/(2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x) - sqrt(4*(b^12*c^4 - 10*a*b^10*c^5 + 37*a^2*b^8*c^6 - 62*a^3*b^6*c^7 + 46*a^4*b^4*c^8 - 12*a^5*b^2*c^9 + a^6*c^10)*x + 2*sqrt(1/2)*(b^18 - 18*a*b^16*c + 135*a^2*b^14*c^2 - 546*a^3*b^12*c^3 + 1288*a^4*b^10*c^4 - 1792*a^5*b^8*c^5 + 1421*a^6*b^6*c^6 - 592*a^7*b^4*c^7 + 114*a^8*b^2*c^8 - 8*a^9*c^9 + (a^7*b^15 - 19*a^8*b^13*c + 148*a^9*b^11*c^2 - 605*a^10*b^9*c^3 + 1374*a^11*b^7*c^4 - 1672*a^12*b^5*c^5 + 928*a^13*b^3*c^6 - 128*a^14*b*c^7)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(-(b^7

```
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2
*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a
^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6
 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*
a^8*b^2*c + 16*a^9*c^2))))) - 3*a*x^(3/2)*sqrt(sqrt(1/2)*sqrt(-(b
^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*
b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*
b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 -
 8*a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^
2*b^2*c^4 - a^3*c^5)*sqrt(x) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2
- 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10*a^8*b^4*c + 32*a^9
*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2
 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a
^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(s
qrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 +
(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 3
7*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
+ a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*
c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) + 3*a*x^(3/2)*sqrt
(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
+ (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^1
7*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 -
 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x) - (b^9 - 9*a*b^7*
c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*c^4 - (a^7*b^6 - 10
*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10
*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^
2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64
*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c
^2 - 7*a^3*b*c^3 + (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^1
2 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^
4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16
*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))))
 - 3*a*x^(3/2)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3
*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b
^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^
16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2))
)*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^4 - a^3*c^5)*sqrt(x
) + (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3*b^3*c^3 + 4*a^4*b*
c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2 - 32*a^10*c^3)*sqr
t((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*
b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a^15*b^4*c + 4
8*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a*b
^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c + 16
*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*
c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12*a
^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b^2
*c + 16*a^9*c^2)))) + 3*a*x^(3/2)*sqrt(sqrt(1/2)*sqrt(-(b^7 - 7*a
*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4 - 8*a^8*b^2*c +
16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^
6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6 - 12
```

```
*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^4 - 8*a^8*b
^2*c + 16*a^9*c^2)))*log(-2*(b^6*c^2 - 5*a*b^4*c^3 + 6*a^2*b^2*c^
4 - a^3*c^5)*sqrt(x) - (b^9 - 9*a*b^7*c + 26*a^2*b^5*c^2 - 25*a^3
*b^3*c^3 + 4*a^4*b*c^4 + (a^7*b^6 - 10*a^8*b^4*c + 32*a^9*b^2*c^2
 - 32*a^10*c^3)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^
3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)/(a^14*b^6
- 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))*sqrt(sqrt(1/2)
*sqrt(-(b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3 - (a^7*b^4
 - 8*a^8*b^2*c + 16*a^9*c^2)*sqrt((b^12 - 10*a*b^10*c + 37*a^2*b^
8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(
a^7*b^4 - 8*a^8*b^2*c + 16*a^9*c^2)))) - 4)/(a*x^(3/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(5/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(5/2)), x)
```

3.1070 $\int \frac{1}{x^{7/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=412

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5ax^{5/2}}$$

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^
2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- (c^(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(
-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.76961, antiderivative size = 412, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2^{3/4}a^2\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(a^2*Sqrt[x]) + (c^(1/4)*(b - (b^2 - 2*a
*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/
4)) + (c^(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^
2*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 2*a*c)/S
qrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
- (c^(1/4)*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(3/4)*a^2*(
-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.104779, size = 107, normalized size = 0.26

$$-\frac{-5\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4bc\log(\sqrt{x}-\#1)-ac\log(\sqrt{x}-\#1)+b^2\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]+\frac{4a}{x^{5/2}}-\frac{20b}{\sqrt{x}}}{10a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(7/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((4*a)/x^(5/2) - (20*b)/Sqrt[x] - 5*RootSum[a + b*#1^4 + c*#1^8
& , (b^2*Log[Sqrt[x] - #1] - a*c*Log[Sqrt[x] - #1] + b*c*Log[Sqrt
[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(10*a^2)
```

Maple [C] time = 0.019, size = 82, normalized size = 0.2

$$\frac{1}{2\,a^2}\sum_{\_R=RootOf(\_Z^8c+\_Z^4b+a)}\frac{bc\_R^6+(-ac+b^2)\,\_R^2}{2\,\_R^7c+\_R^3b}\ln\left(\sqrt{x}-\_R\right)-\frac{2}{5\,a}x^{-\frac{5}{2}}+2\,\frac{b}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(7/2)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2/a^2*sum((b*c*_R^6+(-a*c+b^2)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/
2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\frac{5\,b}{\sqrt{x}}-\frac{a}{x^{\frac{5}{2}}}\right)}{5\,a^2}+\int\frac{bcx^{\frac{5}{2}}+(b^2-ac)\sqrt{x}}{a^2cx^4+a^2bx^2+a^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)),x, algorithm="maxima")
```

```
[Out] 2/5*(5*b/sqrt(x) - a/x^(5/2))/a^2 + integrate((b*c*x^(5/2) + (b^2
 - a*c)*sqrt(x))/(a^2*c*x^4 + a^2*b*x^2 + a^3), x)
```

Fricas [A] time = 5.93168, size = 10322, normalized size = 25.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(20*a^2*x^(5/2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a
^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2
*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 23
0*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*
c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^
20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2
)))*arctan(-1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2
- 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*
a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 - (
a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 +
 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^
16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b
^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8
*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3))
)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3
*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*s
qrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 36
7*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^
7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^2
1*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*
a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^
4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2
*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5
+ 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19
*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c
 + 16*a^11*c^2))/((b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^
3*b^2*c^10 + a^4*c^11)*sqrt(x) + sqrt((b^16*c^14 - 14*a*b^14*c^15
 + 79*a^2*b^12*c^16 - 230*a^3*b^10*c^17 + 367*a^4*b^8*c^18 - 314*
a^5*b^6*c^19 + 130*a^6*b^4*c^20 - 20*a^7*b^2*c^21 + a^8*c^22)*x -
 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^21*c^10 + 230*a^2*b^19*c^11 - 1
311*a^3*b^17*c^12 + 4692*a^4*b^15*c^13 - 10947*a^5*b^13*c^14 + 16
731*a^6*b^11*c^15 - 16380*a^7*b^9*c^16 + 9711*a^8*b^7*c^17 - 3109
*a^9*b^5*c^18 + 425*a^10*b^3*c^19 - 20*a^11*b*c^20 - (a^9*b^18*c^
9 - 22*a^10*b^16*c^10 + 205*a^11*b^14*c^11 - 1050*a^12*b^12*c^12
+ 3206*a^13*b^10*c^13 - 5909*a^14*b^8*c^14 + 6333*a^15*b^6*c^15 -
 3580*a^16*b^4*c^16 + 880*a^17*b^2*c^17 - 64*a^18*c^18)*sqrt((b^1
6 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^
8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*
c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))
*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4
*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a
*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 -
314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^
18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^
```

4 - 8*a^10*b^2*c + 16*a^11*c^2))))) - 20*a^2*x^(5/2)*sqrt(sqrt(1/
2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a
^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14
*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(
a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*arctan(1/2*sqrt(1/2)*(b^18 -
20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10
*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 2
64*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19*a^10*b^11*c + 146*a^
11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3
*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6
*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c +
48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*
b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4
- 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b
^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 +
130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b
^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c +
 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*
b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sq
rt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367
*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7
 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21
*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))/((b^8*c^7 - 7*a*b
^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x) + s
qrt((b^16*c^14 - 14*a*b^14*c^15 + 79*a^2*b^12*c^16 - 230*a^3*b^10
*c^17 + 367*a^4*b^8*c^18 - 314*a^5*b^6*c^19 + 130*a^6*b^4*c^20 -
20*a^7*b^2*c^21 + a^8*c^22)*x - 1/2*sqrt(1/2)*(b^23*c^9 - 23*a*b^
21*c^10 + 230*a^2*b^19*c^11 - 1311*a^3*b^17*c^12 + 4692*a^4*b^15*
c^13 - 10947*a^5*b^13*c^14 + 16731*a^6*b^11*c^15 - 16380*a^7*b^9*
c^16 + 9711*a^8*b^7*c^17 - 3109*a^9*b^5*c^18 + 425*a^10*b^3*c^19
- 20*a^11*b*c^20 + (a^9*b^18*c^9 - 22*a^10*b^16*c^10 + 205*a^11*b
^14*c^11 - 1050*a^12*b^12*c^12 + 3206*a^13*b^10*c^13 - 5909*a^14*
b^8*c^14 + 6333*a^15*b^6*c^15 - 3580*a^16*b^4*c^16 + 880*a^17*b^2
*c^17 - 64*a^18*c^18)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
- 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*
b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 4
8*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b
^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c +
 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^
3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6
- 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b
^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2))))
) - 5*a^2*x^(5/2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*
b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c
+ 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a
^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6
 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*
b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))
*log(1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a
^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6
*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 - (a^9*b^1

3 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a
^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14
*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(
a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(
sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^
3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^
16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b
^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8
*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3))
)/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c
+ 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a
^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c
^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a
^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c
+ 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a
^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*
c^10 + a^4*c^11)*sqrt(x)) + 5*a^2*x^(5/2)*sqrt(sqrt(1/2)*sqrt(-(b
^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 +
(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c +
79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b
^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 -
12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^1
0*b^2*c + 16*a^11*c^2)))*log(-1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c +
168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a
^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^
8 - 16*a^9*c^9 - (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 -
575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^
15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^
10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20
*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c
^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^
2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*
c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230
*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c
^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^2
0*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)
))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a
^4*b*c^4 + (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14
*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4
- 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(
a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*
a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) - 5*a^2*x^(5/2
)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3
*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*s
qrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 36
7*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^
7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^2
1*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*log(1/2*sqrt(1/
2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^2 - 768*a^3*b^12*c^3 + 20
68*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 3024*a^6*b^6*c^6 - 1409*a^7*
b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 + (a^9*b^13 - 19*a^10*b^11
*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3 + 1204*a^13*b^5*c^4 - 12

```
32*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((b^16 - 14*a*b^14*c + 79*a
^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^
5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^
19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))*sqrt(sqrt(1/2)*sqrt(-
(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4
- (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c
 + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5
*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6
- 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a
^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^
2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a
^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^1
0*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*
a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^
2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)) + (b^8
*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10*a^3*b^2*c^10 + a^4*c^11)
*sqrt(x)) + 5*a^2*x^(5/2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c +
 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^1
0*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2
 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6
*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c +
48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^1
1*c^2)))*log(-1/2*sqrt(1/2)*(b^18 - 20*a*b^16*c + 168*a^2*b^14*c^
2 - 768*a^3*b^12*c^3 + 2068*a^4*b^10*c^4 - 3312*a^5*b^8*c^5 + 302
4*a^6*b^6*c^6 - 1409*a^7*b^4*c^7 + 264*a^8*b^2*c^8 - 16*a^9*c^9 +
 (a^9*b^13 - 19*a^10*b^11*c + 146*a^11*b^9*c^2 - 575*a^12*b^7*c^3
 + 1204*a^13*b^5*c^4 - 1232*a^14*b^3*c^5 + 448*a^15*b*c^6)*sqrt((
b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4
*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a
^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3
)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a
^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)
*sqrt((b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 +
367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*
c^7 + a^8*c^8)/(a^18*b^6 - 12*a^19*b^4*c + 48*a^20*b^2*c^2 - 64*a
^21*c^3)))/(a^9*b^4 - 8*a^10*b^2*c + 16*a^11*c^2)))*sqrt(-(b^9 -
9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4 - (a^9*
b^4 - 8*a^10*b^2*c + 16*a^11*c^2)*sqrt((b^16 - 14*a*b^14*c + 79*a
^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^
5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)/(a^18*b^6 - 12*a^
19*b^4*c + 48*a^20*b^2*c^2 - 64*a^21*c^3)))/(a^9*b^4 - 8*a^10*b^2
*c + 16*a^11*c^2)) + (b^8*c^7 - 7*a*b^6*c^8 + 15*a^2*b^4*c^9 - 10
*a^3*b^2*c^10 + a^4*c^11)*sqrt(x)) + 20*b*x^2 - 4*a)/(a^2*x^(5/2)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(7/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*x^(7/2)), x)
```

3.1071 $\int \frac{x^{13/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=544

$$-\frac{bx^{3/2}}{2c(b^2-4ac)}+\frac{x^{7/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -(b*x^(3/2))/(2*c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 1
4*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b -
Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)
*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c - (3*b^2 -
14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
 Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 -
 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3
/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2
 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^
(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 4.76164, antiderivative size = 544, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{bx^{3/2}}{2c\left(b^2-4ac\right)}+\frac{x^{7/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\left(3b^2-14ac\right)\sqrt{b^2-4ac}-20abc+3b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{7/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*x^(3/2))/(2*c*(b^2 - 4*a*c)) + (x^(7/2)*(2*a + b*x^2))/(2*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((3*b^3 - 20*a*b*c + (3*b^2 - 1
4*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b -
Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2)
*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c - (3*b^2 -
14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
 Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3/2
)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - ((3*b^3 - 20*a*b*c + (3*b^2 -
 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^(3
/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((3*b^3 - 20*a*b*c - (3*b^2
 - 14*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(7/4)*(b^2 - 4*a*c)^
(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.325075, size = 144, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{-14\#1^4 ac\log\left(\sqrt{x}-\#1\right)+3\#1^4 b^2\log\left(\sqrt{x}-\#1\right)+3ab\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]-\frac{4x^{3/2}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + R
ootSum[a + b*#1^4 + c*#1^8 & , (3*a*b*Log[Sqrt[x] - #1] + 3*b^2*L
og[Sqrt[x] - #1]*#1^4 - 14*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*
c*#1^5) & ])/(8*c*(b^2 - 4*a*c))
```

Maple [C] time = 0.078, size = 149, normalized size = 0.3

$$2\,\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{\left(2\,ac-b^2\right)x^{7/2}}{\left(4\,ac-b^2\right)c}+1/4\,\frac{abx^{3/2}}{\left(4\,ac-b^2\right)c}\right)+\frac{1}{8\,c}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(14\,ac-3\,b^2\right)\_R^6-3\,\_R^2ab}{\left(4\,ac-b^2\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(7/2)+1/4*a*b/c/(4*a*c-b^2)*x
^(3/2))/(c*x^4+b*x^2+a)+1/8/c*sum(((14*a*c-3*b^2)*_R^6-3*_R^2*a*b
)/(4*a*c-b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_
Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(b^2-2\,ac)\,x^{\frac{7}{2}}+abx^{\frac{3}{2}}}{2\,((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2)}+\int\frac{(3\,b^2-14\,ac)\,x^{\frac{5}{2}}+3\,ab\sqrt{x}}{4\,((b^2c^2-4\,ac^3)x^4+ab^2c-4\,a^2c^2+(b^3c-4\,abc^2)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((b^2 - 2*a*c)*x^(7/2) + a*b*x^(3/2))/((b^2*c^2 - 4*a*c^3)*x
^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + integrate(1
/4*((3*b^2 - 14*a*c)*x^(5/2) + 3*a*b*sqrt(x))/((b^2*c^2 - 4*a*c^3
)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(13/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{\left(cx^4 + bx^2 + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1072 $\int \frac{x^{11/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=520

$$-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{5/2}\left(2a+bx^2\right)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\sqrt{x}}{2c(b^2-4ac)}$$

```
[Out] -(b*Sqrt[x])/(2*c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a
*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c - (b*(b^2 - 12*a*c))/Sq
rt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a
*c])^(3/4)) - ((b^2 - 10*a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))
```

Rubi [A] time = 2.77676, antiderivative size = 520, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{b(b^2-12ac)}{\sqrt{b^2-4ac}}-10ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}c^{5/4}(b^2-4ac)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{5/2}(2a+bx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{b\sqrt{x}}{2c(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(b*Sqrt[x])/(2*c*(b^2 - 4*a*c)) + (x^(5/2)*(2*a + b*x^2))/(2*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a
*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c - (b*(b^2 - 12*a*c))/Sq
rt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2
 - 4*a*c])^(3/4)) - ((b^2 - 10*a*c + (b*(b^2 - 12*a*c))/Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a
*c])^(3/4)) - ((b^2 - 10*a*c - (b*(b^2 - 12*a*c))/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1
/4)])/(4*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.305804, size = 144, normalized size = 0.28

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-10\#1^4ac\log\left(\sqrt{x}-\#1\right)+\#1^4b^2\log\left(\sqrt{x}-\#1\right)+ab\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]-\frac{4\sqrt{x}\left(a\left(b-2cx^2\right)+b^2x^2\right)}{a+bx^2+cx^4}}{8c\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-4*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2)))/(a + b*x^2 + c*x^4) + R
ootSum[a + b*#1^4 + c*#1^8 & , (a*b*Log[Sqrt[x] - #1] + b^2*Log[S
qrt[x] - #1]*#1^4 - 10*a*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*
#1^7) & ])/(8*c*(b^2 - 4*a*c))
```

Maple [C] time = 0.028, size = 146, normalized size = 0.3

$$2\,\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{\left(2\,ac-b^2\right)x^{5/2}}{\left(4\,ac-b^2\right)c}+1/4\,\frac{ab\sqrt{x}}{\left(4\,ac-b^2\right)c}\right)+\frac{1}{8\,c}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(10\,ac-b^2\right)\_R^4-ab}{\left(4\,ac-b^2\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*(2*a*c-b^2)/c/(4*a*c-b^2)*x^(5/2)+1/4*a*b/c/(4*a*c-b^2)*x
^(1/2))/(c*x^4+b*x^2+a)+1/8/c*sum(((10*a*c-b^2)*_R^4-a*b)/(4*a*c-
b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^{\frac{9}{2}}+2\,ax^{\frac{5}{2}}}{2\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)}+\int -\frac{bx^{\frac{7}{2}}+10\,ax^{\frac{3}{2}}}{4\,((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(9/2) + 2*a*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(b*x^(7/2) + 10*a*x
^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*x^2), x)
```

Fricas [A] time = 2.35763, size = 11709, normalized size = 22.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2
*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*
b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
- 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2
571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 262500
0*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 57
6*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 1290
24*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589
824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 +
 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5
*b^2*c^10 + 4096*a^6*c^11)))*arctan(1/2*(b^11 - 47*a*b^9*c + 853*
a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*
c^5 - (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^8
*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c^
11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2
- 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 +
6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12
- 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15
+ 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18
 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 76
5*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 -
24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4
```

*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*
c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2
625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11
 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 -
 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17
+ 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*
c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 614
4*a^5*b^2*c^10 + 4096*a^6*c^11)))/((9*a*b^8 - 451*a^2*b^6*c + 862
5*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + sqr
t((81*a^2*b^16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 9129750*
a^5*b^10*c^3 + 146540625*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 + 9
937500000*a^8*b^4*c^6 - 37500000000*a^9*b^2*c^7 + 62500000000*a^1
0*c^8)*x + 1/2*sqrt(1/2)*(b^22 - 112*a*b^20*c + 5735*a^2*b^18*c^2
 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*c^4 - 52073994*a^5*b^12
*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c^7 + 18208800
000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*b^2
*c^10 - 80000000000*a^11*c^11 - (b^25*c^5 - 91*a*b^23*c^6 + 3641*
a^2*b^21*c^7 - 84776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 132157
44*a^5*b^15*c^10 + 95875584*a^6*b^13*c^11 - 493891584*a^7*b^11*c^
12 + 1798938624*a^8*b^9*c^13 - 4533059584*a^9*b^7*c^14 + 75238604
80*a^10*b^5*c^15 - 7405568000*a^11*b^3*c^16 + 3276800000*a^12*b*c
^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*
c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)
/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*
c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6
*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^
19)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^
3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7
 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096
*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^
3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^
6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3
*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a
^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*
a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)
)))) - 4*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c
- 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^
2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c +
2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26250
00*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 5
76*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129
024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 58
9824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6
+ 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10 + 4096*a^6*c^11)))*arctan(-1/2*(b^11 - 47*a*b^9*c + 85
3*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*
b*c^5 + (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b
^8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*
c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^
2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5
+ 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^1

2 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^1
5 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^
18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c +
765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b
^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^1
0*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^
11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14
- 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^1
7 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^1
0*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10 + 4096*a^6*c^11)))/((9*a*b^8 - 451*a^2*b^6*c + 8
625*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + s
qrt((81*a^2*b^16 - 8118*a^3*b^14*c + 358651*a^4*b^12*c^2 - 912975
0*a^5*b^10*c^3 + 146540625*a^6*b^8*c^4 - 1519250000*a^7*b^6*c^5 +
9937500000*a^8*b^4*c^6 - 37500000000*a^9*b^2*c^7 + 62500000000*a
^10*c^8)*x + 1/2*sqrt(1/2)*(b^22 - 112*a*b^20*c + 5735*a^2*b^18*c
^2 - 176820*a^3*b^16*c^3 + 3634845*a^4*b^14*c^4 - 52073994*a^5*b^
12*c^5 + 527503968*a^6*b^10*c^6 - 3751826400*a^7*b^8*c^7 + 182088
00000*a^8*b^6*c^8 - 56920000000*a^9*b^4*c^9 + 102400000000*a^10*b
^2*c^10 - 80000000000*a^11*c^11 + (b^25*c^5 - 91*a*b^23*c^6 + 364
1*a^2*b^21*c^7 - 84776*a^3*b^19*c^8 + 1280016*a^4*b^17*c^9 - 1321
5744*a^5*b^15*c^10 + 95875584*a^6*b^13*c^11 - 493891584*a^7*b^11*
c^12 + 1798938624*a^8*b^9*c^13 - 4533059584*a^9*b^7*c^14 + 752386
0480*a^10*b^5*c^15 - 7405568000*a^11*b^3*c^16 + 3276800000*a^12*b
*c^17)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^
6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^
6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^1
2*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b
^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*
c^19)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*
c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c
^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 40
96*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*
a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*
a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a
^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064
*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 26214
4*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*
a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^1
1))))) - ((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c
- 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^
2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
- 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c +
2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26250
00*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 5
76*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129
024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 58
9824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6
+ 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10 + 4096*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625
*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + 1/2*

```
(b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a
^4*b^3*c^4 - 40000*a^5*b*c^5 - (b^14*c^5 - 44*a*b^12*c^6 + 720*a^
2*b^10*c^7 - 6080*a^3*b^8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4
*c^10 + 126976*a^6*b^2*c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b
^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4
 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*
c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^
14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c
^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqr
t(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000
*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11
)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b
^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^1
3 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^
16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)
))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8
 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))) + ((b
^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)
*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 
5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (b^12*c^5 - 24*a*b^10*c^6 + 
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*
b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8
*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c
^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*
c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*
c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2
*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^
8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 +
 4096*a^6*c^11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2
 - 75000*a^4*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) - 1/2*(b^11 - 47*a
*b^9*c + 853*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 -
 40000*a^5*b*c^5 - (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 -
 6080*a^3*b^8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 1269
76*a^6*b^2*c^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571
*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a
^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a
^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*
a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824
*a^8*b^2*c^18 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45
*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 +
 (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12
 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^
4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 3
6*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^
4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*
a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5
 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*
b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))) - ((b^2*c^2 - 4*a
*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(s
qrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3
*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*
```

c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4
096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950
*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000
*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*
a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 34406
4*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 2621
44*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280
*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^
11)))*log((9*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2 - 75000*a^4
*b^2*c^3 + 250000*a^5*c^4)*sqrt(x) + 1/2*(b^11 - 47*a*b^9*c + 853
*a^2*b^7*c^2 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b
*c^5 + (b^14*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^
8*c^8 + 29440*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c
^11 - 81920*a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2
 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 +
 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12
 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15
 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^1
8 - 262144*a^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 7
65*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 -
 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^
4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10
*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^1
1 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14
- 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17
 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10
*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 61
44*a^5*b^2*c^10 + 4096*a^6*c^11)))) + ((b^2*c^2 - 4*a*c^3)*x^4 +
a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqr
t(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000
*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11
)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3
 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(b
^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*b^12*c^1
3 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^6*b^6*c^
16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a^9*c^19)
))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8
 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10 + 4096*a^6*c^11)))*log((9
*a*b^8 - 451*a^2*b^6*c + 8625*a^3*b^4*c^2 - 75000*a^4*b^2*c^3 + 2
50000*a^5*c^4)*sqrt(x) - 1/2*(b^11 - 47*a*b^9*c + 853*a^2*b^7*c^2
 - 7324*a^3*b^5*c^3 + 28400*a^4*b^3*c^4 - 40000*a^5*b*c^5 + (b^14
*c^5 - 44*a*b^12*c^6 + 720*a^2*b^10*c^7 - 6080*a^3*b^8*c^8 + 2944
0*a^4*b^6*c^9 - 82944*a^5*b^4*c^10 + 126976*a^6*b^2*c^11 - 81920*
a^7*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3
*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6
*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*b^14*c^12 - 5376*a^3*
b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5*b^8*c^15 + 344064*a^
6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^8*b^2*c^18 - 262144*a
^9*c^19)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c
^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 - (b^12*c^5 - 24*a*b^10*c
^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144
*a^5*b^2*c^10 + 4096*a^6*c^11)*sqrt((b^12 - 78*a*b^10*c + 2571*a^

```
2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*
b^2*c^5 + 6250000*a^6*c^6)/(b^18*c^10 - 36*a*b^16*c^11 + 576*a^2*
b^14*c^12 - 5376*a^3*b^12*c^13 + 32256*a^4*b^10*c^14 - 129024*a^5
*b^8*c^15 + 344064*a^6*b^6*c^16 - 589824*a^7*b^4*c^17 + 589824*a^
8*b^2*c^18 - 262144*a^9*c^19)))/(b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c
^10 + 4096*a^6*c^11)))) + 4*((b^2 - 2*a*c)*x^2 + a*b)*sqrt(x))/((
b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2
)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{(cx^4 + bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1073 $\int \frac{x^{9/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=471

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4
*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 + 12*a*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/
4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.67845, antiderivative size = 471, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(b\sqrt{b^2-4ac}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(b-\frac{12ac+b^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}c^{3/4}\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (
(b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt
[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4
*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + ((b - (b^2 + 12*a*c
)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)*(-b + Sqrt
[b^2 - 4*a*c])^(1/4)) - ((b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(
4*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((b - (b^2 + 12*a*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*c^(3/
4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [A] time = 174.712, size = 428, normalized size = 0.91

$$\frac{x^{\frac{3}{2}}\left(2a+bx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{2}\left(12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{8c^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt[4]{2}\left(12ac+b^2-b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{8c^{\frac{3}{4}}\sqrt[4]{-b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}+\frac{\sqrt[4]{2}\left(12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{8c^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}-\frac{\sqrt[4]{2}\left(12ac+b^2+b\sqrt{-4ac+b^2}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{8c^{\frac{3}{4}}\sqrt[4]{-b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] x**(3/2)*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
 - 2**(1/4)*(12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atan(2**(1/4)
*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(8*c**(3/4)*
(-b + sqrt(-4*a*c + b**2))**(1/4)*(-4*a*c + b**2)**(3/2)) + 2**(1
/4)*(12*a*c + b**2 - b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/
4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(8*c**(3/4)*(-b + s
qrt(-4*a*c + b**2))**(1/4)*(-4*a*c + b**2)**(3/2)) + 2**(1/4)*(12
*a*c + b**2 + b*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/4)*sqrt(
x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(8*c**(3/4)*(-b - sqrt(-4*a
*c + b**2))**(1/4)*(-4*a*c + b**2)**(3/2)) - 2**(1/4)*(12*a*c + b
**2 + b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b
- sqrt(-4*a*c + b**2))**(1/4))/(8*c**(3/4)*(-b - sqrt(-4*a*c + b*
*2))**(1/4)*(-4*a*c + b**2)**(3/2))
```

Mathematica [C] time = 0.234574, size = 124, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4b\log\left(\sqrt{x}-\#1\right)-6a\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]}{8\left(b^2-4ac\right)}-\frac{-2ax^{3/2}-bx^{7/2}}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(-2*a*x^(3/2) - b*x^(7/2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + RootSum[a + b*#1^4 + c*#1^8 & , (-6*a*Log[Sqrt[x] - #1] + b*Lo
g[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.074, size = 121, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{bx^{7/2}}{4\,ac-b^2}-1/2\,\frac{ax^{3/2}}{4\,ac-b^2}\right)+\frac{1}{8}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{-\_R^6b+6\,\_R^2a}{(4\,ac-b^2)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(7/2)-1/2*a/(4*a*c-b^2)*x^(3/2))/(c*x^4+b
*x^2+a)+1/8*sum((-_R^6*b+6*_R^2*a)/(4*a*c-b^2)/(2*_R^7*c+_R^3*b)*
ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bx^{\frac{7}{2}}+2\,ax^{\frac{3}{2}}}{2\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)}-\int-\frac{bx^{\frac{5}{2}}-6\,a\sqrt{x}}{4\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^(7/2) + 2*a*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*
a^2*c + (b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(b*x^(5/2) - 6*a*sq
rt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*x^2), x)
```

Fricas [A] time = 5.42678, size = 14438, normalized size = 30.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 +
 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1
280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*
c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^
3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8
- 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 +
 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14
- 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
- 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a
^6*c^9)))*arctan(1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14
*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^
5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^
2*c^8 + 71663616*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^
19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b
^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 + 1397882
88*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^1
3 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*
c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c
^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 -
 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13
+ 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(
b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 -
 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^
4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6
 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a
^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824
*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^
3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4
*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b
^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c
^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144
*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^
4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16
*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10
 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^1
3 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^1
0*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6
144*a^5*b^2*c^8 + 4096*a^6*c^9))/((343*a^2*b^10 + 14553*a^3*b^8*c
 + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^
4 + 15116544*a^7*c^5)*sqrt(x) + sqrt((117649*a^4*b^20 + 9983358*a
^5*b^18*c + 404714961*a^6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 15
8656107456*a^8*b^12*c^4 + 1707655509504*a^9*b^10*c^5 + 1233881857
3824*a^10*b^8*c^6 + 58812305154048*a^11*b^6*c^7 + 177024646692864
*a^12*b^4*c^8 + 304679870005248*a^13*b^2*c^9 + 228509902503936*a^
14*c^10)*x - 1/2*sqrt(1/2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 1
3335105*a^5*b^21*c^2 + 323354360*a^6*b^19*c^3 + 4269253584*a^7*b^
17*c^4 + 24537890304*a^8*b^15*c^5 - 79436754432*a^9*b^13*c^6 - 16
```

21756588032*a^10*b^11*c^7 - 3506876964864*a^11*b^9*c^8 + 27305557
622784*a^12*b^7*c^9 + 100201644490752*a^13*b^5*c^10 - 14293623531
1104*a^14*b^3*c^11 - 677066377789440*a^15*b*c^12 - (2401*a^3*b^30
*c^3 - 49049*a^4*b^28*c^4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^
24*c^6 + 373184512*a^7*b^22*c^7 - 319185152*a^8*b^20*c^8 - 274088
52992*a^9*b^18*c^9 + 93871525888*a^10*b^16*c^10 + 774145638400*a^
11*b^14*c^11 - 4486009651200*a^12*b^12*c^12 - 5590781263872*a^13*
b^10*c^13 + 81717925773312*a^14*b^8*c^14 - 108093958520832*a^15*b
^6*c^15 - 454721122861056*a^16*b^4*c^16 + 1497904875307008*a^17*b
^2*c^17 - 1283918464548864*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 13
77*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 -
36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*
b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^
7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7
+ 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*
a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^
7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 137
7*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 3
6*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b
^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7
*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9))))) - 4*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqr
t(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^
4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6
*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18
*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 322
56*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 58
9824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^1
2*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840
*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*arctan(-1/2*sqr
t(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3
 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6
 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9
 + (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*
c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b
^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628
928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^1
4)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3
+ 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 -
5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 3
44064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 -
262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a
^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2
*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 1749
6*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a
^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^
5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a
^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*
a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*
c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 +

3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1
280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*
c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^
3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8
- 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 +
344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14
- 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5
- 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a
^6*c^9))/((343*a^2*b^10 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 +
2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sq
rt(x) + sqrt((117649*a^4*b^20 + 9983358*a^5*b^18*c + 404714961*a^
6*b^16*c^2 + 9897860448*a^7*b^14*c^3 + 158656107456*a^8*b^12*c^4
+ 1707655509504*a^9*b^10*c^5 + 12338818573824*a^10*b^8*c^6 + 5881
2305154048*a^11*b^6*c^7 + 177024646692864*a^12*b^4*c^8 + 30467987
0005248*a^13*b^2*c^9 + 228509902503936*a^14*c^10)*x - 1/2*sqrt(1/
2)*(2401*a^3*b^25 + 294294*a^4*b^23*c + 13335105*a^5*b^21*c^2 + 3
23354360*a^6*b^19*c^3 + 4269253584*a^7*b^17*c^4 + 24537890304*a^8
*b^15*c^5 - 79436754432*a^9*b^13*c^6 - 1621756588032*a^10*b^11*c^
7 - 3506876964864*a^11*b^9*c^8 + 27305557622784*a^12*b^7*c^9 + 10
0201644490752*a^13*b^5*c^10 - 142936235311104*a^14*b^3*c^11 - 677
066377789440*a^15*b*c^12 + (2401*a^3*b^30*c^3 - 49049*a^4*b^28*c^
4 - 1432760*a^5*b^26*c^5 - 6473264*a^6*b^24*c^6 + 373184512*a^7*b
^22*c^7 - 319185152*a^8*b^20*c^8 - 27408852992*a^9*b^18*c^9 + 938
71525888*a^10*b^16*c^10 + 774145638400*a^11*b^14*c^11 - 448600965
1200*a^12*b^12*c^12 - 5590781263872*a^13*b^10*c^13 + 817179257733
12*a^14*b^8*c^14 - 108093958520832*a^15*b^6*c^15 - 45472112286105
6*a^16*b^4*c^16 + 1497904875307008*a^17*b^2*c^17 - 12839184645488
64*a^18*c^18)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a
^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*
b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b
^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*
b^2*c^14 - 262144*a^9*c^15)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b
^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8
*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4
096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^
3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b
^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^
8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b
^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*
b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8
+ 4096*a^6*c^9))))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168
*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c
^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17
496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576
*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*
a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824
*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8 + 4096*a^6*c^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 1
46*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096
*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 8957
9520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (b^23*c^3 - 20*a*b^21*c^4 +

432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 19
35360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^1
0 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a
^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 13
77*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 -
36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*
b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^
7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1
/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 +
(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 +
3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 5
4*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4
)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^
9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^
12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)
))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6
 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b
^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 -
24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4
*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c +
1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6
- 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^
4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*
a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3
 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*
b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 145
53*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 1007769
6*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x
^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-
(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (b^12*c^3
- 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b
^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c
+ 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^
6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*
a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 58982
4*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^
4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*log(-1/2*sqrt(1/2)
*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 246
4*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 441
81504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 - (b^
23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 +
195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^
9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^
9*b^5*c^12 + 250609664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqr
t((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 1049
76*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a
^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*
a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144
*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3
*c^2 + 3024*a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c
^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 409
6*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*
b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^1

4*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*
c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2
*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^
8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 +
4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*
a^3*b*c^3 + (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^
3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*s
qrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 10
4976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376
*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 34406
4*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 2621
44*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280
*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9
)) + (343*a^2*b^10 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 24960
96*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)
) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^
2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 302
4*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*
a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)
*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 +
104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 53
76*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344
064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 26
2144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 12
80*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c
^9)))*log(1/2*sqrt(1/2)*(b^18 + 25*a*b^16*c - 146*a^2*b^14*c^2 -
5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1076096*a^5*b^8*c^5 - 104
83200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7 - 89579520*a^8*b^2*c^8 +
71663616*a^9*c^9 + (b^23*c^3 - 20*a*b^21*c^4 + 432*a^2*b^19*c^5
- 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c^7 - 1935360*a^5*b^13*c^8
+ 12214272*a^6*b^11*c^9 - 50823168*a^7*b^9*c^10 + 139788288*a^8*
b^7*c^11 - 245628928*a^9*b^5*c^12 + 250609664*a^10*b^3*c^13 - 113
246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 1
7496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 57
6*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 129024
*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 58982
4*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 2
1*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b
^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 -
6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a
^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a
*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10
*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^
4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*
a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^
7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sqrt(-(b^7 + 21*a*b^5*c +
168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 24
0*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^
2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 +
17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 +
576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 1290
24*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 589
824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 +
240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5

```
*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10 + 14553*a^3*b^8*c + 281
232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 10077696*a^6*b^2*c^4 + 15
116544*a^7*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2
*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c
+ 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*c^3 - 24*a*b^10*c^4 +
240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*
b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2
 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7
+ 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32256*a^4*b^10*c^10 - 12
9024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589824*a^7*b^4*c^13 + 5
89824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^12*c^3 - 24*a*b^10*c^4
 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a
^5*b^2*c^8 + 4096*a^6*c^9)))*log(-1/2*sqrt(1/2)*(b^18 + 25*a*b^16
*c - 146*a^2*b^14*c^2 - 5320*a^3*b^12*c^3 - 2464*a^4*b^10*c^4 + 1
076096*a^5*b^8*c^5 - 10483200*a^6*b^6*c^6 + 44181504*a^7*b^4*c^7
- 89579520*a^8*b^2*c^8 + 71663616*a^9*c^9 + (b^23*c^3 - 20*a*b^21
*c^4 + 432*a^2*b^19*c^5 - 11712*a^3*b^17*c^6 + 195072*a^4*b^15*c^
7 - 1935360*a^5*b^13*c^8 + 12214272*a^6*b^11*c^9 - 50823168*a^7*b
^9*c^10 + 139788288*a^8*b^7*c^11 - 245628928*a^9*b^5*c^12 + 25060
9664*a^10*b^3*c^13 - 113246208*a^11*b*c^14)*sqrt((b^8 + 54*a*b^6*
c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^18*
c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 3225
6*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 589
824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))*sqrt(
sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*
c^3 - (b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*
c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b
^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a
^4*c^4)/(b^18*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b
^12*c^9 + 32256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*
b^6*c^12 - 589824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9
*c^15)))/(b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b
^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)))*sq
rt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (b^12*
c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a
^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)*sqrt((b^8 + 54*a*b^
6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(b^1
8*c^6 - 36*a*b^16*c^7 + 576*a^2*b^14*c^8 - 5376*a^3*b^12*c^9 + 32
256*a^4*b^10*c^10 - 129024*a^5*b^8*c^11 + 344064*a^6*b^6*c^12 - 5
89824*a^7*b^4*c^13 + 589824*a^8*b^2*c^14 - 262144*a^9*c^15)))/(b^
12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 384
0*a^4*b^4*c^7 - 6144*a^5*b^2*c^8 + 4096*a^6*c^9)) + (343*a^2*b^10
 + 14553*a^3*b^8*c + 281232*a^4*b^6*c^2 + 2496096*a^5*b^4*c^3 + 1
0077696*a^6*b^2*c^4 + 15116544*a^7*c^5)*sqrt(x)) - 4*(b*x^3 + 2*a
*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*
a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{(cx^4 + bx^2 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1074 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=483

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (
(3*b^2 + 4*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c
- 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*
(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
 - 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4
*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/
4))
```

Rubi [A] time = 1.85739, antiderivative size = 483, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3

$$\frac{\sqrt{x}\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\left(-3b\sqrt{b^2-4ac}+4ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (
(3*b^2 + 4*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2
- 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c
- 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*
(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((3*b^2 + 4*a*c + 3*b*Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*
c])^(1/4)])/(4*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2
 - 4*a*c])^(3/4)) + ((3*b^2 + 4*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcT
anh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4
*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/
4))
```

Rubi in Sympy [A] time = 172.751, size = 442, normalized size = 0.92

$$\frac{\sqrt{x}\left(2a+bx^{2}\right)}{2\left(-4ac+b^{2}\right)\left(a+bx^{2}+cx^{4}\right)}+\frac{2^{\frac{3}{4}}\left(4ac+3b^{2}-3b\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^{2}}}}\right)}{8\sqrt[4]{c}\left(-b+\sqrt{-4ac+b^{2}}\right)^{\frac{3}{4}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}+\frac{2^{\frac{3}{4}}\left(4ac+3b^{2}-3b\sqrt{-4ac+b^{2}}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^{2}}}}\right)}{8\sqrt[4]{c}\left(-b+\sqrt{-4ac+b^{2}}\right)^{\frac{3}{4}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{2^{\frac{3}{4}}\left(4ac+3b^{2}+3b\sqrt{-4ac+b^{2}}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^{2}}}}\right)}{8\sqrt[4]{c}\left(-b-\sqrt{-4ac+b^{2}}\right)^{\frac{3}{4}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}-\frac{2^{\frac{3}{4}}\left(4ac+3b^{2}+3b\sqrt{-4ac+b^{2}}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^{2}}}}\right)}{8\sqrt[4]{c}\left(-b-\sqrt{-4ac+b^{2}}\right)^{\frac{3}{4}}\left(-4ac+b^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(x)*(2*a + b*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
+ 2**(3/4)*(4*a*c + 3*b**2 - 3*b*sqrt(-4*a*c + b**2))*atan(2**(1/
4)*c**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(8*c**(1/4
)*(-b + sqrt(-4*a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3/2)) + 2**
(3/4)*(4*a*c + 3*b**2 - 3*b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c
**(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/(8*c**(1/4)*(-
b + sqrt(-4*a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3/2)) - 2**(3/4
)*(4*a*c + 3*b**2 + 3*b*sqrt(-4*a*c + b**2))*atan(2**(1/4)*c**(1/
4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(8*c**(1/4)*(-b - s
qrt(-4*a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3/2)) - 2**(3/4)*(4*
a*c + 3*b**2 + 3*b*sqrt(-4*a*c + b**2))*atanh(2**(1/4)*c**(1/4)*s
qrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/(8*c**(1/4)*(-b - sqrt(
-4*a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3/2))
```

Mathematica [C] time = 0.234248, size = 127, normalized size = 0.26

$$\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3\#1^4b\log(\sqrt{x}-\#1)-2a\log(\sqrt{x}-\#1)}{2\#1^7c+\#1^3b}\&\right]}{8(b^2-4ac)}-\frac{-2a\sqrt{x}-bx^{5/2}}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(-2*a*Sqrt[x] - b*x^(5/2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + RootSum[a + b*#1^4 + c*#1^8 & , (-2*a*Log[Sqrt[x] - #1] + 3*b*
Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.025, size = 118, normalized size = 0.2

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\frac{bx^{5/2}}{4ac-b^2}-1/2\frac{a\sqrt{x}}{4ac-b^2}\right)+\frac{1}{8}\sum_{\_R=RootOf(\_Z^8c+\_Z^4b+a)}\frac{-3\_R^4b+2a}{(4ac-b^2)(2\_R^7c+\_R^3b)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/(4*a*c-b^2)*x^(5/2)-1/2*a/(4*a*c-b^2)*x^(1/2))/(c*x^4+b
*x^2+a)+1/8*sum((-3*_R^4*b+2*a)/(4*a*c-b^2)/(2*_R^7*c+_R^3*b)*ln(
x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,cx^{\frac{9}{2}}+bx^{\frac{5}{2}}}{2\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)}-\int-\frac{2\,cx^{\frac{7}{2}}+5\,bx^{\frac{3}{2}}}{4\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(9/2) + b*x^(5/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4
*a^2*c + (b^3 - 4*a*b*c)*x^2) - integrate(-1/4*(2*c*x^(7/2) + 5*b
*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
```

```
*c)*x^2), x)
```

Fricas [A] time = 0.782974, size = 9086, normalized size = 18.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)
*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2
+ (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 +
3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^
4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2
*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b
^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2
*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*
c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 40
96*a^6*c^7)))*arctan(-(81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 -
64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a
^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7
)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^1
6*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6
 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 +
 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(8
1*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 2
40*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b
^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)
/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5
 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b
^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/((1215*b^4 + 26
4*a*b^2*c - 16*a^2*c^2)*sqrt(x) - sqrt((1476225*b^8 + 641520*a*b^
6*c + 30816*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256*a^4*c^4)*x + sqr
t(1/2)*(111537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c^2 - 89
61280*a^3*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 40
96*a^6*c^6 + 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3
- 292096*a^3*b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^
6 + 9519104*a^6*b^7*c^7 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*
c^9 - 262144*a^9*b*c^10)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^
2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c
^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8
 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*
sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10
*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 61
44*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*
a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*
b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b
```

^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^
11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^
4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7))))) - 4*(
(b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sq
rt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*
c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4
*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*
a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^
4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 +
 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 -
262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 12
80*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c
^7)))*arctan((81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^
3 - 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^
4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6
561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 5
76*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024
*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a
^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 7
60*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^
8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 +
4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^
2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*
a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a
^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 2
4*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*
c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))/((1215*b^4 + 264*a*b^2*c
 - 16*a^2*c^2)*sqrt(x) - sqrt((1476225*b^8 + 641520*a*b^6*c + 308
16*a^2*b^4*c^2 - 8448*a^3*b^2*c^3 + 256*a^4*c^4)*x + sqrt(1/2)*(1
11537*b^12 - 1375704*a*b^10*c + 5803760*a^2*b^8*c^2 - 8961280*a^3
*b^6*c^3 + 2522880*a^4*b^4*c^4 - 186368*a^5*b^2*c^5 + 4096*a^6*c^
6 - 8*(81*b^19*c - 2596*a*b^17*c^2 + 36416*a^2*b^15*c^3 - 292096*
a^3*b^13*c^4 + 1465856*a^4*b^11*c^5 - 4716544*a^5*b^9*c^6 + 95191
04*a^6*b^7*c^7 - 11075584*a^7*b^5*c^8 + 5832704*a^8*b^3*c^9 - 262
144*a^9*b*c^10)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*
c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 3225
6*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824
*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(-(81
*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 24
0*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5
+ 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 -
589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^
12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*
a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7))))) - ((b^2*c - 4*
a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2
)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^
10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 -
6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 1
6*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^
3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6
*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*
c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*

c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) - (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 + 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) + (81*b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(b^13*c - 24*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c

```
^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256
*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*
a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a^9*c^11)))*sqrt(sqrt(
1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^12*c - 24*a
*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5
 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 648*a*b^2*c
+ 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 - 5376
*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 344064*
a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262144*a
^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b
^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)))) +
 ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*
sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (b^1
2*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a
^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqrt((6561*b^4 - 64
8*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*
c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7
 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10
- 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 -
1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6
*c^7)))*log(-(1215*b^4 + 264*a*b^2*c - 16*a^2*c^2)*sqrt(x) - (81*
b^6 - 652*a*b^4*c + 1328*a^2*b^2*c^2 - 64*a^3*c^3 - 4*(b^13*c - 2
4*a*b^11*c^2 + 240*a^2*b^9*c^3 - 1280*a^3*b^7*c^4 + 3840*a^4*b^5*
c^5 - 6144*a^5*b^3*c^6 + 4096*a^6*b*c^7)*sqrt((6561*b^4 - 648*a*b
^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3 + 576*a^2*b^14*c^4 -
 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 129024*a^5*b^8*c^7 + 34
4064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 589824*a^8*b^2*c^10 - 262
144*a^9*c^11)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*
a^2*b*c^2 - (b^12*c - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*
b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^2*c^6 + 4096*a^6*c^7)*sqr
t((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(b^18*c^2 - 36*a*b^16*c^3
 + 576*a^2*b^14*c^4 - 5376*a^3*b^12*c^5 + 32256*a^4*b^10*c^6 - 12
9024*a^5*b^8*c^7 + 344064*a^6*b^6*c^8 - 589824*a^7*b^4*c^9 + 5898
24*a^8*b^2*c^10 - 262144*a^9*c^11)))/(b^12*c - 24*a*b^10*c^2 + 24
0*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5 - 6144*a^5*b^
2*c^6 + 4096*a^6*c^7)))) + 4*(b*x^2 + 2*a)*sqrt(x))/((b^2*c - 4*a
*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{(cx^4 + bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1075 $\int \frac{x^{5/2}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=450

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
(c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2
)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1
/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a
*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[
b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rubi [A] time = 1.21996, antiderivative size = 450, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{x^{3/2}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\sqrt{b^2-4ac}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(4b-\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}\left(b^2-4ac\right)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
(c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x
])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2
)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(4*b - Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(
1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1
/4)) + (c^(1/4)*(4*b + Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a
*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(4*b - Sqrt[
b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 -
4*a*c])^(1/4)])/(2*2^(3/4)*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rubi in Sympy [A] time = 147.605, size = 394, normalized size = 0.88

$$\frac{\sqrt[4]{2}\sqrt[4]{c}\left(b-\frac{\sqrt{-4ac+b^2}}{4}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{\sqrt[4]{2}\sqrt[4]{c}\left(b-\frac{\sqrt{-4ac+b^2}}{4}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{\sqrt[4]{2}\sqrt[4]{c}\left(b+\frac{\sqrt{-4ac+b^2}}{4}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} + \frac{\sqrt[4]{2}\sqrt[4]{c}\left(b+\frac{\sqrt{-4ac+b^2}}{4}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}\left(-4ac+b^2\right)^{\frac{3}{2}}} - \frac{x^{\frac{3}{2}}\left(b+2cx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] 2**(1/4)*c**(1/4)*(b - sqrt(-4*a*c + b**2)/4)*atan(2**(1/4)*c**(1
/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/((-b + sqrt(-4*a*c
 + b**2))**(1/4)*(-4*a*c + b**2)**(3/2)) - 2**(1/4)*c**(1/4)*(b -
 sqrt(-4*a*c + b**2)/4)*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b + sqr
t(-4*a*c + b**2))**(1/4))/((-b + sqrt(-4*a*c + b**2))**(1/4)*(-4*
a*c + b**2)**(3/2)) - 2**(1/4)*c**(1/4)*(b + sqrt(-4*a*c + b**2)/
4)*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/
4))/((-b - sqrt(-4*a*c + b**2))**(1/4)*(-4*a*c + b**2)**(3/2)) +
2**(1/4)*c**(1/4)*(b + sqrt(-4*a*c + b**2)/4)*atanh(2**(1/4)*c**(
1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/((-b - sqrt(-4*a*
c + b**2))**(1/4)*(-4*a*c + b**2)**(3/2)) - x**(3/2)*(b + 2*c*x**
2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.191147, size = 109, normalized size = 0.24

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{2\#1^4c\log\left(\sqrt{x}-\#1\right)-3b\log\left(\sqrt{x}-\#1\right)}{2\#1^5c+\#1b}\&\right]+\frac{4x^{3/2}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((4*x^(3/2)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#
1^4 + c*#1^8 & , (-3*b*Log[Sqrt[x] - #1] + 2*c*Log[Sqrt[x] - #1]*
#1^4)/(b*#1 + 2*c*#1^5) & ])/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.026, size = 121, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(1/2\frac{cx^{7/2}}{4ac-b^2}+1/4\frac{bx^{3/2}}{4ac-b^2}\right)+\frac{1}{8}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{2\_R^6c-3\_R^2b}{(4ac-b^2)\left(2\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(7/2)+1/4*b/(4*a*c-b^2)*x^(3/2))/(c*x^4+b*
x^2+a)+1/8*sum((2*_R^6*c-3*_R^2*b)/(4*a*c-b^2)/(2*_R^7*c+_R^3*b)*
ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\,cx^{\frac{7}{2}}+bx^{\frac{3}{2}}}{2\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)}+\int-\frac{2\,cx^{\frac{5}{2}}-3\,b\sqrt{x}}{4\left((b^2c-4\,ac^2)x^4+ab^2-4\,a^2c+(b^3-4\,abc)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*x^(7/2) + b*x^(3/2))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4
*a^2*c + (b^3 - 4*a*b*c)*x^2) + integrate(-1/4*(2*c*x^(5/2) - 3*b
*sqrt(x))/((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b
*c)*x^2), x)
```

Fricas [A] time = 2.22753, size = 12902, normalized size = 28.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2
 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b
^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^
4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*
b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b
^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8
*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4
096*a^7*c^6)))*arctan(1/2*sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c +
 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4
- 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 -
(27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16
*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b
^10*c^6 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*
a^10*b^4*c^9 - 49283072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((
6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c +
576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12902
4*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*
a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 +
760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 +
 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^
18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256
*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*
a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 -
24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b
^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a
^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36
*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^
10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4
*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 -
6144*a^6*b^2*c^5 + 4096*a^7*c^6))/((273375*b^8*c + 205200*a*b^6*c
^2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x)
- sqrt((74733890625*b^16*c^2 + 112193100000*a*b^14*c^3 + 68088600
000*a^2*b^12*c^4 + 20761920000*a^3*b^10*c^5 + 3063744000*a^4*b^8*
c^6 + 113909760*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 - 1179648*a^7*
b^2*c^9 + 65536*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c -
23678649000*a*b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a
^3*b^15*c^4 - 338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c
^6 - 183476674560*a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 7500595
20*a^8*b^5*c^9 + 190316544*a^9*b^3*c^10 - 7340032*a^10*b*c^11 - (
36905625*a*b^28*c - 1159839000*a^2*b^26*c^2 + 15854324400*a^3*b^2
4*c^3 - 122710429440*a^4*b^22*c^4 + 584418357504*a^5*b^20*c^5 - 1
728949905408*a^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 23179832
85248*a^8*b^14*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^
```

10*b^10*c^10 + 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6
*c^12 - 51220840448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 26
8435456*a^15*c^15)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^
2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 3
2256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589
824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(-
(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c +
240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6
*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^
2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c
^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6
- 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/
(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 38
40*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))))) - 4*((b^2*c
- 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqr
t(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24
*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c
^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*
c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 53
76*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 34406
4*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144
*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4
*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*
arctan(-1/2*sqrt(1/2)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^
11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*
b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*a^7*b*c^7 + (27*a*b^22 - 8
20*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57024*a^4*b^16*c^3 + 44544*a
^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 + 3843
6864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 -
49283072*a^11*b^2*c^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648
*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c
^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5
+ 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 -
262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c -
240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*
a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)
*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^
16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4
- 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 +
589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c
+ 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^
6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2
*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6
*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((
6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c +
576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 12902
4*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*
a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a
^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c
^5 + 4096*a^7*c^6))/((273375*b^8*c + 205200*a*b^6*c^2 + 47520*a^2
*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*c^5)*sqrt(x) - sqrt((747338
90625*b^16*c^2 + 112193100000*a*b^14*c^3 + 68088600000*a^2*b^12*c
^4 + 20761920000*a^3*b^10*c^5 + 3063744000*a^4*b^8*c^6 + 11390976
0*a^5*b^6*c^7 - 19021824*a^6*b^4*c^8 - 1179648*a^7*b^2*c^9 + 6553

6*a^8*c^10)*x - 1/2*sqrt(1/2)*(2989355625*b^21*c - 23678649000*a*
b^19*c^2 + 7135160400*a^2*b^17*c^3 + 277460328960*a^3*b^15*c^4 -
338956033536*a^4*b^13*c^5 - 492326940672*a^5*b^11*c^6 - 183476674
560*a^6*b^9*c^7 - 21980119040*a^7*b^7*c^8 + 750059520*a^8*b^5*c^9
 + 190316544*a^9*b^3*c^10 - 7340032*a^10*b*c^11 + (36905625*a*b^2
8*c - 1159839000*a^2*b^26*c^2 + 15854324400*a^3*b^24*c^3 - 122710
429440*a^4*b^22*c^4 + 584418357504*a^5*b^20*c^5 - 1728949905408*a
^6*b^18*c^6 + 2983008514048*a^7*b^16*c^7 - 2317983285248*a^8*b^14
*c^8 - 462348419072*a^9*b^12*c^9 + 1339972648960*a^10*b^10*c^10 +
 254402363392*a^11*b^8*c^11 - 161849802752*a^12*b^6*c^12 - 512208
40448*a^13*b^4*c^13 - 2550136832*a^14*b^2*c^14 + 268435456*a^15*c
^15)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^
3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*
c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^
7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(-(81*b^5 + 760*
a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c
^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 409
6*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 -
 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6
*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*
b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a
^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4
 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))))) - ((b^2*c - 4*a*c^2)*x^4
+ a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81
*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 24
0*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^
2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/
(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3
+ 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 -
589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*
b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*
a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1/2
)*(2187*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3
*b^9*c^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^
6*b^3*c^6 - 180224*a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 1006
4*a^3*b^18*c^2 - 57024*a^4*b^16*c^3 + 44544*a^5*b^14*c^4 + 150528
0*a^6*b^12*c^5 - 10838016*a^7*b^10*c^6 + 38436864*a^8*b^8*c^7 - 7
9233024*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49283072*a^11*b^2*c
^10 + 4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^
2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c
^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6
 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*
sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b
^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a
^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 64
8*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*
c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5
 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8
- 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 -
1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7
*c^6)))*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 2
4*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*
c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2
*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5

376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 3440
64*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 26214
4*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^
4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))
- (273375*b^8*c + 205200*a*b^6*c^2 + 47520*a^2*b^4*c^3 + 2304*a^3
*b^2*c^4 - 256*a^4*c^5)*sqrt(x)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2
- 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 +
760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*b^10*c + 240*a^3*b
^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 +
4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^
18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256
*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*
a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 -
24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/2)*(218
7*b^15 - 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c
^3 + 5177600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*
c^6 - 180224*a^7*b*c^7 - (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*
b^18*c^2 - 57024*a^4*b^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*
b^12*c^5 - 10838016*a^7*b^10*c^6 + 38436864*a^8*b^8*c^7 - 7923302
4*a^9*b^6*c^8 + 92012544*a^10*b^4*c^9 - 49283072*a^11*b^2*c^10 +
4194304*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^
2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 3
2256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589
824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(s
qrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 -
24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4
*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^
2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 -
5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344
064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 2621
44*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a
^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6))
)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 + (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 -
6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 1
6*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^
5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8
*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11
*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273
375*b^8*c + 205200*a*b^6*c^2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c
^4 - 256*a^4*c^5)*sqrt(x)) - ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a
^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*
b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2
- 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*
a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 3
6*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b
^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^
4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2
*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 -
6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*log(1/2*sqrt(1/2)*(2187*b^15
- 47412*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 51
77600*a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 1

```
80224*a^7*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^
2 - 57024*a^4*b^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^
5 - 10838016*a^7*b^10*c^6 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b
^6*c^8 + 92012544*a^10*b^4*c^9 - 49283072*a^11*b^2*c^10 + 4194304
*a^12*c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18
- 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^
6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9
*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2
)*sqrt(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*
b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 -
6144*a^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 1
6*a^2*c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^
5*b^12*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8
*b^6*c^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11
*c^9)))/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*
c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(
-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c
+ 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^
6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c
^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*
c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^
6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))
/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3
840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8
*c + 205200*a*b^6*c^2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 25
6*a^4*c^5)*sqrt(x)) + ((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c +
(b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^5 + 760*a*b^3*c -
 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280
*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6
)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a^3*b
^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10*c^4
 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c^7 +
 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a
^6*b^2*c^5 + 4096*a^7*c^6)))*log(-1/2*sqrt(1/2)*(2187*b^15 - 4741
2*a*b^13*c + 423536*a^2*b^11*c^2 - 1990720*a^3*b^9*c^3 + 5177600*
a^4*b^7*c^4 - 7052288*a^5*b^5*c^5 + 3985408*a^6*b^3*c^6 - 180224*
a^7*b*c^7 + (27*a*b^22 - 820*a^2*b^20*c + 10064*a^3*b^18*c^2 - 57
024*a^4*b^16*c^3 + 44544*a^5*b^14*c^4 + 1505280*a^6*b^12*c^5 - 10
838016*a^7*b^10*c^6 + 38436864*a^8*b^8*c^7 - 79233024*a^9*b^6*c^8
 + 92012544*a^10*b^4*c^9 - 49283072*a^11*b^2*c^10 + 4194304*a^12*
c^11)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a^2*b^18 - 36*a
^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 + 32256*a^6*b^10
*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 589824*a^9*b^4*c
^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))*sqrt(sqrt(1/2)*sqrt
(-(81*b^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c
 + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a
^6*b^2*c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*
c^2)/(a^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12
*c^3 + 32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c
^6 - 589824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9))
)/(a*b^12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 +
3840*a^5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)))*sqrt(-(81*b
^5 + 760*a*b^3*c - 240*a^2*b*c^2 - (a*b^12 - 24*a^2*b^10*c + 240*
```

```
a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^5*b^4*c^4 - 6144*a^6*b^2*
c^5 + 4096*a^7*c^6)*sqrt((6561*b^4 - 648*a*b^2*c + 16*a^2*c^2)/(a
^2*b^18 - 36*a^3*b^16*c + 576*a^4*b^14*c^2 - 5376*a^5*b^12*c^3 +
32256*a^6*b^10*c^4 - 129024*a^7*b^8*c^5 + 344064*a^8*b^6*c^6 - 58
9824*a^9*b^4*c^7 + 589824*a^10*b^2*c^8 - 262144*a^11*c^9)))/(a*b^
12 - 24*a^2*b^10*c + 240*a^3*b^8*c^2 - 1280*a^4*b^6*c^3 + 3840*a^
5*b^4*c^4 - 6144*a^6*b^2*c^5 + 4096*a^7*c^6)) - (273375*b^8*c + 2
05200*a*b^6*c^2 + 47520*a^2*b^4*c^3 + 2304*a^3*b^2*c^4 - 256*a^4*
c^5)*sqrt(x)) + 4*(2*c*x^3 + b*x)*sqrt(x))/((b^2*c - 4*a*c^2)*x^4
 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{\left(cx^4 + bx^2 + a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1076 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=442

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
(c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 -
4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
 + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a
*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^
2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))
```

Rubi [A] time = 1.37225, antiderivative size = 442, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{4b}{\sqrt{b^2-4ac}}+3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(3-\frac{4b}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) +
(c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^2 -
4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])
^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
 + (c^(3/4)*(3 + (4*b)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a
*c)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3 - (4*b)/Sqrt[b^
2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*
a*c])^(1/4)])/(2*2^(1/4)*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(
3/4))
```

Rubi in Sympy [A] time = 143.845, size = 401, normalized size = 0.91

$$-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-\frac{3\sqrt{-4ac+b^2}}{4}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}(-4ac+b^2)^{\frac{3}{2}}}-\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b-\frac{3\sqrt{-4ac+b^2}}{4}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b+\sqrt{-4ac+b^2}}}\right)}{\left(-b+\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}(-4ac+b^2)^{\frac{3}{2}}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b+\frac{3\sqrt{-4ac+b^2}}{4}\right)\operatorname{atan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}(-4ac+b^2)^{\frac{3}{2}}}+\frac{2^{\frac{3}{4}}c^{\frac{3}{4}}\left(b+\frac{3\sqrt{-4ac+b^2}}{4}\right)\operatorname{atanh}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-b-\sqrt{-4ac+b^2}}}\right)}{\left(-b-\sqrt{-4ac+b^2}\right)^{\frac{3}{4}}(-4ac+b^2)^{\frac{3}{2}}}-\frac{\sqrt{x}\left(b+2cx^2\right)}{2\left(-4ac+b^2\right)\left(a+bx^2+cx^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -2**(3/4)*c**(3/4)*(b - 3*sqrt(-4*a*c + b**2)/4)*atan(2**(1/4)*c*
*(1/4)*sqrt(x)/(-b + sqrt(-4*a*c + b**2))**(1/4))/((-b + sqrt(-4*
a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3/2)) - 2**(3/4)*c**(3/4)*(
b - 3*sqrt(-4*a*c + b**2)/4)*atanh(2**(1/4)*c**(1/4)*sqrt(x)/(-b
+ sqrt(-4*a*c + b**2))**(1/4))/((-b + sqrt(-4*a*c + b**2))**(3/4)
*(-4*a*c + b**2)**(3/2)) + 2**(3/4)*c**(3/4)*(b + 3*sqrt(-4*a*c +
 b**2)/4)*atan(2**(1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2
))**(1/4))/((-b - sqrt(-4*a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3
/2)) + 2**(3/4)*c**(3/4)*(b + 3*sqrt(-4*a*c + b**2)/4)*atanh(2**(
1/4)*c**(1/4)*sqrt(x)/(-b - sqrt(-4*a*c + b**2))**(1/4))/((-b - s
qrt(-4*a*c + b**2))**(3/4)*(-4*a*c + b**2)**(3/2)) - sqrt(x)*(b +
 2*c*x**2)/(2*(-4*a*c + b**2)*(a + b*x**2 + c*x**4))
```

Mathematica [C] time = 0.25775, size = 111, normalized size = 0.25

$$-\frac{\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{6\#1^4c\log\left(\sqrt{x}-\#1\right)-b\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+\frac{4\sqrt{x}\left(b+2cx^2\right)}{a+bx^2+cx^4}}{8\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -((4*Sqrt[x]*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#
1^4 + c*#1^8 & , (-(b*Log[Sqrt[x] - #1]) + 6*c*Log[Sqrt[x] - #1]*
#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(8*(b^2 - 4*a*c))
```

Maple [C] time = 0.025, size = 118, normalized size = 0.3

$$2\,\frac{1}{cx^4+bx^2+a}\left(1/2\,\frac{cx^{5/2}}{4\,ac-b^2}+1/4\,\frac{b\sqrt{x}}{4\,ac-b^2}\right)+\frac{1}{8}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{6\,\_R^4c-b}{\left(4\,ac-b^2\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(1/2*c/(4*a*c-b^2)*x^(5/2)+1/4*b/(4*a*c-b^2)*x^(1/2))/(c*x^4+b*
x^2+a)+1/8*sum((6*_R^4*c-b)/(4*a*c-b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1
/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^{\frac{9}{2}}+\left(b^2-2\,ac\right)x^{\frac{5}{2}}}{2\left((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2\right)}+\int-\frac{bcx^{\frac{7}{2}}+\left(b^2+6\,ac\right)x^{\frac{3}{2}}}{4\left((ab^2c-4\,a^2c^2)x^4+a^2b^2-4\,a^3c+(ab^3-4\,a^2bc)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(9/2) + (b^2 - 2*a*c)*x^(5/2))/((a*b^2*c - 4*a^2*c^2)*
x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + integrate(-1
/4*(b*c*x^(7/2) + (b^2 + 6*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*x
^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2), x)
```

Fricas [A] time = 1.35533, size = 10425, normalized size = 23.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 +
 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1
280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*
c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^
3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2
- 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 +
 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
- 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2
- 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a
^9*c^6)))*arctan(-1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*
a^3*b^3*c^3 + 5184*a^4*b*c^4 - (a^3*b^14 - 12*a^4*b^12*c - 48*a^5
*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*
c^5 - 69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c
+ 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^1
8 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*
a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 58982
4*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sq
rt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^
3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8
 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^1
2*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b
^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*
c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6
*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))/((7*
b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x)
 + sqrt((49*b^12*c^2 + 3150*a*b^10*c^3 + 95985*a^2*b^8*c^4 + 1621
296*a^3*b^6*c^5 + 15746400*a^4*b^4*c^6 + 75582720*a^5*b^2*c^7 + 1
36048896*a^6*c^8)*x + 1/2*sqrt(1/2)*(b^18 + 52*a*b^16*c + 1269*a^
2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48608*a^4*b^10*c^4 - 679392*a^5
*b^8*c^5 - 4209408*a^6*b^6*c^6 - 4105728*a^7*b^4*c^7 + 214990848*
a^8*b^2*c^8 - 483729408*a^9*c^9 - (a^3*b^23 + 7*a^4*b^21*c - 152*
a^5*b^19*c^2 - 2960*a^6*b^17*c^3 + 44032*a^7*b^15*c^4 + 60928*a^8
*b^13*c^5 - 4444160*a^9*b^11*c^6 + 36855808*a^10*b^9*c^7 - 153681
920*a^11*b^7*c^8 + 363528192*a^12*b^5*c^9 - 467140608*a^13*b^3*c^
10 + 254803968*a^14*b*c^11)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4
*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^1
6*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4
- 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7
 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(-(b^7 + 21*a*b^5
*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
```

^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*
c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16
*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 -
 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7
+ 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^1
0*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 614
4*a^8*b^2*c^5 + 4096*a^9*c^6))))) - 4*((b^2*c - 4*a*c^2)*x^4 + a*
b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^7 +
21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^
4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
- 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*
a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*
a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^
10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*
b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24
*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c
^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*arctan(1/2*(b^9 + 19*a*b^
7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^4 + (a^3*
b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^3 - 11520
*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 49152*a^10
*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c
^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2
 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5
+ 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8
 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 16
8*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10*c + 240*
a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*
c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 1
7496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 57
6*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024
*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 58982
4*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 2
40*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b
^2*c^5 + 4096*a^9*c^6)))/((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2
*c^3 + 11664*a^3*c^4)*sqrt(x) + sqrt((49*b^12*c^2 + 3150*a*b^10*c
^3 + 95985*a^2*b^8*c^4 + 1621296*a^3*b^6*c^5 + 15746400*a^4*b^4*c
^6 + 75582720*a^5*b^2*c^7 + 136048896*a^6*c^8)*x + 1/2*sqrt(1/2)*
(b^18 + 52*a*b^16*c + 1269*a^2*b^14*c^2 + 14294*a^3*b^12*c^3 + 48
608*a^4*b^10*c^4 - 679392*a^5*b^8*c^5 - 4209408*a^6*b^6*c^6 - 410
5728*a^7*b^4*c^7 + 214990848*a^8*b^2*c^8 - 483729408*a^9*c^9 + (a
^3*b^23 + 7*a^4*b^21*c - 152*a^5*b^19*c^2 - 2960*a^6*b^17*c^3 + 4
4032*a^7*b^15*c^4 + 60928*a^8*b^13*c^5 - 4444160*a^9*b^11*c^6 + 3
6855808*a^10*b^9*c^7 - 153681920*a^11*b^7*c^8 + 363528192*a^12*b^
5*c^9 - 467140608*a^13*b^3*c^10 + 254803968*a^14*b*c^11)*sqrt((b^
8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^
4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^
12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*
b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15
*c^9)))*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^
3 - (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^
3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8
 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4
*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^1
2*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b

```
^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*
c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6
*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6))))) +
((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*s
qrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^
3*b*c^3 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*
b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqr
t((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 1049
76*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a
^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*
a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144
*a^15*c^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a
^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6))
)*log((7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4
)*sqrt(x) + 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^
3*c^3 + 5184*a^4*b*c^4 - (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*
c^2 + 1600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 -
69632*a^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36
*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b
^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2
)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a
^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 38
40*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*
a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/
(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3
+ 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6
 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))
/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 +
 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) - ((b^2*c
 - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqr
t(1/2)*sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3
 + (a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8
+ 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*
c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12
*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^
6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c
^9)))/(a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*
c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((
7*b^6*c + 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(
x) - 1/2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 +
 5184*a^4*b*c^4 - (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1
600*a^6*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a
^9*b^2*c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^
4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^
16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4
 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^
7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(
-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 + (a^3*b^12
 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*
b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c
 + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^
```

18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256
*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 5898
24*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b
^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a
^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) + ((b^2*c - 4*a*
c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*
sqrt(-(b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3
*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840
*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*
b^6*c + 1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a
^6*b^18 - 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 +
32256*a^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 -
589824*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(
a^3*b^12 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3
840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c
+ 225*a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) + 1/
2*(b^9 + 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a
^4*b*c^4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6
*b^8*c^3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*
c^6 + 49152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 +
17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c +
576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 1290
24*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589
824*a^14*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 +
21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a
^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4
- 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377
*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36
*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b
^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13
*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 2
4*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*
c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))) - ((b^2*c - 4*a*c^2)*x^
4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(
b^7 + 21*a*b^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 -
24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^
4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c +
1377*a^2*b^4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18
- 36*a^7*b^16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a
^10*b^10*c^4 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824
*a^13*b^4*c^7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^1
2 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5 + 4096*a^9*c^6)))*log((7*b^6*c + 225*
a*b^4*c^2 + 3240*a^2*b^2*c^3 + 11664*a^3*c^4)*sqrt(x) - 1/2*(b^9
+ 19*a*b^7*c + 124*a^2*b^5*c^2 - 2160*a^3*b^3*c^3 + 5184*a^4*b*c^
4 + (a^3*b^14 - 12*a^4*b^12*c - 48*a^5*b^10*c^2 + 1600*a^6*b^8*c^
3 - 11520*a^7*b^6*c^4 + 39936*a^8*b^4*c^5 - 69632*a^9*b^2*c^6 + 4
9152*a^10*c^7)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^4*c^2 + 17496*
a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^16*c + 576*a^8
*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4 - 129024*a^11
*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^7 + 589824*a^1
4*b^2*c^8 - 262144*a^15*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^7 + 21*a*b
^5*c + 168*a^2*b^3*c^2 + 3024*a^3*b*c^3 - (a^3*b^12 - 24*a^4*b^10
*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144

```
*a^8*b^2*c^5 + 4096*a^9*c^6)*sqrt((b^8 + 54*a*b^6*c + 1377*a^2*b^
4*c^2 + 17496*a^3*b^2*c^3 + 104976*a^4*c^4)/(a^6*b^18 - 36*a^7*b^
16*c + 576*a^8*b^14*c^2 - 5376*a^9*b^12*c^3 + 32256*a^10*b^10*c^4
 - 129024*a^11*b^8*c^5 + 344064*a^12*b^6*c^6 - 589824*a^13*b^4*c^
7 + 589824*a^14*b^2*c^8 - 262144*a^15*c^9)))/(a^3*b^12 - 24*a^4*b
^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6
144*a^8*b^2*c^5 + 4096*a^9*c^6)))) + 4*(2*c*x^2 + b)*sqrt(x))/((b
^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^4 + bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1077 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=489

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) + (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*
(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c
)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*a*c)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]
)/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 1.9734, antiderivative size = 489, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(b-\frac{b^2-20ac}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{b^2-20ac}{\sqrt{b^2-4ac}}+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a\left(b^2-4ac\right)\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) + (c^(1/4)*(b - (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*
(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sq
rt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c
)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (c^(1/4)*(b - (b^2 - 20*a*c)/
Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(b + (b^2 - 20*a*c)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)]
)/(4*2^(3/4)*a*(b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.208369, size = 149, normalized size = 0.3

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{\#1^4bc\log(\sqrt{x}-\#1)-10ac\log(\sqrt{x}-\#1)+b^2\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]+4x^{3/2}\left(-2ac+b^2+bcx^2\right)}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(4*x^(3/2)*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum
[a + b*#1^4 + c*#1^8 & , (b^2*Log[Sqrt[x] - #1] - 10*a*c*Log[Sqrt
[x] - #1] + b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(8
*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4))
```

Maple [C] time = 0.077, size = 149, normalized size = 0.3

$$2\frac{1}{cx^4+bx^2+a}\left(-1/4\frac{bcx^{7/2}}{a(4ac-b^2)}+1/4\frac{(2ac-b^2)x^{3/2}}{a(4ac-b^2)}\right)+\frac{1}{8a}\sum_{\_R=RootOf(\_Z^8c+\_Z^4b+a)}\frac{-bc\_R^6+(10ac-b^2)\_R^2}{(4ac-b^2)(2\_R^7c+\_R^3b)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(7/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x
^(3/2))/(c*x^4+b*x^2+a)+1/8/a*sum((-b*c*_R^6+(10*a*c-b^2)*_R^2)/(
4*a*c-b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4
*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{bcx^{\frac{7}{2}}+(b^2-2ac)x^{\frac{3}{2}}}{2((ab^2c-4a^2c^2)x^4+a^2b^2-4a^3c+(ab^3-4a^2bc)x^2)}-\int-\frac{bcx^{\frac{5}{2}}+(b^2-10ac)\sqrt{x}}{4((ab^2c-4a^2c^2)x^4+a^2b^2-4a^3c+(ab^3-4a^2bc)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x^(7/2) + (b^2 - 2*a*c)*x^(3/2))/((a*b^2*c - 4*a^2*c^2)*
x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) - integrate(-1
/4*(b*c*x^(5/2) + (b^2 - 10*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*
x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2), x)
```

Fricas [A] time = 10.3935, size = 16226, normalized size = 33.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 -
 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2
*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2
571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 262500
0*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 57
6*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 1290
24*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589
824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5 + 4096*a^11*c^6)))*arctan(-1/2*sqrt(1/2)*(b^22 - 91*a*b
^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14
*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072
*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 +
4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (a^5*b^25 - 70*
a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*
b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 11447
5008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7*
c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 41943
0400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 4
5950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 625
0000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5
376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 3
44064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 -
262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a
^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*
a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^
4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c +
 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625
000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c +
576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 12
9024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 5
89824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c
```

+ 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a
^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*
b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 25
71*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000
*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576
*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 12902
4*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 5898
24*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c +
240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5 + 4096*a^11*c^6))/((729*b^12*c^4 - 52731*a*b^10*c^5 + 16
00425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8
- 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x) + sqrt((5
31441*b^24*c^8 - 76881798*a*b^22*c^9 + 5113978011*a^2*b^20*c^10 -
206852401350*a^3*b^18*c^11 + 5667080000625*a^4*b^16*c^12 - 11079
2866500000*a^5*b^14*c^13 + 1584936775000000*a^6*b^12*c^14 - 16715
805000000000*a^7*b^10*c^15 + 128988375000000000*a^8*b^8*c^16 - 71
0150000000000000*a^9*b^6*c^17 + 2647500000000000000*a^10*b^4*c^18
- 6000000000000000000*a^11*b^2*c^19 + 6250000000000000000*a^12*c
^20)*x - 1/2*sqrt(1/2)*(6561*b^31*c^5 - 1032993*a*b^29*c^6 + 7563
4965*a^2*b^27*c^7 - 3414264975*a^3*b^25*c^8 + 106186248955*a^4*b^
23*c^9 - 2407919378459*a^5*b^21*c^10 + 41083864936232*a^6*b^19*c^
11 - 536376931701360*a^7*b^17*c^12 + 5394460343808000*a^8*b^15*c^
13 - 41720627697600000*a^9*b^13*c^14 + 245614092480000000*a^10*b^
11*c^15 - 1078472304000000000*a^11*b^9*c^16 + 3410524800000000000
*a^12*b^7*c^17 - 7314160000000000000*a^13*b^5*c^18 + 948800000000
0000000*a^14*b^3*c^19 - 5600000000000000000*a^15*b*c^20 - (6561*a
^5*b^34*c^5 - 895212*a^6*b^32*c^6 + 56697732*a^7*b^30*c^7 - 22120
69617*a^8*b^28*c^8 + 59497163992*a^9*b^26*c^9 - 1169816993840*a^1
0*b^24*c^10 + 17397456159488*a^11*b^22*c^11 - 199763116583168*a^1
2*b^20*c^12 + 1791922585643008*a^13*b^18*c^13 - 12624164431147008
*a^14*b^16*c^14 + 69835076189159424*a^15*b^14*c^15 - 301610411758
387200*a^16*b^12*c^16 + 1004700278784000000*a^17*b^10*c^17 - 2527
971917824000000*a^18*b^8*c^18 + 4641908326400000000*a^19*b^6*c^19
- 5864652800000000000*a^20*b^4*c^20 + 4554752000000000000*a^21*b
^2*c^21 - 1638400000000000000*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c
+ 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 26
25000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c
+ 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 -
129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 +
589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c
+ 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^
12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^
9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*
b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^
4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b
^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*
c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4
*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5 + 4096*a^11*c^6))))) - 4*((a*b^2*c - 4*a^2*c^
2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1
/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3

+ 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 -
 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a
^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*
b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*
c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*
b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^1
6*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^
19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*
b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))
*arctan(1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 8
7230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5
+ 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8
*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 25
60000000*a^11*c^11 + (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^
2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*
c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 36195532
8*a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^1
0 - 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12
- 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4
*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36
*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^1
4*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a
^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(
1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3
 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2
- 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*
a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3
*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6
*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13
*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^
16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a
^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6))
)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 +
18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^1
1*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^
6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^
6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^
12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*
b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19
*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^
6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6))/((
729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*
a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 25
00000000*a^6*c^10)*sqrt(x) + sqrt((531441*b^24*c^8 - 76881798*a*b
^22*c^9 + 5113978011*a^2*b^20*c^10 - 206852401350*a^3*b^18*c^11 +
 5667080000625*a^4*b^16*c^12 - 110792866500000*a^5*b^14*c^13 + 15
84936775000000*a^6*b^12*c^14 - 16715805000000000*a^7*b^10*c^15 +
128988375000000000*a^8*b^8*c^16 - 710150000000000000*a^9*b^6*c^17
 + 2647500000000000000*a^10*b^4*c^18 - 6000000000000000000*a^11*b
^2*c^19 + 6250000000000000000*a^12*c^20)*x - 1/2*sqrt(1/2)*(6561*
b^31*c^5 - 1032993*a*b^29*c^6 + 75634965*a^2*b^27*c^7 - 341426497
5*a^3*b^25*c^8 + 106186248955*a^4*b^23*c^9 - 2407919378459*a^5*b^

21*c^10 + 41083864936232*a^6*b^19*c^11 - 536376931701360*a^7*b^17
*c^12 + 5394460343808000*a^8*b^15*c^13 - 41720627697600000*a^9*b^
13*c^14 + 245614092480000000*a^10*b^11*c^15 - 1078472304000000000
*a^11*b^9*c^16 + 3410524800000000000*a^12*b^7*c^17 - 731416000000
0000000*a^13*b^5*c^18 + 9488000000000000000*a^14*b^3*c^19 - 56000
00000000000000*a^15*b*c^20 + (6561*a^5*b^34*c^5 - 895212*a^6*b^32
*c^6 + 56697732*a^7*b^30*c^7 - 2212069617*a^8*b^28*c^8 + 59497163
992*a^9*b^26*c^9 - 1169816993840*a^10*b^24*c^10 + 17397456159488*
a^11*b^22*c^11 - 199763116583168*a^12*b^20*c^12 + 179192258564300
8*a^13*b^18*c^13 - 12624164431147008*a^14*b^16*c^14 + 69835076189
159424*a^15*b^14*c^15 - 301610411758387200*a^16*b^12*c^16 + 10047
00278784000000*a^17*b^10*c^17 - 2527971917824000000*a^18*b^8*c^18
 + 4641908326400000000*a^19*b^6*c^19 - 5864652800000000000*a^20*b
^4*c^20 + 4554752000000000000*a^21*b^2*c^21 - 1638400000000000000
*a^22*c^22)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a
^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a
^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^
13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*
a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144
*a^19*c^9)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3
*b^3*c^3 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*
b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5
 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 4
5950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 625
0000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5
376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 3
44064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 -
262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 -
1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^
11*c^6))))) - ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a
*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 7
65*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 -
 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^
4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10
*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 -
2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*
c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4
- 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7
 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^
10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 61
44*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(1/2*sqrt(1/2)*(b^22 - 91*a
*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^
14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 4976510
72*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9
+ 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (a^5*b^25 - 7
0*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^
9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 114
475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^
7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419
430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 -
 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6
250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 -
 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 +
 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8

- 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x)) + ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(-1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11 - (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*

c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*
c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2
*c^8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8
*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 +
4096*a^11*c^6)))*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880
*a^3*b^3*c^3 + 18000*a^4*b*c^4 + (a^5*b^12 - 24*a^6*b^10*c + 240*
a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2
*c^5 + 4096*a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2
 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 +
 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2
 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5
 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^
8 - 262144*a^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 409
6*a^11*c^6)) + (729*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8
*c^6 - 26110000*a^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*
a^5*b^2*c^9 + 2500000000*a^6*c^10)*sqrt(x)) - ((a*b^2*c - 4*a^2*c
^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(
1/2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3
 + 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2
- 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*
a^11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3
*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6
*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13
*b^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^
16*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a
^19*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8
*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6))
)*log(1/2*sqrt(1/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 872
30*a^3*b^16*c^3 + 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 +
100253344*a^6*b^10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b
^6*c^8 - 3627264000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560
000000*a^11*c^11 + (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2
- 40672*a^8*b^19*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^
5 + 25976832*a^11*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*
a^13*b^9*c^8 - 802029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10
- 1046478848*a^16*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 -
78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b
^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a
^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*
b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^1
7*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/
2)*sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 +
 18000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 -
1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^
11*c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b
^6*c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c
^6)/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b
^12*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16
*b^6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^1
9*c^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b
^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*
sqrt(-(b^9 - 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18
000*a^4*b*c^4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 128

0*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*
c^6)*sqrt((b^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*
c^3 + 470625*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)
/(a^10*b^18 - 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12
*c^3 + 32256*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^
6*c^6 - 589824*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c
^9)))/(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*
c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (7
29*b^12*c^4 - 52731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a
^3*b^6*c^7 + 241500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 250
0000000*a^6*c^10)*sqrt(x)) + ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(b^9 -
 45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^
4 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^
3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b
^12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625
*a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18
- 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256
*a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 5898
24*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b
^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a
^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*log(-1/2*sqrt(1
/2)*(b^22 - 91*a*b^20*c + 3683*a^2*b^18*c^2 - 87230*a^3*b^16*c^3
+ 1338850*a^4*b^14*c^4 - 13940024*a^5*b^12*c^5 + 100253344*a^6*b^
10*c^6 - 497651072*a^7*b^8*c^7 + 1672046080*a^8*b^6*c^8 - 3627264
000*a^9*b^4*c^9 + 4582400000*a^10*b^2*c^10 - 2560000000*a^11*c^11
 + (a^5*b^25 - 70*a^6*b^23*c + 2192*a^7*b^21*c^2 - 40672*a^8*b^19
*c^3 + 498432*a^9*b^17*c^4 - 4254720*a^10*b^15*c^5 + 25976832*a^1
1*b^13*c^6 - 114475008*a^12*b^11*c^7 + 361955328*a^13*b^9*c^8 - 8
02029568*a^14*b^7*c^9 + 1183842304*a^15*b^5*c^10 - 1046478848*a^1
6*b^3*c^11 + 419430400*a^17*b*c^12)*sqrt((b^12 - 78*a*b^10*c + 25
71*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4*b^4*c^4 - 2625000
*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36*a^11*b^16*c + 576
*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^14*b^10*c^4 - 12902
4*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a^17*b^4*c^7 + 5898
24*a^18*b^2*c^8 - 262144*a^19*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b^9 -
45*a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4
 - (a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^
12 - 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*
a^4*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 -
 36*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*
a^14*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 58982
4*a^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^
12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^
9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)))*sqrt(-(b^9 - 45*
a*b^7*c + 765*a^2*b^5*c^2 - 5880*a^3*b^3*c^3 + 18000*a^4*b*c^4 -
(a^5*b^12 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 +
3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)*sqrt((b^12
- 78*a*b^10*c + 2571*a^2*b^8*c^2 - 45950*a^3*b^6*c^3 + 470625*a^4
*b^4*c^4 - 2625000*a^5*b^2*c^5 + 6250000*a^6*c^6)/(a^10*b^18 - 36
*a^11*b^16*c + 576*a^12*b^14*c^2 - 5376*a^13*b^12*c^3 + 32256*a^1
4*b^10*c^4 - 129024*a^15*b^8*c^5 + 344064*a^16*b^6*c^6 - 589824*a
^17*b^4*c^7 + 589824*a^18*b^2*c^8 - 262144*a^19*c^9)))/(a^5*b^12

```
- 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b
^4*c^4 - 6144*a^10*b^2*c^5 + 4096*a^11*c^6)) + (729*b^12*c^4 - 52
731*a*b^10*c^5 + 1600425*a^2*b^8*c^6 - 26110000*a^3*b^6*c^7 + 241
500000*a^4*b^4*c^8 - 1200000000*a^5*b^2*c^9 + 2500000000*a^6*c^10
)*sqrt(x)) - 4*(b*c*x^3 + (b^2 - 2*a*c)*x)*sqrt(x))/((a*b^2*c - 4
*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2+a)**2,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(cx^4 + bx^2 + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")

[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^2, x)
```

3.1078 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=503

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) + (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcT
an[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (
c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b
^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3*b
^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)
^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c
 + 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
 Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b
+ Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 2.37338, antiderivative size = 503, normalized size of antiderivative = 1., number of

steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(-3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(3b\sqrt{b^2-4ac}-28ac+3b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\sqrt[4]{2}a\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)) + (c^(3/4)*(3*b^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcT
an[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*
2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (
c^(3/4)*(3*b^2 - 28*a*c + 3*b*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b
^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(3*b
^2 - 28*a*c - 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqr
t[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)
^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(3*b^2 - 28*a*c
 + 3*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b +
 Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^(1/4)*a*(b^2 - 4*a*c)^(3/2)*(-b
+ Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.275197, size = 153, normalized size = 0.3

$$-\frac{\left(a+bx^2+cx^4\right)\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{3\#1^4bc\log\left(\sqrt{x}-\#1\right)-14ac\log\left(\sqrt{x}-\#1\right)+3b^2\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]+4\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{8a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(4*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2) + (a + b*x^2 + c*x^4)*RootSum
[a + b*#1^4 + c*#1^8 & , (3*b^2*Log[Sqrt[x] - #1] - 14*a*c*Log[Sq
rt[x] - #1] + 3*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) &
 ])/(8*a*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4))
```

Maple [C] time = 0.026, size = 144, normalized size = 0.3

$$2\,\frac{1}{cx^4+bx^2+a}\left(-1/4\,\frac{bcx^{5/2}}{a\left(4\,ac-b^2\right)}+1/4\,\frac{\left(2\,ac-b^2\right)\sqrt{x}}{a\left(4\,ac-b^2\right)}\right)+\frac{1}{8\,a}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{-3\,\_R^4bc+14\,ac-3\,b^2}{\left(4\,ac-b^2\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 2*(-1/4*b/a/(4*a*c-b^2)*c*x^(5/2)+1/4*(2*a*c-b^2)/a/(4*a*c-b^2)*x
^(1/2))/(c*x^4+b*x^2+a)+1/8/a*sum((-3*_R^4*b*c+14*a*c-3*b^2)/(4*a
*c-b^2)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+
a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,b^2c-14\,ac^2\right)x^{\frac{9}{2}}+\left(3\,b^3-13\,abc\right)x^{\frac{5}{2}}+4\left(ab^2-4\,a^2c\right)\sqrt{x}}{2\,(a^3b^2-4\,a^4c+(a^2b^2c-4\,a^3c^2)x^4+(a^2b^3-4\,a^3bc)x^2)}-\int\frac{\left(3\,b^2c-14\,ac^2\right)x^{\frac{7}{2}}+\left(3\,b^3-17\,abc\right)x^{\frac{3}{2}}}{4\,(a^3b^2-4\,a^4c+(a^2b^2c-4\,a^3c^2)x^4+(a^2b^3-4\,a^3bc)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*sqrt(x)),x, algorithm="maxima")
```

```
[Out] 1/2*((3*b^2*c - 14*a*c^2)*x^(9/2) + (3*b^3 - 13*a*b*c)*x^(5/2) +
4*(a*b^2 - 4*a^2*c)*sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*
a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((3*b^2
*c - 14*a*c^2)*x^(7/2) + (3*b^3 - 17*a*b*c)*x^(3/2))/(a^3*b^2 - 4
*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)
, x)
```

Fricas [A] time = 5.13735, size = 13199, normalized size = 26.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*sqrt(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 -
4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20
790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 18110
4*a^5*b*c^5 + (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*
a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*
c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 3
8499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*
c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^
8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^1
7*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a
^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*
a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^
10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^
6)))*arctan(-1/2*(243*b^14 - 7857*a*b^12*c + 105732*a^2*b^10*c^2
- 760311*a^3*b^8*c^3 + 3104898*a^4*b^6*c^4 - 6982136*a^5*b^4*c^5
+ 7430752*a^6*b^2*c^6 - 2151296*a^7*c^7 - (3*a^7*b^15 - 92*a^8*b^
13*c + 1200*a^9*b^11*c^2 - 8640*a^10*b^9*c^3 + 37120*a^11*b^7*c^4
 - 95232*a^12*b^5*c^5 + 135168*a^13*b^3*c^6 - 81920*a^14*b*c^7)*s
qrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 3849946
2*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 +
```

898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)
/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12
*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^
6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c
^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^
7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^
5 + (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c
^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt(
(6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^
3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 8987
83137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^
14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3
 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^
6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9))
)/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3
 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)))/((267
3*b^10*c^2 - 68445*a*b^8*c^3 + 666846*a^2*b^6*c^4 - 2974545*a^3*b
^4*c^5 + 5474280*a^4*b^2*c^6 - 1882384*a^5*c^7)*sqrt(x) - sqrt((7
144929*b^20*c^4 - 365906970*a*b^18*c^5 + 8249676741*a^2*b^16*c^6
- 107186466510*a^3*b^14*c^7 + 881134553646*a^4*b^12*c^8 - 4726564
284204*a^5*b^10*c^9 + 16406600944545*a^6*b^8*c^10 - 3507750488692
8*a^7*b^6*c^11 + 41166213348960*a^8*b^4*c^12 - 20609394167040*a^9
*b^2*c^13 + 3543369523456*a^10*c^14)*x + 1/2*sqrt(1/2)*(59049*b^2
8 - 3818502*a*b^26*c + 112685175*a^2*b^24*c^2 - 2006183214*a^3*b^
22*c^3 + 23996183904*a^4*b^20*c^4 - 203172719904*a^5*b^18*c^5 + 1
249344303669*a^6*b^16*c^6 - 5630371599204*a^7*b^14*c^7 + 18522020
088594*a^8*b^12*c^8 - 43727956655856*a^9*b^10*c^9 + 7171140007292
8*a^10*b^8*c^10 - 77372920636928*a^11*b^6*c^11 + 50337719029248*a
^12*b^4*c^12 - 17166378299392*a^13*b^2*c^13 + 2314037239808*a^14*
c^14 - (729*a^7*b^29 - 45927*a^8*b^27*c + 1331640*a^9*b^25*c^2 -
23536197*a^10*b^23*c^3 + 283046346*a^11*b^21*c^4 - 2447287920*a^1
2*b^19*c^5 + 15665468896*a^13*b^17*c^6 - 75268700672*a^14*b^15*c^
7 + 272035456000*a^15*b^13*c^8 - 733077069824*a^16*b^11*c^9 + 144
1307017216*a^17*b^9*c^10 - 1985616084992*a^18*b^7*c^11 + 17812486
88128*a^19*b^5*c^12 - 899513581568*a^20*b^3*c^13 + 176234168320*a
^21*b*c^14)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*
c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a
^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 9223
6816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5
376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 3
44064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 -
262144*a^23*c^9)))*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*
c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5
+ (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3
 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6
561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*
b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783
137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14
*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 +
 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6
- 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/
(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 +
 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6))))) - 4*(
(a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*

c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*
b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*
c^5 - (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6
*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqr
t((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*
a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 89
8783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(
a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c
^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*
c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9
)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c
^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)))*arc
tan(1/2*(243*b^14 - 7857*a*b^12*c + 105732*a^2*b^10*c^2 - 760311*
a^3*b^8*c^3 + 3104898*a^4*b^6*c^4 - 6982136*a^5*b^4*c^5 + 7430752
*a^6*b^2*c^6 - 2151296*a^7*c^7 + (3*a^7*b^15 - 92*a^8*b^13*c + 12
00*a^9*b^11*c^2 - 8640*a^10*b^9*c^3 + 37120*a^11*b^7*c^4 - 95232*
a^12*b^5*c^5 + 135168*a^13*b^3*c^6 - 81920*a^14*b*c^7)*sqrt((6561
*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^1
0*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137
*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^
18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32
256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 5
89824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))*sqr
t(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 1
00023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 - (a^7*
b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^1
6 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^
3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6
*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 -
 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*
a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 58982
4*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^
12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)))/((2673*b^10*c^
2 - 68445*a*b^8*c^3 + 666846*a^2*b^6*c^4 - 2974545*a^3*b^4*c^5 +
5474280*a^4*b^2*c^6 - 1882384*a^5*c^7)*sqrt(x) - sqrt((7144929*b^
20*c^4 - 365906970*a*b^18*c^5 + 8249676741*a^2*b^16*c^6 - 1071864
66510*a^3*b^14*c^7 + 881134553646*a^4*b^12*c^8 - 4726564284204*a^
5*b^10*c^9 + 16406600944545*a^6*b^8*c^10 - 35077504886928*a^7*b^6
*c^11 + 41166213348960*a^8*b^4*c^12 - 20609394167040*a^9*b^2*c^13
 + 3543369523456*a^10*c^14)*x + 1/2*sqrt(1/2)*(59049*b^28 - 38185
02*a*b^26*c + 112685175*a^2*b^24*c^2 - 2006183214*a^3*b^22*c^3 +
23996183904*a^4*b^20*c^4 - 203172719904*a^5*b^18*c^5 + 1249344303
669*a^6*b^16*c^6 - 5630371599204*a^7*b^14*c^7 + 18522020088594*a^
8*b^12*c^8 - 43727956655856*a^9*b^10*c^9 + 71711400072928*a^10*b^
8*c^10 - 77372920636928*a^11*b^6*c^11 + 50337719029248*a^12*b^4*c
^12 - 17166378299392*a^13*b^2*c^13 + 2314037239808*a^14*c^14 + (7
29*a^7*b^29 - 45927*a^8*b^27*c + 1331640*a^9*b^25*c^2 - 23536197*
a^10*b^23*c^3 + 283046346*a^11*b^21*c^4 - 2447287920*a^12*b^19*c^
5 + 15665468896*a^13*b^17*c^6 - 75268700672*a^14*b^15*c^7 + 27203
5456000*a^15*b^13*c^8 - 733077069824*a^16*b^11*c^9 + 144130701721
6*a^17*b^9*c^10 - 1985616084992*a^18*b^7*c^11 + 1781248688128*a^1
9*b^5*c^12 - 899513581568*a^20*b^3*c^13 + 176234168320*a^21*b*c^1

```
4)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 384
99462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^
5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*
c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*
b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^2
0*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^
23*c^9)))*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 100
023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 - (a^7*b^
12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^16
- 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3
+ 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b
^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 3
6*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^
18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*
a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^1
1*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6))))) - ((a*b^2*c -
4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 1
00023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 + (a^7*
b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^1
6 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^
3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6
*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 -
 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*
a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 58982
4*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^
12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)))*log(-(2673*b^1
0*c^2 - 68445*a*b^8*c^3 + 666846*a^2*b^6*c^4 - 2974545*a^3*b^4*c^
5 + 5474280*a^4*b^2*c^6 - 1882384*a^5*c^7)*sqrt(x) + 1/2*(243*b^1
4 - 7857*a*b^12*c + 105732*a^2*b^10*c^2 - 760311*a^3*b^8*c^3 + 31
04898*a^4*b^6*c^4 - 6982136*a^5*b^4*c^5 + 7430752*a^6*b^2*c^6 - 2
151296*a^7*c^7 - (3*a^7*b^15 - 92*a^8*b^13*c + 1200*a^9*b^11*c^2
- 8640*a^10*b^9*c^3 + 37120*a^11*b^7*c^4 - 95232*a^12*b^5*c^5 + 1
35168*a^13*b^3*c^6 - 81920*a^14*b*c^7)*sqrt((6561*b^16 - 258066*a
*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 20086558
2*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 5
05420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^1
6*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^
4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c
^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))*sqrt(sqrt(1/2)*sqrt
(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c^
3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 + (a^7*b^12 - 24*a^8*b^
10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^16 - 258066*a*b^1
4*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^
4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 50542
0104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c
+ 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 -
129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 +
 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10
```

*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 61
44*a^12*b^2*c^5 + 4096*a^13*c^6)))) + ((a*b^2*c - 4*a^2*c^2)*x^4
+ a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqr
t(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c
^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 + (a^7*b^12 - 24*a^8*b
^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561*b^16 - 258066*a*b^
14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a
^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 5054
20104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c
 + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 -
 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7
+ 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^1
0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6
144*a^12*b^2*c^5 + 4096*a^13*c^6)))*log(-(2673*b^10*c^2 - 68445*a
*b^8*c^3 + 666846*a^2*b^6*c^4 - 2974545*a^3*b^4*c^5 + 5474280*a^4
*b^2*c^6 - 1882384*a^5*c^7)*sqrt(x) - 1/2*(243*b^14 - 7857*a*b^12
*c + 105732*a^2*b^10*c^2 - 760311*a^3*b^8*c^3 + 3104898*a^4*b^6*c
^4 - 6982136*a^5*b^4*c^5 + 7430752*a^6*b^2*c^6 - 2151296*a^7*c^7
- (3*a^7*b^15 - 92*a^8*b^13*c + 1200*a^9*b^11*c^2 - 8640*a^10*b^9
*c^3 + 37120*a^11*b^7*c^4 - 95232*a^12*b^5*c^5 + 135168*a^13*b^3*
c^6 - 81920*a^14*b*c^7)*sqrt((6561*b^16 - 258066*a*b^14*c + 42785
01*a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 -
 596117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^
2*c^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*
b^14*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19
*b^8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^2
2*b^2*c^8 - 262144*a^23*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 20
79*a*b^9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*
b^3*c^4 - 181104*a^5*b*c^5 + (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*
b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c
^5 + 4096*a^13*c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a
^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596
117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^
7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14
*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8
*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^
2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^
8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5
 + 4096*a^13*c^6)))) - ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a
^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2
079*a*b^9*c + 20790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4
*b^3*c^4 - 181104*a^5*b*c^5 - (a^7*b^12 - 24*a^8*b^10*c + 240*a^9
*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*
c^5 + 4096*a^13*c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*
a^2*b^12*c^2 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 59
6117340*a^5*b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c
^7 + 92236816*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^1
4*c^2 - 5376*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^
8*c^5 + 344064*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b
^2*c^8 - 262144*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b
^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^
5 + 4096*a^13*c^6)))*log(-(2673*b^10*c^2 - 68445*a*b^8*c^3 + 6668
46*a^2*b^6*c^4 - 2974545*a^3*b^4*c^5 + 5474280*a^4*b^2*c^6 - 1882

384*a^5*c^7)*sqrt(x) + 1/2*(243*b^14 - 7857*a*b^12*c + 105732*a^2
*b^10*c^2 - 760311*a^3*b^8*c^3 + 3104898*a^4*b^6*c^4 - 6982136*a^
5*b^4*c^5 + 7430752*a^6*b^2*c^6 - 2151296*a^7*c^7 + (3*a^7*b^15 -
 92*a^8*b^13*c + 1200*a^9*b^11*c^2 - 8640*a^10*b^9*c^3 + 37120*a^
11*b^7*c^4 - 95232*a^12*b^5*c^5 + 135168*a^13*b^3*c^6 - 81920*a^1
4*b*c^7)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2
 - 38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*
b^6*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 9223681
6*a^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376
*a^17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 3440
64*a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262
144*a^23*c^9)))*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20
790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 18110
4*a^5*b*c^5 - (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*
a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*
c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 3
8499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*
c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^
8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^1
7*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a
^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*
a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^
10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^
6)))) + ((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 -
 4*a^2*b*c)*x^2)*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 2
0790*a^2*b^7*c^2 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 1811
04*a^5*b*c^5 - (a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280
*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13
*c^6)*sqrt((6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 -
38499462*a^3*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6
*c^5 + 898783137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a
^8*c^8)/(a^14*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^
17*b^12*c^3 + 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*
a^20*b^6*c^6 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144
*a^23*c^9)))/(a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a
^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c
^6)))*log(-(2673*b^10*c^2 - 68445*a*b^8*c^3 + 666846*a^2*b^6*c^4
- 2974545*a^3*b^4*c^5 + 5474280*a^4*b^2*c^6 - 1882384*a^5*c^7)*sq
rt(x) - 1/2*(243*b^14 - 7857*a*b^12*c + 105732*a^2*b^10*c^2 - 760
311*a^3*b^8*c^3 + 3104898*a^4*b^6*c^4 - 6982136*a^5*b^4*c^5 + 743
0752*a^6*b^2*c^6 - 2151296*a^7*c^7 + (3*a^7*b^15 - 92*a^8*b^13*c
+ 1200*a^9*b^11*c^2 - 8640*a^10*b^9*c^3 + 37120*a^11*b^7*c^4 - 95
232*a^12*b^5*c^5 + 135168*a^13*b^3*c^6 - 81920*a^14*b*c^7)*sqrt((
6561*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3
*b^10*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 89878
3137*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^1
4*b^18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3
+ 32256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6
 - 589824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))
*sqrt(sqrt(1/2)*sqrt(-(81*b^11 - 2079*a*b^9*c + 20790*a^2*b^7*c^2
 - 100023*a^3*b^5*c^3 + 226072*a^4*b^3*c^4 - 181104*a^5*b*c^5 - (
a^7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 +
3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)*sqrt((6561
*b^16 - 258066*a*b^14*c + 4278501*a^2*b^12*c^2 - 38499462*a^3*b^1

```
0*c^3 + 200865582*a^4*b^8*c^4 - 596117340*a^5*b^6*c^5 + 898783137
*a^6*b^4*c^6 - 505420104*a^7*b^2*c^7 + 92236816*a^8*c^8)/(a^14*b^
18 - 36*a^15*b^16*c + 576*a^16*b^14*c^2 - 5376*a^17*b^12*c^3 + 32
256*a^18*b^10*c^4 - 129024*a^19*b^8*c^5 + 344064*a^20*b^6*c^6 - 5
89824*a^21*b^4*c^7 + 589824*a^22*b^2*c^8 - 262144*a^23*c^9)))/(a^
7*b^12 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 38
40*a^11*b^4*c^4 - 6144*a^12*b^2*c^5 + 4096*a^13*c^6)))) + 4*(b*c*
x^2 + b^2 - 2*a*c)*sqrt(x))/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^2 \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*sqrt(x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^2*sqrt(x)), x)
```

3.1079 $\int \frac{1}{x^{3/2}(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=573

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}\,(b^2-4ac)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\,(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$
$$-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\,(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$
$$-\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\,(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$
$$+\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2\,(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$
$$+\frac{-2ac+b^2+bcx^2}{2a\sqrt{x}\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

```
[Out] -(5*b^2 - 18*a*c)/(2*a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c +
b*c*x^2)/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/
4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (
c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - (c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])
^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rubi [A] time = 4.34032, antiderivative size = 573, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{5b^2-18ac}{2a^2\sqrt{x}(b^2-4ac)}+\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$
$$-\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$
$$-\frac{\sqrt[4]{c}\left(-\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$
$$+\frac{\sqrt[4]{c}\left(\left(5b^2-18ac\right)\sqrt{b^2-4ac}-28abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{4\,2^{3/4}a^2(b^2-4ac)^{3/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$
$$+\frac{-2ac+b^2+bcx^2}{2a\sqrt{x}(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(5*b^2 - 18*a*c)/(2*a^2*(b^2 - 4*a*c)*Sqrt[x]) + (b^2 - 2*a*c +
b*c*x^2)/(2*a*(b^2 - 4*a*c)*Sqrt[x]*(a + b*x^2 + c*x^4)) + (c^(1/
4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(4*2^
(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (
c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*c])*A
rcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/
(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - (c^(1/4)*(5*b^3 - 28*a*b*c - (5*b^2 - 18*a*c)*Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b - Sqrt[b^2 - 4*a*c])
^(1/4)) + (c^(1/4)*(5*b^3 - 28*a*b*c + (5*b^2 - 18*a*c)*Sqrt[b^2
- 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*
c])^(1/4)])/(4*2^(3/4)*a^2*(b^2 - 4*a*c)^(3/2)*(-b + Sqrt[b^2 - 4
*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.409386, size = 190, normalized size = 0.33

$$-\frac{\frac{\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{-18\#1^4 ac^2\log\left(\sqrt{x}-\#1\right)+5\#1^4 b^2 c\log\left(\sqrt{x}-\#1\right)-23abc\log\left(\sqrt{x}-\#1\right)+5b^3\log\left(\sqrt{x}-\#1\right)}{2\#1^5 c+\#1 b}\&\right]}{b^2-4ac}+\frac{4x^{3/2}\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{16}{\sqrt{x}}}{8a^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(x^(3/2)*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(16/Sqrt[x] + (4*x^(3/2)*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^
2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + RootSum[a + b*#1^4 + c*
#1^8 & , (5*b^3*Log[Sqrt[x] - #1] - 23*a*b*c*Log[Sqrt[x] - #1] +
5*b^2*c*Log[Sqrt[x] - #1]*#1^4 - 18*a*c^2*Log[Sqrt[x] - #1]*#1^4)
/(b*#1 + 2*c*#1^5) & ]/(b^2 - 4*a*c))/(8*a^2)
```

Maple [C] time = 0.035, size = 245, normalized size = 0.4

$$-\frac{c^2}{a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{7}{2}}+\frac{b^2c}{2a^2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{7}{2}}-\frac{3bc}{2a\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{3}{2}}+\frac{b^3}{2a^2\left(cx^4+bx^2+a\right)\left(4ac-b^2\right)}x^{\frac{3}{2}}-\frac{1}{8a^2}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{c\left(18ac-5b^2\right)\_R^6+b\left(23ac-5b^2\right)\_R^2}{\left(4ac-b^2\right)\left(2\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)-2\frac{1}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(3/2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^(7/2)+1/2/a^2/(c*x^4+b*x^2
+a)*c/(4*a*c-b^2)*x^(7/2)*b^2-3/2/a/(c*x^4+b*x^2+a)*b/(4*a*c-b^2)
```

```
*x^(3/2)*c+1/2/a^2/(c*x^4+b*x^2+a)*b^3/(4*a*c-b^2)*x^(3/2)-1/8/a^
2*sum((c*(18*a*c-5*b^2)*_R^6+b*(23*a*c-5*b^2)*_R^2)/(4*a*c-b^2)/(
2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))-2/a^2
/x^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(5\,b^{2}c-18\,ac^{2}\right)x^{\frac{7}{2}}+\left(5\,b^{3}-19\,abc\right)x^{\frac{3}{2}}+\frac{4\,\left(ab^{2}-4\,a^{2}c\right)}{\sqrt{x}}}{2\,(a^{3}b^{2}-4\,a^{4}c+(a^{2}b^{2}c-4\,a^{3}c^{2})x^{4}+(a^{2}b^{3}-4\,a^{3}bc)x^{2})}-\int\frac{\left(5\,b^{2}c-18\,ac^{2}\right)x^{\frac{5}{2}}+\left(5\,b^{3}-23\,abc\right)\sqrt{x}}{4\,(a^{3}b^{2}-4\,a^{4}c+(a^{2}b^{2}c-4\,a^{3}c^{2})x^{4}+(a^{2}b^{3}-4\,a^{3}bc)x^{2})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^(3/2)),x, algorithm="maxima")
```

```
[Out] -1/2*((5*b^2*c - 18*a*c^2)*x^(7/2) + (5*b^3 - 19*a*b*c)*x^(3/2) +
 4*(a*b^2 - 4*a^2*c)/sqrt(x))/(a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4
*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2) - integrate(1/4*((5*b^
2*c - 18*a*c^2)*x^(5/2) + (5*b^3 - 23*a*b*c)*sqrt(x))/(a^3*b^2 -
4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2
), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^(3/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(3/2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^4 + bx^2 + a\right)^2 x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^2*x^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^2*x^(3/2)), x)
```

3.1080 $\int \frac{x^{15/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=621

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}(b^2-4ac)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}(b^2-4ac)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}(b^2-4ac)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}(b^2-4ac)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{9/2}(2a+bx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x^{5/2}\left(x^2(12ac+b^2)+8ab\right)}{16(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{3\sqrt{x}(12ac+b^2)}{16c(b^2-4ac)^2}$$

```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*
a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*
(8*a*b + (b^2 + 12*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt
[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^
2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*
a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*
(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*
a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b
^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*
c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2
^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 3.75799, antiderivative size = 621, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{-24a^2c^2-30ab^2c+b^4}{\sqrt{b^2-4ac}}-28abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}c^{5/4}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{x^{9/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3x^{5/2}\left(x^2\left(12ac+b^2\right)+8ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{3\sqrt{x}\left(12ac+b^2\right)}{16c\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-3*(b^2 + 12*a*c)*Sqrt[x])/(16*c*(b^2 - 4*a*c)^2) + (x^(9/2)*(2*
a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (3*x^(5/2)*
(8*a*b + (b^2 + 12*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt
[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b - Sqrt[b^
2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c - (b^4 - 30*a*b^2*c - 24*
a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b^2 - 4*a*c)^2*
(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*c + (b^4 - 30*
a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)
*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(5/4)*(b
^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(b^3 - 28*a*b*
c - (b^4 - 30*a*b^2*c - 24*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2
^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1
/4)*c^(5/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.67206, size = 254, normalized size = 0.41

$$\frac{3c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-28\#1^4abc\log(\sqrt{x}-\#1)+\#1^4b^3\log(\sqrt{x}-\#1)+12a^2c\log(\sqrt{x}-\#1)+ab^2\log(\sqrt{x}-\#1)}{2\#1^7c+\#1^3b}\&\right]+1}{64c^2\left(b^2-4ac\right)^2(a}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(15/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(-4*b^4 + 21*a*b^2*c - 68*a^2*c^2 + b^3*c*x^2 - 28*a*b
*c^2*x^2)*(a + b*x^2 + c*x^4) + 16*(b^2 - 4*a*c)*Sqrt[x]*(-2*a^2*
c + b^3*x^2 + a*b*(b - 3*c*x^2)) + 3*c*(a + b*x^2 + c*x^4)^2*Root
Sum[a + b*#1^4 + c*#1^8 & , (a*b^2*Log[Sqrt[x] - #1] + 12*a^2*c*L
og[Sqrt[x] - #1] + b^3*Log[Sqrt[x] - #1]*#1^4 - 28*a*b*c*Log[Sqrt
[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*c^2*(b^2 - 4*a*c)^2*
(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.078, size = 275, normalized size = 0.4

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(-\frac{3\,a^2\left(12\,ac+b^2\right)\sqrt{x}}{\left(512\,a^2c^2-256\,ab^2c+32\,b^4\right)c}-3/16\,\frac{ab\left(8\,ac+b^2\right)x^{5/2}}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)c}-1/32\,\frac{\left(68\,a^2c^2+7\,ab^2c+3\,b^4\right)x^{9/2}}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)c}-\right.$$
$$+\frac{3}{64\,c}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{b\left(-28\,ac+b^2\right)\_R^4+12\,a^2c+ab^2}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(15/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-3/32*a^2*(12*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^(1/2)-3/
16*a/c*b*(8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)-1/32*(68*
a^2*c^2+7*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c*x^(9/2)-1/3
2*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^
2+a)^2+3/64/c*sum((b*(-28*a*c+b^2)*_R^4+12*a^2*c+a*b^2)/(16*a^2*c
^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8
*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^{2}c+12\,ac^{2}\right)x^{\frac{17}{2}}+\left(7\,b^{3}+44\,abc\right)x^{\frac{13}{2}}+24\,a^{2}bx^{\frac{5}{2}}+\left(35\,ab^{2}+4\,a^{2}c\right)x^{\frac{9}{2}}}{16\,((b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{8}+2\,(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{6}+a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+(b^{6}-6\,ab^{4}c+32\,a^{3}c^{3})x^{4}+2\,(a}$$
$$-\int\frac{3\left(\left(b^{2}+12\,ac\right)x^{\frac{7}{2}}+40\,abx^{\frac{3}{2}}\right)}{32\,(ab^{4}-8\,a^{2}b^{2}c+16\,a^{3}c^{2}+(b^{4}c-8\,ab^{2}c^{2}+16\,a^{2}c^{3})x^{4}+(b^{5}-8\,ab^{3}c+16\,a^{2}bc^{2})x^{2})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c + 12*a*c^2)*x^(17/2) + (7*b^3 + 44*a*b*c)*x^(13/2)
 + 24*a^2*b*x^(5/2) + (35*a*b^2 + 4*a^2*c)*x^(9/2))/((b^4*c^2 - 8
*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*
c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c
+ 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2) -
 integrate(3/32*((b^2 + 12*a*c)*x^(7/2) + 40*a*b*x^(3/2))/(a*b^4
- 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x
^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(15/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{15}{2}}}{(cx^4 + bx^2 + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(15/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(15/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1081 $\int \frac{x^{13/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=569

$$\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^{3/2}\left(x^2\left(28ac+5b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 (x^(3/2)*(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b
^2 + 28*a*c))*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3 + 172*a*b*c)/Sq
rt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[
b^2 - 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5
*b^2 + 28*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2 + 28*a*c - (5*b^3 + 172*a*b*c)
/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + S
qrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 3.78662, antiderivative size = 569, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{x^{7/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x^{3/2}\left(x^2\left(28ac+5b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$+\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$
$$+\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}$$
$$-\frac{\left(\sqrt{b^2-4ac}\left(28ac+5b^2\right)+172abc+5b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$
$$-\frac{\left(-\frac{172abc+5b^3}{\sqrt{b^2-4ac}}+28ac+5b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}c^{3/4}\left(b^2-4ac\right)^2\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(7/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 (x^(3/2)*(24*a*b + (5*b^2 + 28*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) + ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5*b
^2 + 28*a*c))*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4
*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt
[b^2 - 4*a*c])^(1/4)) + ((5*b^2 + 28*a*c - (5*b^3 + 172*a*b*c)/Sq
rt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2
- 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + Sqrt[
b^2 - 4*a*c])^(1/4)) - ((5*b^3 + 172*a*b*c + Sqrt[b^2 - 4*a*c]*(5
*b^2 + 28*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2
- 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(1/4)) - ((5*b^2 + 28*a*c - (5*b^3 + 172*a*b*c)
/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[
b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*c^(3/4)*(b^2 - 4*a*c)^2*(-b + S
qrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.618805, size = 216, normalized size = 0.38

$$\frac{c\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{28\#1^4ac\log(\sqrt{x}-\#1)+5\#1^4b^2\log(\sqrt{x}-\#1)-72ab\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]-16x^{3/2}\left(b^2-4ac\right)}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(13/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*x^(3/2)*(4*b^3 + 8*a*b*c + 5*b^2*c*x^2 + 28*a*c^2*x^2)*(a + b*
x^2 + c*x^4) - 16*(b^2 - 4*a*c)*x^(3/2)*(b^2*x^2 + a*(b - 2*c*x^2
)) + c*(a + b*x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-72
*a*b*Log[Sqrt[x] - #1] + 5*b^2*Log[Sqrt[x] - #1]*#1^4 + 28*a*c*Lo
g[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(64*c*(b^2 - 4*a*c)^
2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.076, size = 242, normalized size = 0.4

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(3/4\frac{a^2bx^{3/2}}{16a^2c^2-8ab^2c+b^4}-1/32\frac{a\left(4ac-37b^2\right)x^{7/2}}{16a^2c^2-8ab^2c+b^4}+\frac{9b\left(4ac+b^2\right)x^{11/2}}{512a^2c^2-256ab^2c+32b^4}+1/32\frac{c\left(28ac+5\right.}{16a^2c^2-8}\right.$$
$$+\frac{1}{64}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(28ac+5b^2\right)\_R^6-72\_R^2ab}{\left(16a^2c^2-8ab^2c+b^4\right)\left(2\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(13/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32*a*(4*a*c-37*
b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+9/32*b*(4*a*c+b^2)/(16*a^
```

```
2*c^2-8*a*b^2*c+b^4)*x^(11/2)+1/32*c*(28*a*c+5*b^2)/(16*a^2*c^2-8
*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2+1/64*sum(((28*a*c+5*b^2
)*_R^6-72*_R^2*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*
ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,b^2c+28\,ac^2\right)x^{\frac{15}{2}}+9\left(b^3+4\,abc\right)x^{\frac{11}{2}}+24\,a^2bx^{\frac{3}{2}}+\left(37\,ab^2-4\,a^2c\right)x^{\frac{7}{2}}}{16\left((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^8+2\left(b^5c-8\,ab^3c^2+16\,a^2bc^3\right)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^6-6\,ab^4c+32\,a^3c^3)x^4+2\,(a\right.}$$
$$+\int\frac{\left(5\,b^2+28\,ac\right)x^{\frac{5}{2}}-72\,ab\sqrt{x}}{32\left(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c + 28*a*c^2)*x^(15/2) + 9*(b^3 + 4*a*b*c)*x^(11/2)
+ 24*a^2*b*x^(3/2) + (37*a*b^2 - 4*a^2*c)*x^(7/2))/((b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c +
 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2) +
integrate(1/32*((5*b^2 + 28*a*c)*x^(5/2) - 72*a*b*sqrt(x))/(a*b^4
 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(13/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{13}{2}}}{(cx^4 + bx^2 + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1082 $\int \frac{x^{11/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=569

$$\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 (Sqrt[x]*(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*
b^2 + 20*a*c))*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqr
t[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/
Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^
2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]
*(7*b^2 + 20*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b^3 + 36*a*
b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b
 + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 3.49101, antiderivative size = 569, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{\sqrt{x}\left(x^2\left(20ac+7b^2\right)+24ab\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{5/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{3\left(\sqrt{b^2-4ac}\left(20ac+7b^2\right)+36abc+7b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}} - \frac{3\left(-\frac{36abc+7b^3}{\sqrt{b^2-4ac}}+20ac+7b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}\sqrt[4]{c}\left(b^2-4ac\right)^2\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(5/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) +
 (Sqrt[x]*(24*a*b + (7*b^2 + 20*a*c)*x^2))/(16*(b^2 - 4*a*c)^2*(a
 + b*x^2 + c*x^4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]*(7*
b^2 + 20*a*c))*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 -
4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqr
t[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b^3 + 36*a*b*c)/
Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^
2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) - (3*(7*b^3 + 36*a*b*c + Sqrt[b^2 - 4*a*c]
*(7*b^2 + 20*a*c))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b
^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b
- Sqrt[b^2 - 4*a*c])^(3/4)) - (3*(7*b^2 + 20*a*c - (7*b^3 + 36*a*
b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*c^(1/4)*(b^2 - 4*a*c)^2*(-b
 + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.594087, size = 219, normalized size = 0.38

$$\frac{3c\left(a+bx^2+cx^4\right)^2 \text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{20\#1^4ac\log(\sqrt{x}-\#1)+7\#1^4b^2\log(\sqrt{x}-\#1)-8ab\log(\sqrt{x}-\#1)}{2\#1^7c+\#1^3b}\&\right]-16\sqrt{x}\left(b^2-4ac\right)\left(\right.}{64c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(11/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (4*Sqrt[x]*(4*b^3 + 8*a*b*c + 7*b^2*c*x^2 + 20*a*c^2*x^2)*(a + b*
x^2 + c*x^4) - 16*(b^2 - 4*a*c)*Sqrt[x]*(b^2*x^2 + a*(b - 2*c*x^2
)) + 3*c*(a + b*x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (-
8*a*b*Log[Sqrt[x] - #1] + 7*b^2*Log[Sqrt[x] - #1]*#1^4 + 20*a*c*L
og[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*c*(b^2 - 4*a*
c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.046, size = 241, normalized size = 0.4

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(3/4\frac{a^2b\sqrt{x}}{16a^2c^2-8ab^2c+b^4}-\frac{\left(12ac-39b^2\right)ax^{5/2}}{512a^2c^2-256ab^2c+32b^4}+1/32\frac{b\left(28ac+11b^2\right)x^{9/2}}{16a^2c^2-8ab^2c+b^4}+1/32\frac{c\left(20ac+\right.}{16a^2c^2-}\right.$$
$$+\frac{3}{64}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(20ac+7b^2\right)\_R^4-8ab}{\left(16a^2c^2-8ab^2c+b^4\right)\left(2\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(11/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/4*a^2*b/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-3/32*(4*a*c-13*b^
2)*a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*b*(28*a*c+11*b^2)/(1
```

```
6*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c*(20*a*c+7*b^2)/(16*a^2*c^
2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64*sum(((20*a*c+7*
b^2)*_R^4-8*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(
x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{24\,bc^2x^{\frac{17}{2}} + \left(41\,b^2c - 20\,ac^2\right)x^{\frac{13}{2}} + \left(13\,b^3 + 20\,abc\right)x^{\frac{9}{2}} + 3\left(3\,ab^2 + 4\,a^2c\right)x^{\frac{5}{2}}}{16\left((b^4c^2 - 8\,ab^2c^3 + 16\,a^2c^4)x^8 + 2\,(b^5c - 8\,ab^3c^2 + 16\,a^2bc^3)x^6 + a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (b^6 - 6\,ab^4c + 32\,a^3c^3)x^4 + 2\right.}$$
$$+\int \frac{3\left(8\,bcx^{\frac{7}{2}} + 5\left(3\,b^2 + 4\,ac\right)x^{\frac{3}{2}}\right)}{32\left(ab^4 - 8\,a^2b^2c + 16\,a^3c^2 + (b^4c - 8\,ab^2c^2 + 16\,a^2c^3)x^4 + (b^5 - 8\,ab^3c + 16\,a^2bc^2)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(17/2) + (41*b^2*c - 20*a*c^2)*x^(13/2) + (13*b
^3 + 20*a*b*c)*x^(9/2) + 3*(3*a*b^2 + 4*a^2*c)*x^(5/2))/((b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^
2) + integrate(3/32*(8*b*c*x^(7/2) + 5*(3*b^2 + 4*a*c)*x^(3/2))/(
a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [A] time = 4.5942, size = 16012, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(12*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c -
8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c
 + 16*a^3*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^
7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4
+ (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4
+ 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 -
 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10
 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 1363
80000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b
```

^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))*arctan((9604*b^11 - 77648*a*b^9*c + 49792*a^2*b^7*c^2 + 710144*a^3*b^5*c^3 - 486400*a^4*b^3*c^4 - 2560000*a^5*b*c^5 + (11*b^22*c - 420*a*b^20*c^2 + 7120*a^2*b^18*c^3 - 70080*a^3*b^16*c^4 + 437760*a^4*b^14*c^5 - 1763328*a^5*b^12*c^6 + 4300800*a^6*b^10*c^7 - 4423680*a^7*b^8*c^8 - 6881280*a^8*b^6*c^9 + 30146560*a^9*b^4*c^10 - 40894464*a^10*b^2*c^11 + 20971520*a^11*c^12)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))/((252105*b^8 + 2197104*a*b^6*c + 6748000*a^2*b^4*c^2 + 8800000*a^3*b^2*c^3 + 4000000*a^4*c^4)*sqrt(x) + sqrt((63556931025*b^16 + 1107801807840*a*b^14*c + 8229675066816*a^2*b^12*c^2 + 34089163584000*a^3*b^10*c^3 + 86221374400000*a^4*b^8*c^4 + 136341632000000*a^5*b^6*c^5 + 131424000000000*a^6*b^4*c^6 + 70400000000000*a^7*b^2*c^7 + 16000000000000*a^8*c^8)*x + sqrt(1/2)*(789777737*b^22 - 7443973964*a*b^20*c - 2705008400*a^2*b^18*c^2 + 166642188480*a^3*b^16*c^3 - 23017121280*a^4*b^14*c^4 - 1866033297408*a^5*b^12*c^5 - 803898138624*a^6*b^10*c^6 + 11168850739200*a^7*b^8*c^7 + 14678630400000*a^8*b^6*c^8 - 23490560000000*a^9*b^4*c^9 - 64307200000000*a^10*b^2*c^10 - 40960000000000*a^11*c^11 + 8*(26411*b^33*c - 1221952*a*b^31*c^2 + 25385088*a^2*b^29*c^3 - 309750784*a^3*b^27*c^4 + 2424181760*a^4*b^25*c^5 - 12295815168*a^5*b^23*c^6 + 36966465536*a^6*b^21*c^7 - 34375204864*a^7*b^19*c^8 - 198547734528*a^8*b^17*c^9 + 848696442880*a^9*b^15*c^10 - 948860616704*a^10*b^13*c^11 - 2216807104512*a^11*b^11*c^12 + 8103865090048*a^12*b^9*c^13 - 62609885

75744*a^13*b^7*c^14 - 10597831802880*a^14*b^5*c^15 + 236223201280
00*a^15*b^3*c^16 - 13421772800000*a^16*b*c^17)*sqrt((5764801*b^8
+ 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^
3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*
c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20
*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160
*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*
c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046
430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15
*c^17)))*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 8
33280*a^3*b^3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 +
720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 25804
8*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 29491
20*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((
5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 18800000
0*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 16
80*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075
072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9
 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 314887372
8*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6
*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 107
3741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 -
 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 8
60160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2
621440*a^9*b^2*c^10 + 1048576*a^10*c^11))))) - 12*((b^4*c^2 - 8*a
*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^
3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c +
32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqr
t(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2
+ 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 - (b^20*c - 40*a*b^18*c^2
 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 25
8048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 29
49120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqr
t((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 18800
0000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 +
 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3
075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*
c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 314887
3728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*
b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 -
1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^
3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6
+ 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9
- 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))*arctan(-(9604*b^11
- 77648*a*b^9*c + 49792*a^2*b^7*c^2 + 710144*a^3*b^5*c^3 - 486400
*a^4*b^3*c^4 - 2560000*a^5*b*c^5 - (11*b^22*c - 420*a*b^20*c^2 +
7120*a^2*b^18*c^3 - 70080*a^3*b^16*c^4 + 437760*a^4*b^14*c^5 - 17
63328*a^5*b^12*c^6 + 4300800*a^6*b^10*c^7 - 4423680*a^7*b^8*c^8 -
 6881280*a^8*b^6*c^9 + 30146560*a^9*b^4*c^10 - 40894464*a^10*b^2*
c^11 + 20971520*a^11*c^12)*sqrt((5764801*b^8 + 45138800*a*b^6*c +
 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^
4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24
*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*
b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 131

2030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^
11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15
 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt(sqrt(1
/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280
*a^3*b^3*c^3 + 672000*a^4*b*c^4 - (b^20*c - 40*a*b^18*c^2 + 720*a
^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5
*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^
8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((57648
01*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3
*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^
2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a
^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 42
1724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^1
0*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14
 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 10737418
24*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680
*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160
*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 262144
0*a^9*b^2*c^10 + 1048576*a^10*c^11)))/((252105*b^8 + 2197104*a*b^
6*c + 6748000*a^2*b^4*c^2 + 8800000*a^3*b^2*c^3 + 4000000*a^4*c^4
)*sqrt(x) + sqrt((63556931025*b^16 + 1107801807840*a*b^14*c + 822
9675066816*a^2*b^12*c^2 + 34089163584000*a^3*b^10*c^3 + 862213744
00000*a^4*b^8*c^4 + 136341632000000*a^5*b^6*c^5 + 131424000000000
*a^6*b^4*c^6 + 70400000000000*a^7*b^2*c^7 + 16000000000000*a^8*c^
8)*x + sqrt(1/2)*(789777737*b^22 - 7443973964*a*b^20*c - 27050084
00*a^2*b^18*c^2 + 166642188480*a^3*b^16*c^3 - 23017121280*a^4*b^1
4*c^4 - 1866033297408*a^5*b^12*c^5 - 803898138624*a^6*b^10*c^6 +
11168850739200*a^7*b^8*c^7 + 14678630400000*a^8*b^6*c^8 - 2349056
0000000*a^9*b^4*c^9 - 64307200000000*a^10*b^2*c^10 - 409600000000
00*a^11*c^11 - 8*(26411*b^33*c - 1221952*a*b^31*c^2 + 25385088*a^
2*b^29*c^3 - 309750784*a^3*b^27*c^4 + 2424181760*a^4*b^25*c^5 - 1
2295815168*a^5*b^23*c^6 + 36966465536*a^6*b^21*c^7 - 34375204864*
a^7*b^19*c^8 - 198547734528*a^8*b^17*c^9 + 848696442880*a^9*b^15*
c^10 - 948860616704*a^10*b^13*c^11 - 2216807104512*a^11*b^11*c^12
 + 8103865090048*a^12*b^9*c^13 - 6260988575744*a^13*b^7*c^14 - 10
597831802880*a^14*b^5*c^15 + 23622320128000*a^15*b^3*c^16 - 13421
772800000*a^16*b*c^17)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136
380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(
b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5
 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18
*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030
720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b
^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4
026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt(-(2401*b^9
 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 6720
00*a^4*b*c^4 - (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*
a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*
a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440
*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a
*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 10000000
0*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*
a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500
480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^
10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 57252

24960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11))))) - 3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))*log(243*(252105*b^8 + 2197104*a*b^6*c + 6748000*a^2*b^4*c^2 + 8800000*a^3*b^2*c^3 + 4000000*a^4*c^4)*sqrt(x) + 243*(9604*b^11 - 77648*a*b^9*c + 49792*a^2*b^7*c^2 + 710144*a^3*b^5*c^3 - 486400*a^4*b^3*c^4 - 2560000*a^5*b*c^5 + (11*b^22*c - 420*a*b^20*c^2 + 7120*a^2*b^18*c^3 - 70080*a^3*b^16*c^4 + 437760*a^4*b^14*c^5 - 1763328*a^5*b^12*c^6 + 4300800*a^6*b^10*c^7 - 4423680*a^7*b^8*c^8 - 6881280*a^8*b^6*c^9 + 30146560*a^9*b^4*c^10 - 40894464*a^10*b^2*c^11 + 20971520*a^11*c^12)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8

```
*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))) + 3*((b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 1
6*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*
a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2
)*x^2)*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 413280*a^
2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 + (b^20*c - 40*
a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^1
2*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^
6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10
*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c
^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*
b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^
22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 105431040
*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^1
1 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 763363
3280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^14*b
^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a
^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5
*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^
8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))*log(243*(
252105*b^8 + 2197104*a*b^6*c + 6748000*a^2*b^4*c^2 + 8800000*a^3*
b^2*c^3 + 4000000*a^4*c^4)*sqrt(x) - 243*(9604*b^11 - 77648*a*b^9
*c + 49792*a^2*b^7*c^2 + 710144*a^3*b^5*c^3 - 486400*a^4*b^3*c^4
- 2560000*a^5*b*c^5 + (11*b^22*c - 420*a*b^20*c^2 + 7120*a^2*b^18
*c^3 - 70080*a^3*b^16*c^4 + 437760*a^4*b^14*c^5 - 1763328*a^5*b^1
2*c^6 + 4300800*a^6*b^10*c^7 - 4423680*a^7*b^8*c^8 - 6881280*a^8*
b^6*c^9 + 30146560*a^9*b^4*c^10 - 40894464*a^10*b^2*c^11 + 209715
20*a^11*c^12)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^
2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2
- 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440
*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 10
5431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b
^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 +
 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840
*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt(sqrt(1/2)*sqrt(-(24
01*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3
+ 672000*a^4*b*c^4 + (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 -
 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 8
60160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2
621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 4513
8800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 10
0000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 -
29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 +
 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b
^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 -
 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 7046430720
*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17)
))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4
 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7
- 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^1
0 + 1048576*a^10*c^11)))) - 3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^
4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8
*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*
(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(2
```

401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3*c^3
+ 672000*a^4*b*c^4 - (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3
- 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 +
860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 -
2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 + 451
38800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 1
00000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 -
29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7
+ 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*a^8*
b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12
- 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 704643072
0*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*c^17
)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^
4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7
- 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^
10 + 1048576*a^10*c^11)))*log(243*(252105*b^8 + 2197104*a*b^6*c +
6748000*a^2*b^4*c^2 + 8800000*a^3*b^2*c^3 + 4000000*a^4*c^4)*sqr
t(x) + 243*(9604*b^11 - 77648*a*b^9*c + 49792*a^2*b^7*c^2 + 71014
4*a^3*b^5*c^3 - 486400*a^4*b^3*c^4 - 2560000*a^5*b*c^5 - (11*b^22
*c - 420*a*b^20*c^2 + 7120*a^2*b^18*c^3 - 70080*a^3*b^16*c^4 + 43
7760*a^4*b^14*c^5 - 1763328*a^5*b^12*c^6 + 4300800*a^6*b^10*c^7 -
4423680*a^7*b^8*c^8 - 6881280*a^8*b^6*c^9 + 30146560*a^9*b^4*c^1
0 - 40894464*a^10*b^2*c^11 + 20971520*a^11*c^12)*sqrt((5764801*b^
8 + 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*
c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^2
6*c^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^
20*c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 4217241
60*a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^1
0*c^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 70
46430720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^
15*c^17)))*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 41328
0*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 - (b^20*c -
40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4
*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^
7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*
a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*b
^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 - 6
0*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a^
4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 10543
1040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^12
*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 76
33633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a^
14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 + 7
20*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048
*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 294912
0*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))) + 3*
((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^
2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6
- 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*
b*c^2)*x^2)*sqrt(sqrt(1/2)*sqrt(-(2401*b^9 + 86640*a*b^7*c + 4132
80*a^2*b^5*c^2 + 833280*a^3*b^3*c^3 + 672000*a^4*b*c^4 - (b^20*c
- 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^
4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a
^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576

```
*a^10*c^11)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 136380000*a^2*
b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30*c^2 -
60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 349440*a
^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8 - 1054
31040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*a^9*b^1
2*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c^13 + 7
633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 4026531840*a
^14*b^2*c^16 - 1073741824*a^15*c^17)))/(b^20*c - 40*a*b^18*c^2 +
720*a^2*b^16*c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 25804
8*a^5*b^10*c^6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 29491
20*a^8*b^4*c^9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)))*log(
243*(252105*b^8 + 2197104*a*b^6*c + 6748000*a^2*b^4*c^2 + 8800000
*a^3*b^2*c^3 + 4000000*a^4*c^4)*sqrt(x) - 243*(9604*b^11 - 77648*
a*b^9*c + 49792*a^2*b^7*c^2 + 710144*a^3*b^5*c^3 - 486400*a^4*b^3
*c^4 - 2560000*a^5*b*c^5 - (11*b^22*c - 420*a*b^20*c^2 + 7120*a^2
*b^18*c^3 - 70080*a^3*b^16*c^4 + 437760*a^4*b^14*c^5 - 1763328*a^
5*b^12*c^6 + 4300800*a^6*b^10*c^7 - 4423680*a^7*b^8*c^8 - 6881280
*a^8*b^6*c^9 + 30146560*a^9*b^4*c^10 - 40894464*a^10*b^2*c^11 + 2
0971520*a^11*c^12)*sqrt((5764801*b^8 + 45138800*a*b^6*c + 1363800
00*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3 + 100000000*a^4*c^4)/(b^30
*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c^4 - 29120*a^3*b^24*c^5 + 3
49440*a^4*b^22*c^6 - 3075072*a^5*b^20*c^7 + 20500480*a^6*b^18*c^8
 - 105431040*a^7*b^16*c^9 + 421724160*a^8*b^14*c^10 - 1312030720*
a^9*b^12*c^11 + 3148873728*a^10*b^10*c^12 - 5725224960*a^11*b^8*c
^13 + 7633633280*a^12*b^6*c^14 - 7046430720*a^13*b^4*c^15 + 40265
31840*a^14*b^2*c^16 - 1073741824*a^15*c^17)))*sqrt(sqrt(1/2)*sqrt
(-(2401*b^9 + 86640*a*b^7*c + 413280*a^2*b^5*c^2 + 833280*a^3*b^3
*c^3 + 672000*a^4*b*c^4 - (b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*
c^3 - 7680*a^3*b^14*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^
6 + 860160*a^6*b^8*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^
9 - 2621440*a^9*b^2*c^10 + 1048576*a^10*c^11)*sqrt((5764801*b^8 +
 45138800*a*b^6*c + 136380000*a^2*b^4*c^2 + 188000000*a^3*b^2*c^3
 + 100000000*a^4*c^4)/(b^30*c^2 - 60*a*b^28*c^3 + 1680*a^2*b^26*c
^4 - 29120*a^3*b^24*c^5 + 349440*a^4*b^22*c^6 - 3075072*a^5*b^20*
c^7 + 20500480*a^6*b^18*c^8 - 105431040*a^7*b^16*c^9 + 421724160*
a^8*b^14*c^10 - 1312030720*a^9*b^12*c^11 + 3148873728*a^10*b^10*c
^12 - 5725224960*a^11*b^8*c^13 + 7633633280*a^12*b^6*c^14 - 70464
30720*a^13*b^4*c^15 + 4026531840*a^14*b^2*c^16 - 1073741824*a^15*
c^17)))/(b^20*c - 40*a*b^18*c^2 + 720*a^2*b^16*c^3 - 7680*a^3*b^1
4*c^4 + 53760*a^4*b^12*c^5 - 258048*a^5*b^10*c^6 + 860160*a^6*b^8
*c^7 - 1966080*a^7*b^6*c^8 + 2949120*a^8*b^4*c^9 - 2621440*a^9*b^
2*c^10 + 1048576*a^10*c^11)))) - 4*((7*b^2*c + 20*a*c^2)*x^6 + (1
1*b^3 + 28*a*b*c)*x^4 + 24*a^2*b + 3*(13*a*b^2 - 4*a^2*c)*x^2)*sq
rt(x))/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2
 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c +
16*a^3*b*c^2)*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(11/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{11}{2}}}{(cx^4 + bx^2 + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1083 $\int \frac{x^{9/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=533

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) -
 (3*x^(3/2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) - (3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4
*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^
(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(1
6*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)
*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)
*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4
*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 2.70076, antiderivative size = 533, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{3x^{3/2}\left(-4ac+5b^2+8bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x^{3/2}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(-4b\sqrt{b^2-4ac}+20ac+11b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\,2^{3/4}\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) -
 (3*x^(3/2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) - (3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4
*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^
(1/4)])/(16*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^
(1/4)) + (3*c^(1/4)*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 - 4*a*c])*Arc
Tan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(1
6*2^(3/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (
3*c^(1/4)*(11*b^2 + 20*a*c + 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1
/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)
*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)
*(11*b^2 + 20*a*c - 4*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/
4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(3/4)*(b^2 - 4
*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.380375, size = 176, normalized size = 0.33

$$\frac{-3\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{8\#1^4 bc\log(\sqrt{x}-\#1)-20ac\log(\sqrt{x}-\#1)-7b^2\log(\sqrt{x}-\#1)}{2\#1^5 c+\#1 b}\&\right]-\frac{12x^{3/2}(-4ac+5b^2+8bcx^2)}{a+bx^2+cx^4}+\frac{16x^{3/2}(b^2-4ac)(2\cdots}{(a+bx^2+cx^4\cdots}}{64(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(9/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((16*(b^2 - 4*a*c)*x^(3/2)*(2*a + b*x^2))/(a + b*x^2 + c*x^4)^2 -
 (12*x^(3/2)*(5*b^2 - 4*a*c + 8*b*c*x^2))/(a + b*x^2 + c*x^4) - 3
*RootSum[a + b*#1^4 + c*#1^8 & , (-7*b^2*Log[Sqrt[x] - #1] - 20*a
*c*Log[Sqrt[x] - #1] + 8*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*
#1^5) & ])/(64*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.045, size = 244, normalized size = 0.5

$$2\frac{1}{(cx^4+bx^2+a)^2}\left(-1/32\frac{a(20ac+7b^2)x^{3/2}}{16a^2c^2-8ab^2c+b^4}-1/32\frac{b(28ac+11b^2)x^{7/2}}{16a^2c^2-8ab^2c+b^4}+\frac{(12ac-39b^2)cx^{11/2}}{512a^2c^2-256ab^2c+32b^4}-3/4\frac{bc\cdots}{16a^2c^2-\cdots}\right.$$
$$+\frac{3}{64}\sum_{\_R=RootOf(\_Z^8c+\_Z^4b+a)}\frac{-8bc\_R^6+(20ac+7b^2)\_R^2}{(16a^2c^2-8ab^2c+b^4)(2\_R^7c+\_R^3b)}\ln(\sqrt{x}-\_R)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(9/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(20*a*c+7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)-1/32
*b*(28*a*c+11*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(7/2)+3/32*(4*a*c
-13*b^2)*c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)-3/4*b*c^2/(16*a^2*
```

```
c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2+3/64*sum((-8*b*c*_
R^6+(20*a*c+7*b^2)*_R^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^
3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{24\,bc^2x^{\frac{15}{2}}+3\left(13\,b^2c-4\,ac^2\right)x^{\frac{11}{2}}+\left(11\,b^3+28\,abc\right)x^{\frac{7}{2}}+\left(7\,ab^2+20\,a^2c\right)x^{\frac{3}{2}}}{16\,((b^4c^2-8\,ab^2c^3+16\,a^2c^4)x^8+2\,(b^5c-8\,ab^3c^2+16\,a^2bc^3)x^6+a^2b^4-8\,a^3b^2c+16\,a^4c^2+(b^6-6\,ab^4c+32\,a^3c^3)x^4+2}$$
$$-\int\frac{3\left(8\,bcx^{\frac{5}{2}}-\left(7\,b^2+20\,ac\right)\sqrt{x}\right)}{32\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(24*b*c^2*x^(15/2) + 3*(13*b^2*c - 4*a*c^2)*x^(11/2) + (11*
b^3 + 28*a*b*c)*x^(7/2) + (7*a*b^2 + 20*a^2*c)*x^(3/2))/((b^4*c^2
 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^
2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^
4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^
2) - integrate(3/32*(8*b*c*x^(5/2) - (7*b^2 + 20*a*c)*sqrt(x))/(a
*b^4 - 8*a^2*b^2*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c
^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(9/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{9}{2}}}{(cx^4 + bx^2 + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(9/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1084 $\int \frac{x^{7/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=533

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) -
 (Sqrt[x]*(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) + (c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*
2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^
(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b
^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*
b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c
)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 2.37046, antiderivative size = 533, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{c^{3/4}\left(-36b\sqrt{b^2-4ac}+28ac+41b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{16\sqrt[4]{2}\,(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(2a+bx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}-\frac{\sqrt{x}\left(-4ac+13b^2+24bcx^2\right)}{16\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (Sqrt[x]*(2*a + b*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) -
 (Sqrt[x]*(13*b^2 - 4*a*c + 24*b*c*x^2))/(16*(b^2 - 4*a*c)^2*(a +
 b*x^2 + c*x^4)) + (c^(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*
a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(
1/4)])/(16*2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) - (c^(3/4)*(41*b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*c])*ArcTa
n[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*
2^(1/4)*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (c^
(3/4)*(41*b^2 + 28*a*c + 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b
^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (c^(3/4)*(41*
b^2 + 28*a*c - 36*b*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*S
qrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(16*2^(1/4)*(b^2 - 4*a*c
)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.472641, size = 177, normalized size = 0.33

$$-\frac{\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{72\#1^4 bc\log(\sqrt{x}-\#1)-28ac\log(\sqrt{x}-\#1)-5b^2\log(\sqrt{x}-\#1)}{2\#1^7 c+\#1^3 b}\&\right]+\frac{4\sqrt{x}\left(28a^2c+a\left(5b^2+36bcx^2-4c^2x^4\right)+bx^2\left(9b^2+37b\right.\right.}{\left(a+bx^2+cx^4\right)^2}}{64\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(7/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -((4*Sqrt[x]*(28*a^2*c + a*(5*b^2 + 36*b*c*x^2 - 4*c^2*x^4) + b*x
^2*(9*b^2 + 37*b*c*x^2 + 24*c^2*x^4)))/(a + b*x^2 + c*x^4)^2 + Ro
otSum[a + b*#1^4 + c*#1^8 & , (-5*b^2*Log[Sqrt[x] - #1] - 28*a*c*
Log[Sqrt[x] - #1] + 72*b*c*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*
#1^7) & ])/(64*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.045, size = 237, normalized size = 0.4

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(-1/32\,\frac{a\left(28\,ac+5\,b^2\right)\sqrt{x}}{16\,a^2c^2-8\,ab^2c+b^4}-\frac{9\,b\left(4\,ac+b^2\right)x^{5/2}}{512\,a^2c^2-256\,ab^2c+32\,b^4}+1/32\,\frac{c\left(4\,ac-37\,b^2\right)x^{9/2}}{16\,a^2c^2-8\,ab^2c+b^4}-3/4\,\frac{c^2b}{16\,a^2c^2-}\right.$$
$$+\frac{1}{64}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{-72\,\_R^4bc+28\,ac+5\,b^2}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(7/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(-1/32*a*(28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)-9/32
*b*(4*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(5/2)+1/32*c*(4*a*c-3
7*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-3/4*b*c^2/(16*a^2*c^2-8
```

```
*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+1/64*sum((-72*_R^4*b*c+
28*a*c+5*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(x^(
1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,b^2c^2 + 28\,ac^3\right)x^{\frac{17}{2}} + 2\left(5\,b^3c + 16\,abc^2\right)x^{\frac{13}{2}} + \left(5\,b^4 + ab^2c + 60\,a^2c^2\right)x^{\frac{9}{2}} + \left(ab^3 + 20\,a^2bc\right)}{16\left((ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4)x^8 + a^3b^4 - 8\,a^4b^2c + 16\,a^5c^2 + 2\,(ab^5c - 8\,a^2b^3c^2 + 16\,a^3bc^3)x^6 + (ab^6 - 6\,a^2b^4c + 32\,a^4c^3)x\right.}$$
$$- \int \frac{\left(5\,b^2c + 28\,ac^2\right)x^{\frac{7}{2}} + 5\left(b^3 + 20\,abc\right)x^{\frac{3}{2}}}{32\left(a^2b^4 - 8\,a^3b^2c + 16\,a^4c^2 + (ab^4c - 8\,a^2b^2c^2 + 16\,a^3c^3)x^4 + (ab^5 - 8\,a^2b^3c + 16\,a^3bc^2)x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^2*c^2 + 28*a*c^3)*x^(17/2) + 2*(5*b^3*c + 16*a*b*c^2)*
x^(13/2) + (5*b^4 + a*b^2*c + 60*a^2*c^2)*x^(9/2) + (a*b^3 + 20*a
^2*b*c)*x^(5/2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 +
a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 +
 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(
a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) - integrate(1/32*((5*b
^2*c + 28*a*c^2)*x^(7/2) + 5*(b^3 + 20*a*b*c)*x^(3/2))/(a^2*b^4 -
 8*a^3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3
)*x^4 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(7/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{7}{2}}}{(cx^4 + bx^2 + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1085 $\int \frac{x^{5/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=594

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\left(12ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\left(12ac+b^2\right)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
+ (3*x^(3/2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2
 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3
/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a
*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(b^
3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(
b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(
b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c
))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rubi [A] time = 4.33535, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{3x^{3/2}\left(cx^2\left(12ac+b^2\right)+b\left(4ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{x^{3/2}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\left(12ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+12ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^2\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{3\sqrt[4]{c}\left(\sqrt{b^2-4ac}\left(12ac+b^2\right)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(x^(3/2)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
+ (3*x^(3/2)*(b*(b^2 + 4*a*c) + c*(b^2 + 12*a*c)*x^2))/(16*a*(b^2
 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*c^(1/4)*(b^2 + 12*a*c - b^3
/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4
)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a
*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4)) + (3*c^(1/4)*(b^
3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c))*ArcTan[(2^(1/4)*
c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a*(
b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) - (3*c^(1/4)*(
b^2 + 12*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*
c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1
/4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(1/4
)) - (3*c^(1/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 12*a*c
))*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/
4)])/(32*2^(3/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(
1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.62793, size = 222, normalized size = 0.37

$$\frac{3\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{12\#1^4ac^2\log(\sqrt{x}-\#1)+\#1^4b^2c\log(\sqrt{x}-\#1)-28abc\log(\sqrt{x}-\#1)+b^3\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]-16a}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(5/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*x^(3/2)*(b + 2*c*x^2) + 12*x^(3/2)*(b^3 + 4*
a*b*c + b^2*c*x^2 + 12*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*
x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] -
 #1] - 28*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4
+ 12*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ])/(64*a*(
b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.077, size = 277, normalized size = 0.5

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(1/32\frac{b\left(28\,ac-b^2\right)x^{3/2}}{16\,a^2c^2-8\,ab^2c+b^4}+1/32\frac{\left(68\,a^2c^2+7\,ab^2c+3\,b^4\right)x^{7/2}}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)a}+3/16\frac{bc\left(8\,ac+b^2\right)x^{11/2}}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)a}+\frac{3}{(512}\right.$$
$$+\frac{3}{64\,a}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{c\left(12\,ac+b^2\right)\_R^6+b\left(-28\,ac+b^2\right)\_R^2}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(5/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*b*(28*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(3/2)+1/32*(6
8*a^2*c^2+7*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(7/2)+3
/16/a*c*b*(8*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)+3/32*c^
2*(12*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(15/2))/(c*x^4+b*x^
2+a)^2+3/64/a*sum((c*(12*a*c+b^2)*_R^6+b*(-28*a*c+b^2)*_R^2)/(16*
a^2*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf
(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^{2}c^{2}+12\,ac^{3}\right)x^{\frac{15}{2}}+6\left(b^{3}c+8\,abc^{2}\right)x^{\frac{11}{2}}+\left(3\,b^{4}+7\,ab^{2}c+68\,a^{2}c^{2}\right)x^{\frac{7}{2}}-\left(ab^{3}-28\,a^{2}bc\right)}{16\left((ab^{4}c^{2}-8\,a^{2}b^{2}c^{3}+16\,a^{3}c^{4})x^{8}+a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}+2\,(ab^{5}c-8\,a^{2}b^{3}c^{2}+16\,a^{3}bc^{3})x^{6}+(ab^{6}-6\,a^{2}b^{4}c+32\,a^{4}c^{3})x\right.}$$

$$+\int\frac{3\left(\left(b^{2}c+12\,ac^{2}\right)x^{\frac{5}{2}}+\left(b^{3}-28\,abc\right)\sqrt{x}\right)}{32\,(a^{2}b^{4}-8\,a^{3}b^{2}c+16\,a^{4}c^{2}+(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3})x^{4}+(ab^{5}-8\,a^{2}b^{3}c+16\,a^{3}bc^{2})x^{2})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^2*c^2 + 12*a*c^3)*x^(15/2) + 6*(b^3*c + 8*a*b*c^2)*x^(
11/2) + (3*b^4 + 7*a*b^2*c + 68*a^2*c^2)*x^(7/2) - (a*b^3 - 28*a^
2*b*c)*x^(3/2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 +
16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2) + integrate(3/32*((b^2*
c + 12*a*c^2)*x^(5/2) + (b^3 - 28*a*b*c)*sqrt(x))/(a^2*b^4 - 8*a^
3*b^2*c + 16*a^4*c^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*x^4
 + (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(5/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{5}{2}}}{(cx^4 + bx^2 + a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1086 $\int \frac{x^{3/2}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=594

$$-\frac{\sqrt{x}\,(b+2cx^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{\sqrt{x}\,(cx^2(44ac+b^2)+b(20ac+b^2))}{16a(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a(b^2-4ac)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\,(44ac+b^2)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a(b^2-4ac)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\,(44ac+b^2)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a(b^2-4ac)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
+ (Sqrt[x]*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/
Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*
(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3
 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b
^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b
^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/
4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c)
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))
```

Rubi [A] time = 4.68936, antiderivative size = 594, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{\sqrt{x}\left(b+2cx^2\right)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{\sqrt{x}\left(cx^2\left(44ac+b^2\right)+b\left(20ac+b^2\right)\right)}{16a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$
$$-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$-\frac{3c^{3/4}\left(\frac{68abc}{\sqrt{b^2-4ac}}-\frac{b^3}{\sqrt{b^2-4ac}}+44ac+b^2\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a\left(b^2-4ac\right)^2\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$
$$-\frac{3c^{3/4}\left(\sqrt{b^2-4ac}\left(44ac+b^2\right)-68abc+b^3\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -(Sqrt[x]*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2)
+ (Sqrt[x]*(b*(b^2 + 20*a*c) + c*(b^2 + 44*a*c)*x^2))/(16*a*(b^2
- 4*a*c)^2*(a + b*x^2 + c*x^4)) - (3*c^(3/4)*(b^2 + 44*a*c - b^3/
Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*
(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b^3
 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c))*ArcTan[(2^(1/4)*c
^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a*(b
^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(b
^2 + 44*a*c - b^3/Sqrt[b^2 - 4*a*c] + (68*a*b*c)/Sqrt[b^2 - 4*a*c
])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/
4)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^2*(-b - Sqrt[b^2 - 4*a*c])^(3/4)
) - (3*c^(3/4)*(b^3 - 68*a*b*c + Sqrt[b^2 - 4*a*c]*(b^2 + 44*a*c)
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4
)])/(32*2^(1/4)*a*(b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.612484, size = 224, normalized size = 0.38

$$\frac{3\left(a+bx^2+cx^4\right)^2\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{44\#1^4ac^2\log(\sqrt{x}-\#1)+\#1^4b^2c\log(\sqrt{x}-\#1)-12abc\log(\sqrt{x}-\#1)+b^3\log(\sqrt{x}-\#1)}{2\#1^7c+\#1^3b}\&\right]-16a}{64a\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x^(3/2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (-16*a*(b^2 - 4*a*c)*Sqrt[x]*(b + 2*c*x^2) + 4*Sqrt[x]*(b^3 + 20*
a*b*c + b^2*c*x^2 + 44*a*c^2*x^2)*(a + b*x^2 + c*x^4) + 3*(a + b*
x^2 + c*x^4)^2*RootSum[a + b*#1^4 + c*#1^8 & , (b^3*Log[Sqrt[x] -
 #1] - 12*a*b*c*Log[Sqrt[x] - #1] + b^2*c*Log[Sqrt[x] - #1]*#1^4
+ 44*a*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(64*a
*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)^2)
```

Maple [C] time = 0.048, size = 270, normalized size = 0.5

$$2\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(\frac{3b\left(12ac-b^2\right)\sqrt{x}}{512a^2c^2-256ab^2c+32b^4}+1/32\frac{\left(76a^2c^2+13ab^2c+b^4\right)x^{5/2}}{\left(16a^2c^2-8ab^2c+b^4\right)a}+1/16\frac{bc\left(32ac+b^2\right)x^{9/2}}{\left(16a^2c^2-8ab^2c+b^4\right)a}+1/32\right.$$

$$+\frac{3}{64a}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{c\left(44ac+b^2\right)\_R^4-12abc+b^3}{\left(16a^2c^2-8ab^2c+b^4\right)\left(2\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(3/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*b*(12*a*c-b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(1/2)+1/32*(7
6*a^2*c^2+13*a*b^2*c+b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(5/2)+1/
16/a*c*b*(32*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)+1/32*c^2
*(44*a*c+b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)/a*x^(13/2))/(c*x^4+b*x^2
+a)^2+3/64/a*sum((c*(44*a*c+b^2)*_R^4-12*a*b*c+b^3)/(16*a^2*c^2-8
*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z^8*c+_
Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(b^3c^2-12\,abc^3\right)x^{\frac{17}{2}}+\left(6\,b^4c-71\,ab^2c^2+44\,a^2c^3\right)x^{\frac{13}{2}}+\left(3\,b^5-28\,ab^3c-8\,a^2bc^2\right)x^{\frac{9}{2}}+\left(7\,ab^4-59\,a\right.}{16\left((a^2b^4c^2-8\,a^3b^2c^3+16\,a^4c^4)x^8+a^4b^4-8\,a^5b^2c+16\,a^6c^2+2\,(a^2b^5c-8\,a^3b^3c^2+16\,a^4bc^3)x^6+(a^2b^6-6\,a^3b^4c+32\,a^5c\right.}+\int-\frac{3\left(\left(b^3c-12\,abc^2\right)x^{\frac{7}{2}}+\left(b^4-13\,ab^2c-44\,a^2c^2\right)x^{\frac{3}{2}}\right)}{32\,(a^3b^4-8\,a^4b^2c+16\,a^5c^2+(a^2b^4c-8\,a^3b^2c^2+16\,a^4c^3)x^4+(a^2b^5-8\,a^3b^3c+16\,a^4bc^2)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(b^3*c^2 - 12*a*b*c^3)*x^(17/2) + (6*b^4*c - 71*a*b^2*c^2
 + 44*a^2*c^3)*x^(13/2) + (3*b^5 - 28*a*b^3*c - 8*a^2*b*c^2)*x^(9
/2) + (7*a*b^4 - 59*a^2*b^2*c + 76*a^3*c^2)*x^(5/2))/((a^2*b^4*c^
2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c + 16*
a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2
*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c +
 16*a^5*b*c^2)*x^2) + integrate(-3/32*((b^3*c - 12*a*b*c^2)*x^(7/
2) + (b^4 - 13*a*b^2*c - 44*a^2*c^2)*x^(3/2))/(a^3*b^4 - 8*a^4*b^
2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 +
 (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(3/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{\frac{3}{2}}}{(cx^4 + bx^2 + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(x^(3/2)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1087 $\int \frac{\sqrt{x}}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=658

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b
^2 - 44*a*c)*x^2))/(16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*
Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqr
t[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c
^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2
- 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4
- 54*a*b^2*c + 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^
(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b*(5*b^2 -
44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 10.3774, antiderivative size = 658, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{x^{3/2}\left(52a^2c^2+bcx^2\left(5b^2-44ac\right)-45ab^2c+5b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c-b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(520a^2c^2-54ab^2c+b\left(5b^2-44ac\right)\sqrt{b^2-4ac}+5b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\,2^{3/4}a^2\left(b^2-4ac\right)^{5/2}\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{x^{3/2}\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + b*c*(5*b
^2 - 44*a*c)*x^2))/(16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) -
 (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*
Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^
2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqr
t[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c
^2 + b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4
)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2
- 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(5*b^4
- 54*a*b^2*c + 520*a^2*c^2 - b*(5*b^2 - 44*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c])^(1/4
)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*a*c])^
```

```
(1/4)) - (c^(1/4)*(5*b^4 - 54*a*b^2*c + 520*a^2*c^2 + b*(5*b^2 -
44*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(3/4)*a^2*(b^2 - 4*a*c)^(5/2)*
(-b + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.797828, size = 254, normalized size = 0.39

$$\frac{\text{RootSum}\left[\#1^8 c+\#1^4 b+a\&,\frac{-44\#1^4 abc^2\log(\sqrt{x}-\#1)+5\#1^4 b^3 c\log(\sqrt{x}-\#1)+260a^2c^2\log(\sqrt{x}-\#1)-49ab^2c\log(\sqrt{x}-\#1)+5b^4\log(\sqrt{x}-\#1)}{2\#1^5c+\#1b}\&\right]+}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[x]/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*x^(3/2)*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^
2 + c*x^4)^2 + (4*x^(3/2)*(5*b^4 - 45*a*b^2*c + 52*a^2*c^2 + 5*b^
3*c*x^2 - 44*a*b*c^2*x^2))/(a + b*x^2 + c*x^4) + RootSum[a + b*#1
^4 + c*#1^8 & , (5*b^4*Log[Sqrt[x] - #1] - 49*a*b^2*c*Log[Sqrt[x]
 - #1] + 260*a^2*c^2*Log[Sqrt[x] - #1] + 5*b^3*c*Log[Sqrt[x] - #1
]*#1^4 - 44*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]
)/(64*a^2*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.077, size = 321, normalized size = 0.5

$$2\,\frac{1}{\left(cx^4+bx^2+a\right)^2}\left(\frac{\left(84\,a^2c^2-69\,ab^2c+9\,b^4\right)x^{3/2}}{\left(512\,a^2c^2-256\,ab^2c+32\,b^4\right)a}-1/32\,\frac{b\left(8\,a^2c^2+36\,ab^2c-5\,b^4\right)x^{7/2}}{a^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)}+1/32\,\frac{c\left(52\,a^2c^2-89\,ab^2c+10\right.}{a^2\left(16\,a^2c^2-8\,ab^2c+\right.}\right.$$

$$+\frac{1}{64\,a^2}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{bc\left(-44\,ac+5\,b^2\right)\_R^6+\left(260\,a^2c^2-49\,ab^2c+5\,b^4\right)\_R^2}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(3/32*(28*a^2*c^2-23*a*b^2*c+3*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/
a*x^(3/2)-1/32*b*(8*a^2*c^2+36*a*b^2*c-5*b^4)/a^2/(16*a^2*c^2-8*a
*b^2*c+b^4)*x^(7/2)+1/32/a^2*c*(52*a^2*c^2-89*a*b^2*c+10*b^4)/(16
*a^2*c^2-8*a*b^2*c+b^4)*x^(11/2)-1/32*c^2*b*(44*a*c-5*b^2)/a^2/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^(15/2))/(c*x^4+b*x^2+a)^2+1/64/a^2*sum
((b*c*(-44*a*c+5*b^2)*_R^6+(260*a^2*c^2-49*a*b^2*c+5*b^4)*_R^2)/(
16*a^2*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=Roo
tOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(5\,b^{3}c^{2}-44\,abc^{3}\right)x^{\frac{15}{2}}+\left(10\,b^{4}c-89\,ab^{2}c^{2}+52\,a^{2}c^{3}\right)x^{\frac{11}{2}}+\left(5\,b^{5}-36\,ab^{3}c-8\,a^{2}bc^{2}\right)x^{\frac{7}{2}}+3\left(3\,ab^{4}-23\right.}{16\left(\left(a^{2}b^{4}c^{2}-8\,a^{3}b^{2}c^{3}+16\,a^{4}c^{4}\right)x^{8}+a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}+2\left(a^{2}b^{5}c-8\,a^{3}b^{3}c^{2}+16\,a^{4}bc^{3}\right)x^{6}+\left(a^{2}b^{6}-6\,a^{3}b^{4}c+32\,a^{5}c^{3}\right.\right.}$$
$$-\int -\frac{\left(5\,b^{3}c-44\,abc^{2}\right)x^{\frac{5}{2}}+\left(5\,b^{4}-49\,ab^{2}c+260\,a^{2}c^{2}\right)\sqrt{x}}{32\left(a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}+\left(a^{2}b^{4}c-8\,a^{3}b^{2}c^{2}+16\,a^{4}c^{3}\right)x^{4}+\left(a^{2}b^{5}-8\,a^{3}b^{3}c+16\,a^{4}bc^{2}\right)x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*b^3*c^2 - 44*a*b*c^3)*x^(15/2) + (10*b^4*c - 89*a*b^2*c^
2 + 52*a^2*c^3)*x^(11/2) + (5*b^5 - 36*a*b^3*c - 8*a^2*b*c^2)*x^(
7/2) + 3*(3*a*b^4 - 23*a^2*b^2*c + 28*a^3*c^2)*x^(3/2))/((a^2*b^4
*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c +
16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (
a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*
c + 16*a^5*b*c^2)*x^2) - integrate(-1/32*((5*b^3*c - 44*a*b*c^2)*
x^(5/2) + (5*b^4 - 49*a*b^2*c + 260*a^2*c^2)*sqrt(x))/(a^3*b^4 -
8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^
3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(cx^4 + bx^2 + a)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(c*x^4 + b*x^2 + a)^3, x)
```

3.1088 $\int \frac{1}{\sqrt{x}(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=658

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b
^2 - 52*a*c)*x^2))/(16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c
)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a
^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^
(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(
b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(
7*b^4 - 66*a*b^2*c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2*c^2 + b*(
7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
```

```
)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 10.5664, antiderivative size = 658, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35

$$\frac{\sqrt{x}\left(60a^2c^2+bcx^2\left(7b^2-52ac\right)-55ab^2c+7b^4\right)}{16a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{3c^{3/4}\left(280a^2c^2-66ab^2c-b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{3c^{3/4}\left(280a^2c^2-66ab^2c+b\left(7b^2-52ac\right)\sqrt{b^2-4ac}+7b^4\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\sqrt{x}}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{32\sqrt[4]{2}a^2\left(b^2-4ac\right)^{5/2}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{\sqrt{x}\left(-2ac+b^2+bcx^2\right)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(4*a*(b^2 - 4*a*c)*(a + b*x^2 +
 c*x^4)^2) + (Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + b*c*(7*b
^2 - 52*a*c)*x^2))/(16*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) +
 (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c
)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[
b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - S
qrt[b^2 - 4*a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a
^2*c^2 + b*(7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^
(1/4)*Sqrt[x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(
```

```
b^2 - 4*a*c)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) + (3*c^(3/4)*(
7*b^4 - 66*a*b^2*c + 280*a^2*c^2 - b*(7*b^2 - 52*a*c)*Sqrt[b^2 -
4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[x])/(-b - Sqrt[b^2 - 4*a*c]
)^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c)^(5/2)*(-b - Sqrt[b^2 - 4*
a*c])^(3/4)) - (3*c^(3/4)*(7*b^4 - 66*a*b^2*c + 280*a^2*c^2 + b*(
7*b^2 - 52*a*c)*Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[
x])/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(32*2^(1/4)*a^2*(b^2 - 4*a*c
)^(5/2)*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi in Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Mathematica [C] time = 0.764115, size = 258, normalized size = 0.39

$$\frac{3\text{RootSum}\left[\#1^8c+\#1^4b+a\&,\frac{-52\#1^4abc^2\log\left(\sqrt{x}-\#1\right)+7\#1^4b^3c\log\left(\sqrt{x}-\#1\right)+140a^2c^2\log\left(\sqrt{x}-\#1\right)-59ab^2c\log\left(\sqrt{x}-\#1\right)+7b^4\log\left(\sqrt{x}-\#1\right)}{2\#1^7c+\#1^3b}\&\right]}{64a^2\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/(Sqrt[x]*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-16*a*(-b^2 + 4*a*c)*Sqrt[x]*(b^2 - 2*a*c + b*c*x^2))/(a + b*x^
2 + c*x^4)^2 + (4*Sqrt[x]*(7*b^4 - 55*a*b^2*c + 60*a^2*c^2 + 7*b^
3*c*x^2 - 52*a*b*c^2*x^2))/(a + b*x^2 + c*x^4) + 3*RootSum[a + b*
#1^4 + c*#1^8 & , (7*b^4*Log[Sqrt[x] - #1] - 59*a*b^2*c*Log[Sqrt[
x] - #1] + 140*a^2*c^2*Log[Sqrt[x] - #1] + 7*b^3*c*Log[Sqrt[x] -
#1]*#1^4 - 52*a*b*c^2*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7)
 & ])/(64*a^2*(b^2 - 4*a*c)^2)
```

Maple [C] time = 0.049, size = 316, normalized size = 0.5

$$2\frac{1}{(cx^4+bx^2+a)^2}\left(1/32\,\frac{\left(92\,a^2c^2-79\,ab^2c+11\,b^4\right)\sqrt{x}}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)a}-1/32\,\frac{b\left(8\,a^2c^2+44\,ab^2c-7\,b^4\right)x^{5/2}}{a^2\left(16\,a^2c^2-8\,ab^2c+b^4\right)}+1/32\,\frac{c\left(60\,a^2c^2-107\,ab^2\right.}{a^2\left(16\,a^2c^2-8\right.}\right.$$
$$+\frac{3}{64\,a^2}\sum_{\_R=RootOf\left(\_Z^8c+\_Z^4b+a\right)}\frac{bc\left(-52\,ac+7\,b^2\right)\_R^4+140\,a^2c^2-59\,ab^2c+7\,b^4}{\left(16\,a^2c^2-8\,ab^2c+b^4\right)\left(2\,\_R^7c+\_R^3b\right)}\ln\left(\sqrt{x}-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/x^(1/2)/(c*x^4+b*x^2+a)^3,x)
```

```
[Out] 2*(1/32*(92*a^2*c^2-79*a*b^2*c+11*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)
/a*x^(1/2)-1/32*b*(8*a^2*c^2+44*a*b^2*c-7*b^4)/a^2/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^(5/2)+1/32/a^2*c*(60*a^2*c^2-107*a*b^2*c+14*b^4)/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^(9/2)-1/32*c^2*b*(52*a*c-7*b^2)/a^2/(
16*a^2*c^2-8*a*b^2*c+b^4)*x^(13/2))/(c*x^4+b*x^2+a)^2+3/64/a^2*su
m((b*c*(-52*a*c+7*b^2)*_R^4+140*a^2*c^2-59*a*b^2*c+7*b^4)/(16*a^2
*c^2-8*a*b^2*c+b^4)/(2*_R^7*c+_R^3*b)*ln(x^(1/2)-_R),_R=RootOf(_Z
^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3\left(7\,b^4c^2-59\,ab^2c^3+140\,a^2c^4\right)x^{\frac{17}{2}}+\left(42\,b^5c-347\,ab^3c^2+788\,a^2bc^3\right)x^{\frac{13}{2}}+\left(21\,b^6-121\,ab^4c-41\,a^2b^2c^2+900\,a^3c^3\right)x^{\frac{9}{2}}+}{16\,(a^5b^4-8\,a^6b^2c+16\,a^7c^2+(a^3b^4c^2-8\,a^4b^2c^3+16\,a^5c^4)x^8+2\,(a^3b^5c-8\,a^4b^3c^2+16\,a^5bc^3)x^6+(a^3b^6-}$$
$$-\int\frac{3\left(\left(7\,b^4c-59\,ab^2c^2+140\,a^2c^3\right)x^{\frac{7}{2}}+\left(7\,b^5-66\,ab^3c+192\,a^2bc^2\right)x^{\frac{3}{2}}\right)}{32\,(a^4b^4-8\,a^5b^2c+16\,a^6c^2+(a^3b^4c-8\,a^4b^2c^2+16\,a^5c^3)x^4+(a^3b^5-8\,a^4b^3c+16\,a^5bc^2)x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*sqrt(x)),x, algorithm="maxima")
```

```
[Out] 1/16*(3*(7*b^4*c^2 - 59*a*b^2*c^3 + 140*a^2*c^4)*x^(17/2) + (42*b
^5*c - 347*a*b^3*c^2 + 788*a^2*b*c^3)*x^(13/2) + (21*b^6 - 121*a*
b^4*c - 41*a^2*b^2*c^2 + 900*a^3*c^3)*x^(9/2) + (49*a*b^5 - 398*a
^2*b^3*c + 832*a^3*b*c^2)*x^(5/2) + 32*(a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2)*sqrt(x))/(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2 + (a^3*b^
4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*x^8 + 2*(a^3*b^5*c - 8*a^4*b^
3*c^2 + 16*a^5*b*c^3)*x^6 + (a^3*b^6 - 6*a^4*b^4*c + 32*a^6*c^3)*
x^4 + 2*(a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^2) - integrate(3
/32*((7*b^4*c - 59*a*b^2*c^2 + 140*a^2*c^3)*x^(7/2) + (7*b^5 - 66
*a*b^3*c + 192*a^2*b*c^2)*x^(3/2))/(a^4*b^4 - 8*a^5*b^2*c + 16*a^
6*c^2 + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^4 + (a^3*b^5 -
 8*a^4*b^3*c + 16*a^5*b*c^2)*x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*sqrt(x)),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/x**(1/2)/(c*x**4+b*x**2+a)**3,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^3 \sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^3*sqrt(x)),x, algorithm="giac")

[Out] integrate(1/((c*x^4 + b*x^2 + a)^3*sqrt(x)), x)
```

3.1089 $\int (dx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2,
9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(5*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.552309, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2,
9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(5*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 31.355, size = 131, normalized size = 0.89

$$\frac{2\,(dx)^{\frac{5}{2}}\,\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},-\frac{1}{2},-\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*(d*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, -1/2, -1/2
, 9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-
```

```
4*a*c + b**2)))/(5*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 5.33786, size = 1048, normalized size = 7.13

$$d\sqrt{dx}\left(-\frac{25b\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)a^2}{5aF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{3}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx}{\sqrt{b^2-4a}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*Sqrt[d*x]*(10*c*(2*b + 5*c*x^2)*(a + b*x^2 + c*x^4)^2 - (25*a^
2*b*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*a*AppellF1[1/4, 1/
2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2
, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])) + (90*a^2*c*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c
*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9
/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/(9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b +
Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sq
rt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (27*a*b
^2*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-9*a*AppellF1[5/4
, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9
/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4
, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]))))/(225*c^2*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)

[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\sqrt{dx}dx, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}}\sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2), x)
```

3.1090 $\int \sqrt{dx}\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2,
7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.440227, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2,
7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 31.1868, size = 131, normalized size = 0.89

$$\frac{2\,(dx)^{\frac{3}{2}}\,\sqrt{a+bx^2+cx^4}\,\operatorname{appellf_1}\left(\frac{3}{4},-\frac{1}{2},-\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*(d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, -1/2, -1/2
, 7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-
```

```
4*a*c + b**2)))/(3*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 4.05014, size = 706, normalized size = 4.8

$$2x\sqrt{dx}\left(\frac{49a^2\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{c\left(7aF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{3}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\cdots\right)\right)\right)}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[d*x]*(21*(a + b*x^2 + c*x^4)^2 + (49*a^2*(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/
4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])])/(c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))) + (33*a*b*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(
22*a*c*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*c*x^2*((b + Sqrt[
b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b
^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(147*(a +
b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \sqrt{dx}\sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x),x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x),x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\sqrt{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x),x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x), x)
```

3.1091 $\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/
4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.443972, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/
4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
+ (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 31.5619, size = 129, normalized size = 0.89

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] 2*sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, -1/2, -1/2, 5
/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a
```

```
*c + b**2)))/(d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt
(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.11498, size = 709, normalized size = 4.89

$$2x\left(\frac{25a^2\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{c\left(5aF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{3}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*(a + b*x^2 + c*x^4)^2 + (25*a^2*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])])/(c*(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*
((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b
 - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (9
*a*b*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(2*c*(9*a*Appell
F1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*Appe
llF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])])))))/(25*Sqrt[d*x]*(a + b*x^2 + c*
x^4)^(3/2))
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int 1\sqrt{cx^4+bx^2+a}\frac{1}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/sqrt(d*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/sqrt(d*x), x)
```

3.1092 $\int \frac{\sqrt{a+bx^2+cx^4}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
])/(d*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.448933, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{2\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
])/(d*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 31.9225, size = 133, normalized size = 0.92

$$-\frac{2\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(-\frac{1}{4},-\frac{1}{2},-\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] -2*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, -1/2, -1/2, 3/4, -2*c
*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**
```

```
2)))/(d*sqrt(d*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.2384, size = 707, normalized size = 4.88

$$2x\left(\frac{49abx^2\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{2c\left(7aF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{3}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4a}}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[a + b*x^2 + c*x^4]/(d*x)^(3/2),x]
```

```
[Out] (2*x*(-21*(a + b*x^2 + c*x^4)^2 + (49*a*b*x^2*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/
2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(2*c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)]))) + (33*a*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(11*a*
AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c
])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c]
)*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(21*(d*x)^(3/2)*(a +
b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int 1\sqrt{cx^4+bx^2+a}\,(dx)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(1/2)/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^4 + bx^2 + a}}{\sqrt{dx}dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)/(sqrt(d*x)*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^2 + cx^4}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/(d*x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(d*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^4 + bx^2 + a}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/(d*x)^(3/2), x)
```

3.1093 $\int (dx)^{3/2} \left(a+bx^2+cx^4\right)^{3/2} \, dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2
, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(5*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.445966, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2a(dx)^{5/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*(d*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2
, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(5*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.9858, size = 133, normalized size = 0.9

$$\frac{2a\left(dx\right)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\operatorname{appellf_1}\left(\frac{5}{4},-\frac{3}{2},-\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*a*(d*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, -3/2, -3
/2, 9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt
```

```
(-4*a*c + b**2)))/(5*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 5.00813, size = 1751, normalized size = 11.83

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*Sqrt[d*x]*(5*c*(a + b*x^2 + c*x^4)^2*(-28*b^3 + 20*b^2*c*x^2
 + 65*c^2*x^2*(7*a + 3*c*x^4) + b*c*(176*a + 285*c*x^4)) + (175*a
^2*b^3*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*a*AppellF1[1/4,
 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/
4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4,
3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])])) - (1100*a^3*b*c*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])])/(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b
 + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (189*a
*b^4*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(9*a*AppellF1[5/
4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[
9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/
4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])])) + (2340*a^3*c^2*x^2*(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4,
1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])])/(9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] -
 x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
) + (1413*a^2*b^2*c*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(
```

```
-9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4
*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a
*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(16575*c^3*(a + b*x
^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int (dx)^{\frac{3}{2}} \left(c x^4 + b x^2 + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^4 + b x^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(c d x^5 + b d x^3 + a d x\right)\sqrt{c x^4 + b x^2 + a}\sqrt{dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*d*x^5 + b*d*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt
(d*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(3/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2), x)
```

3.1094 $\int \sqrt{dx}\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=148

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2
, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(3*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.435376, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2a(dx)^{3/2}\sqrt{a+bx^2+cx^4}F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```


```
[Out] (2*a*(d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2
, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c])])/(3*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*S
qrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 38.0809, size = 133, normalized size = 0.9

$$\frac{2a\,(dx)^{\frac{3}{2}}\,\sqrt{a+bx^2+cx^4}\,\operatorname{appellf}_1\left(\frac{3}{4},-\frac{3}{2},-\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*a*(d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, -3/2, -3
/2, 7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt
```

```
(-4*a*c + b**2)))/(3*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.28571, size = 1395, normalized size = 9.43

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[d*x]*(7*c*(a + b*x^2 + c*x^4)^2*(12*b^2 + 119*b*c*x^2 +
 11*c*(19*a + 7*c*x^4)) - (147*a^2*b^2*(b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2,
 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])])/(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b
+ Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b -
Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (2156
*a^3*c*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(7*a*AppellF1[3/4,
 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/
4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4,
 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])) + (1188*a^2*b*c*x^2*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/
2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])])/(11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
- x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)])) + (165*a*b^3*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-
11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 -
4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4
*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(8085*c^2*(a + b
*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \sqrt{dx}\,\left(c x^{4}+b x^{2}+a \right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)

[Out] int((d*x)^(1/2)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\sqrt{dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}\sqrt{dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x),x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx}\,\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} \sqrt{dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x), x)
```

3.1095 $\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{dx}}\,dx$

Optimal. Leaf size=146

$$\frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2,
5/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.439445, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*a*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2,
5/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[
1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.1618, size = 131, normalized size = 0.9

$$\frac{2a\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] 2*a*sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, -3/2, -3/2,
 5/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4
*a*c + b**2)))/(d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.10833, size = 1395, normalized size = 9.55

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/Sqrt[d*x],x]
```

```
[Out] (2*x*(5*c*(a + b*x^2 + c*x^4)^2*(4*b^2 + 25*b*c*x^2 + 3*c*(17*a +
 5*c*x^4)) - (25*a^2*b^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/
(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4
*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (900*a^3*c*(b - Sqrt[
b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF
1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])])/(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])])) + (252*a^2*b*c*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
/(9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 -
4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*
a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])) + (27*a*b^3*x^2*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*A
ppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-9*a*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/
2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2,
 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])]))))/(975*c^2*Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int 1 \left(c x^{4}+b \,x^{2}+a \right)^{\frac{3}{2}} \frac{1}{\sqrt{d x}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

```
[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}}{\sqrt{dx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x),x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(1/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/sqrt(d*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/sqrt(d*x), x)
```

3.1096 $\int \frac{\left(a+bx^2+cx^4\right)^{3/2}}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=146

$$-\frac{2a\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(d*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.450764, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$-\frac{2a\sqrt{a+bx^2+cx^4}F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(d*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 37.5935, size = 134, normalized size = 0.92

$$-\frac{2a\sqrt{a+bx^2+cx^4}\,\operatorname{appellf}_1\left(-\frac{1}{4},-\frac{3}{2},-\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] -2*a*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, -3/2, -3/2, 3/4, -2
*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b
**2)))/(d*sqrt(d*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*
sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.19354, size = 1059, normalized size = 7.25

$$2x\left(\frac{924a^2\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)x^4}{11aF_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{11}{4};\frac{1}{2},\frac{3}{2};\frac{15}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{11}{4};\frac{3}{2},\frac{1}{2};\frac{15}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4a}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^(3/2)/(d*x)^(3/2),x]
```

```
[Out] (2*x*(7*(a + b*x^2 + c*x^4)^2*(-77*a + 13*b*x^2 + 7*c*x^4) + (784
*a^2*b*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(c*(7*a*Appell
F1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*Appe
llF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*Appell
F1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) + (924*a^2*x^4*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4,
 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/(11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])])) + (33*a*b^2*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/
(c*(11*a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^
2 - 4*a*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))))/(539*(d*x)^
(3/2)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int 1 \left(c x^4+b x^2+a\right)^{\frac{3}{2}} (d x)^{-\frac{3}{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)

[Out] int((c*x^4+b*x^2+a)^(3/2)/(d*x)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^4+b x^2+a\right)^{\frac{3}{2}}}{(d x)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2),x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(c x^4+b x^2+a\right)^{\frac{3}{2}}}{\sqrt{d x} d x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2),x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^(3/2)/(sqrt(d*x)*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+b x^2+c x^4\right)^{\frac{3}{2}}}{(d x)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**(3/2)/(d*x)**(3/2),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(d*x)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4 + bx^2 + a\right)^{\frac{3}{2}}}{(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/(d*x)^(3/2), x)
```

3.1097 $\int \frac{(dx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(5*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.435469, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(5*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 31.9078, size = 129, normalized size = 0.88

$$\frac{2\,(dx)^{\frac{5}{2}}\,\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{5}{4},\frac{1}{2},\frac{1}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5ad\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*(d*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, 1/2, 1/2,
9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*
```

```
a*c + b**2)))/(5*a*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.298719, size = 386, normalized size = 2.63

$$-\frac{18a^2x(dx)^{3/2}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\cdots}{5\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^2+cx^4\right)^{3/2}\left(x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{9}{4};\frac{1}{2},\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^(3/2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-18*a^2*x*(d*x)^(3/2)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqr
t[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(5
*(b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x
^4)^(3/2)*(-9*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + S
qrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqr
t[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int 1\,(dx)^{\frac{3}{2}}\,\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}dx}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{3}{2}}}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**(3/2)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^{\frac{3}{2}}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.1098 $\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=147

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(3*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.445494, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(3*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 31.917, size = 129, normalized size = 0.88

$$\frac{2\,(dx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3ad\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*(d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, 1/2, 1/2,
7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*
```

```
a*c + b**2)))/(3*a*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.329395, size = 386, normalized size = 2.63

$$-\frac{14a^2x\sqrt{dx}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{3\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^2+cx^4\right)^{3/2}\left(x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[d*x]/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-14*a^2*x*Sqrt[d*x]*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*(
b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(a + b*x^2 + c*x^4
)^(3/2)*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqr
t[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[
b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.026, size = 0, normalized size = 0.

$$\int 1\sqrt{dx}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x)/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

3.1099 $\int \frac{1}{\sqrt{dx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4,
(-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/(d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.441832, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4,
(-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/(d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 32.3441, size = 128, normalized size = 0.88

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\text{appellf}_1\left(\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{ad\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] 2*sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, 1/2, 1/2, 5/4
, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sqrt
(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.310001, size = 384, normalized size = 2.65

$$-\frac{10a^2x\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F}{\sqrt{dx}\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(a+bx^2+cx^4)^{3/2}\left(x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-10*a^2*x*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2
 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*Sqrt[d*x]*(a + b*x^2 + c*x^4)^
(3/2)*(-5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[
b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2
 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt{dx}}\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4 + bx^2 + a}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)), x)
```

3.1100 $\int \frac{1}{(dx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=145

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/
(d*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.447976, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/
(d*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 32.4651, size = 131, normalized size = 0.9

$$-\frac{2\sqrt{a+bx^2+cx^4}\,\operatorname{appellf}_1\left(-\frac{1}{4},\frac{1}{2},\frac{1}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{ad\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -2*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, 1/2, 1/2, 3/4, -2*c*x
**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)
```

```
))/(a*d*sqrt(d*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*sq
rt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.23925, size = 710, normalized size = 4.9

$$2x\left(\frac{49abx^2\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{4c\left(7aF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{3}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4a\ldots}}\right.\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((d*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*(-21*(a + b*x^2 + c*x^4)^2 + (49*a*b*x^2*(b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/
2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])])/(4*c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)]))) + (99*a*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(44*a*
AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*x^2*((b + Sqrt[b^2 - 4*a
*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(21*a*(d*x)^(3/2)*(
a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int 1\,(dx)^{-\frac{3}{2}}\,\frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^4+bx^2+a}\sqrt{dx}dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/(sqrt(c*x^4 + b*x^2 + a)*sqrt(d*x)*d*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^4+bx^2+a}\,(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(d*x)^(3/2)), x)
```

3.1101 $\int \frac{(dx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(5*a*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.456447, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2(dx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(5*a*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 43.9088, size = 131, normalized size = 0.87

$$\frac{2\,(dx)^{\frac{5}{2}}\sqrt{a+bx^2+cx^4}\,\operatorname{appellf}_1\left(\frac{5}{4},\frac{3}{2},\frac{3}{2},\frac{9}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5a^2d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*(d*x)**(5/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(5/4, 3/2, 3/2,
9/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*
```

```
a*c + b**2)))/(5*a**2*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 1.79187, size = 720, normalized size = 4.8

$$d\sqrt{dx}\left(\frac{9ax^2\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)}{18aF_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{9}{4};\frac{1}{2},\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{9}{4};\frac{3}{2},\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2c}{\sqrt{b^2-}}\right.\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^(3/2)/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*Sqrt[d*x]*(-5*(b + 2*c*x^2)*(a + b*x^2 + c*x^4) + (25*a*b*(b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*A
ppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (
2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(4*c*(5*a*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] - x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2
, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1
/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))) + (9*a*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])])/(18*a*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt
[b^2 - 4*a*c])*AppellF1[9/4, 1/2, 3/2, 13/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b
^2 - 4*a*c])*AppellF1[9/4, 3/2, 1/2, 13/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(5*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int 1\,(dx)^{\frac{3}{2}}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}dx}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)*d*x/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**(3/2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^{\frac{3}{2}}}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.1102 $\int \frac{\sqrt{dx}}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.441549, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2(dx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*(d*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4
, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(3*a*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 43.4117, size = 131, normalized size = 0.87

$$\frac{2\,(dx)^{\frac{3}{2}}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{3}{4},\frac{3}{2},\frac{3}{2},\frac{7}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3a^2d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*(d*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)*appellf1(3/4, 3/2, 3/2,
7/4, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*
```

```
a*c + b**2)))/(3*a**2*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) +
 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.20803, size = 1058, normalized size = 7.05

$$x\sqrt{dx}\left(\frac{196\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)a^2}{14aF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{1}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{7}{4};\frac{3}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\cdots\right)\cdots\right)}\cdots\right)$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[Sqrt[d*x]/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[d*x]*(-84*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4) + (
196*a^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(14*a*AppellF1[3/
4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF
1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x
^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[
7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])])) + (49*a*b^2*(b - Sqrt[b^2 - 4*a*c]
+ 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1
/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])])/(c*(7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^
2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] +
 (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
- (99*a*b*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-11*a*App
ellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2
*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*
AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*A
ppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))/(84*a*(-b^2 + 4*a*c)*(a
+ b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int 1\sqrt{dx}\left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(cx^4 + bx^2 + a)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{dx}}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(a + bx^2 + cx^4)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(d*x)/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{dx}}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.1103 $\int \frac{1}{\sqrt{dx}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=148

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4,
(-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/(a*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.441563, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083

$$\frac{2\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*Sqrt[d*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4,
(-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c])])/(a*d*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 41.9585, size = 129, normalized size = 0.87

$$\frac{2\sqrt{dx}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{5}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2d\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] 2*sqrt(d*x)*sqrt(a + b*x**2 + c*x**4)*appellf1(1/4, 3/2, 3/2, 5/4
, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c
```

```
 + b**2)))/(a**2*d*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)*s
qrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.03135, size = 1058, normalized size = 7.15

$$x\left(\frac{300\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)a^2}{10aF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{1}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{5}{4};\frac{3}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/(Sqrt[d*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(-20*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4) + (300*a^2*(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(10*a*AppellF1[1/4, 1/2, 1/
2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 1/2
, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[5/4, 3/2, 1
/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])])) - (25*a*b^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[1/4, 1/2, 1/2, 5/4, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])])/(c*(5*a*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - x^2*((b + Sqrt
[b^2 - 4*a*c])*AppellF1[5/4, 1/2, 3/2, 9/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^
2 - 4*a*c])*AppellF1[5/4, 3/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (9*a*b*x^2*(
b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
)*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(-9*a*AppellF1[5/4, 1/2, 1
/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 1/2,
 3/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[9/4, 3/2, 1
/2, 13/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])]))))/(20*a*(-b^2 + 4*a*c)*Sqrt[d*x]*(a + b*x^2 +
c*x^4)^(3/2))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int 1\frac{1}{\sqrt{dx}}\left(c x^{4}+b x^{2}+a\right)^{-\frac{3}{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(c x^{4}+b x^{2}+a\right)^{\frac{3}{2}}\sqrt{dx}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)),x, algorithm="fricas")
```

```
[Out] integral(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{dx}\left(a+b x^{2}+c x^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(d*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}}\sqrt{dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(d*x)), x)
```

3.1104 $\int \frac{1}{(dx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=148

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/
(a*d*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.445524, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{2\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad\sqrt{dx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^
2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/
(a*d*Sqrt[d*x]*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 42.7389, size = 133, normalized size = 0.9

$$-\frac{2\sqrt{a+bx^2+cx^4}\,\operatorname{appellf}_1\left(-\frac{1}{4},\frac{3}{2},\frac{3}{2},\frac{3}{4},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2d\sqrt{dx}\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] -2*sqrt(a + b*x**2 + c*x**4)*appellf1(-1/4, 3/2, 3/2, 3/4, -2*c*x
**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)
```

```
))/(a**2*d*sqrt(d*x)*sqrt(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)
*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 3.84863, size = 1600, normalized size = 10.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In]  Integrate[1/((d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*((7*x^2*(b^3 - 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2)*(a + b*x^2 +
 c*x^4))/(a^2*(-b^2 + 4*a*c)) - (14*(a + b*x^2 + c*x^4)^2)/a^2 +
(49*b^3*x^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a
*c] + 2*c*x^2)*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]])/((b^2 - 4*a*c
)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-7*a*AppellF1
[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Appell
F1[7/4, 1/2, 3/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1
[7/4, 3/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (147*a*b*c*x^2*(b - Sqrt[b^2 -
4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[3/4,
 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
 + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(
b + Sqrt[b^2 - 4*a*c])*(-7*a*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 1/2, 3/2, 11/4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[7/4, 3/2, 1/2, 11/4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
))) + (99*b^2*c*x^4*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((b^
2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-11*
a*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a
*c])*AppellF1[11/4, 1/2, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[11/4, 3/2, 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))) - (330*a*c^2*x^4*(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*
AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((b^2 - 4*a*c)*(-b + Sqrt[b
^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(-11*a*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 1/2
, 3/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[11/4, 3/2,
```

```
 1/2, 15/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])]))))))/(7*(d*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int 1\,(dx)^{-\frac{3}{2}}\left(cx^{4}+bx^{2}+a\right)^{-\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int(1/(d*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^{4}+bx^{2}+a\right)^{\frac{3}{2}}(dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cdx^{5}+bdx^{3}+adx)\sqrt{cx^{4}+bx^{2}+a}\sqrt{dx}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)),x, algorithm="fricas")
```

```
[Out] integral(1/((c*d*x^5 + b*d*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*s
qrt(d*x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx)^{\frac{3}{2}} (a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(d*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/((d*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^4 + bx^2 + a)^{\frac{3}{2}} (dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)),x, algorithm="giac")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^(3/2)), x)
```

3.1105 $\int (dx)^m \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=156

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(dx)^{m+13}}{d^{13}(m+13)}$$

```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3
 + m)) + (3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2
+ 6*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9
 + m))/(d^9*(9 + m)) + (3*b*c^2*(d*x)^(11 + m))/(d^11*(11 + m)) +
 (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi [A] time = 0.215905, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{a^3(dx)^{m+1}}{d(m+1)}+\frac{3a^2b(dx)^{m+3}}{d^3(m+3)}+\frac{3c\left(ac+b^2\right)(dx)^{m+9}}{d^9(m+9)}+\frac{b\left(6ac+b^2\right)(dx)^{m+7}}{d^7(m+7)}+\frac{3a\left(ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{3bc^2(dx)^{m+11}}{d^{11}(m+11)}+\frac{c^3(dx)^{m+13}}{d^{13}(m+13)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*(d*x)^(1 + m))/(d*(1 + m)) + (3*a^2*b*(d*x)^(3 + m))/(d^3*(3
 + m)) + (3*a*(b^2 + a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (b*(b^2
+ 6*a*c)*(d*x)^(7 + m))/(d^7*(7 + m)) + (3*c*(b^2 + a*c)*(d*x)^(9
 + m))/(d^9*(9 + m)) + (3*b*c^2*(d*x)^(11 + m))/(d^11*(11 + m)) +
 (c^3*(d*x)^(13 + m))/(d^13*(13 + m))
```

Rubi in Sympy [A] time = 35.4592, size = 143, normalized size = 0.92

$$\frac{a^3\left(dx\right)^{m+1}}{d\left(m+1\right)}+\frac{3a^2b\left(dx\right)^{m+3}}{d^3\left(m+3\right)}+\frac{3a\left(dx\right)^{m+5}\left(ac+b^2\right)}{d^5\left(m+5\right)}+\frac{3bc^2\left(dx\right)^{m+11}}{d^{11}\left(m+11\right)}+\frac{b\left(dx\right)^{m+7}\left(6ac+b^2\right)}{d^7\left(m+7\right)}+\frac{c^3\left(dx\right)^{m+13}}{d^{13}\left(m+13\right)}+\frac{3c\left(dx\right)^{m+9}\left(ac+b^2\right)}{d^9\left(m+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] a**3*(d*x)**(m + 1)/(d*(m + 1)) + 3*a**2*b*(d*x)**(m + 3)/(d**3*(
m + 3)) + 3*a*(d*x)**(m + 5)*(a*c + b**2)/(d**5*(m + 5)) + 3*b*c*
*2*(d*x)**(m + 11)/(d**11*(m + 11)) + b*(d*x)**(m + 7)*(6*a*c + b
**2)/(d**7*(m + 7)) + c**3*(d*x)**(m + 13)/(d**13*(m + 13)) + 3*c
*(d*x)**(m + 9)*(a*c + b**2)/(d**9*(m + 9))
```

Mathematica [A] time = 0.153286, size = 111, normalized size = 0.71

$$(dx)^m\left(\frac{a^3x}{m+1}+\frac{3a^2bx^3}{m+3}+\frac{3cx^9\left(ac+b^2\right)}{m+9}+\frac{bx^7\left(6ac+b^2\right)}{m+7}+\frac{3ax^5\left(ac+b^2\right)}{m+5}+\frac{3bc^2x^{11}}{m+11}+\frac{c^3x^{13}}{m+13}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (d*x)^m*((a^3*x)/(1 + m) + (3*a^2*b*x^3)/(3 + m) + (3*a*(b^2 + a*
c)*x^5)/(5 + m) + (b*(b^2 + 6*a*c)*x^7)/(7 + m) + (3*c*(b^2 + a*c
)*x^9)/(9 + m) + (3*b*c^2*x^11)/(11 + m) + (c^3*x^13)/(13 + m))
```

Maple [B] time = 0.011, size = 782, normalized size = 5.

$$\frac{\left(c^3m^6x^{12}+36\,c^3m^5x^{12}+3\,bc^2m^6x^{10}+505\,c^3m^4x^{12}+114\,bc^2m^5x^{10}+3480\,c^3m^3x^{12}+3\,ac^2m^6x^8+3\,b^2cm^6x^8+1665\,bc^2m^4x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] x*(c^3*m^6*x^12+36*c^3*m^5*x^12+3*b*c^2*m^6*x^10+505*c^3*m^4*x^12
+114*b*c^2*m^5*x^10+3480*c^3*m^3*x^12+3*a*c^2*m^6*x^8+3*b^2*c*m^6
*x^8+1665*b*c^2*m^4*x^10+12139*c^3*m^2*x^12+120*a*c^2*m^5*x^8+120
*b^2*c*m^5*x^8+11820*b*c^2*m^3*x^10+19524*c^3*m*x^12+6*a*b*c*m^6*
x^6+1839*a*c^2*m^4*x^8+b^3*m^6*x^6+1839*b^2*c*m^4*x^8+42117*b*c^2
*m^2*x^10+10395*c^3*x^12+252*a*b*c*m^5*x^6+13584*a*c^2*m^3*x^8+42
*b^3*m^5*x^6+13584*b^2*c*m^3*x^8+68706*b*c^2*m*x^10+3*a^2*c*m^6*x
^4+3*a*b^2*m^6*x^4+4074*a*b*c*m^4*x^6+49881*a*c^2*m^2*x^8+679*b^3
*m^4*x^6+49881*b^2*c*m^2*x^8+36855*b*c^2*x^10+132*a^2*c*m^5*x^4+1
32*a*b^2*m^5*x^4+31752*a*b*c*m^3*x^6+83064*a*c^2*m*x^8+5292*b^3*m
^3*x^6+83064*b^2*c*m*x^8+3*a^2*b*m^6*x^2+2259*a^2*c*m^4*x^4+2259*
a*b^2*m^4*x^4+122010*a*b*c*m^2*x^6+45045*a*c^2*x^8+20335*b^3*m^2*
x^6+45045*b^2*c*x^8+138*a^2*b*m^5*x^2+18840*a^2*c*m^3*x^4+18840*a
```

```
*b^2*m^3*x^4+209916*a*b*c*m*x^6+34986*b^3*m*x^6+a^3*m^6+2505*a^2*
b*m^4*x^2+77937*a^2*c*m^2*x^4+77937*a*b^2*m^2*x^4+115830*a*b*c*x^
6+19305*b^3*x^6+48*a^3*m^5+22620*a^2*b*m^3*x^2+142308*a^2*c*m*x^4
+142308*a*b^2*m*x^4+925*a^3*m^4+104277*a^2*b*m^2*x^2+81081*a^2*c*
x^4+81081*a*b^2*x^4+9120*a^3*m^3+219162*a^2*b*m*x^2+48259*a^3*m^2
+135135*a^2*b*x^2+129072*a^3*m+135135*a^3)*(d*x)^m/(13+m)/(11+m)/
(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.317971, size = 802, normalized size = 5.14

$$\frac{\left(\left(c^3m^6 + 36\,c^3m^5 + 505\,c^3m^4 + 3480\,c^3m^3 + 12139\,c^3m^2 + 19524\,c^3m + 10395\,c^3\right)x^{13} + 3\,\left(bc^2m^6 + 38\,bc^2m^5 + 555\,bc^2m^4 + 3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^3*m^6 + 36*c^3*m^5 + 505*c^3*m^4 + 3480*c^3*m^3 + 12139*c^3*m
^2 + 19524*c^3*m + 10395*c^3)*x^13 + 3*(b*c^2*m^6 + 38*b*c^2*m^5
+ 555*b*c^2*m^4 + 3940*b*c^2*m^3 + 14039*b*c^2*m^2 + 22902*b*c^2*
m + 12285*b*c^2)*x^11 + 3*((b^2*c + a*c^2)*m^6 + 40*(b^2*c + a*c^
2)*m^5 + 613*(b^2*c + a*c^2)*m^4 + 4528*(b^2*c + a*c^2)*m^3 + 150
15*b^2*c + 15015*a*c^2 + 16627*(b^2*c + a*c^2)*m^2 + 27688*(b^2*c
 + a*c^2)*m)*x^9 + ((b^3 + 6*a*b*c)*m^6 + 42*(b^3 + 6*a*b*c)*m^5
+ 679*(b^3 + 6*a*b*c)*m^4 + 5292*(b^3 + 6*a*b*c)*m^3 + 19305*b^3
+ 115830*a*b*c + 20335*(b^3 + 6*a*b*c)*m^2 + 34986*(b^3 + 6*a*b*c
)*m)*x^7 + 3*((a*b^2 + a^2*c)*m^6 + 44*(a*b^2 + a^2*c)*m^5 + 753*
(a*b^2 + a^2*c)*m^4 + 6280*(a*b^2 + a^2*c)*m^3 + 27027*a*b^2 + 27
027*a^2*c + 25979*(a*b^2 + a^2*c)*m^2 + 47436*(a*b^2 + a^2*c)*m)*
x^5 + 3*(a^2*b*m^6 + 46*a^2*b*m^5 + 835*a^2*b*m^4 + 7540*a^2*b*m^
3 + 34759*a^2*b*m^2 + 73054*a^2*b*m + 45045*a^2*b)*x^3 + (a^3*m^6
 + 48*a^3*m^5 + 925*a^3*m^4 + 9120*a^3*m^3 + 48259*a^3*m^2 + 1290
72*a^3*m + 135135*a^3)*x)*(d*x)^m/(m^7 + 49*m^6 + 973*m^5 + 10045
*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [A] time = 20.6481, size = 4451, normalized size = 28.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3/(12*x**12) - 3*a**2*b/(10*x**10) - 3*a**2*c/(8*
x**8) - 3*a*b**2/(8*x**8) - a*b*c/x**6 - 3*a*c**2/(4*x**4) - b**3
/(6*x**6) - 3*b**2*c/(4*x**4) - 3*b*c**2/(2*x**2) + c**3*log(x))/
d**13, Eq(m, -13)), ((-a**3/(10*x**10) - 3*a**2*b/(8*x**8) - a**2
*c/(2*x**6) - a*b**2/(2*x**6) - 3*a*b*c/(2*x**4) - 3*a*c**2/(2*x*
*2) - b**3/(4*x**4) - 3*b**2*c/(2*x**2) + 3*b*c**2*log(x) + c**3*
x**2/2)/d**11, Eq(m, -11)), ((-a**3/(8*x**8) - a**2*b/(2*x**6) -
3*a**2*c/(4*x**4) - 3*a*b**2/(4*x**4) - 3*a*b*c/x**2 + 3*a*c**2*l
og(x) - b**3/(2*x**2) + 3*b**2*c*log(x) + 3*b*c**2*x**2/2 + c**3*
x**4/4)/d**9, Eq(m, -9)), ((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) -
3*a**2*c/(2*x**2) - 3*a*b**2/(2*x**2) + 6*a*b*c*log(x) + 3*a*c**2
*x**2/2 + b**3*log(x) + 3*b**2*c*x**2/2 + 3*b*c**2*x**4/4 + c**3*
x**6/6)/d**7, Eq(m, -7)), ((-a**3/(4*x**4) - 3*a**2*b/(2*x**2) +
3*a**2*c*log(x) + 3*a*b**2*log(x) + 3*a*b*c*x**2 + 3*a*c**2*x**4/
4 + b**3*x**2/2 + 3*b**2*c*x**4/4 + b*c**2*x**6/2 + c**3*x**8/8)/
d**5, Eq(m, -5)), ((-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a**2*c*x
**2/2 + 3*a*b**2*x**2/2 + 3*a*b*c*x**4/2 + a*c**2*x**6/2 + b**3*x
**4/4 + b**2*c*x**6/2 + 3*b*c**2*x**8/8 + c**3*x**10/10)/d**3, Eq
(m, -3)), ((a**3*log(x) + 3*a**2*b*x**2/2 + 3*a**2*c*x**4/4 + 3*a
*b**2*x**4/4 + a*b*c*x**6 + 3*a*c**2*x**8/8 + b**3*x**6/6 + 3*b**
2*c*x**8/8 + 3*b*c**2*x**10/10 + c**3*x**12/12)/d, Eq(m, -1)), (a
**3*d**m*m**6*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57
379*m**3 + 177331*m**2 + 264207*m + 135135) + 48*a**3*d**m*m**5*x
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 1773
31*m**2 + 264207*m + 135135) + 925*a**3*d**m*m**4*x*x**m/(m**7 +
49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2642
07*m + 135135) + 9120*a**3*d**m*m**3*x*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 48259*a**3*d**m*m**2*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 129072*a
**3*d**m*m*x*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379
*m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**3*d**m*x*x**
m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m
**2 + 264207*m + 135135) + 3*a**2*b*d**m*m**6*x**3*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
7*m + 135135) + 138*a**2*b*d**m*m**5*x**3*x**m/(m**7 + 49*m**6 +
973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135
135) + 2505*a**2*b*d**m*m**4*x**3*x**m/(m**7 + 49*m**6 + 973*m**5
 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 2
2620*a**2*b*d**m*m**3*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 1004
5*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 104277*a
**2*b*d**m*m**2*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
```

+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 219162*a**2*b*
d**m*m*x**3*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*
m**3 + 177331*m**2 + 264207*m + 135135) + 135135*a**2*b*d**m*x**3
*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 1773
31*m**2 + 264207*m + 135135) + 3*a**2*c*d**m*m**6*x**5*x**m/(m**7
 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 2
64207*m + 135135) + 132*a**2*c*d**m*m**5*x**5*x**m/(m**7 + 49*m**
6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m +
 135135) + 2259*a**2*c*d**m*m**4*x**5*x**m/(m**7 + 49*m**6 + 973*
m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135)
 + 18840*a**2*c*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 +
10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 7793
7*a**2*c*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 142308*a**2
*c*d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 81081*a**2*c*d**m*x*
*5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + 3*a*b**2*d**m*m**6*x**5*x**m/(m*
*7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
 264207*m + 135135) + 132*a*b**2*d**m*m**5*x**5*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135) + 2259*a*b**2*d**m*m**4*x**5*x**m/(m**7 + 49*m**6 + 97
3*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 13513
5) + 18840*a*b**2*d**m*m**3*x**5*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 77
937*a*b**2*d**m*m**2*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045
*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 142308*a*
b**2*d**m*m*x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5
7379*m**3 + 177331*m**2 + 264207*m + 135135) + 81081*a*b**2*d**m*
x**5*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 6*a*b*c*d**m*m**6*x**7*x**m/(m
7 + 49*m6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
+ 264207*m + 135135) + 252*a*b*c*d**m*m**5*x**7*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
 + 135135) + 4074*a*b*c*d**m*m**4*x**7*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 31752*a*b*c*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 +
10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1220
10*a*b*c*d**m*m**2*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 209916*a*b*
c*d**m*m*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 115830*a*b*c*d**m*x**
7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177
331*m**2 + 264207*m + 135135) + 3*a*c**2*d**m*m**6*x**9*x**m/(m**
7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 +
264207*m + 135135) + 120*a*c**2*d**m*m**5*x**9*x**m/(m**7 + 49*m*
*6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m
+ 135135) + 1839*a*c**2*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 13584*a*c**2*d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 +
 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 498
81*a*c**2*d**m*m**2*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*
m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 83064*a*c*
*2*d**m*m*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573

79*m**3 + 177331*m**2 + 264207*m + 135135) + 45045*a*c**2*d**m*x*
*9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 17
7331*m**2 + 264207*m + 135135) + b**3*d**m*m**6*x**7*x**m/(m**7 +
 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264
207*m + 135135) + 42*b**3*d**m*m**5*x**7*x**m/(m**7 + 49*m**6 + 9
73*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 679*b**3*d**m*m**4*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 1
0045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 5292*
b**3*d**m*m**3*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 20335*b**3*d**m
*m**2*x**7*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m
3 + 177331*m2 + 264207*m + 135135) + 34986*b**3*d**m*m*x**7*x
m/(m7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331
*m**2 + 264207*m + 135135) + 19305*b**3*d**m*x**7*x**m/(m**7 + 49
*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207
*m + 135135) + 3*b**2*c*d**m*m**6*x**9*x**m/(m**7 + 49*m**6 + 973
*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135
) + 120*b**2*c*d**m*m**5*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 1
0045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 1839*
b**2*c*d**m*m**4*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**
4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 13584*b**2*c*
d**m*m**3*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 49881*b**2*c*d**m*m*
*2*x**9*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
 + 177331*m**2 + 264207*m + 135135) + 83064*b**2*c*d**m*m*x**9*x*
*m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*
m**2 + 264207*m + 135135) + 45045*b**2*c*d**m*x**9*x**m/(m**7 + 4
9*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 26420
7*m + 135135) + 3*b*c**2*d**m*m**6*x**11*x**m/(m**7 + 49*m**6 + 9
73*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 1351
35) + 114*b*c**2*d**m*m**5*x**11*x**m/(m**7 + 49*m**6 + 973*m**5
+ 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 16
65*b*c**2*d**m*m**4*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045
*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 11820*b*c
2*dm*m**3*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4
+ 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 42117*b*c**2*d*
*m*m**2*x**11*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 5737
9*m**3 + 177331*m**2 + 264207*m + 135135) + 68706*b*c**2*d**m*m*x
11*xm/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 +
177331*m**2 + 264207*m + 135135) + 36855*b*c**2*d**m*x**11*x**m/(
m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2
 + 264207*m + 135135) + c**3*d**m*m**6*x**13*x**m/(m**7 + 49*m**6
 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m +
135135) + 36*c**3*d**m*m**5*x**13*x**m/(m**7 + 49*m**6 + 973*m**5
 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m + 135135) + 5
05*c**3*d**m*m**4*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m
4 + 57379*m3 + 177331*m**2 + 264207*m + 135135) + 3480*c**3*d
m*m3*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 573
79*m**3 + 177331*m**2 + 264207*m + 135135) + 12139*c**3*d**m*m**2
*x**13*x**m/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3
+ 177331*m**2 + 264207*m + 135135) + 19524*c**3*d**m*m*x**13*x**m
/(m**7 + 49*m**6 + 973*m**5 + 10045*m**4 + 57379*m**3 + 177331*m*
*2 + 264207*m + 135135) + 10395*c**3*d**m*x**13*x**m/(m**7 + 49*m
6 + 973*m5 + 10045*m**4 + 57379*m**3 + 177331*m**2 + 264207*m

```
+ 135135), True))
```

GIAC/XCAS [A] time = 0.275012, size = 1, normalized size = 0.01

$$Done$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^3*(d*x)^m,x, algorithm="giac")
```

```
[Out] Done
```

3.1106 $\int (dx)^m \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=101

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 +
 m)) + ((b^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)
^(7 + m))/(d^7*(7 + m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rubi [A] time = 0.119178, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{\left(2ac+b^2\right)(dx)^{m+5}}{d^5(m+5)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*(d*x)^(1 + m))/(d*(1 + m)) + (2*a*b*(d*x)^(3 + m))/(d^3*(3 +
 m)) + ((b^2 + 2*a*c)*(d*x)^(5 + m))/(d^5*(5 + m)) + (2*b*c*(d*x)
^(7 + m))/(d^7*(7 + m)) + (c^2*(d*x)^(9 + m))/(d^9*(9 + m))
```

Rubi in Sympy [A] time = 23.2288, size = 90, normalized size = 0.89

$$\frac{a^2(dx)^{m+1}}{d(m+1)}+\frac{2ab(dx)^{m+3}}{d^3(m+3)}+\frac{2bc(dx)^{m+7}}{d^7(m+7)}+\frac{c^2(dx)^{m+9}}{d^9(m+9)}+\frac{(dx)^{m+5}\left(2ac+b^2\right)}{d^5(m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] a**2*(d*x)**(m + 1)/(d*(m + 1)) + 2*a*b*(d*x)**(m + 3)/(d**3*(m +
 3)) + 2*b*c*(d*x)**(m + 7)/(d**7*(m + 7)) + c**2*(d*x)**(m + 9)/
(d**9*(m + 9)) + (d*x)**(m + 5)*(2*a*c + b**2)/(d**5*(m + 5))
```

Mathematica [A] time = 0.0788425, size = 70, normalized size = 0.69

$$(dx)^m\left(\frac{a^2x}{m+1}+\frac{x^5\left(2ac+b^2\right)}{m+5}+\frac{2abx^3}{m+3}+\frac{2bcx^7}{m+7}+\frac{c^2x^9}{m+9}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (d*x)^m*((a^2*x)/(1 + m) + (2*a*b*x^3)/(3 + m) + ((b^2 + 2*a*c)*x
^5)/(5 + m) + (2*b*c*x^7)/(7 + m) + (c^2*x^9)/(9 + m))
```

Maple [B] time = 0.009, size = 301, normalized size = 3.

$$\frac{\left(c^2m^4x^8+16\,c^2m^3x^8+2\,bcm^4x^6+86\,c^2m^2x^8+36\,bcm^3x^6+176\,c^2mx^8+2\,acm^4x^4+b^2m^4x^4+208\,bcm^2x^6+105\,c^2x^8+40}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(c^2*m^4*x^8+16*c^2*m^3*x^8+2*b*c*m^4*x^6+86*c^2*m^2*x^8+36*b*c
*m^3*x^6+176*c^2*m*x^8+2*a*c*m^4*x^4+b^2*m^4*x^4+208*b*c*m^2*x^6+
105*c^2*x^8+40*a*c*m^3*x^4+20*b^2*m^3*x^4+444*b*c*m*x^6+2*a*b*m^4
*x^2+260*a*c*m^2*x^4+130*b^2*m^2*x^4+270*b*c*x^6+44*a*b*m^3*x^2+6
00*a*c*m*x^4+300*b^2*m*x^4+a^2*m^4+328*a*b*m^2*x^2+378*a*c*x^4+18
9*b^2*x^4+24*a^2*m^3+916*a*b*m*x^2+206*a^2*m^2+630*a*b*x^2+744*a^
2*m+945*a^2)*(d*x)^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.298807, size = 325, normalized size = 3.22

$$\frac{\left(\left(c^2m^4 + 16\,c^2m^3 + 86\,c^2m^2 + 176\,c^2m + 105\,c^2\right)x^9 + 2\left(bcm^4 + 18\,bcm^3 + 104\,bcm^2 + 222\,bcm + 135\,bc\right)x^7 + \left(\left(b^2 + 2\,ac\right)m^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c^2*m^4 + 16*c^2*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^9 +
2*(b*c*m^4 + 18*b*c*m^3 + 104*b*c*m^2 + 222*b*c*m + 135*b*c)*x^7
+ ((b^2 + 2*a*c)*m^4 + 20*(b^2 + 2*a*c)*m^3 + 130*(b^2 + 2*a*c)*m
^2 + 189*b^2 + 378*a*c + 300*(b^2 + 2*a*c)*m)*x^5 + 2*(a*b*m^4 +
22*a*b*m^3 + 164*a*b*m^2 + 458*a*b*m + 315*a*b)*x^3 + (a^2*m^4 +
24*a^2*m^3 + 206*a^2*m^2 + 744*a^2*m + 945*a^2)*x)*(d*x)^m/(m^5 +
 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [A] time = 8.39635, size = 1486, normalized size = 14.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2/(8*x**8) - a*b/(3*x**6) - a*c/(2*x**4) - b**2/(
4*x**4) - b*c/x**2 + c**2*log(x))/d**9, Eq(m, -9)), ((-a**2/(6*x*
*6) - a*b/(2*x**4) - a*c/x**2 - b**2/(2*x**2) + 2*b*c*log(x) + c*
*2*x**2/2)/d**7, Eq(m, -7)), ((-a**2/(4*x**4) - a*b/x**2 + 2*a*c*
log(x) + b**2*log(x) + b*c*x**2 + c**2*x**4/4)/d**5, Eq(m, -5)),
((-a**2/(2*x**2) + 2*a*b*log(x) + a*c*x**2 + b**2*x**2/2 + b*c*x*
*4/2 + c**2*x**6/6)/d**3, Eq(m, -3)), ((a**2*log(x) + a*b*x**2 +
a*c*x**4/2 + b**2*x**4/4 + b*c*x**6/3 + c**2*x**8/8)/d, Eq(m, -1)
), (a**2*d**m*m**4*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
 1689*m + 945) + 24*a**2*d**m*m**3*x*x**m/(m**5 + 25*m**4 + 230*m
**3 + 950*m**2 + 1689*m + 945) + 206*a**2*d**m*m**2*x*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**2*d**m*m
*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 9
45*a**2*d**m*x*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*
m + 945) + 2*a*b*d**m*m**4*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 +
 950*m**2 + 1689*m + 945) + 44*a*b*d**m*m**3*x**3*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 328*a*b*d**m*m**2*x
**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
916*a*b*d**m*m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 630*a*b*d**m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3
 + 950*m**2 + 1689*m + 945) + 2*a*c*d**m*m**4*x**5*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 40*a*c*d**m*m**3*x
```

```
**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
260*a*c*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 600*a*c*d**m*m*x**5*x**m/(m**5 + 25*m**4 + 230
*m**3 + 950*m**2 + 1689*m + 945) + 378*a*c*d**m*x**5*x**m/(m**5 +
 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + b**2*d**m*m**4*x
**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) +
20*b**2*d**m*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2
 + 1689*m + 945) + 130*b**2*d**m*m**2*x**5*x**m/(m**5 + 25*m**4 +
 230*m**3 + 950*m**2 + 1689*m + 945) + 300*b**2*d**m*m*x**5*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 189*b**2*
d**m*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 9
45) + 2*b*c*d**m*m**4*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*
m**2 + 1689*m + 945) + 36*b*c*d**m*m**3*x**7*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 208*b*c*d**m*m**2*x**7*x
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 444*b
*c*d**m*m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*
m + 945) + 270*b*c*d**m*x**7*x**m/(m**5 + 25*m**4 + 230*m**3 + 95
0*m**2 + 1689*m + 945) + c**2*d**m*m**4*x**9*x**m/(m**5 + 25*m**4
 + 230*m**3 + 950*m**2 + 1689*m + 945) + 16*c**2*d**m*m**3*x**9*x
**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 86*c*
*2*d**m*m**2*x**9*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 16
89*m + 945) + 176*c**2*d**m*m*x**9*x**m/(m**5 + 25*m**4 + 230*m**
3 + 950*m**2 + 1689*m + 945) + 105*c**2*d**m*x**9*x**m/(m**5 + 25
*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945), True))
```

GIAC/XCAS [A] time = 0.273654, size = 687, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^2*(d*x)^m,x, algorithm="giac")
```

```
[Out] (c^2*m^4*x^9*e^(m*ln(d*x)) + 16*c^2*m^3*x^9*e^(m*ln(d*x)) + 2*b*c
*m^4*x^7*e^(m*ln(d*x)) + 86*c^2*m^2*x^9*e^(m*ln(d*x)) + 36*b*c*m^
3*x^7*e^(m*ln(d*x)) + 176*c^2*m*x^9*e^(m*ln(d*x)) + b^2*m^4*x^5*e
^(m*ln(d*x)) + 2*a*c*m^4*x^5*e^(m*ln(d*x)) + 208*b*c*m^2*x^7*e^(m
*ln(d*x)) + 105*c^2*x^9*e^(m*ln(d*x)) + 20*b^2*m^3*x^5*e^(m*ln(d*
x)) + 40*a*c*m^3*x^5*e^(m*ln(d*x)) + 444*b*c*m*x^7*e^(m*ln(d*x))
+ 2*a*b*m^4*x^3*e^(m*ln(d*x)) + 130*b^2*m^2*x^5*e^(m*ln(d*x)) + 2
60*a*c*m^2*x^5*e^(m*ln(d*x)) + 270*b*c*x^7*e^(m*ln(d*x)) + 44*a*b
*m^3*x^3*e^(m*ln(d*x)) + 300*b^2*m*x^5*e^(m*ln(d*x)) + 600*a*c*m*
x^5*e^(m*ln(d*x)) + a^2*m^4*x*e^(m*ln(d*x)) + 328*a*b*m^2*x^3*e^(
m*ln(d*x)) + 189*b^2*x^5*e^(m*ln(d*x)) + 378*a*c*x^5*e^(m*ln(d*x)
) + 24*a^2*m^3*x*e^(m*ln(d*x)) + 916*a*b*m*x^3*e^(m*ln(d*x)) + 20
6*a^2*m^2*x*e^(m*ln(d*x)) + 630*a*b*x^3*e^(m*ln(d*x)) + 744*a^2*m
*x*e^(m*ln(d*x)) + 945*a^2*x*e^(m*ln(d*x)))/(m^5 + 25*m^4 + 230*m
^3 + 950*m^2 + 1689*m + 945)
```

3.1107 $\int (dx)^m \left(a+bx^2+cx^4\right)\, dx$

Optimal. Leaf size=52

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) +
 (c*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rubi [A] time = 0.0435398, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056

$$\frac{a(dx)^{m+1}}{d(m+1)}+\frac{b(dx)^{m+3}}{d^3(m+3)}+\frac{c(dx)^{m+5}}{d^5(m+5)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*(d*x)^(1 + m))/(d*(1 + m)) + (b*(d*x)^(3 + m))/(d^3*(3 + m)) +
 (c*(d*x)^(5 + m))/(d^5*(5 + m))
```

Rubi in Sympy [A] time = 10.7902, size = 42, normalized size = 0.81

$$\frac{a\,(dx)^{m+1}}{d\,(m+1)}+\frac{b\,(dx)^{m+3}}{d^3\,(m+3)}+\frac{c\,(dx)^{m+5}}{d^5\,(m+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

```
[Out] a*(d*x)**(m + 1)/(d*(m + 1)) + b*(d*x)**(m + 3)/(d**3*(m + 3)) +
c*(d*x)**(m + 5)/(d**5*(m + 5))
```

Mathematica [A] time = 0.0336494, size = 35, normalized size = 0.67

$$(dx)^m\left(\frac{ax}{m+1}+\frac{bx^3}{m+3}+\frac{cx^5}{m+5}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(d*x)^m*(a + b*x^2 + c*x^4),x]
```

```
[Out] (d*x)^m*((a*x)/(1 + m) + (b*x^3)/(3 + m) + (c*x^5)/(5 + m))
```

Maple [A] time = 0.004, size = 78, normalized size = 1.5

$$\frac{\left(cm^{2}x^{4}+4\,cmx^{4}+bm^{2}x^{2}+3\,cx^{4}+6\,bmx^{2}+am^{2}+5\,bx^{2}+8\,am+15\,a\right)\,x\,(dx)^{m}}{(5+m)\,(3+m)\,(1+m)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(c*m^2*x^4+4*c*m*x^4+b*m^2*x^2+3*c*x^4+6*b*m*x^2+a*m^2+5*b*x^2+
8*a*m+15*a)*(d*x)^m/(5+m)/(3+m)/(1+m)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.302217, size = 96, normalized size = 1.85

$$\frac{\left(\left(cm^{2}+4\,cm+3\,c\right)x^{5}+\left(bm^{2}+6\,bm+5\,b\right)x^{3}+\left(am^{2}+8\,am+15\,a\right)x\right)\,(dx)^{m}}{m^{3}+9\,m^{2}+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] ((c*m^2 + 4*c*m + 3*c)*x^5 + (b*m^2 + 6*b*m + 5*b)*x^3 + (a*m^2 +
 8*a*m + 15*a)*x)*(d*x)^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [A] time = 2.44005, size = 314, normalized size = 6.04

$$\begin{cases} \frac{-\frac{a}{4x^4}-\frac{b}{2x^2}+c\log(x)}{d^5} \\ \frac{-\frac{a}{2x^2}+b\log(x)+\frac{cx^2}{2}}{d^3} \\ \frac{a\log(x)+\frac{bx^2}{2}+\frac{cx^4}{4}}{d} \\ \frac{ad^m m^2 x x^m}{m^3+9m^2+23m+15} + \frac{8ad^m m x x^m}{m^3+9m^2+23m+15} + \frac{15ad^m x x^m}{m^3+9m^2+23m+15} + \frac{bd^m m^2 x^3 x^m}{m^3+9m^2+23m+15} + \frac{6bd^m m x^3 x^m}{m^3+9m^2+23m+15} + \frac{5bd^m x^3 x^m}{m^3+9m^2+23m+15} + \frac{cd^m m^2 x^5 x^m}{m^3+9m^2+23m+15} + \frac{4}{m^3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise((((-a/(4*x**4) - b/(2*x**2) + c*log(x))/d**5, Eq(m, -5))
, ((-a/(2*x**2) + b*log(x) + c*x**2/2)/d**3, Eq(m, -3)), ((a*log(
x) + b*x**2/2 + c*x**4/4)/d, Eq(m, -1)), (a*d**m*m**2*x*x**m/(m**
3 + 9*m**2 + 23*m + 15) + 8*a*d**m*m*x*x**m/(m**3 + 9*m**2 + 23*m
 + 15) + 15*a*d**m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + b*d**m*m*
*2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 6*b*d**m*m*x**3*x**m/(
m**3 + 9*m**2 + 23*m + 15) + 5*b*d**m*x**3*x**m/(m**3 + 9*m**2 +
23*m + 15) + c*d**m*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) +
4*c*d**m*m*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*c*d**m*x**5*
x**m/(m**3 + 9*m**2 + 23*m + 15), True))
```

GIAC/XCAS [A] time = 0.265979, size = 185, normalized size = 3.56

$$\frac{cm^2x^5e^{(m\ln(dx))} + 4\,cmx^5e^{(m\ln(dx))} + bm^2x^3e^{(m\ln(dx))} + 3\,cx^5e^{(m\ln(dx))} + 6\,bmx^3e^{(m\ln(dx))} + am^2xe^{(m\ln(dx))} + 5\,bx^3e^{(m\ln(dx))} +}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)*(d*x)^m,x, algorithm="giac")
```

```
[Out] (c*m^2*x^5*e^(m*ln(d*x)) + 4*c*m*x^5*e^(m*ln(d*x)) + b*m^2*x^3*e^
(m*ln(d*x)) + 3*c*x^5*e^(m*ln(d*x)) + 6*b*m*x^3*e^(m*ln(d*x)) + a
*m^2*x*e^(m*ln(d*x)) + 5*b*x^3*e^(m*ln(d*x)) + 8*a*m*x*e^(m*ln(d*
x)) + 15*a*x*e^(m*ln(d*x)))/(m^3 + 9*m^2 + 23*m + 15)
```

3.1108 $\int \frac{(dx)^m}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=173

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^
2 - 4*a*c])*d*(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 0.514942, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1

$$\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(dx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^
2 - 4*a*c])*d*(1 + m)) - (2*c*(d*x)^(1 + m)*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[
b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi in Sympy [A] time = 28.4358, size = 148, normalized size = 0.86

$$-\frac{2c\,(dx)^{m+1}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\left(b+\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}+\frac{2c\,(dx)^{m+1}\,{}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b-\sqrt{-4ac+b^2}}\right)}{d\left(b-\sqrt{-4ac+b^2}\right)(m+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**4+b*x**2+a),x)
```

```
[Out] -2*c*(d*x)**(m + 1)*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -2*c*x**2
/(b + sqrt(-4*a*c + b**2)))/(d*(b + sqrt(-4*a*c + b**2))*(m + 1)*
sqrt(-4*a*c + b**2)) + 2*c*(d*x)**(m + 1)*hyper((1, m/2 + 1/2), (
m/2 + 3/2,), -2*c*x**2/(b - sqrt(-4*a*c + b**2)))/(d*(b - sqrt(-4
*a*c + b**2))*(m + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 0.0883319, size = 82, normalized size = 0.47

$$\frac{(dx)^m \text{RootSum}\left[\#1^4c+\#1^2b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^3c+\#1b}\&\right]}{2m}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d*x)^m*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2F1[-m, -m
, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ])/
(2*m)
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^4+b*x^2+a),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{cx^4 + bx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a), x)
```

3.1109 $\int \frac{(dx)^m}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=315

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a +
 b*x^2 + c*x^4)) + (c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m)
- 4*a*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^
(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c*(b^2*(1 - m) - b*Sq
rt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi [A] time = 1.30641, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{c(dx)^{m+1}\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(dx)^{m+1}\left(-b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((d*x)^(1 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d*(a +
 b*x^2 + c*x^4)) + (c*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m)
- 4*a*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^
(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*(1 + m)) - (c*(b^2*(1 - m) - b*Sq
```

```
rt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 - m))*(d*x)^(1 + m)*Hypergeome
tric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d*(1 + m))
```

Rubi in Sympy [A] time = 101.333, size = 264, normalized size = 0.84

$$-\frac{c(dx)^{m+1}\left(-4ac(-m+3)+b^2(-m+1)-b(-m+1)\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{2ad\left(b+\sqrt{-4ac+b^2}\right)(m+1)(-4ac+b^2)^{\frac{3}{2}}}+\frac{c(dx)^{m+1}\left(-4ac(-m+3)+b^2(-m+1)+b(-m+1)\sqrt{-4ac+b^2}\right){}_2F_1\left(\begin{matrix}1,\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\middle|-\frac{2cx^2}{b-\sqrt{-4ac+b^2}}\right)}{2ad\left(b-\sqrt{-4ac+b^2}\right)(m+1)(-4ac+b^2)^{\frac{3}{2}}}+\frac{(dx)^{m+1}\left(-2ac+b^2+bcx^2\right)}{2ad(-4ac+b^2)(a+bx^2+cx^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -c*(d*x)**(m + 1)*(-4*a*c*(-m + 3) + b**2*(-m + 1) - b*(-m + 1)*s
qrt(-4*a*c + b**2))*hyper((1, m/2 + 1/2), (m/2 + 3/2,), -2*c*x**2
/(b + sqrt(-4*a*c + b**2)))/(2*a*d*(b + sqrt(-4*a*c + b**2))*(m +
 1)*(-4*a*c + b**2)**(3/2)) + c*(d*x)**(m + 1)*(-4*a*c*(-m + 3) +
 b**2*(-m + 1) + b*(-m + 1)*sqrt(-4*a*c + b**2))*hyper((1, m/2 +
1/2), (m/2 + 3/2,), -2*c*x**2/(b - sqrt(-4*a*c + b**2)))/(2*a*d*(
b - sqrt(-4*a*c + b**2))*(m + 1)*(-4*a*c + b**2)**(3/2)) + (d*x)*
*(m + 1)*(-2*a*c + b**2 + b*c*x**2)/(2*a*d*(-4*a*c + b**2)*(a + b
*x**2 + c*x**4))
```

Mathematica [C] time = 1.72907, size = 376, normalized size = 1.19

$$\frac{a(m+3)x(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)F_1\left(m\right.}{4c(m+1)\left(a+bx^2+cx^4\right)^3\left(a(m+3)F_1\left(\frac{m+1}{2};2,2;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-2x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{m+3}{2};2,3;\frac{m+5}{2};-\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*(3 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[
b^2 - 4*a*c] + 2*c*x^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*
```

```
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)])/(4*c*(1 + m)*(a + b*x^2 + c*x^4)^3*(a*(3 + m)*AppellF1[(1 + m
)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])] - 2*x^2*((b + Sqrt[b^2 - 4*a*c])*App
ellF1[(3 + m)/2, 2, 3, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c]
)*AppellF1[(3 + m)/2, 3, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.034, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^4 + bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^4+b*x^2+a)^2,x)

[Out] int((d*x)^m/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^4 + bx^2 + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="maxima")

[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{c^2x^8 + 2\,bcx^6 + (b^2 + 2\,ac)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b
*x^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.1110 $\int (dx)^m \left(a+bx^2+cx^4\right)^{3/2} dx$

Optimal. Leaf size=158

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2
, -3/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.478585, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{a(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*(d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2
, -3/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqr
t[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 36.0902, size = 139, normalized size = 0.88

$$\frac{a\,(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{m}{2}+\frac{1}{2},-\frac{3}{2},-\frac{3}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\,(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] a*(d*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, -3
/2, -3/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x*
```

```
*2/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1)*sqrt(2*c*x**2/(b - sqrt(
-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)
)
```

Mathematica [B] time = 10.4398, size = 1080, normalized size = 6.84

$$\frac{\left(b-\sqrt{b^2-4ac}\right)\left(b+\sqrt{b^2-4ac}\right)x(dx)^m\left(2cx^2+b-\sqrt{b^2-4ac}\right)\left(2cx^2+b+\sqrt{b^2-4ac}\right)\left(\frac{\cdots}{(m+5)\left(\left(\left(b+\sqrt{b^2-4ac}\right)F_1\left(\frac{m+7}{2};-\frac{1}{2},\frac{1}{2};\frac{m}{\cdots}\right.\right.\right.}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*x*(d*x)^m*(b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*((a
*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((1 +
 m)*(2*a*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2, -1/2, 1/2, (
5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2, 1/
2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])]))) + (b*(5 + m)*x^2*AppellF1[(3 + m)/2
, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((3 + m)*(2*a*(5 + m)*AppellF1[(
3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c
]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*
a*c])*AppellF1[(5 + m)/2, -1/2, 1/2, (7 + m)/2, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqr
t[b^2 - 4*a*c])*AppellF1[(5 + m)/2, 1/2, -1/2, (7 + m)/2, (-2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])
)) + (c*(7 + m)*x^4*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*
c])])/((5 + m)*(2*a*(7 + m)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 +
m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^
2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*AppellF1[(7 + m)/2, -
1/2, 1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[(7
 + m)/2, 1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))))/(8*c^2*Sqrt[a + b*x^2
+ c*x^4])
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^4+bx^2+a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4+bx^2+a\right)^{\frac{3}{2}} (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m*(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^{\frac{3}{2}} (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(d*x)^m, x)
```

3.1111 $\int (dx)^m\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2,
-1/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi [A] time = 0.462792, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2,
-1/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[
b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rubi in Sympy [A] time = 31.2228, size = 138, normalized size = 0.88

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{m}{2}+\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{d\,(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, -1/2
, -1/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2
```

```
/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1)*sqrt(2*c*x**2/(b - sqrt(-4
*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 0.291371, size = 423, normalized size = 2.69

$$\frac{(m+3)x\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4ac}\right.}{8c^2(m+1)\sqrt{a+bx^2+cx^4}\left(x^2\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{m+3}{2};-\frac{1}{2},\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{m+3}{2};\frac{1}{2}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(3 + m)*x*(d*x)^
m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)*AppellF1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + S
qrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(8*c^2*(1
 + m)*Sqrt[a + b*x^2 + c*x^4]*(2*a*(3 + m)*AppellF1[(1 + m)/2, -1
/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Appell
F1[(3 + m)/2, -1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*
c])*AppellF1[(3 + m)/2, 1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4+bx^2+a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m,x, algorithm="maxima")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^4 + bx^2 + a}\,(dx)^m\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m,x, algorithm="fricas")

[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m \sqrt{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**4+b*x**2+a)**(1/2),x)

[Out] Integral((d*x)**m*sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^4 + bx^2 + a}\,(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m,x, algorithm="giac")

[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(d*x)^m, x)
```

3.1112 $\int \frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=157

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/
2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.475939, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/
2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])])/(d*(1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 32.1691, size = 136, normalized size = 0.87

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf_1}\left(\frac{m}{2}+\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{ad\,(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, 1/2,
 1/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(
```

```
b + sqrt(-4*a*c + b**2)))/(a*d*(m + 1)*sqrt(2*c*x**2/(b - sqrt(-4
*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1))
```

Mathematica [B] time = 2.18508, size = 425, normalized size = 2.71

$$-\frac{2a^2(m+3)x(dx)^m\Big(-\sqrt{b^2-4ac}+b+2cx^2\Big)\Big(\sqrt{b^2-4}}{(m+1)\Big(b-\sqrt{b^2-4ac}\Big)\Big(\sqrt{b^2-4ac}+b\Big)(a+bx^2+cx^4)^{3/2}\Big(x^2\Big(\Big(\sqrt{b^2-4ac}+b\Big)F_1\Big(\frac{m+3}{2};\frac{1}{2},\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (-2*a^2*(3 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m
)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1
+ m)*(a + b*x^2 + c*x^4)^(3/2)*(-2*a*(3 + m)*AppellF1[(1 + m)/2,
1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2
)/(-b + Sqrt[b^2 - 4*a*c])] + x^2*((b + Sqrt[b^2 - 4*a*c])*Appell
F1[(3 + m)/2, 1/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[(3 + m)/2, 3/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int (dx)^m \frac{1}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{\sqrt{cx^4+bx^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d*x)**m/sqrt(a + b*x**2 + c*x**4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx)^m}{\sqrt{cx^4+bx^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

3.1113 $\int \frac{(dx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/
2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])])/(a*d*(1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi [A] time = 0.467935, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{(dx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{ad(m+1)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] ((d*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1
 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/
2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c])])/(a*d*(1 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rubi in Sympy [A] time = 42.2231, size = 138, normalized size = 0.86

$$\frac{(dx)^{m+1}\sqrt{a+bx^2+cx^4}\,\mathrm{appellf}_1\left(\frac{m}{2}+\frac{1}{2},\frac{3}{2},\frac{3}{2},\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{a^2d\,(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1}\sqrt{\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] (d*x)**(m + 1)*sqrt(a + b*x**2 + c*x**4)*appellf1(m/2 + 1/2, 3/2,
 3/2, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/(
```

```
b + sqrt(-4*a*c + b**2)))/(a**2*d*(m + 1)*sqrt(2*c*x**2/(b - sqrt
(-4*a*c + b**2)) + 1)*sqrt(2*c*x**2/(b + sqrt(-4*a*c + b**2)) + 1
))
```

Mathematica [B] time = 2.83284, size = 426, normalized size = 2.66

$$\frac{2a^2(m+3)x(dx)^m\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\sqrt{b^2-4a\right.}{(m+1)\left(b-\sqrt{b^2-4ac}\right)\left(\sqrt{b^2-4ac}+b\right)\left(a+bx^2+cx^4\right)^{5/2}\left(2a(m+3)F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-3x^2\left(\left(\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a^2*(3 + m)*x*(d*x)^m*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])])/((b - Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c])*(1 +
 m)*(a + b*x^2 + c*x^4)^(5/2)*(2*a*(3 + m)*AppellF1[(1 + m)/2, 3/
2, 3/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])] - 3*x^2*((b + Sqrt[b^2 - 4*a*c])*Appell
F1[(3 + m)/2, 3/2, 5/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b - Sqrt[b^2 - 4*a*c
])*AppellF1[(3 + m)/2, 5/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.013, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^4+bx^2+a\right)^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((d*x)^m/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(cx^4+bx^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx)^m}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + bx^2 + cx^4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d*x)**m/(a + b*x**2 + c*x**4)**(3/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(cx^4 + bx^2 + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

3.1114 $\int (dx)^m \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=155

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

```
[Out] ((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])])/(d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.302921, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{m+1}{2};-p,-p;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*x^2 + c*x^4)^p*AppellF1[(1 + m)/2, -p, -p,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c])])/(d*(1 + m)*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*
c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 29.6263, size = 129, normalized size = 0.83

$$\frac{(dx)^{m+1}\left(\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{appellf}_1\left(\frac{m}{2}+\frac{1}{2},-p,-p,\frac{m}{2}+\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b}}\right.}{d\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((d*x)**m*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] (d*x)**(m + 1)*(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*
c*x**2/(b + sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**2 + c*x**4)
**p*appellf1(m/2 + 1/2, -p, -p, m/2 + 3/2, -2*c*x**2/(b - sqrt(-4
```

```
*a*c + b**2)), -2*c*x**2/(b + sqrt(-4*a*c + b**2)))/(d*(m + 1))
```

Mathematica [B] time = 4.99231, size = 499, normalized size = 3.22

$$-\frac{c(m+3)2^{-p-2}x\left(\sqrt{b^2-4ac}+b\right)(dx)^m\left(x^2\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right.}{(m+1)\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(px^2\left(\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{m+3}{2};1-p,-p;\frac{m+5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(d*x)^m*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -((2^(-2 - p)*c*(b + Sqrt[b^2 - 4*a*c])*(3 + m)*x*(d*x)^m*((b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4
*a*c])*x^2)^2*(a + b*x^2 + c*x^4)^(-1 + p)*AppellF1[(1 + m)/2, -p
, -p, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*(1 + m)*((b -
Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2)*(a*(3 + m)*AppellF1[(1 + m)/2, -p, -p, (3 + m)/2, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + p*x
^2*((b - Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2, 1 - p, -p, (5 + m
)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
 - 4*a*c])] + (b + Sqrt[b^2 - 4*a*c])*AppellF1[(3 + m)/2, -p, 1 -
 p, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])]))))
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int (dx)^m \left(cx^4+bx^2+a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int((d*x)^m*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p (dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((d*x)**m*(c*x**4+b*x**2+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p (dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^p*(d*x)^m, x)
```

3.1115 $\int x^7 \left(a+bx^2+cx^4\right)^p \,dx$

Optimal. Leaf size=257

$$-\frac{b2^{p-2}\left(6ac-b^2(p+3)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{\left(-2ac(2p+3)+b^2(p+2)(p+3)-2bc(p+1)(p+3)x^2\right)\left(a+bx^2+cx^4\right)^{p+1}}{8c^3(p+1)(p+2)(2p+3)}+\frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+2)}$$

```
[Out] (x^4*(a + b*x^2 + c*x^4)^(1 + p))/(4*c*(2 + p)) + ((b^2*(2 + p)*(
3 + p) - 2*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2
+ c*x^4)^(1 + p))/(8*c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-2 + p)
*b*(6*a*c - b^2*(3 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqr
t[b^2 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi [A] time = 0.695351, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{b2^{p-2}\left(6ac-b^2(p+3)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^3(p+1)(2p+3)\sqrt{b^2-4ac}}+\frac{\left(-2ac(2p+3)+b^2(p+2)(p+3)-2bc(p+1)(p+3)x^2\right)\left(a+bx^2+cx^4\right)^{p+1}}{8c^3(p+1)(p+2)(2p+3)}+\frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+2)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^7*(a + b*x^2 + c*x^4)^p,x]
```


```
[Out] (x^4*(a + b*x^2 + c*x^4)^(1 + p))/(4*c*(2 + p)) + ((b^2*(2 + p)*(
3 + p) - 2*a*c*(3 + 2*p) - 2*b*c*(1 + p)*(3 + p)*x^2)*(a + b*x^2
+ c*x^4)^(1 + p))/(8*c^3*(1 + p)*(2 + p)*(3 + 2*p)) - (2^(-2 + p)
*b*(6*a*c - b^2*(3 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqr
t[b^2 - 4*a*c])])/(c^3*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi in Sympy [A] time = 52.4285, size = 230, normalized size = 0.89

$$\frac{b\left(\frac{-\frac{b}{2}-cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-6ac+b^2p+3b^2\right)\,{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|\frac{\frac{b}{2}+cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{8c^3\left(p+1\right)\left(2p+3\right)\sqrt{-4ac+b^2}} + \frac{x^4\left(a+bx^2+cx^4\right)^{p+1}}{4c\left(p+2\right)} - \frac{\left(a+bx^2+cx^4\right)^{p+1}\left(2ac\left(2p+3\right)-b^2\left(p+2\right)\left(p+3\right)+2bcx^2\left(p+1\right)\left(p+3\right)\right)}{8c^3\left(p+1\right)\left(p+2\right)\left(2p+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**7*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] b*((-b/2 - c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**
(-p - 1)*(a + b*x**2 + c*x**4)**(p + 1)*(-6*a*c + b**2*p + 3*b**2
)*hyper((-p, p + 1), (p + 2,), (b/2 + c*x**2 + sqrt(-4*a*c + b**2
)/2)/sqrt(-4*a*c + b**2))/(8*c**3*(p + 1)*(2*p + 3)*sqrt(-4*a*c +
 b**2)) + x**4*(a + b*x**2 + c*x**4)**(p + 1)/(4*c*(p + 2)) - (a
+ b*x**2 + c*x**4)**(p + 1)*(2*a*c*(2*p + 3) - b**2*(p + 2)*(p +
3) + 2*b*c*x**2*(p + 1)*(p + 3))/(8*c**3*(p + 1)*(p + 2)*(2*p + 3
))
```

Mathematica [C] time = 3.87806, size = 440, normalized size = 1.71

$$\frac{5c2^{-p-4}x^8\left(\sqrt{b^2-4ac}+b\right)\left(x^2\left(b-\sqrt{b^2-4ac}\right)+2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{c}\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(px^2\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(5;1-p,-p;6;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4ac}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^7*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (5*2^(-4 - p)*c*(b + Sqrt[b^2 - 4*a*c])*x^8*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/c)^(1 + p)*(2*a + (b - Sqrt[b^2 - 4*a*c])*x^2)^2*(a
 + x^2*(b + c*x^2))^(-1 + p)*AppellF1[4, -p, -p, 5, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b
+ Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^p*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(-10*a*AppellF1[4, -p, -p, 5, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])
] + p*x^2*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[5, 1 - p, -p, 6, (-2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[5, -p, 1 - p, 6, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int x^{7} \left(c x^{4}+b x^{2}+a \right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^7*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^7*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^{4} + bx^{2} + a)^{p} x^{7} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^7,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((cx^{4} + bx^{2} + a)^{p} x^{7}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^7,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**7*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^7 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^7,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^7, x)
```

3.1116 $\int x^5 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=223

$$\frac{2^{p-1}\left(2ac-b^2(p+2)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a+bx^2+cx^4\right)^{p+1}}{4c^2(p+1)(2p+3)} + \frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{2c(2p+3)}$$

```
[Out] -(b*(2 + p)*(a + b*x^2 + c*x^4)^(1 + p))/(4*c^2*(1 + p)*(3 + 2*p)
) + (x^2*(a + b*x^2 + c*x^4)^(1 + p))/(2*c*(3 + 2*p)) + (2^(-1 +
p)*(2*a*c - b^2*(2 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqr
t[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi [A] time = 0.429162, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{2^{p-1}\left(2ac-b^2(p+2)\right)\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c^2(p+1)(2p+3)\sqrt{b^2-4ac}} - \frac{b(p+2)\left(a+bx^2+cx^4\right)^{p+1}}{4c^2(p+1)(2p+3)} + \frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{2c(2p+3)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^5*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -(b*(2 + p)*(a + b*x^2 + c*x^4)^(1 + p))/(4*c^2*(1 + p)*(3 + 2*p)
) + (x^2*(a + b*x^2 + c*x^4)^(1 + p))/(2*c*(3 + 2*p)) + (2^(-1 +
p)*(2*a*c - b^2*(2 + p))*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]))^(-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqr
t[b^2 - 4*a*c])])/(c^2*Sqrt[b^2 - 4*a*c]*(1 + p)*(3 + 2*p))
```

Rubi in Sympy [A] time = 40.7271, size = 192, normalized size = 0.86

$$-\frac{b(p+2)\left(a+bx^2+cx^4\right)^{p+1}}{4c^2(p+1)(2p+3)}+\frac{x^2\left(a+bx^2+cx^4\right)^{p+1}}{2c(2p+3)}+\frac{\left(\frac{-\frac{b}{2}-cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(2ac-b^2(p+2)\right)\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|\frac{\frac{b}{2}+cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{4c^2(p+1)(2p+3)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**5*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] -b*(p + 2)*(a + b*x**2 + c*x**4)**(p + 1)/(4*c**2*(p + 1)*(2*p +
3)) + x**2*(a + b*x**2 + c*x**4)**(p + 1)/(2*c*(2*p + 3)) + ((-b/
2 - c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**(-p - 1
)*(2*a*c - b**2*(p + 2))*(a + b*x**2 + c*x**4)**(p + 1)*hyper((-p
, p + 1), (p + 2,), (b/2 + c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-
4*a*c + b**2))/(4*c**2*(p + 1)*(2*p + 3)*sqrt(-4*a*c + b**2))
```

Mathematica [C] time = 3.64026, size = 395, normalized size = 1.77

$$\frac{x^6\left(\sqrt{b^2-4ac}+b\right)\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(x^2\left(b-\sqrt{b^2-4ac}\right)+2a\right)^2\left(a+x^2\right.}{3\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(px^2\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(4;1-p,-p;5;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4a\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^5*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] ((b + Sqrt[b^2 - 4*a*c])*x^6*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(2
*a + (b - Sqrt[b^2 - 4*a*c])*x^2)^2*(a + x^2*(b + c*x^2))^(-1 + p
)*AppellF1[3, -p, -p, 4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c
*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*(-b + Sqrt[b^2 - 4*a*c])*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(-8*a*AppellF1[3, -p, -p, 4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
 + p*x^2*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[4, 1 - p, -p, 5, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[4, -p, 1 - p, 5, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int x^{5} \left(c\,x^{4}+b\,x^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^5*(c*x^4+b*x^2+a)^p,x)

[Out] int(x^5*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{4}+bx^{2}+a\right)^{p} x^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^5,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(cx^{4}+bx^{2}+a\right)^{p} x^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^5,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p*x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**5*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^5,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^5, x)
```

3.1117 $\int x^3 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=160

$$\frac{b2^{p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+1)}$$

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(4*c*(1 + p)) + (2^(-1 + p)*b*(-((b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b
*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 -
4*a*c]*(1 + p))
```

Rubi [A] time = 0.255272, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{b2^{p-1}\left(a+bx^2+cx^4\right)^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)\sqrt{b^2-4ac}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(4*c*(1 + p)) + (2^(-1 + p)*b*(-((b -
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(-1 - p)*(a + b
*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c*Sqrt[b^2 -
4*a*c]*(1 + p))
```

Rubi in Sympy [A] time = 19.3745, size = 136, normalized size = 0.85

$$\frac{b\left(\frac{-\frac{b}{2}-cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(\begin{matrix}-p,p+1\\p+2\end{matrix}\middle|\frac{\frac{b}{2}+cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{4c\left(p+1\right)\sqrt{-4ac+b^2}}+\frac{\left(a+bx^2+cx^4\right)^{p+1}}{4c\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**3*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] b*((-b/2 - c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**
(-p - 1)*(a + b*x**2 + c*x**4)**(p + 1)*hyper((-p, p + 1), (p + 2
,), (b/2 + c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))/(
4*c*(p + 1)*sqrt(-4*a*c + b**2)) + (a + b*x**2 + c*x**4)**(p + 1)
/(4*c*(p + 1))
```

Mathematica [C] time = 3.7756, size = 440, normalized size = 2.75

$$\frac{3c2^{-p-3}x^4\left(\sqrt{b^2-4ac}+b\right)\left(x^2\left(b-\sqrt{b^2-4ac}\right)+2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\Big(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{c}}{\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\Big(px^2\Big(\left(\sqrt{b^2-4ac}-b\right)F_1\left(3;1-p,-p;4;-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-\Big(\sqrt{b^2-4ac}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^3*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (3*2^(-3 - p)*c*(b + Sqrt[b^2 - 4*a*c])*x^4*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/c)^(1 + p)*(2*a + (b - Sqrt[b^2 - 4*a*c])*x^2)^2*(a
 + x^2*(b + c*x^2))^(-1 + p)*AppellF1[2, -p, -p, 3, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b
+ Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^p*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(-6*a*AppellF1[2, -p, -p, 3, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
 + p*x^2*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[3, 1 - p, -p, 4, (-2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[3, -p, 1 - p, 4, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int x^3\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^3*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^3*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2+a\right)^p x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**3*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^3, x)
```

3.1118 $\int x\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=126

$$-\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(
-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/
(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi [A] time = 0.155393, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{2^p\left(-\frac{-\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{2cx^2+b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[x*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] -((2^p*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c]))^(
-1 - p)*(a + b*x^2 + c*x^4)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/
(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi in Sympy [A] time = 10.3605, size = 112, normalized size = 0.89

$$-\frac{\left(\frac{-\frac{b}{2}-cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(a+bx^2+cx^4\right)^{p+1}{}_2F_1\left(\left.\begin{matrix}-p,p+1\\p+2\end{matrix}\right|\frac{\frac{b}{2}+cx^2+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{2\left(p+1\right)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] -((-b/2 - c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**(
-p - 1)*(a + b*x**2 + c*x**4)**(p + 1)*hyper((-p, p + 1), (p + 2,
```

```
), (b/2 + c*x**2 + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))/(2
*(p + 1)*sqrt(-4*a*c + b**2))
```

Mathematica [A] time = 0.188044, size = 135, normalized size = 1.07

$$\frac{2^{p-2}\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^2+cx^4\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-2cx^2-b+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (2^(-2 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)
^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] -
2*c*x^2)/(2*Sqrt[b^2 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/Sqrt[b^2 - 4*a*c])^p)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int x\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^4 + bx^2 + a)^p x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p*x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x*(c*x**4+b*x**2+a)**p,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^4 + bx^2 + a)^p x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^p*x, x)
```

3.1119 $\int \frac{(a+bx^2+cx^4)^p}{x} \, dx$

Optimal. Leaf size=152

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p,
 -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*
c*x^2)])/(p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p)
```

Rubi [A] time = 0.337592, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-2p;-p,-p;1-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{p}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^p/x,x]
```

```
[Out] (4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[-2*p, -p, -p, 1 - 2*p,
 -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*c])/(2*
c*x^2)])/(p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*((b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p)
```

Rubi in Sympy [A] time = 24.0463, size = 128, normalized size = 0.84

$$\frac{\left(\frac{b+2cx^2-\sqrt{-4ac+b^2}}{2cx^2}\right)^{-p}\left(\frac{b+2cx^2+\sqrt{-4ac+b^2}}{2cx^2}\right)^{-p}(a+bx^2+cx^4)^p \operatorname{appellf}_1\left(-2p,-p,-p,-2p+1,-\frac{b-\sqrt{-4ac+b^2}}{2cx^2},-\frac{b+\sqrt{-4ac+b^2}}{2cx^2}\right)}{4p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**p/x,x)
```

```
[Out] ((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(2*c*x**2))**(-p)*((b + 2*c
*x**2 + sqrt(-4*a*c + b**2))/(2*c*x**2))**(-p)*(a + b*x**2 + c*x*
*4)**p*appellf1(-2*p, -p, -p, -2*p + 1, -(b - sqrt(-4*a*c + b**2)
```

```
)/(2*c*x**2), -(b + sqrt(-4*a*c + b**2))/(2*c*x**2))/(4*p)
```

Mathematica [B] time = 2.86055, size = 497, normalized size = 3.27

$$\frac{c2^{-2p-3}(2p-1)x^2\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{b-\sqrt{b^2-4ac}}{2cx^2}+1\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{c}\right)^{p+1}\left(\frac{-\sqrt{b^2-4\cdots}}{\cdots}\right.}{p\left(2c(2p-1)x^2F_1\left(-2p;-p,-p;1-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)-p\left(\sqrt{b^2-4ac}+b\right)F_1\left(1-2p;1-p,-p;2-2p;-\frac{b+\sqrt{b^2-4\cdots}}{2cx^2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^p/x,x]
```

```
[Out] (2^(-3 - 2*p)*c*(-1 + 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/c)^(1 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)^(-1 + p)*AppellF1[
-2*p, -p, -p, 1 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b +
Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/(p*(1 + (b - Sqrt[b^2 - 4*a*c])/(2
*c*x^2))^p*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^p*(-((b + Sqrt[b
^2 - 4*a*c])*p*AppellF1[1 - 2*p, 1 - p, -p, 2 - 2*p, -(b + Sqrt[b
^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)]) + (-
b + Sqrt[b^2 - 4*a*c])*p*AppellF1[1 - 2*p, -p, 1 - p, 2 - 2*p, -(
b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x
^2)] + 2*c*(-1 + 2*p)*x^2*AppellF1[-2*p, -p, -p, 1 - 2*p, -(b + S
qrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])
)
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^p/x,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^4+bx^2+a\right)^p}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**p/x,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^p/x, x)
```

3.1120 $\int \frac{(a+bx^2+cx^4)^p}{x^3}\,dx$

Optimal. Leaf size=166

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

```
[Out] -((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2
*(1 - p), -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*
a*c])/(2*c*x^2)])/((1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rubi [A] time = 0.325214, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2^{2p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}(a+bx^2+cx^4)^p F_1\left(1-2p;-p,-p;2(1-p);-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-2p)x^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^p/x^3,x]
```

```
[Out] -((2^(-1 + 2*p)*(a + b*x^2 + c*x^4)^p*AppellF1[1 - 2*p, -p, -p, 2
*(1 - p), -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*
a*c])/(2*c*x^2)])/((1 - 2*p)*x^2*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rubi in Sympy [A] time = 24.7386, size = 151, normalized size = 0.91

$$-\frac{\left(\frac{b+2cx^2-\sqrt{-4ac+b^2}}{2cx^2}\right)^{-p}\left(\frac{b+2cx^2+\sqrt{-4ac+b^2}}{2cx^2}\right)^{-p}(a+bx^2+cx^4)^p\left(\frac{1}{x^2}\right)^{2p}\left(\frac{1}{x^2}\right)^{-2p+1}\operatorname{appellf}_1\left(-2p+1,-p,-p,-2p+2,-\frac{b-\sqrt{-4ac}}{2cx}\right.}{2(-2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**p/x**3,x)
```

```
[Out] -((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(2*c*x**2))**(-p)*((b + 2*
c*x**2 + sqrt(-4*a*c + b**2))/(2*c*x**2))**(-p)*(a + b*x**2 + c*x
**4)**p*(x**(-2))**(2*p)*(x**(-2))**(-2*p + 1)*appellf1(-2*p + 1,
```

```
 -p, -p, -2*p + 2, -(b - sqrt(-4*a*c + b**2))/(2*c*x**2), -(b + s
qrt(-4*a*c + b**2))/(2*c*x**2))/(2*(-2*p + 1))
```

Mathematica [B] time = 3.24957, size = 516, normalized size = 3.11

$$\frac{2^{-2p-1}(p-1)\left(\sqrt{b^2-4ac}-b-2cx^2\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{b-\sqrt{b^2-4ac}}{2cx^2}+1\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c}{c}\right.}{(2p-1)\left(-4c(p-1)x^2F_1\left(1-2p;-p,-p;2-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)+p\left(\sqrt{b^2-4ac}+b\right)F_1\left(2-2p;1-p,-p;3-2p;\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^p/x^3,x]
```

```
[Out] (2^(-1 - 2*p)*(-1 + p)*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*((b - S
qrt[b^2 - 4*a*c] + 2*c*x^2)/c)^p*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2)/(c*x^2))^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^
4)^(-1 + p)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(b + Sqrt[b^2 - 4
*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1 + 2*p
)*(1 + (b - Sqrt[b^2 - 4*a*c])/(2*c*x^2))^p*((b - Sqrt[b^2 - 4*a*
c])/(2*c) + x^2)^p*(-4*c*(-1 + p)*x^2*AppellF1[1 - 2*p, -p, -p, 2
 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*
c])/(2*c*x^2)] + (b + Sqrt[b^2 - 4*a*c])*p*AppellF1[2 - 2*p, 1 -
p, -p, 3 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^
2 - 4*a*c])/(2*c*x^2)] + (b - Sqrt[b^2 - 4*a*c])*p*AppellF1[2 - 2
*p, -p, 1 - p, 3 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b +
 Sqrt[b^2 - 4*a*c])/(2*c*x^2)]))
```

Maple [F] time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^p/x^3,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^4 + bx^2 + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**p/x**3,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^p}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^3, x)
```

3.1121 $\int \frac{(a+bx^2+cx^4)^p}{x^5}\,dx$

Optimal. Leaf size=164

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-p)x^4}$$

```
[Out] -((4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2*(1 - p), -p, -p, 3
 - 2*p, -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*
c])/(2*c*x^2)])/((1 - p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rubi [A] time = 0.327224, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{4^{p-1}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^2}{cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(2(1-p);-p,-p;3-2p;-\frac{b-\sqrt{b^2-4ac}}{2cx^2},-\frac{b+\sqrt{b^2-4ac}}{2cx^2}\right)}{(1-p)x^4}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] -((4^(-1 + p)*(a + b*x^2 + c*x^4)^p*AppellF1[2*(1 - p), -p, -p, 3
 - 2*p, -(b - Sqrt[b^2 - 4*a*c])/(2*c*x^2), -(b + Sqrt[b^2 - 4*a*
c])/(2*c*x^2)])/((1 - p)*x^4*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
c*x^2))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p))
```

Rubi in Sympy [A] time = 24.8475, size = 150, normalized size = 0.91

$$-\frac{\left(\frac{b+2cx^2-\sqrt{-4ac+b^2}}{2cx^2}\right)^{-p}\left(\frac{b+2cx^2+\sqrt{-4ac+b^2}}{2cx^2}\right)^{-p}\left(a+bx^2+cx^4\right)^p\left(\frac{1}{x^2}\right)^{2p}\left(\frac{1}{x^2}\right)^{-2p+2}\operatorname{appellf}_1\left(-2p+2,-p,-p,-2p+3,-\frac{b-\sqrt{-4a}}{2cx}\right.}{4(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**p/x**5,x)
```

```
[Out] -((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(2*c*x**2))**(-p)*((b + 2*
c*x**2 + sqrt(-4*a*c + b**2))/(2*c*x**2))**(-p)*(a + b*x**2 + c*x
**4)**p*(x**(-2))**(2*p)*(x**(-2))**(-2*p + 2)*appellf1(-2*p + 2,
```

```
 -p, -p, -2*p + 3, -(b - sqrt(-4*a*c + b**2))/(2*c*x**2), -(b + s
qrt(-4*a*c + b**2))/(2*c*x**2))/(4*(-p + 1))
```

Mathematica [B] time = 3.22874, size = 504, normalized size = 3.07

$$\frac{c2^{-2p-3}(2p-3)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(\frac{b-\sqrt{b^2-4ac}}{2cx^2}+1\right)^{-p}\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{c}\right)^{p+1}\left(-\sqrt{b\right.}{(p-1)x^2\left(2c(2p-3)x^2F_1\left(2-2p;-p,-p;3-2p;-\frac{b+\sqrt{b^2-4ac}}{2cx^2},\frac{\sqrt{b^2-4ac}-b}{2cx^2}\right)-p\left(\left(\sqrt{b^2-4ac}+b\right)F_1\left(3-2p;1-p,-p;4-2\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^p/x^5,x]
```

```
[Out] (2^(-3 - 2*p)*c*(-3 + 2*p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/c)^
(1 + p)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(c*x^2))^p*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2)*(a + b*x^2 + c*x^4)^(-1 + p)*AppellF1[2 -
2*p, -p, -p, 3 - 2*p, -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + S
qrt[b^2 - 4*a*c])/(2*c*x^2)])/((-1 + p)*(1 + (b - Sqrt[b^2 - 4*a*
c])/(2*c*x^2))^p*x^2*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^p*(2*c
*(-3 + 2*p)*x^2*AppellF1[2 - 2*p, -p, -p, 3 - 2*p, -(b + Sqrt[b^2
 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)] - p*((b
 + Sqrt[b^2 - 4*a*c])*AppellF1[3 - 2*p, 1 - p, -p, 4 - 2*p, -(b +
 Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*c*x^2)
] + (b - Sqrt[b^2 - 4*a*c])*AppellF1[3 - 2*p, -p, 1 - p, 4 - 2*p,
 -(b + Sqrt[b^2 - 4*a*c])/(2*c*x^2), (-b + Sqrt[b^2 - 4*a*c])/(2*
c*x^2)])))
```

Maple [F] time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^p/x^5,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^5,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^4 + bx^2 + a)^p}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^5,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p/x^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**p/x**5,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^p}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^5,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^p/x^5, x)
```

3.1122 $\int x^4 \left(a+bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.34518, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{1}{5}x^5\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{5}{2};-p,-p;\frac{7}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^5*(a + b*x^2 + c*x^4)^p*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 26.1896, size = 116, normalized size = 0.84

$$\frac{x^5\left(\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{appellf}_1\left(\frac{5}{2},-p,-p,\frac{7}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**4*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] x**5*(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**2/(b
+ sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**2 + c*x**4)**p*appell
f1(5/2, -p, -p, 7/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x*
*2/(b + sqrt(-4*a*c + b**2)))/5
```

Mathematica [B] time = 3.43206, size = 457, normalized size = 3.31

$$\frac{7c2^{-p-2}x^5\left(\sqrt{b^2-4ac}+b\right)\left(x^2\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c}{c}\right.}{5\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(px^2\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{7}{2};1-p,-p;\frac{9}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^4*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (7*2^(-2 - p)*c*(b + Sqrt[b^2 - 4*a*c])*x^5*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^2)^2*
(a + b*x^2 + c*x^4)^(-1 + p)*AppellF1[5/2, -p, -p, 7/2, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(
5*(-b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^
p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(-7*a*AppellF1[5/2, -p, -p, 7
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + p*x^2*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[7/2, 1 - p,
 -p, 9/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[7/2, -p, 1 -
p, 9/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])])))
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int x^4\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^4*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^4*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^4+bx^2+a\right)^p x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**4*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^4, x)
```

3.1123 $\int x^2 (a+bx^2+cx^4)^p \, dx$

Optimal. Leaf size=138

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.307954, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{1}{3}x^3\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(\frac{3}{2};-p,-p;\frac{5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^3*(a + b*x^2 + c*x^4)^p*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 27.9291, size = 116, normalized size = 0.84

$$\frac{x^3\left(\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}(a+bx^2+cx^4)^p \operatorname{appellf_1}\left(\frac{3}{2},-p,-p,\frac{5}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x**2*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] x**3*(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**2/(b
+ sqrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**2 + c*x**4)**p*appell
f1(3/2, -p, -p, 5/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x*
*2/(b + sqrt(-4*a*c + b**2)))/3
```

Mathematica [B] time = 3.3655, size = 457, normalized size = 3.31

$$\frac{5c2^{-p-2}x^3\left(\sqrt{b^2-4ac}+b\right)\left(x^2\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2c}{c}\right.}{3\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(px^2\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{5}{2};1-p,-p;\frac{7}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[x^2*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (5*2^(-2 - p)*c*(b + Sqrt[b^2 - 4*a*c])*x^3*((b - Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^2)^2*
(a + b*x^2 + c*x^4)^(-1 + p)*AppellF1[3/2, -p, -p, 5/2, (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(
3*(-b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^
p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(-5*a*AppellF1[3/2, -p, -p, 5
/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
- 4*a*c])] + p*x^2*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[5/2, 1 - p,
 -p, 7/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[5/2, -p, 1 -
p, 7/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])])))
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int x^2\left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x^2*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] int(x^2*(c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p*x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x**2*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p*x^2, x)
```

3.1124 $\int \left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=133

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]))^p)
```

Rubi [A] time = 0.178851, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$x\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p F_1\left(\frac{1}{2};-p,-p;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (x*(a + b*x^2 + c*x^4)^p*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/((1 +
 (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 28.8024, size = 112, normalized size = 0.84

$$x\left(\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}\left(a+bx^2+cx^4\right)^p \operatorname{appellf}_1\left(\frac{1}{2},-p,-p,\frac{3}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**p,x)
```

```
[Out] x*(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**2/(b + s
qrt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**2 + c*x**4)**p*appellf1(
1/2, -p, -p, 3/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/
(b + sqrt(-4*a*c + b**2)))
```

Mathematica [B] time = 3.59406, size = 487, normalized size = 3.66

$$\frac{3\,4^{-p-1}x\left(\sqrt{b^2-4ac}+b\right)\left(x^2\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b}{2c}+x^2\right)^{-p}\left(\frac{-\sqrt{b^2-4ac}+b+2cx^2}{c}\right)^{p+1}\left(\sqrt{b^2-\right.}{\left(\sqrt{b^2-4ac}-b\right)\left(px^2\left(\left(\sqrt{b^2-4ac}-b\right)F_1\left(\frac{3}{2};1-p,-p;\frac{5}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)-\left(\sqrt{b^2-4ac}+b\right)F_1\left(\frac{3}{2};-p,1-p;\frac{5}{2}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (3*4^(-1 - p)*(b + Sqrt[b^2 - 4*a*c])*x*((b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/c)^(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/c)^(-1 +
p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^2)^2*(a + b*x^2 + c*x^4)^(-
1 + p)*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c
])*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^p*((b + Sqrt[b^2 - 4*a*c
])/(2*c) + x^2)^p*(-3*a*AppellF1[1/2, -p, -p, 3/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + p*x^2
*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[3/2, 1 - p, -p, 5/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] -
 (b + Sqrt[b^2 - 4*a*c])*AppellF1[3/2, -p, 1 - p, 5/2, (-2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^p,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4+bx^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^4 + bx^2 + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^4 + bx^2 + a\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p, x)
```

3.1125 $\int \frac{(a+bx^2+cx^4)^p}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]))^p))
```

Rubi [A] time = 0.272928, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{1}{2};-p,-p;\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{x}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -(((a + b*x^2 + c*x^4)^p*AppellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(x*(
1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]))^p))
```

Rubi in Sympy [A] time = 26.3104, size = 116, normalized size = 0.85

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}(a+bx^2+cx^4)^p\,\text{appellf}_1\left(-\frac{1}{2},-p,-p,\frac{1}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**p/x**2,x)
```

```
[Out] -(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**2/(b + sq
rt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**2 + c*x**4)**p*appellf1(-
1/2, -p, -p, 1/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2/
```

```
(b + sqrt(-4*a*c + b**2)))/x
```

Mathematica [B] time = 3.57003, size = 472, normalized size = 3.47

$$-\frac{2^{-p-2}\left(\sqrt{b^2-4ac}+b\right)\left(\sqrt{b^2-4ac}-b-2cx^2\right)\left(x^2\left(\sqrt{b^2-4ac}-b\right)-2a\right)^2\left(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\right)^{-p}\left(\cdots\right.}{x\left(\sqrt{b^2-4ac}-b\right)\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(px^2\left(\left(b-\sqrt{b^2-4ac}\right)F_1\left(\frac{1}{2};1-p,-p;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\right)+\left(\sqrt{b^2-\cdots}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^p/x^2,x]
```

```
[Out] -((2^(-2 - p)*(b + Sqrt[b^2 - 4*a*c])*(-b + Sqrt[b^2 - 4*a*c] - 2
*c*x^2)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/c)^p*(-2*a + (-b + Sqr
t[b^2 - 4*a*c])*x^2)^2*(a + b*x^2 + c*x^4)^(-1 + p)*AppellF1[-1/2
, -p, -p, 1/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c])])/((-b + Sqrt[b^2 - 4*a*c])*x*((b - Sqrt[b^2
 - 4*a*c])/(2*c) + x^2)^p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a*Ap
pellF1[-1/2, -p, -p, 1/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + p*x^2*((b - Sqrt[b^2 - 4*a*c])
*AppellF1[1/2, 1 - p, -p, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (b + Sqrt[b^2 - 4*a*c])*A
ppellF1[1/2, -p, 1 - p, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))))
```

Maple [F] time = 0.036, size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^p/x^2,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^4+bx^2+a\right)^p}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^2,x, algorithm="maxima")

[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^4 + bx^2 + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^2,x, algorithm="fricas")

[Out] integral((c*x^4 + b*x^2 + a)^p/x^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**p/x**2,x)

[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^p}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^2,x, algorithm="giac")

[Out] integrate((c*x^4 + b*x^2 + a)^p/x^2, x)
```

3.1126 $\int \frac{(a+bx^2+cx^4)^p}{x^4}\,dx$

Optimal. Leaf size=138

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

```
[Out] -((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*x
^3*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.275932, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^2+cx^4)^p F_1\left(-\frac{3}{2};-p,-p;-\frac{1}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] -((a + b*x^2 + c*x^4)^p*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(
b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*x
^3*(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^p)
```

Rubi in Sympy [A] time = 26.1248, size = 121, normalized size = 0.88

$$-\frac{\left(\frac{2cx^2}{b-\sqrt{-4ac+b^2}}+1\right)^{-p}\left(\frac{2cx^2}{b+\sqrt{-4ac+b^2}}+1\right)^{-p}(a+bx^2+cx^4)^p \operatorname{appellf}_1\left(-\frac{3}{2},-p,-p,-\frac{1}{2},-\frac{2cx^2}{b-\sqrt{-4ac+b^2}},-\frac{2cx^2}{b+\sqrt{-4ac+b^2}}\right)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**4+b*x**2+a)**p/x**4,x)
```

```
[Out] -(2*c*x**2/(b - sqrt(-4*a*c + b**2)) + 1)**(-p)*(2*c*x**2/(b + sq
rt(-4*a*c + b**2)) + 1)**(-p)*(a + b*x**2 + c*x**4)**p*appellf1(-
3/2, -p, -p, -1/2, -2*c*x**2/(b - sqrt(-4*a*c + b**2)), -2*c*x**2
```

```
/(b + sqrt(-4*a*c + b**2)))/(3*x**3)
```

Mathematica [B] time = 3.18373, size = 456, normalized size = 3.3

$$\frac{c2^{-p-2}\Big(\sqrt{b^2-4ac}+b\Big)\Big(x^2\Big(\sqrt{b^2-4ac}-b\Big)-2a\Big)^2\Big(\frac{b-\sqrt{b^2-4ac}}{2c}+x^2\Big)^{-p}\Big(\frac{-\sqrt{b^2-4ac}+b+2c}{c}}{3x^3\Big(\sqrt{b^2-4ac}-b\Big)\Big(\sqrt{b^2-4ac}+b+2cx^2\Big)\Big(px^2\Big(\Big(\sqrt{b^2-4ac}-b\Big)F_1\Big(-\frac{1}{2};1-p,-p;\frac{1}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{\sqrt{b^2-4ac}-b}\Big)-\Big(\sqrt{b^2}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(a + b*x^2 + c*x^4)^p/x^4,x]
```

```
[Out] (2^(-2 - p)*c*(b + Sqrt[b^2 - 4*a*c])*((b - Sqrt[b^2 - 4*a*c] + 2
*c*x^2)/c)^(1 + p)*(-2*a + (-b + Sqrt[b^2 - 4*a*c])*x^2)^2*(a + b
*x^2 + c*x^4)^(-1 + p)*AppellF1[-3/2, -p, -p, -1/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])])/(3*(-
b + Sqrt[b^2 - 4*a*c])*x^3*((b - Sqrt[b^2 - 4*a*c])/(2*c) + x^2)^
p*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)*(a*AppellF1[-3/2, -p, -p, -1/
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 -
 4*a*c])] + p*x^2*((-b + Sqrt[b^2 - 4*a*c])*AppellF1[-1/2, 1 - p,
 -p, 1/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])] - (b + Sqrt[b^2 - 4*a*c])*AppellF1[-1/2, -p, 1 -
 p, 1/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])])))
```

Maple [F] time = 0.045, size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^4+b*x^2+a)^p/x^4,x)
```

```
[Out] int((c*x^4+b*x^2+a)^p/x^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4+bx^2+a)^p}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^4,x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^4 + bx^2 + a)^p}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^4,x, algorithm="fricas")
```

```
[Out] integral((c*x^4 + b*x^2 + a)^p/x^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**4+b*x**2+a)**p/x**4,x)
```

```
[Out] Timed out
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^4 + bx^2 + a)^p}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^4 + b*x^2 + a)^p/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^p/x^4, x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```